institute for pure & applied mathematics

Workshop I: Mathematical Foundations of Traffic

Capacity drop at freeway merges: Analytical integration of microscopic behaviors

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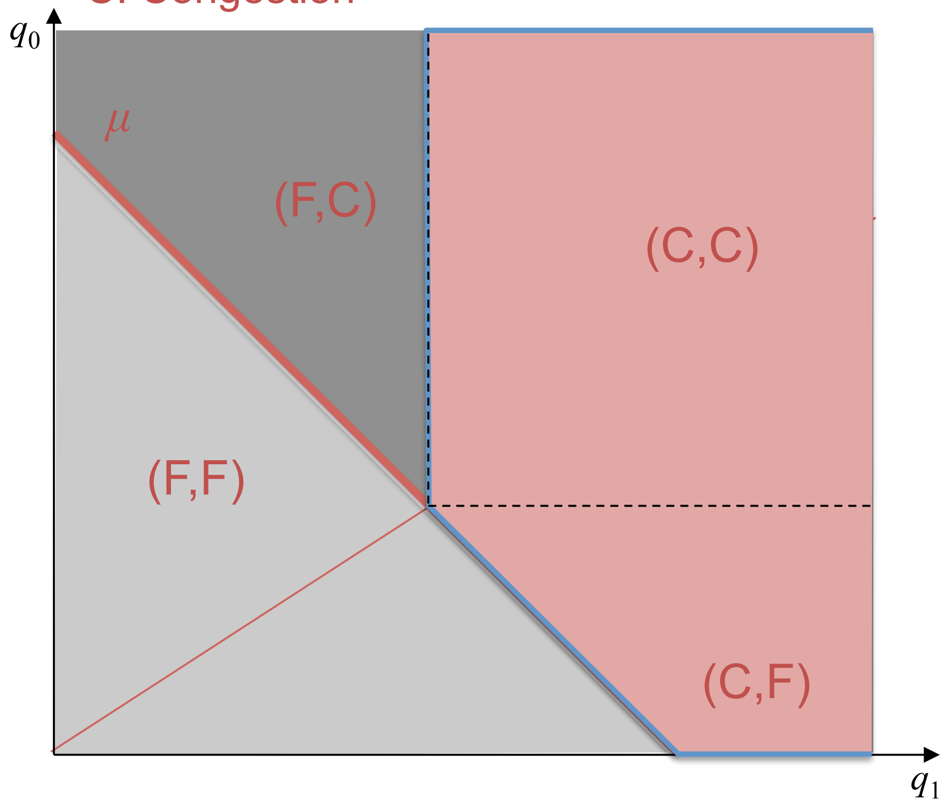
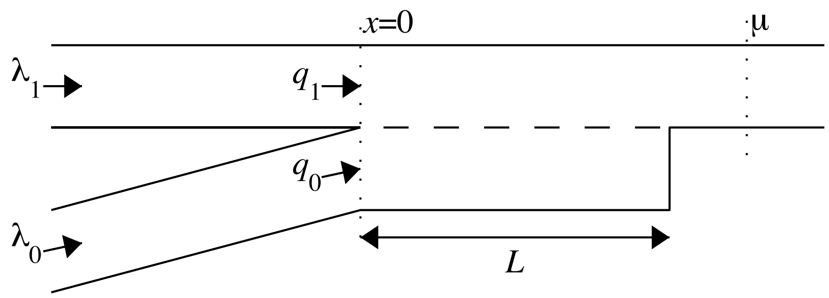
Newell-Daganzo's merge model





- Strengths
  - Simple
  - Good agreement with experimental findings
- Weakness
  - Not relevant for active bottleneck

References

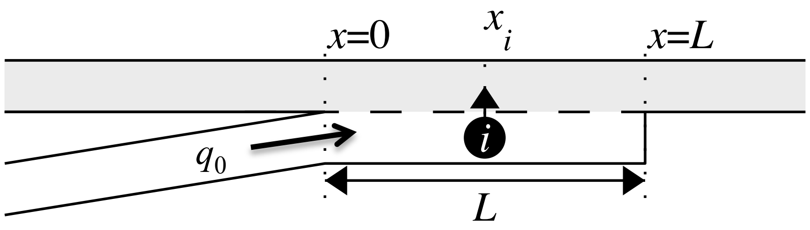
- Leclercq, L., Laval, J.A., Chiabaut, N., 2011. Capacity Drops at Merges: an endogenous model, *Transportation Research Part B,* 45(9):1302-1313.
- Leclercq, L., Knoop, V., Marczak, F., Hoogendoorn, S.P. Capacity Drops at Merges: New Analytical Investigations, *Transportation Research part C*, *in press*.
- Leclercq, L., Marczak, F., Knoop, V., Hoogendoorn, S.P. Capacity drops at merges: analytical expressions for multilane freeways, *Submitted to Transportation Research Records*.

Outline

- Capacity drop on the right lane
  - Presentation of the analytical framework
  - Numerical results
- Capacity drop for multilane freeways
  - Extension of the analytical framework
  - Numerical results and sensitivity analysis
  - Comparison with traffic simulation
  - Experimental validation
- Conclusion

ANALYTICAL FRAMEWORK:
ONE SINGLE LANE ON FREEWAY

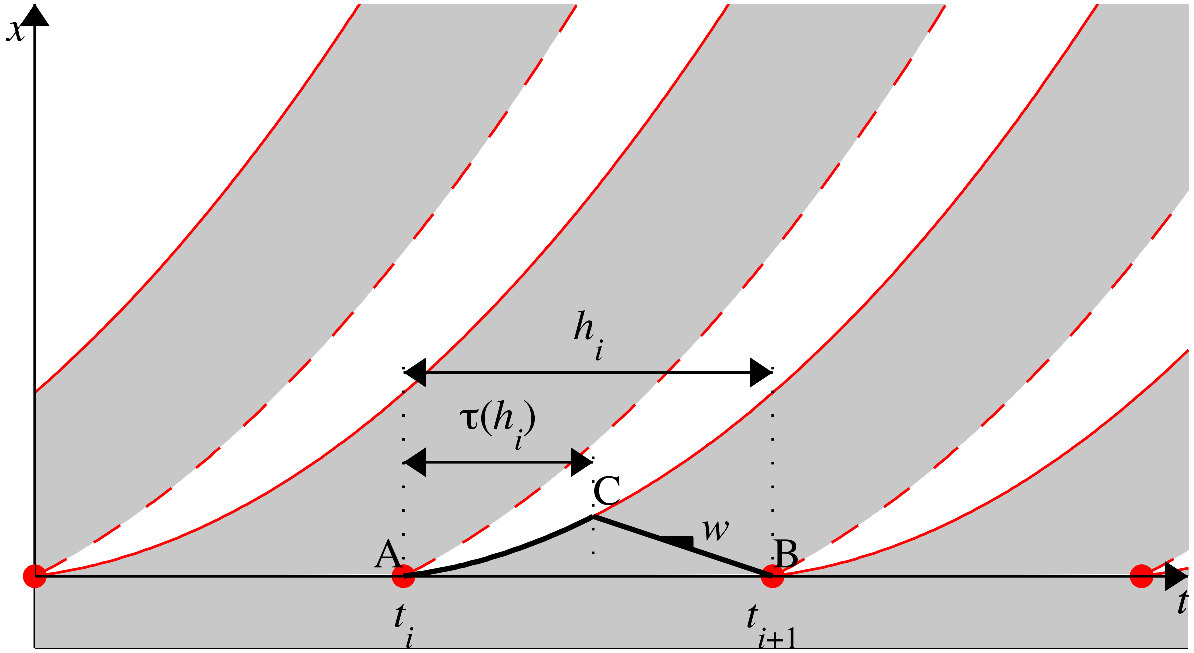
Hypothesis



- Network:
  - The main road has only one lane
  - The inserting flow is equal to q_0
  - LWR model and triangular fundamental diagram (free-flow speed u wave speed w and jam density κ)

- Insertions:
  - Time between two insertions: $H(h_0=1/q_0, s_H)$
  - Inserting positions are uniformly distributed between 0 and L
  - Vehicles insert at speed v_0 with an acceleration $A(a,s_A)$ and a jam density $K(\kappa,s_K)$
  - Inserting vehicles behave as moving bottlenecks on target lane

Case 1: $L=0$ and $s_H>0$



The effective capacity C is given by :

$$C = \sum_{i=1}^{n} \left(N_{i+1} - N_i\right) / \sum_{i=1}^{n} h_i \quad \text{with} \quad n \rightarrow +\infty$$

