Robot Algorithms for Medical Simulation & Virtual Prototyping

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Motion

- Ubiquitous in physical world and virtual environments
- Arising from dynamical systems: biological entities or man-made structures or mechanisms

Robotics & Automation

- Robot/Computer Assisted Surgery (JHU)
- Dream Robot (Sony)
- Assembly Planning (UNC)

Medical Applications

- AR for Surgery (UNC/Ultrasound)
- Third Ventriculostomy (HT/Immersion Medical)
- da Vinci Surgical System (Intuitive Surgery, Inc.)
- Human Simulation (BDI)
- Imaging (UNC/MIDAG)
**Engineering Design**

Interactive Prototyping of Massive Structures
Walkthrough Research Group (UNC)

**Scientific Visualization**

Interactive Visualization & Manipulation of nanoStructures
nanoManipulator Research Group (UNC)

**Virtual Environments**

Interactive Agents and Avatars (UPENN/HMS)
Walking Experiment & the Pit
EVE Research Group (UNC)

**Training & Education**

VET (ISI/USC)
Team Training (NRL)
Games & Entertainment

- Medal of Honor™ (Pacific Assault)
- Monster Inc. (Pixar Animation Studio)

Robot Algorithms

- Sensing and vision
- Motion control
- Dynamics and kinematics
- Motion planning
- Manipulation
- Others….

Organization

- Articulated Bodies
- Deformable Models

Organization

- Articulated Bodies
- Deformable Models
Algorithms for Articulated Bodies

- Continuous Collision Detection for Articulated Models
- Adaptive Forward Dynamics (AD)
- Planning of Highly Articulated Body Using AD + Contact Handling

Collision & Proximity Queries

Geometric reasoning of spatial relationships among objects (in a dynamic environment)

- Collision Detection
- Contact Points & Normals
- Closest Points & Separation Distance
- Penetration Depth

⇒ Can take a high fraction of the running time in simulations

SWIFT++: Distance & Collision Queries

Use of Multi-resolution Reps. & Coherence
[EHmann & Lin, EG 2001]

PIVOT: Proximity Computations

Proximity Queries Using Graphics Hardware Acceleration
[Hoff, et al, I3D 2001]
Real-Time Simulation of Complex Objects Using PIVOT on GPU

http://gamma.cs.unc.edu/PIVOT

Penetration Depth Computation

Dynamic Simulation
Virtual Prototyping
Haptic Rendering

DEEP: [Kim et al, ICRA 2002; HS 2002]
PD: [Kim et al, SCA 2002]

Continuous Collision Detection for Articulated Models Using GPUs

Motivation: Prior discrete algorithms only check for collision at each time instance, not in between steps. Collisions can be missed for fast moving and/or thin objects, such as an avatar rapidly moving around in a VE.

http://gamma.cs.unc.edu/Articulate
[Redon, Kim, Lin, Manocha; SM 2004]

Overview

Motion interpolation
BVH Construction
BVH Cutting
SV Generation
Collision Checking and TDC Estimation

\[ M^{-1}(t) = \{ P^{-1}(t), T^{-1}(t) \} \]
\[ T^{-1}(t) = e + s \]
\[ P^{-1}(t) = \cos(u), A, + \sin(u), B, + C \]
\[ M(t) = M(t), M(t), M(t) \]
We use the dynamic AABBs to cull away the links far from the environment.

We use a line-swept sphere volume to represent each link and compute its swept volume for all links of the avatar, as shown on the right (blue/green indicating initial/final position).
Overview

Local Planning in Contact Space

Planning the motion of a Puma Robot (800 triangles, 7 dofs)
in a partial Auxiliary Machine Room model (117,000 triangles)
Two orders of magnitude performance improvement

Mobile Interaction in VE

An avatar moving around rapidly in a living room;
Real-time continuous collision detection is used to check for possible interference with the furniture.
[IEEE VR 2004]
Adaptive Forward Dynamics

- Adaptive computation of dynamic (or quasi-static) simulations of complex linkages or articulated bodies:
  - Error-bounded Approximation
    Introducing “total acceleration equations” to approximate Linkage Acceleration
  - Progressive Refinement
    Using hierarchical reference frames to lazily update the linkage states
  - Efficiency
    Avoiding linear-time update of all linkages

[Redon et al, SIGGRAPH 2005]

Main Results

- Hybrid bodies
  - Novel articulated-body representation
  - To reduce the no. of degrees of freedom (DOFs)
- Adaptive joint selection
  - Novel customizable motion metrics
  - To determine the most important DOFs
- Adaptive update mechanisms

