Deformation Computation via Statistical Models

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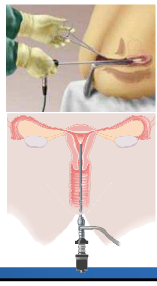
Outline

- Overview hysteroscopy simulator *HystSim*
- Statistical model of hydrometra
- Live demonstration

Application Area – Hysteroscopy

- Endoscopic inspection of uterus
- Extension of cavum uteri via distension fluid (*hydrometra*)
- Complications: Bleeding, uterus wall perforation, fluid intravasation



Components of Hysteroscopy Simulator

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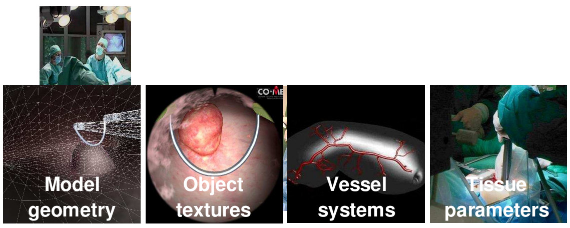

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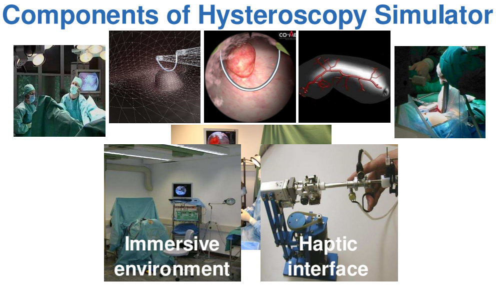

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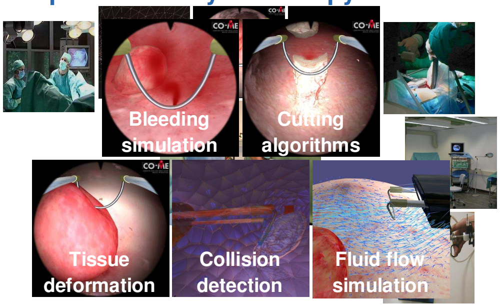
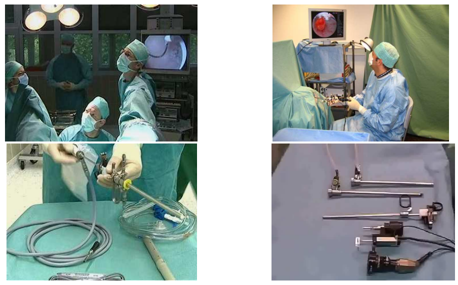
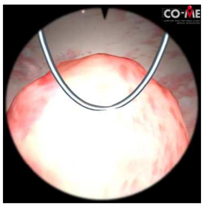


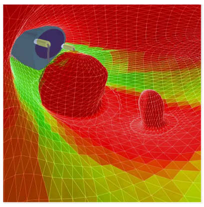
Real and Simulated Hysteroscopy



Quantitative Analysis



Camera view



Outside view
Visibility

Integration into Education

§ Swiss OB/GYN course

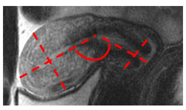
gynécologie suisse

§ Foundation of ETH start-up

VIRTAMED SIMULATED REALISM

Model of Anatomical Variation

§ Creation of new triangle meshes of healthy uteri
§ Avoid time-intensive manual segmentation
§ Model including natural organ variability
§ Usable by clinical experts

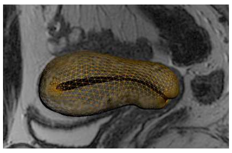
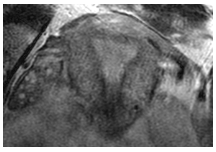


Principal Component Analysis

§ Orthogonal transformation into new basis
§ Principal components ordered by variance
§ Minimization of correlation
§ Possibility of dimensionality reduction

Medical Data Acquisition

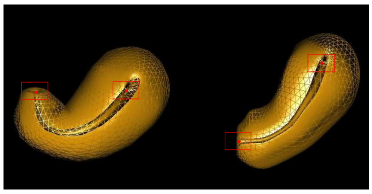
- MRI datasets from volunteer study
- 16-25 slices (0.46^2x3.0 mm^3), 1.5 T scanner
- 19 pre- & 7 post-menopausal (11 & 1 nullipara)

