

# Why License Plate Rationing Does Not Work and How to Fix It?

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# Outline

- 1 Introduction
  - Traffic congestion problem
  - Solutions
- 2 Model
- 3 Why not work?
  - Analysis
  - Numerical results
- 4 How to fix it?
  - LPR+NVQ
  - LPR+Trading with Auto Owners
  - Permit rationing and trading with all travelers
- 5 Conclusions



# The traffic congestion problem



NORTHWESTERN  
UNIVERSITY

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- Costed urban Americans approximately \$121 billion in 2012.
- A daunting challenge for the developing countries due to rapid urbanization.



## TTI report

## National Congestion Tables

Table 1. What Congestion Means to You, 2011

Urban Area	Yearly Delay per Auto Commuter		Travel Time Index	
	Hours	Rank	Value	Rank
<b>Very Large Average (15 areas)</b>	<b>52</b>		<b>1.27</b>	
Washington DC-VA-MD	67	1	1.32	4
Los Angeles-Long Beach-Santa Ana CA	61	2	1.37	1
San Francisco-Oakland CA	61	2	1.22	23
New York-Newark NY-NJ-CT	59	4	1.33	3
Boston MA-NH-RI	53	5	1.28	6
Houston TX	52	6	1.26	10
Atlanta GA	51	7	1.24	17
Chicago IL-IN	51	7	1.25	14
Philadelphia PA-NJ-DE-MD	48	9	1.26	10
Seattle WA	48	9	1.26	10
Miami FL	47	11	1.25	14
Dallas-Fort Worth-Arlington TX	45	13	1.26	10
Detroit MI	40	25	1.18	37
San Diego CA	37	37	1.18	37
Phoenix-Mesa AZ	35	40	1.18	37

Very Large Urban Areas—over 3 million population.

Large Urban Areas—over 1 million and less than 3 million population.

Yearly Delay per Auto Commuter—Extra travel time during the year divided by the number of people who commute in private Travel Time Index—The ratio of travel time in the peak period to the travel time at free-flow conditions. A value of 1.30 indicates

Medium Urban Areas—over 50

Small Urban Areas—less than 1



# Autonavi report: Top 10 most congested Chinese cities

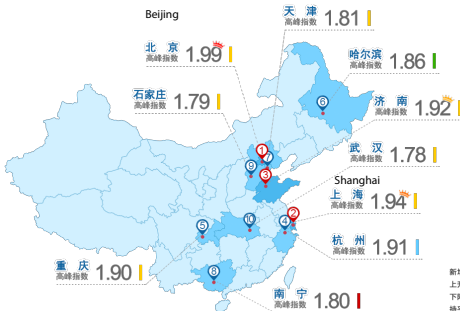
2015 Q1 中国主要城市交通分析报告



1

2015 First Quarter

2015Q1 Most congested city TOP10



数据说明:

早高峰: 07:00-09:00

晚高峰: 17:00-19:00

全天: 06:00-22:00

图例说明:

2015.1.1-2015.3.31

图例说明:

我图取城市区划的中心城区或建成区  
作图城市整体道路网的图价范围

排名说明:

目前, 高德支持全国 114 城市交通信息  
服务, 我图取 45 个重要城市参与排名

新增 ■  
上升 ■  
下降 ■  
持平 ■

(图 1) 2015Q1中国主要城市拥堵排名分布图



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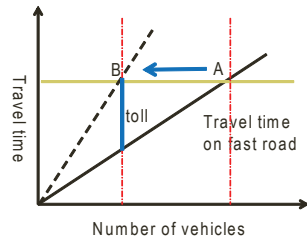
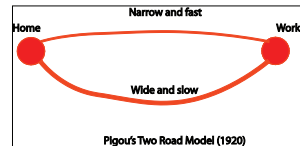
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  - May face financial and physical limits.
  - May be self-defeating as it induces demand.
- Manage demand: reduce total VMT by automobiles.
  - Sticks: pricing or rationing car ownership and/or use
  - Carrots: incentivizing efficient and green travel modes (sharing, walking, biking).



# Congestion pricing

The basic economic theory is compelling

- If nothing is done, everybody will travel at the low speed.
- If some drivers are "forced" out the fast road, the total travel time will be reduced.



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"Yet another tax!!!"



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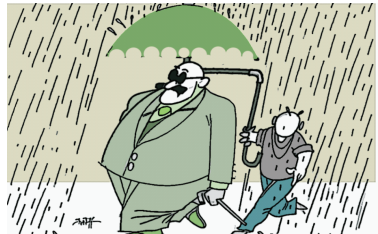
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# Why nobody likes it?

- Successful stories of congestion pricing are limited to a handful of cities (Singapore, London, Stockholm)
- High-profile public rejections (Hong Kong, Edinburgh, New York)
- Politically too expensive even for very powerful governments.

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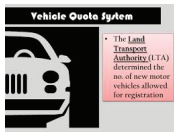
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1000



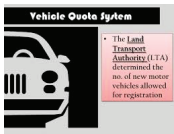
# New vehicle quota: a low-hanging fruit?



- VQS was first implemented in Singapore (New license plates were sold through auction)



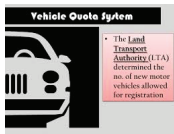
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- Shanghai adopted Singapore's VQS
- Beijing 2010, license plates are distributed by lottery
- Guangzhou (2012), Tianjin (2013), Hangzhou and ShengZhen (2014) - mixed distribution schemes.



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- Bogota, Columbia (2000)
- Beijing, China (2011)
- Chengdu, Tianjin, Hangzhou.... (since 2012)



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- Revenue neutral
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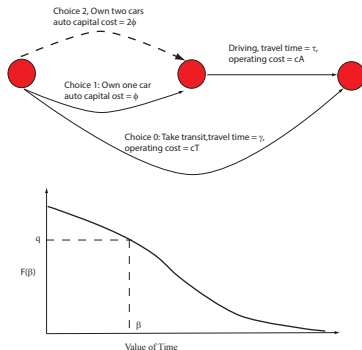
## Appeal of LPR

- Easy to implement and enforce
- Revenue neutral
- Perceived as fair (since restrictions apply to all)

- 1 First, I will explain why LPR is a not a good policy
- 2 Second, I will propose and analyze a few alternative policies that retain these advantages of LPR as much as possible.