[Redon, Galoppo, Lin; SIGGRAPH 2005]

Hybrid bodies

Featherstone’s DCA [1999]

- Recursive definition

An articulated body is formed by assembling two articulated bodies

Hybrid bodies

Featherstone’s DCA

- Recursive definition

An articulated body is formed by assembling two articulated bodies
Recursive definition

Hybrid bodies
Featherstone’s DCA

The assembly tree of an articulated body

Pairs of rigid bodies

Rigid bodies

The complete articulated body

Articulated-body equation

Inverse inertias and cross-inertias

Applied Forces

Bias accelerations

Body Accelerations

The cross-coupling inverse inertia describes the effect of a force applied to body 2, on the acceleration of body 1.

The bias acceleration is the acceleration of body 1 when no forces are applied.
Hybrid bodies
Featherstone’s DCA

- Recursive definition
- Articulated-body equation
- Two main steps
  - Compute the articulated-body coefficients (↑)

\[
\begin{align*}
\Phi^{(c)}_{ij} &= \Phi^{(c)}_{ij} - \Phi^{(c)}_{ij} W \Phi^{(c)}_{ij} \\
\Phi^{(e)}_{ij} &= \Phi^{(e)}_{ij} - \Phi^{(e)}_{ij} W \Phi^{(e)}_{ij} \\
\Phi^{(f)}_{ij} &= \Phi^{(f)}_{ij} W \Phi^{(f)}_{ij} = (\Phi^{(f)}_{ij})^T \\
b^{(c)}_i &= b^{(c)}_i - \Phi^{(c)}_{ij} \gamma \\
b^{(e)}_i &= b^{(e)}_i + \Phi^{(e)}_{ij} \gamma \\
b^{(f)}_i &= b^{(f)}_i + \Phi^{(f)}_{ij} \gamma
\end{align*}
\]

Hybrid bodies
Definitions

- Active region

The active region contains the mobile joints

Hybrid bodies
Definitions

- Active region
- Hybrid-body coefficients

Articulated-body coefficients

\[
\begin{align*}
\Phi^{(c)} &= \Phi^{(c)} - \Phi^{(c)} W \Phi^{(c)} \\
\Phi^{(e)} &= \Phi^{(e)} - \Phi^{(e)} W \Phi^{(e)} \\
\Phi^{(f)} &= \Phi^{(f)} W \Phi^{(f)} = (\Phi^{(f)})^T \\
b^{(c)} &= b^{(c)} - \Phi^{(c)} \gamma \\
b^{(e)} &= b^{(e)} + \Phi^{(e)} \gamma
\end{align*}
\]

Kinematic constraint forces

\[
\begin{align*}
q &= (S^T V S)^{-1} (Q - S^T V (\Phi^{(f)} f^{(f)} - \Phi^{(e)} f^{(e)} + \beta)) \\
f^{(c)} &= W \Phi^{(c)} f^{(c)} \\
f^{(e)} &= -f^{(e)}
\end{align*}
\]

Rigidify joint

Hybrid-body coefficients
Hybrid bodies
Definitions
- Active region
- Hybrid-body coefficients
- Hybrid-body simulation
  - Same steps as articulated-body simulation
  - Computations restricted to a sub-tree (cf. paper)

Adaptive joint selection
Motion metrics
- Acceleration metric
  \[ \mathcal{A}(C) = \sum_{i \in C} \ddot{q}_i^T A \dddot{q}_i \]
- Velocity metric
  \[ \mathcal{V}(C) = \sum_{i \in C} \dot{q}_i^T V_i \dot{q}_i \]

Theorem
The acceleration metric value of an articulated body can be computed before computing its joint accelerations:
\[ \mathcal{A}(C) = \left[ \begin{array}{c} f_1^T \\ f_2^T \\ \Psi_1^T \\ \Psi_2^T \\ f_1^T \\ f_2^T \\ p_1^T \\ p_2^T \end{array} \right]^T \left[ \begin{array}{cccc} \Psi_1^T & \Psi_2^T & f_1^T & f_2^T \\ p_1^T & p_2^T \end{array} \right] + \mathcal{A}_C \]

Adaptive joint selection
Acceleration simplification
- Four subassemblies with joint accelerations implicitly set to zero
  \[ \mathcal{A}(C) = 96 \quad 3 \quad \mathcal{A}(C) = 6 \]