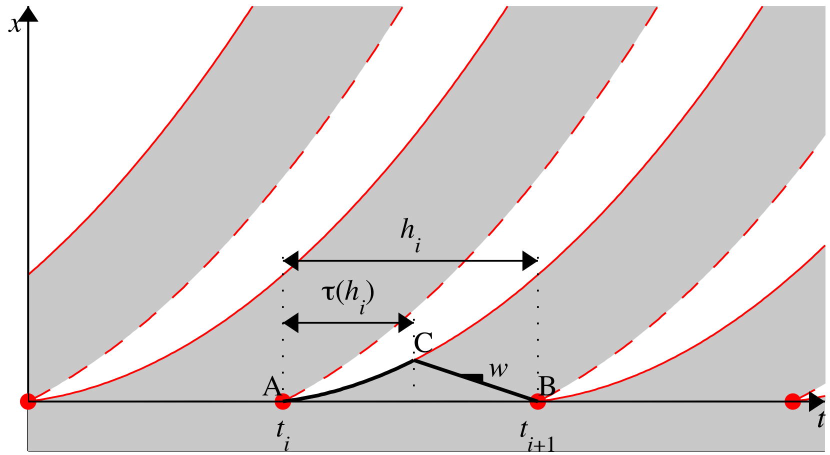
Simplest homogeneous case
(L=0, s_H>0, s_A=0, s_K=0)

- Effective capacity

$$C = \sum_{i=1}^{n} w\kappa\left(\tau\left(h_i\right) - h_i\right) / \sum_{i=1}^{n} h_i$$

$$C = w\kappa\left(1 - \sum_{i=1}^{n} \tau(h_i) / \sum_{i=1}^{n} h_i\right)$$

$$\tau(h_i) = -\frac{w + v_0}{a} + \frac{1}{a}\sqrt{(w + v_0)^2 + 2awh_i}$$



- Law of large numbers

$$(1/n)\sum_{i=1}^{n} h_i \to E(h_i) \text{ and } (1/n)\sum_{i=1}^{n} \tau(h_i) \to E\left(\tau(h_i)\right)$$

- Second order Taylor approximation

$$E\left(\tau(h_i)\right) \simeq \tau\left(E\left(h_i\right)\right) + \frac{1}{2} s^2 \frac{\partial^2 \tau}{\partial h_i^2}\left(E(h_i)\right) = \tau\left(h_0\right) - \frac{as^2 w^2}{2\left(\left(w + v_0\right)^2 + 2awh_0\right)^{3/2}}$$

Estimating q_0 with the priority ratio

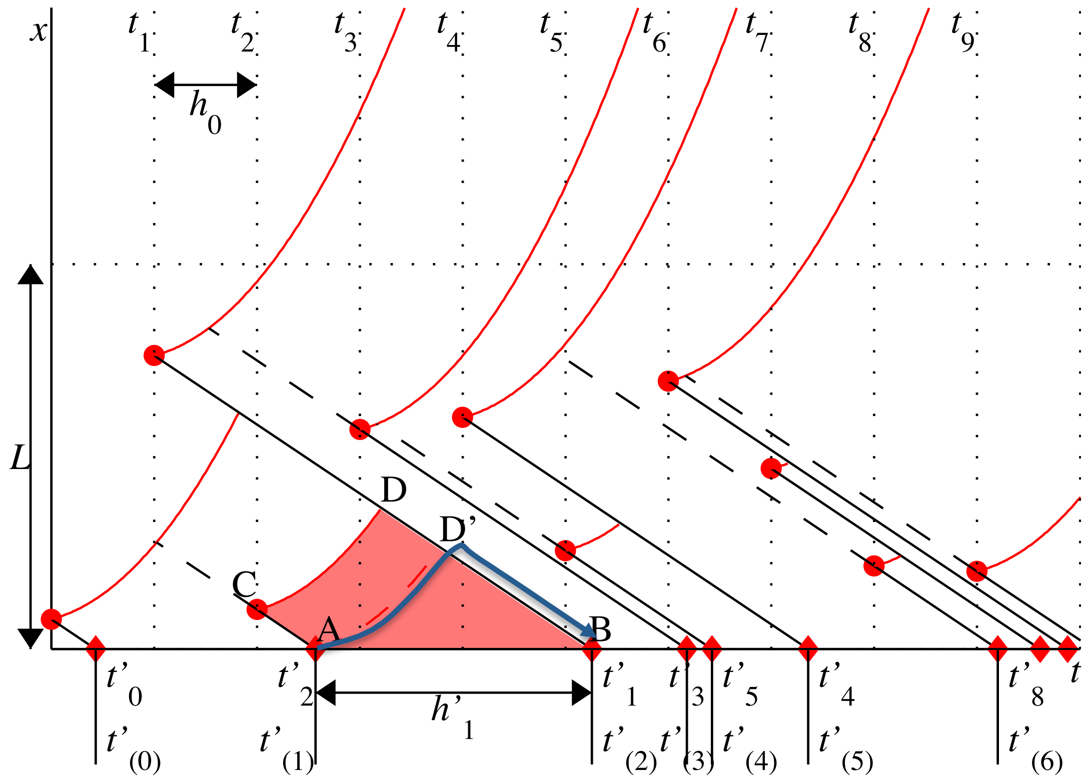
- Expression of the effective capacity

$$\begin{cases} C \simeq w\kappa + \dfrac{w\kappa}{h_0}\left(\dfrac{w+v_0}{a} - \dfrac{1}{a}\sqrt{(w+v_0)^2 + 2awh_0} + \dfrac{as^2w^2}{2\left((w+v_0)^2 + 2awh_0\right)^{3/2}} \right) \\ C = q_0 + q_1 = (1 + 1/\alpha) q_0 \end{cases}$$

- By applying the FD, it appears an unique equation in q_0:

$$w\kappa + q_0\left(\frac{w^3\kappa^2}{a(w\kappa - q_0)} - \frac{w\kappa}{a}\sqrt{\frac{w^4\kappa^2}{(w\kappa - q_0)^2} + \frac{2aw}{q_0}} + \frac{as^2w^3\kappa}{2\left(w^4\kappa^2/(w\kappa - q_0)^2 + 2aw/q_0\right)^{3/2}} \right) - \left(1 + \frac{1}{\alpha}\right) q_0 = 0$$

Case 2: (L>0, s_A=0, s_K=0) – no interactions



Same problem as case 1 by switching inserting times and $t'_{()}$ at x=0 !

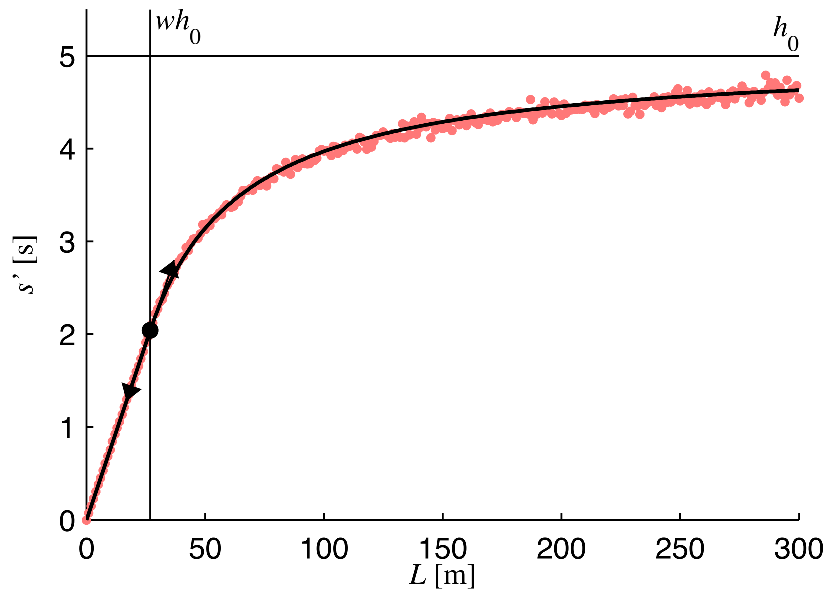
H'-distribution
when $L>0$ and $s_H=0$, $s_A=0$, no interaction

1/ when $L<wh_0$, $h'_i=t'_{(i+1)}-t'_{(i)}=h_0+(x_{i+1}-x_i)/w$.
Thus s' can be analytically derived.
➔ $s'(L)=L/\sqrt{6}w$ (analytical)

2/ $t'_{()}$ distribution is asymptotically exponential when X is uniform.
➔ $s'(\infty)=h_0$

