

# Simplifying Modeling Complexity In Dynamic Transportation Systems: A State-space-time Network-based Framework

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Arizona State University

Prepared for Workshop IV: Decision Support for Traffic

Long Program [New Directions in Mathematical Approaches for Traffic Flow Management](#)

Institute for Pure & Applied Mathematics, UCLA

# Outline

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## 1. Introduction

large-scale dynamic traffic assignment and **simulation**

## 2. From simulation to **optimization**:

- Modeling next-generation of transportation systems
- Key Questions: modeling challenges
- Problem Definition: VRPPDTW
- Methodology: combination of dynamic programming and Lagrangian relaxation

## 3. Extensions

Traffic flow state **estimation**, and traffic signal control and train timetabling...

# Background

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Xuesong Zhou

- Pronounced as “Su-song Joe”

Acronym for **e**xtending traffic **u**ser **e**quilibrium and **s**ystem **o**ptimum models to **n**ext **g**eneration

## Research areas

Simulation-based mesoscopic dynamic traffic assignment

DYNASMART

DTALite (based on simplified kinematic wave model)

Traffic state estimation and prediction

Train routing and scheduling

→ Vehicle routing and scheduling (**new**)

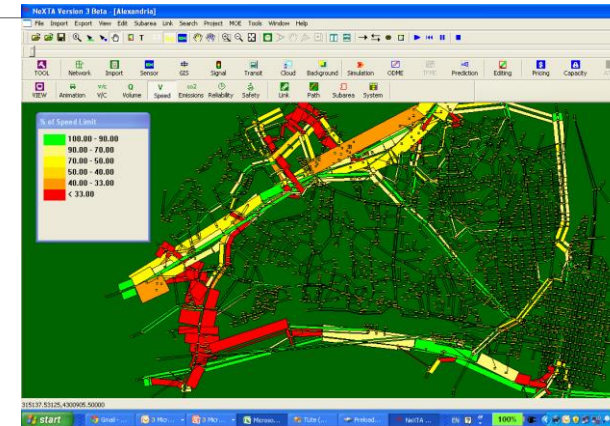
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Topic 1:  
Introduction: large-scale dynamic traffic  
assignment and simulation

# Open-source Free Software Package

## NEXTA: front-end Graphical User Interface GUI (C++)

- [https://github.com/xzhou99/dtalite\\_beta\\_test](https://github.com/xzhou99/dtalite_beta_test)



## DTALite: Open-source computational engine (C++)

- Light-weight and agent-based DTA
- Simplified kinematic wave model (Newell)
- Built-in OD demand matrix estimation (ODME) program
- Emission prediction (light-weight MOVES interface)
  - Simplified car follow modeling (Newell)

```
C:\NEXTA_OpenSource\Internal_release\DTALite.exe
Converting demand flow to vehicles...
---- Network Loading For Iteration 16 ----
Free global path set...
start simulation process...
simulation clock:0 min. # of vehicles -- Generated: 0. In network: 0
simulation clock:5 min. # of vehicles -- Generated: 721. In network: 721
simulation clock:10 min. # of vehicles -- Generated: 1442. In network: 1280
simulation clock:15 min. # of vehicles -- Generated: 2163. In network: 1279
simulation clock:20 min. # of vehicles -- Generated: 2885. In network: 1279
simulation clock:25 min. # of vehicles -- Generated: 3606. In network: 1279
simulation clock:30 min. # of vehicles -- Generated: 4328. In network: 1280
simulation clock:35 min. # of vehicles -- Generated: 5049. In network: 1279
simulation clock:40 min. # of vehicles -- Generated: 5771. In network: 1280
simulation clock:45 min. # of vehicles -- Generated: 6492. In network: 1280
simulation clock:50 min. # of vehicles -- Generated: 7213. In network: 1279
simulation clock:55 min. # of vehicles -- Generated: 7934. In network: 1278
simulation clock:60 min. # of vehicles -- Generated: 8657. In network: 1280
simulation clock:65 min. # of vehicles -- Generated: 8657. In network: 559
--Simulation completes as all the vehicles are out of the network.
CPU Clock: 00:00:02 --Iteration: 16. Average Travel Time: 8.86283. Average Distanc
e: 4.66305. Switch: 250. Number of Vehicles Complete Their Trips: 8657. 00%
Avg Gap: 0.142547. Demand Dev: 0. Avg volume error: 0.81. Avg % error: 17.4991
Iteration = 17
Processor 0 is working on shortest path calculation..
```

# Learning Traffic Network Modeling using Open-source tools

[www.learning-transportation.org](http://www.learning-transportation.org)

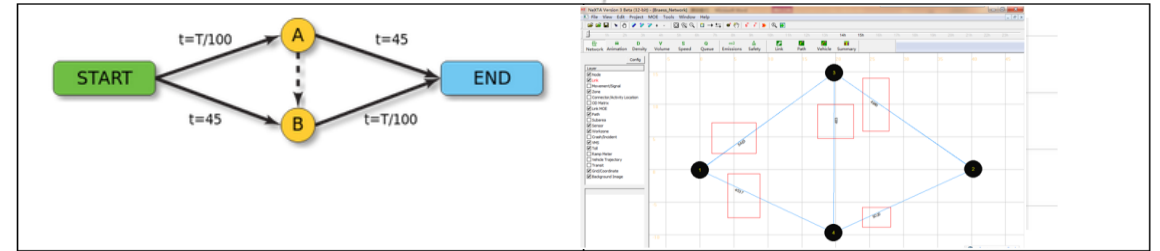
12 lessons:

Lesson 1: Let Us Create a Transportation Network

Lesson 2: From Population to Driving Trips:

Lesson 3: Remove Roads to Speed Travel?

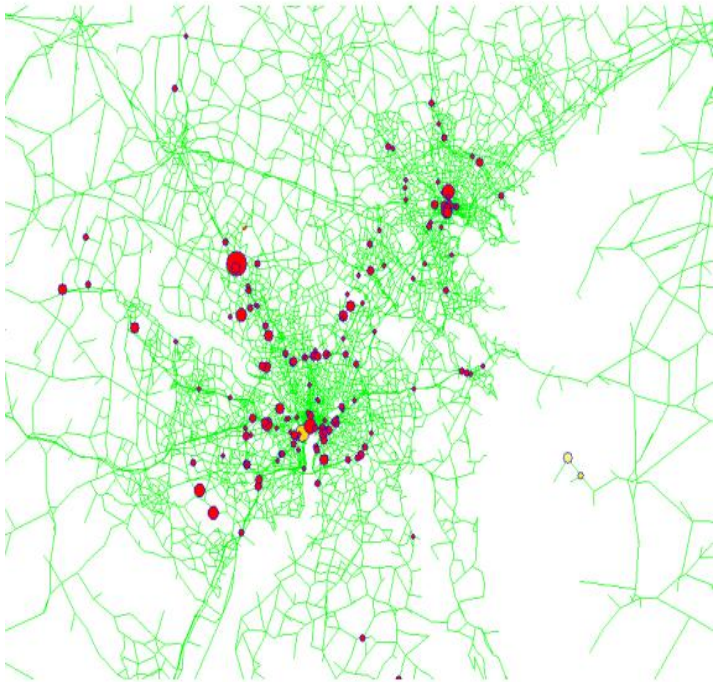
Lesson 4: Optimize Traffic Signal Lights



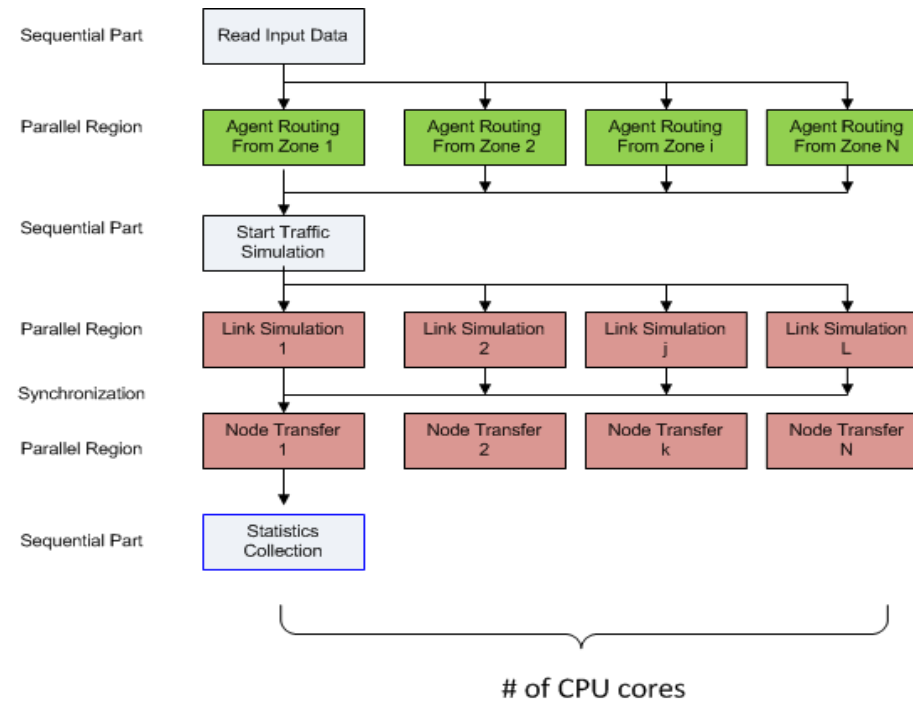
Learning Goals:

1. Understand modeling principles of user equilibrium, and gap functions
2. Know how to setup BPR parameters for special link types
3. Understand the impact of random incidents and analyze traffic at link, path and network levels
4. Understand different network equilibrium method: method of successive average vs. day to day learning
5. Understand the impact of road pricing to resolve Braess's Paradox

# Computational Challenges



Shared memory-based parallel computing for agent-based path finding and mesoscopic traffic simulation (based on OpenMP)



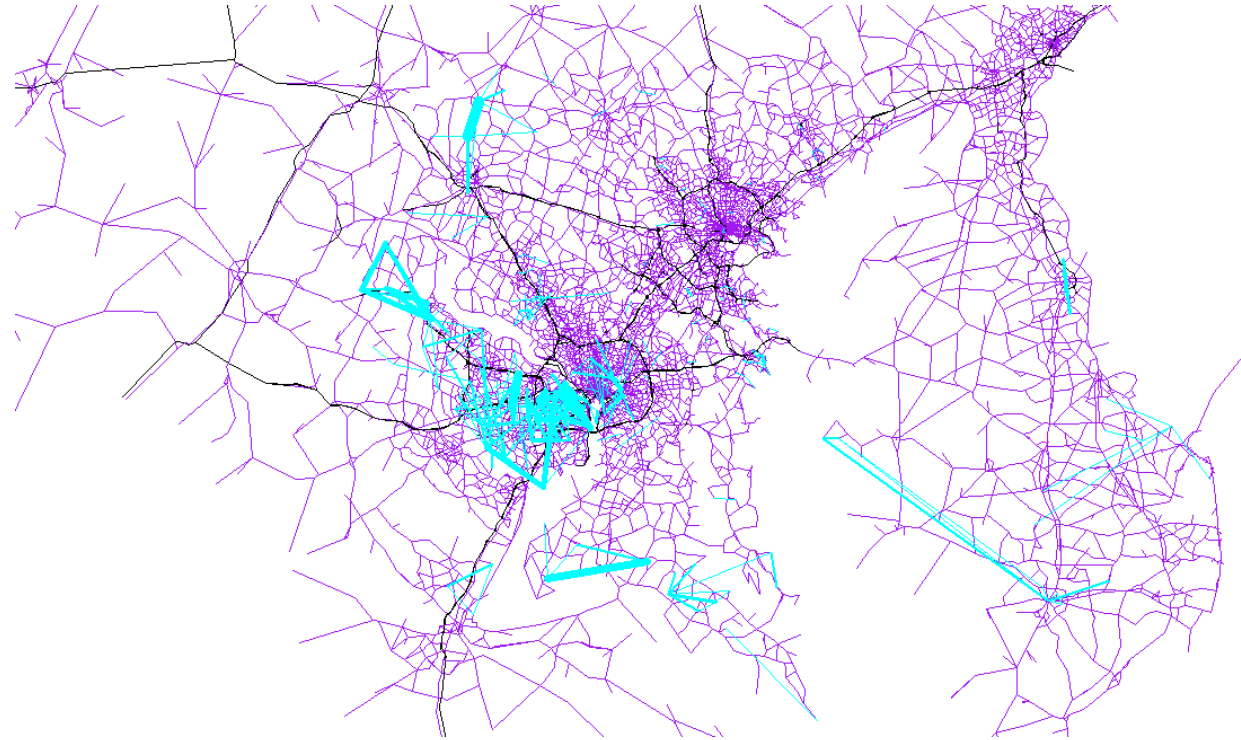
Maryland State-wide model:

20 K nodes, 47K links, 3,000 zones, 18 M agents

CPU time: 30 min per UE iteration on a 20-core workstation with 194 GB RAM

# Origin-Destination Demand Spatial Distribution Pattern

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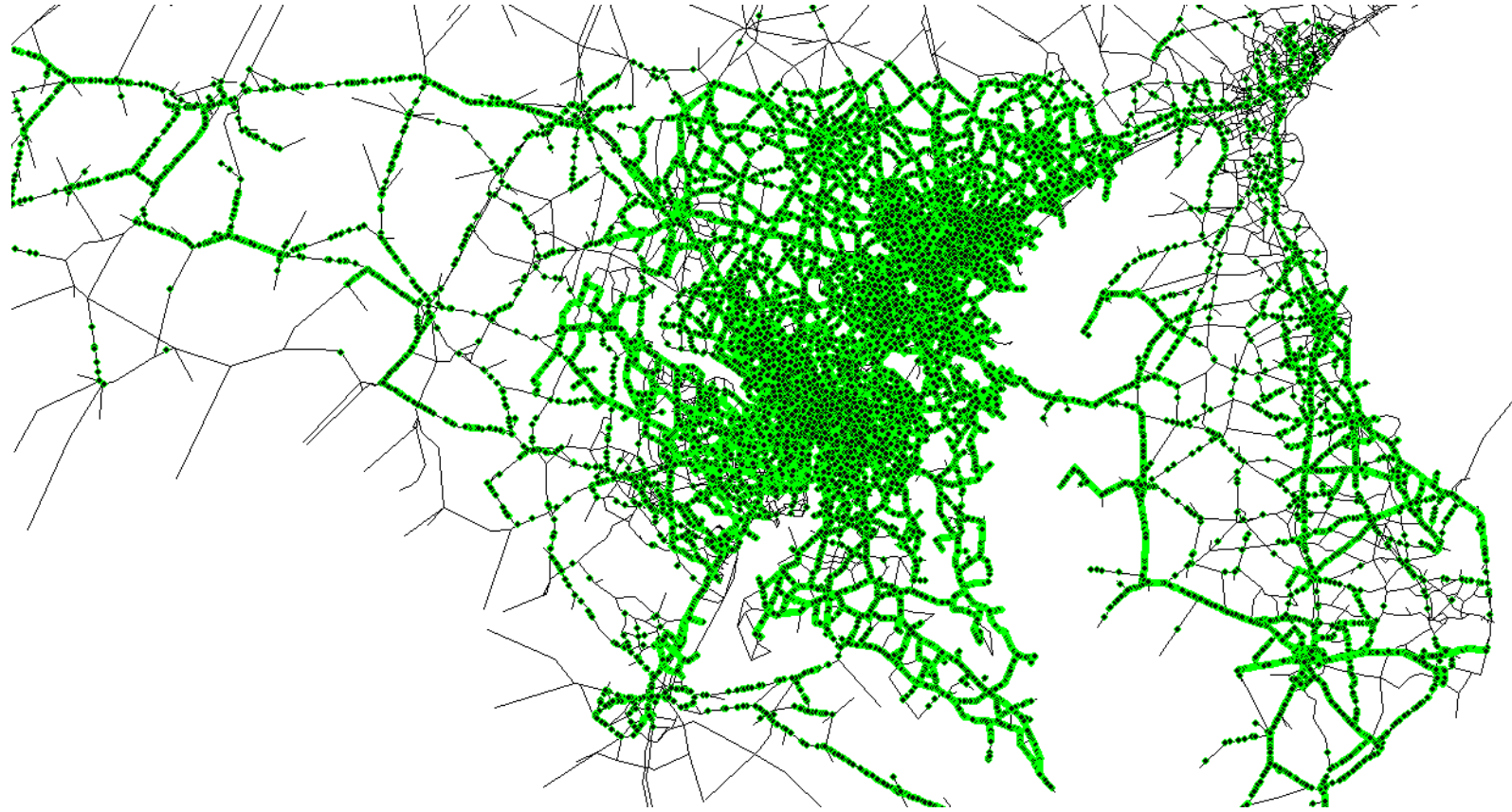


Collaboration with University of Maryland and Maryland [State Highway Administration](#)  
Supported by TRB SHRP II Program