- Theorem
  The acceleration metric value of an articulated body can be computed before computing its joint accelerations:
  \[ \mathcal{A}(C) = \left[ \begin{array}{c} f_1^T \\ f_2^T \\ \Psi_1^T \\ \Psi_2^T \\ f_1^T \\ f_2^T \\ p_1^T \\ p_2^T \end{array} \right]^T \left[ \begin{array}{cccc} \Psi_1^T & \Psi_2^T & f_1^T & f_2^T \\ p_1^T & p_2^T \end{array} \right] + \mathcal{A}_C \]
**Adaptive update mechanisms**

- Position-dependent coefficients
- Hierarchical state representation

[Redon and Lin; SPM 2005]

**Adaptive update mechanisms**

- Velocity-dependent coefficients
- Linear coefficients tensors

\[
\begin{align*}
(b_1^c)_a &= (B_1^c)_{abc} (v_1^c)_b (v_1^c)_c \\
(b_2^c)_a &= (B_2^c)_{abc} (v_2^c)_b (v_2^c)_c \\
(p_1^c)_a &= (P_1^c)_{abc} (v_1^c)_b (v_1^c)_c \\
(p_2^c)_a &= (P_2^c)_{abc} (v_2^c)_b (v_2^c)_c \\
\eta^c &= (E^c)_{abc} (v^c)_b (v^c)_c (v^c)_d
\end{align*}
\]

**Results**

**Adaptive joint selection**

Achieved 10x speed-up

**Progressive dynamics**

Progressive dynamics of a 300-link pendulum

- Average cost per time step
- Time
- N = 300
- N = 20
- N = 100
- N = 50
- N = 1

Adaptive determination of the active region
Design of Mechanical Chains

Simulation of Articulated Robots

Interactive Manipulation of Molecular Chains

Results
Test application

L: Individual component
R: The millipede robot

L: The entire chain
R: A close-up view

Achieved two orders of magnitude performance gain
For 200 avatars with 17,800 rigid bodies and 19,000 dofs
Adaptive Dynamics (AD) with Contact Handling

- Use impulse-based dynamics
- Solve $p$ analytical constraints:
  $$h(q(t')) - h(q(t)) = k_f \Rightarrow h - h' = K_f$$
- Derive a new hybrid-body Jacobian:

$$J_P(q(t)) = \begin{pmatrix}
\frac{\delta x}{\delta q_1} & \cdots & \frac{\delta x}{\delta q_{df}} \\
\frac{\delta y}{\delta q_1} & \cdots & \frac{\delta y}{\delta q_{df}} \\
\frac{\delta z}{\delta q_1} & \cdots & \frac{\delta z}{\delta q_{df}}
\end{pmatrix}$$

[Gayle et al; RSS 2006]

Cable Routing on Bridge

A snake robot of 500 links and 500 DOFs with only 70 DOFs used in this bridge scene

Pipe Inspection

A snake robot of 2000 links with only 200 DOFs used

Search & Rescue in Debris

A snake robot of 2000 links with only 200 DOFs used
Catheterization Procedures

- In medical and surgical procedures, flexible catheters are often inserted in human vessels to
  - Obtain diagnostic information (blood pressure or flow)
  - Enhance imaging with the injection of contrast agents
  - Provide a mechanism to deliver treatment to a specific area

Liver Chemoembolization

- Catheter is used to inject chemotherapy drugs directly to the blood vessel supplying a liver tumor
- Catheter is inserted into the femoral artery (near the groin) and advanced into the selected liver artery
  - A fluoroscopic display and the resistance felt from the catheter are used to determine how it should be advanced, withdrawn, or rotated
- Chemotherapy drugs followed by embolizing agents are injected through the catheter into the liver tumor

Motion Planning Application

- Application to plan the path of a flexible catheter, inserted at the femoral artery, to a specific liver artery supplying a tumor
  - Environment: 3D models of the liver and blood vessels obtained from the 4D NCAT phantom, a realistic computer model of the human body
  - Catheter was modeled as a snake robot with 2,500 joints with only 10% of joints simulated to achieve 10x speed up.