Organ Shape Description

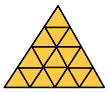
- Point-distribution-model via triangle vertices

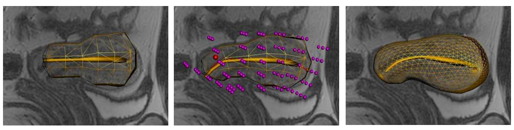
$$\mathbf{p}_i = [x_{1,i}, y_{1,i}, z_{1,i}, \ldots, x_{M,i}, y_{M,i}, z_{M,i}] \qquad i = 1, \ldots, N$$



Organ Segmentation Approach

- Identical process initialization
- Operations maintain correspondences
- Automatic local mesh adjustment





Statistical Model

- Average uterus shape

$$\bar{\mathbf{p}} = \frac{1}{N} \sum_{i=1}^{N} \mathbf{p}_i$$

- Instance-specific difference vectors

$$\Delta\mathbf{p}_i = \mathbf{p}_i - \bar{\mathbf{p}} \qquad \Delta\mathbf{P} = [\Delta\mathbf{p}_1, \ldots, \Delta\mathbf{p}_N]$$

- Covariance matrix

$$\mathbf{\Sigma} = \frac{1}{N-1} \Delta\mathbf{P}\Delta\mathbf{P}^T = \frac{1}{N-1} \sum_{i=1}^{N} \Delta\mathbf{p}_i \Delta\mathbf{p}_i^T$$

Statistical Model

- Eigenvalue decomposition

$$\mathbf{\Sigma} \overset{PCA}{=} \mathbf{U}\mathbf{\Lambda}\mathbf{U}^T$$

- Basis transformation

$$\mathbf{b}_i = \mathbf{U}^T \Delta\mathbf{p}_i \qquad \mathbf{b}_i = [b_{1,i}, \ldots, b_{N-1,i}]$$

$$\mathbf{p}_i = \bar{\mathbf{p}} + \mathbf{U}\mathbf{b}_i$$

Statistical Model

- Alternative derivation

$$\hat{\mathbf{\Sigma}} = \frac{1}{N-1} \Delta\mathbf{P}^T \Delta\mathbf{P} \overset{PCA}{=} \hat{\mathbf{U}}\hat{\mathbf{\Lambda}}\hat{\mathbf{U}}^T$$

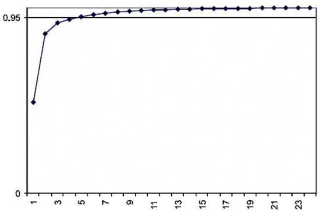
$$\mathbf{\Sigma}(\Delta\mathbf{P}\hat{\mathbf{u}}_i) = \hat{\lambda}_i (\Delta\mathbf{P}\hat{\mathbf{u}}_i)$$

$$\Rightarrow \varphi(\Delta\mathbf{P}\hat{\mathbf{u}}_i) = \mathbf{u}_i \wedge \hat{\lambda}_i = \lambda_i$$

Statistical Model

- Shape variability of eigenmodes

$$i \rightarrow \left(\sum_{k=1}^{i} \lambda_k \Big/ \sum_{k=1}^{N-1} \lambda_k \right)$$



Statistical Model

- Reduction of dimensionality

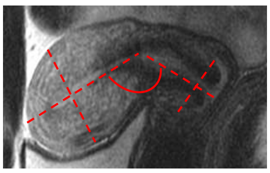
$$\mathbf{p}_i \approx \bar{\mathbf{p}} + \sum_{j=1}^{k} b_j \mathbf{u}_j \qquad k << N$$

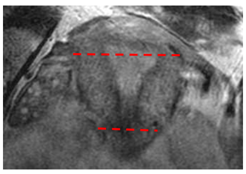
- New organ instances

$$\tilde{\mathbf{p}} = \bar{\mathbf{p}} + \sum_{j=1}^{k} \tilde{b}_j \mathbf{u}_j \qquad k << N$$