# Model

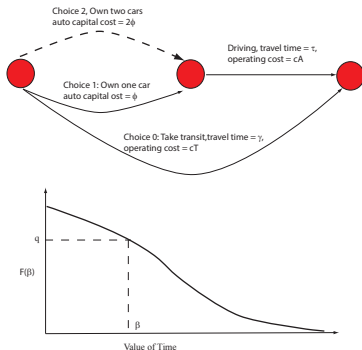


## Assumptions

- The travel demand is fixed;



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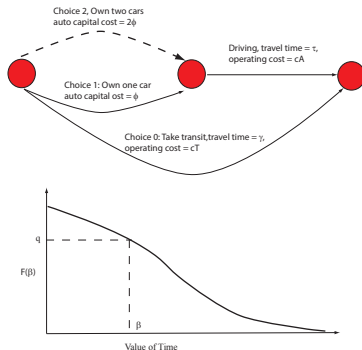


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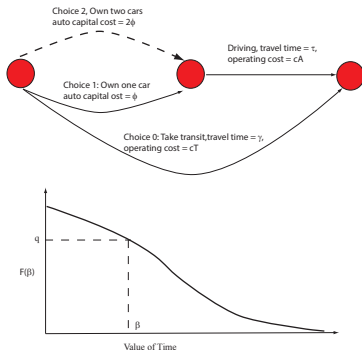


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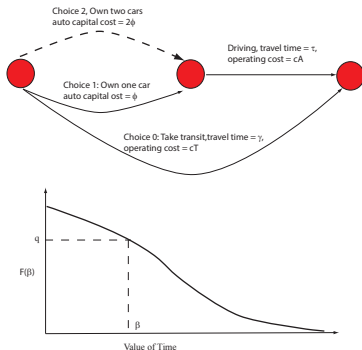


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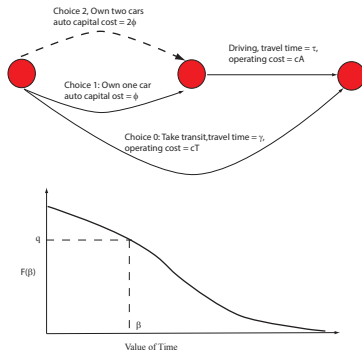


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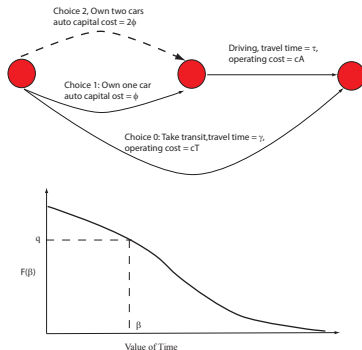


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- One car is sufficient to meet travel needs (drivers would buy the second car only to avoid use restriction).



# Model



The travel cost is represented as

$$u_A = \beta\tau(q) + c_A + \phi,$$

$$u_T = \beta\gamma + c_T.$$



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# User equilibrium

Ignoring corner solutions, the equilibrium is achieved when  $u_A = u_T$ , i.e.

$$F^{-1}(q_e)\tau(q_e) + c_A + \phi = \gamma F^{-1}(q_e) + c_T.$$

$$(\gamma - \tau(q_e))\beta_e = \Delta c \quad (1)$$

where  $\beta_e = F^{-1}(q_e)$ ,  $\Delta c = c_A + \phi - c_T > 0$



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- Travelers with  $\beta > \beta_e$  will drive
- Travelers with  $\beta < \beta_e$  will ride transit.



# System optimum

The total system cost can be written as

$$\hat{G} \equiv \int_0^q F^{-1}(w)\tau(q)dw + \int_q^d F^{-1}(w)\gamma dw + (c_A + \phi)q + c_T(d - q)$$

The first-order optimality condition leads to

$$\frac{d\hat{G}}{dq} = 0 \rightarrow (\gamma - \tau(q))F^{-1}(q) = \Delta c + \tau(q)' \int_0^q F^{-1}(w)dw$$

If  $q_s$  is solution to the above equation, then the system optimal toll is

$$\mu_s = \tau(q_s)' \int_0^{q_s} F^{-1}(w)dw$$



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- A traveler may respond to rationing by purchasing another vehicle, if it reduces the travel cost.
- There are three choices: 0 (taking transit), 1 (owning one car), and 2 (owning two cars).



# User equilibrium (UE) solutions

## User cost

$$\begin{aligned}u_1 &= \lambda(\beta\tau(q) + c_A) + (1 - \lambda)(\beta\gamma + c_T) + \phi, \\u_2 &= \beta\tau(q) + c_A + 2\phi, \\u_0 &= \beta\gamma + c_T.\end{aligned}$$

Also note that highway flow  $q = f_2 + \lambda f_1$ .

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- When  $\lambda$  is reduced to 0.5, all drivers would have two cars.



# System optimum (SO) solution

$$\begin{aligned} \min \hat{G} = & \int_0^{f_2} F^{-1}(w) \tau(q) dw + (c_A + 2\phi) f_2 + \int_{f_2}^{f_1+f_2} F^{-1}(w) (\lambda \tau(q) + (1-\lambda) \gamma) dw \\ & + \lambda f_1 (c_A + \phi) + (1-\lambda) f_1 (\phi + c_T) + \int_{f_1+f_2}^d F^{-1}(w) \gamma dw + c_T (d - f_1 - f_2) \end{aligned}$$

subject to:  $f \in [0, d], \lambda \in [0, 1]$



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$$\frac{\partial \hat{G}}{\partial f_1} = \lambda F^{-1}(f_1 + f_2) (\tau(q) - \gamma) + \lambda \pi + \lambda \Delta c + (1-\lambda) \phi$$

$$\frac{\partial \hat{G}}{\partial f_2} = (\lambda F^{-1}(f_1 + f_2) - (1-\lambda) F^{-1}(f_2)) (\tau(q) - \gamma) + \pi + \Delta c + \phi$$

$$\frac{\partial \hat{G}}{\partial \lambda} = \int_{f_2}^{f_1+f_2} F^{-1}(w) dw (\tau(q) - \gamma) + f_1 \pi + f_1 (c_A - c_T),$$

where

$$\pi = \tau(q)' \left( \lambda \int_{f_2}^{f_1+f_2} F^{-1}(w) dw + \int_0^{f_2} F^{-1}(w) dw \right)$$



# Main result I: cost at UE

## Proposition

*Let  $[f_1^a, f_2^a]$  and  $[f_1^b, f_2^b]$  be UE solutions corresponding to  $\lambda_a$  and  $\lambda_b$ . (1) If  $1 \geq \lambda_a > \lambda_b \geq \hat{\lambda}$ ,  $\tau(q^a) > \tau(q^b)$ ; and (2) If  $\hat{\lambda} > \lambda_a > \lambda_b \geq 0.5$  and  $f_1^a + f_2^a < f_1^b + f_2^b$ ,  $\tau(q^a) > \tau(q^b)$ .*



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- Highway travel time decreases with tighter rationing policies until travelers begin to buy the second car.



