Implied Systemic Risk Index

(work in progress, still at an early stage)

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Workshop I: Systemic risk and financial networks

Systemic Risk, Contagion, Dependence

- **Contagion** linked to coincidence of extreme returns. Studies on "coexceedances" using bank data reject the assumption that the modelling can be done using a multivariate Gaussian assumption. They find "non linearities" in the tail. Large common shocks are highly correlated compared to small shocks.
- **Systemic risk measures**: CoVaR, CoES, SRISK... are "conditional" risk measures.

Existing studies on contagion and systemic risk measures are under the **real-world** measure.

▶Our objective is to measure contagion/systemic risk using option prices (thus information on the **risk-neutral** probability).

Outline

1. **The CBOE implied correlation**: use information on implied volatilities of *at-the-money* options
2. Toy **examples** with factor models
   - to show that it may fail to capture changes in the dependence among assets.
   - Importance of using the full marginal distributions
3. An **algorithm** to describe the set of possible dependence structures (copulas) that are consistent with the information
   - marginal distribution of each individual asset return X_i
   - distribution of aggregated risk $\omega_1 X_1 + ... + \omega_d X_d$
4. Algorithm useful
   - to detect changes in "**implied dependence**."
   - **forward looking** measure of contagion/of systemic risk.
   - to compute **conditional risk measures** (correlation in the tail), systemic risk measures under the risk neutral probability (**model-free**)

CBOE implied correlation

- $S_T = \sum_i \omega_i X_{i,T}$ with $\sum_i \omega_i = 1$
- For observed at-the-money call option prices with maturity T, define σ_S and σ_i as follows

$$C_{\text{index,observed}} = \text{BlackScholesCall}(\sigma_S, S_0)$$

$$E\left[(S_T - S_0)^+\right] = E\left[\left(S_0 e^{(r-\frac{\sigma_S^2}{2})T+\sigma_S W_T} - S_0\right)^+\right]$$

$$C_{X_{i,\text{observed}}} = \text{BlackScholesCall}(\sigma_i, X_{i,0})$$

$$E\left[(X_{i,T} - X_{i,0})^+\right] = E\left[\left(X_{i,0} e^{(r-\frac{\sigma_i^2}{2})T+\sigma_i W_{i,T}} - X_{i,0}\right)^+\right]$$

- Use these implied volatilities to define an "implied correlation index."

CBOE implied correlation

- The CBOE correlation index is defined by

$$\rho_{\mathbf{cboe}} = \frac{\sigma_S^2 - \sum_{i=1}^{d} \omega_i^2 \sigma_i^2}{2 \sum_{i=1}^{d-1} \sum_{j>i} \omega_i \omega_j \sigma_i \sigma_j}$$

- ... assuming that the index implied volatility σ_S and the individual implied volatilities σ_i for $i = 1, ..., d$ are such that

$$\sigma_S^2 = \sum_{i=1}^{d} \omega_i^2 \sigma_i^2 + 2 \sum_{i=1}^{d-1} \sum_{j>i} \omega_i \omega_j \sigma_i \sigma_j \rho_{\mathbf{ij}}$$

where $\rho_{\mathbf{ij}}$ is **constant** equal to $\rho_{\mathbf{cboe}}$

- Given this assumption,

$$\rho_{\mathbf{cboe}} = \frac{\sum_{i=1}^{d-1} \sum_{j>i} \omega_i \omega_j \sigma_i \sigma_j \rho_{\mathbf{ij}}}{\sum_{i=1}^{d-1} \sum_{j>i} \omega_i \omega_j \sigma_i \sigma_j}$$

Comments on the CBOE implied correlation

- not always between -1 and 1.
- not always a feasible correlation parameter.
- The **CBOE implied correlation index can be very far from the weighted average of pairwise correlations**.
- **No setting** in which the assumption of a constant pairwise correlation ρ among assets logreturns allows us to find that the CBOE implied correlation is equal to ρ exactly.
- But it **works well in a multivariate Black Scholes** with constant pairwise correlation ρ_{ij}.
- It is **affected by changes in marginal** distributions and not only in the dependence.
- It does not give any information on the dependence in the tail (**global measure**).

