Error Control for Real-Time Streaming Applications

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Joint work with:

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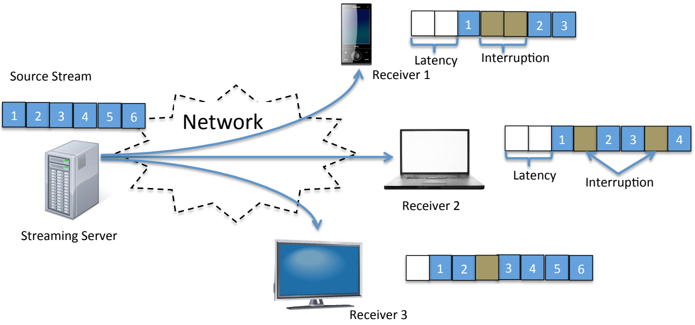
Xiaoqing Zhu (Cisco), John Apostolopoulos (Cisco)

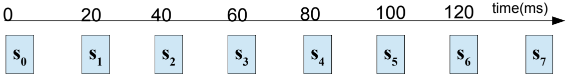
Multimedia Streaming - Overview



Applications

Application	Bit-Rate (Mbps)	MSDU (B)	Delay (ms)	PLR
Video Conf.	2 Mbps	1500	200 ms	10^{-4}
Interactive Gaming	1 Mbps	512	50 ms	10^{-4}
Video Streaming	4 Mbps	1500	1 s	10^{-6}

Streaming Audio



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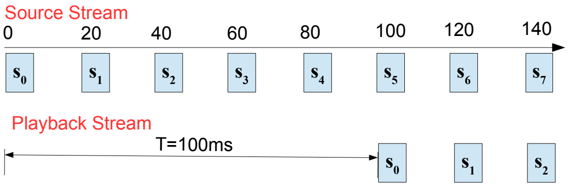

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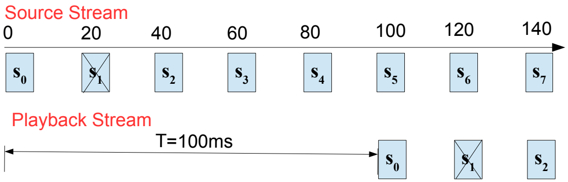

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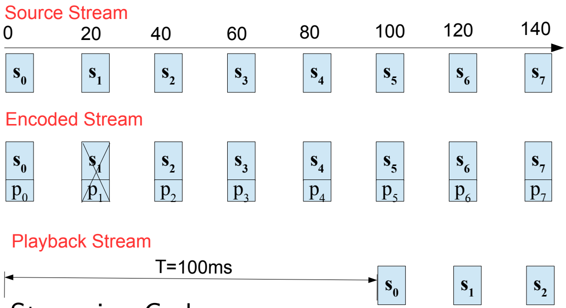
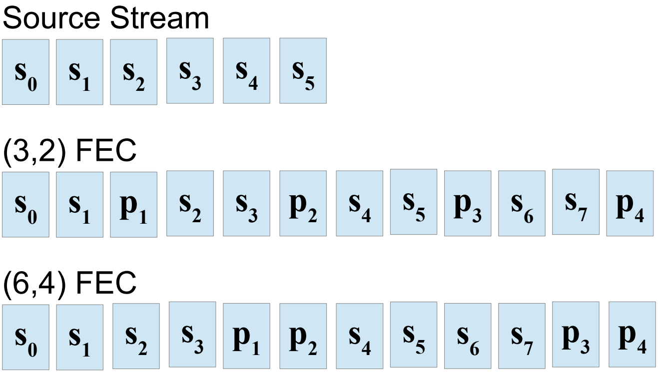


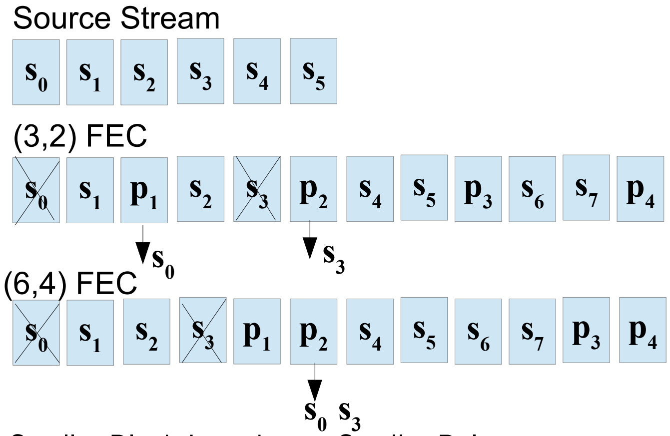
- (n, k) Streaming Code
- Source Packet: $\mathbf{s}_i \in \mathbb{F}_q^k$, Parity Packet: $\mathbf{p}_i \in \mathbb{F}_q^{n-k}$
- Channel Packet: $\mathbf{x}_i = \begin{pmatrix} \mathbf{s}_i \\ \mathbf{p}_i \end{pmatrix} \in \mathbb{F}_q^n$
- Rate $R = \frac{k}{n}$.

Block Length and Error Correction



- Smaller Block Length $\Longrightarrow$ Smaller Delay
- Longer Block Length $\Longrightarrow$ Better Error Correction
- Channel Dynamics affect Delay

Block Length and Error Correction



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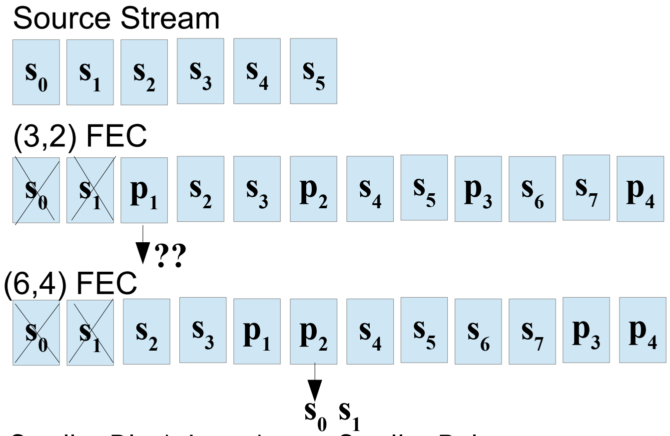

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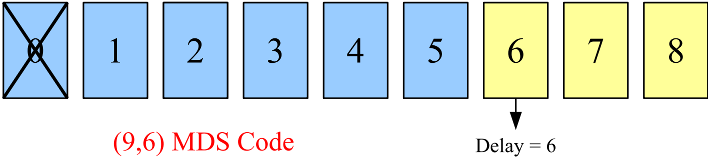
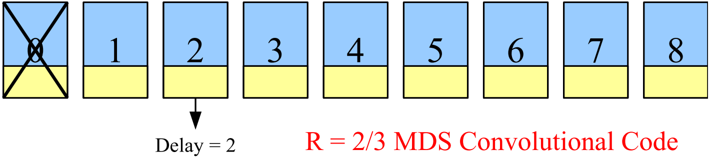
Non-Block Codes

- Block Codes



- Convolutional/Streaming Codes



Real-Time Communication System

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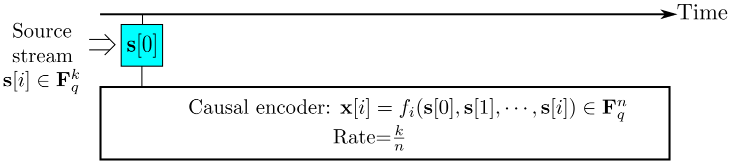

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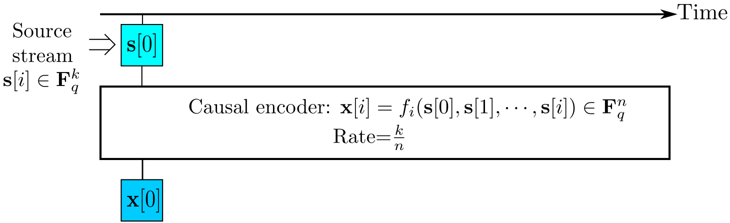