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- A sufficient condition is that the share of transit mode must increase in response to a tighter rationing policy (a very strong condition)
- Unexpected result:  $\tau$  may increase when  $\lambda$  is reduced!
- The total system cost at UE MAY increase under LPR.



# Main result II: cost at SO

## Proposition

*Let  $[f^*, \lambda^*]$  be the solution to SO problem. Ignoring trivial corner solutions,  $\lambda^* = 1$ .*

For any given  $\lambda < 1$ , the system cost can always be minimized with  $\lambda$  being treated as a parameter instead of a variable.

## Implications



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## Implications

- Proposition 2 asserts that the solutions for those parametric problems would be always inferior to that with  $\lambda = 1$ .
- **The total system cost will always increase at SO!**
- Even if a first-best policy can be implemented, it cannot minimize the system cost under LPR.



# Main result III: SO toll

## SO toll under LPR

Under LPR, to decentralize the SO we will need to charge one-car travelers a toll equal  $\lambda\pi$  and two-car travelers a toll equal  $\pi$ , where

$$\pi = \tau(q)' \left( \lambda \int_{f_2}^{f_1+f_2} F^{-1}(w) dw + \int_0^{f_2} F^{-1}(w) dw \right)$$



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- Those who opt to buy a second car need to pay an extra toll equal to  $(1 - \lambda)\pi$



# Main result III: SO toll

## SO toll under LPR

Under LPR, to decentralize the SO we will need to charge one-car travelers a toll equal  $\lambda\pi$  and two-car travelers a toll equal  $\pi$ , where

$$\pi = \tau(q)' \left( \lambda \int_{f_2}^{f_1+f_2} F^{-1}(w) dw + \int_0^{f_2} F^{-1}(w) dw \right)$$

- Those who opt to buy a second car need to pay an extra toll equal to  $(1 - \lambda)\pi$
- This additional toll may be collected as an extra “sales tax” (or an additional registration fee) upon the purchase of the second car.



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- This SO toll is progressive

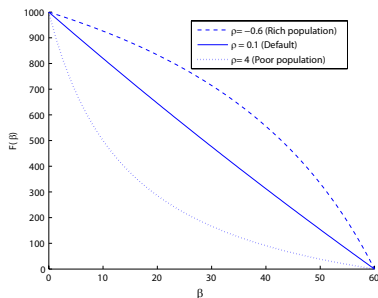


# Experimental setting

$$\tau(q) = \tau_0 \left( 1 + 0.15 \left( \frac{q}{C} \right)^4 \right),$$



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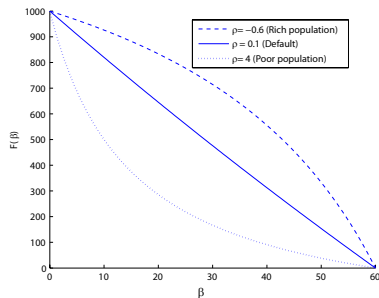
$\rho$  is called the index of wealth

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$$F(\beta) = \frac{d(\beta_U - \beta)}{\rho\beta + \beta_U},$$



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$$F(\beta) = \frac{d(\beta_U - \beta)}{\rho\beta + \beta_U},$$

- $\rho = 0$ : a uniform distribution between 0 and  $\beta_U$
- $\rho \in (-1, 0)$ , skewed to individuals with higher VOT
- $\rho \in (1, \infty)$ , skewed to individuals with lower VOT



# Experimental setting

**Table:** Description of model parameters

Parameters	Default value	Unit	Description
$\gamma$	1	hour	Transit travel time/trip
$c_T$	5	\$	Transit operating cost/trip
$\tau_0$	0.5	hour	Highway free flow travel time/trip
$C$	500	veh/hour	Highway capacity
$d$	1000	person	Total demand
$c_A$	6	\$	Auto operating cost/trip
$\phi$	5	\$	Auto capital cost/trip
$\beta_U$	60	\$/hour	Highest VOT
$\rho$	0.1	-	Index of wealth

**Scenario D** All parameters take default values.

**Scenario P** All parameters take default values except  $\rho = 4$  (poor population)

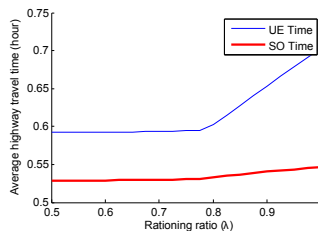
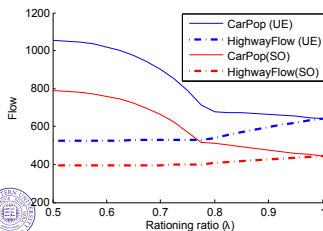
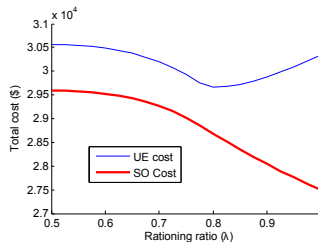
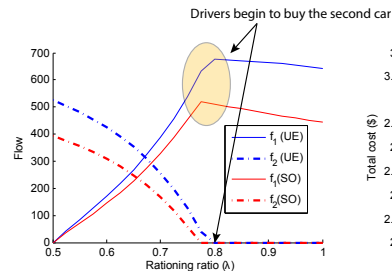
**Scenario R** All parameters take default values except  $\rho = -0.6$  (rich population)

**Scenario L** All parameters take default values except  $\phi = 2.5$  (low auto capital cost)

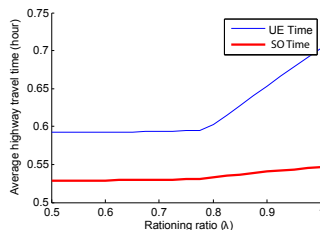
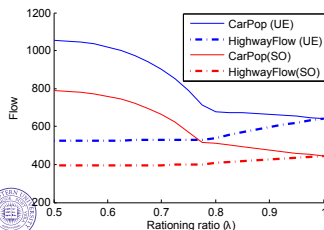
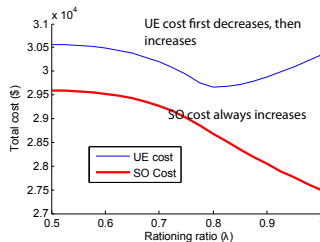
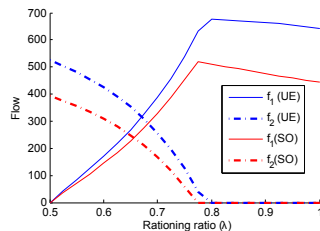
**Scenario H** All parameters take default values except  $\rho = 10$  (high auto capital cost)