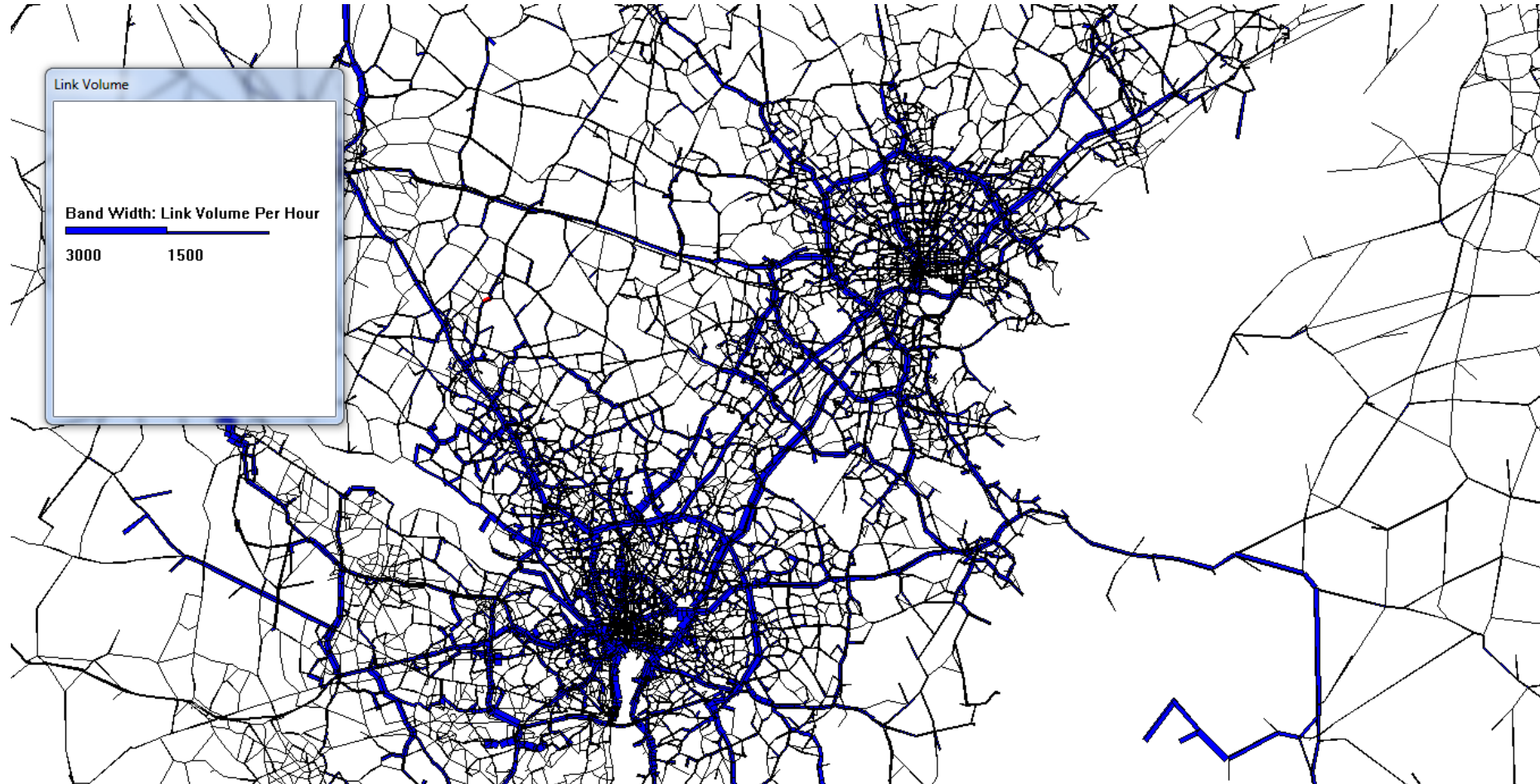


# Vehicle Animation at Network Level

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# Volume at Network Level



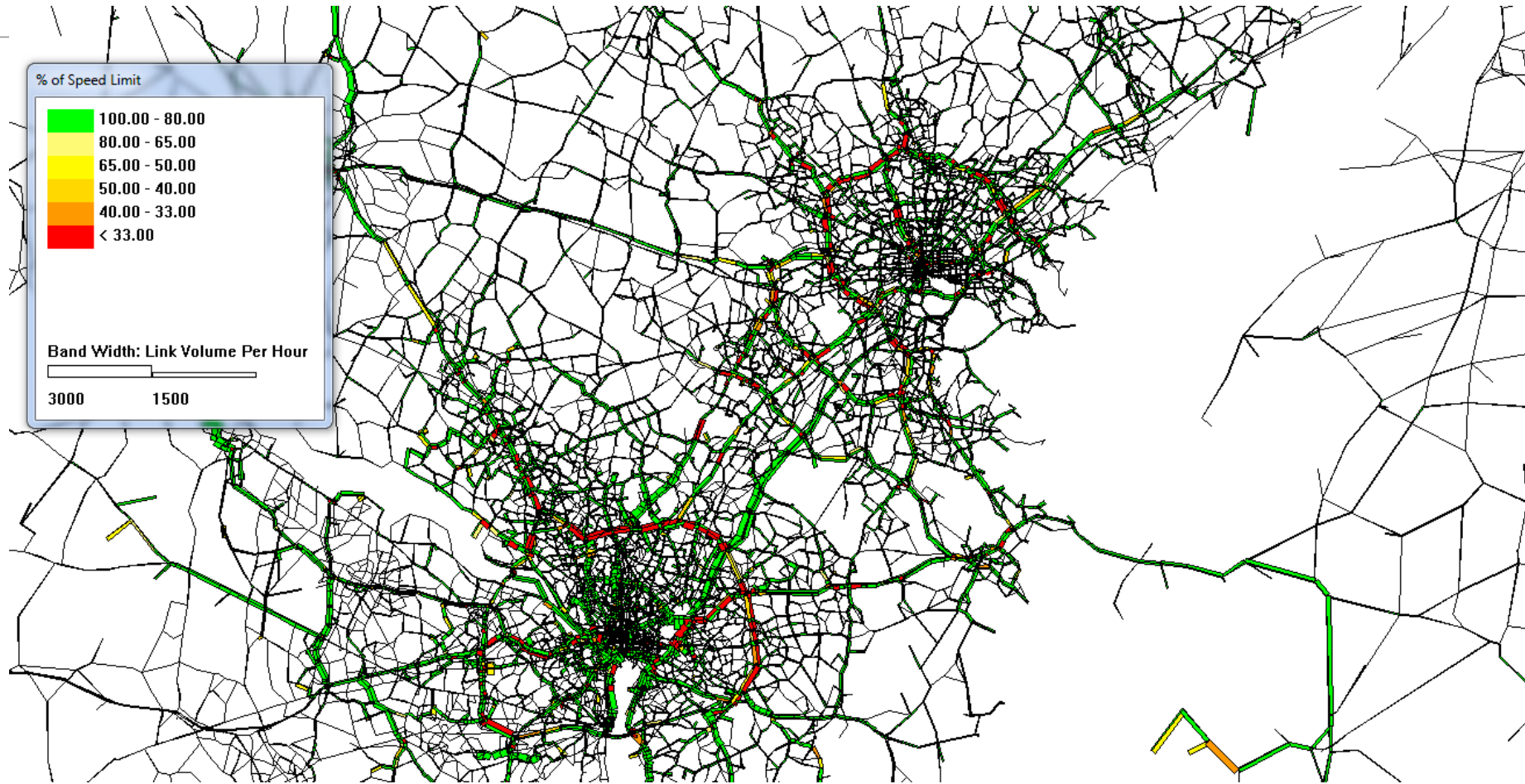
Band width of a link is proportional to link volume

# Density at Network Level



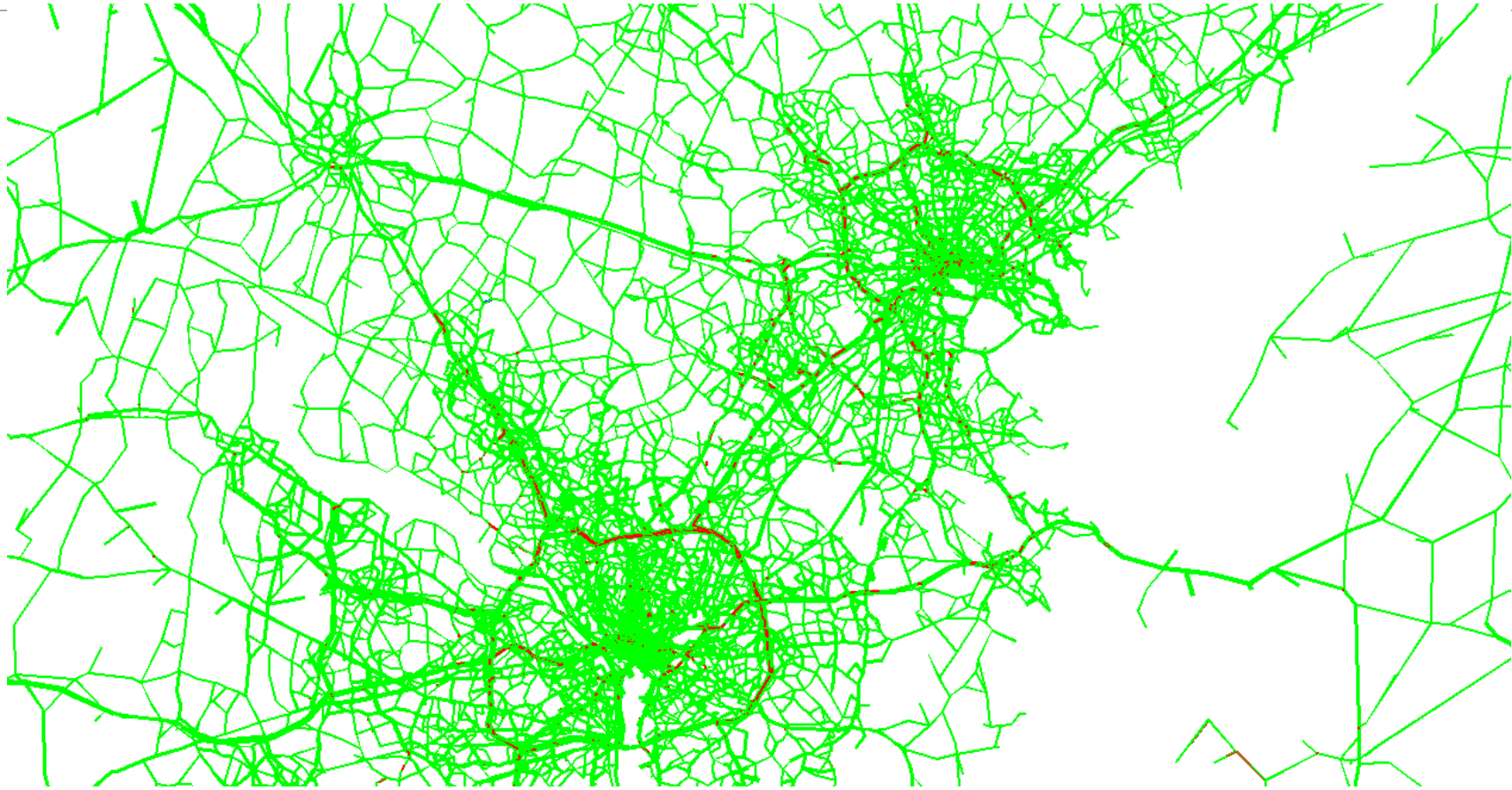


# Speed at Network Level



# Queue at Network Level

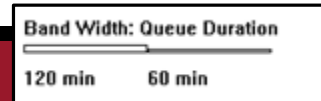
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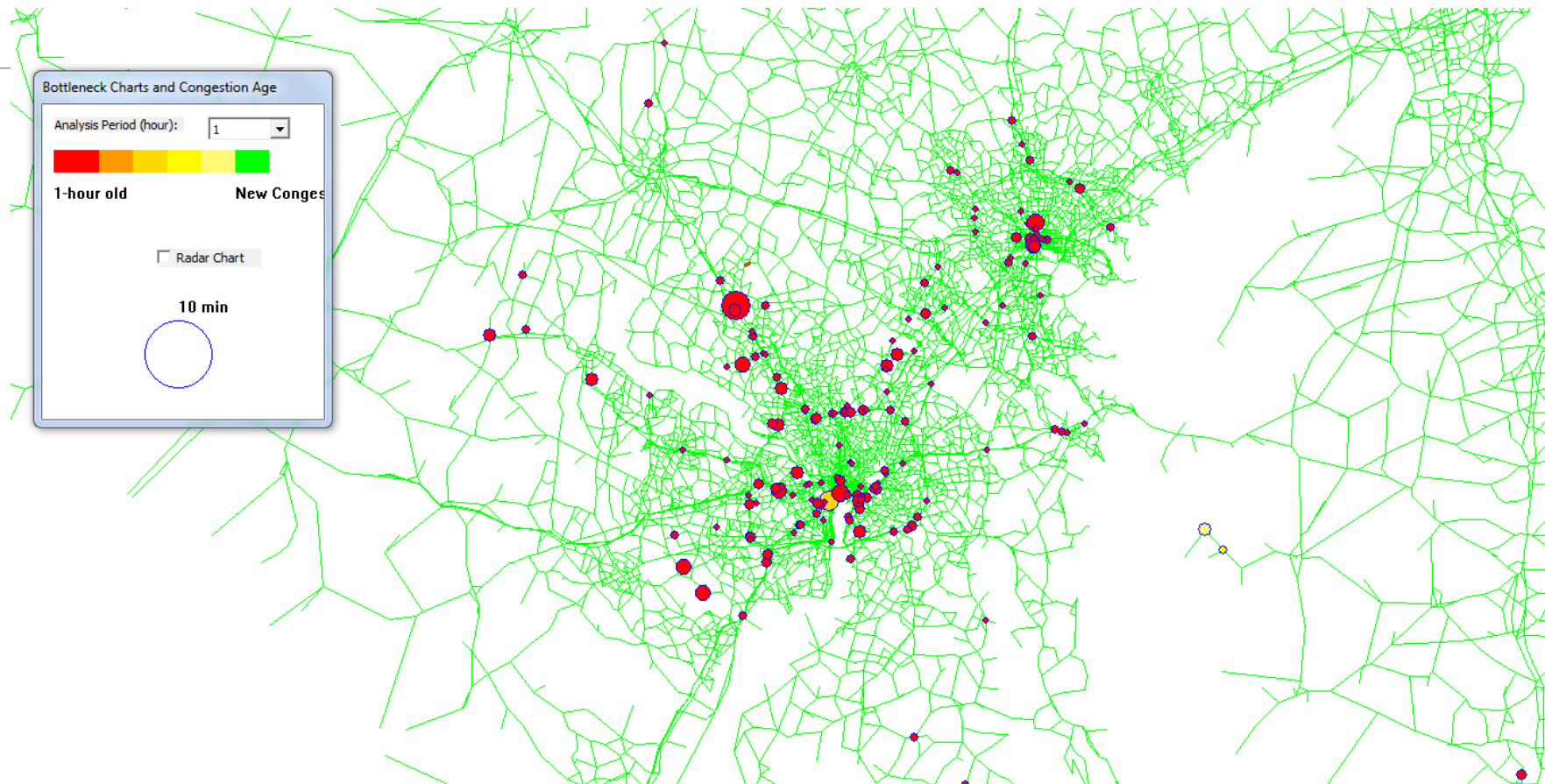
# Queue Duration at Network Level



Link width represents duration of congestion (e.g. 60 min vs. 120 min)



# Time-dependent Bottleneck Locations

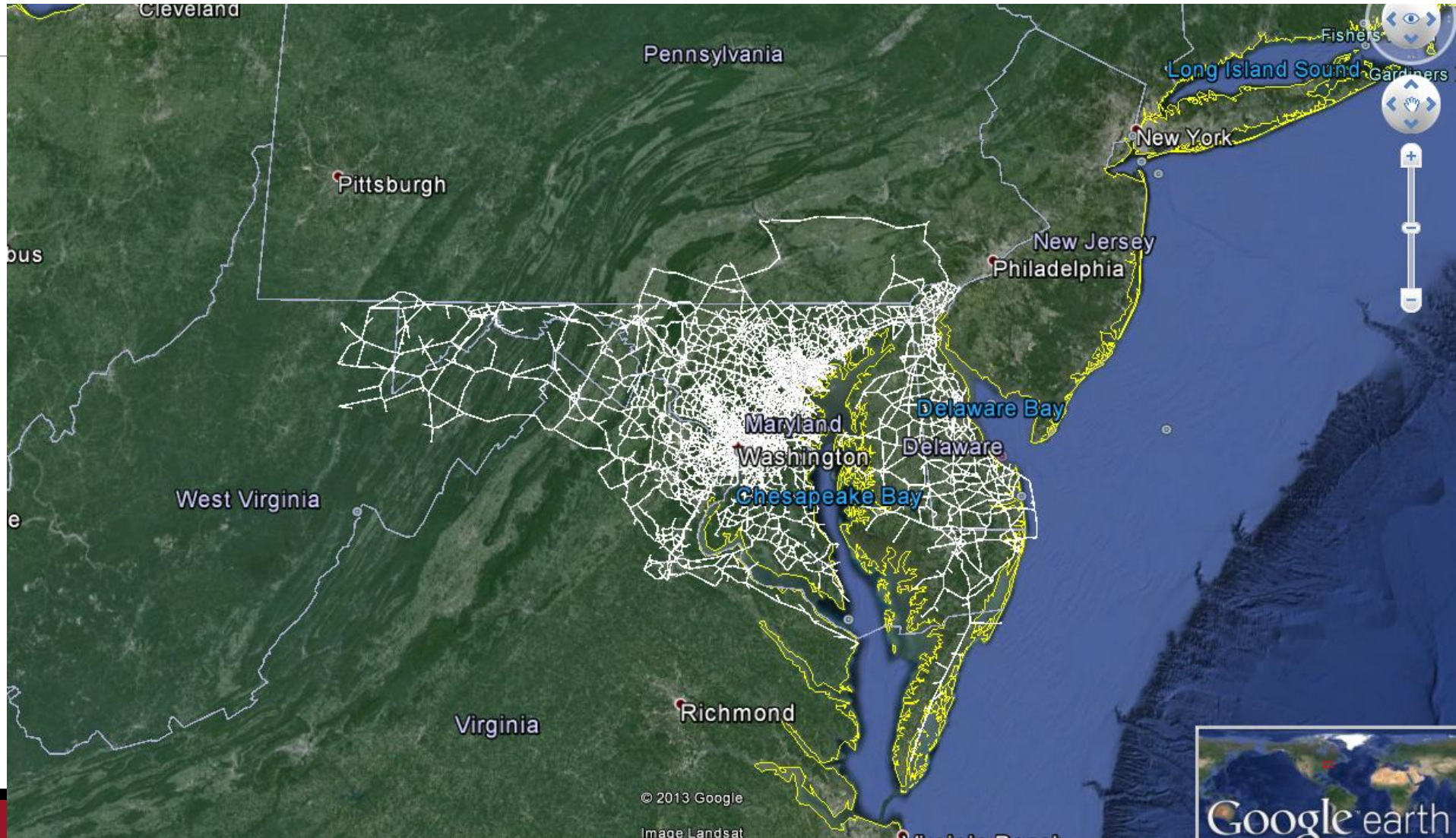


Size of a circle represents the total delay at one node

Color of a circle represents the age of congestion (to identify the congestion propagation sequence)



# Statewide Network Coverage in Google Earth





# Volume Display in Google Earth

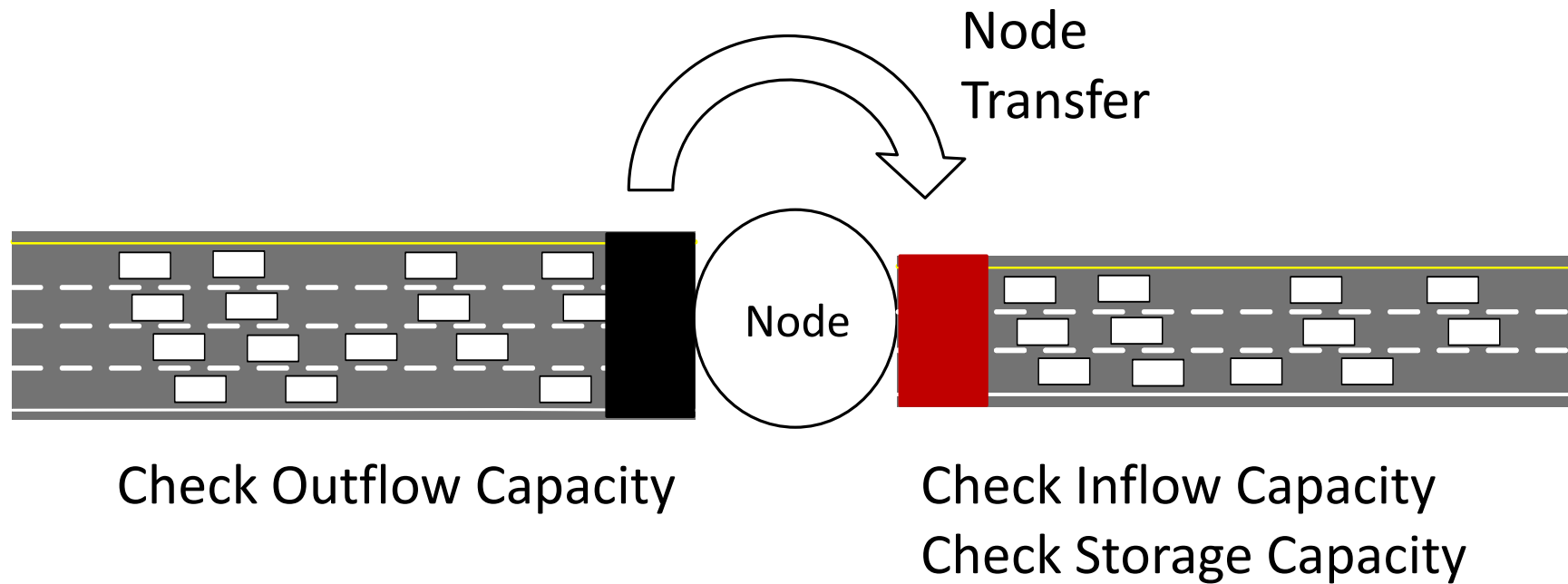
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Height as volume

# Inside: Simplified Event-based Traffic Simulator

Node transfer



# Multiple Traffic Flow Models

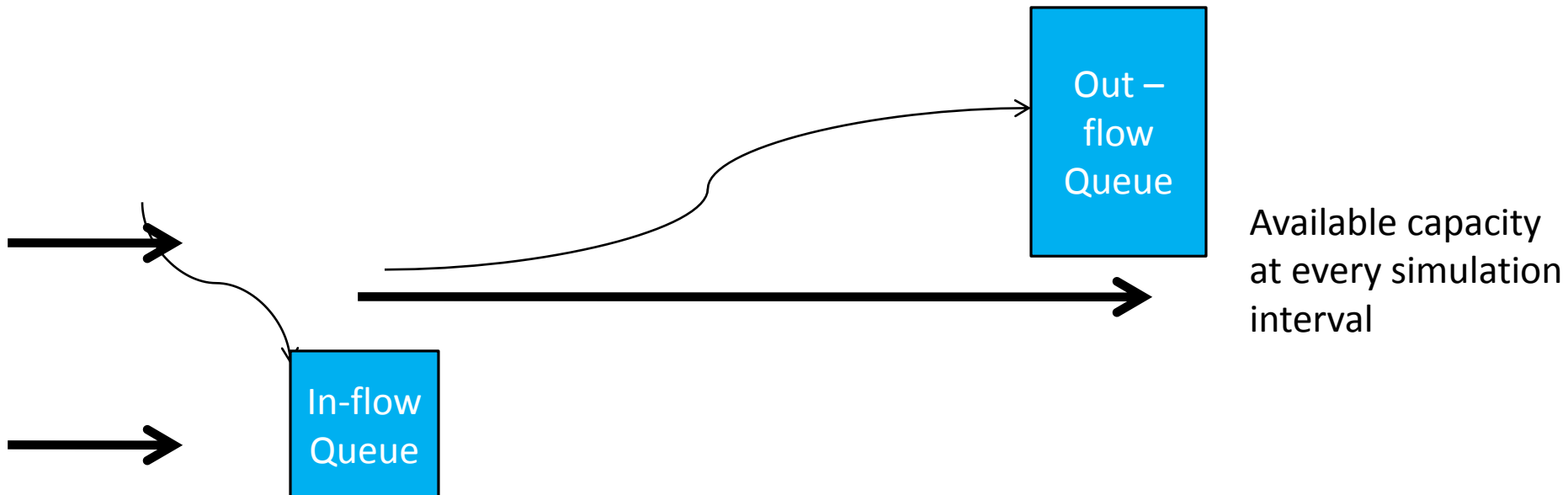
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Point queue (relaxed storage constraints)