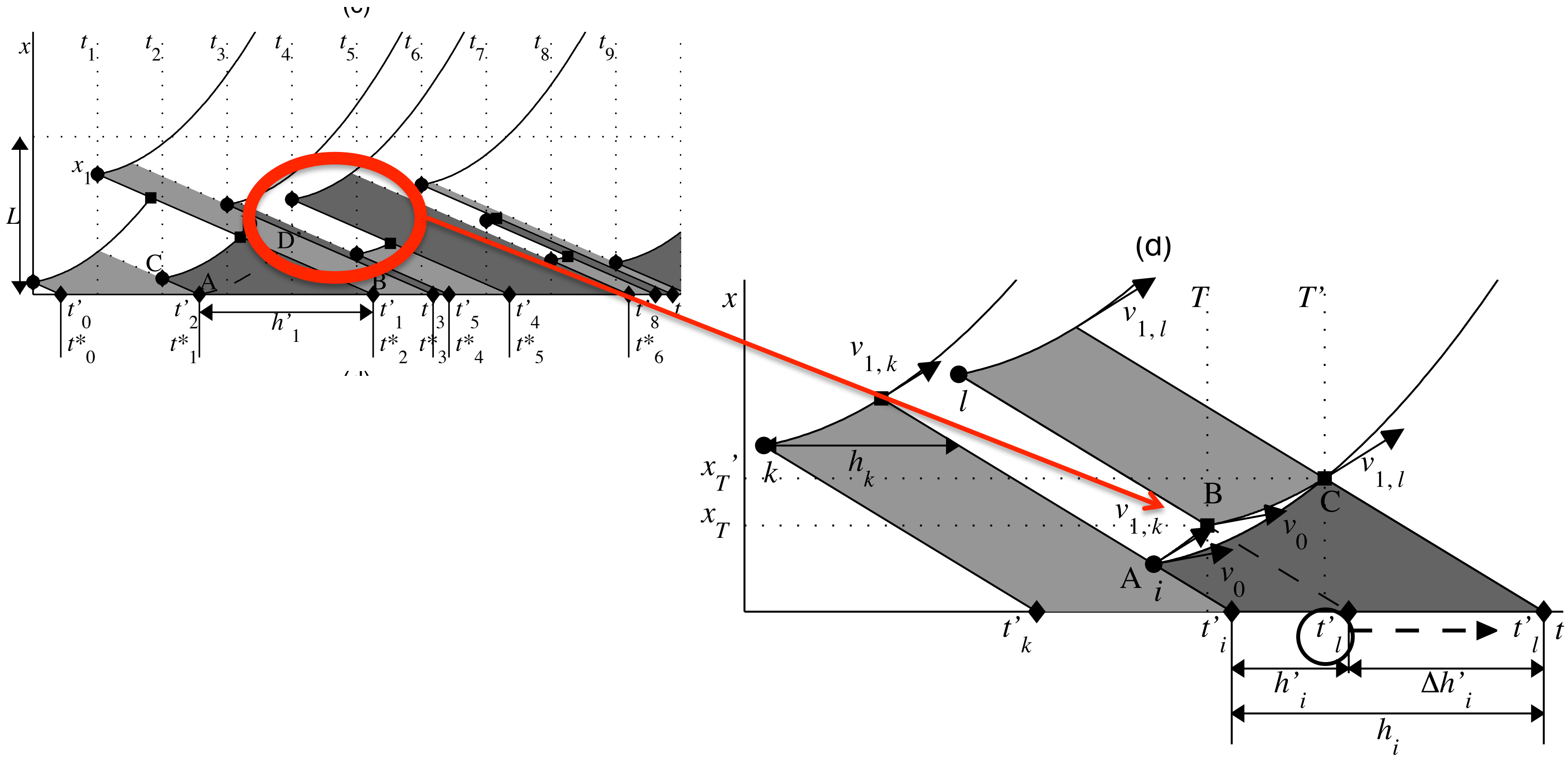
3/ when $L>wh_0$, the best fit for s' is given by
➔ $s'(L)=h_0(L+c)/(L+d)$

4/ Assuming s' smooth and derivable at wh_0 let c and d be analytically derived:



$$s'(L)=\begin{cases} L/\sqrt{6}w & \text{if } L<wh_0 \\ h_0\left(L-wh_0/\sqrt{6}\right)/\left(L+\left(\sqrt{6}-2\right)wh_0\right) & \text{if } L\geq wh_0 \end{cases}$$

Case 3: (L>0, s_A=0, s_K=0) - with interactions waves and voids



The intial speed at x=0 is no longer constant

We have to characterize the probability for interactions to appear

Extended analytical derivations

- Extended formulation of the effective capacity

$$C = w \sum_{i=1}^{n\to\infty} \kappa_i \left(h_i - \tau(h_i, v_{0,i}, a_i)\right) \Big/ \sum_{i=1}^{n\to\infty} h_i$$

$$\tau(h_i, v_{0,i}, a_i) = \frac{1}{a_i}\left(-w - v_{0,i} + \sqrt{(w + v_{0,i})^2 + 2wa_ih_i}\right)$$

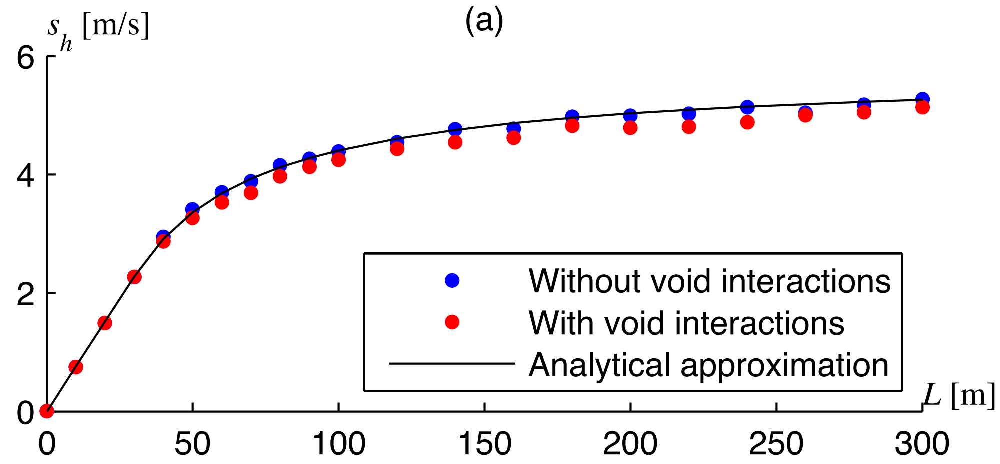
- Introduction the second order Taylor development for τ

$$C = \left(w\kappa \big/ h_0\right)\left(\begin{array}{l} h_0 - \tau(h_0, v_0, a) - \frac{1}{2} s_H^2 \frac{\partial^2 \tau}{\partial H^2} - \frac{1}{2} s_{V_0}^2 \frac{\partial^2 \tau}{\partial V_0{}^2} \\ - \frac{1}{2} s_A^2 \frac{\partial^2 \tau}{\partial A^2} - \theta_{H,V_0} \frac{\partial^2 \tau}{\partial H\, \partial V_0} - \frac{\theta_{A,K}}{\kappa} \frac{\partial \tau}{\partial A} \end{array}\right)$$

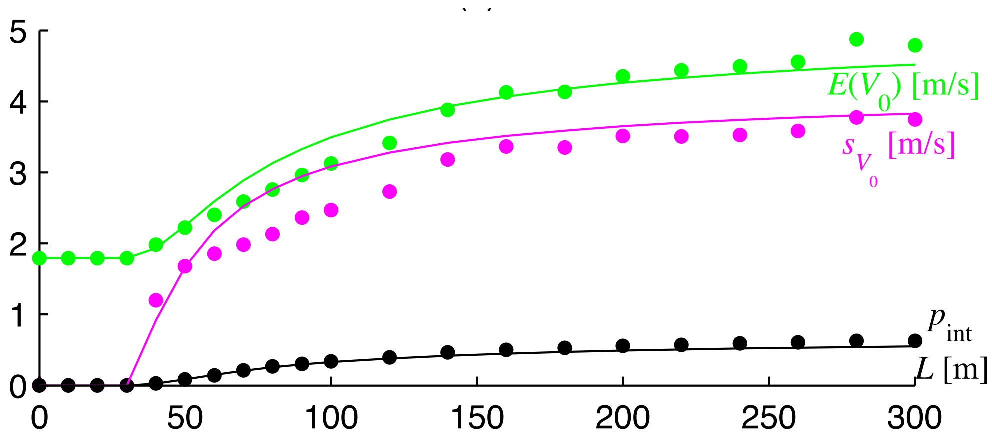
- One have now to characterize the distributions of the key parameters

Nota: the K-distribution plays no role

Characterizing the distributions with interactions (L>0 and s_H=0, s_A=0)