Proposal

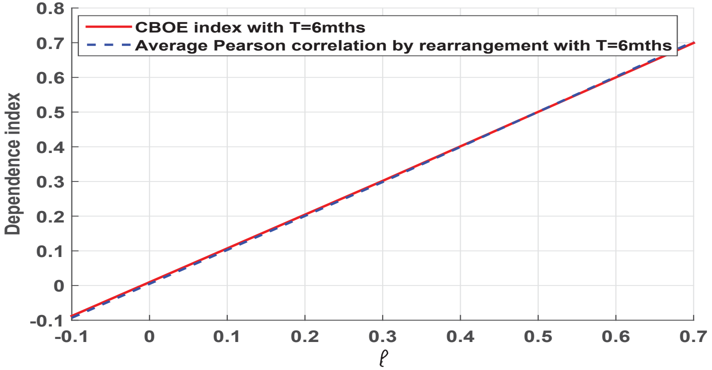
- The CBOE implied correlation index makes use of the implied volatilities of **at-the-money** option prices only
- Use **all** strikes to get the **full marginal distribution** of X_i and S and **infer the dependence** structures that are compatible with this information. Method based on the "**Rearrangement Algorithm**." Using this approach, we can compute for example the average pairwise Pearson correlation

$$\bar{\rho} := \frac{\sum_{i=1}^{d-1} \sum_{j>i} \omega_i \omega_j \sigma_i \sigma_j \widehat{\rho_{ij}}}{\sum_{i=1}^{d-1} \sum_{j>i} \omega_i \omega_j \sigma_i \sigma_j}$$

- Then we **compare** $\bar{\rho}$ with the CBOE index and with the true correlation in toy examples...

In a multidimensional Black Scholes model

with heterogeneous volatilities (from 20% to 70%)
with logreturns that are **multivariate Gaussian** with homogeneous correlation matrix $\rho_{ij} = \rho$ for all $i \neq j$.
$d = 10$ assets, weights are all equal to $1/d$



▶**The CBOE index is roughly equal to ρ and $\bar{\rho}$.**

Factor model with 2 assets returns

Define

$$X_i = 100e^{r-\frac{v_i^2}{2}+v_i W_i(1)}, \quad Z = 100e^{r-\frac{\sigma_Z^2}{2}+\sigma_Z W_Z(1)}$$

W_1, W_2 and W_Z are Brownian motions. W_Z is **independent** of W_1 and W_2 and W_1 and W_2 have **correlation** ρ_{12}.

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W_1, W_2 and W_Z are Brownian motions. W_Z is **independent** of W_1 and W_2 and W_1 and W_2 have **correlation** ρ_{12}.
Let $\mathbb{I}$ be a variable indicating in which **regime** we are.

$$S_1 = (1-\mathbb{I})X_1 + \mathbb{I}Z$$

$$S_2 = (1-\mathbb{I})X_2 + \mathbb{I}Z$$

In one regime (when $\mathbb{I} = 1$), S_1 and S_2 are perfectly dependent (equal here) and in the other regime, 2-dimensional Black-Scholes.

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$$\text{Index:} = \frac{S_1}{2} + \frac{S_2}{2}.$$

$\mathbb{I} = \mathbb{1}_{Z<z_q}$ where z_q is the Value-at-Risk at level q of Z.

