Data–fitted second order models for traffic flow

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joint work
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Objective: Include available data in the modeling to improve model predictions

- Example 1: Density–Flux data used to improve fundamental diagram (B. Seibold, S. Fan)
- Example 2: Model validation based on accident data (S. Moutari, V. Schleper)

Interest in mathematical properties of *macroscopic* models for traffic flow

Summary

Data–fitted second order model

Accident modeling

Macroscopic second order model

$$\rho_t + (\rho u)_x = 0,\ (\rho v)_t + (\rho v w)_x = 0,\ 0 = V(w, v, \rho)$$

- Aw–Rascle–Zhang and Lebacque introduce the models as extension the Lighthill–Whitham–Richards model
- Possibly to add a relaxation term towards a (desired) equilibrium velocity (skipped for this talk, see talk of Rosales, MON)
- Original relation gives a Temple system: $V(w, v, \rho) = w + p(\rho) - v$ with $p'(\rho) > 0$
- System strictly hyperbolic except for $\rho = 0$ with GNL and LD field

Phase diagram and characteristic fields in ARZ model

$$\rho_t + (\rho u)_x = 0,\ (\rho v)_t + (\rho v w)_x = 0,\ 0 = V(w, v, \rho) = w + p(\rho) - v$$

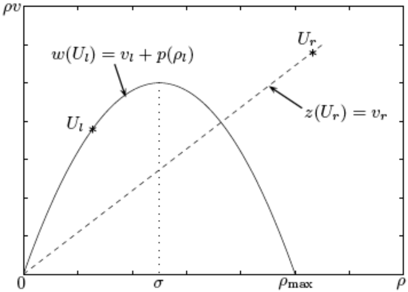
- First characteristic field coincides with (shifted) LWR
- Second characteristic field is LD and propagates information with speed v
- $\rho_{\max}$ is not fixed across the fields
- Slope close to zero is stronger than linear to ensure strict hyperbolicity

Mathematical properties of ARZ equations

$$\rho_t + (\rho u)_x = 0,\ (\rho v)_t + (\rho v w)_x = 0,\ w = v + p(\rho)$$

- Reformulation of equations yields $w_t + v w_x = 0$
- Two characteristic families: GNL (as in Lighthill–Whitham–Richards) and LD (transport)
- Eigenvalues $\lambda = v - \rho p'(\rho)$ and $\lambda = v$

Figure of Riemann invariants for $p(\rho) = \rho^\gamma$ and $\gamma > 1$



Data and phase diagram

$$\rho_t + (\rho u)_x = 0,\ (\rho v)_t + (\rho v w)_x = 0,\ 0 = w - v - p(\rho)$$

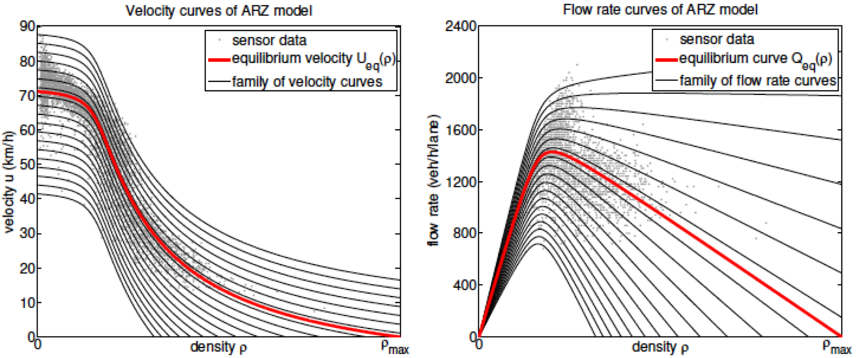


Figure 2. Velocity vs. density (left panel) and flow rate vs. density (right panel) curves of the smooth three-parameter model (12), fitted with historic fundamental diagram data (gray dots), for the ARZ model.