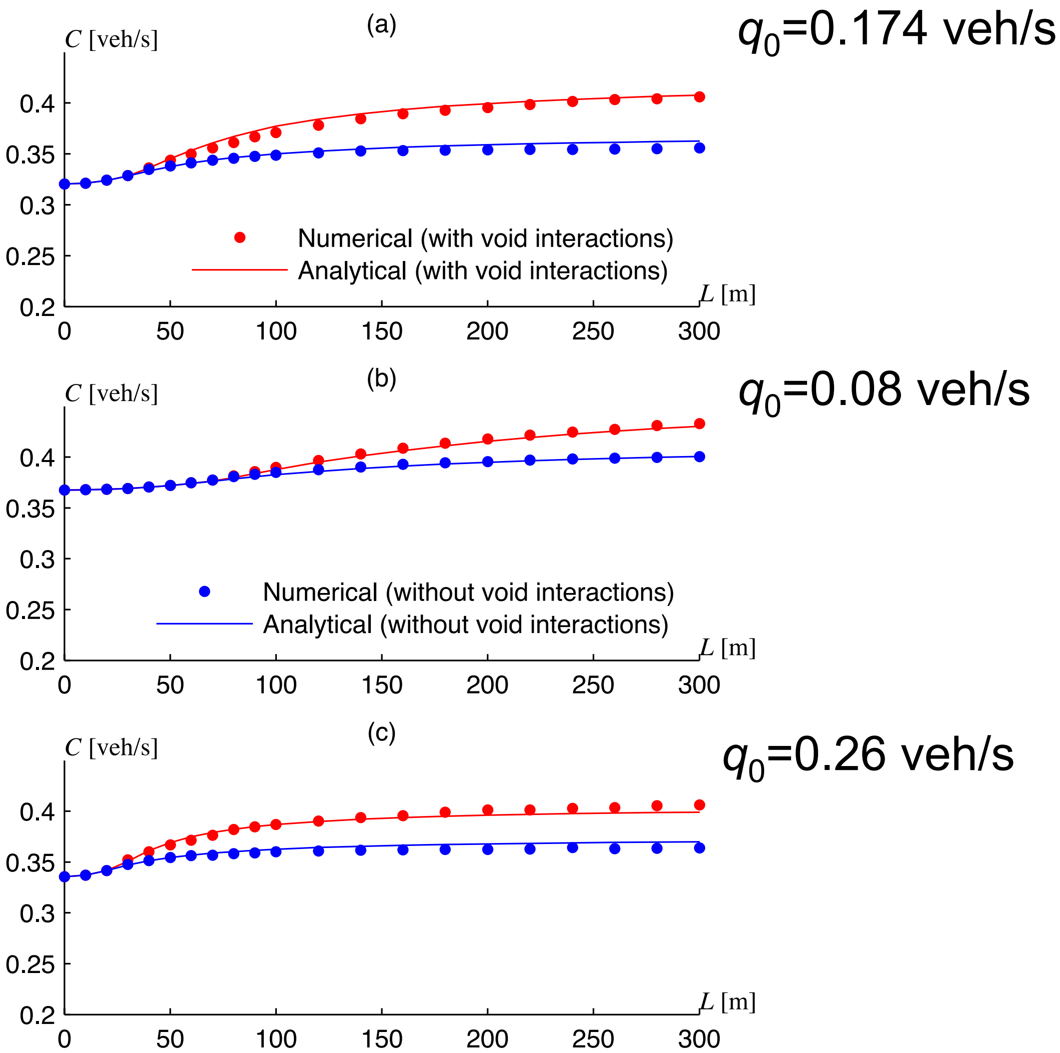
H-distribution



V_0-distribution

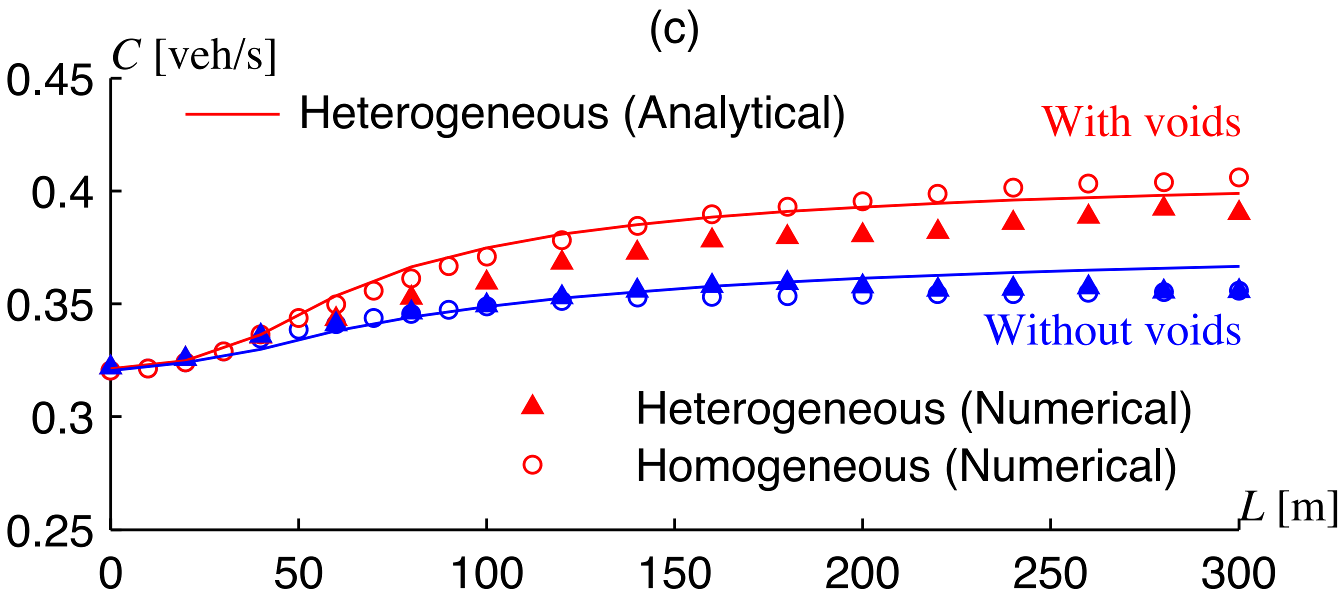
NUMERICAL RESULTS

Resulting analytical curves



The heterogeneous case (s_A>0, s_K>0)

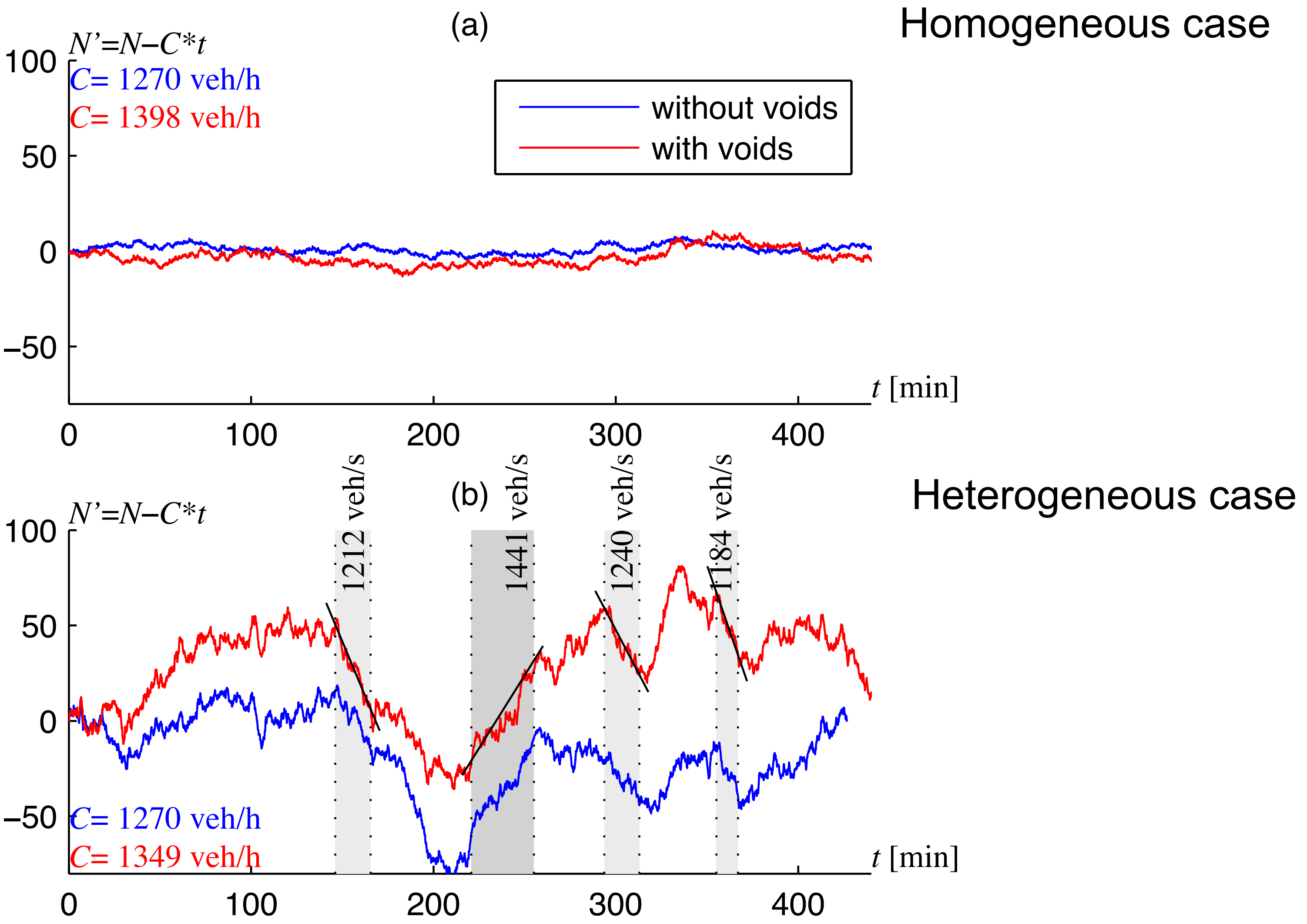
- Some voids may never disappear (the wave then never reaches x=0) => new probability to characterize
- The distribution of v_0 is harder to characterize…



