Game Theoretic Learning and Social Influence

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IPAM Workshop on Decision Support for Traffic
November 16–20, 2015
Traffic decision makers:

- Users
- Fleet managers
- Infrastructure planners

cf., Ritter: “Traffic decision support”, Thursday AM
*Traffic decision makers:*
  - Users
  - Fleet managers
  - Infrastructure planners

*cf., Ritter: “Traffic decision support”, Thursday AM*

*Satisfaction with decision depends on decisions of others.*
Illustrations

- **Commuting:**
  - Decision = Path
  - Satisfaction depends on paths of others

- **Planning:**
  - Decision = Infrastructure allocation
  - Satisfaction depends on utilization
“... the study of mathematical models of conflict and cooperation between intelligent rational decision-makers”

Myerson (1991), *Game Theory.*
“... the study of mathematical models of conflict and cooperation between intelligent rational decision-makers”

Myerson (1991), *Game Theory*.

- *Extensive literature*...

- **Game elements:**
  - Players/Agents/Actors
  - Actions/Strategies/Choices
  - Preferences over *joint* choices
- Game models
- Influence models
Viewpoint: Game theory

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- Actions/Strategies/Choices
- Preferences over *joint* choices

**Solution concept:** What to expect?
Single decision maker:

- Choice set: $x \in \mathcal{X}
- Utility function: $U(x)$

$$x = \arg \max_{x' \in \mathcal{X}} U(x')$$

Linear/Convex/Semidefinite/Integer/Dynamic...Programming
Making decisions vs modeling decisions

- **Single decision maker:**
  - Choice set: \( x \in \mathcal{X} \)
  - Utility function: \( U(x) \)

\[
x = \arg \max_{x' \in \mathcal{X}} U(x')
\]

Linear/Convex/Semidefinite/Integer/Dynamic...Programming

- Good *model*?
**Single decision maker:**
- Choice set: $x \in \mathcal{X}$
- Utility function: $U(x)$

$$x = \arg \max_{x' \in \mathcal{X}} U(x')$$

Linear/Convex/Semidefinite/Integer/Dynamic...Programming

**Good model?**

**Issues:**
- Complexity, randomness, incompleteness, framing...
- Furthermore...are preferences even consistent with utility function?
Elements: Players, choices, and preferences over joint choices:

\[ U_i(x) = U_i(x_i, x_{-i}) \]

Solution concept: What to expect?
Elements: Players, choices, and preferences over joint choices:

\[ U_i(x) = U_i(x_i, x_{-i}) \]

Solution concept: What to expect?

Nash Equilibrium

Everyone’s choice is optimal given the choices of others.
Elements: Players, choices, and preferences over joint choices:

\[ U_i(x) = U_i(x_i, x_{-i}) \]

Solution concept: What to expect?

Nash Equilibrium

Everyone’s choice is optimal given the choices of others.

Alternatives:
- Bounded rationality models
- Hannan consistency, correlated equilibrium, ...
Setup:
- Players: 1, 2, ..., n
- Set of resources: $\mathcal{R} = \{r_1, ..., r_m\}$ (roads)
- Action sets: $A_i \subset 2^\mathcal{R}$ (paths)
- Joint action: $(a_1, a_2, ..., a_n)$
**Setup:**
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- Joint action: $(a_1, a_2, ..., a_n)$

**Cost:** (vs Utility)
- Resource level:
  \[
  c_r(a) = \phi_r(\sigma_r(a))
  \]
  \[
  #\text{users}
  \]
- User level:
  \[
  C_i(a_i, a_{-i}) = \sum_{r \in a_i} c_r(a)
  \]
**Setup:**
- Players: 1, 2, ..., n
- Set of resources: \( R = \{r_1, ..., r_m\} \) (roads)
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- User level:
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  C_i(a_i, a_{-i}) = \sum_{r \in a_i} c_r(a)
  \]