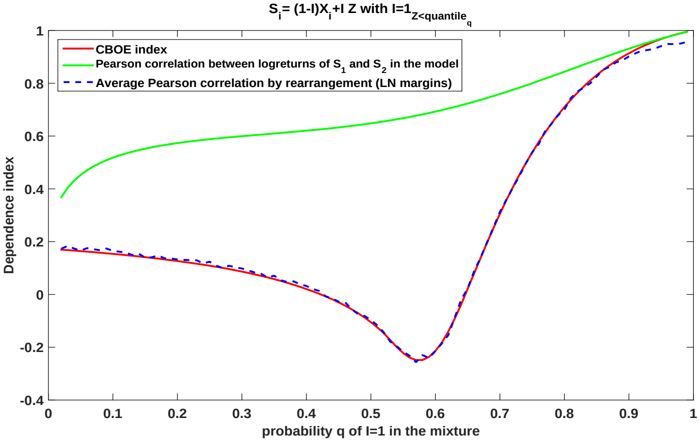
Factor model with 2 assets returns

Within this toy model

- By simulation, get **prices** for at-the-money calls on S and X_i.
- Estimate **implied volatilities** σ_i and σ_S.
- Compute **CBOE index** from implied volatilities.
- For our approach (that I will describe later) we need to specify the marginal distributions (and not just the implied volatilities)
  1. Use **lognormal distribution** for X_1, X_2 and S with logmean $r - \frac{\sigma_i^2}{2}$ and $r - \frac{\sigma_S^2}{2}$ respectively and logvariance σ_i^2 and σ_S^2.
  2. Use empirical distributions obtained by simulation (**correct margins**).

Change of regime driven by Z

$\mathbb{I} = \mathbb{1}_{Z<z_q}$ where z_q is the Value-at-Risk at level q of Z.



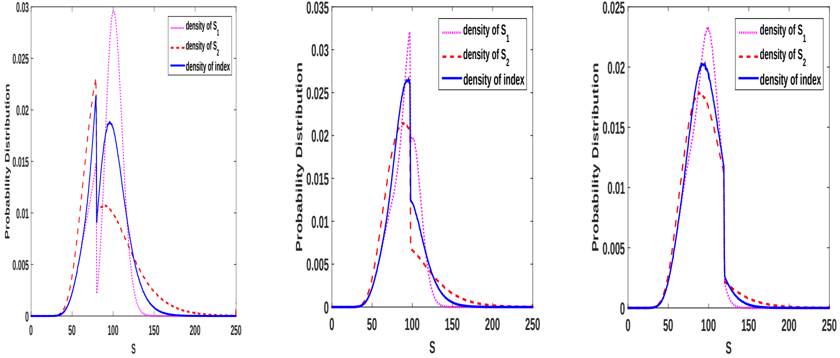
Change of regime driven by Z

How to explain this graph?

Change of regime driven by Z

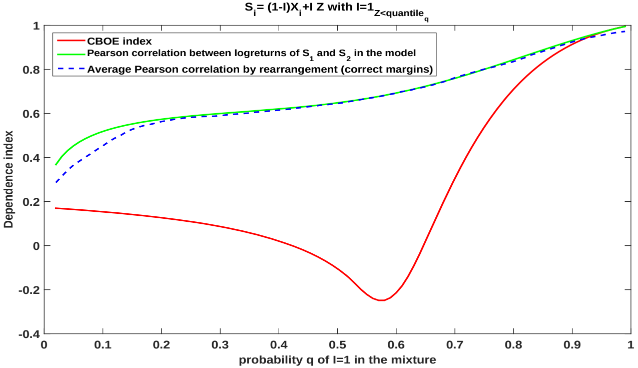
How to explain this graph? S_1, S_2, $S_1 + S_2$ are far from lognormally distributed...

$q = 0.25$ $\quad$ $q = 0.5$ $\quad$ $q = 0.75$



Change of regime driven by Z - Correct margins

$\mathbb{I} = \mathbb{1}_{Z<z_q}$ where z_q is the Value-at-Risk at level q of Z.
We apply our method with the **correct** margins (and not with Lognormal). Better than CBOE!



Other example with $\mathbb{I}$ independent (indep)

Consequences

- **Marginal distributions** matter a lot
- The CBOE implied correlation is model-free, but it is roughly equal to the average pairwise correlation **assuming**
  - logreturns are normal
  - Gaussian dependence
- Our approach allows us to compute the **average pairwise correlation** with the information about margins of the index components and of the index.
- In fact, our approach finds the set of dependence structures consistent with margin informations (i.e. **full joint distribution** of $(X_1, X_2, ..., X_d)$). We can thus compute anything...