Partial conclusion

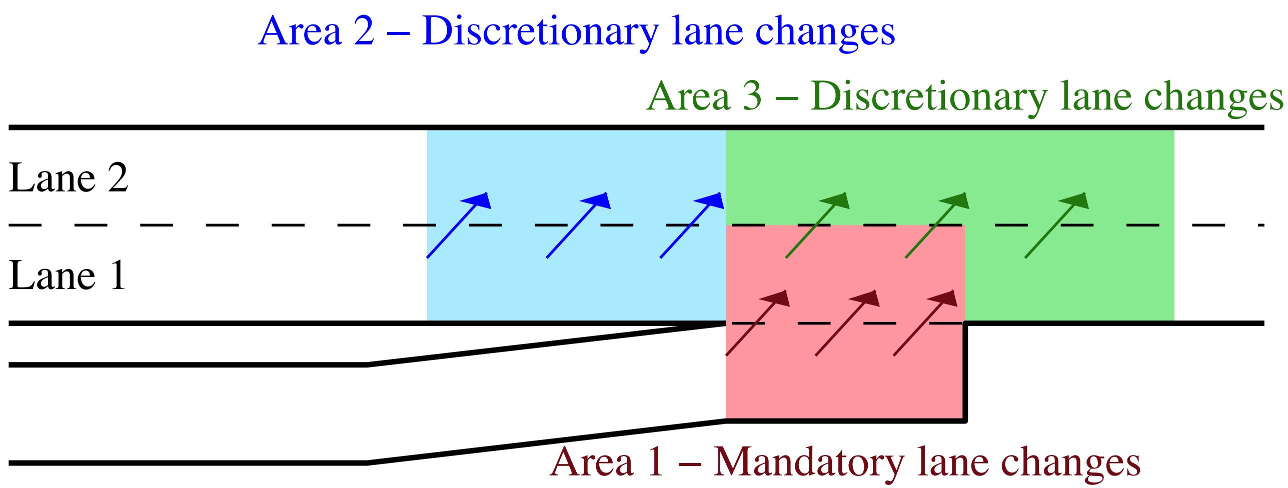
- Accounting for heterogeneous vehicle characteristics is not necessary to properly estimate the mean effective capacity
- Void and wave interactions have a significant influence on the capacity value
- The capacity formulae combined with the Newell-Daganzo's model can be used to derive the full behavior of the merge, i.e. q_0 can be determined as an implicit function (Leclercq *et al*, 2010)

Traffic dynamics just upstream of a merge

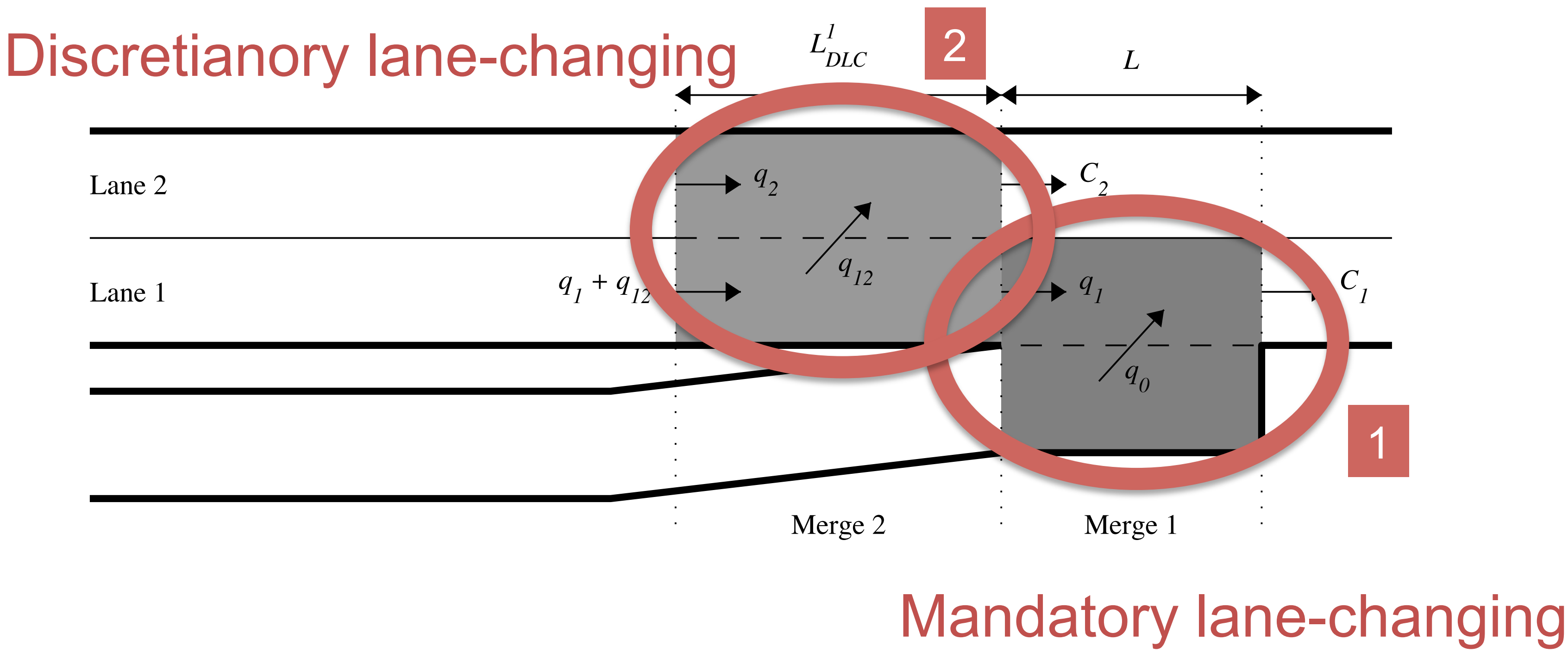


EXTENSION TO MULTILANE FREEWAYS

Sketch of the merge (1)



Sketch of the merge (2)



We will put together previous analytical results to fully describe the merge behavior in congestion

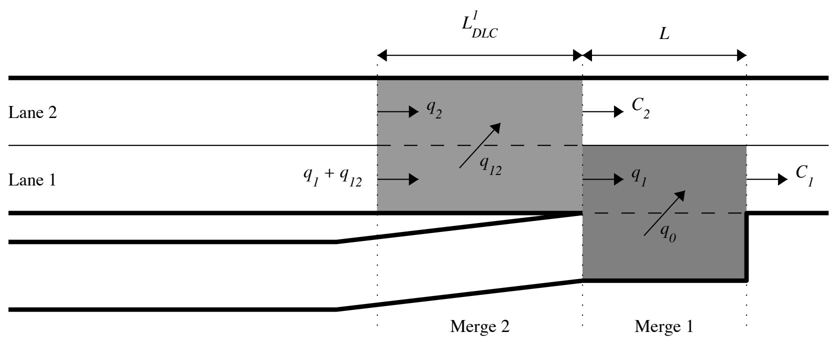
Discretionary lane changing (1)

- Lane changing flow ϕ triggers by the positive speed difference between lane *i* and *j*

$$\Phi(k_i, k_j) = \min\left(1, \frac{\mu_j(k_j)}{\lambda_j(k_j)}\right)\frac{\lambda_i(k_i)\max(v_j - v_i, 0)}{u^2\tau}$$

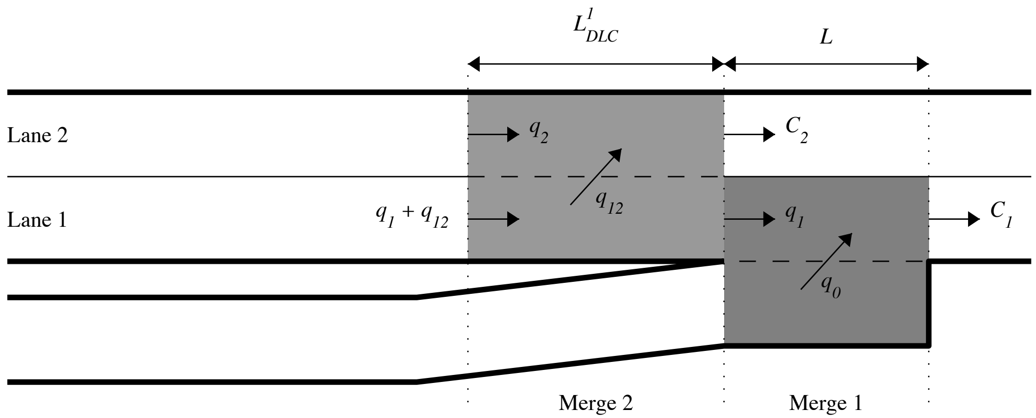
(Laval and Leclercq, 2008)