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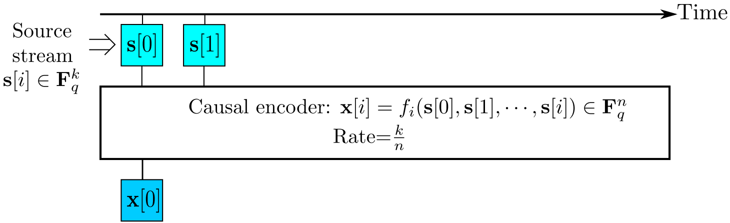

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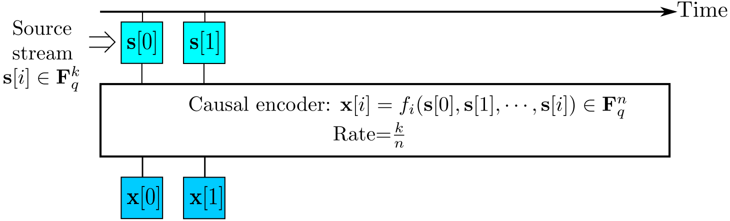

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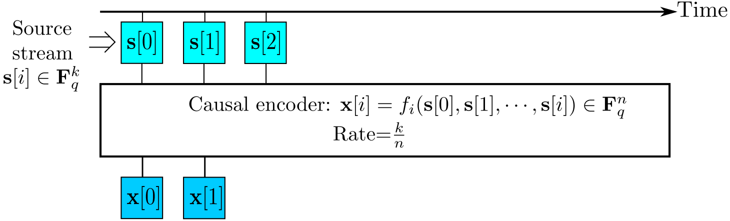

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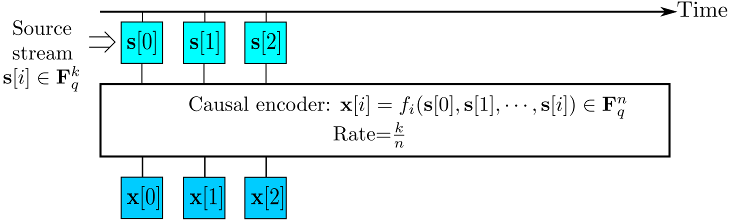

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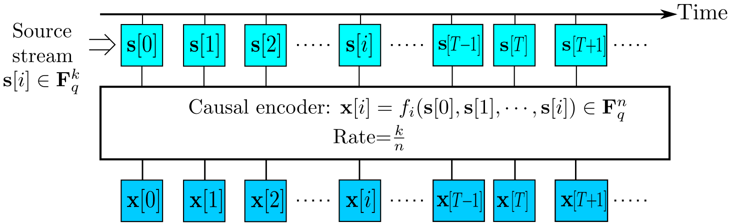

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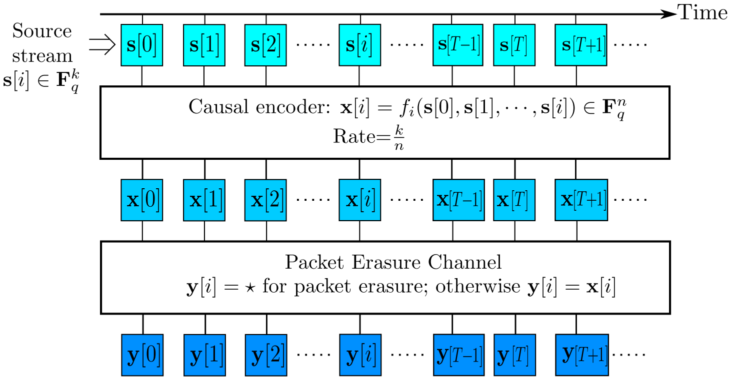

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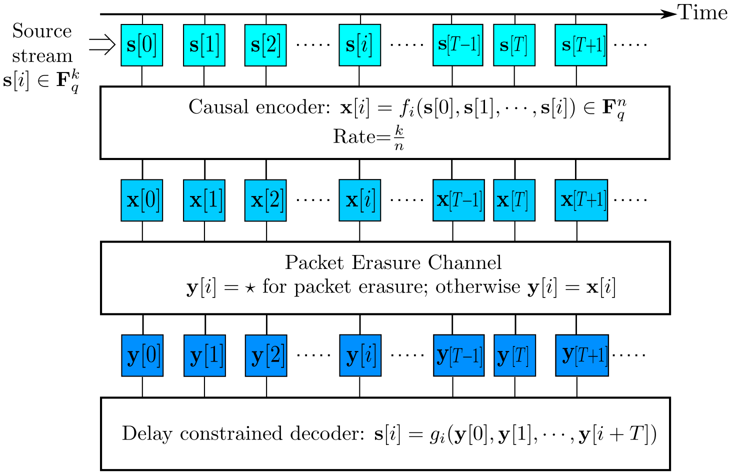

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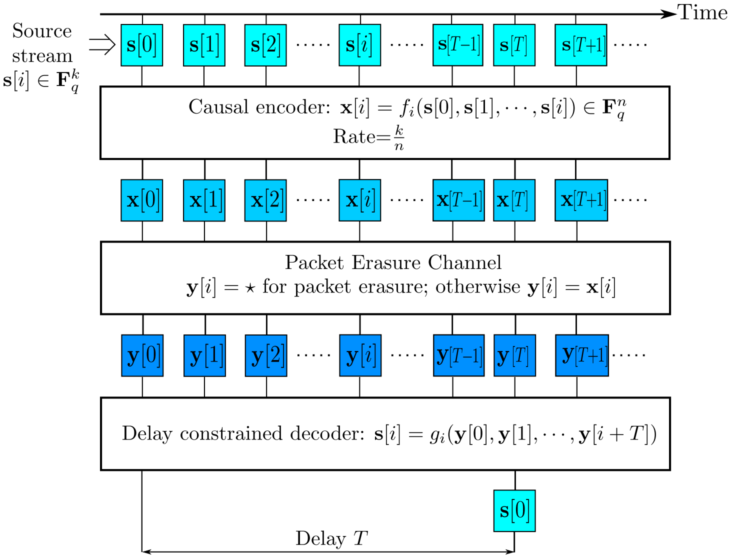

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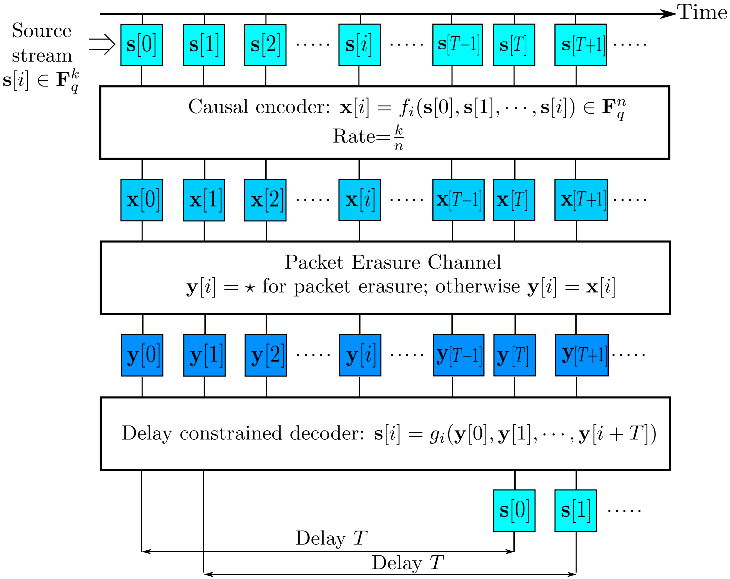
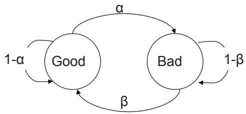


Proposed Channel Model

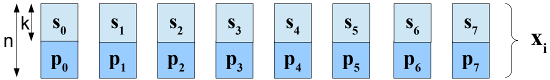


Gilbert-Elliott Model

- Practical Channels introduce both burst and isolated losses.
- Gilbert-Elliott Channel Models

Our Contribution: Error Control Codes for Real-Time Streaming over Channels with Burst and Isolated Erasures.