Benchmark: Liver (Courtesy of JHU)

- A catheter enters the left artery.
- A closer view of liver and the internal arteries
Catheter Insertion

System Demonstration

[Gayle et al; ICRA 2007]

Organization

- Articulated Bodies
- Deformable Models

Algorithms for Deformable Models

- Collision Detection for Deformable Models using Chromatic Decomposition
- Constraint-based Planning of Deformable Robots
- Modeling of Soft Articulated Bodies in Contact Using Dynamic Deformation Textures

Collision Detection for Deformable Models using Chromatic Decomposition

- General collision detection algorithm
  - Robust and efficient
  - Continuous self-collision detection
  - Works on all triangulated meshes with fixed mesh connectivity
- Up to an order of magnitude speedup over prior algorithms
Hierarchical Approaches

- Spatial partitioning or bounding volume hierarchies
  - AABBs [Bridson et al. 02, Baraff et al. 03, DeRose et al. 98], OBBs, k-DOPs [Mezger et al. 02, Volino and Thalmann 00]

- Issues
  - Trade-off between speed and culling efficiency
  - Overlap tests are conservative, resulting in many false positives

Self-Collision Detection: Complexity

Overlaps with many adjacent triangles

Overlaps using AABBs (Axis Aligned Bounding Box)
**GPU-based Algorithms**

- Use rasterization and involve no pre-processing
  - Applicable to deformable models
    - Baciu and Wong 2002, Hiedelberger et al. 04, Govindaraju et al. 05, Rossignac et al. 92, Vassilev et al. 01

- Issues
  - Self-collision computations at image resolution

**Algorithm Overview**

- Problem decomposition
  - Non-adjacent collision detection (NACD)
  - Adjacent collision detection (ACD)

**Problem Decomposition**

**Problem Decomposition: NACD**
Problem Decomposition: NACD

Separation

Problem Decomposition: ACD

Based on mesh connectivity

Problem Decomposition

Decomposition into NACD and ACD
- Significant collision culling
- Fewer false positives

Algorithm: Overview

Stage I: AABB-based NACD
Stage II: 2.5-D test based NACD
Stage III: Exact tests for non-adjacent primitives
Stage IV: Exact tests for adjacent primitives

Broad Phase

Narrow Phase
Algorithm: Overview

Stage I       Stage II       Stage III       Stage IV
AABB-based NACD  2.5-D test based NACD  Exact tests for non-adjacent primitives  Exact tests for adjacent primitives

Stage I       Stage II       Stage III       Stage IV
AABB-based NACD  2.5-D test based NACD  Exact tests for non-adjacent primitives  Exact tests for adjacent primitives

Stage I       Stage II       Stage III       Stage IV
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Stage I       Stage II       Stage III       Stage IV
AABB-based NACD  2.5-D test based NACD  Exact tests for non-adjacent primitives  Exact tests for adjacent primitives
Algorithm: Overview

Stage I  Stage II  Stage III  Stage IV
AABB-based NACD  2.5-D test based NACD  Exact tests for non-adjacent primitives  Exact tests for adjacent primitives
AABB-hierarchy construction  Mesh decomposition

Runtime
Pre-process

Mesh Decomposition vs. Coloring

<table>
<thead>
<tr>
<th></th>
<th>Mesh Decomposition</th>
<th>Vertex Coloring</th>
</tr>
</thead>
<tbody>
<tr>
<td>Partitions</td>
<td>Triangles</td>
<td>Vertices</td>
</tr>
<tr>
<td>Sets are</td>
<td>disjoint</td>
<td>disjoint</td>
</tr>
<tr>
<td>Partitioned Sets have</td>
<td>Non-adjacent triangles</td>
<td>Non-adjacent vertices</td>
</tr>
</tbody>
</table>

Mesh Decomposition

- Mesh decomposition is similar to vertex coloring
- Vertex Coloring: Color vertices such that adjacent vertices have different colors

Mesh

Extended Dual Graph
Mesh Decomposition

Vertex Coloring

- Minimum vertex coloring is NP-complete
- Use greedy heuristic algorithms based on vertex degree (DSATUR)
  - Typically around 10-20 sets

Stage I: AABB-based NACD

- Update AABB hierarchy
- Check AABB hierarchy against itself
  - Only check for NACD

Culling Efficiency of Stage I

- Potentially overlapping

With Stage I AABBs: 20K elementary tests
Stage II: 2.5-D based NACD

- Uses 2.5-D visibility tests on GPUs [Govindaraju et al. 2003]
- 2.5-D tests: Objects are tested for full visibility against a set of objects
- Used for set-based collision culling

2.5-D Tests: Geometric Interpretation

- Checks for existence of separating surface
- Performed on GPU
- Sufficiently fatten to account for precision errors [Govindaraju et al; VRST 04]