Spatial queue

Newell's model (i.e. Link Transmission Model)

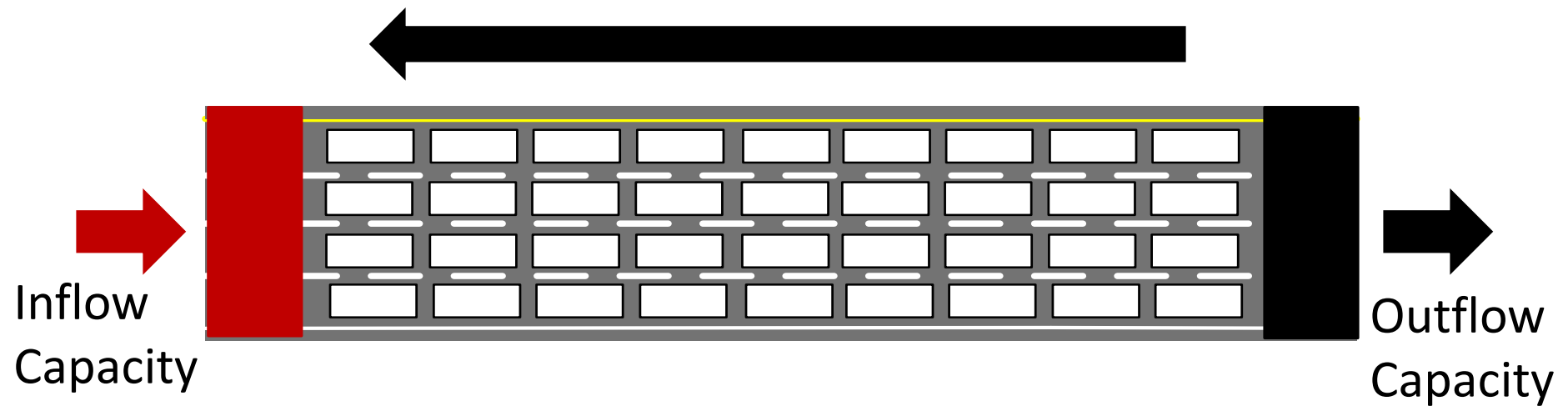
- Shockwave propagation



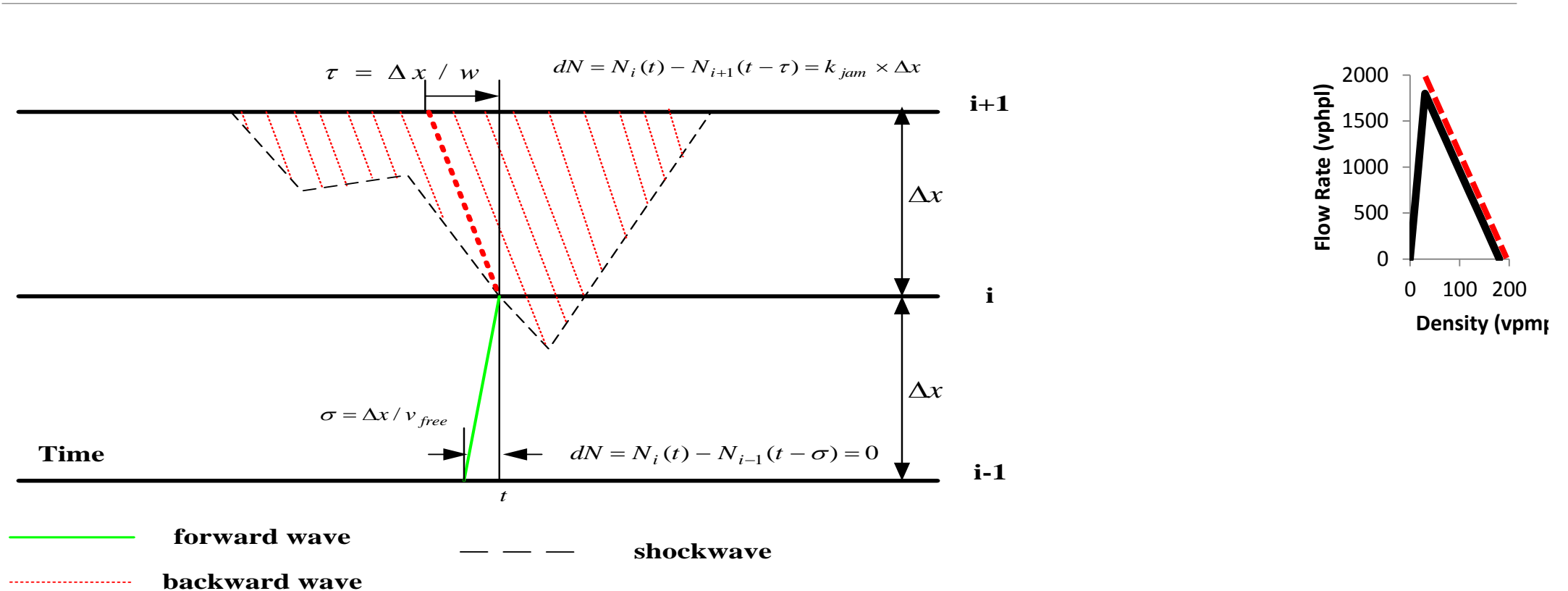
# Traffic Flow Model (on the Link)

## Queue propagation

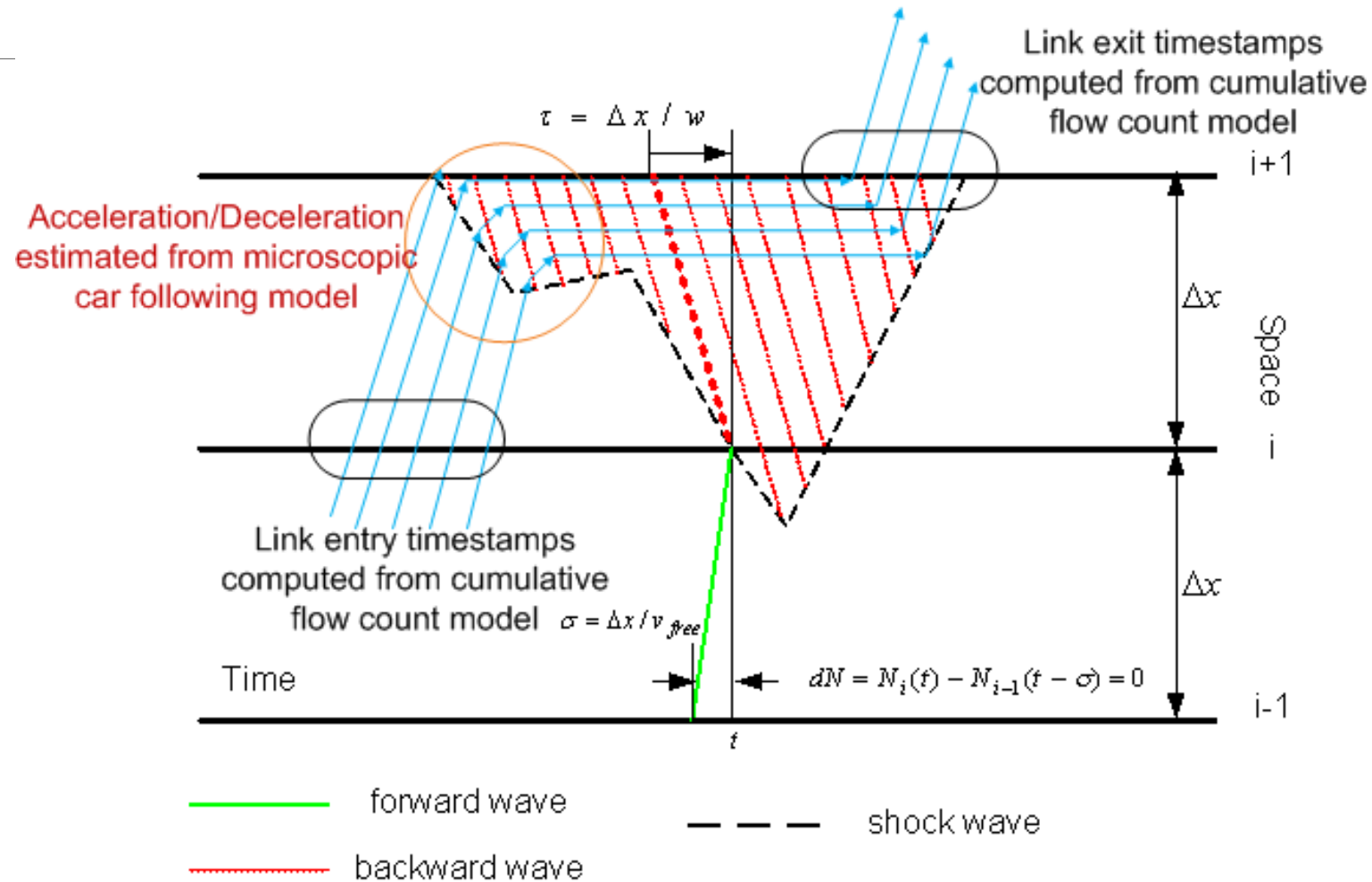
- Inflow capacity = outflow capacity



# Illustration of N-Curve Computation For Tracking Queue Spillback

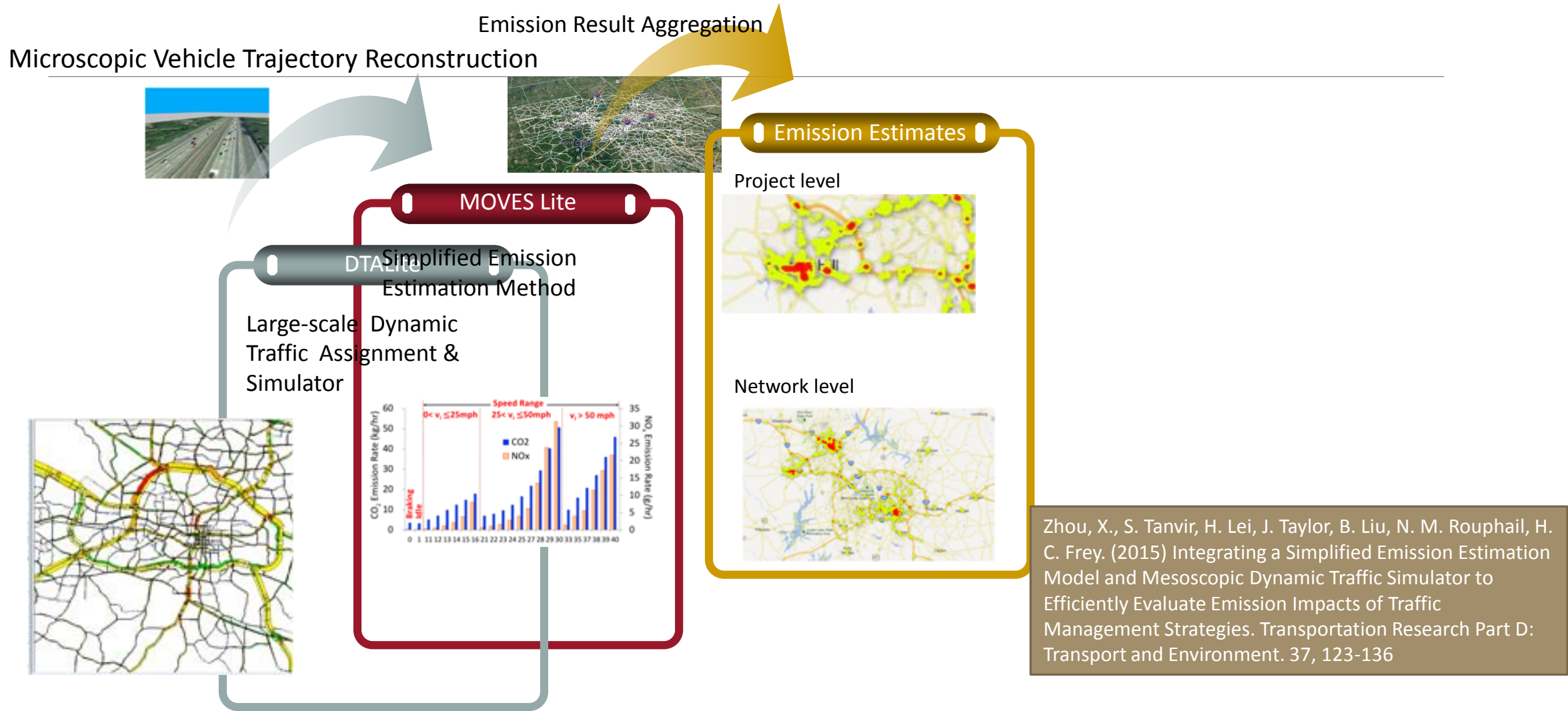


# Construct Microscopic Vehicle Trajectory from Mesoscopic Simulation Results using Consistent Simplified Kinematic Wave Model and Simplified Car following Model





# Mesoscopic Dynamic Traffic Assignment for Emission Evaluation



Zhou, X., S. Tanvir, H. Lei, J. Taylor, B. Liu, N. M. Rouphail, H. C. Frey. (2015) Integrating a Simplified Emission Estimation Model and Mesoscopic Dynamic Traffic Simulator to Efficiently Evaluate Emission Impacts of Traffic Management Strategies. *Transportation Research Part D: Transport and Environment*. 37, 123-136

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## Topic 2: Modeling next-generation of transportation systems: from simulation to optimization

Based on Paper titled “Finding Optimal Solutions for Vehicle Routing Problem with Pickup and Delivery Services with Time Windows: A Dynamic Programming Approach Based on State-space-time Network Representations”

Monirehalsadat Mahmoudi, Xuesong Zhou

Submitted Transportation Research Part B; <http://arxiv.org/abs/1507.02731>

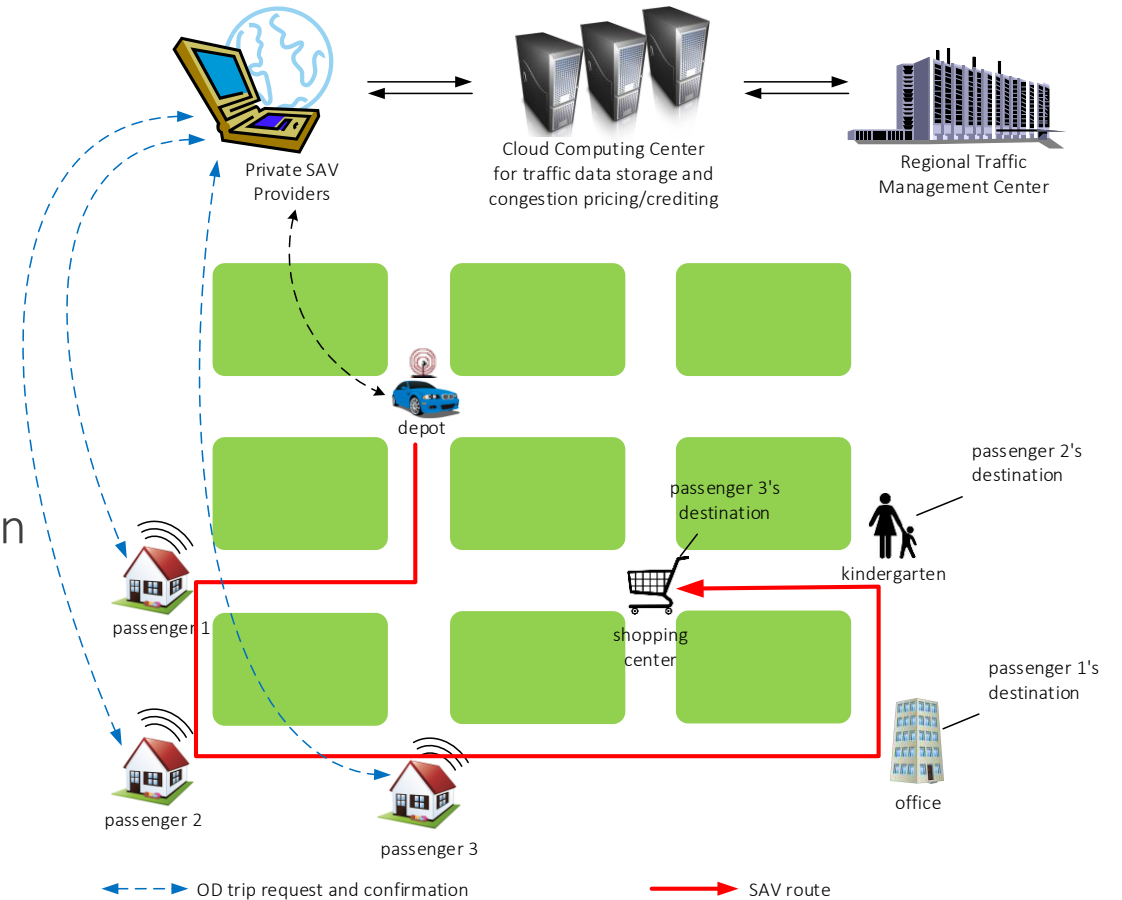


# Motivation

Concept of Ride Sharing:

Advantages of Ride Sharing:

- Reducing driver stress and driving cost
- Increasing safety
- Increasing road capacity and reducing costs
- Increasing fuel efficiency and reducing pollution



# Ride Sharing Companies

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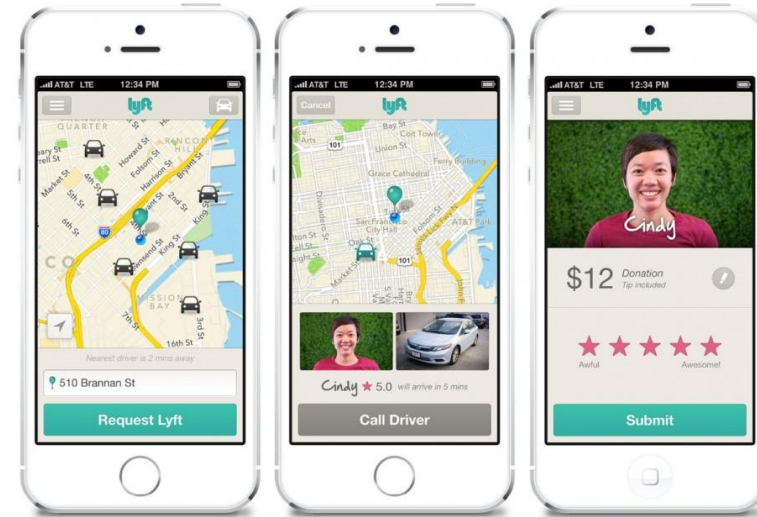


Side•car  
a whole new way to get around



# Ridesharing Apps

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One-to-One Matching  
Innovative Pricing Mechanism  
Accessibility for Low-income Families

# Key Questions

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- **How many cars** a city should use to support the overall transportation activity demand, at different levels of coordination and pre-trip scheduling?
- **How much energy** is used at the optimal state (optimal state is a condition in which 100% of travelling is supported by ride sharing)?
- **How much emissions** can be minimized?

To address the first question:

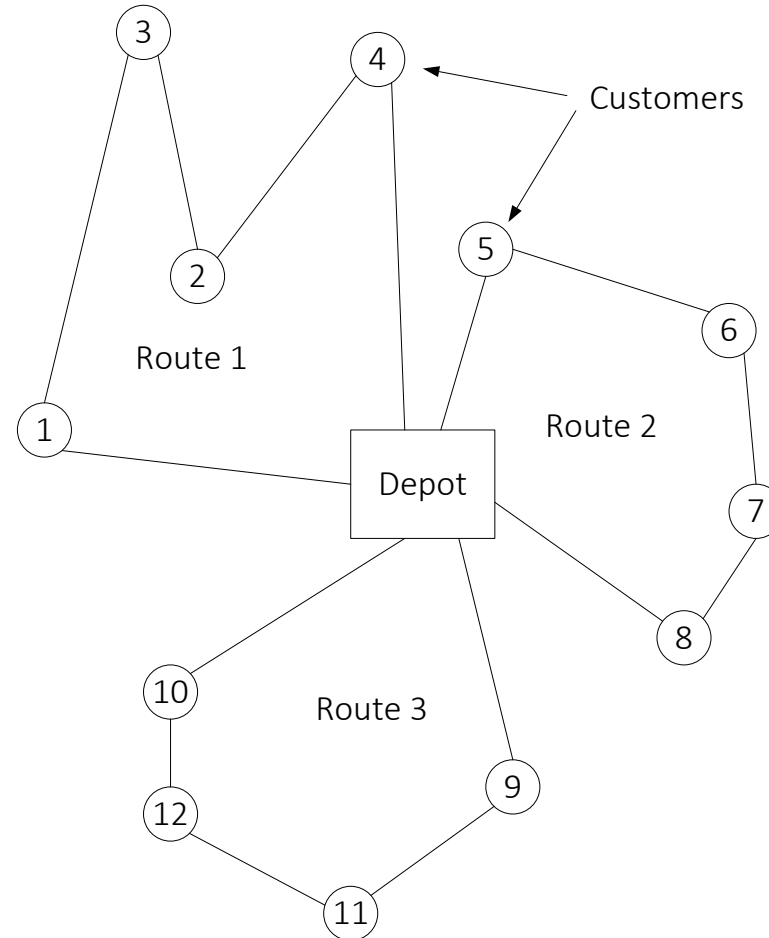
we propose a new mathematical model for ***pickup and delivery problem with time windows (PDPTW)*** to present a holistic optimization approach for synchronizing travel activity schedules, transportation services, and infrastructure on urban networks.