Baseline Approach: “Random Linear Codes”



$$\mathbf{p}_i = \mathbf{s}_{i-1} \cdot \mathbf{H}_1 + \ldots + \mathbf{s}_{i-M} \cdot \mathbf{H}_M, \qquad \mathbf{H}_i \in \mathbb{F}_q^{k \times n-k}$$

Erasure Codes:

- Random Linear Codes
- Convolutional-MDS Codes (Gabidulin’88, G.-Luerssen’06)

$$\begin{bmatrix} \mathbf{p}_4 \\ \mathbf{p}_5 \\ \mathbf{p}_6 \\ \mathbf{p}_7 \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{H}_4 & \mathbf{H}_3 & \mathbf{H}_2 & \mathbf{H}_1 \\ \mathbf{H}_5 & \mathbf{H}_4 & \mathbf{H}_3 & \mathbf{H}_2 \\ 0 & \mathbf{H}_5 & \mathbf{H}_4 & \mathbf{H}_3 \\ 0 & 0 & \mathbf{H}_5 & \mathbf{H}_4 \end{bmatrix}}_{\text{full rank}} \begin{bmatrix} \mathbf{s}_0 \\ \mathbf{s}_1 \\ \mathbf{s}_2 \\ \mathbf{s}_3 \end{bmatrix}$$

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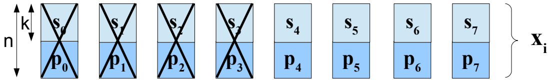

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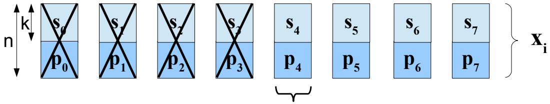

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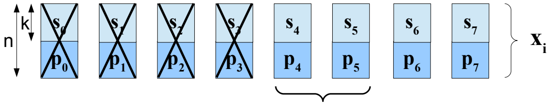

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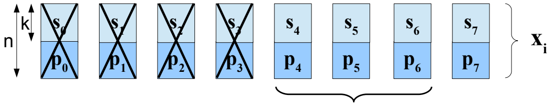

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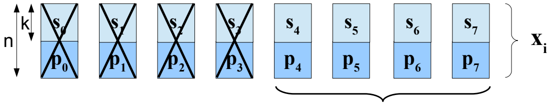

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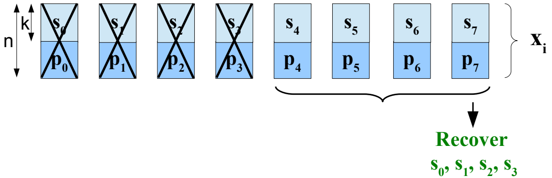

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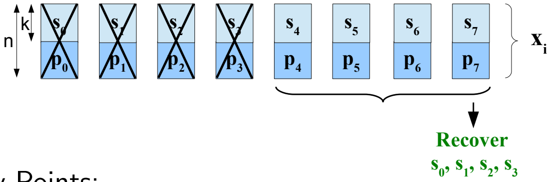


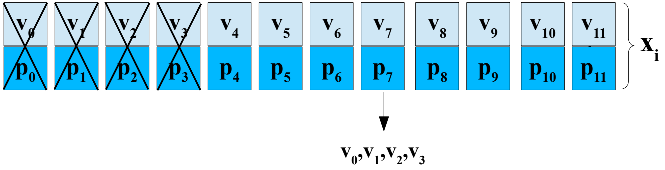
Take Away Points:

- Recovery Condition:
  # of erased symbols $<$ # of received parity equations.
- Adaptive to Erasure Pattern
- Weak Burst Correction (R=1/2 $\Longrightarrow$, $T \approx 2B$)

Layered Coding Scheme

$B = 4$, $T = 8$, $R = \frac{T}{T+B} = \frac{2}{3}$

Rate $1/2$ Baseline Erasure Codes, $T = 7$



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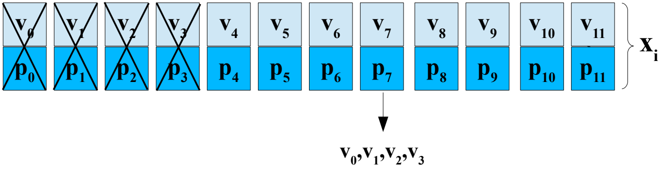
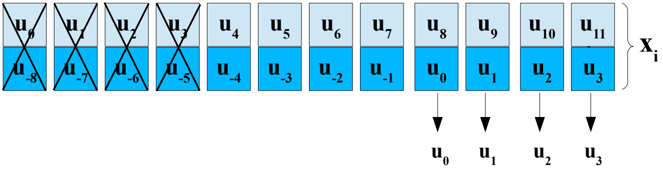


Rate $1/2$ Repetition Code, $T = 8$



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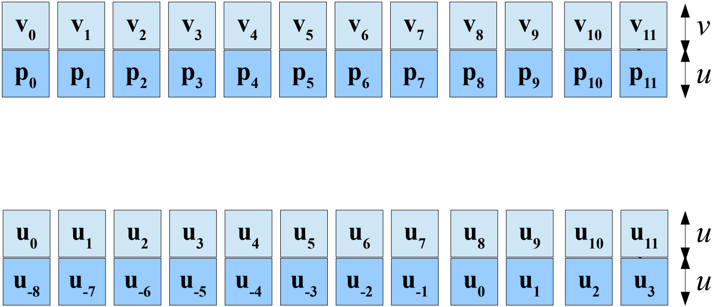

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u_0	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}	u_{11}	u
v_0	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9	v_{10}	v_{11}	v
p_0	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}	p_{11}	u
u_{-8}	u_{-7}	u_{-6}	u_{-5}	u_{-4}	u_{-3}	u_{-2}	u_{-1}	u_0	u_1	u_2	u_3	u

$$R = \frac{u+v}{3u+v} = \frac{1}{2}$$

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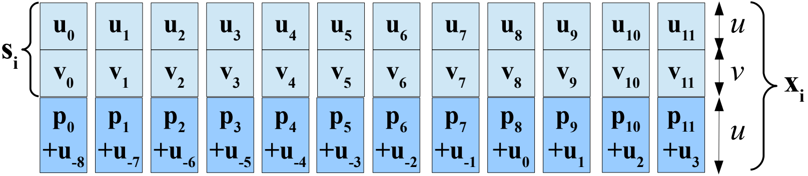

$$R = \frac{u+v}{2u+v} = \frac{2}{3}$$

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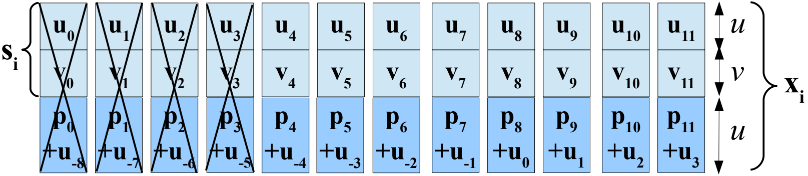

$$R = \frac{u+v}{2u+v}$$

Encoding:

1. **Source Splitting**: $\mathbf{s}_i = (\mathbf{u}_i, \mathbf{v}_i)$, $\mathbf{u}_i \in \mathbb{F}_q^B$, $\mathbf{v}_i \in \mathbb{F}_q^{T-B}$
2. **Erasure Code on $\mathbf{v}_i$**: Generate $\mathbf{v}_i \to (\mathbf{v}_i, \mathbf{p}_i)$ where $\mathbf{p}_i \in \mathbb{F}_q^B$ is obtained from a RLC.
3. **Repetition Code on $\mathbf{u}_i$**: Repeat the $\mathbf{u}_i$ symbols with a shift of T
4. **Merging**: Combine the repeated $\mathbf{u}_i$'s with the $\mathbf{p}_i$'s
5. **Rate**: $R = \frac{T}{T+B}$

Layered Coding Scheme

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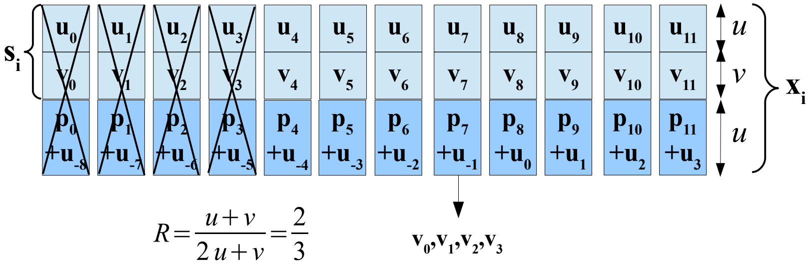