- Classical ARZ $p(\rho) = \rho^\gamma$ does *not* capture well the spread in the data and there is no bound on maximal density

Family of functions modifying the density velocity relation in ARZ

$$\rho_t + (\rho u)_x = 0,\ (\rho v)_t + (\rho v w)_x = 0,\ v = V(w, \rho)$$

Well–posedness of the hyperbolic system provided the following properties of V are fulfilled

- $V \geq 0$
- $\partial_{\rho^2}(\rho V(\rho, w)) < 0$ ensuring concavity of the flux function
- $\partial_w V(\rho, w) > 0$, i.e., faster empty road velocity ensure faster velocity for all densities
- $V(0, w) = w$ normalizes the empty road velocity to w

Riemann problem well–posed and construction of solutions similar to ARZ

$$Q = \rho V(\rho, w) = \alpha \left(a + (b - a)\frac{\rho}{\rho_{\max}} - \sqrt{1 + y^2} \right)$$

$$a = \sqrt{1 + (\lambda p)^2}, b = \sqrt{1 + (\lambda(1 - p))^2}, y = \lambda(\frac{\rho}{\rho_{\max}} - p).$$

Family of functions for modifying the density velocity relation in ARZ

$$\rho_t + (\rho u)_x = 0, \ (\rho v)_t + (\rho v w)_x = 0, \ v = V_{\alpha,p,\lambda}(\rho, w)$$

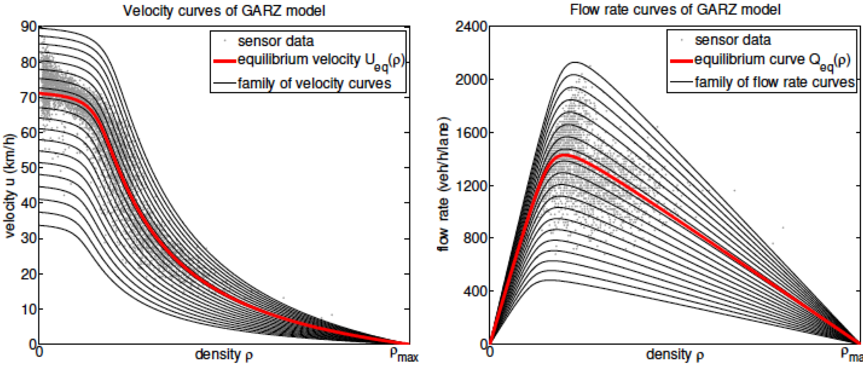
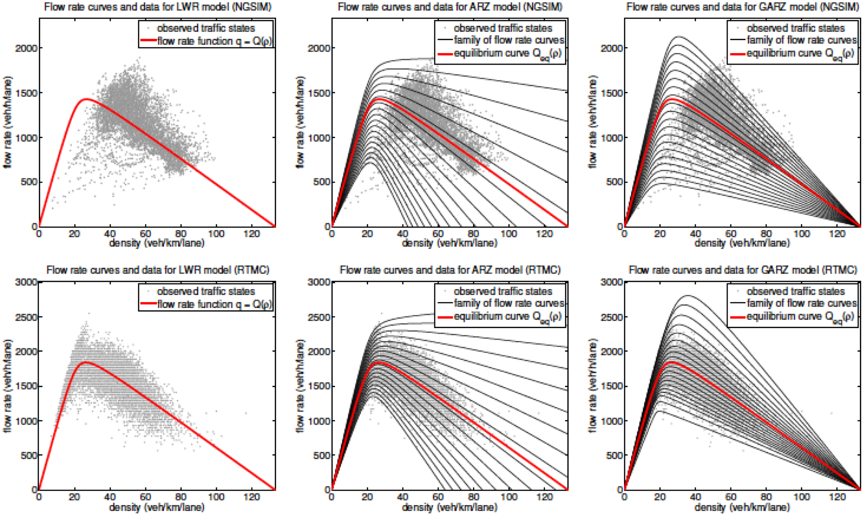