# Vehicle Routing Problem (VRP) & Vehicle Routing Problem with Time Windows (VRPTW)

Inputs:

- ❑ Passengers' Location
- ❑ Passengers' Preferred Service Time Window

Output: Routing



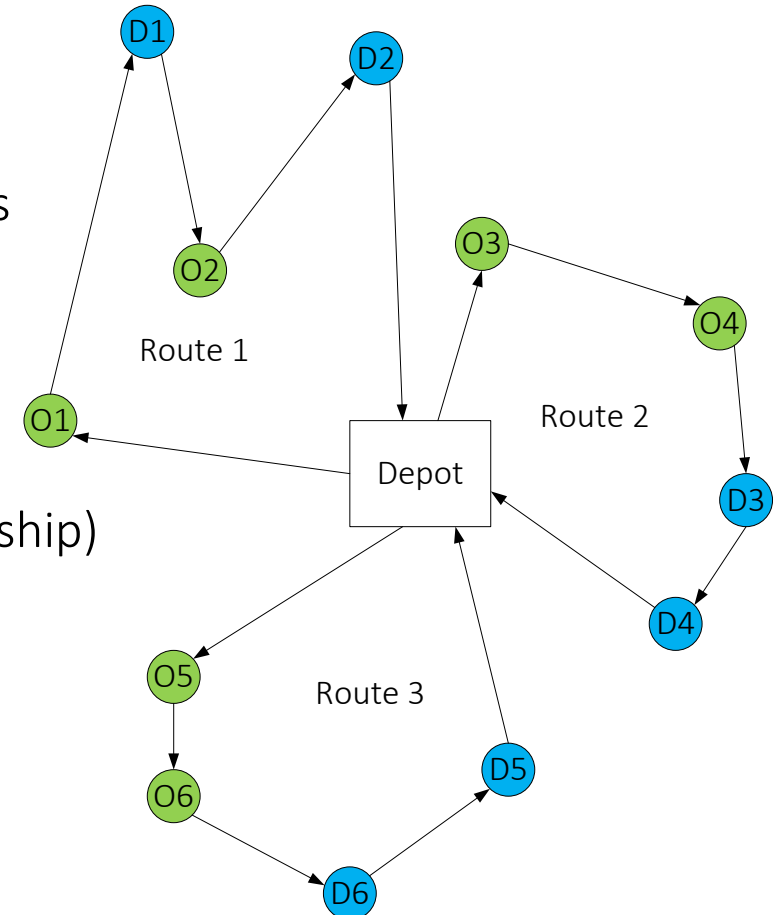
# Vehicle Routing Problem with Pick up and Delivery with Time Windows (VRPPDTW)

Inputs:

- Passengers' Origin and Destination Location
- Passengers' Preferred Pick up and Delivery Time Windows

Outputs:

- Vehicle Routing
- Vehicle Scheduling
- Assigning Passengers to Vehicles (Many-to- One Relationship)
- Pricing



# Optimization-based Approaches for Solving VRPPDTW

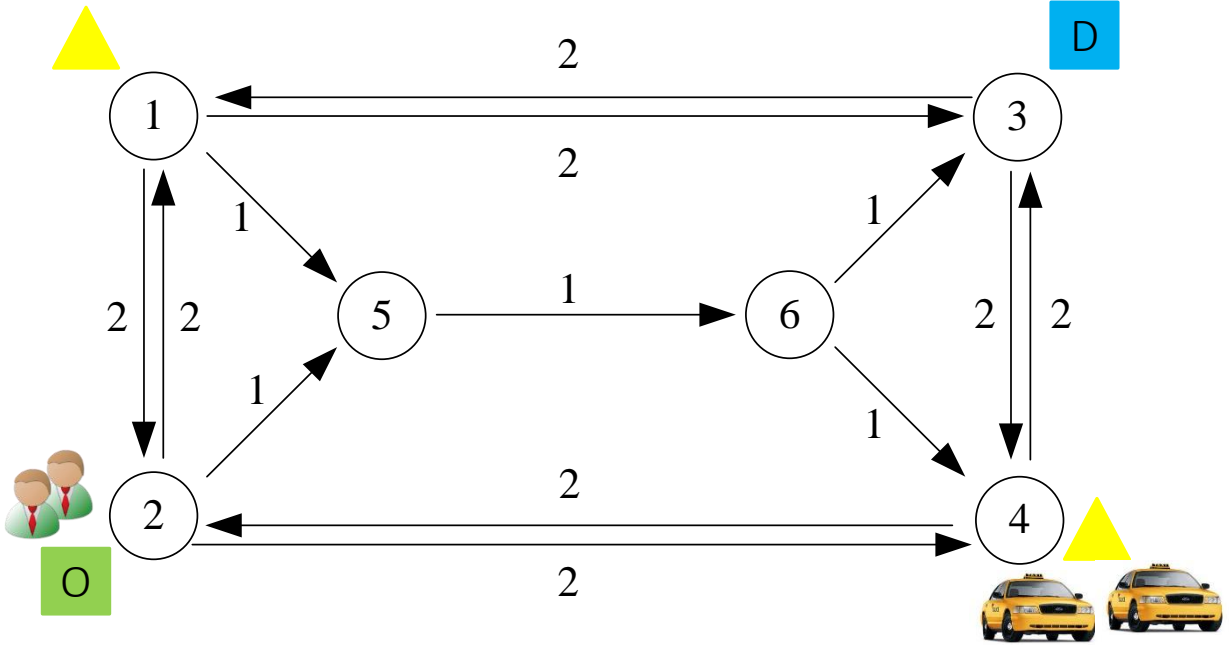
Reference	Method (Algorithm)	Type of problem	Objective Function
Psaraftis (1980)	Exact backward dynamic programming	Single vehicle VRPPD	Weighted combination of the total service time and the total customer inconvenience
Psaraftis (1983)	Forward dynamic programming	Single vehicle VRPPDTW	Sum of waiting and riding times
Sexton and Bodin (1985)	Benders' decomposition	Single vehicle VRPPD with one sided windows	Total customer inconvenience
Desrosiers, Dumas, and Soumis (1986)	Exact forward dynamic programming	single vehicle VRPPDTW	Total distance traveled
Dumas, Desrosiers, and Soumis (1991)	Column Generation	Multiple vehicle VRPPDTW	Total travel cost

## Optimization-based Approaches for Solving VRPPDTW (contd.)

Ruland (1995, 1997)	Polyhedral approach	Single Vehicle VRPPD without capacity constraints	Total travel cost
Savelsbergh and Sol (1998)	Branch-and-price	Multiple vehicle VRPPDTW	Primary objective function: Total number of vehicles, secondary: Total distance traveled
Lu and Dessouky (2004)	Branch-and-cut	Multiple vehicle VRPPDTW	Total travel cost and the fixed vehicle cost
Ropke, Cordeau, Laporte (2007)	Branch-and-cut-and-price	Multiple vehicle VRPPDTW	Total traveled distance
Ropke and Cordeau (2009)	Branch-and-cut-and-price	Multiple vehicle VRPPDTW	Total traveled distance
Baldacci, Bartolini, Mingozzi (2011)	Set-partitioning formulation improved by additional cuts	Multiple vehicle VRPPDTW	Primary objective function: Route costs, secondary: Sum of vehicle fixed costs and then sum of route costs



# Problem Statement by a Simple 6-node Transportation Network



- O Origin
- D Destination
- ▲ Depot

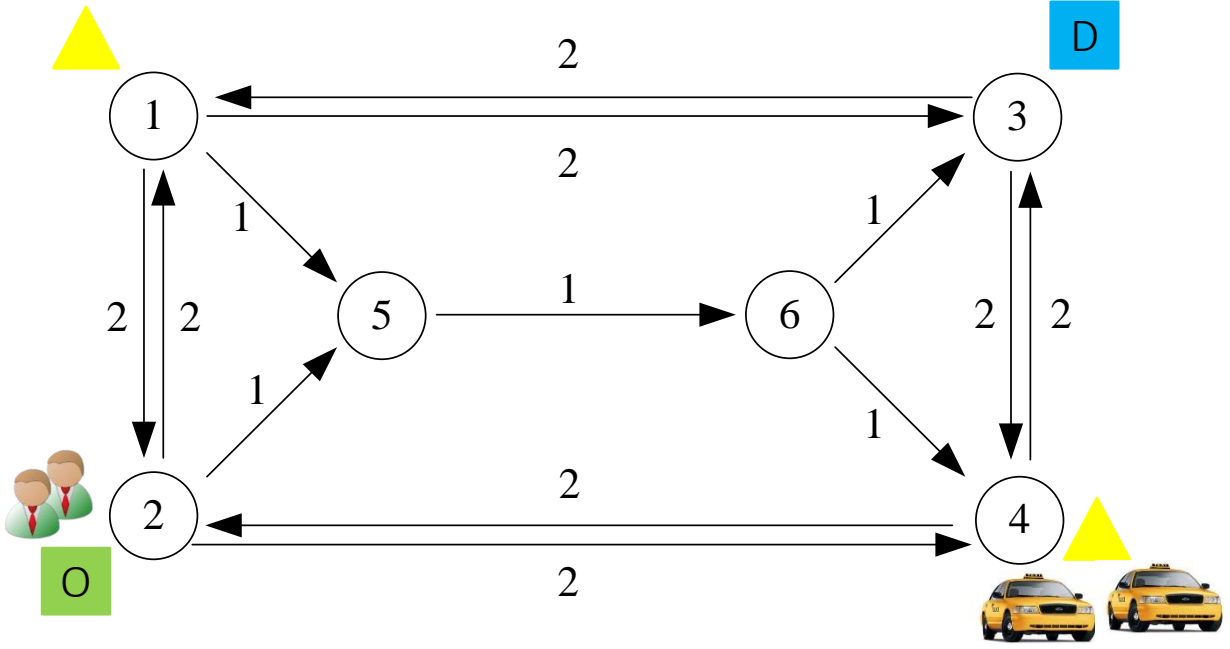
Two passengers

Two vehicles

Passenger 1 and 2's origin: node 2  
 Passenger 1 and 2's destination: node 3

Vehicle 1 and 2's origin depot: node 4  
 Vehicle 1 and 2's destination depot: node 1

# Problem Statement by a Simple 6-node Transportation Network

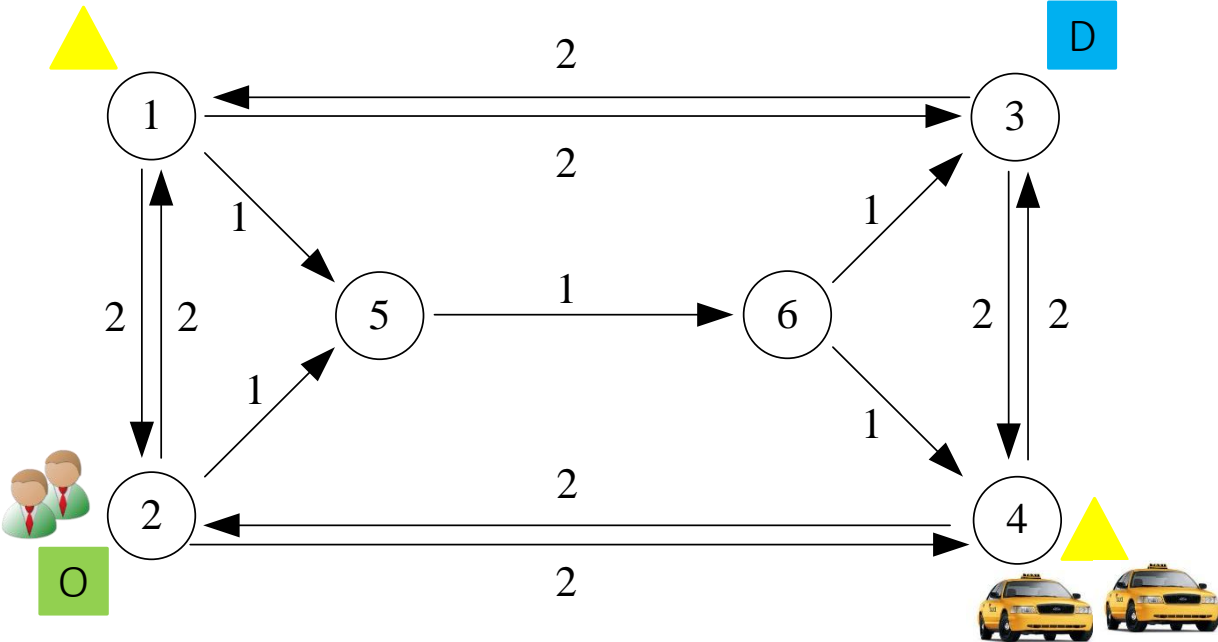


- Origin
- Destination
- Depot

Passenger 1's preferred time window for departure from origin: [4,7]  
Passenger 1's preferred time for arrival at destination: [9,12]

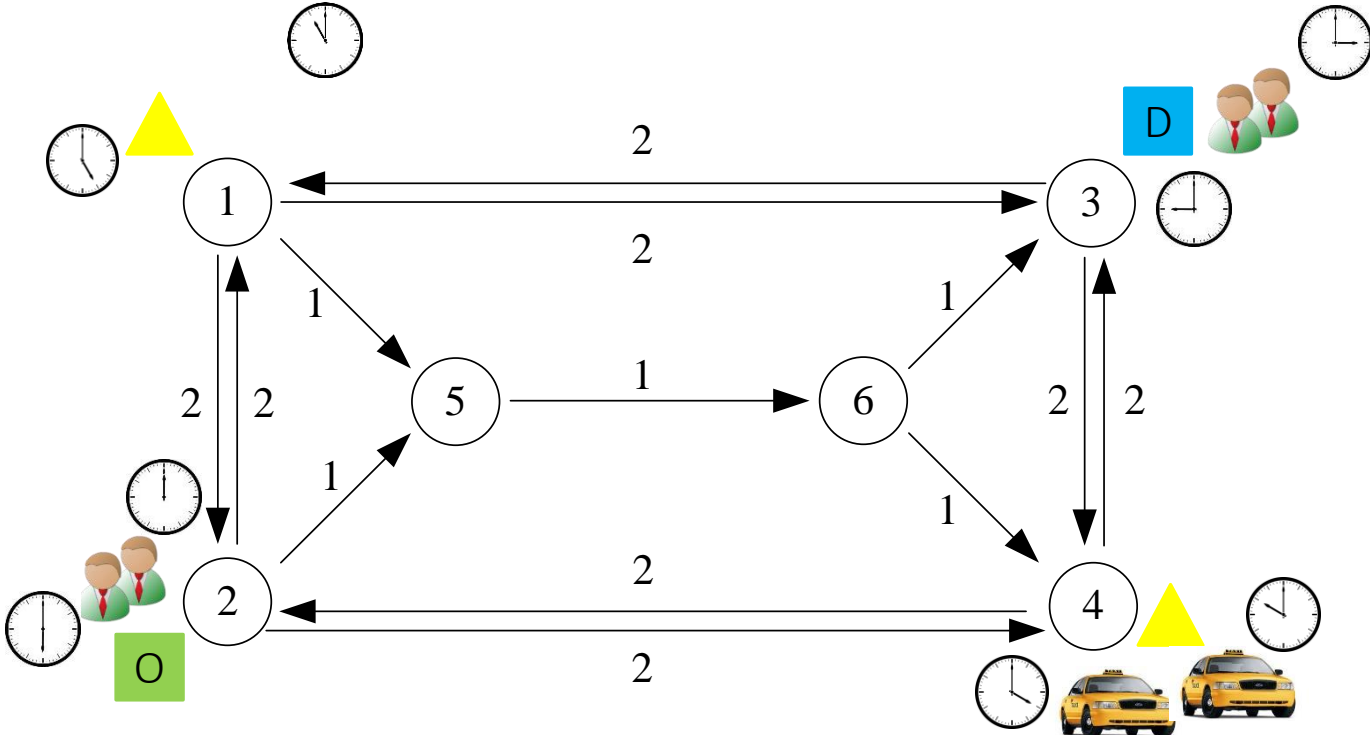
Passenger 2's preferred time window for departure from origin: [10,12]  
Passenger 2's preferred time for arrival at destination: [15,17]

# Problem Statement by a Simple 6-node Transportation Network



- Origin
- Destination
- Depot

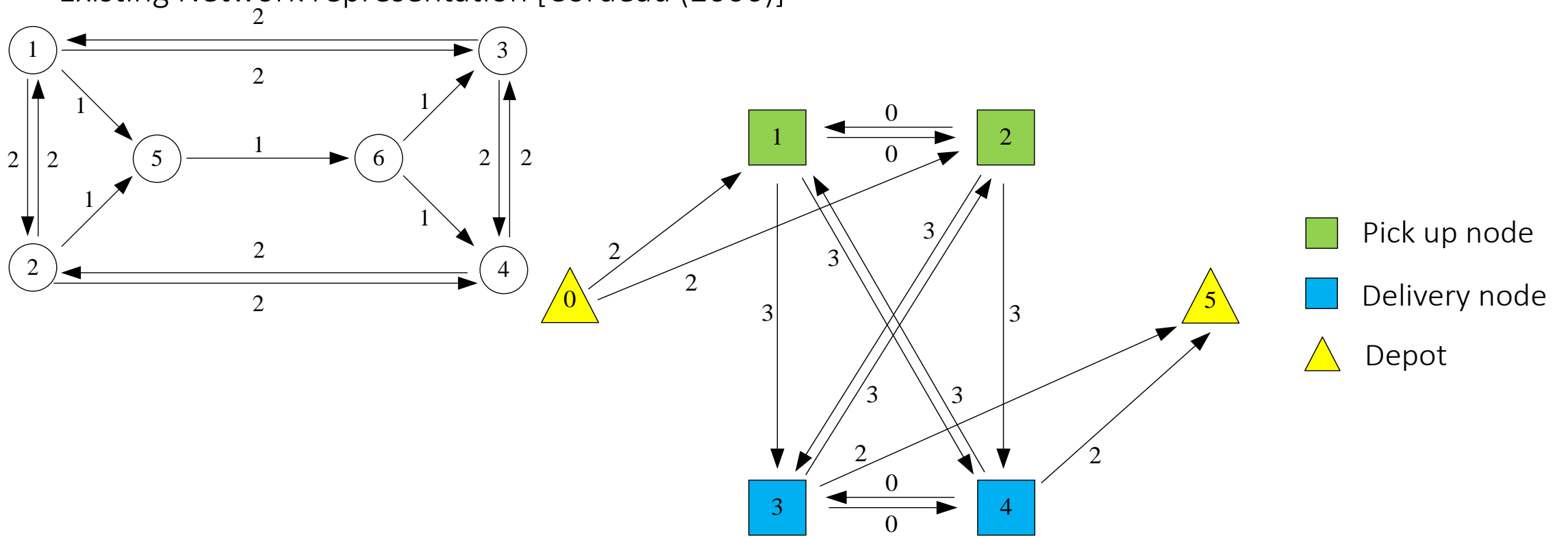
# Problem Statement by a Simple 6-node Transportation Network



- Origin
- Destination
- Depot

# Opening Statement about Our Method

Existing Network representation [Cordeau (2006)]