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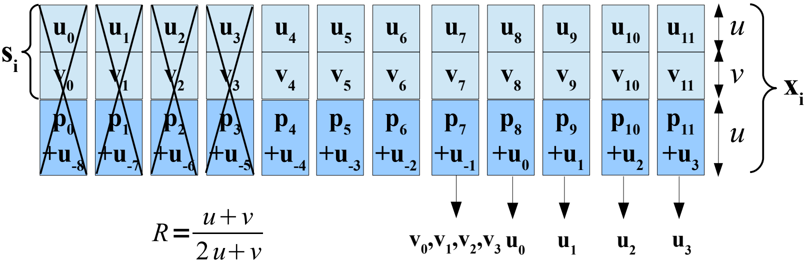

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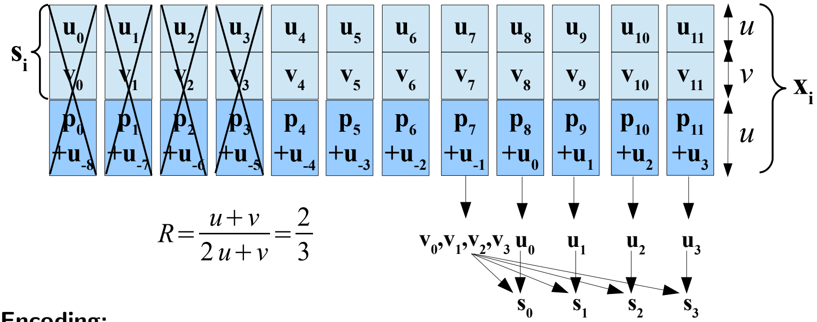


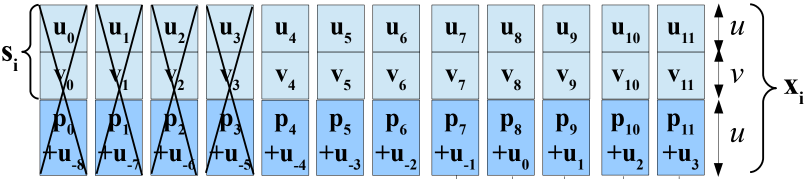
$$R = \frac{u+v}{2u+v} = \frac{2}{3}$$

Encoding:

1. **Source Splitting**: $\mathbf{s}_i = (\mathbf{u}_i, \mathbf{v}_i)$, $\mathbf{u}_i \in \mathbb{F}_q^B$, $\mathbf{v}_i \in \mathbb{F}_q^{T-B}$
2. **Erasure Code on $\mathbf{v}_i$**: Generate $\mathbf{v}_i \rightarrow (\mathbf{v}_i, \mathbf{p}_i)$ where $\mathbf{p}_i \in \mathbb{F}_q^B$ is obtained from a RLC.
3. **Repetition Code on $\mathbf{u}_i$**: Repeat the $\mathbf{u}_i$ symbols with a shift of T
4. **Merging**: Combine the repeated $\mathbf{u}_i$'s with the $\mathbf{p}_i$'s
5. **Rate**: $R = \frac{T}{T+B}$

Layered Coding Scheme

$B = 4$, $T = 8$, $R = \frac{T}{T+B} = \frac{2}{3}$



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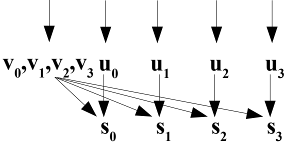

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5. **Rate**: $R = \frac{T}{T+B}$

Convolutional Codes

$\mathbf{s}_i = (s_{i,1}, \ldots, s_{i,k})$ → **(n,k,m) Convolutional Code** → $\mathbf{x}_i = (x_{i,1}, \ldots, x_{i,n})$

Source Symbol: s_i | Channel Symbol: x_i

$$\mathbf{x}_i = \mathbf{s}_i \cdot \mathbf{G}_0 + \mathbf{s}_{i-1} \cdot \mathbf{G}_1 + \ldots + \mathbf{s}_{i-m} \cdot \mathbf{G}_m, \quad \mathbf{G}_j \in \mathbb{F}_q^{k \times n}$$

Column Distance: d_T:

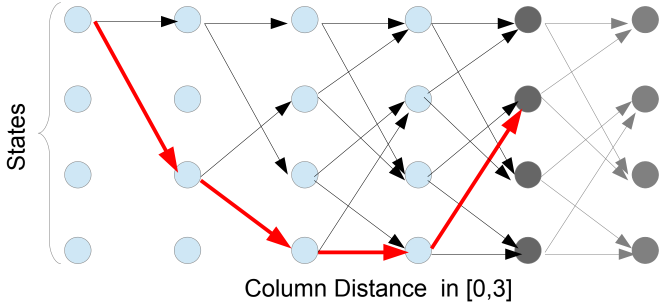
$$d_T = \min_{\substack{[\mathbf{s}_0, \ldots, \mathbf{s}_T] \\ \mathbf{s}_0 \neq 0}} \hat{\mathrm{wt}} \left(\begin{bmatrix} \mathbf{s}_0 & \ldots & \mathbf{s}_T \end{bmatrix} \begin{bmatrix} \mathbf{G}_0 & \mathbf{G}_1 & \ldots & \mathbf{G}_T \\ 0 & \mathbf{G}_0 & \ldots & \mathbf{G}_{T-1} \\ \vdots & & \ddots & \vdots \\ 0 & & \ldots & \mathbf{G}_0 \end{bmatrix} \right)$$

$\hat{\mathrm{wt}}(\cdot)$: Hamming weight (Packet Level).

Convolutional Codes

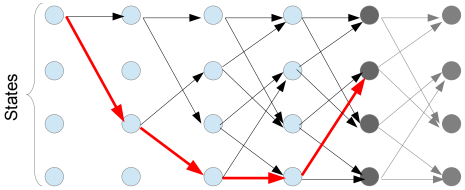
Column-Distance (Costello '1969):
Trellis Path with the following properties:

- Diverges from the all-zero state at $t = 0$
- Min. Weight in the interval $[0, T]$



Strongly-MDS Convolutional Codes

H. Gluesing-Luerssen, J. Rosenthal and R. Smarandache (IT-2006)

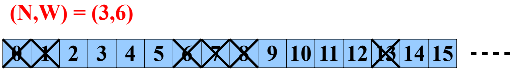


- Generalized Singleton Bound: $d_T \leq (1 - R)(T + 1) + 1.$
- Counterpart of MDS Block Codes
- Systematic Codes: $\mathbf{x}_i = (\mathbf{s}_i, \mathbf{p}_i)$, where $\mathbf{p}_i \in \mathbb{F}_q^{n-k}$

Column Distance & Error Correction - Packet Erasures

Sliding Window Erasure Channel of Type I: $\mathcal{C}_I(N, W)$

In any sliding window of length W there are no more than N erasures



A Convolutional Code with column distance d_T recovers every source packet with delay of T if:

- $W \geq T + 1$
- $d_T \geq N + 1$

Column Distance & Error Correction - Packet Erasures

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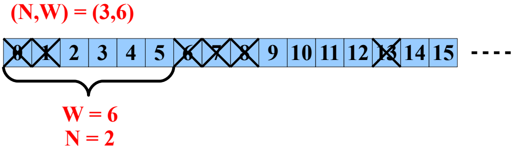

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Column Distance & Error Correction - Packet Erasures

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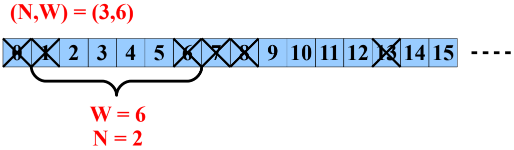

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- $W \geq T + 1$
- $d_T \geq N + 1$