- *μ* and λ are respectively the supply and the demand derived from the triangular FD
- τ is the time for a lane-changing maneuver to complete

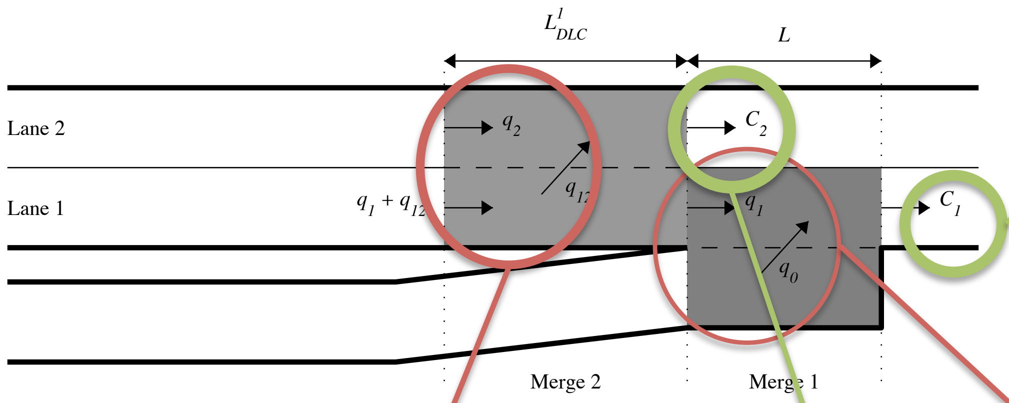


Discretionary lane changing (2)

- Lanes *i* and *j* are congested, so
  - $\mu(k_j)=C_j$
  - $\lambda(k_j)=\lambda(k_i)=Q_{max}$
- It comes that: $$q_{ij} = \Phi(k_i, k_j)L^i_{DLC} = \frac{\max(v_j - v_i, 0)C_j}{u^2\tau}L^i_{DLC}$$



Agregating the different components



Capacity formula (1):

$$q_0 + q_1 = C_1(q_0, v_0)$$

$$v_0 = \frac{wq_0}{w\kappa - q_0} \qquad \text{(FD)}$$

Daganzo's merge model

$$q_0 = \alpha q_1$$

Discretionary lane-changing flow :

$$q_{12} = \frac{\max(v_2 - v_1, 0) L_{DLC}}{u^2 \tau} C_2$$

$$v_2 = \frac{wq_2}{w\kappa - q_2} \qquad \text{(FD)}$$

Capacity formula (2):

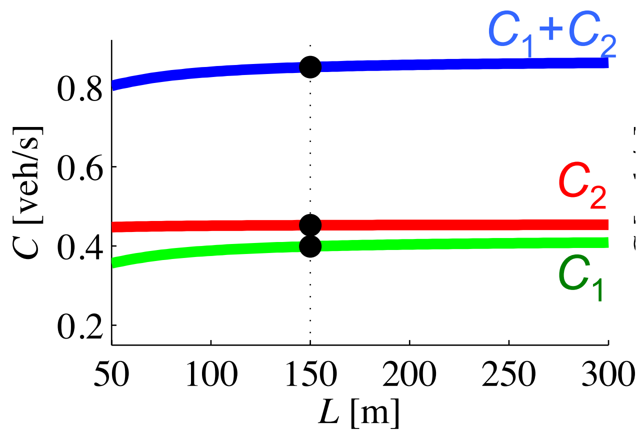
$$q_{12} + q_2 = C_2(q_{12}, v_1)$$

$$v_1 = \frac{w(q_{12} + q_1)}{w\kappa - (q_{12} + q_1)} \qquad \text{(FD)}$$

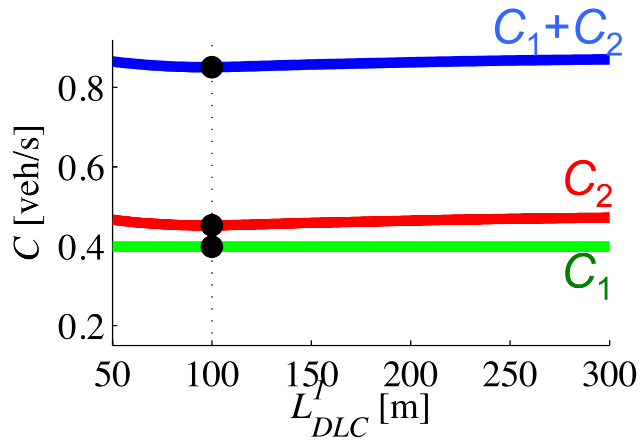
System of 4 equations with 4 unknowns:
q_0, q_{12}, q_1, q_2