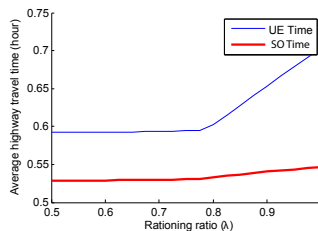
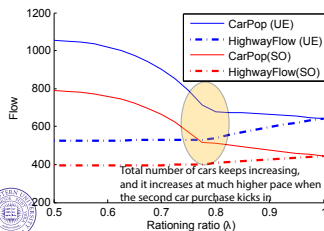
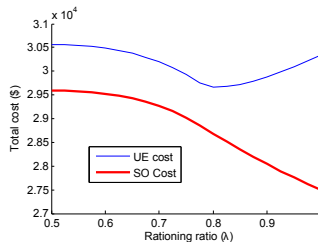
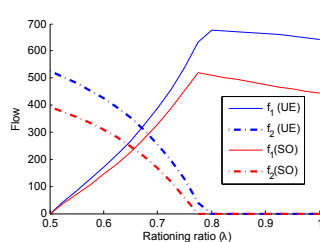
# Default scenario



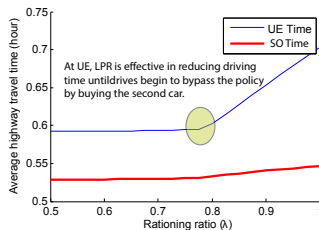
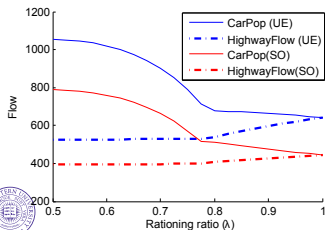
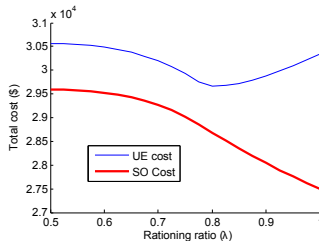
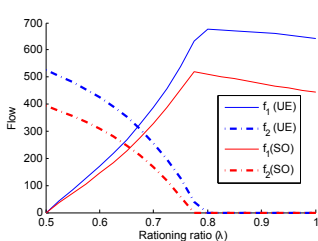
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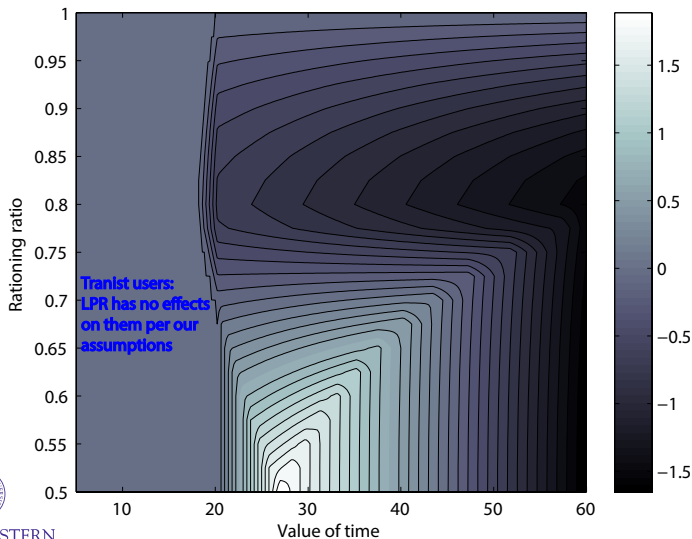
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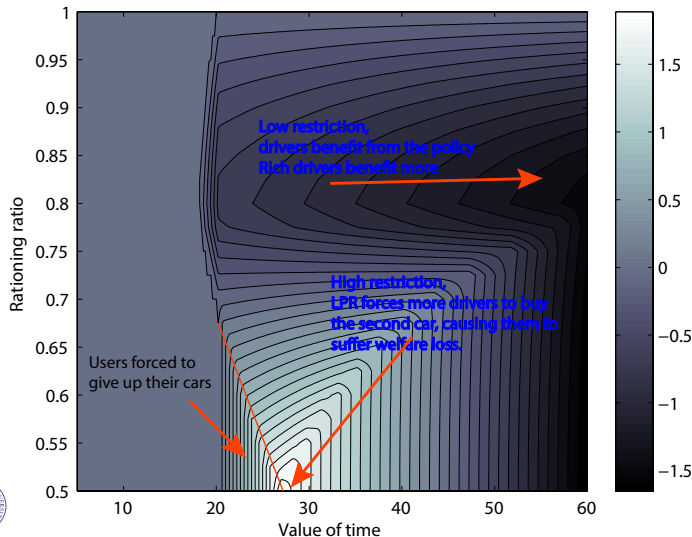
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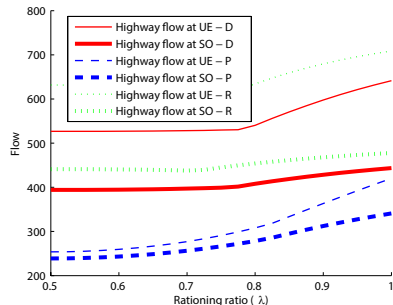
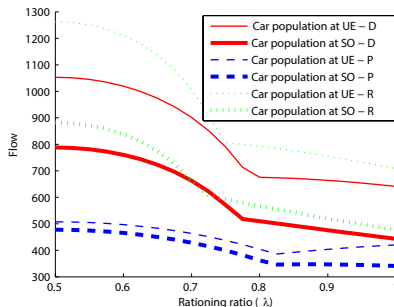
# Welfare effects: cost increases compared to UE



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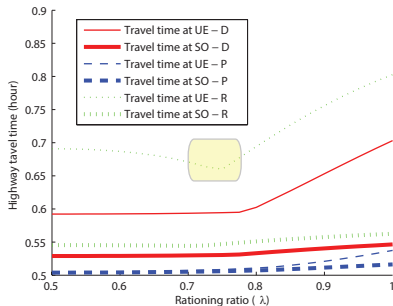
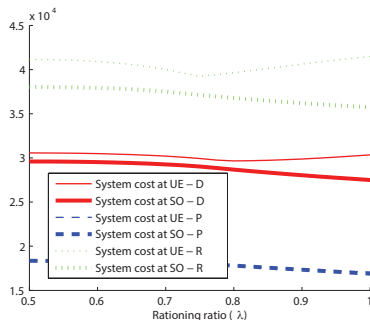
# Sensitivity to index of wealth $\rho$



- The total number of cars is higher for the richer population.
- The highway flow is higher for the richer population.