**Nash equilibrium:** \( a^* = (a_1^*, ..., a_n^*) \)
\[
C_i(a_i^*, a_{-i}) \leq C_i(a_i', a_{-i}^*)
\]
Claim: For such congestion games, NE minimizes

\[ P(a) = \sum_r \sum_{k=0}^{\sigma_r(a)} \phi_r(k) \]
Claim: For such congestion games, NE minimizes

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Overall congestion:

\[ G(a) = \sum_r \phi_r(\sigma_r(a)) \cdot \sigma_r(a) \]

\# users

\textit{cost}
Claim: For such congestion games, NE minimizes

\[ P(a) = \sum_r \sum_{k=0}^{\sigma_r(a)} \phi_r(k) \]

Overall congestion:

\[ G(a) = \sum_r \phi_r(\sigma_r(a)) \cdot \sigma_r(a) \]

Claim: Modified resource cost

\[ \phi_r(\sigma_r(a)) + (\sigma_r(a) - 1) \cdot \left( \phi_r(\sigma_r(a)) - \phi_r(\sigma_r(a) - 1) \right) \]

results in NE that minimizes overall congestion.
Discussion:

- Model presumes NE as outcome
- How does NE compare to social planner optimal measured by $G(\cdot)$?
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- How does NE compare to social planner optimal measured by $G(\cdot)$?

Price-of-Anarchy (PoA):

$$\text{PoA} = \frac{\max_{a \in \text{NE}} G(a)}{\min_a G(a)}$$

pessimistic ratio of performance at NE vs optimal performance
Discussion:
- Model presumes NE as outcome
- How does NE compare to social planner optimal measured by $G(\cdot)$?

**Price-of-Anarchy (PoA):**

$$\text{PoA} = \frac{\max_{a \in NE} G(a)}{\min_a G(a)}$$

pessimistic ratio of performance at NE vs optimal performance

**Price-of-Stability (PoS):**

$$\text{PoS} = \frac{\min_{a \in NE} G(a)}{\min_a G(a)}$$

optimistic ratio of performance at NE vs optimal performance
**Setup:**
- Equally shared cost of resource:
  \[
  \frac{\alpha}{\#\ users}
  \]
  (vs increasing congestion)
- High road: \(n - \epsilon\)
- Low road: 1
**Setup:**
- Equally shared cost of resource:
  \[
  \frac{\alpha}{\# \text{ users}}
  \]
  (vs increasing congestion)
- High road: \( n - \epsilon \)
- Low road: 1

**NE:**
- All use High road at individual cost \( \frac{n-\epsilon}{n} < 1 \)
- All use Low road at individual cost \( \frac{1}{n} \)
**Setup:**
- Equally shared cost of resource:
  \[
  \frac{\alpha}{\# \text{ users}}
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  (vs increasing congestion)
- High road: \( n - \epsilon \)
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**NE:**
- All use High road at individual cost \( \frac{n - \epsilon}{n} < 1 \)
- All use Low road at individual cost \( \frac{1}{n} \)

**PoX:** \( G(a) \) is sum of individual costs

\[
\text{PoA} \approx n \quad \text{&} \quad \text{PoS} = 1
\]
Setup:
- Equally shared cost as before
- User specific starting points
Illustration: Equal cost sharing, cont.

- **Setup:**
  - Equally shared cost as before
  - User specific starting points

- **NE:**
  - All use private resource at individual cost 1
  - All use shared resource at individual cost \( \frac{k}{n} \)

Balcan, Blum, & Mansour (2013), “Circumventing the Price of Anarchy: Leading Dynamics to Good Behavior”
Illustration: Equal cost sharing, cont.

