Decentralized Optimal Control for Online Coordination of Connected and Automated Vehicles

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  - LDRD Award
Why we should be concerned about transportation?

- Two-thirds of the oil used around the world currently goes to power transportation vehicles, of which half goes to passenger cars and light trucks.
- In emerging Asia alone, the total number of vehicles is expected to rise from 55 million in 2003 to 420 million in 2030.
Traffic jam in China
What is the impact on petroleum displacement?

- Until 1989 the US produced enough petroleum to meet the needs of the transportation sector although petroleum needs of all other sectors were not met.

What is the impact on GHG emissions?

- The transportation sector accounted for about 28% of total US greenhouse gas (GHG) emissions in 2006 making it the second behind electricity generation (34%).

Source: Transportation for America
Many stakeholders see the benefits of connected and automated vehicles (CAVs) and have started to develop business cases for their respective domains, including the automotive and insurance industries, government, and service providers.
The focus has been on safety[1]

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[1] Video: courtesy of UMTRI.
Can we use CAVs to improve efficiency?
Example of an "emergent pattern" through coordination
Can we coordinate vehicles...?
Can we then scale up...?
1939 New York World’s Fair

- Norman Bel Geddes designed an exhibit at the 1939 New York World’s Fair, Futurama. The idea was to present a possible model of the world 20 years into the future (1959-1960).
1939 New York World’s Fair
General Motors future car

- In 1956, General Motors featured a film that looked into the far 20 year distant future 1976.
Outline

- Coordination of CAVs
  - Intersections
  - Merging roadways
- Centralized problem formulation
- Decentralized problem formulation and solution approach
- Illustrative examples
- Concluding remarks
Transportation network
Step 1: Address specific transportation segments
Step 2: Scale up to the whole network
Vehicle coordination: state of the art

Various approaches have been proposed in the literature\textsuperscript{[2]}.

**Centralized:**
- There is at least one task in the system that is globally decided for all vehicles by a single central controller.

**Decentralized:**
- Each vehicle obtains information from other vehicles and roadside infrastructure to optimize specific performance criteria (e.g., efficiency, travel time).

Vehicle coordination in intersections
Optimal control problem
Local information and observation

- **Assumption:** Each vehicle $i$ has proximity sensors and can observe and/or estimate local information that can be shared with other vehicles.
The model

\[ \mathcal{N}(t) = \{1, \ldots, N(t)\} \]

\[ \dot{p}_i = v_i(t) \]
\[ \dot{v}_i = u_i(t) \]  \hspace{1cm} (1)

where \( p_i(t) \in \mathcal{P}_i \), \( v_i(t) \in \mathcal{V}_i \), and \( u_i(t) \in \mathcal{U}_i \), and \( t \in \mathbb{R}^+ \).

\( \mathcal{P}_i \), \( \mathcal{V}_i \) and \( \mathcal{U}_i \), \( i \in \mathcal{N}(t) \), are complete and totally bounded subsets of \( \mathbb{R} \).
The model

\[ x_i(t) = [ p_i(t) \ v_i(t) ]^T, \text{ with } x_i^0 = [ 0 \ v_i^0 ]^T \]

\[ \dot{x}_i = f(t, x_i, u_i), \quad x_i(t_i^0) = x_i^0, \quad (2) \]

\( \forall (t_i^0, x_i^0) \) and \( \forall u(t) \in U_i \), (2) has a unique solution \( x(t) \) on some \([t_i^0, t_i^f]\).

- \( f \) is continuous in \( u \) and continuously differentiable in \( x \).
- \( f_x \), is continuous in \( u \).
- \( u(t) \) is continuous in \( t \).
The model

- **Control and state constraints**

  \[
  u_{\text{min}} \leq u_i(t) \leq u_{\text{max}}, \quad \text{and}
  \]
  \[
  0 \leq v_{\text{min}} \leq v_i(t) \leq v_{\text{max}}, \quad \forall t \in [t_i^0, t_i^f],
  \]

- **Assumption:** The vehicle speed inside the merging zone is constant.
Feasibility sets

**Definition**

**Rear-end control interval**

\[ R_i \triangleq \left\{ u_i(t) \in [u_{\text{min}}, u_{\text{max}}] \mid s_i(t) = p_k(t) - p_i(t) \geq \delta, \right. \]
\[ \left. v_i(t) \in [v_{\text{min}}, v_{\text{max}}], \forall i, k \in \mathcal{N}, i \neq k, |\mathcal{N}| > 1, \forall t \in [t_i^0, t_i^f] \right\}. \]  
(4)

**Definition**

**Lateral zone set**

\[ M_i \triangleq \left\{ p_i(t) \mid p_i(t) \in [L, L + S], \forall i \in \mathcal{N}, |\mathcal{N}| > 1, \forall t \in [t_i^0, t_i^f] \right\}. \]  
(5)
Feasibility sets