Let us explain "how"?

Algorithm to infer dependence

Inputs

- Distributions of X_i for $i = 1, 2, ..., d$ (discretized)
- Distribution of the index S (discretized)

Output

- The joint distribution of $(X_1, X_2, ..., X_d)$

Algorithm to infer dependence

$N = 4$ observations of $d = 3$ variables: X_1, X_2, X_3

$$\mathbf{M} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 6 & 1 \\ 4 & 0 & 0 \\ 6 & 3 & 4 \end{bmatrix}$$

Each column: **marginal** distribution

Interaction among columns: **dependence**

Rearrange the order of the elements per column ⇒ Same margins but effect on the sum! Find the "right" rearrangement.

- Use of the *Rearrangement Algorithm* first used to minimize

$$\text{var}(X_1 + X_2 + ... + X_d)$$

Why do we need an algorithm?

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Why do we need an algorithm?

- **When** $d = 2$, then the minimum variance is the lower Fréchet-Hoeffding bound or "extreme negative dependence" (antimonotonic)

$$var(F_1^{-1}(U) + F_2^{-1}(1 - U)) \leqslant var(X_1 + X_2)$$

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$$var(F_1^{-1}(U) + F_2^{-1}(1-U)) \leqslant var(X_1 + X_2)$$

- **When** $d \geqslant 2$**,** the Fréchet lower bound does not exist:
  - Wang and Wang (2011) study "complete mixability" ($X_1 \sim F_1, ..., X_d \sim F_d$ are completely mixable if there exists a dependence structure between X_1,... X_d such that $X_1 + X_2 + ... + X_d = cst$)
  - Puccetti and Rüschendorf (2012): algorithm (RA) useful to approximate the minimum variance.

Solving for the minimum variance

Inputs:

- $X_1 \sim F_1$, ..., $X_d \sim F_d$
- Goal: look for copulas such that

$$\min \operatorname{var}(X_1 + X_2 + \dots + X_d)$$

It's a NP complete problem: there are no efficient algorithms but we develop an heuristic that performs very well in practice.

Rearrangement Algorithm to solve the minimum variance

$N = 4$ observations of $d = 3$ variables: X_1, X_2, X_3

$$\mathbf{M} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 6 & 1 \\ 4 & 0 & 0 \\ 6 & 3 & 4 \end{bmatrix}$$

Each column: **marginal** distribution
Interaction among columns: **dependence**

Rearrangement Algorithm: Sum with Minimum Variance

minimum variance with $d = 2$ risks X_1 and X_2

Antimonotonicity: $var(\mathbf{X_1^a} + X_2) \leqslant var(\mathbf{X_1} + X_2)$

How about in d dimensions?

Rearrangement Algorithm: Sum with Minimum Variance

minimum variance with $d = 2$ risks X_1 and X_2

Antimonotonicity: $var(\mathbf{X_1^a} + X_2) \leqslant var(\mathbf{X_1} + X_2)$

How about in d dimensions?
Use of the rearrangement algorithm on the original matrix M.

Aggregate Risk with Minimum Variance

- Columns of M are rearranged such that they become anti-monotonic with the sum of all other columns.

$$\forall k \in \{1, 2, ..., d\}, \mathbf{X_k^a} \text{ antimonotonic with } \sum_{j \neq k} X_j$$

- After each step, $var\left(\mathbf{X_k^a} + \sum_{j \neq k} X_j\right) \leqslant var\left(\mathbf{X_k} + \sum_{j \neq k} X_j\right)$ where $\mathbf{X_k^a}$ is antimonotonic with $\sum_{j \neq k} X_j$

Aggregate risk with minimum variance
Step 1: First column

$$\begin{array}{c} \downarrow \\ \begin{bmatrix} \mathbf{6} & 6 & 4 \\ \mathbf{4} & 3 & 2 \\ \mathbf{1} & 1 & 1 \\ \mathbf{0} & 0 & 0 \end{bmatrix} \end{array} \quad \begin{array}{c} X_2 + X_3 \\ 10 \\ 5 \\ 2 \\ 0 \end{array} \quad \text{becomes} \quad \begin{bmatrix} \mathbf{0} & 6 & 4 \\ \mathbf{1} & 3 & 2 \\ \mathbf{4} & 1 & 1 \\ \mathbf{6} & 0 & 0 \end{bmatrix}$$