Column Distance & Error Correction - Packet Erasures

Sliding Window Erasure Channel of Type I: $\mathcal{C}_I(N, W)$

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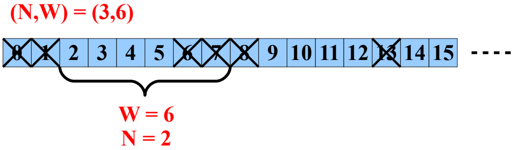
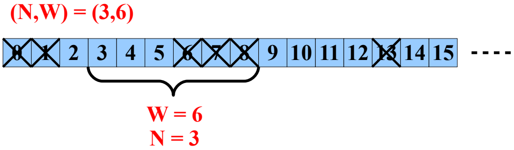


A Convolutional Code with column distance d_T recovers every source packet with delay of T if:

- $W \geq T + 1$
- $d_T \geq N + 1$

Sliding Window Erasure Channel of Type I: $\mathcal{C}_I(N, W)$

In any sliding window of length W there are no more than N erasures



A Convolutional Code with column distance d_T recovers every source packet with delay of T if:

- $W \geq T + 1$
- $d_T \geq N + 1$

(Packet-Level) Distance and Span Properties

Packet Level Column Distance: d_T

$$d_T = \min_{\substack{[\mathbf{s}_0, \ldots, \mathbf{s}_T] \\ \mathbf{s}_0 \neq 0}} \mathrm{wt}\left(\begin{bmatrix} \mathbf{s}_0 & \ldots & \mathbf{s}_T \end{bmatrix} \begin{bmatrix} \mathbf{G}_0 & \mathbf{G}_1 & \ldots & \mathbf{G}_T \\ 0 & \mathbf{G}_0 & \ldots & \mathbf{G}_{T-1} \\ \vdots & & \ddots & \vdots \\ 0 & & \ldots & \mathbf{G}_0 \end{bmatrix} \right)$$

Packet-Level Column Span: c_T

$$c_T = \min_{\substack{[\mathbf{s}_0, \ldots, \mathbf{s}_T] \\ \mathbf{s}_0 \neq 0}} \mathrm{span}\left(\begin{bmatrix} \mathbf{s}_0 & \ldots & \mathbf{s}_T \end{bmatrix} \begin{bmatrix} \mathbf{G}_0 & \mathbf{G}_1 & \ldots & \mathbf{G}_T \\ 0 & \mathbf{G}_0 & \ldots & \mathbf{G}_{T-1} \\ \vdots & & \ddots & \vdots \\ 0 & & \ldots & \mathbf{G}_0 \end{bmatrix} \right)$$

$$c_T = \min_{\substack{[\mathbf{s}_0,\dots,\mathbf{s}_T] \\ \mathbf{s}_0 \neq 0}} \text{span}\left(\begin{bmatrix} \mathbf{s}_0 & \dots & \mathbf{s}_T \end{bmatrix} \begin{bmatrix} \mathbf{G}_0 & \mathbf{G}_1 & \dots & \mathbf{G}_T \\ 0 & \mathbf{G}_0 & \dots & \mathbf{G}_{T-1} \\ \vdots & & \ddots & \vdots \\ 0 & & \dots & \mathbf{G}_0 \end{bmatrix} \right)$$

$\mathbf{x}[0]$	$\mathbf{x}[1]$	$\mathbf{x}[2]$	$\mathbf{x}[3]$	$\mathbf{x}[4]$	$\mathbf{x}[5]$
0	0	0	0	0	0
1	1	0	0	0	1
1	0	1	0	0	1
1	0	0	1	0	0
1	0	0	0	1	1
1	0	0	0	0	1
1	0	1	1	0	1
1	1	0	0	1	1
1	1	1	1	0	1
⋮	⋮	⋮	⋮	⋮	⋮

$$\underbrace{\qquad\qquad}_{\substack{j+1=6 \\ c_j = c_5 = 6}}$$

$$\mathbf{x}_{[0,T]} = \left[\mathbf{x}[0], \mathbf{x}[1], \dots, \mathbf{x}[T]\right]$$

Column-Span:

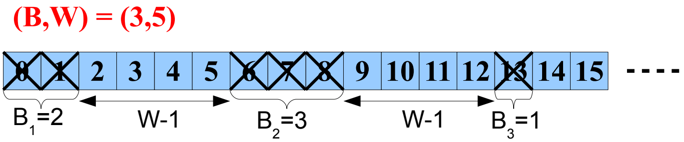
$$c_T \triangleq \min_{\mathbf{x} \in \mathcal{C}} \{span(\mathbf{x}_{[0:T]} | \mathbf{s}[0] \neq 0\}$$
$$\leq \min\left(1, \frac{1-R}{R}\right) T + 1$$

$$c_T \geq d_T$$

Column Span & Error Correction - Packet Erasures

Sliding Window Erasure Channel of Type II: $\mathcal{C}_{II}(B, W)$

In any sliding window of length W there is a single erasure burst of length B



A Convolutional Code with column distance c_T recovers every source packet with delay of T if:

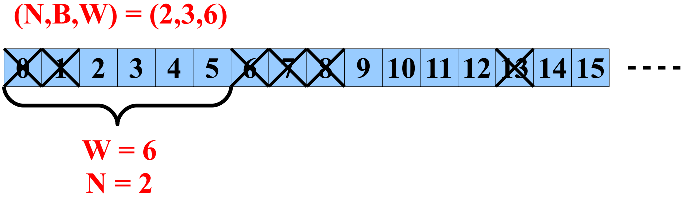
- $W \geq T + 1$
- $c_T \geq B + 1$

Column-Distance & Column-Span Tradeoff

Badr-Patil-Khisti-Tan-Apostolopoulos (T-IT 2017)

Theorem

Consider a $\mathcal{C}(N, B, W)$ channel with delay T and $W \geq T + 1$. A streaming code is feasible over this channel if and only if it satisfies: $d_T \geq N + 1$ and $c_T \geq B + 1$



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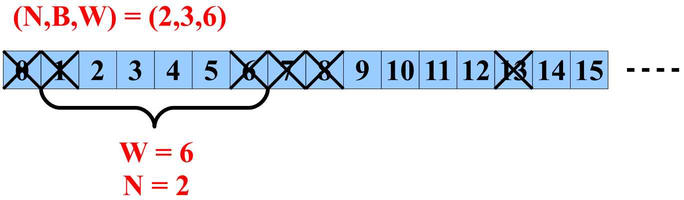

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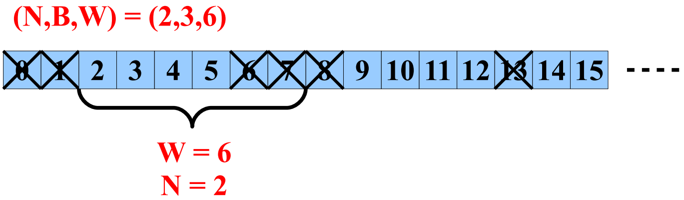

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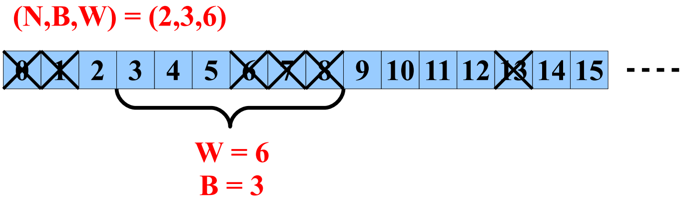

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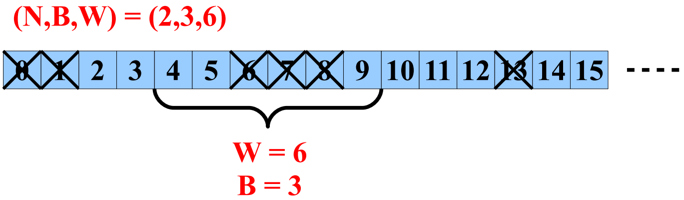


Column-Distance & Column-Span Tradeoff

Badr-Patil-Khisti-Tan-Apostolopoulos (T-IT 2017)

Theorem

Consider a $\mathcal{C}(N, B, W)$ channel with delay T and $W \geq T + 1$. A streaming code is feasible over this channel if and only if it satisfies: $d_T \geq N + 1$ and $c_T \geq B + 1$