Culling Efficiency after Stage II

- After Stage I: 20K elementary tests
- After Stage II: 1K elementary tests

Improved Culling: 2.5D tests
Dancing Girl: Cloth Simulation

- **Skirt** – 12.5 K triangles
- 0.55 sec self-collision detection time

Walking Girl: Cloth Simulation

- **Skirt** – 12.5 K triangles
- 0.55 sec self-collision detection time

Application: Surgical Simulation

- Path planning for a deformable catheter
- Liver – 83K triangles; Catheter – 10K triangles
- 60~90 msec collision detection time between catheter and arteries

Speed Improvement

[Graph showing speed improvement with different collision detection methods]
**Speed Scales Linearly with Polygon Count**

![Graph showing speed scales linearly with polygon count](image)

**Multiresolution Collision Handling**

- Precompute multiresolution meshes
  - Subdivision Hierarchy
  - Chromatic Decomposition
  - Use of Graphics Processor Units (GPUs)
- Adaptively select appropriate CLODs
- Refine simulation on the fly
- Accelerate overall computation by 7x

[Jain et al; CASA 2005]

**Multiresolution Hierarchy**

- Subdivision Hierarchy
- Adaptive Mesh Refinement

**Benchmarks**

- Squatting human with a pair of pants (10K tris)
- Folding curtains (33 K tris total)
- Cloth (33 K tris) draping over a bunny

[Jain et al; CASA 2005]
Planning of Deformable Robots

**Motivations**
- Surgical planning
- Layout for mechanical/electrical systems in complex structures
- Planning of reconfigurable robots
- Character animation

Motion Planning to Aid Catheterization Procedure

- Accurate motion planning studies with deformable models can provide a vital tool to aid in the catheterization procedure
- Preoperatively, they may be used as part of surgical planning techniques to help choose the size and properties of the catheter used
- During the actual procedure, they can be used to compute the optimal path of the catheter to the targeted area, ensuring the best possible outcome for the patient

Problem Description

- Extend the classical motion planning problem by allowing the robot to deform in order to follow a path while maintaining physical constraints

Constraint-based Planning for Deformable Robots

- Formulate motion planning as a constrained dynamical system
  - Constraints are transformed to virtual forces acting on the simulated system
  - Solve the problem as a constrained minimization
- Introduce both hard and soft constraints
  - Goal seeking
  - Collision avoidance
  - Volume preservation
  - Non-penetration constraint (hard)

[Gayle, Lin, Manocha; ICRA 2005]
**General Framework**

**INPUT:** Robots, Obstacles, Goals

- **Constraints:** $C_1, C_2, C_3, \ldots$

  - **Simulation Loop**
    - **Constraint Force:** $f_c = -\frac{\partial E(C_i(q))}{\partial q}$
    - **Energy Function:** $E(C_i(q)) = \frac{1}{2}k_i \cdot C_i(q) \cdot C_i(q)$

**Simulation Loop**

**Algorithm Overview**

- **Roadmap Generation**
- **Path Estimation**
- **Path Query**
  - Advance simulation while satisfy constraints
    - $\min E(x)$ subject to $\forall V(x) \leq \varepsilon$
    - $E_i(X) - \sum \frac{1}{2}||d_j||^2 - L_j^2$
  - **Non-penetration**
  - **Volume Preservation**

**Benchmark**

The Serial Walls benchmark. Note that the robot is slightly larger than the holes through which it must pass.
Benchmark

Steps along the planned path. We capture the robot at various stages during path traversal.

Final position
Starting position

Motion Planning Application

- Application to plan the path of a flexible catheter, inserted at the femoral artery, to a specific liver artery supplying a tumor
  - Environment: 3D models of the liver and blood vessels obtained from the 4D NCAT phantom, a realistic computer model of the human body
  - Deformable robot: Catheter was modeled as a cylinder with a length of 100 cm and a diameter of 1.35 mm

System Demonstration

[Gayle et al; ICRA 2005]
Modeling of Soft Articulated Bodies in Contact Using Dynamic Deformation Textures

Motivation

Global deformations ↔ Detailed deformations

Barbić & James ’05  Müller et al. ’05  Ours

Layered Models Motivation

- Detailed, small-scale deformations
- Global (skeletal) deformations
- Dynamic interplay between skeletal motion and surface deformation during contact

Motivation

- Dynamic simulation of deformable solids
- Highly detailed surface geometry
- Large contact area: objects bounce, roll, slide,…
Layered Models

- Overview
  - Dynamic Deformation Textures
- Fast Coupled Layered Dynamics
- Fast Condensed Skeleton Dynamics