Definition

Critical control interval

\[ \Gamma_i \triangleq \left\{ u_i(t) \in R_i \mid p_i(t) \in M_i, \forall i \in \mathcal{N}, \right. \]
\[ \left. |\mathcal{N}| > 1, \forall t \in [t_i^0, t_i^f] \right\}. \tag{6} \]

- Lateral collision constraint

\[ \Gamma_i \cap \Gamma_j = \emptyset, \forall t \in [t_i^0, t_i^f]. \tag{7} \]
Centralized problem formulation

Centralized control problem

\[
\min_{u_i} \left( \frac{1}{2} \sum_{i=1}^{N(t)} \int_{t^0_i}^{t^f_i} u_i^2 \, dt + \sum_{i=2}^{N(t)} (t^m_{i-1} - t^m_i) \right)
\]  \hspace{1cm} (8)

Subject to:  
vehicle dynamics 
state and control bounds 
rear-end collision 
lateral collision,

where \( t^m_i \) is the time for each vehicle \( i \) that enters the merging zone.
Centralized problem formulation
Centralized problem formulation

![Diagram showing control and merging zones with time and distance axes, illustrating vehicle trajectories.](image-url)
Centralized problem formulation
Centralized problem formulation

\[ t_{i-1}^m = t_i^m \]

\[ t_{i-1}^f = t_i^f \]

Distance

Control Zone

Merging Zone

Vehicle \( i-1 \)

Vehicle \( i \)

L

S

Time
Challenges

- The **computational burden** associated with deriving the optimal solution in (8).
- **Online** implementation requirement.
- **Problem dimensionality** as the number of vehicles inside the control zone increases.
CAV coordination: decentralized control
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CAV coordination: decentralized control
Coordinator’s identity

<table>
<thead>
<tr>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>The <strong>unique identity</strong> that the coordinator assigns to each vehicle ( i \in \mathcal{N} ), at the time ( t ) when the vehicle arrives at <strong>control zone</strong>, is a pair ((i, j)), where</td>
</tr>
</tbody>
</table>

- \( i = N_z(t) + 1 \) is an integer representing the **location** of the vehicle in the FIFO queue \( \mathcal{N}(t) \) and
- \( j \in \{1, \ldots, 4\} \) is an integer based on a **one-to-one mapping** from \( \{\mathcal{C}(t), \mathcal{R}(t), \mathcal{O}(t), \mathcal{L}(t)\} \) onto \( \{1, \ldots, 4\} \). |
Subsets of $\mathcal{N}(t)$

- $C(t)$ contains $i-1$ and $i$ when traveling on different roads with destinations that can cause collision, e.g., #2 and #3.
Subsets of $\mathcal{N}(t)$

- $\mathcal{R}(t)$ contains $i-1$ and $i$ when traveling on the same road, towards the same direction but on different lanes, e.g., #4 and #5.
Subsets of $\mathcal{N}(t)$

- $\mathcal{O}(t)$ contains $i-1$ and $i$ when traveling on the same road and opposite destinations, e.g., #3 and #4.
Subsets of $\mathcal{N}(t)$

- $\mathcal{L}(t)$ contains $i-1$ and $i$ when traveling on the same road and lane e.g., #5 and #6.
Local observation set

Definition

Local observation set

\[ Y_i(t) \triangleq \left\{ p_i(t), v_i(t), Q_j, j = 1, \ldots, 4, s_i(t), t_i^f \right\}, \quad \forall t \in [t_i^0, t_i^f], \tag{9} \]

where \( Q_j \in \{ R(t), L(t), C(t), O(t) \} \), is the subset assigned to vehicle \( i \) by the coordinator.
Time to exit the merging zone

- $t_i^f$ is computed as follows:

\[
    t_i^f = \begin{cases} 
    t_1^f, & \text{if } i = 1, \\
    t_{i-1}^f, & \text{if } i - 1 \in R(t) \text{ (or } O(t)), \\
    t_{i-1}^f + \frac{\delta}{v_i(t_{i-1}^f)}, & \text{if } i - 1 \in L(t), \\
    t_{i-1}^f + \frac{S}{v_i(t_{i-1}^f)}, & \text{if } i - 1 \in C(t). 
    \end{cases}
\]

(10)
Decentralized problem formulation

Lemma

The decentralized communication structure aims each vehicle $i$ to solve an optimal problem for $t \in [t_i^0, t_i^f]$ the solution of which depends only on the solution of the vehicle $i-1$.

It is shown$^3$ by induction that for each vehicle $i$, $t_i^f$ depends only on $t_{i-1}^f$.