**Setup:**
- Equally shared cost as before
- User specific starting points

**NE:**
- All use private resource at individual cost 1
- All use shared resource at individual cost $\frac{k}{n}$

**PoX:** $G(a)$ is sum of individual costs

$$
PoA = \frac{n}{k} \quad \& \quad PoS = 1
$$

Balcan, Blum, & Mansour (2013), “Circumventing the Price of Anarchy: Leading Dynamics to Good Behavior”
**Extensions:** Broader solution concepts, various families of games, price-of-uncertainty, price-of-byzantine, price-of-...
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• **$(\lambda, \mu)$-smoothness:** For any two action profiles, $a^*$ & $a'$

\[
\sum_i C_i(a_i^*, a_{-i}') \leq \lambda \sum_i C_i(a^*) + \mu \sum_i C(a')
\]

Theorem: Under $(\lambda, \mu)$-smoothness, \(\text{PoA} \leq \frac{\lambda}{1 - \mu}\)}
- **Extensions:** Broader solution concepts, various families of games, price-of-uncertainty, price-of-byzantine, price-of- ...

- **$(\lambda, \mu)$-smoothness:** For any two action profiles, $a^*$ & $a'$

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\]

- **Theorem:** Under $(\lambda, \mu)$-smoothness,

\[
\text{PoA} \leq \frac{\lambda}{1 - \mu}
\]

*Think of $a'$ as NE and $a^*$ as central optimum*

Roughgarden (2009), “Intrinsic robustness of the price of anarchy”
• **Game models**
  - Setup & equilibrium
  - Price-of-X

• **Influence models**
  - Equilibrium shaping

*Lingering issues:*
- Uncertain landscapes
- Equilibrium analysis
Marginal contribution utility:

- Assume global objective, \( G(\cdot) \)
- Define “null” action \( \emptyset \)
- Set

\[
U(a_i, a_{-i}) = G(a_i, a_{-i}) - G(\emptyset, a_{-i})
\]

Claim: PoS = 1
Marginal contribution utility:  
- Assume global objective, $G(\cdot)$  
- Define “null” action $\emptyset$  
- Set  

$$U(a_i, a_{-i}) = G(a_i, a_{-i}) - G(\emptyset, a_{-i})$$  

Claim: $\text{PoS} = 1$

Recall:  

$$\phi_r(\sigma_r(a)) + \beta \left( \sigma_r(a) - 1 \right) \cdot \left( \phi_r(\sigma_r(a)) - \phi_r(\sigma_r(a) - 1) \right)$$

imposition toll: $\tau(a)$
**Marginal contribution utility:**
- Assume global objective, $G(\cdot)$
- Define “null” action $\emptyset$
- Set
  \[ U(a_i, a_{-i}) = G(a_i, a_{-i}) - G(\emptyset, a_{-i}) \]
- **Claim:** PoS = 1

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\phi_r(\sigma_r(a)) + \beta (\sigma_r(a) - 1) \cdot \left( \phi_r(\sigma_r(a)) - \phi_r(\sigma_r(a) - 1) \right)
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**imposition toll:** $\tau(a)$

**Uncertain landscape:** What if users have different $\beta$’s? many more sources of uncertainty...
Equilibrium shaping under uncertainty

\[ \phi_r(\sigma_r(a)) + \beta \left( \sigma_r(a) - 1 \right) \cdot \left( \phi_r(\sigma_r(a)) - \phi_r(\sigma_r(a) - 1) \right) \]

imposition toll: \( \tau(a) \)

- **Theorem:** As \( \kappa \to \infty \)

\[ \kappa \cdot (\phi_r(\sigma_r(a)) + \tau(a)) \]

leads to \( \text{PoA} \to 1 \)
Equilibrium shaping under uncertainty

\[ \phi_r(\sigma_r(a)) + \begin{array}{c} \text{new term} \\ \beta \end{array} \left( \sigma_r(a) - 1 \right) \cdot \left( \phi_r(\sigma_r(a)) - \phi_r(\sigma_r(a) - 1) \right) \]

imposition toll: \( \tau(a) \)

- **Theorem:** As \( \kappa \to \infty \)
  \[ \kappa \cdot \left( \phi_r(\sigma_r(a)) + \tau(a) \right) \]

leads to \( \text{PoA} \to 1 \)

- **Extensions:** Optimal bounded tolls in special case of parallel links & affine costs.