# Sensitivity to index of wealth $\rho$



- The difference between SO and UE diminishes as the population becomes poorer
- Highway travel increases as  $\lambda$  becomes more restrictive, for the rich population.



# Proposed strategies

The key is to encourage travelers to cope with the restriction by switching to transit, not by getting the second car.

Proposed policy	Rationale
LPR coupled with new vehicle quota	



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LPR coupled with trading among auto owners	Inspired by the recent studies on tradable credit schemes (TCS), desirable access to driving may be achieved at a lower cost by purchasing permits than another car
Permit rationing and trading among all travelers	Avoid making the right to drive as a de facto “entitlement” of auto owners



# New vehicle quota (NVQ)

## Recall

$$u_1(\beta) = \lambda(\beta\tau(q) + c_A) + (1 - \lambda)(\beta\gamma + c_T) + \phi,$$

$$u_2(\beta) = \beta\tau(q) + c_A + 2\phi,$$

$$u_0(\beta) = \beta\gamma + c_T.$$

The NVQ scheme will introduce the following constraint:

$$f_1 + 2f_2 \leq K_0 f_e$$

where  $K_0 \geq 1$  is the desired vehicle control target and  $f_e$  is the UE flow when  $\lambda = 1.0$ .



# New vehicle quota (NVQ)

Let  $\nu$  be the multiplier associated with the capacity constraints, the complementarity requires

$$\nu \geq 0; \nu(f_1 + 2f_2 - K_0 f_e) = 0$$

The UE conditions that incorporate this complementarity condition are

$$f_1 \in (0, d) \rightarrow \exists \beta_1 \in [\beta_L, \beta_U], s.t. \quad u_1(\beta_1) + \nu = u_0(\beta_1)$$

$$f_2 > 0 \rightarrow \exists \beta_2 \in [\beta_L, \beta_U], s.t. \quad u_1(\beta_2) + \nu = u_2(\beta_2) + 2\nu$$



# Model trading with auto owners (TAO)

## rationale

- Buying another vehicle to gain more access to the highway could be more expensive than acquiring permits
- Facilitate efficient allocation of permits among auto owners



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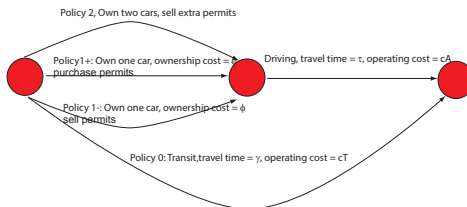
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- Permits can then be traded in a virtual market and linked to registered vehicles through an on-board unit.
- Transaction and enforcement may be done via vehicle-to-infrastructure (V2I) communication.



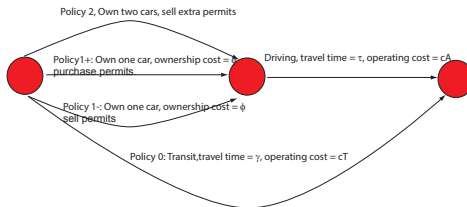
# LPR-TAO

Travelers face four choices: transit (0), own one car and sell permits (1-), own one car and buy permits (1+), and own two cars and sell extra permits (2).



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$$u_{1+}(\beta) = (\lambda + \delta(\beta))(\beta\tau + c_A) + (1 - \lambda - \delta(\beta))(\beta\gamma + c_T) + \phi + \delta(\beta)P$$

$$u_{1-}(\beta) = (\lambda - \delta(\beta))(\beta\tau + c_A) + (1 - \lambda + \delta(\beta))(\beta\gamma + c_T) + \phi - \delta(\beta)P$$

$$u_2(\beta) = \beta\tau + c_A + 2\phi - P\delta(\beta)$$

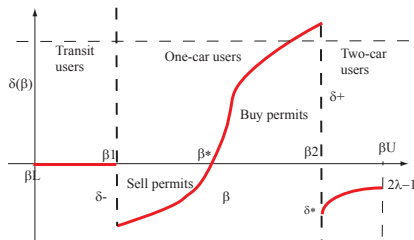
$$u_0(\beta) = \beta\gamma + c_T$$

where  $P$  is the price of permits required to gain full driving access.



# Trading function

A traveler may purchase or sell certain amount of permits, which is assumed to be a function of  $\beta$ , denoted as  $\delta(\beta)$



$$\beta^* = F^{-1}(f_2 + f_{1+})$$

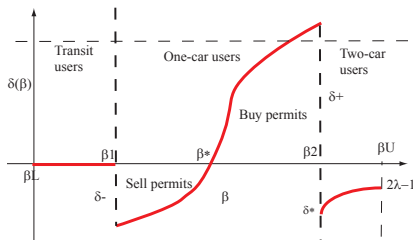
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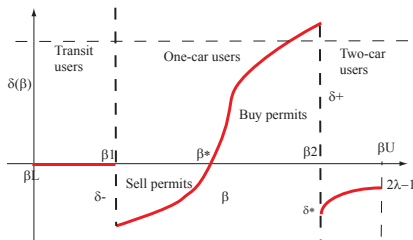
$$\beta_2 = F^{-1}(f_2); f_1 = f_{1+} + f_{1-}$$

## Lemma

Consider two travelers  $a$  and  $b$ , each with a VOT  $\beta_a$  and  $\beta_b$  such that  $\beta_a > \beta_b$  and permits  $\lambda_a, \lambda_b \in (0, 1)$ . Traveler  $a$  would always gain more than what traveler  $b$  would lose if  $\epsilon \in (0, \min(\lambda_b, 1 - \lambda_a))$  permit is transferred from  $b$  to  $a$ .

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- Trading will always occur when  $\lambda$  is restricted below 1.
- Since trading is mutually beneficial, the permit price must be positive.