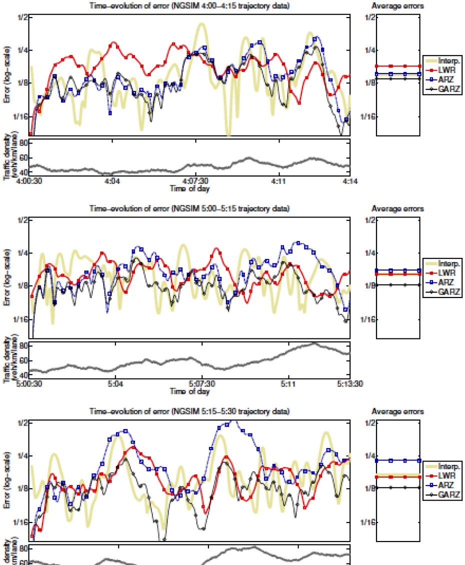
Figure 3. Family of velocity vs. density (left panel) and flow rate vs. density (right panel) curv generated from the weighted least square (WLSQ) algorithm in constructing the velocity functi $u = V(\rho, w)$ in the GARZ model.

- Parameters α, p, λ optimized according to data

Numerical results for data–fitted models: fundamental diagrams



NGSIM data: Error is sum of normalized density and velocity mismatchs in L^1 to reconstructed density and velocity from trajectory data



RTMC data: Aggregate densities and flows at three positions over 30 sec

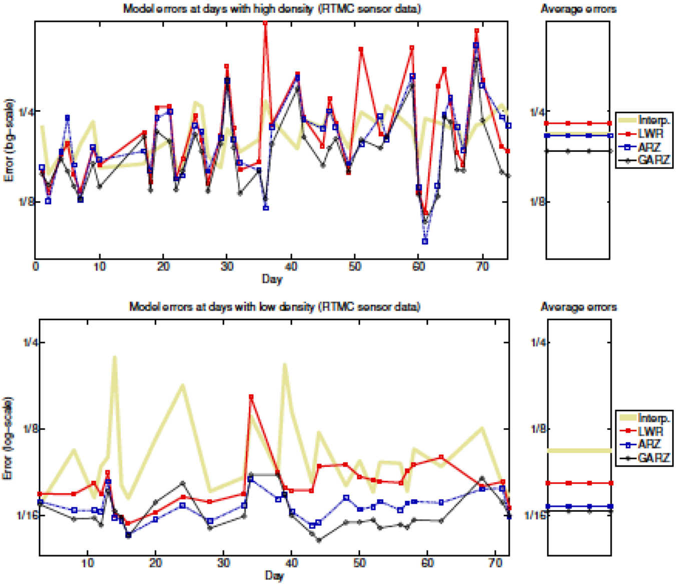


Table with numerical errors

Data set		Interp.		LWR		ARZ		GARZ
NGSIM	4:00–4:15	0.151	(+10%)	0.181	(+31%)	0.153	(+11%)	0.138
NGSIM	5:00–5:15	0.160	(+25%)	0.161	(+26%)	0.174	(+35%)	0.129
NGSIM	5:15–5:30	0.168	(+30%)	0.162	(+25%)	0.228	(+76%)	0.129
RTMC	congested	0.203	(+14%)	0.228	(+24%)	0.208	(+13%)	0.184
RTMC	non-cong.	0.108	(+63%)	0.081	(+26%)	0.067	(+4%)	0.064

Table 2. Spatio-temporal average errors of the traffic models and the Interpolation predictor for the NGSIM data sets (rows 2–4) and the RTMC data (rows 5–6), separated into congested and non-congested days. In each row, the parentheses denote the excess error relative to the best model (GARZ in all cases).