A “mechanism” $\mathcal{M}$ is a rule from reports to decisions. Mechanism $\mathcal{M}$ induces a game in reporting strategies. Seek to implement $\mathcal{D}$ as solution of game, i.e.,

$$\mathcal{D} = \mathcal{M} \circ S?$$
Standard illustration: Sealed bid/second price auctions

Private info $\xrightarrow{D} \text{Social decision}$

vs

Private info $\xrightarrow{S} \text{Messages} \xrightarrow{M} \text{Social decision}$

- **Planner objective:** Assign item to highest private valuation
- **Agent objective:** Item value minus payment
- **Messages:** Bids
- **Social decision:**
  - Item to high bidder
  - Payment from high bidder = Second highest bid
- **Claim:** Truthful bidding is a NE.
- **Special case of broad discussion...**
Private info $\xrightarrow{D} \text{Social decision}$
vs
Private info $\xrightarrow{S} \text{Messages} \xrightarrow{M} \text{Social decision}$

- Private info is revealed *sequentially*
- Agents do not know own valuations in advance
- Decisions based on sequential messages
Two users: \{1, 2\}
User’s need for resource is low or high: \(\theta_i \in \{L, H\}\)
Example

- Two users: \( \{1, 2\} \)
- User’s need for resource is low or high: \( \theta_i \in \{L, H\} \)
- Planner allocates resource \( a \in \{1, 2\} \)
Two users: \( \{1, 2\} \)

User’s need for resource is low or high: \( \theta_i \in \{L, H\} \)

Planner allocates resource \( a \in \{1, 2\} \)

Planner objective: Fair allocation with User 2 priority
Example

- Two users: \( \{1, 2\} \)
- User’s need for resource is low or high: \( \theta_i \in \{L, H\} \)
- Planner allocates resource \( a \in \{1, 2\} \)
- Planner objective: Fair allocation with User 2 priority
- Dynamics: Coupled state transitions according to \( 4 \times 4 \) matrices over set
  \[ \{(L, L), (H, L), (L, H), (H, H)\} \]
Sequential resource allocation, cont

Private info $\xrightarrow{D}$ Social decision

vs

Private info $\xrightarrow{S}$ Messages $\xrightarrow{M}$ Social decision

- **Planner objective:** Induce truthful reporting
- **Agent objective:** Future discounted resource access minus payments

$$\sum_{t=0}^{\infty} \delta^t \left( v_i(\pi(r_1^t, ..., r_n^t), \theta_i^t) - q_i^t(r_1^t, ..., r_n^t) \right)$$

- **Messages:** High/Low resource need
- **Theorem:** LP computations so that truthful reporting is a NE.

Kotsalis & Shamma (2013): “Dynamic mechanism design in correlated environments”
Optimal efficient policy: Favor Agent 2

\[ \pi(L, L) = 1/2, \quad \pi(H, L) = 1, \quad \pi(L, H) = 2, \quad \pi(H, H) = 2 \]

Agent 2 can monopolize by misreporting

Payment rule:

\[ q_1(\cdot, \cdot) = 0 \]

\[ q_2(L, L) < 0, \quad q_2(L, H) < 0, \quad q_2(H, L) < 0 \]

\[ q_2(H, H) > 0 \]

Ex ante payment from agent 2 = 0
Critique

Lingering issues:
Uncertain landscapes
Equilibrium analysis

- Tuned for behavior only at specific Nash equilibrium
- Presumes agents solve coordinated (dynamic) optimization
- Presumes knowledge of full system dynamics available to all
- Neglects model mismatch
Game models
- Setup & equilibrium
- Price-of-X

Influence models
- Equilibrium shaping
- Mechanism design

Lingering issues:
Uncertain landscapes
Equilibrium analysis
Shift of focus:

- Away from equilibrium—Nash equilibrium
- Towards how players might arrive to solution—i.e., dynamics

"The attainment of equilibrium requires a disequilibrium process."