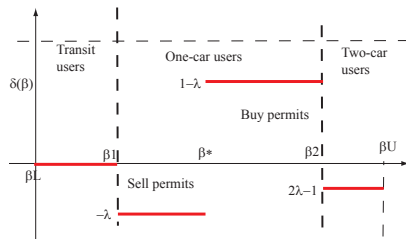


# Main result: characteristics of the trading function

## Proposition

If  $\lambda \in [0.5, 1]$  and  $\beta_1 < \beta^* < \beta_2 < \beta_U$ , then at user equilibrium, the permit trading function

$$\delta(\beta) = \begin{cases} 1 - 2\lambda & \beta \in [\beta_2, \beta_U] \\ 1 - \lambda & \beta \in [\beta^*, \beta_2] \\ -\lambda & \beta \in [\beta_1, \beta^*] \end{cases}$$

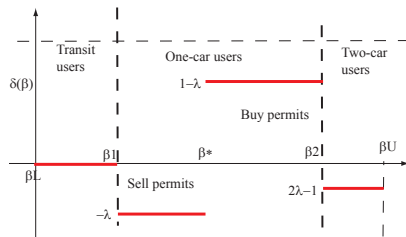


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The amount of permits traded jumps abruptly, and its change coincides with the change in the primary travel choices



# Main results: characteristics of UE solution

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- As  $\lambda$  becomes more restrictive, the permit will become more valuable, and more zero-car travelers will become permit suppliers.
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The UE solution may be summarized as follows.

- When  $\lambda \in [\max(0.5, \hat{\lambda}), 1)$ , travelers may choose policy 0, 1+ or 1-, but not 2.
- When  $\lambda \in [0.5, \max(0.5, \hat{\lambda})]$ , travelers may choose policy 1+, 1- or 2, but not 0.

where  $\hat{\lambda}$  is the threshold where travelers begin to acquire the second car.



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- For a restrictive LPR, all travelers would choose to own at least one vehicle.
- There would be many who own cars but never use them - a waste of social resources.
- The overall effectiveness of the policy is questionable.



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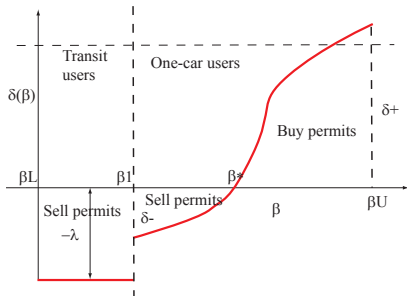


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- Permits are given to travelers, not to vehicles, so no incentive to buy extra vehicles.



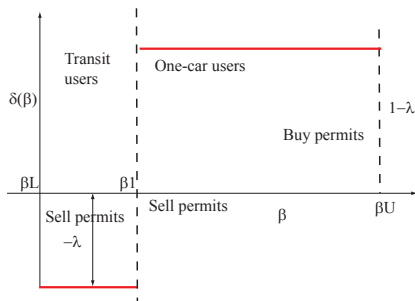
# Main result



Three choices: use transit and sell all permits to auto owners (0), own one car and sell portion of the permit to other car owners (1-), and own one-car and buy options (1+).



# Main result



## Proposition

With the proposed PRA-TAT scheme, (1) no traveler would choose to own a car but sell permits at UE, i.e.,  $f_{1-} = 0$ . (2) One-car travelers must purchase  $1 - \lambda$  permit at UE, i.e.,

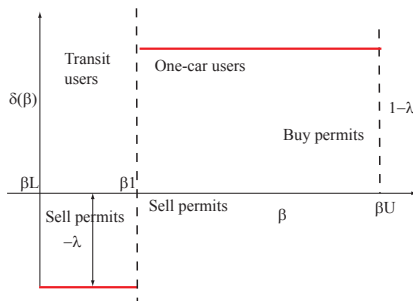
$$\delta(\beta) = \begin{cases} 1 - \lambda & \beta \in [\beta_1, \beta_U] \\ -\lambda & \beta \in [\beta_L, \beta_1] \end{cases}$$

(3) For target highway flow  $q_0$ , driving restriction  $\lambda = q_0/d$ ; and (4) the permit price  $P = \phi/\lambda$ .

$$\beta_1 = F^{-1}(f_{1+} + f_{1-}), \beta^* = F^{-1}(f_{1+}).$$



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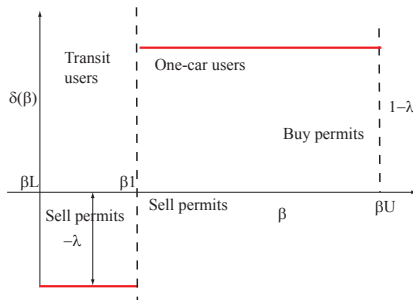
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- Permit trading in PRA-TAT leads to a surprisingly simple equilibrium solution!



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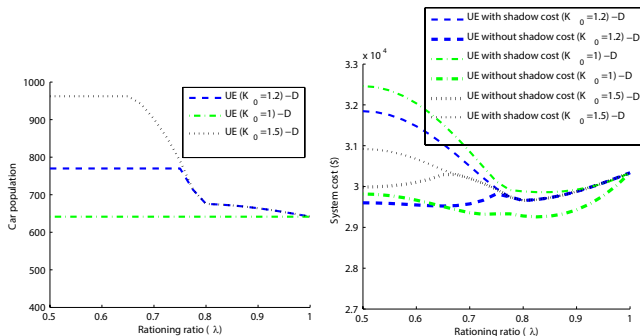
$$\delta(\beta) = \begin{cases} 1 - \lambda & \beta \in [\beta_1, \beta_U] \\ -\lambda & \beta \in [\beta_L, \beta_1] \end{cases}$$

(3) For target highway flow  $q_0$ , driving restriction  $\lambda = q_0/d$ ; and (4) the permit price  $P = \phi/\lambda$ .

- Permit trading in PRA-TAT leads to a surprisingly simple equilibrium solution!
- Trading behavior is defined by auto ownership, independent of user heterogeneity.



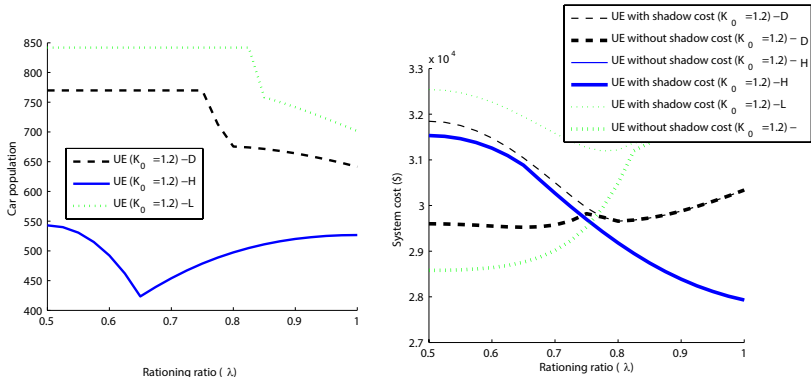
# LPR-NVQ: Result



- Each NVQ policy effectively restricts the total number of automobiles at the level dictated by  $K_0$
- When the shadow cost is excluded, LPR-NVQ improve the system cost
- With the shadow cost, the system costs under LPR-NVQ becomes worse.