# Current Mathematical Model for VRPPDTW[Cordeau (2006)]

$$\text{Min} \sum_{v \in V} \sum_{i \in N} \sum_{j \in N} c_{ij}^v x_{ij}^v$$

objective function: minimizing the total routing cost

$$\sum_{v \in V} \sum_{j \in N} x_{ij}^v = 1$$

$$\forall i \in P$$

guarantees that each passenger is definitely picked up

$$\sum_{j \in N} x_{ij}^v - \sum_{j \in N} x_{n+i,j}^v = 0$$

$$\forall i \in P, v \in V$$

ensure that each passenger's origin and destination are visited exactly once by the same vehicle

$$\sum_{j \in N} x_{0j}^v = 1$$

$$\forall v \in V$$

each vehicle  $v$  starts its route from the origin depot

$$\sum_{j \in N} x_{ji}^v - \sum_{j \in N} x_{ij}^v = 0$$

$$\forall i \in P \cup D, v \in V$$

flow balance on each node

$$\sum_{i \in N} x_{i,2n+1}^v = 1$$

$$\forall v \in V$$

each vehicle  $v$  ends its route to the destination depot

# Current Mathematical Model for VRPPDTW[Cordeau (2006)] (contd.)

$x_{ij}^v (B_i^v + d_i + t_{ij}) \leq B_j^v$	<b>Non-linear Constraint</b>	$\forall i \in N, j \in N, v \in V$	validity of the time variables
--	------------------------------	-------------------------------------	--------------------------------

$x_{ij}^v (Q_i^v + q_j) \leq Q_j^v$	<b>Non-linear Constraint</b>	$\forall i \in N, j \in N, v \in V$	validity of the load variables
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$L_i^v = B_{n+i}^v - (B_i^v + d_i)$	$\forall i \in P, v \in V$	defines each passenger's ride time
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$B_{2n+1}^v - B_0^v \leq T_v$	$\forall v \in V$	impose maximal duration of each route
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$e_i \leq B_i^v \leq l_i$	$\forall i \in N, v \in V$	impose time windows constraints
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$t_{i,n+i} \leq L_i^v \leq L$	$\forall i \in P, v \in V$	impose the ride time of each passenger constraints
-------------------------------	----------------------------	--

$\max\{0, q_i\} \leq Q_i^v \leq \min\{Q_v, Q_v + q_i\}$	$\forall i \in N, v \in V$	impose capacity constraints
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$x_{ij}^v \in \{0,1\}$	$\forall i \in N, j \in N, v \in V$	
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# Current Theoretical and Computational Challenges

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- Single vs multiple vehicles
  - The focus of most research was on solving the PDPTW for a single vehicle (simpler case)
- Single vs multiple depots
- Limited number of transportation requests (passengers)
  - The most Current Algorithm: Baldacci et al. (2011) based on a set-partitioning formulation solved instances of approximately 500 requests with tight time windows.
- Time windows
  - Some preprocessing steps to find feasible transportation requests are needed
  - Research only focus on tight time windows to prevent some fluctuation in results



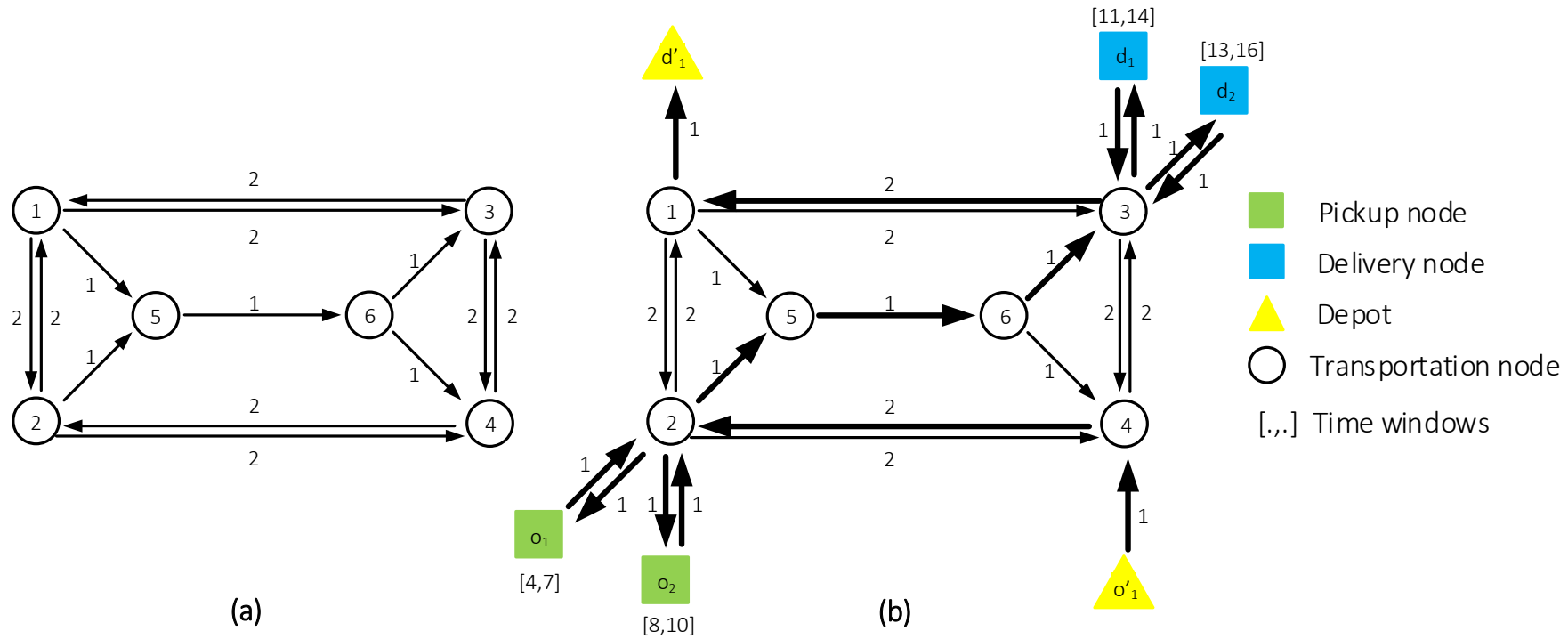
## Current Theoretical and Computational Challenges (contd.)

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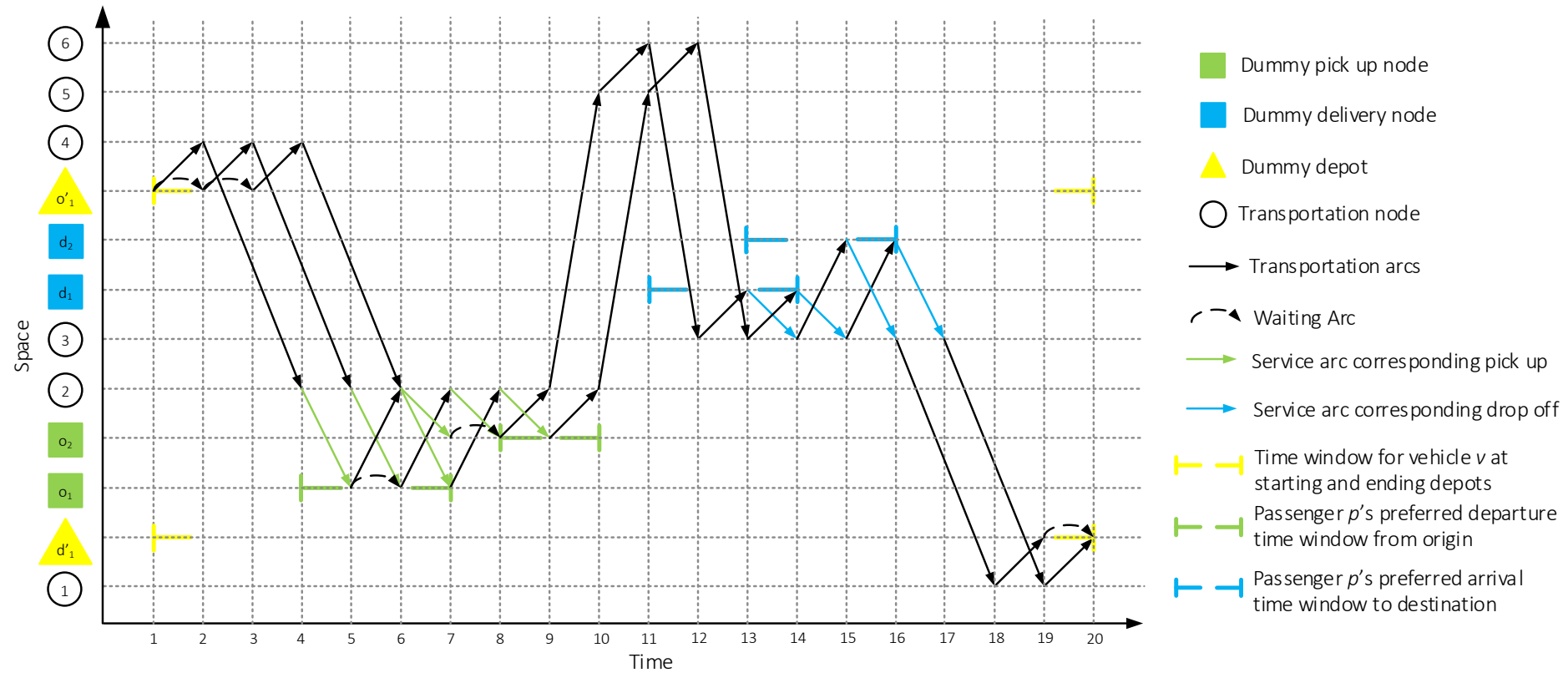
- Fixed routing cost (travel time) over time
  - Existing network for PDPTW: an offline network in which each link has a fixed routing cost (travel time)
- Existence of sub-tour in the optimal solution
  - Some additional constraints are needed to avoid the existence of any sub-tour in the optimal solution

# Opening Statement about Our Method (contd.)

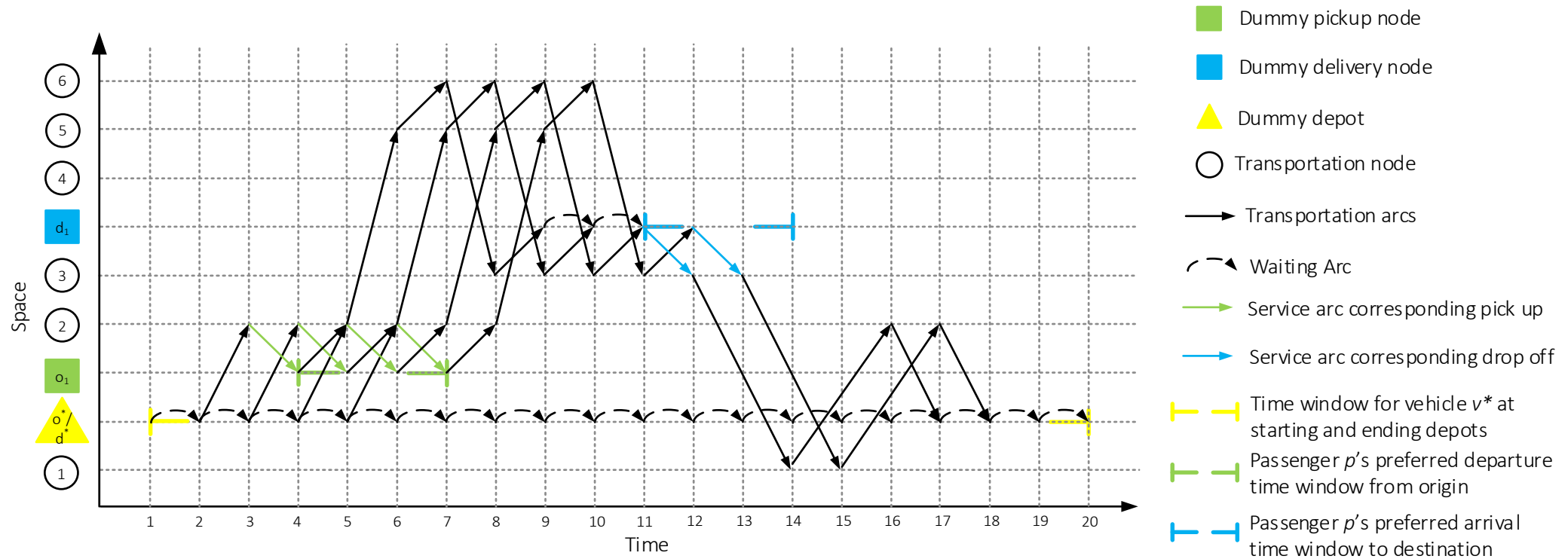
Description of the PDPTW in Space-Time Transportation Network [Mahmoudi, M. & Zhou, X. (2014)]



# Physical Vehicle's Space-time Transportation Network

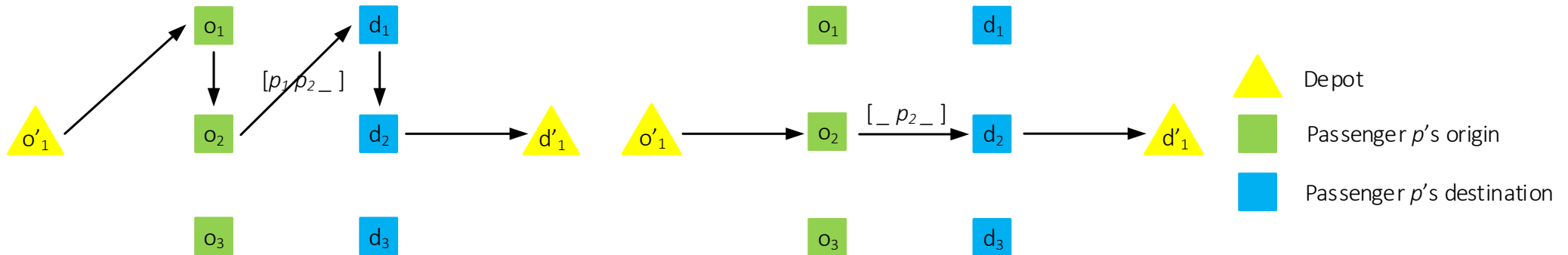


# Virtual Vehicle's Space-time Transportation Network



# Binary representation and equivalent character-based representation for passenger carrying states

Binary representation	Equivalent character-based representation
[0,0,0]	[- - -]
[1,0,0]	[ $p_1$ - -]
[0,1,1]	[- $p_2$ $p_3$ ]



(a) Shared ride

(b) Serving single passenger once a time

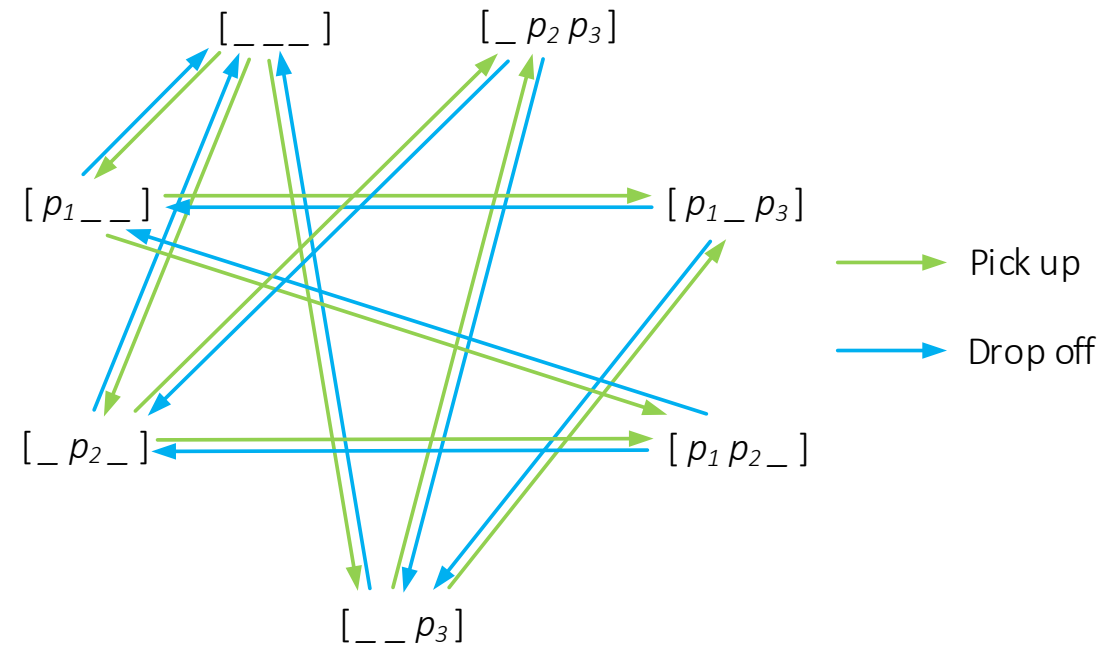
# All possible combinations of passenger carrying states

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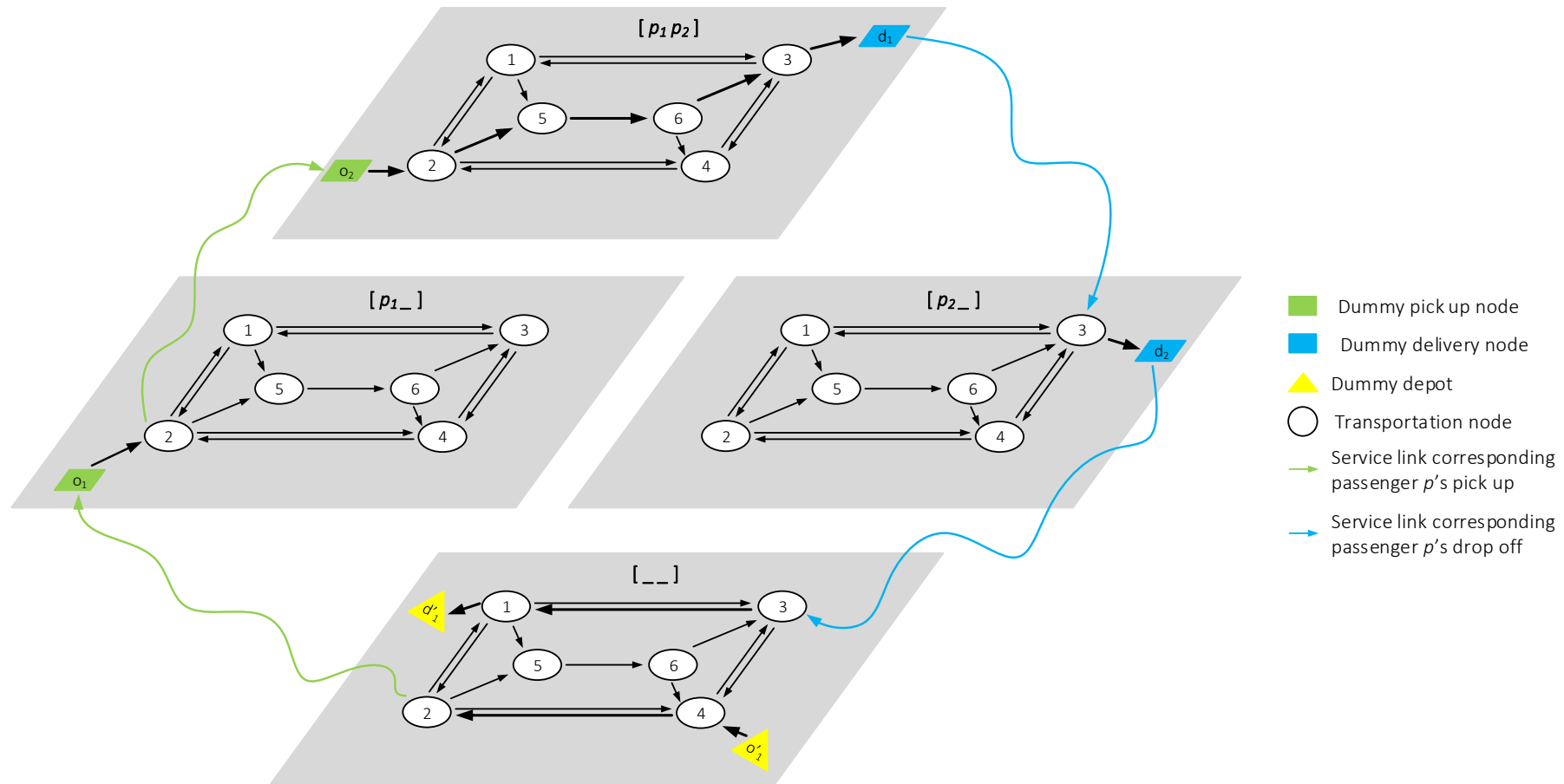
$w \backslash w'$	$[- - -]$	$[p_1 - -]$	$[- p_2 -]$	$[- - p_3]$	$[p_1 p_2 -]$	$[p_1 - p_3]$	$[- p_2 p_3]$
$[- - -]$	no change	pickup	pickup	pickup			
$[p_1 - -]$	drop-off	no change			pickup	pickup	
$[- p_2 -]$	drop-off		no change		pickup		pickup
$[- - p_3]$	drop-off			no change		pickup	pickup
$[p_1 p_2 -]$		drop-off	drop-off		no change		
$[p_1 - p_3]$		drop-off		drop-off		no change	
$[- p_2 p_3]$			drop-off	drop-off			no change

Finite states graph showing all possible passenger carrying state transition (pickup or drop-off)

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# Projection on state-space network representation for ride-sharing path (pick up passenger $p_1$ and then $p_2$ ).