Theorem

For any rate R convolutional code and any $T \geq 0$ the Column-Distance d_T and Column-Span c_T satisfy the following:

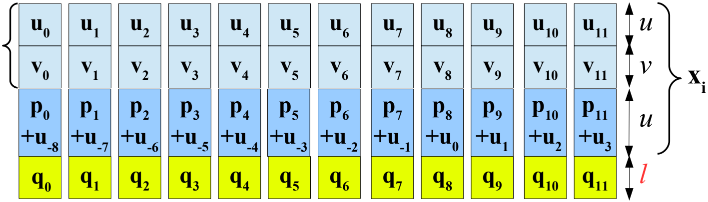
$$\left(\frac{R}{1-R}\right) c_T + d_T \leq T + 1 + \frac{1}{1-R}$$

Any (c_T, d_T) pair with $c_T \geq d_T$ is achievable over a sufficiently large field provided that:

$$\left(\frac{R}{1-R}\right) c_T + d_T \leq T + \frac{1}{1-R}$$

MiDAS Construction

Layered Code Design



- **Burst-Erasure Streaming Code**: $(\mathbf{u}_i, \mathbf{v}_i, \mathbf{p}_i + \mathbf{u}_{i-T})$
- **Erasure Code**: $\mathbf{q}_i = \sum_{t=1}^{M} \mathbf{u}_{i-t} \cdot \mathbf{H}_t^u, \qquad \mathbf{q}_i \in \mathbb{F}_q^l$
- **Concatenation**: $(\mathbf{u}_i, \mathbf{v}_i, \mathbf{p}_i + \mathbf{u}_{i-T}, \mathbf{q}_i)$

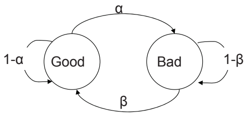
$$R = \frac{T}{T+B+l}$$

- Attains the lower bound

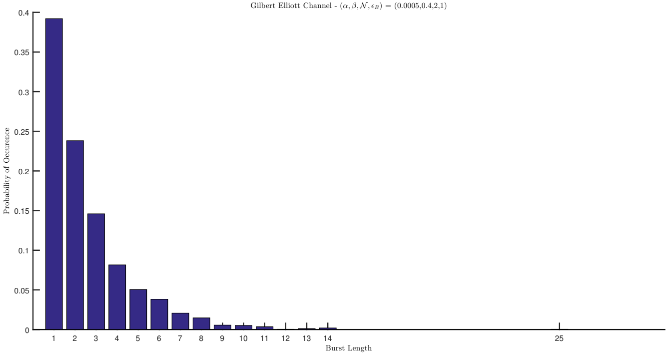
Simulation Results

Gilbert-Elliott Channel $(\alpha, \beta) = (5 \times 10^{-4}, 0.5)$, $T = 12$ and $R \approx 0.5$

Gilbert Elliott Channel



- Good State: $\Pr(\text{loss}) = \varepsilon$
- Bad State: $\Pr(\text{loss}) = 1$



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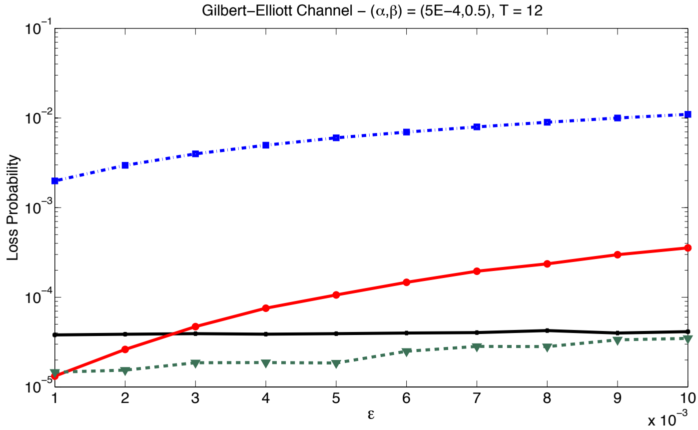
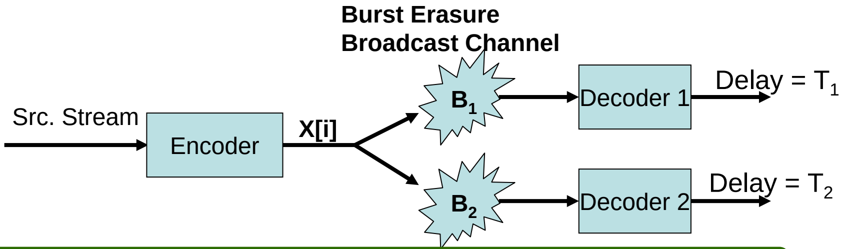


Code	N	B	Code	N	B
RLC	6	6	MiDAS	2	9
Burst-Erasure	1	11			

Multicast Streaming Codes

Badr-Khisti-Martinian (JSAC 2011) Badr-Khisti-Lui (IT Trans. 2015)

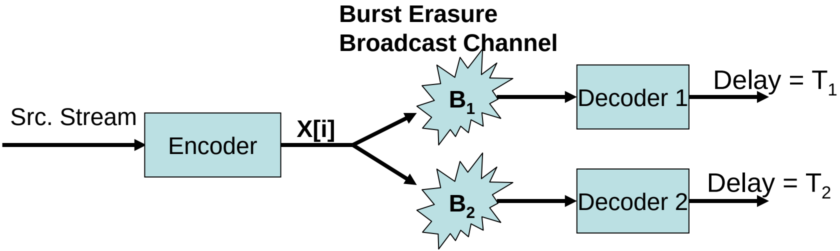


Motivation

- Common Source Stream, Multiple Receivers
- $B_1 < B_2$
- Receiver 1 : Good Channel State
- Receiver 2: Weaker Channel State
- Delay adapts to Channel State

Multicast Streaming Codes

Badr-Khisti-Martinian (JSAC 2011) Badr-Khisti-Lui (IT Trans. 2015)



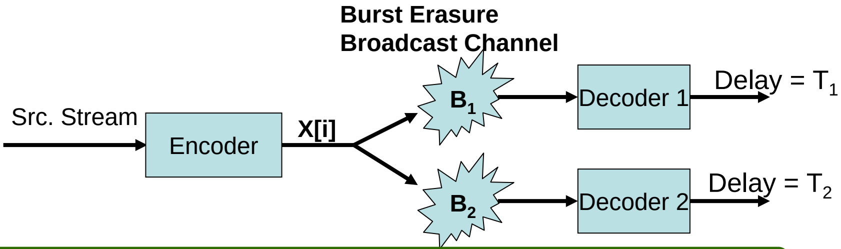
Column-Span

A streaming code is feasible on the (B_1, T_1, B_2, T_2) multicast channel if and only if:

- $c_{T_1} \geq B_1 + 1$
- $c_{T_2} \geq B_2 + 1$

Diversity Embedded Streaming Codes: DE-SCo

Badr-Khisti-Martinian (JSAC, March 2011)



Theorem

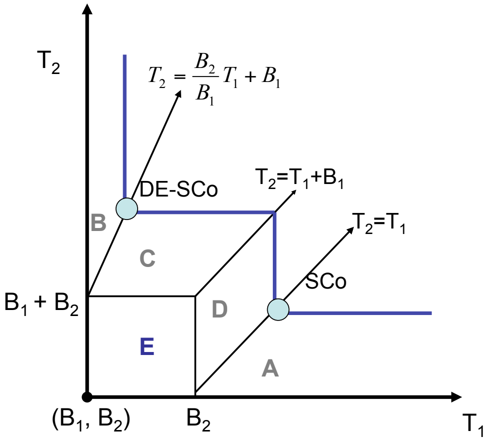
There exists a $\{(B_1, T_1), (B_2, T_2)\}$ *multicast streaming code of rate* $R = \frac{T_1}{T_1+B_1}$ *provided*

$$T_2 \geq T_2^\star \triangleq \frac{B_2}{B_1}T_1 + B_1.$$

$T_2^\star$ *is the minimum such threshold.*

Multicast Streaming Capacity
Badr-Khisti-Lui (IT Trans. 2015)