Summary

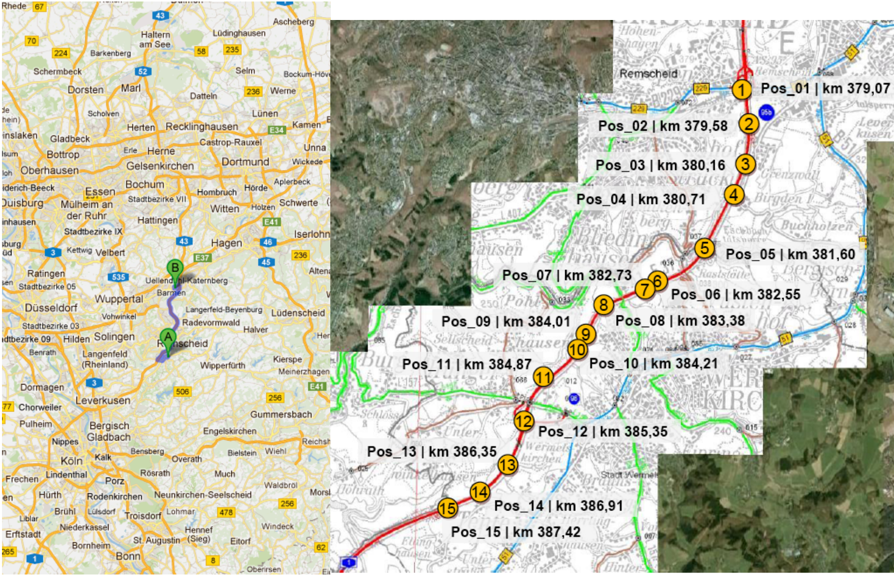
Data–fitted second order model

Accident modeling

Motivation: Present model to improve design of construction sites

- Observation: Average of one accident per day on construction site on highway A1 in Germany
- Current construction site will be up for at least two years; lane reduction of 33%; no on- and off–ramps
- Ministry interested in prediction and improvement of highway safety
- Camera installments available along the construction site
- Available data: density and velocity by car tracking (trajectory data) – data is still being acquired and processed

Map of highway A1 and camera locations



Questions on mathematical modeling

- Modeling question: Under what traffic conditions are accidents likely?
- Different modeling perspectives to adress the topic

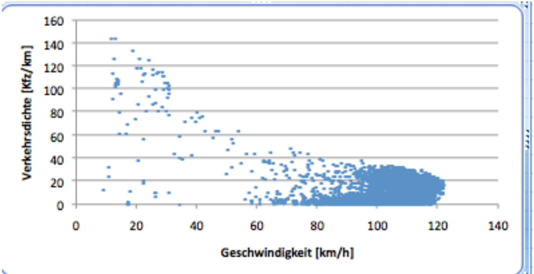
1. - Accidents are stochastic and requires stochasticity in the equation
   - Data available in the literature on likelihood of accidents
2. - Predict accidents based on deterministic models
   - Allows for falsifiable claims and mechanism leading to accidents
   - Models break down as soon as accidents occurs

- Advantages and disadvantages for both approaches

ARZ equations and properties

$$\rho_t + (\rho u)_x = 0,\ (\rho v)_t + (\rho v w)_x = 0,\ v = V(w, \rho)$$

- w is a Lagrangian marker (quantity not changing in (mass-)Lagrangian coordinates)
- Non–negative velocities can not appear (in contrast to Payne–Whitham)
- Interpretation as microscopic model possible and $V(\rho, w)$ obtained through data
- $A1$ fundamental diagram data available



Microscopic interpretation of ARZ equation

$$\rho_t + (\rho u)_x = 0,\ (\rho v)_t + (\rho v w)_x = 0,\ w = v + p(\rho)$$

- $X = \int^x \rho(s,t)ds$ is mass–Lagrangian coordinate and (X,t) a new moving coordinate system

$$\partial_t \frac{1}{\rho} - \partial_X v = 0,\ \partial_t w = 0,\ w = v + p(\rho)$$