Layered Models

- Overview
  - Dynamic Deformation Textures
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Layered Models

Dynamic Deformation Textures

Dynamic Deformation Texture
Layered Models

- Overview
  - Dynamic Deformation Textures
- Fast Coupled Layered Dynamics
- Fast Condensed Skeleton Dynamics

Fast Coupled Layered Dynamics

- Generalized Lagrangian framework
- Linear elasticity in moving frame of reference [Terzopoulos&Witkin'88]
- FEM discretization
- Implicit integration
- Physically based approximations

Motion interaction  Heterogeneous material

Dynamic Formulation

\[
\tilde{M} = M + \Delta v_e
\]

- Exploit matrix structure + parallelize

Cylinder: 161K tetrahedra
**Dynamic Formulation**

**Decoupled Solution**

1. Project surface forces to update core velocity

2. Distribute core velocity to update surface velocities

**Fast & Responsive Contact**

- Hierarchical Collision Queries
  - Texture-based Collision Detection
- Responsive Contact Handling
- Fast Coupled Contact Response
  - Skeleton Response Anticipation

**Texture-based Collision Detection**

- Hierarchical Collision Queries
  - Texture-based Collision Detection
- Responsive Contact Handling
- Fast Coupled Contact Response
  - Skeleton Response Anticipation
Texture-based Collision Detection
Stage 1: Low-resolution
- Fast proximity tests between proxies
- Contact plane D per contact region

Texture-based Collision Detection
Stage 2: Detailed Surface Interference
- Surface projection onto contact plane D
- Identify contacts in D from separation distances

Texture-based Collision Detection
Collision Information Transfer
- Contact response in dynamic deformation texture
- Transfer collision information

Hierarchical Pruning
- Exploit skeletal nature of deformation
  - Locality
Fast & Responsive Contact

- Hierarchical Collision Queries
  - Texture-based Collision Detection
- Responsive Contact Handling
- Fast Coupled Contact Response
  - Skeleton Response Anticipation

Contact Response

\[ J^T M^{-1} J^T \lambda = -J^T v \]

- Implicit matrices, hard to parallelize
- Decoupled solution

Articulated Contact Response

Snake: 3,102 surface points 16 bones
Articulated Contact Response

- Joint constraint complications

\[
\begin{pmatrix}
\tilde{M}_b & \tilde{M}_{bs} & -J^T_j & -J_b^T \\
\tilde{M}_{bs} & \tilde{M}_s & 0 & -J^T_j \\
-J_j & 0 & 0 & 0 \\
-J_b & -J_s & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\delta v_b \\
\delta v_s \\
\mu \\
\lambda
\end{pmatrix}
= \begin{pmatrix}
0 \\
0 \\
b^*_\mu \\
b^*_\lambda
\end{pmatrix},
\]

Complexity: \(O(mnk)\)

- Contact-consistent Articulated Dynamics

\[
\begin{pmatrix}
\tilde{M}_b & -J^T_j \\
-J_j & 0
\end{pmatrix}
\begin{pmatrix}
\delta v_b \\
\mu
\end{pmatrix}
= \begin{pmatrix}
b^*_b \\
b^*_\mu
\end{pmatrix},
\]

with \(M^*_b = \tilde{M}_b + J^T_{cond} M^{-1}_{\lambda} J_{cond}\)

\[
b^*_b = -J^T_{cond} M^{-1}_{\lambda} b_{\lambda}
\]

Complexity: \(O(m + n + k)\)

Rubbing, rolling, sliding

Tire: 162K tetrahedra

Highly detailed, dynamic deformations

Head: 240K tetrahedra 40K vertices
Exploiting Parallelism

- All surface-related computations on GPU
- Communication between core and surface computations minimized
- Dynamic deformation textures for immediate rendering

Fast Contact Modeling for Soft Characters

Running deer

Future Research Directions

- Multi-scale Dynamics Simulator for Virtual Prototyping and Design Automation
- Modeling and Simulation of Nano-structures and Nano-systems
- Functional Endoscopic Sinus Surgical Training System with Haptic Interfaces
- Planning and Control of Multiple Intelligent Agents using GPU Accelerated Computations
- Enabling Real-Time Interaction between Avatars and Complex Virtual Environments

Collaborators

- Dinesh Manocha
- Russell Gayle
- Nico Galoppo
- Naga Govindaraju
- Markus Gross
- Nitin Jain
- Kenneth Hoff
- Young Kim
- David Knotts
- Miguel Otaduy
- Stephane Redon
**Research Sponsors**
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