Aggregate risk with minimum variance

$$\begin{array}{ccc} \downarrow & & \\ \left[\begin{array}{ccc} \mathbf{6} & 6 & 4 \\ \mathbf{4} & 3 & 2 \\ \mathbf{1} & 1 & 1 \\ \mathbf{0} & 0 & 0 \end{array}\right] & \begin{array}{c} X_2 + X_3 \\ 10 \\ 5 \\ 2 \\ 0 \end{array} & \text{becomes} \left[\begin{array}{ccc} \mathbf{0} & 6 & 4 \\ \mathbf{1} & 3 & 2 \\ \mathbf{4} & 1 & 1 \\ \mathbf{6} & 0 & 0 \end{array}\right] \end{array}$$

$$\begin{array}{ccc} \quad\downarrow & & \\ \left[\begin{array}{ccc} 0 & \mathbf{6} & 4 \\ 1 & \mathbf{3} & 2 \\ 4 & \mathbf{1} & 1 \\ 6 & \mathbf{0} & 0 \end{array}\right] & \begin{array}{c} X_1 + X_3 \\ 4 \\ 3 \\ 5 \\ 6 \end{array} & \text{becomes} \left[\begin{array}{ccc} 0 & \mathbf{3} & 4 \\ 1 & \mathbf{6} & 2 \\ 4 & \mathbf{1} & 1 \\ 6 & \mathbf{0} & 0 \end{array}\right] \end{array}$$

$$\begin{array}{ccc} \qquad\downarrow & & \\ \left[\begin{array}{ccc} 0 & 3 & \mathbf{4} \\ 1 & 6 & \mathbf{2} \\ 4 & 1 & \mathbf{1} \\ 6 & 0 & \mathbf{0} \end{array}\right] & \begin{array}{c} X_1 + X_2 \\ 3 \\ 7 \\ 5 \\ 6 \end{array} & \text{becomes} \left[\begin{array}{ccc} 0 & 3 & \mathbf{4} \\ 1 & 6 & \mathbf{0} \\ 4 & 1 & \mathbf{2} \\ 6 & 0 & \mathbf{1} \end{array}\right] \end{array}$$

Aggregate risk with minimum variance

Each column is antimonotonic with the sum of the others:

$$\begin{array}{c} \downarrow \\ \begin{bmatrix} 0 & 3 & 4 \\ 1 & 6 & 0 \\ 4 & 1 & 2 \\ 6 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} X_2+X_3 \\ 7 \\ 6 \\ 3 \\ 1 \end{array}, \quad \begin{array}{c} \downarrow \\ \begin{bmatrix} 0 & 3 & 4 \\ 1 & 6 & 0 \\ 4 & 1 & 2 \\ 6 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} X_1+X_3 \\ 4 \\ 1 \\ 6 \\ 7 \end{array}, \quad \begin{array}{c} \downarrow \\ \begin{bmatrix} 0 & 3 & 4 \\ 1 & 6 & 0 \\ 4 & 1 & 2 \\ 6 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} X_1+X_2 \\ 3 \\ 7 \\ 5 \\ 6 \end{array}$$