# Problem Definition

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$$\text{Min } Z = \sum_{v \in (V \cup V^*)} \sum_{(i,j,t,s,w,w') \in B_v} c(v, i, j, t, s, w, w') y(v, i, j, t, s, w, w')$$

s.t.

Flow balance constraints at vehicle  $v$ 's origin vertex

$$\sum_{(i,j,t,s,w,w') \in B_v} y(v, i, j, t, s, w, w') = 1 \quad i = o'_v, t = e_v, w = w' = w_0, \forall v \in (V \cup V^*)$$

Flow balance constraint at vehicle  $v$ 's destination vertex

$$\sum_{(i,j,t,s,w,w') \in B_v} y(v, i, j, t, s, w, w') = 1 \quad j = d'_v, s = l_v, w = w' = w_0, \forall v \in (V \cup V^*)$$

Flow balance constraint at intermediate vertex

$$\sum_{(j,s,w'')} y(v, i, j, t, s, w, w'') - \sum_{(j',s',w')} y(v, j', i, s', t, w', w) = 0 \quad (i, t, w) \notin \{(o'_v, e_v, w_0), (d'_v, l_v, w_0)\}, \forall v \in (V \cup V^*)$$

## Problem Definition (contd.)

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Passenger  $p$ 's pick-up request constraint

$$\sum_{v \in (V \cup V^*)} \sum_{(i,j,t,s,w,w') \in \Psi_{p,v}} y(v, i, j, t, s, w, w') = 1 \quad \forall p \in P$$

Passenger  $p$ 's drop-off request constraint

$$\sum_{v \in (V \cup V^*)} \sum_{(i,j,t,s,w,w') \in \Phi_{p,v}} y(v, i, j, t, s, w, w') = 1 \quad \forall p \in P$$

Binary definitional constraint

$$y(v, i, j, t, s, w, w') \in \{0, 1\} \quad \forall (i, j, t, s, w, w') \in B_v, \forall v \in (V \cup V^*)$$

# Lagrangian Relaxation-based Solution Approach

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- Pickup and drop-off constraints : each passenger is picked up and dropped off exactly once by a vehicle (either physical or virtual).
- Flow balance constraints on intermediate nodes force the vehicle to end its route at the destination depot with the empty passenger carrying state.
- If vehicle  $v$  picks up passenger  $p$  from his origin, to maintain the flow balance constraints on intermediate nodes , the vehicle must drop-off the passenger at his destination node so that the vehicle comes back to its ending depot with the empty passenger carrying state.
- As a result, constraint (6) is redundant

$$L = \sum_{v \in (V \cup V^*)} \sum_{(i,j,t,s,w,w') \in B_v} c(v, i, j, t, s, w, w') y(v, i, j, t, s, w, w')$$

$$+ \sum_{p \in P} \lambda(p) \left[ \sum_{v \in (V \cup V^*)} \sum_{(i,j,t,s,w,w') \in \Psi_{p,v}} y(v, i, j, t, s, w, w') - 1 \right]$$

## New Relaxed Problem

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*Min L*

s.t.

$$\begin{aligned} \sum_{(i,j,t,s,w,w') \in B_v} y(v, i, j, t, s, w, w') &= 1 & i = o'_v, t = e_v, w = w' = w_0, \forall v \in (V \cup V^*) \\ \sum_{(i,j,t,s,w,w') \in B_v} y(v, i, j, t, s, w, w') &= 1 & j = d'_v, s = l_v, w = w' = w_0, \forall v \in (V \cup V^*) \\ \sum_{(j,s,w'')} y(v, i, j, t, s, w, w'') - \\ \sum_{(j',s',w')} y(v, j', i, s', t, w', w) &= 0 & (i, t, w) \notin \{(o'_v, e_v, w_0), (d'_v, l_v, w_0)\}, \forall v \in (V \cup V^*) \\ y(v, i, j, t, s, w, w') &\in \{0, 1\} & \forall (i, j, t, s, w, w') \in B_v, \forall v \in (V \cup V^*) \end{aligned}$$

The simplified Lagrangian function  $L$ :

$$L = \sum_{v \in (V \cup V^*)} \sum_{(i,j,t,s,w,w') \in B_v} \xi(v, i, j, t, s, w, w') y(v, i, j, t, s, w, w') - \sum_{p \in P} \lambda(p)$$

# Time-dependent Forward Dynamic Programming

---

**for** each vehicle  $v \in (V \cup V^*)$  **do**

$L(.,.,.) := +\infty;$

node pred of vertex  $(.,.,.) := -1;$  time pred of vertex  $(.,.,.) := -1;$  state pred of vertex  $(.,.,.) := -1;$

$L(o'_v, e_v, w_0) := 0;$

**for** each time  $t \in [e_v, l_v]$  **do**

**for** each link  $(i, j)$  **do**

**for** each state  $w$  **do**

derive downstream state  $w'$  based on the possible state transition on link  $(i, j);$

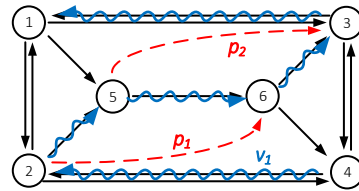
derive arrival time  $s = t + TT(i, j, t);$

**if**  $(L(i, t, w) + \xi(v, i, j, t, s, w, w') < L(j, s, w'))$  **then**

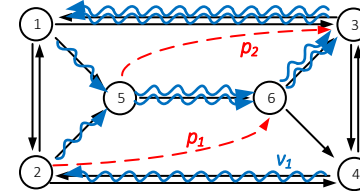
$L(j, s, w') := L(i, t, w) + \xi(v, i, j, t, s, w, w');$  //label update

node pred of vertex  $(j, s, w') := i;$  time pred of vertex  $(j, s, w') := t;$  state pred of vertex  $(j, s, w') := w;$

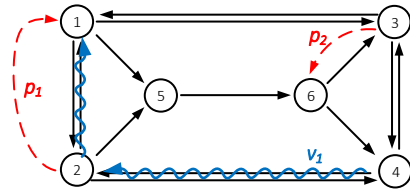
# Computational Results for Testing Different Cases



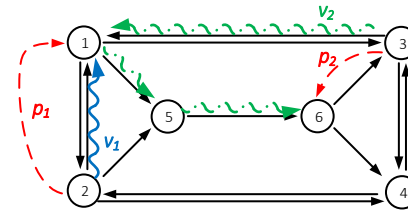
Scenario I. Two passengers are served by one vehicle through ride-sharing mode.



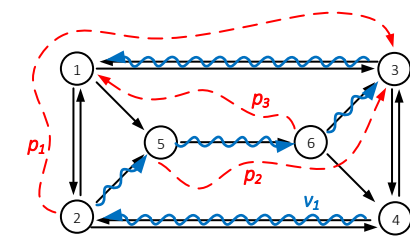
Scenario II. Two passengers are served by one vehicle (not through ride-sharing mode).



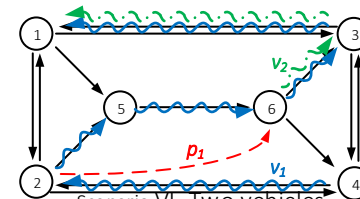
Scenario III. Two passengers and one vehicle; one passenger remains unserved.



Scenario IV. Two passengers and two vehicles; each vehicle is assigned to a passenger



Scenario V. Three passengers are served by one vehicle through ride-sharing mode

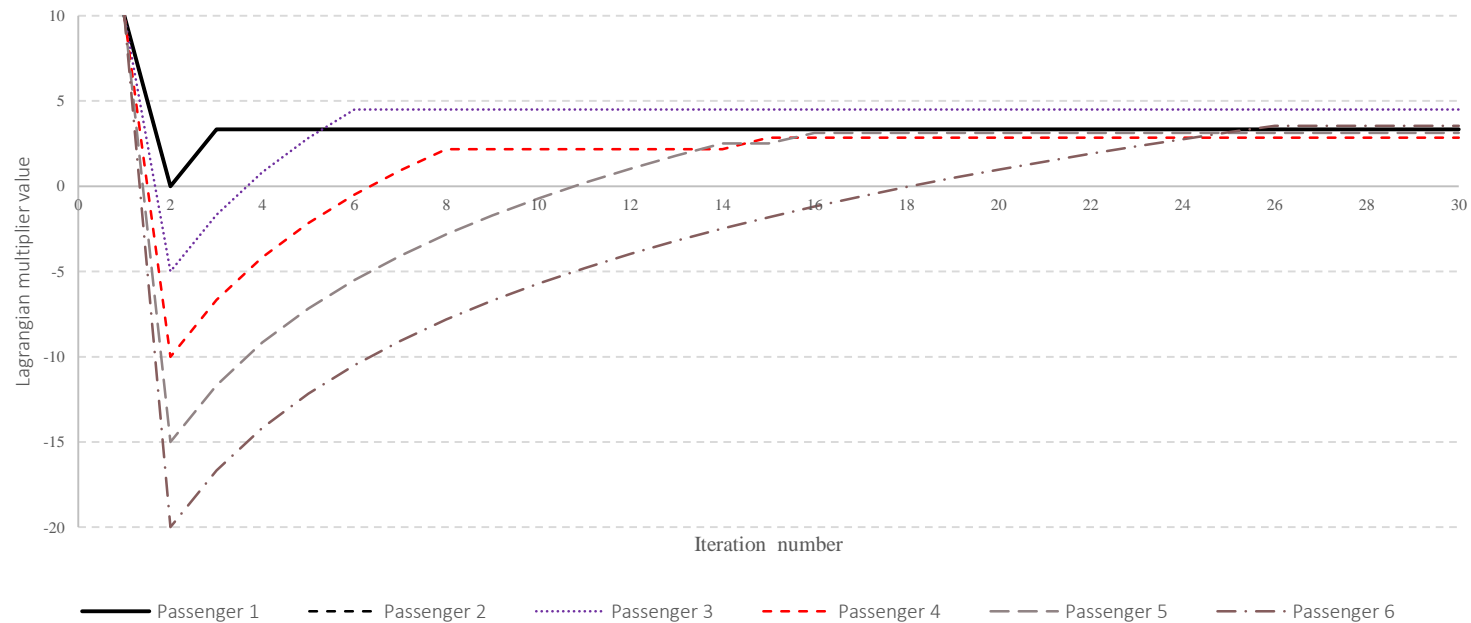


Scenario VI. Two vehicles compete for serving a passenger

Transportation link     
  Representative of passenger  $p$ 's origin-destination pair     
  Representative of vehicle routing

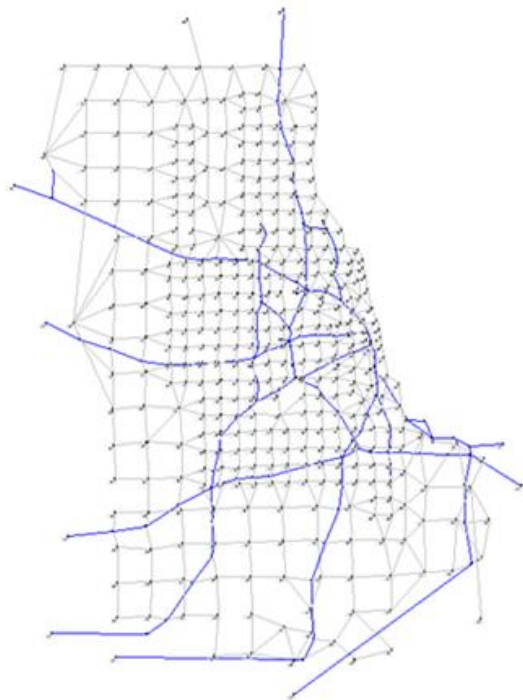
# Lagrangian multipliers along 30 iterations in test case 1 for the six-node transportation network: System Marginal Cost!

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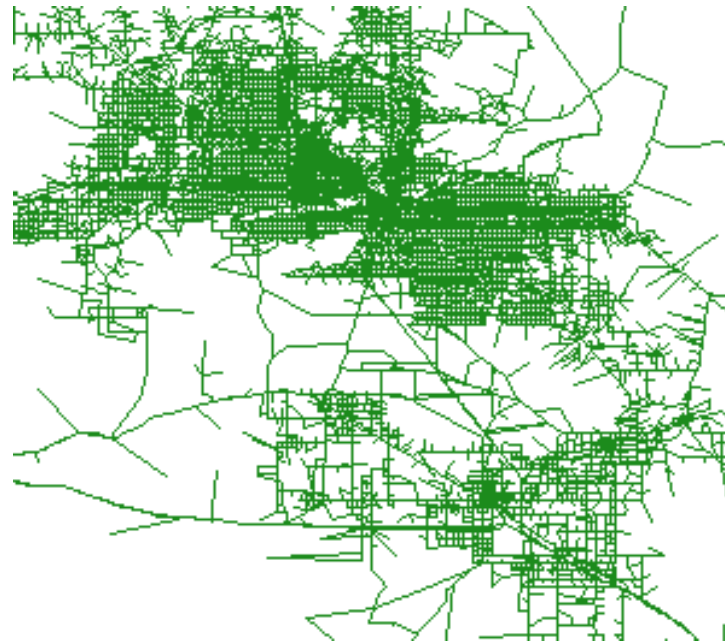


# Medium and large-scale transportation networks for computational performance testing

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(a) Chicago sketch network



(b) Phoenix metropolitan regional network



# Results for the Chicago network with 933 transportation nodes and 2,967 links

---

Test case number	Number of iterations	Number of passengers	Number of vehicles	<i>LB*</i>	<i>UB*</i>	Gap (%)	Number of passengers not served	CPU running time (sec)
1	20	2	2	108.43	108.43	0.00%	0	17.43
2	20	11	3	352.97	352.97	0.00%	0	91.87
3	20	20	5	616.66	626.18	1.52%	1	327.51
4	20	46	15	1586.81	1664.07	4.64%	2	4681.52
5	20	60	15	1849.98	1878.55	1.52%	3	7096.50

---

# Results for the Phoenix network with 13,777 transportation nodes and 33,879 links

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---

Test case number	Number of iterations	Number of passengers	Number of vehicles	<i>LB</i> *	<i>UB</i> *	Gap (%)	Number of passengers not served	CPU running time (sec)
1	6	4	2	70.95	70.95	0.00%	0	110.39
2	6	10	5	191.55	207.05	7.49%	1	398.37
3	6	20	6	310.37	310.37	0.00%	0	1323.18
4	6	40	12	622.23	622.23	0.00%	0	3756.505
5	6	50	15	784.07	784.07	0.00%	0	6983.189

---

## Short Summary

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- We propose a new mathematical formulation in state-space-time network for PDPTW
- Based on time-dependent forward dynamic programming approach in the Lagrangian reformulation framework, the main problem is transformed to easy sub-problems (Time-dependent least cost path sub-problems) which is solved independently without much effort
- Unlike former proposed models for PDPTW, this model is now able to solve PDPTW in large scale transportation networks.

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Topic 3:

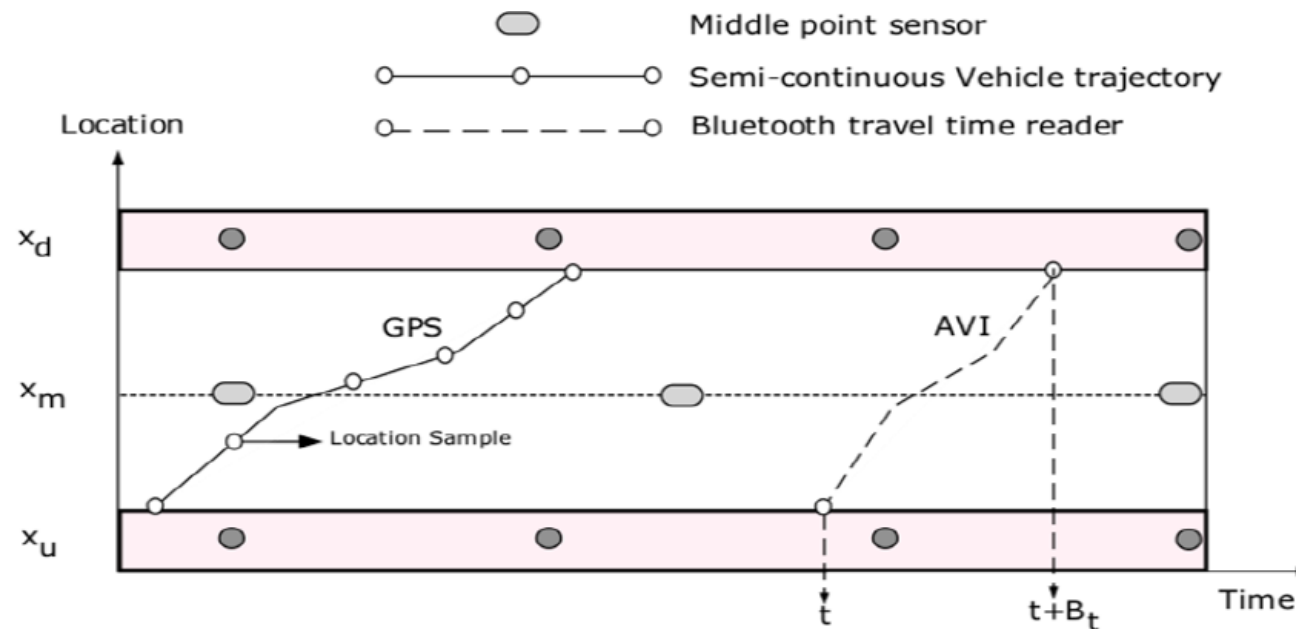
Extensions of state-space-time modeling  
framework:

Traffic flow state estimation, and traffic signal  
control and train timetabling...

# Application 1: Traffic State Estimation

How much information is sufficient?

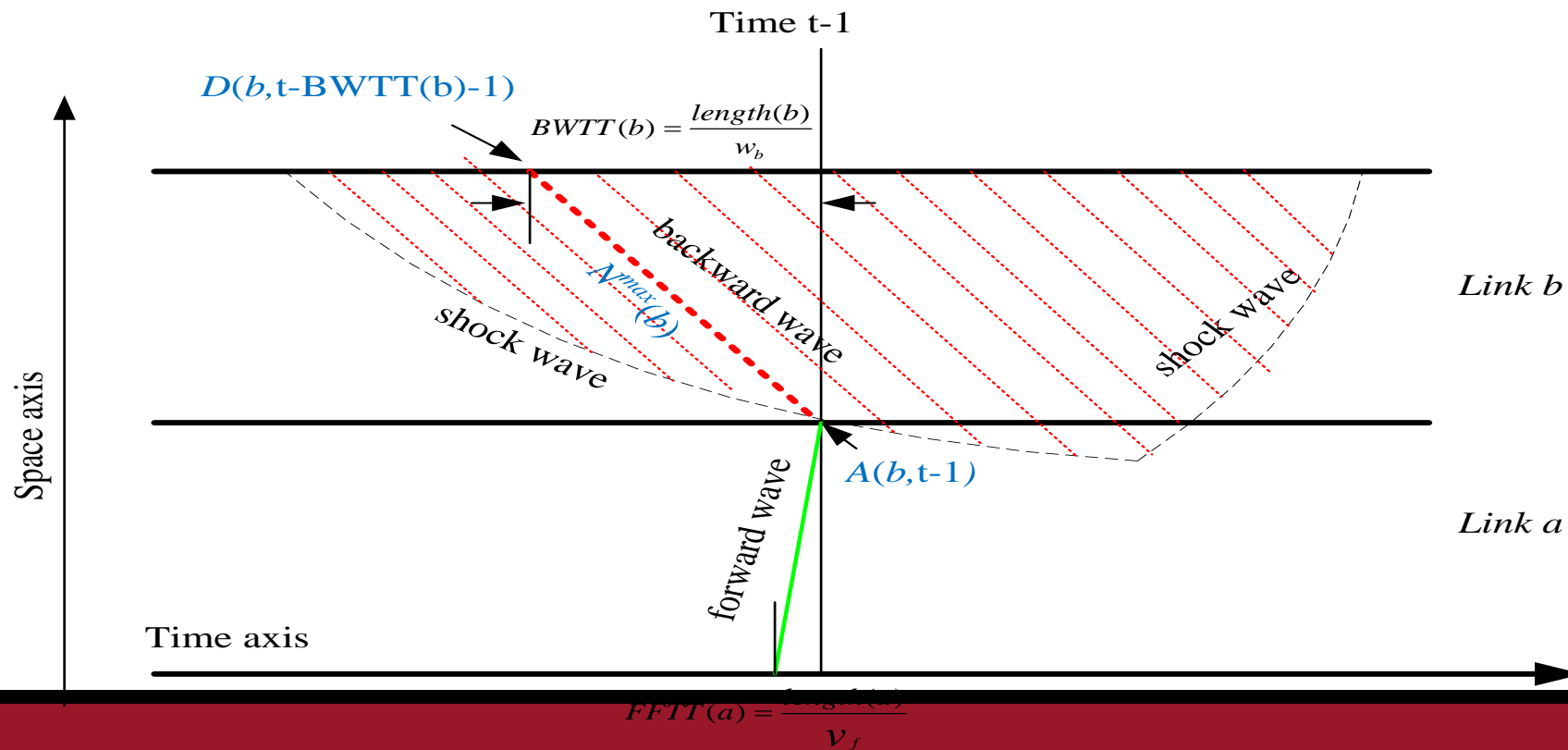
- How to locate point sensors on a traffic segment?
- How to locate Bluetooth reader locations?
- How much AVI/GPS market penetration rate is sufficient?