# LPR-NVQ: Result



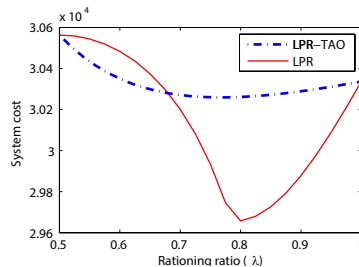
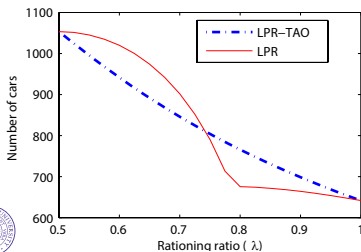
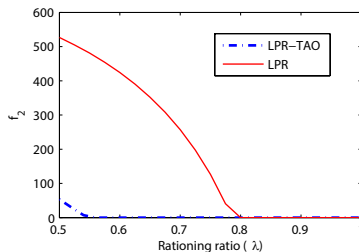
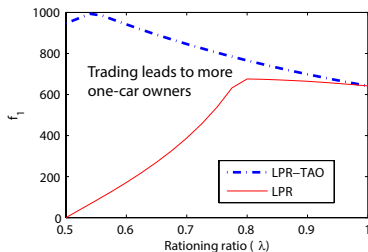
- Higher auto capital cost leads to lower auto ownership

- Low auto capital cost leads to high shadow price.

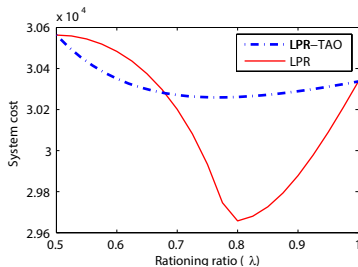
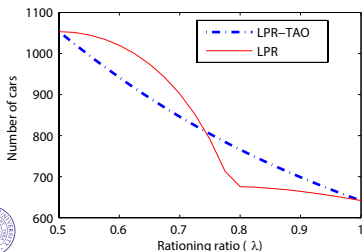
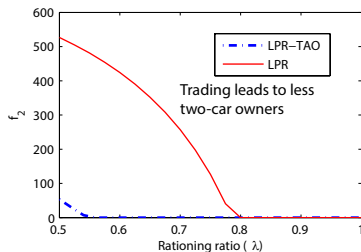
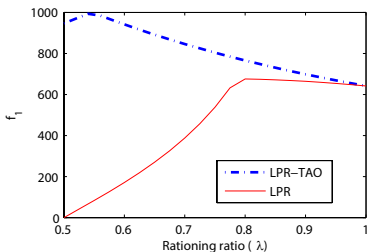
- With shadow price, the system is better off with high auto capacity cost!



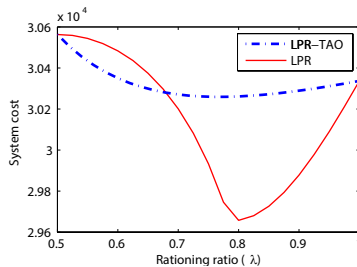
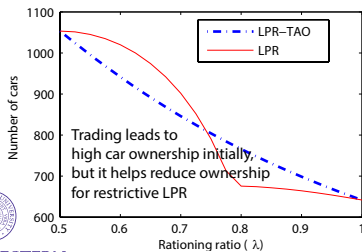
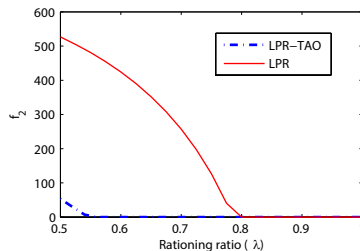
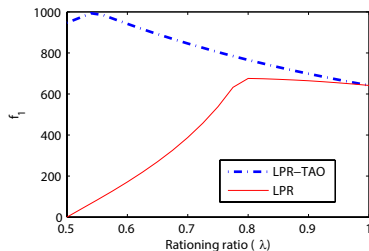
# LPR-TAO: Result



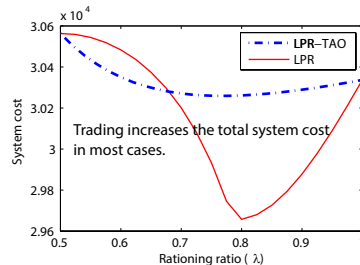
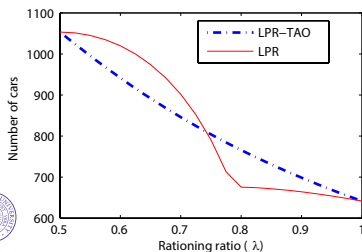
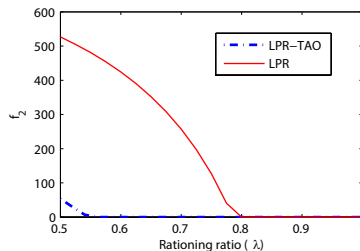
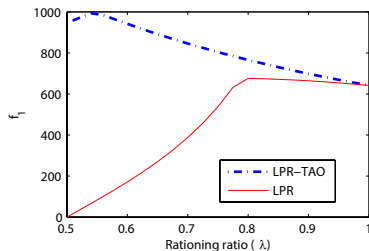
# LPR-TAO: Result