Aggregate risk with minimum variance

Each column is antimonotonic with the sum of the others:

$$\begin{bmatrix} \color{blue}{0} & \color{red}{3} & 4 \\ \color{blue}{1} & \color{red}{6} & 0 \\ \color{blue}{4} & \color{red}{1} & 2 \\ \color{blue}{6} & \color{red}{0} & 1 \end{bmatrix} \quad \begin{matrix} \color{red}{X_2} + X_3 \\ 7 \\ 6 \\ 3 \\ 1 \end{matrix}, \quad \begin{bmatrix} \color{blue}{0} & \color{red}{3} & 4 \\ \color{blue}{1} & \color{red}{6} & 0 \\ \color{blue}{4} & \color{red}{1} & 2 \\ \color{blue}{6} & \color{red}{0} & 1 \end{bmatrix} \quad \begin{matrix} \color{blue}{X_1} + X_3 \\ 4 \\ 1 \\ 6 \\ 7 \end{matrix}, \quad \begin{bmatrix} \color{blue}{0} & \color{red}{3} & 4 \\ \color{blue}{1} & \color{red}{6} & 0 \\ \color{blue}{4} & \color{red}{1} & 2 \\ \color{blue}{6} & \color{red}{0} & 1 \end{bmatrix} \quad \begin{matrix} \color{blue}{X_1} + \color{red}{X_2} \\ 3 \\ 7 \\ 5 \\ 6 \end{matrix}$$

$$\begin{bmatrix} \color{blue}{\mathbf{0}} & \color{red}{\mathbf{3}} & \mathbf{4} \\ \color{blue}{\mathbf{1}} & \color{red}{\mathbf{6}} & \mathbf{0} \\ \color{blue}{\mathbf{4}} & \color{red}{\mathbf{1}} & \mathbf{2} \\ \color{blue}{\mathbf{6}} & \color{red}{\mathbf{0}} & \mathbf{1} \end{bmatrix} \qquad S_N = \overset{\color{blue}{X_1} + \color{red}{X_2} + X_3}{\begin{bmatrix} 7 \\ 7 \\ 7 \\ 7 \end{bmatrix}}$$

The minimum variance of the sum is equal to 0! (ideal case of a constant sum (*complete mixability*, see Wang and Wang (2011))

Block Rearrangement Algorithm

With more than 3 variables, we can **improve the standard algorithm** (which proceeds column by column) by proceeding by block:

▶Split d columns into two subsets Π and $\bar{\Pi}$. and make sure that $\sum_{i\in\Pi}\mathbf{X}_i$ is in reverse order with $\sum_{i\in\bar{\Pi}}\mathbf{X}_i$
(working paper with D. McLeish)

1. In general, many local minima for the variance of the sum:
2. By starting with a **random** initial matrix, and reproducing the experience several times, we are able to approximate the set of all copulas that minimize the variance of the sum.

Using the Block RA to infer the dependence

Inputs:

- $X_1 \sim F_1$, ... $X_d \sim F_d$
- the cdf of $\omega_1 X_1 + ... + \omega_d X_d \sim G$ is known for some $\omega_i \in \mathbb{R}$

Question

Describe the set of possible dependence structures (copulas) that are consistent with this information.

Method: Block RA to infer the dependence

- Inputs:
  - $X_1 \sim F_1$, ... $X_d \sim F_d$
  - $X_1 + ... + X_d \sim G$
- Method:
  - Matrix of n rows (for discretization step) by $d + 1$ columns.
  - In each of the first d columns

$$F_j^{-1}\left(\frac{i}{n+1}\right), \qquad i = 1, 2, ..., n$$

  - In the last column

$$-G^{-1}\left(\frac{i}{n+1}\right), \qquad i = 1, 2, ..., n$$

  - Apply the Block RA on the full matrix
- Output: Extract the d first columns, and they describe a discrete copula that is consistent with the information on the cdfs of the risks and of their sum.

A proposed global dependence measure

- Instead of Pearson correlation, we can use **Spearman's rho**

$$\varrho_{ij} := \text{Spearman's rho}(X_i, X_j) = \rho(F_i(X_i), F_j(X_j))$$

  (correlation between the ranks)
- It is not affected by changes in marginal distributions (and thus not sensitive to changes in the volatility parameter)
- We can consider the **average pairwise Spearman's rho**

$$\bar{\varrho} := \frac{\sum_{i=1}^{d-1} \sum_{j>i} \omega_i \omega_j \varrho_{ij}}{\sum_{i=1}^{d-1} \sum_{j>i} \omega_i \omega_j}$$

Compared to the CBOE, it is not affected by changes in the volatilities of the individual components of the index.