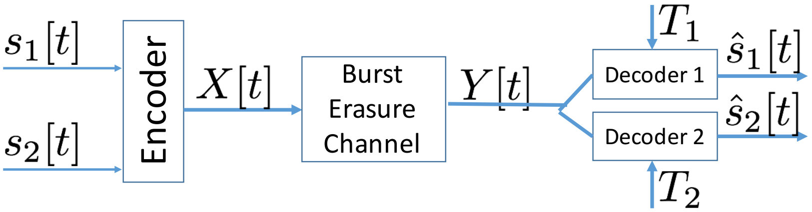
Assume w.l.o.g. $B_2 \geq B_1$



Region	Capacity
A	$\frac{T_2}{T_2 + B_2}$
B	$\frac{T_1}{T_1 + B_1}$
C	$\frac{T_2 - B_1}{T_2 - B_1 + B_2}$
D	$\frac{T_1}{T_1 + B_2}$
E	Partial Characterization

Multiple Multiplexed Streams

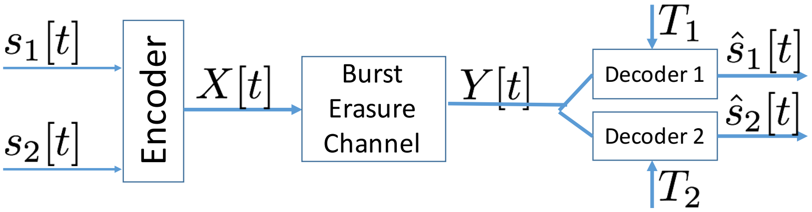
Badr-Lui-Khisti-Tan-Zhu-Apostolopoulos (T-IT (Submitted), Netcod 2016)



- Source Streams: $s_1[t] \in \mathbb{F}_q^{k_1}$ $s_2[t] \in \mathbb{F}_q^{k_2}$
- Encoder: $x[t] = f_t(s_1[0], s_2[0], \ldots, s_1[t], s_2[t]) \in \mathbb{F}_q^n$
- $R_1 = k_1/n$ and $R_2 = k_2/n$
- Burst Erasure Channel (B): $x[t] \longrightarrow y[t]$
- Stream 1: $\hat{s}_1[t] = \gamma_{1,t}(y[0], \ldots, y[t+T_1])$
- Stream 2: $\hat{s}_2[t] = \gamma_{2,t}(y[0], \ldots, y[t+T_2])$
- Assume $T_2 > T_1$ (wlog)

Multiple Multiplexed Streams

Badr-Lui-Khisti-Tan-Zhu-Apostolopoulos (T-IT (Submitted), Netcod 2016)



Capacity Region

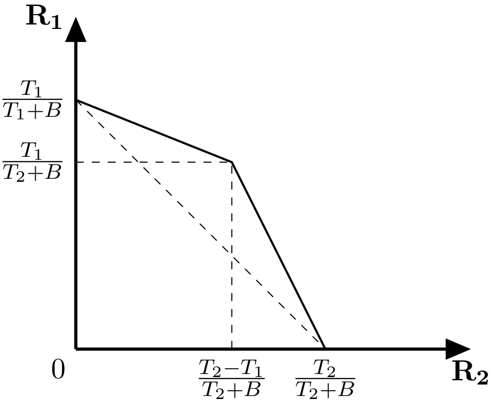
A rate pair (R_1, R_2) is achievable if there is a streaming code with $R_1 = k_1/n$ and $R_2 = k_2/n$ over some $\mathbb{F}_q$ such that:

- Each source packet in stream 1 is recovered with a delay of T_1
- Each source packet in stream 2 is recovered with a delay of T_2

The union of all achievable rate pairs (R_1, R_2) is the capacity region.

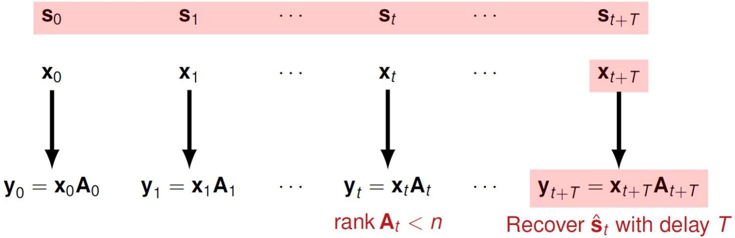
Capacity Region

$T_2 \geq T_1 + 2B$



$$R_1 + R_2 \leq \frac{T_2}{T_2 + B}, \qquad (T_1 + B)R_1 + T_1 R_2 \leq T_1$$

Network Streaming
Mahmood-Badr-Khisti (IT-Trans 2016)



- Channel Input: $\mathbf{x}_t \in (\mathbb{F}_{q^M})^n$ (Extension Field)
- Channel Matrix: $\mathbf{A}_t \in \mathbb{F}_q{}^{n\times n}$ (Base Field)
- Chanel Output $\mathbf{y}_t = \mathbf{x}_t\mathbf{A}_t$
- Delay constraint T

$$\hat{\mathbf{s}}_t = f_t(\mathbf{y}_0, \ldots, \mathbf{y}_{t+T})$$

Column Sum Rank:

$$d_R(T, \mathcal{C}) = \min_{\substack{(\mathbf{x}_0, \ldots, \mathbf{x}_T) \in \mathcal{C} \\ \mathbf{s}_0 \neq 0}} \sum_{t=0}^{T} \mathrm{rank}\ \phi(\mathbf{x}_t)$$

Theorem

Within the interval $[0, T]$ the source symbol $\mathbf{s}_0$ is recoverable at time $t = T$ if and only if

$$\sum_{j=0}^{T} \mathrm{rankdef}\ \mathbf{A}_j < d_R(T, \mathcal{C})$$

Construction of Maximum-Sum-Rank (MSR) convolutional codes.

Other Extensions

- Field Size Constraints: Diagonally Interleaved Codes (Badr-Khisti-Tan-Apostolopoulos 2014)
- Mismatched Streaming Codes (Patil-Badr-Khisti-Tan Asilomar 2013)
- Partial Recovery Streaming Codes (Badr-Khisti-Tan-Apostolopoulos JSTSP 2014)
- Multiple Erasure Bursts (Li-Khisti-Girod Asilomar 2011) - Interleaved Low-Delay Codes
- Multiple Links (Lui-Badr-Khisti CWIT 2011) - Layered coding for burst erasure channels

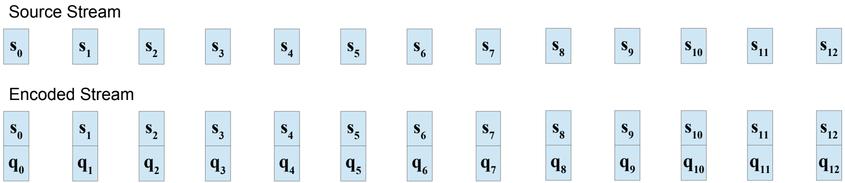
Other Works

- Martinian and Sundberg (2004)
- Leong-Ho (ISIT 2012), Leong-Qureshi-Ho (ISIT 2013), N. Adler and Y. Cassuto (ISIT 2015)

Dual-Delay Streaming Codes

Badr-Khisti-Zhu-Tan-Apostopolous (Infocom 2017)

- Short-Memory RLC Code
- Repetition Code with longer Memory

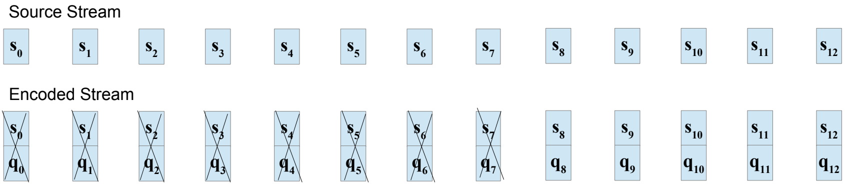


$$\mathbf{q}_i = \mathbf{s}_{i-1} + \mathbf{s}_{i-2} + \mathbf{s}_{i-10}$$

Dual-Delay Streaming Codes

Badr-Khisti-Zhu-Tan-Apostopolous (Infocom 2017)

