Quasisymmetries of Sierpiński carpet Julia sets

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UIUC

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Sierpiński carpets

Sierpiński carpets

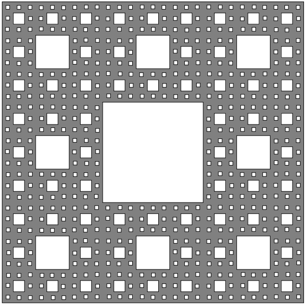

Figure: The standard Sierpiński carpet S_3.

Whyburn's characterization

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Sierpiński carpet—

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Sierpiński carpet—

planar continuum

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topdim=1

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locally connected

Whyburn's characterization

Sierpiński carpet—

- planar continuum
- topdim=1
- locally connected
- no local cut points

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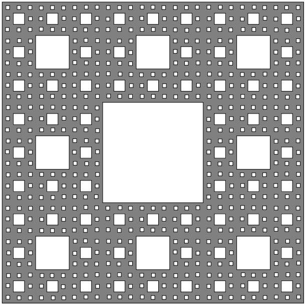

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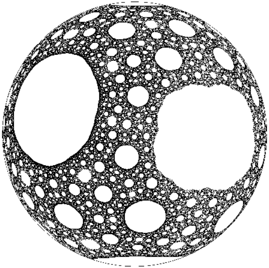

Figure: http://www.knowledgesutra.com/forums/user/105975-bikerman/

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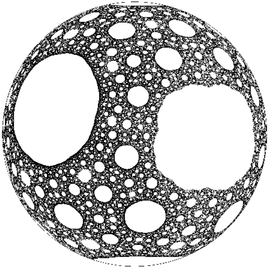

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$\overline{B_i}$—disjoint topological discs

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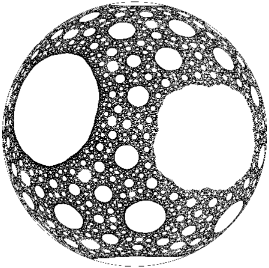

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$S = \mathbb{S}^2 \setminus \cup B_i$ has no interior

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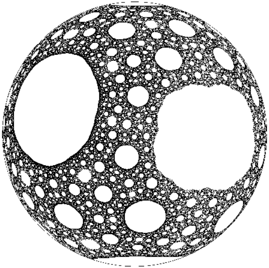

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$\mathrm{diam}(B_i) \to 0$

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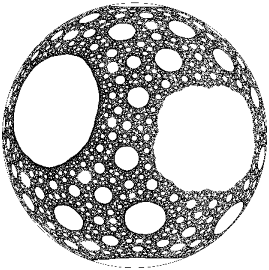

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$\overline{B_i}$—disjoint topological discs

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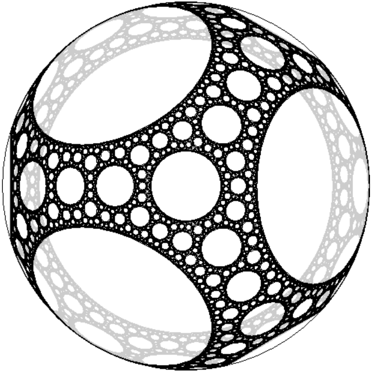
$\mathrm{diam}(B_i) \to 0$

$\Rightarrow S$—Sierpiński carpet

Schottky sets

Schottky sets



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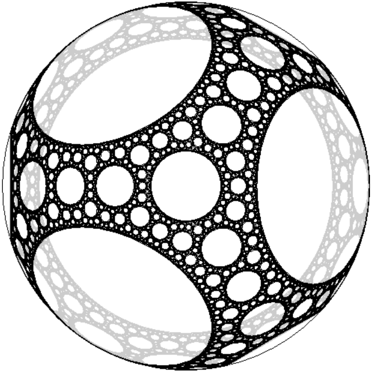

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diam(B_i) $\to$ 0—automatic

Standard Sierpiński p-carpet

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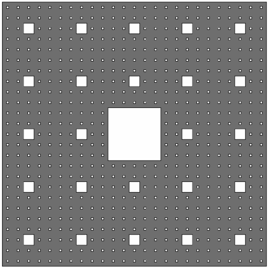

Figure: $S_p, \quad p = 5.$

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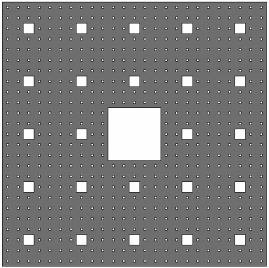

Figure: $S_p, \quad p = 5.$

S_p homeomorphic to S_3

Julia sets

Julia sets

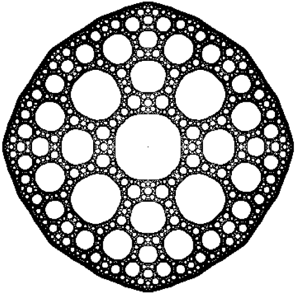

Figure: $f_\lambda(z) = z^2 + \frac{\lambda}{z^2}, \quad \lambda = -\frac{1}{16}$.

Quasisymmetric deformations

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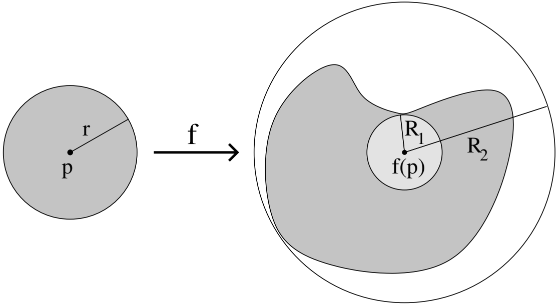

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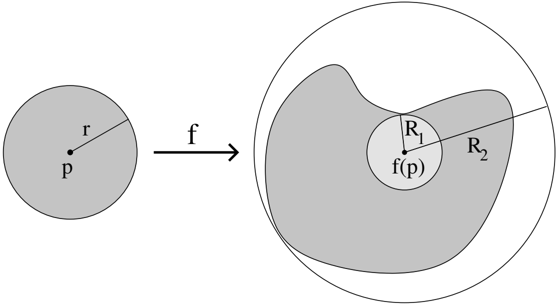

$R_2/R_1 \leq Const$

Standard Sierpiński carpets

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Theorem (M. Bonk, S. M.)