Empirical work (coming soon)

- From option prices on Dow Jones 30 (use all strikes) to estimate the marginal distribution of index
- From option prices on components of Dow Jones 30 to estimate their marginal distributions
- Compare this proposal with the CBOE index.
  1. Not affected by changes in volatility
  2. Use full information from option prices
  3. Same empirical conclusions? *"Similar to the VIX, implied correlation exhibits a tendency to increase when the S&P 500 decreases."*

An Implied Systemic Risk Measure

Of interest to go beyond a "global" measure of dependence.
Systemic risk measurement is closely related to

- contagion effects in the tail
- extreme events / coexceedances (tail dependence)

▶ **Our approach allows to study the dependence in the tail.**

A natural measure to study is for example a pairwise average of

$$\varrho_{ij}^{tail} := \text{Spearman's rho}(X_i, X_j|\text{scenarios})$$

These scenarios can be driven by the aggregate risk or some sector, or some individual institutions.

More empirical work (coming soon)

- From option prices on DJ 30 and on its components (use all strikes) estimate marginal distributions
- Study systemic risk contribution of each of the 30 institutions within the DJ 30

$$E_Q\left[Xi\left|\sum_i X_i < \text{quantile}\right.\right]$$

and check whether the order is consistent with what is found under the real-world probability measure (same spirit as SES of Acharya et al. (2010), SRisk of Brownlees and Engle (2014)...)

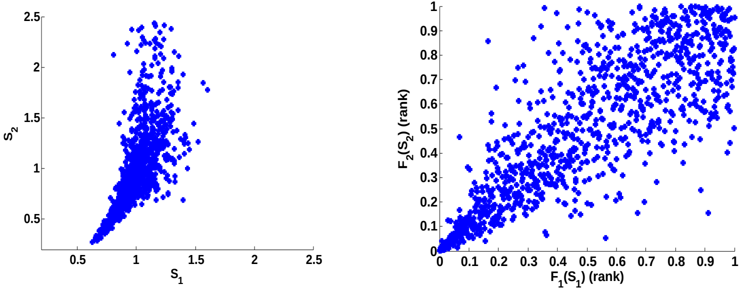
Using the Block RA to infer the dependence

Example: start with a situation for which we know the dependence, and see if we can "recover" this information.

A one-period financial market

- with maturity T.
- with two assets LogNormally distributed (as in Black-Scholes) with $r = 0.01$, $\sigma_1 = 15\%$ and $\sigma_2 = 40\%$. $S_0^1 = S_0^2 = 1$.
- a Clayton copula with parameter 3.
- obtain the distribution of the sum G by simulation

Information obtained by simulation



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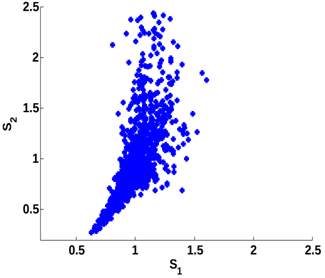

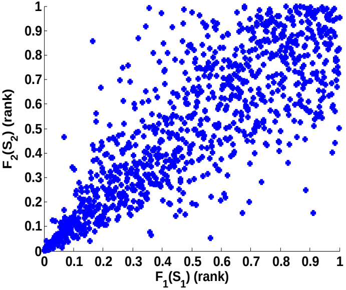

$$\text{Pearson correlation} = \rho\,(X_1, X_2) \approx 0.78$$

Denote $q^S_\alpha = \text{Quantile}_\alpha(X_1 + X_2)$

$$\rho_{\{X_1,X_2 | X_1+X_2 \leqslant q^S_{25\%}\}} \approx 0.81 \quad ; \quad \rho_{\{X_1,X_2 | X_1+X_2 \geqslant q^S_{75\%}\}} \approx -0.15$$

$$\rho_{\{X_1,X_2 | X_1+X_2 \in [q^S_{25\%}, q^S_{75\%}]\}} \approx 0.26$$

Information that we obtain using the information on F_1, F_2 and G *ONLY* and the BRA ran 500 times