- Short-Memory RLC Code
- Repetition Code with longer Memory

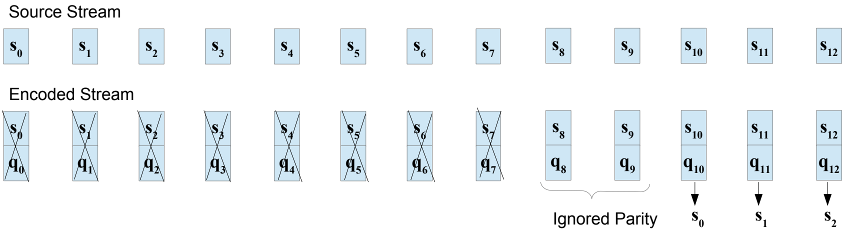


$$\mathbf{q}_i = \mathbf{s}_{i-1} + \mathbf{s}_{i-2} + \color{red}{\mathbf{s}_{i-10}}$$

Dual-Delay Streaming Codes

Badr-Khisti-Zhu-Tan-Apostopolous (Infocom 2017)

- Short-Memory RLC Code
- Repetition Code with longer Memory

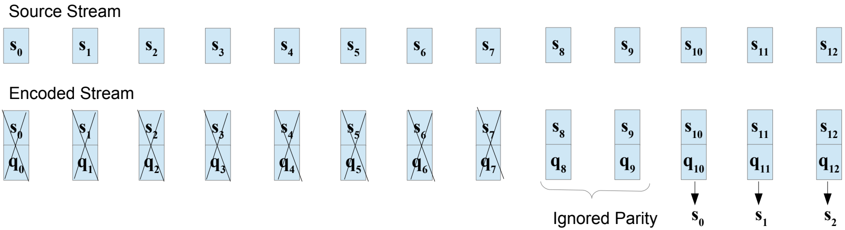


$$\mathbf{q}_{10} = \underbrace{\mathbf{s}_8 + \mathbf{s}_9}_{\text{not erased}} + \mathbf{s}_0$$

Dual-Delay Streaming Codes

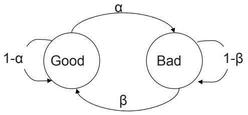
Badr-Khisti-Zhu-Tan-Apostopolous (Infocom 2017)

- Short-Memory RLC Code
- Repetition Code with longer Memory

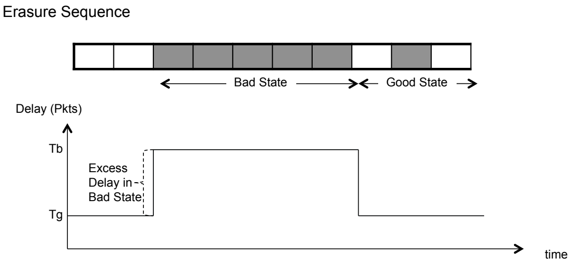


- Short Bursts $B = 1, 2$, delay $T = 2B$, like RLC
- Longer Bursts $B \in [3, 8]$, delay $T = 10$, Like Repetition Code
- $B > 8$, partial recovery with $T > 10$
- RLC Threshold $B = 5$

Gilbert Elliott Channel Model



Gilbert-Elliott Model



Chanel-Adaptive Delay

- Good State: $T_g = 3$ packets
- Bad State: $T_b = 12$ packets

Comparisons

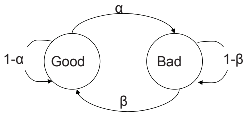
- Baseline (RLC) Codes
- Non-Adaptive Streaming Codes
- Adaptive (Dual-Delay) Streaming Codes

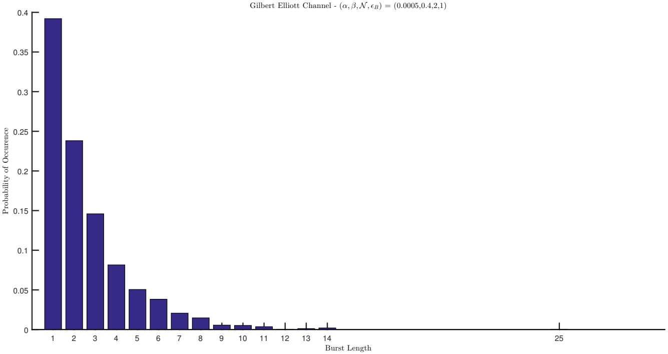
Simulation Result

Gilbert-Elliott Channel $(\alpha, \beta) = (5 \times 10^{-4}, 0.4)$ and $R \approx 0.5$

Gilbert Elliott Channel

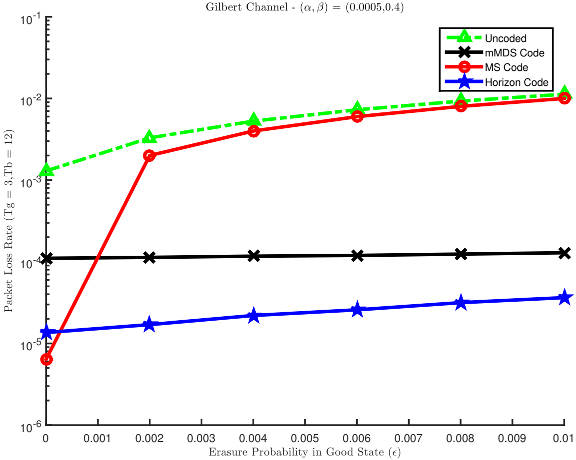
- Good State: $\Pr(\text{loss}) = \varepsilon$
- Bad State: $\Pr(\text{loss}) = 1$





Simulation Result

Gilbert-Elliott Channel $(\alpha, \beta) = (5 \times 10^{-4}, 0.4)$ and $R \approx 0.5$



- Black Line: Random Linear Codes
- Red Line: Non-Adaptive Burst Erasure Codes
- Blue Line: Proposed Codes

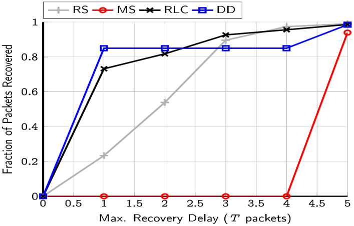
Real-World Traces

- `http://rawdad.org/due/paketdelivery/20150401`
- These are 18.75 million packets
- We divide them into windows of 5 minutes (15000 packets)
- total of 1,250 windows.
- Delay $T = 5$.

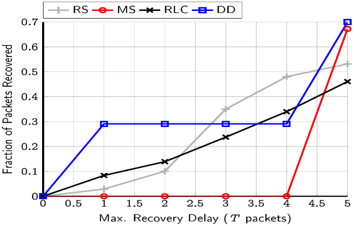
	$\bar{B} \leq 3$	$\bar{B} \in [4, 20]$
PLR $\leq$ 1%	363	307
PLR $>$ 1%	63	339

- 178 windows with very long bursts were not considered.

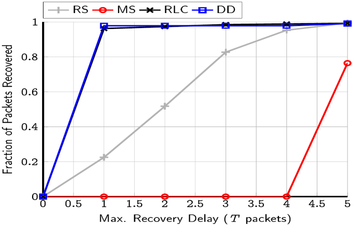
Real-World Traces-II



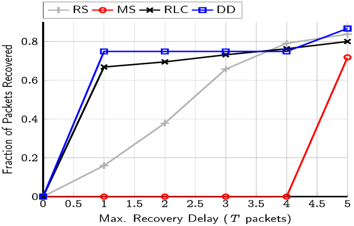
(a) Mild: PLR $< 1\%$, $B \leq 3$



(b) Bursty: PLR $< 1\%$, $B \in [4, 20]$



(c) Isolated: PLR $\geq 1\%$, $B \leq 3$



(d) Mixed: PLR $\geq 1\%$, $B \in [4, 20]$

Demo

Conclusions

Error Control Coding for Multimedia Streaming

- Real-Time Communication over Channels with Burst and Isolated Erasures
  - Sequential Recovery
  - Baseline Schemes are not ideal
  - New Coding Schemes and Performance Gains

Other Directions:

- Channel Dispersion in Streaming Setup (joint work with S. Lee and V. Tan)
- Source Coding in a streaming setup