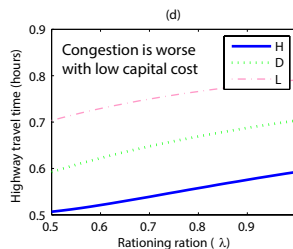
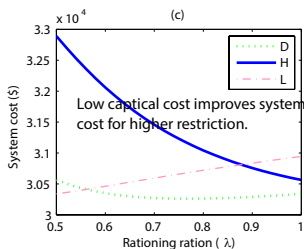
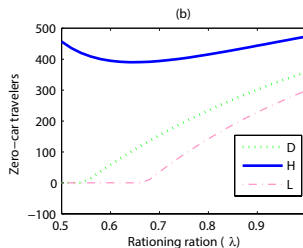
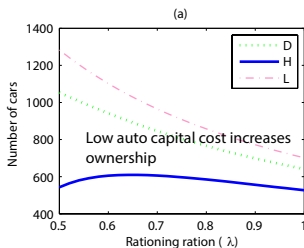
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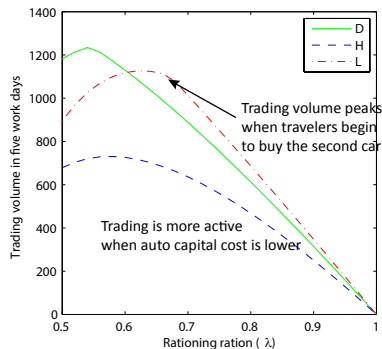
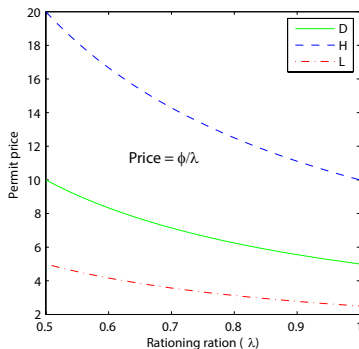
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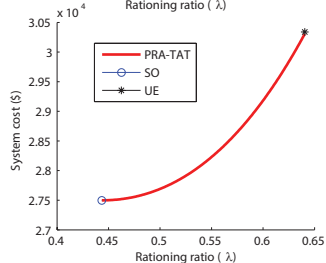
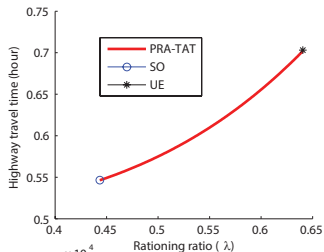
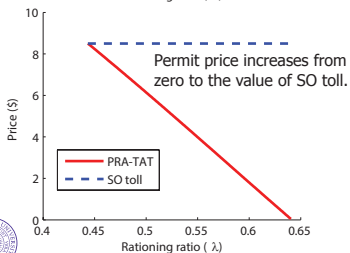
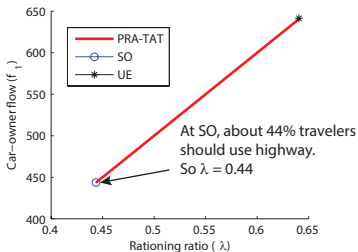
# LPR-TAO: Sensitivity to auto capital cost



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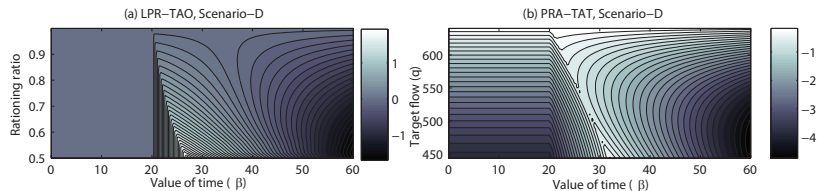


# PRA-TAT: Result



# Welfare effects of LPR-TAO vs. PRA-TAT

## Default population

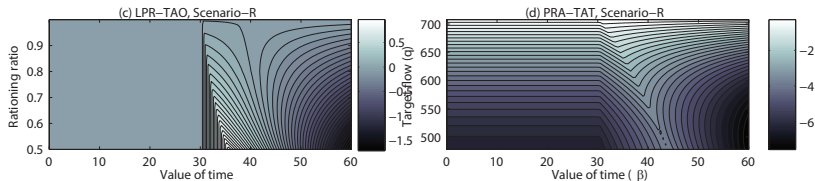


- LPR-TAO benefits the travelers with high value of time at the expense of those with medium value of time.
- Under PRA-TAT all travelers benefit (Pareto-improving), though the benefits of “middle class” are the lowest.
- Equity issue generally is worse when rationing is more restrictive.



# Welfare effects of LPR-TAO vs. PRA-TAT

## Rich population

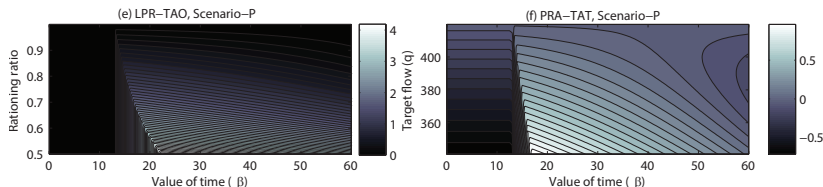


- Benefits of both policies are improved with a rich population



# Welfare effects of LPR-TAO vs. PRA-TAT

## Poor population



- Benefits of both policies are worsened with a poor population
- Even PRA-TAT does not achieve Pareto-improving.
- Whether or not such a policy is effective depends on the distribution of VOT.



# Summary of findings

## Shortcomings

- LPR is neither first-best nor second-best.

## Possible solutions



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- Allowing auto owners to trade their permit to drive is generally a worse policy than LPR itself.
- Allowing all travelers to trade permits is more efficient than other alternatives.
  - A *revenue-neutral first-best policy* with our assumptions.
  - can be introduced as an amendment in cities where LPR is already in place



# Future studies

- Generalize the analysis to determine the optimal control target in PRA-TAT in real-world applications



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- Validating the trading behavioral with day-to-day dynamics models or agent-simulation model
- Combine PRA-TAT with other TDM policies, e.g. NVQ (many cities have both)...
- Implementation issues?



# Thank you!

## Questions and comments?

### Acknowledgement

Support of National Science Foundation (the award number CMMI-1256021).

### Related publications

Nie, Y. (2015) Why License Plate Rationing Is a Bad Transport Policy?. Transportation Research Part A, under review.

Nie, Y. (2015) On the Potential Remedies for License Plate Rationing. Transportation Research Part B, under review.



- Davis, L. W. (2008), 'The effect of driving restrictions on air quality in mexico city', *Journal of Political Economy* **116**(1), 38–81.
- Eskeland, G. S. & Feyzioglu, T. (1997), 'Rationing can backfire: the day without a car in mexico city', *The World Bank Economic Review* **11**(3), 383–408.
- GUETA, G. P. & GUETA, L. B. (2013), 'How travel pattern changes after number coding scheme as a travel demand management measure was implemented?', *Journal of the Eastern Asia Society for Transportation Studies* **10**(0), 412–426.
- Nie, Y. M. (2015), 'Why license plate rationing is a bad transport policy?', *Transportation Research Part A* **Under review**.
- Zupan, J., de Cerreno, A. & Paaswell, R. (2007), 'An evaluation of alternatives to the new york city congestion pricing plan', *Regional Plan Association* .