"The explanatory significance of the equilibrium concept depends on the underlying dynamics."
Shift of focus:

- Away from equilibrium—Nash equilibrium
- Towards how players might arrive to solution—i.e., dynamics

“The attainment of equilibrium requires a disequilibrium process.”


“The explanatory significance of the equilibrium concept depends on the underlying dynamics.”

Literature

Monographs:

Surveys:
Caution! Single agent learning $\neq$ Multiagent learning

Best reply dynamics (with inertia)

\[ a_i(t) \in \begin{cases} 
B_i(a_{-i}(t - 1)) & \text{w.p. } 0 < \rho < 1 \\
 a_i(t - 1) & \text{w.p. } 1 - \rho 
\end{cases} \]
Best reply dynamics (with inertia)

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\end{cases} \]

Features:
- Pure NE is a stationary point
- Based on greedy response to myopic forecast:
  \[ a_{\text{guess}}(t) = a_{-i}(t - 1)? \]
- Need *not* converge to NE
Best reply dynamics (with inertia)

\[ a_i(t) \in \begin{cases} B_i(a_{-i}(t-1)) & \text{w.p. } 0 < \rho < 1 \\ a_i(t-1) & \text{w.p. } 1 - \rho \end{cases} \]

Features:
- Pure NE is a stationary point
- Based on greedy response to myopic forecast:
  \[ a_{\text{guess}}^{i}(t) = a_{-i}(t - 1) \]
- Need *not* converge to NE

Theorem
For *finite-improvement-property games* under *best reply with inertia*, player strategies *converge to NE*.

*(Includes anonymous congestion games...)*
Each player:

- Maintain empirical frequencies (histograms) of opposing actions
- Forecasts (incorrectly) that others play independently according to observed empirical frequencies
- Selects an action that maximizes expected payoff
Each player:
- Maintain empirical frequencies (histograms) of opposing actions
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Compare to best reply:

\[ a_{-i}^{\text{guess}}(t) = a_{-i}(t - 1) \quad \text{vs} \quad a_{-i}^{\text{guess}}(t) \sim q_{-i}(t) \in \Delta(A_{-i}) \]
Theorem

For zero-sum games (1951), $2 \times 2$ games (1961), potential games (1996), and $2 \times N$ games (2003) under fictitious play, player empirical frequencies converge to NE.
Fictitious play: Convergence results

Theorem

For zero-sum games (1951), 2 × 2 games (1961), potential games (1996), and 2 × N games (2003) under fictitious play, player empirical frequencies converge to NE.

Theorem

For Shapley “fashion game” (1964), Jordan anti-coordination game (1993), Foster & Young merry-go-round game (1998) under fictitious play, player empirical frequencies DO NOT converge to NE.

Detail: Discussion extended to mixed/randomized NE
FP simulations

Rock-Paper-Scissors
FP simulations

Shapley "Fashion Game"

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**FP bookkeeping**

- Observe actions of **all** players
- Construct probability distribution of **all possible** opponent configurations

![Game theory matrices](image)
**FP bookkeeping**

- Observe actions of all players
- Construct probability distribution of all possible opponent configurations

![Game matrix diagrams](diagram.png)

- *Prohibitive* for large games
Modification:

- Maintain empirical frequencies (histograms) of opposing actions
- Forecasts (incorrectly) that others play independently according to observed empirical frequencies
- Selects an action that maximizes expected payoff
Virtual payoff vector: The payoffs that could have been obtained

\[
U_i(t) = \begin{pmatrix}
    u_i(1, a_{-i}(t)) \\
    u_i(2, a_{-i}(t)) \\
    \vdots \\
    u_i(m, a_{-i}(t))
\end{pmatrix}
\]

Time averaged virtual payoff:

\[
V_i(t + 1) = (1 - \rho)V_i(t) + \rho U_i(t)
\]