NUMERICAL RESULTS

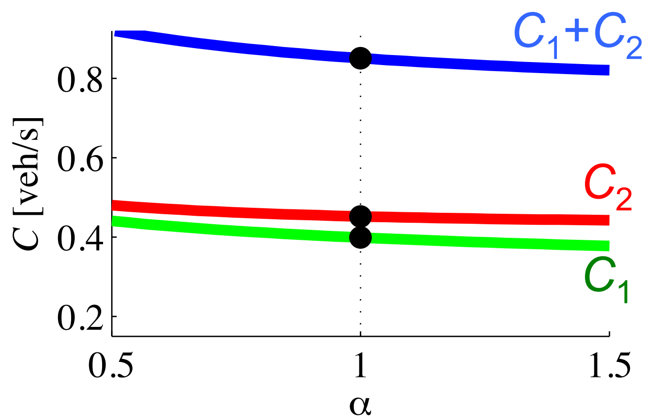
Sensitivity to road parameters



Length of the insertion area

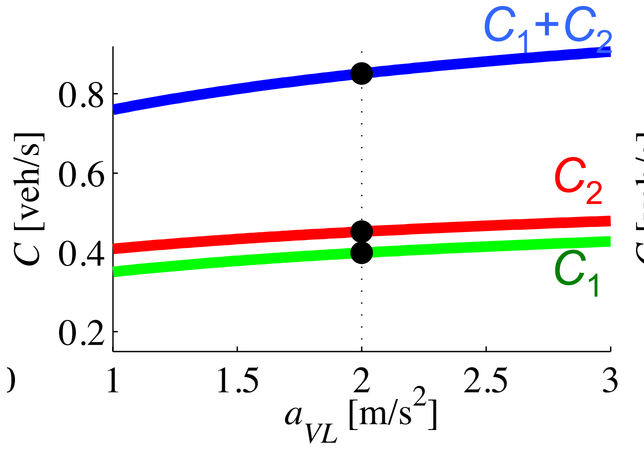


Length of the discretionary lane-changing area

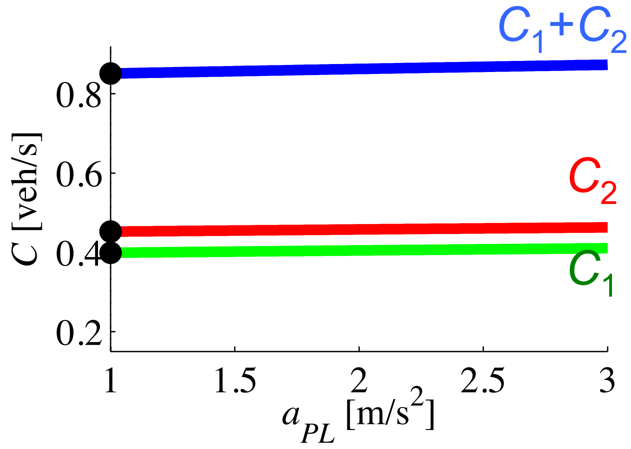


Merge ratio

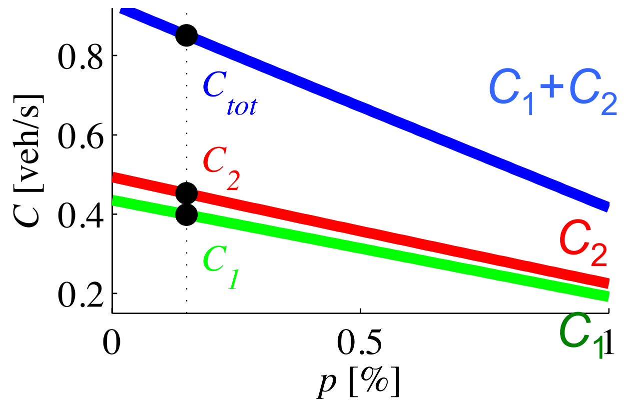
Sensitivity to vehicle characteristics



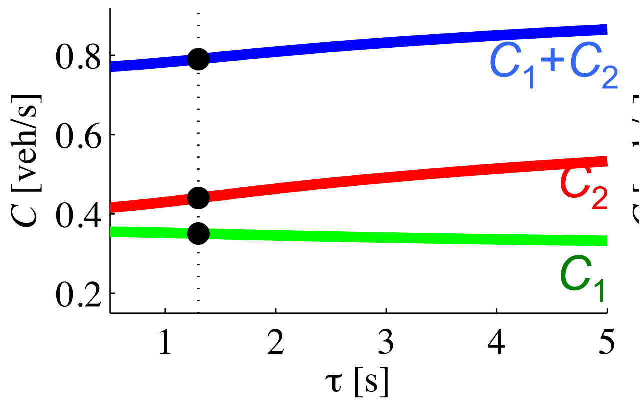
Car acceleration



Truck acceleration



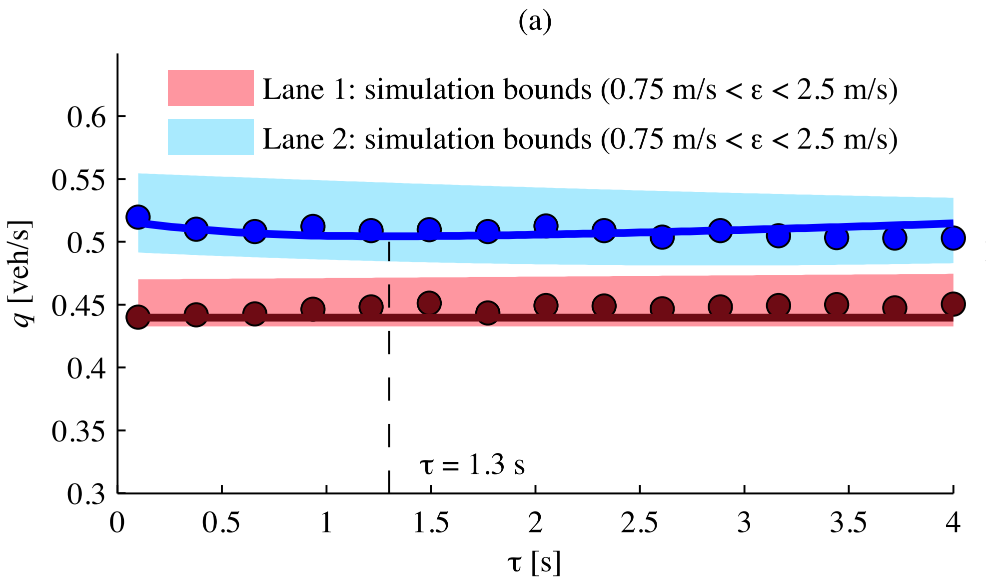
Truck proportion

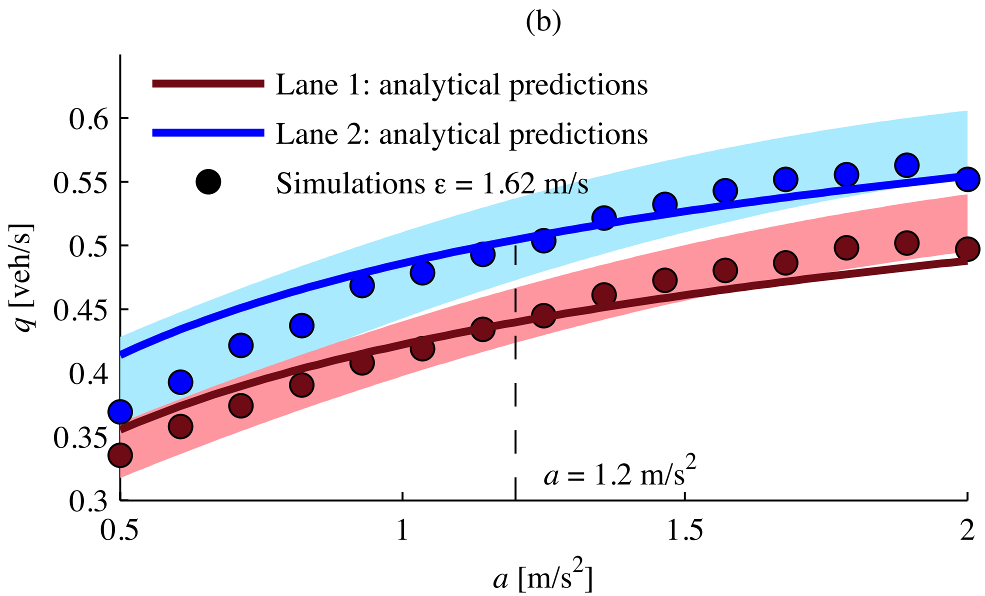


Time to perform a discretionary lane-change

Comparison with a traffic simulator

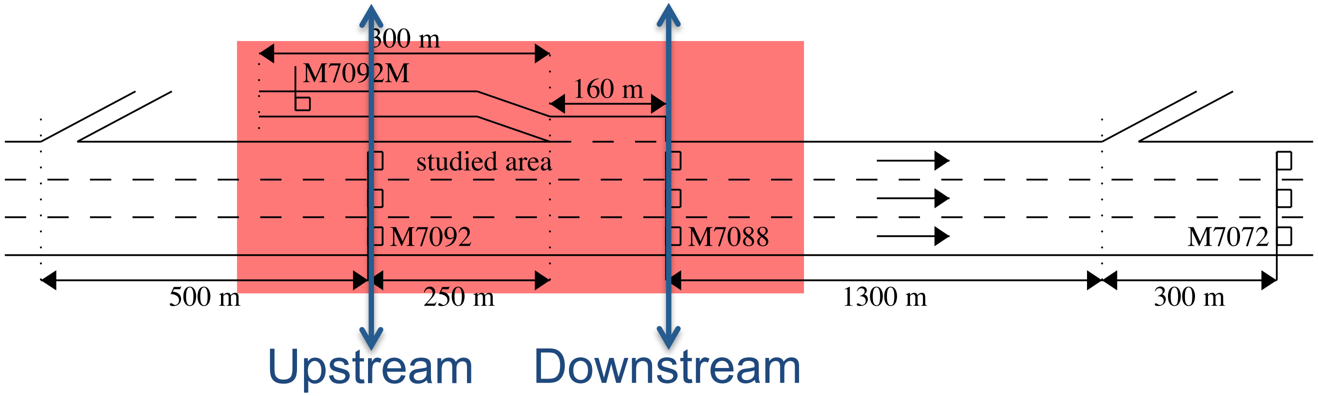
ε is the relaxation parameter

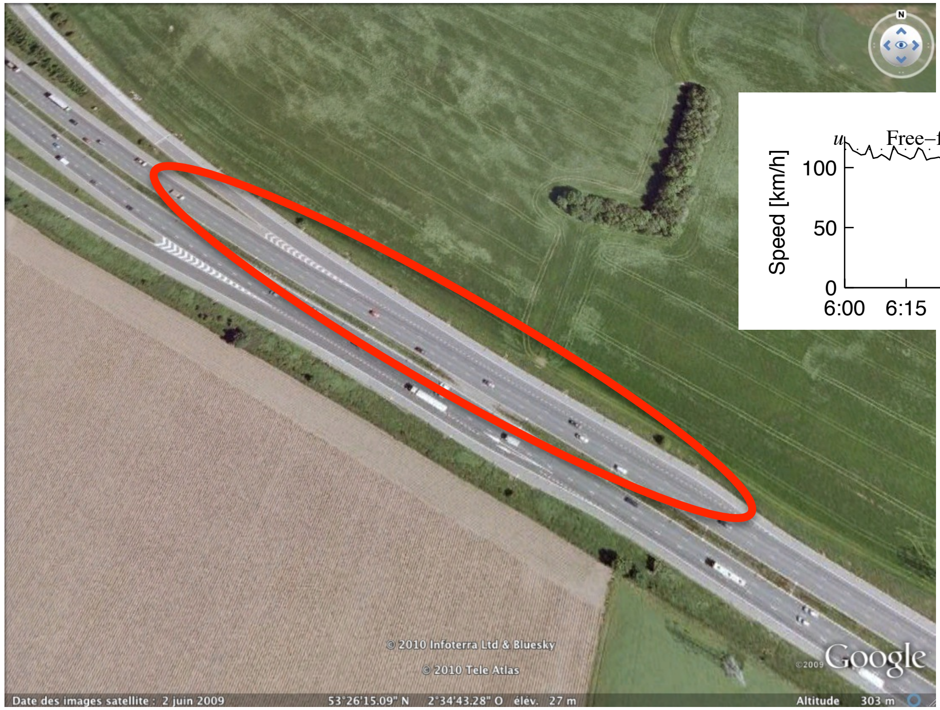


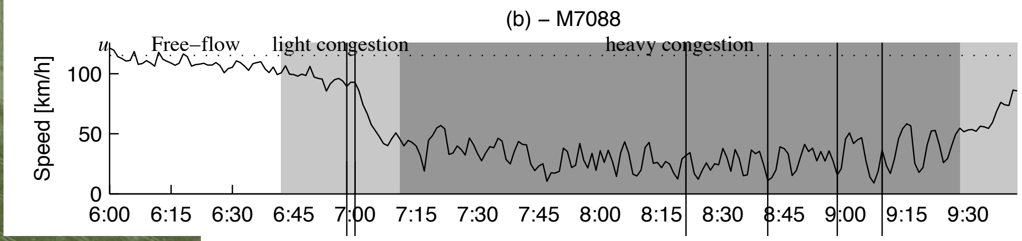


EXPERIMENTAL VALIDATION

Experimental site (M6 – England)