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Sketch of proof with intuition
Sketch of proof with intuition
Sketch of proof with intuition
Decentralized problem formulation

- We formulate $N(t)$ decentralized tractable problems that can be solved online.

$$\min_{u_i} \frac{1}{2} \int_{t_i}^{t_{f_i}} u_i^2 dt$$

Subject to: vehicle dynamics

state and control bounds,

where $t_{f_i}$ is given by (10), $\forall i \in \mathcal{N}$. 
Analytical solution

\[ H_i(t, x(t), u(t)) = L_i(t, x(t), u(t)) + \lambda^T \cdot f_i(t, x(t), u(t)) + \mu^T \cdot g_i(t, x(t), u(t)), \] (12)

where

\[ g_i(t, x(t), u(t)) = \begin{cases} 
  u_i(t) - u_{max} \leq 0, \\
  u_{min} - u_i(t) \leq 0, \\
  v_i(t) - v_{max} \leq 0, \\
  v_{min} - v_i(t) \leq 0, \\
  p_i(t) - p_{i-1}(t) + \delta \leq 0, i = 1, 2, \ldots, i \in \mathcal{N}. 
\] (13)

Optimal control input

\[ u_i^*(t, p_i(t), v_i(t)) = a_i(t, p_i(t), v_i(t)) \cdot t + b_i(t, p_i(t), v_i(t)). \] (13)
Comparison between centralized and decentralized solution

Theorem

The optimal solution of the centralized problem (8) preserves the queue $N$.

The analysis of the centralized solution shows\textsuperscript{[4]} that the queue of the vehicles is preserved.

Corollary

The solution of the decentralized control problem is equivalent to the solution of the centralized problem.

Vehicle coordination with decentralized control
Vehicle coordination with decentralized control
Impact on fuel consumption and travel time

- Fuel consumption (55% improvement) and travel time (17% improvement)
Vehicle coordination with decentralized control\textsuperscript{[5]}

Vehicle coordination with decentralized control
Impact on fuel consumption and travel time

- Fuel consumption (65% improvement) and travel time (6% improvement)
Vehicle coordination - VISSIM

- With traffic lights and with decentralized control of CAVs
Impact on fuel consumption and travel time

- For **470 vehicles** that crossed the intersection
- Fuel consumption (52% improvement) and travel time (21% improvement)
Decentralized stochastic control: intersection

- Co-investigator: Professor Christos Cassandras (BU)
With traffic lights and with decentralized control of CAVs
Impact on fuel consumption and travel time

- For **448 vehicles** that crossed the intersection
- Fuel consumption (42.4% improvement) and travel time (37.3% improvement)
Automated intersection: futuristic movie\textsuperscript{[7]}

\textsuperscript{7}Courtesy of Fernando Livschitz
Traffic flow merging with vehicle coordination\cite{8,9}

- Coordination of 30 vehicles (15 vehicles on each road)

\begin{itemize}
  \item Control Zone
  \item Merging Zone
  \item L
  \item S
  \item in
  \item out
\end{itemize}

\begin{itemize}
  \item \textit{Hierarchical crossing sequence:}
  \item $x_1 > x_2 > x_3 > x_4 > x_5$
\end{itemize}

\begin{itemize}
\end{itemize}
Traffic flow merging **without** vehicle coordination
Traffic flow merging with vehicle coordination
Traffic flow merging with vehicle coordination

- Fuel consumption (50% improvement) and travel time (7% improvement)
Traffic flow merging with vehicle coordination and different speed
Desired traffic flow "emergence" with CAVs[^10]

Penetration of CAVs in the transportation network

11 Toolbox for Urban Mobility Simulation (TUMS)
How about connected vehicles with a driver...?

- Each individual driving style is different.
- Variation in fuel economy can be as much as 30%\(^{[12]}\).

Driver’s feedback system

- We identified the driving-style factors that have a major impact on fuel economy and developed an optimization framework to optimize individual driving styles with respect to these factors\[^{13}\],\[^{14}\].


Technology can be integrated in an app

Concluding remarks

- The results presented here address the problem of coordinating online a continuous flow of CAVs crossing an intersection.
- We presented a decentralized control framework and a closed-form solution that yields for each vehicle the optimal acceleration/deceleration at any time aimed at minimizing fuel consumption.
- What is next? In this "new world" of massive amount of data how we can improve the efficiency in mega cities?
Future research direction
What else we should be expecting to see...?\textsuperscript{[16]}

\textsuperscript{16}Courtesy of Fernando Livschitz
Connected cities

- Connected people, vehicles and infrastructure
Complex urban systems

- Develop system-wide optimal solutions using scalable data and informatics
System-wide optimal solutions in complex urban systems[17]

17 Synthesis of various videos: courtesy of Siemens.
Thank you for your Attention!

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