# Space-state-time network: $N(x,t)$ : state $N$ as cumulative flow counts

Dr. Newell's three-detector model provides a unified framework

- $N(t,x) = \text{Min} \{ N_{\text{upstream}}(t - \text{BWTT}) + K_{\text{jam}} * \text{distance}, N_{\text{downstream}}(t - \text{FFTT}) \}$

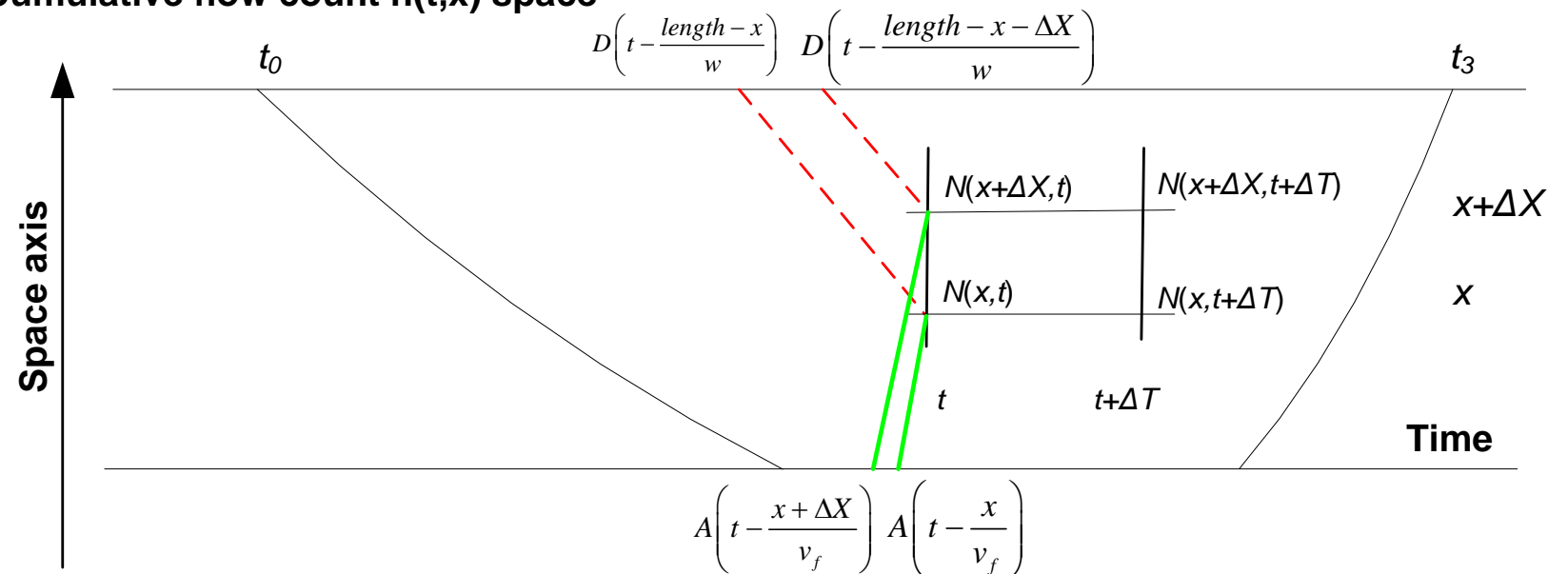


# 1: From Point Sensor Data to Boundary N-curves

Cell density and flow are all functions of cumulative flow counts

$$k(t, x) = \frac{n(t, x) - n(t, x + \Delta x)}{\Delta x}$$

Cumulative flow count  $n(t, x)$  space

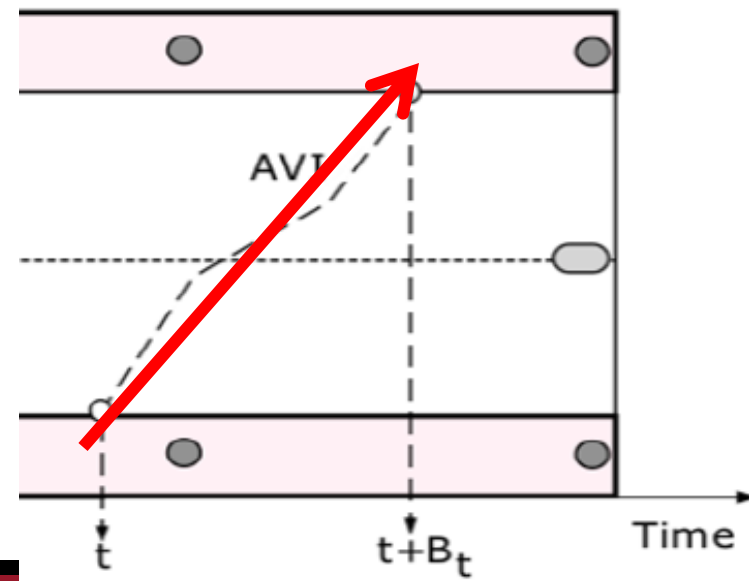


## 2: From Bluetooth Travel Time to Boundary N-curves

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Downstream and upstream N-Curves between two time stamps are connected

$$n_u(t) = n_d(t + B_t)$$

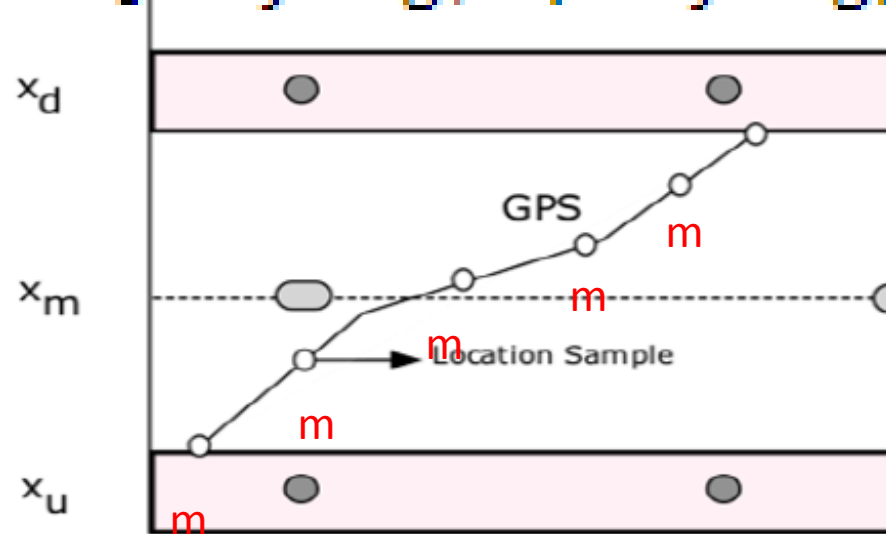




### 3: From to GPS Trajectory Data to Boundary N-curves

Under FIFO conditions, GPS probe vehicle keeps the same N-Curve number (say m)

$$m = n[t + j'' \Delta g, x(t + j'' \Delta g)]$$



# Stochastic 3-detector Model

---

All sensors have errors → error propagation

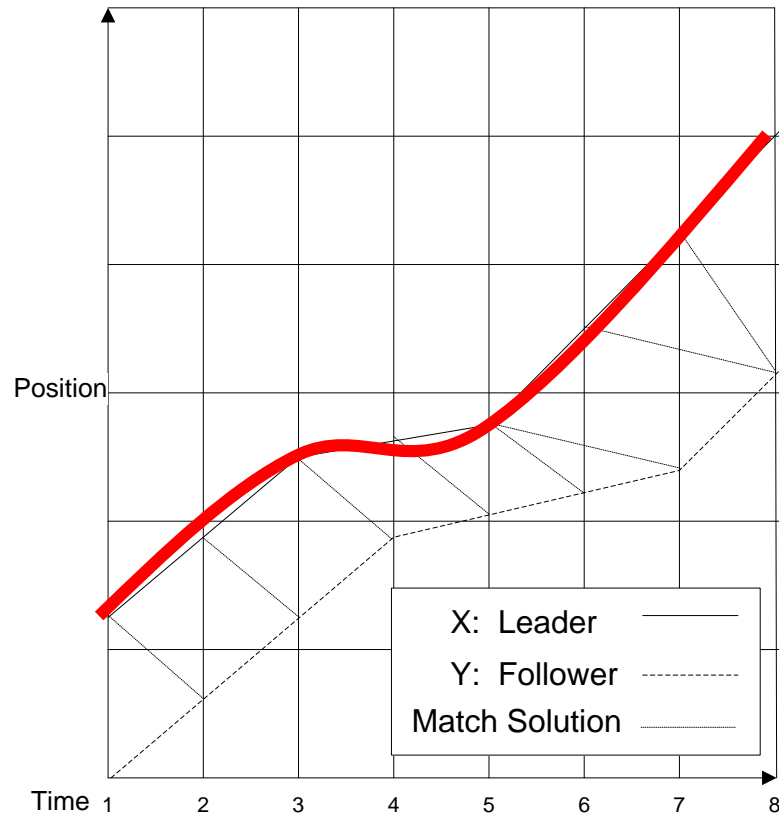
$$\text{Min } \{N_{\text{upstream}}(*) + e_u, N_{\text{downstream}}(*) + e_d\}$$

Deng, W. Lei H. ,Zhou, X. (2013) Freeway Traffic State Estimation and Uncertainty Quantification based on Heterogeneous Data Sources: A Three Detector Approach. Transportation Research Part B. 57, 132-157

From single segment to corridor

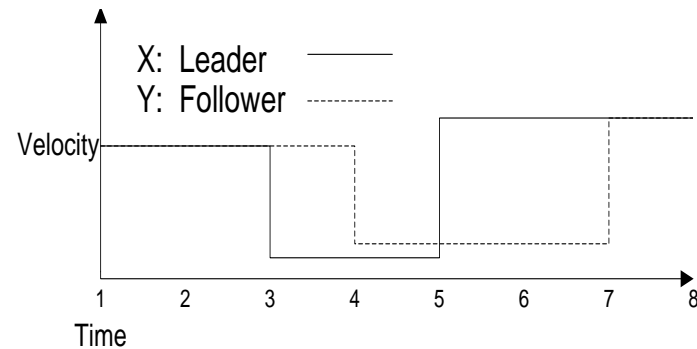
Lei, H., & Zhou, X. (2014). Linear Programming Model for Estimating High-Resolution Freeway Traffic States from Vehicle Identification and Location Data. Transportation Research Record: Journal of the Transportation Research Board, 2421, 151-160.

Application 2: State-space-time path → State as speed for trajectory  
Use Dynamic Time Warping (DTW) to Estimate dynamic car following model



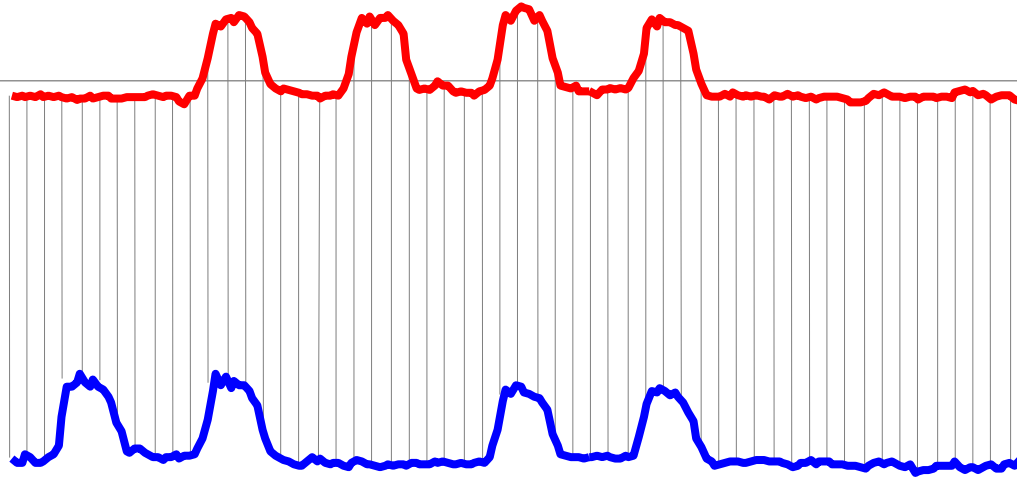
(A) Vehicle Trajectories with DTW Match Solution

- Matches points by measure of similarity



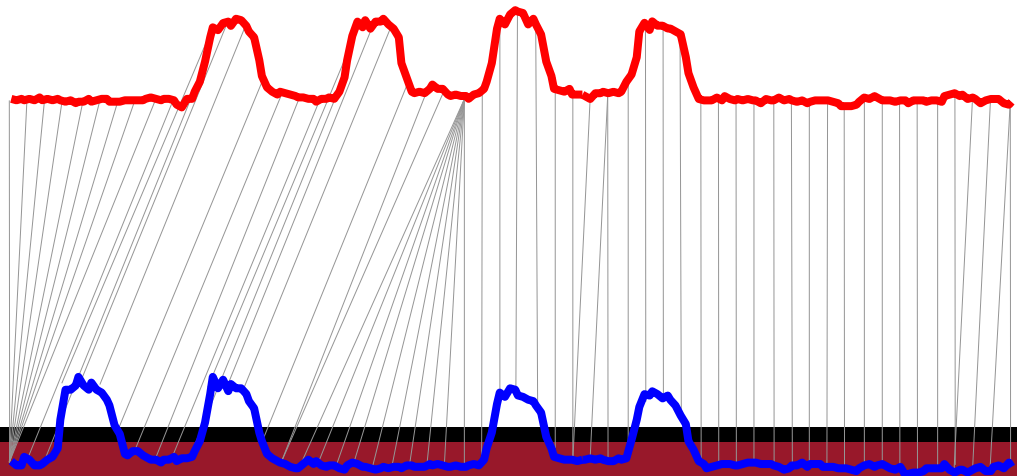
(B) Vehicle Velocity Time Series

# Euclidean Vs Dynamic Time Warping



## Euclidean Distance

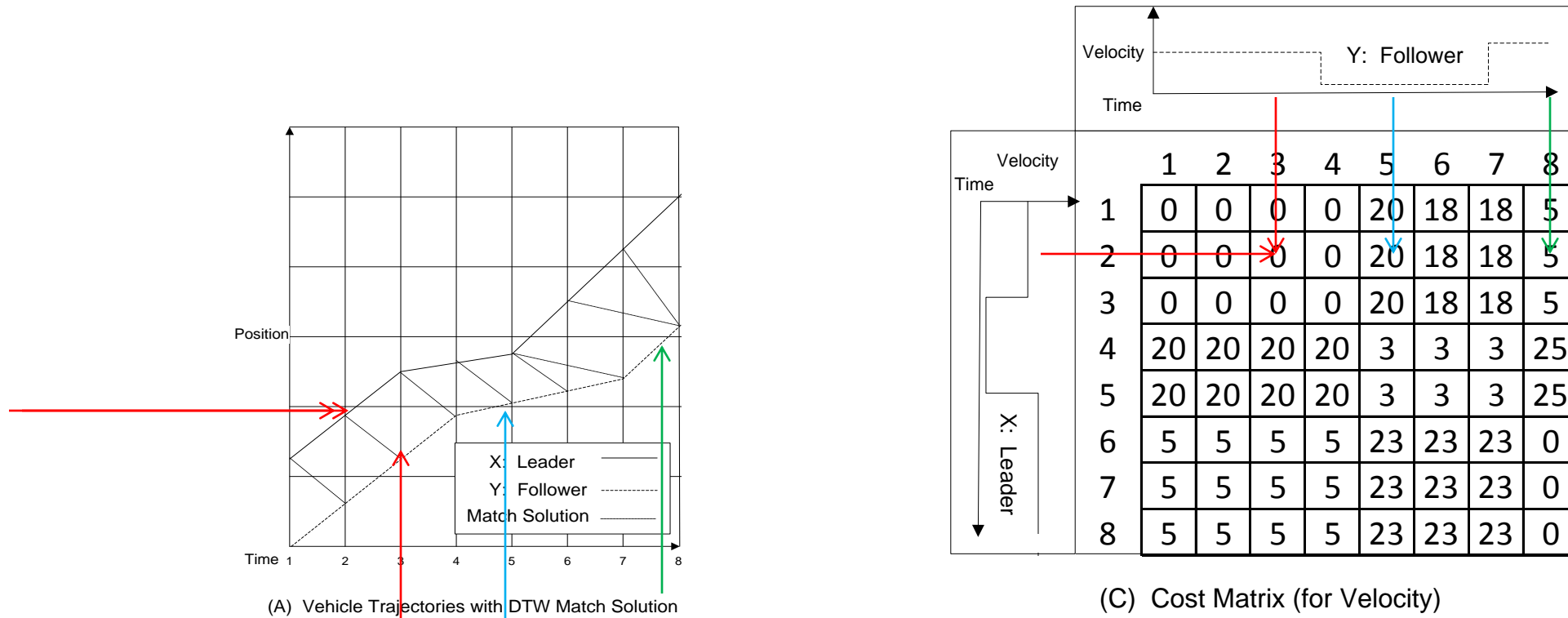
*Sequences are aligned "one to one".*



## "Warped" Time Axis

*Nonlinear alignments are possible.*

# Construct Cost Matrix for Traffic Trajectory Matching



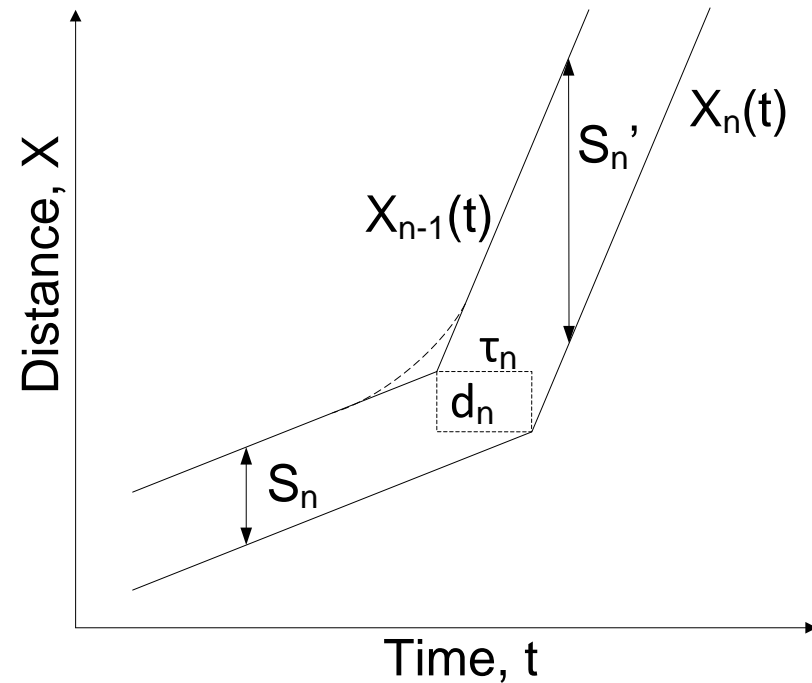
$$C(X_i, Y_j) = |X_i - Y_j| + \alpha |j - i - \tau^-| + \beta |x_L(i) - x_F(j) - d^-| + r \left| \frac{x_L(i) - x_F(j)}{j - i} - w^- \right|$$