Stepsize \(\rho\) is either constant (fading) or diminishing (averaging)
Virtual payoff vector: The payoffs that could have been obtained

\[ U_i(t) = \begin{pmatrix} u_i(1, a_{-i}(t)) \\ u_i(2, a_{-i}(t)) \\ \vdots \\ u_i(m, a_{-i}(t)) \end{pmatrix} \]

Time averaged virtual payoff:

\[ V_i(t + 1) = (1 - \rho) V_i(t) + \rho U_i(t) \]

Stepsize \( \rho \) is either constant (fading) or diminishing (averaging)

Equivalent JSFP: At each stage, select best virtual payoff action

Viewpoint: Bookkeeping is oracle based (cf., traffic reports)
Anonymous congestion
• **Game models**
  - Setup & equilibrium
  - Price-of-X
  - Learning/evolutionary games

• **Influence models**
  - Equilibrium shaping
  - Mechanism design

*Lingering issues:*
Uncertain landscapes
Equilibrium analysis
Public service advertising

Phase I:
- Receptive agents: Follow planner’s advice (e.g., with probability $\alpha$)
- Non-receptive agents: Unilateral best-response dynamics

Phase II: Receptive agents may revert to best-response dynamics
• **Public service advertising**
  - Phase I:
    - Receptive agents: Follow planner’s advice (e.g., with probability $\alpha$)
    - Non-receptive agents: Unilateral best-response dynamics
  - Phase II: Receptive agents may revert to best-response dynamics

• **Main results**: Desirable bounds on resulting PoA for various settings (anonymous congestion, shared cost, set coverage...)

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*Social influence: Sparse seeding*

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*Jeff S. Shamma*

*Game Theoretic Learning and Social Influence ~ 38/42*
Public service advertising

Phase I:
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Phase II: Receptive agents may revert to best-response dynamics

Main results: Desirable bounds on resulting PoA for various settings (anonymous congestion, shared cost, set coverage...)

Compare: Nash equilibrium vs learning agents

Balcan, Blum, & Mansour (2013), “Circumventing the Price of Anarchy: Leading Dynamics to Good Behavior”
Balcan, Krehbiel, Piliouras, and Shin (2012), “Minimally invasive mechanism design: Distributed covering with carefully chosen advice”
**Game models**
- Setup & equilibrium
- Price-of-X
- Learning/evolutionary games

**Influence models**
- Equilibrium shaping
- Mechanism design
- Dynamic incentives

*Lingering issues:*
- Uncertain landscapes
- Equilibrium analysis
Social influence & feedback control

```
Actuate
Gas/Brake

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Sense
Velocity

VS

Actuate

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Sense

```

Jeff S. Shamma

Game Theoretic Learning and Social Influence ~ 40/42
Social influence & feedback control

Benefits of feedback (Astrom)
- Reliable behavior from unreliable components humans
- Mitigate disturbances
- Shape dynamic behavior
Dynamic incentive challenges

**Modeling:**
- Order?
- Time-scale?
- Heterogeneity?
- Non-stationarity?
- Resolution?
- Social network effects?
Dynamic incentive challenges

- **Modeling:**
  - Order?
  - Time-scale?
  - Heterogeneity?
  - Non-stationarity?
  - Resolution?
  - Social network effects?

“Thus unless we know quite a lot about the topology of interaction and the agents’ decision-making processes, estimates of the speed of adjustment could be off by many orders of magnitude.”

Dynamic incentive challenges

- **Modeling:**
  - Order?
  - Time-scale?
  - Heterogeneity?
  - Non-stationarity?
  - Resolution?
  - Social network effects?

*Reinforcement learning vs Trend-based reinforcement learning*
Dynamic incentive challenges

- **Modeling:**
  - Order?
  - Time-scale?
  - Heterogeneity?
  - Non-stationarity?
  - Resolution?
  - Social network effects?

- **Measurement & actuation**
Game models
- Setup & equilibrium
- Price-of-X
- Learning/evolutionary games

Influence models
- Equilibrium shaping
- Mechanism design
- Dynamic incentives

Challenges
- Modeling
- Measurement & actuation