- Non-intuitive shape variability by eigenvectors

Instance Derivation via Medical Metrics





Instance Derivation via Medical Metrics

- Constrained optimization

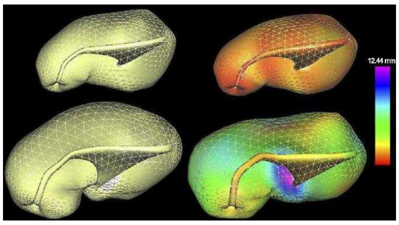
$$d_q(\tilde{\mathbf{p}}) = \left\| \tilde{p}_{s_q} - \tilde{p}_{t_q} \right\| = \xi_q \qquad q = 1, \dots, 7$$

$$D_m(\tilde{\mathbf{b}}) = \sqrt{\tilde{\mathbf{b}}^T \boldsymbol{\Lambda}^{-1} \tilde{\mathbf{b}}}$$

$$L(\tilde{\mathbf{b}}, \mathbf{l}) = \sqrt{\tilde{\mathbf{b}}^T \boldsymbol{\Lambda}^{-1} \tilde{\mathbf{b}}} - \mathbf{l}^T \left[d_q(\bar{\mathbf{p}} + \mathbf{U}\tilde{\mathbf{b}}) - \xi_q \right]$$

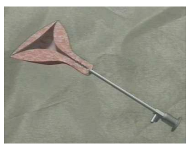
Model Analysis

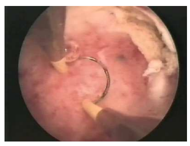
- *Leave-one-out* experiment



Hydrometra Simulation

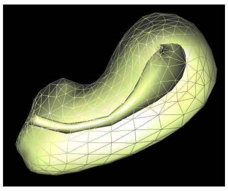
- Nonlinear deformation
- Fast pressure changes
- Minor modification of boundary conditions





Hydrometra Simulation

- Offline deformation calculation/online animation



Extended Statistical Model

- New instance vectors, adding js deformations

$$\mathbf{p}_i^* = \left[\mathbf{p}_i, \mathbf{q}_i^{j1}, \mathbf{q}_i^{j2}, \ldots, \mathbf{q}_i^{js}\right]^T$$

- Principal component analysis

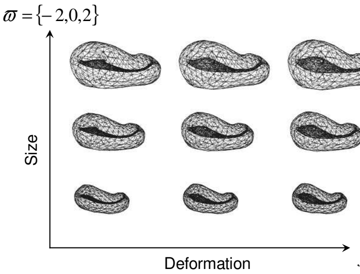
$$\mathbf{\Sigma}^* = \frac{1}{N-1}\left(\Delta\mathbf{P}^*\right)^T \Delta\mathbf{P}^* \overset{PCA}{=} \mathbf{U}^*\Lambda^*\left(\mathbf{U}^*\right)^T$$

- New undeformed as well as deformed instances

$$\tilde{\mathbf{p}}^* = \mathbf{p}^* + \sum_{j=1}^{4} \tilde{b}_j \mathbf{u}_j^*$$

Extended Statistical Model

$\varpi = \{-2,0,2\}$

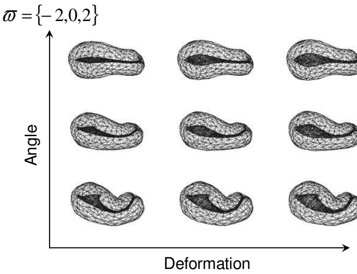


$\mathbf{p}^* + \varpi\sqrt{\lambda_1^*}\mathbf{u}_1^*$

$js = \{0,10,20\}$

Extended Statistical Model

$\varpi = \{-2,0,2\}$

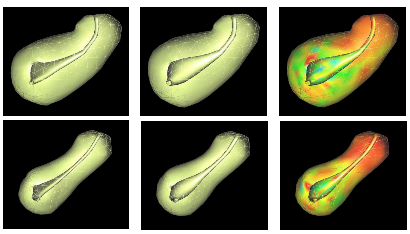


$\mathbf{p}^* + \varpi\sqrt{\lambda_2^*}\mathbf{u}_2^*$

$js = \{0,10,20\}$

Model Analysis

- Comparison of computation and synthesis

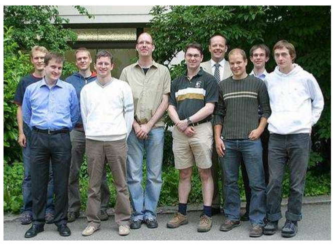


d_{max} (mm) = 0.48

d_{max} (mm) = 0.39

∅ d_{max} (mm) = 0.35 (σ = 0.08)

Acknowledgements



CO-ME
COMPUTER AIDED AND IMAGE GUIDED MEDICAL INTERVENTIONS