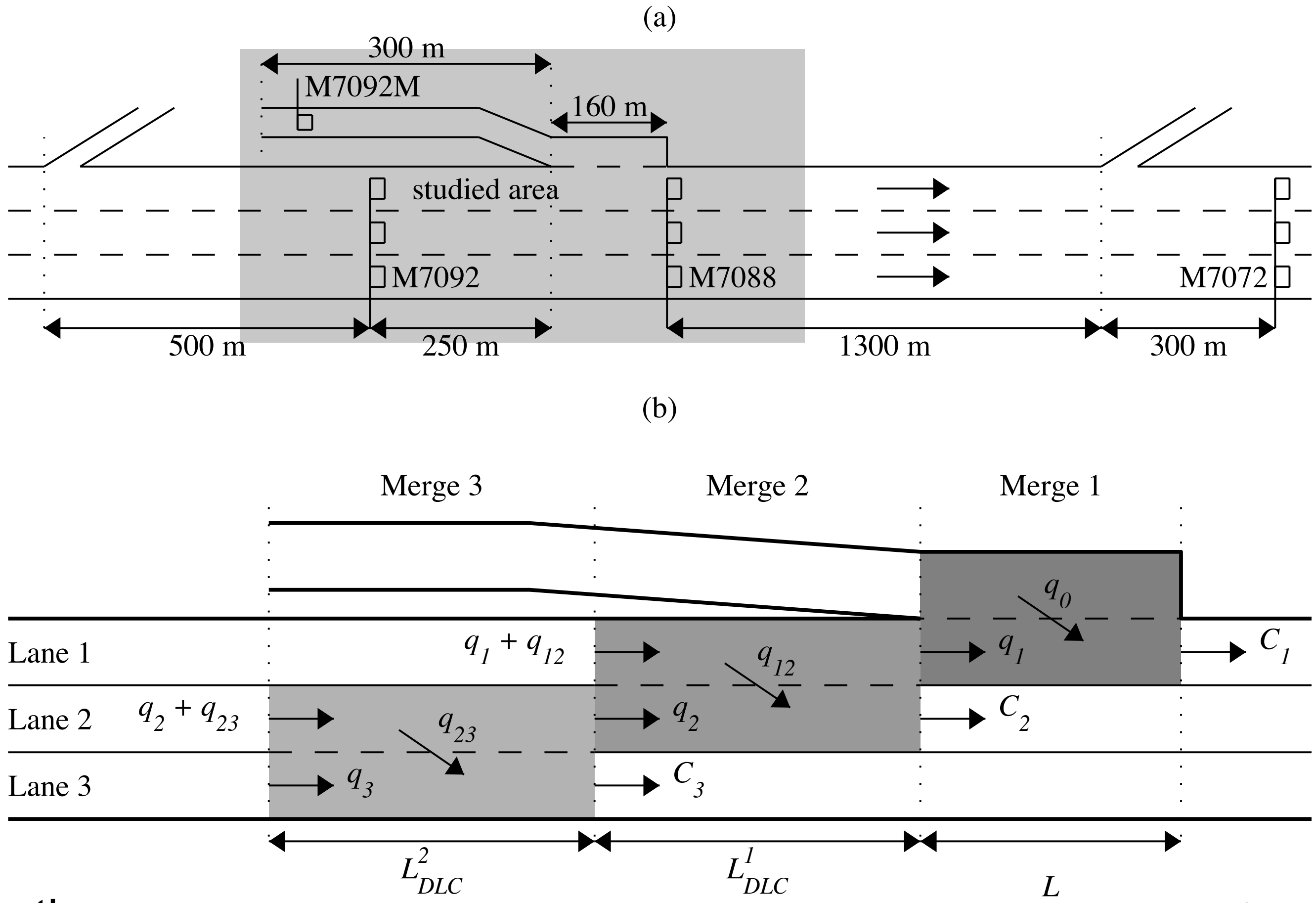




6 days of observations

17 periods (20 min) of heavy congestion

Extended sketch of the model



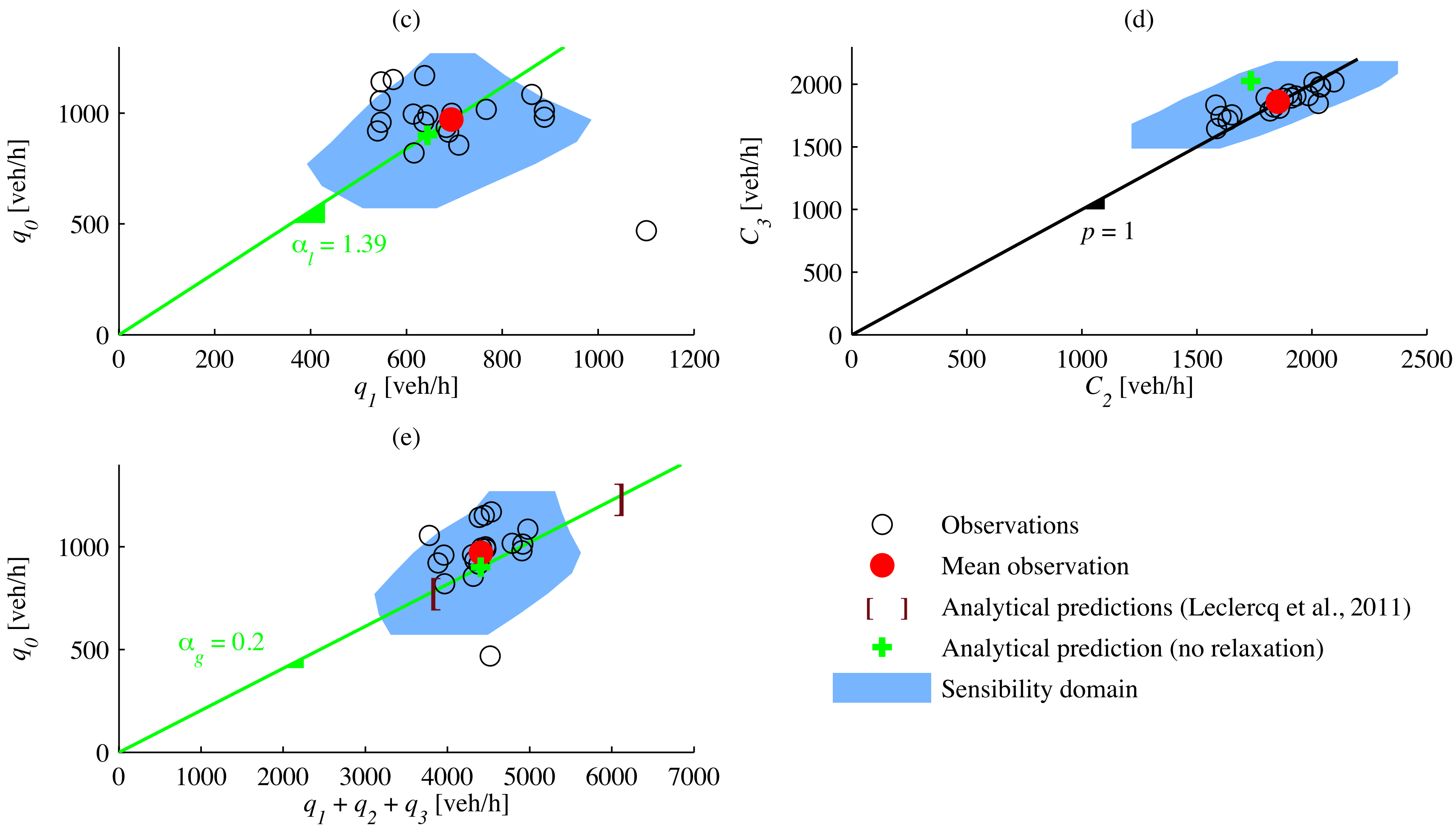
Rough calibration:

-FD (per lane): u=115 km/h, w=20 km/h, κ=145 veh/km
-a=1.8 m/s^2; τ_1=τ_2=3 s;
-L=160 m ; L^2_{DLC}=L^1_{DLC}=100 m

L^2_{DLC}=L^1_{DLC}

τ_1=τ_2

Experimental results



CONCLUSION

Conclusion

- Combining different analytical formulae designed for local problems (local merge, discretionary lane-changing,…) leads to a global analytical model for multilane freeways
- Fast (low computational cost) estimation can be obtained for the total effective capacity and the capacity per lane
- The proposed framework can account for vehicle heterogeneity
- First experimental results are promising

- Of course, this is only an estimate of the mean capacity value for a large time period (20 min). This approach is not able to estimate the short-term evolution of the flow (traffic dynamics)

Thank you for your attention

Are you interested in collaborating on an very exiting project ?......

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