# Application to Newell's Model

Follower separated by leader by reaction time and critical jam spacing

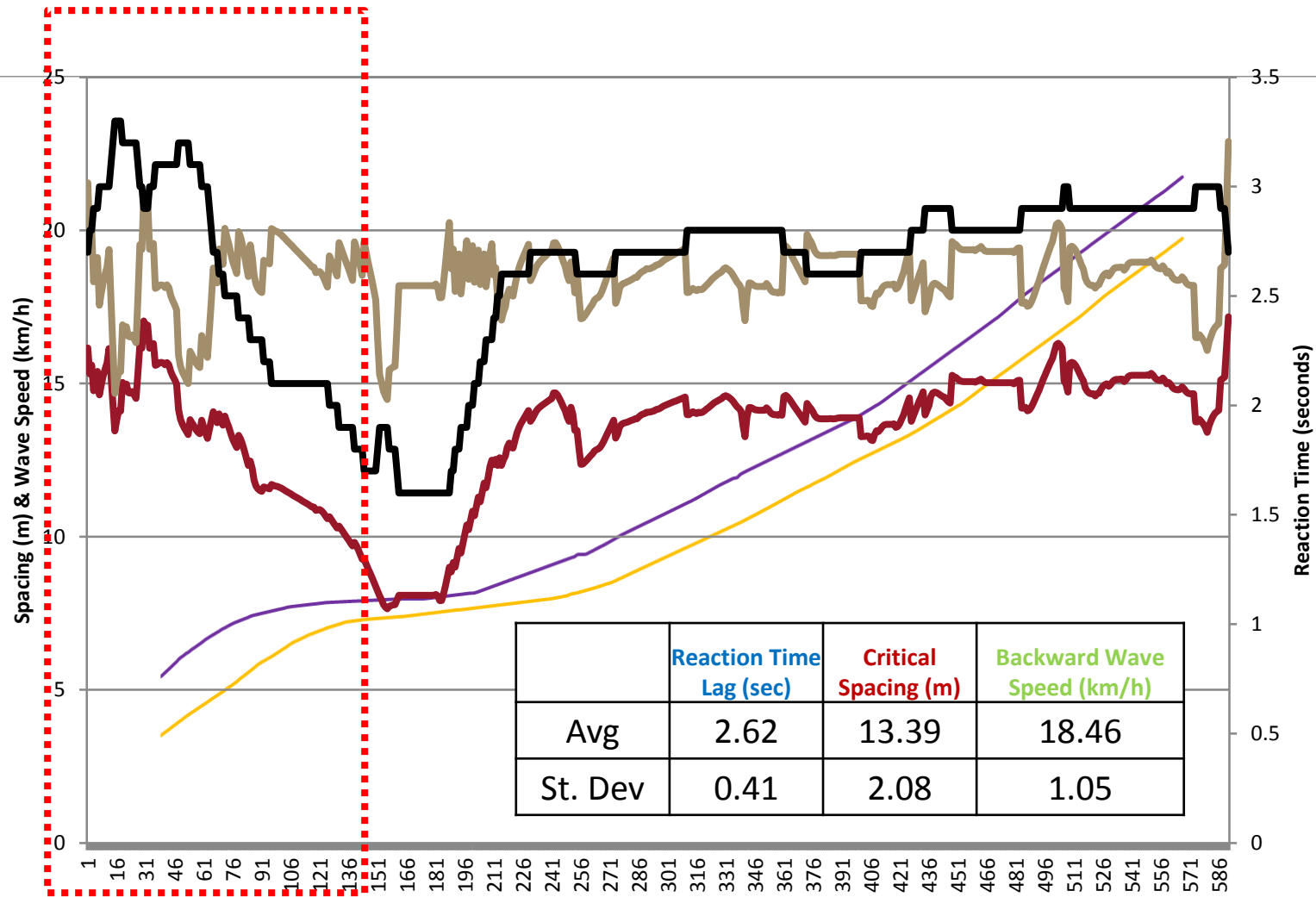
Algorithm finds optimal  $\tau_n$  (time lag) for best velocity match

- Calculate  $d_n$  for all time steps along the trajectory



$$x_n(t + \tau_n) = x_{n-1}(t) - d_n$$

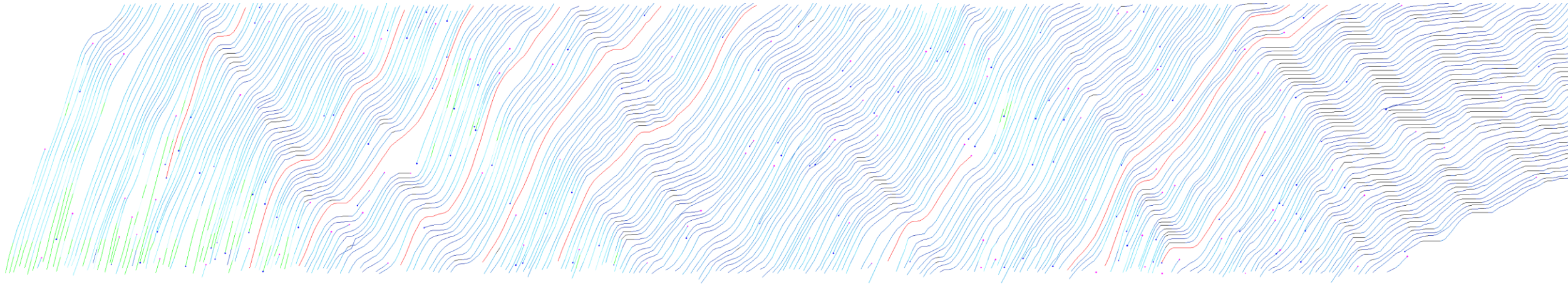
# Calibrated Parameters: Car 1737



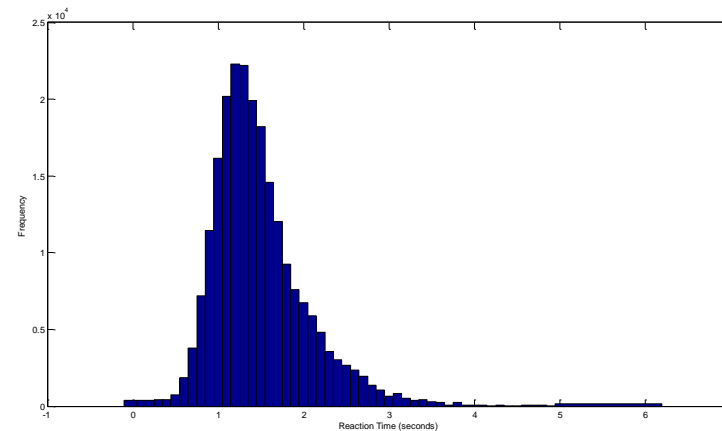
Critical Jam Spacing    Backward Wave Speed    Reaction Time



# NGSIM Data: I-80 Lane 4



Taylor, J., Zhou, X. Roupail, N., Porter, R.J. (2015) Method for investigating intradriver heterogeneity using vehicle trajectory data: A Dynamic Time Warping approach. Transportation Research Part B, 73, 59-80

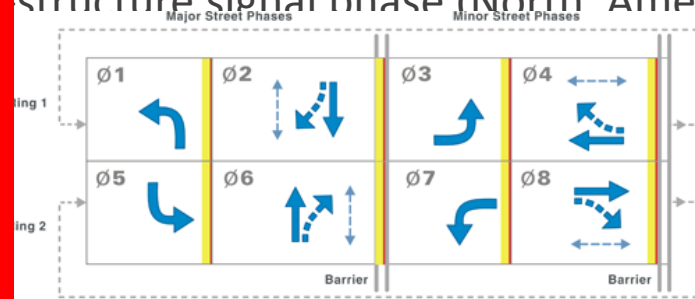


Reaction Time Distribution

# Application 3: Phase-time network for Signal Optimization

## Traffic signal phasing sequence representation

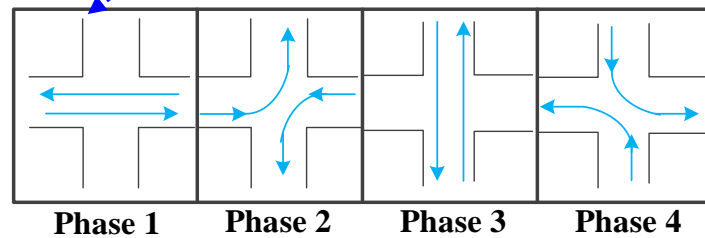
- NEMA ring structure signal phase (North America)



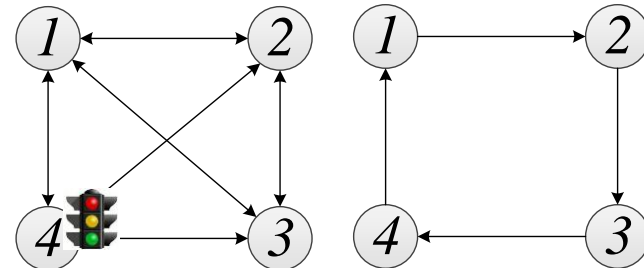
P. Li., P. Mirchandani, X. Zhou, Solving Simultaneous Route Guidance and Traffic Signal Optimization Problem Using Coupled Space-time and Phase-time Networks. Transportation Research Part B. 81, 103-130.

- Movement-based signal phase (same control flexibility, fewer variables)

Source: Traffic Signal Timing Manual: FHWA



Signal Phases



Flexible Phasing Sequence

Cyclic Phasing Sequence

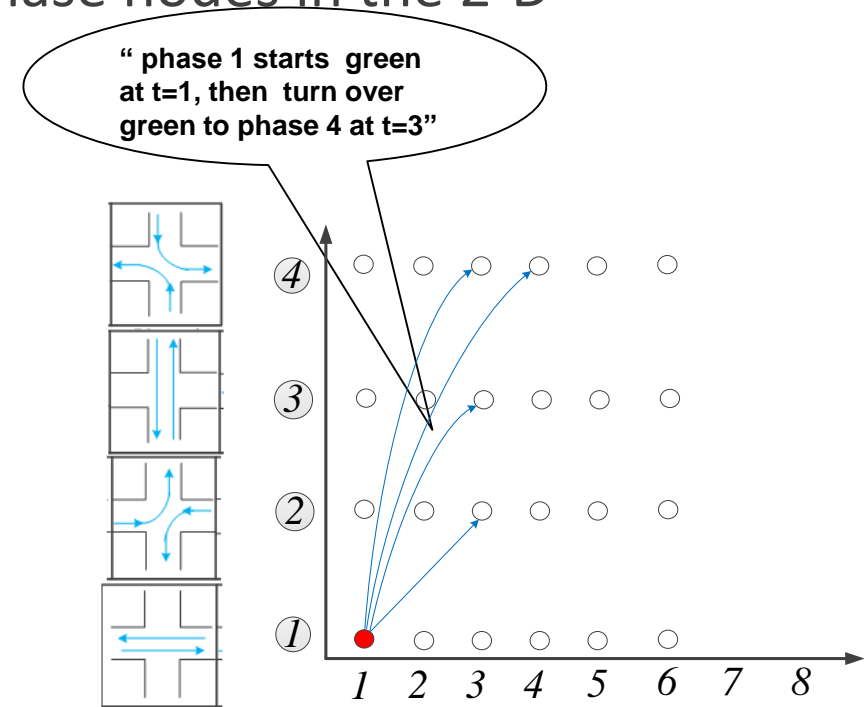
# Phase-time network for Signal Optimization

- A signal timing plan is composed of:
  - Phasing sequence
  - Phase duration
- Signal timings can also be represented with a series of phase nodes in the 2-D phase-time network

- **Similar structure with a space-time network!**

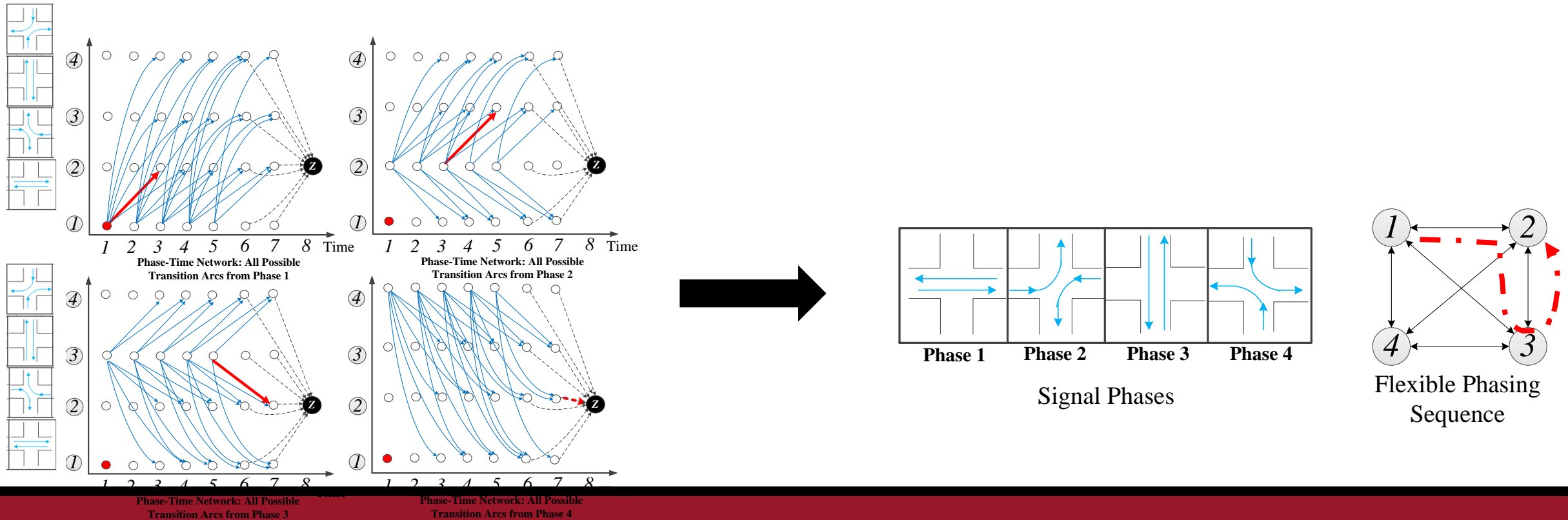
- Solution is provided like:

$$W_{(m,k,\tau,h)} = \begin{cases} 1, & \text{if signal phase } m \text{ is green at } \tau \\ & \text{and then turns green to signal phase } k \text{ at } h \\ 0, & \text{otherwise} \end{cases}$$



# Optimize traffic signal in phase-time network

- Each green phase will generate cost on other phases (e.g., delays)
- Find a least-cost path from origin (starting phase) to destination (end of horizon)

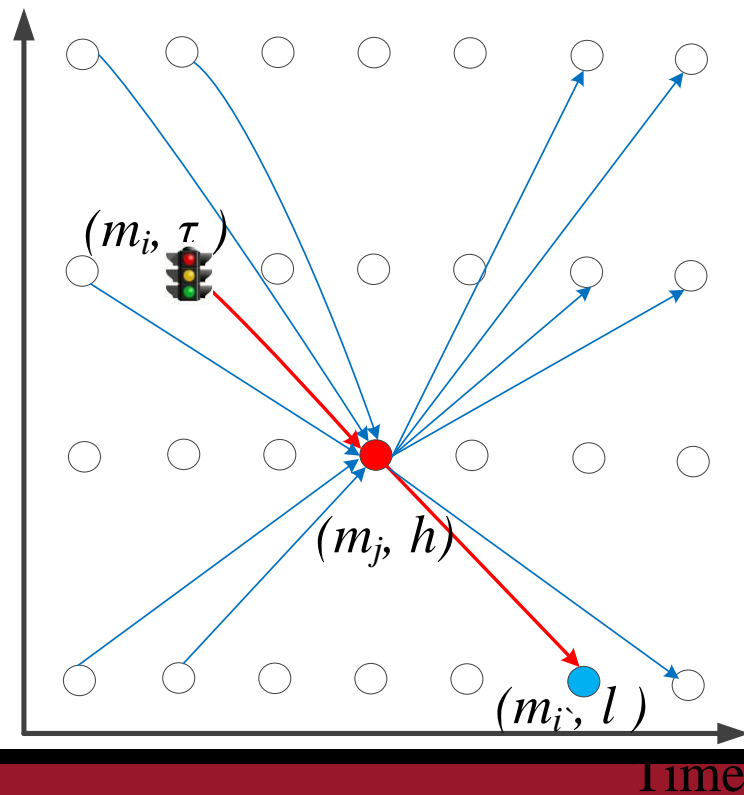


# Intersection control constraints

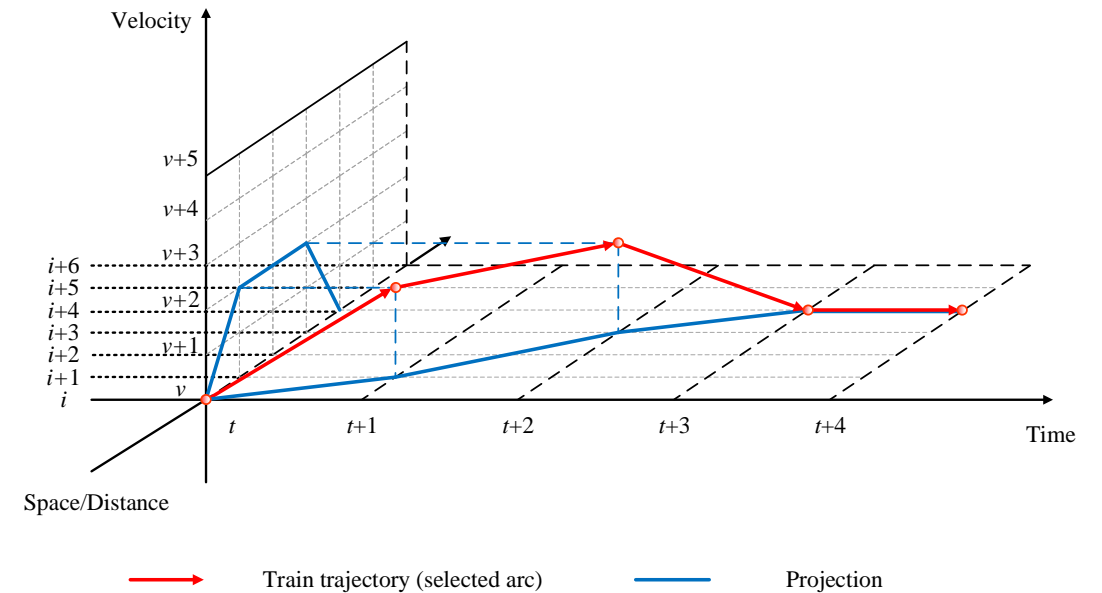
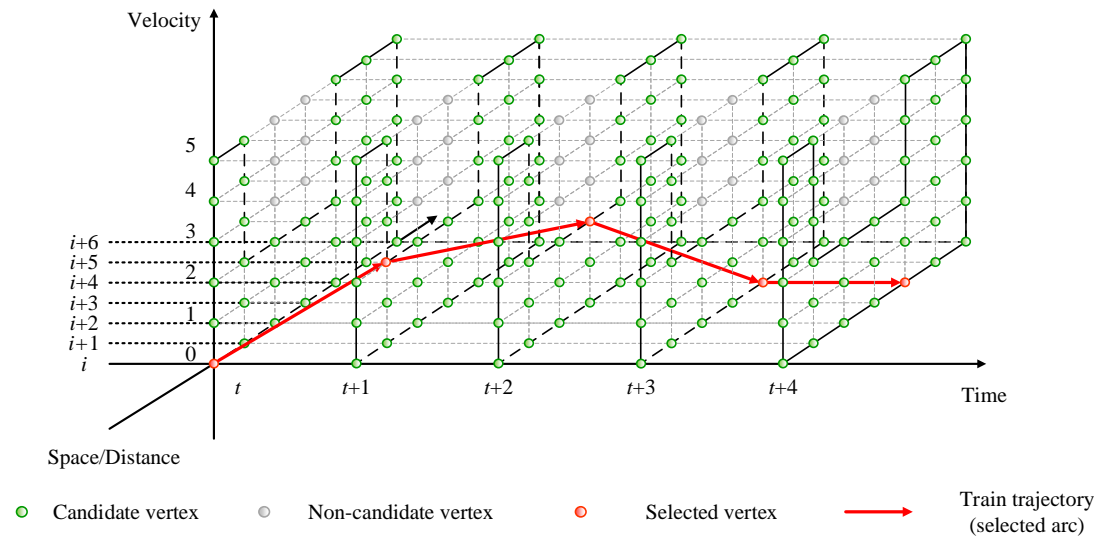
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## Mutual exclusiveness of signal phases

- Any signal phase has one and only one predecessor phase and successor phase at one time.



# Application 4: Speed-space-time network for high-speed train timetabling and speed control



# Conclusions

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Present a state-space-time (SST) based modeling framework

By adding additional state dimensions, we **prebuild many complex constraints** into a multi-dimensional network

- **Computationally efficient solution** algorithm: forward dynamic programming + Lagrangian relaxation
- Wide range of applications
- (i) how to estimate macroscopic and microscopic freeway traffic states from heterogeneous measurements,
- (ii) how to optimize transportation systems and ride-sharing services involving vehicular routing decisions with pickup and delivery time windows (VRPPDTW).

## Challenges

- Selecting state is an art...
- How to overcome curse of dimensionality
- Dynamics and uncertainty (demand/supply)
- Smart search space reduction and metaheuristics algorithms for real-time applications