Quasisymmetry group of S_3 *is its symmetry group.*

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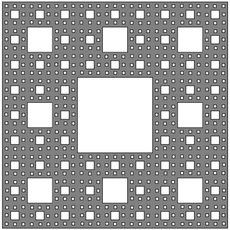

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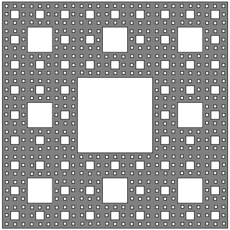

Theorem (M. Bonk, S. M.)

The quasisymmetry group of S_p *is finite dihedral.*

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$p \neq q \Rightarrow S_p$ and S_q are quasisymmetrically distinct.

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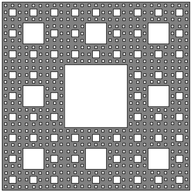

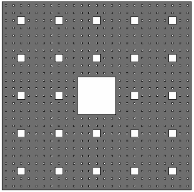

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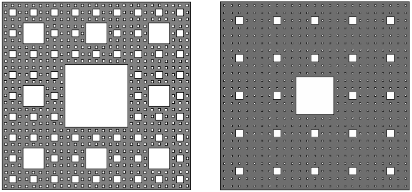

Theorem (M. Bonk, J. Tyson)

$$\frac{\log(p^2-1)}{\log p} < 1 + \frac{\log(q-1)}{\log q}$$

$\Rightarrow S_p$ and S_q are quasisymmetrically distinct.

Schottky sets

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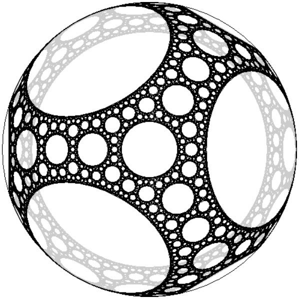

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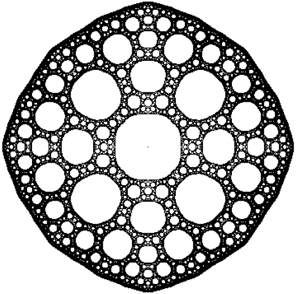

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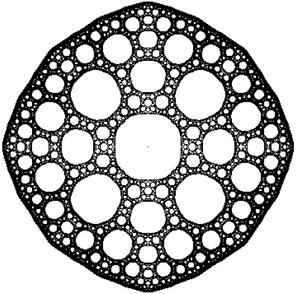

Figure: $f_\lambda(z) = z^2 + \frac{\lambda}{z^2}, \quad \lambda = -\frac{1}{16}$.

Quasi-self-similarity:

$$\exists\ f\colon \frac{1}{r}(J \cap B(p,r)) \hookrightarrow J \quad - L\text{-bi-Lipschitz}.$$

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$f, g \colon \hat{\mathbb{C}} \to \hat{\mathbb{C}}$—*postcritically finite rational maps*

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Assumptions:

- *Julia sets* $\mathcal{J}(f)$ *and* $\mathcal{J}(g)$—*Sierpiński carpets*

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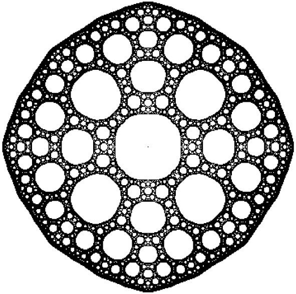

Corollary

f—postcritically finite rational map
$\mathcal{J}(f)$—Sierpiński carpet

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$\Rightarrow QS(\mathcal{J}(f))$—finite

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$\Rightarrow$ *$QS(\mathcal{J}(f))$—finite*

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f—postcritically finite rational map
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f—postcritically finite rational map
$\mathcal{J}(f)$—Sierpiński carpet
G—Kleinian group
$\Rightarrow \mathcal{J}(f) \not\approx \Lambda(G)$

Outline of the proof

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S—Schottky set.

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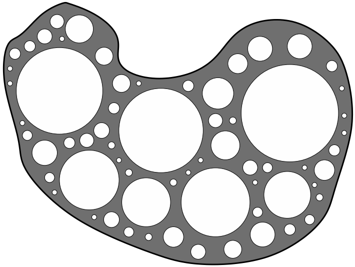

Conformality

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Theorem (S. M.)

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$$\forall p \in S, \quad f'(p) = \lim_{q \in S,\, q \to p} \frac{f(q) - f(p)}{q - p}$$

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Moreover: f—*locally bi-Lipschitz in* S;
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Assumptions:

- *S—locally porous*

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$S \subset D$—relative Schottky set
$U \subset D$—open

Theorem (S. M.)

$f\colon\ S \cap U \to S$—*local homeomorphism*
Assumptions:

- S—*locally porous*
- $S \cap U$—*connected*

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- *f'—continuous in $S \cap U$*
- *$f(p) = p$ and $f'(p) = 1$*

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Conclusion: $f = \mathrm{id}$.

Idea of proof

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$h_k \equiv h,\ \forall k \geq N.$

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$\Rightarrow \xi$ has a quasiconformal extension, conformal a.e. $\Rightarrow \xi$—Möbius. □