$$\text{Pearson correlation} = \rho\left(X_1, X_2\right) \in [0.7800, 0.7801]$$

$$q_\alpha^S = \text{Quantile}_\alpha(X_1 + X_2)$$

$$\rho_{\left\{X_1, X_2 | X_1+X_2 \leqslant q^S_{25\%}\right\}} \approx [0.813, 0.818]$$

$$\rho_{\left\{X_1, X_2 | X_1+X_2 \geqslant q^S_{75\%}\right\}} \in [-0.15, -0.14]$$

$$\rho_{\left\{X_1, X_2 | X_1+X_2 \in [q^S_{25\%}, q^S_{75\%}]\right\}} \in [0.24, 0.27]$$

The method works in higher dimensions

but

- Not able to reproduce a single pairwise correlation, especially if X_1, ... X_d have same marginal distributions.
- But able to reproduce an average correlation, an average tail correlation...
- Intervals are wider in higher dimensions than in two dimensions because of uncertainty on the copula even if one knows the distribution of the sum.

Conclusions & Research Directions

- Develop an **efficient algorithm** for inferring the dependence among variables for which we know the marginal distributions and the distribution of a weighted sum.
- Use it to develop new indicators of implied dependence among assets, richer than the implied correlation from CBOE.
- We hope to find an indicator that is **forward looking** and can have some predictive power...

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Thank You

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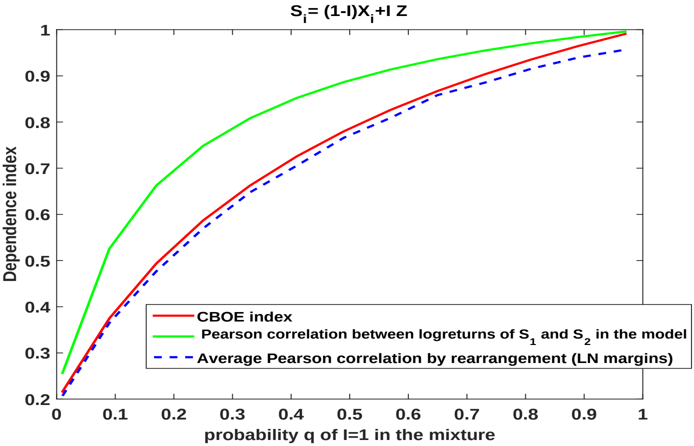
Change of regime independent from X_1, X_2 and Z

$\mathbb{I}$ takes the value 1 with probability q (independent from X_1, X_2 and Z), $\sigma_1 = 0.1$, $\sigma_2 = 0.2$, $\sigma_Z = 0.4$, $\rho_{12} = 0.2$.



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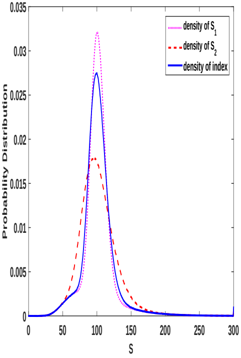
Change of regime independent from X_1, X_2 and Z

How to explain the discrepancy between the CBOE index and the actual correlation in the model?

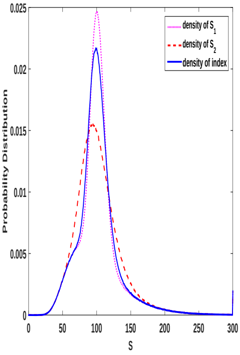
Change of regime independent from X_1, X_2 and Z

How to explain the discrepancy between the CBOE index and the actual correlation in the model? S_1, S_2, $S_1 + S_2$ are not lognormally distributed...

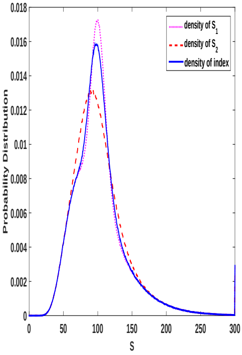
$q = 0.25$



$q = 0.5$



$q = 0.75$



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