- $\frac{1}{\rho}$ is the average car spacing and a semi–discretization in space X with index i and average car length ΔX and $\frac{x_{i+1}-x_i}{\Delta X} = \rho(X_i, t)$

$$\frac{d}{dt}x_i = v_i,\ \frac{d}{dt}w_i = 0 \implies \frac{d}{dt}v_i = -p'(\rho_i)\Delta X \frac{v_{i+1} - v_i}{(x_{i+1} - x_i)^2}$$

- Microscopic equation are a follow–the–leader model where acceleration is governed by dynamic pressure $p = p(\rho_i)$

Accident model

$$\frac{d}{dt}x_i = v_i, \quad \frac{d}{dt}v_i = -p'(\rho_i)\Delta X \frac{v_{i+1} - v_i}{(x_{i+1} - x_i)^2}$$

- Modeling hypothesis: accidents might occur when drivers become reckless, or unobservant
- Reckless drivers do not react with respect to to distance of cars in front
- Equivalent to assume $p' \equiv 0$
- Reformulation in Eulerian–coordinates (x, t) yields pressure–less gas dynamics (PGD)

$$\rho_t + (\rho v)_x = 0, \; (\rho u)_t + (\rho u^2)_x = 0$$

Properties of pressureless gas dynamic systems

$$\rho_t + (\rho v)_x = 0,\ (\rho u)_t + (\rho u^2)_x = 0$$

- System is (for smooth solutions) equivalent to conservation law and Burgers equation for velocity v
- Shock in v induces a $\delta-$concentration in the density ρ
- Occurence of δ concentration at position (x, t) in Eulerian coordinates
- δ concentration is seen as accident
- Model for accidents: *coupling of ARZ and PGD equations and predict occurence of $\delta-$shocks*
  - in Eulerian coordinates at a fixed position: some drivers due to possibly reduced lane change behavior (ARZ $\to$ PGD)
  - in Lagrangian coordinates: we know a priori which car label is going to be in the accident

Coupling of ARZ and PGD in Eulerian coordinates

Assume drivers change behavior at a certain point in the road

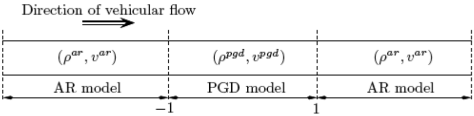


FIG. 2. Modeling road accidents using different traffic regimes.

- Mathematical problem is a bounday value problem for a coupled system
- Well–posed problem for constant initial data provided additional coupling conditions are fulfilled
- Coupling conditions at $x_0 \in \{\pm 1\}$
  - Conservation of mass

$$(\rho v)^{ARZ}(x_0, t) = (\rho v)^{PGD}(x_0, t)$$

  - Equality of dynamic traffic pressure

$$(v + p(\rho))^{ARZ}(x_0, t) = v^{PGD}(x_0, t)$$

Well-posedness result

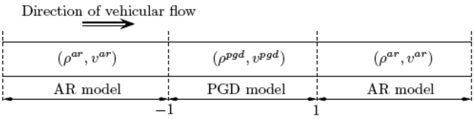


FIG. 2. Modeling road accidents using different traffic regimes.

$$
\begin{aligned}
&(ARZ) \quad \rho_t + (\rho v)_x = 0,\ (\rho v)_t + (\rho v w)_x = 0,\ w = v + p(\rho)\\
&(PGD) \qquad\qquad \rho_t + (\rho v)_x = 0,\ (\rho u)_t + (\rho u^2)_x = 0
\end{aligned}
$$

- *ARZ/PGD* : Conservation of mass, equal pressure, constant initial data with non–vacuum initial data in the ARZ phase is a well–posed boundary value problem of the given system and exhibits a weak self–similar solution.
- *ARZ/PGD* : Conservation of mass, equal pressure, constant initial data with non–vacuum initial data in the ARZ phase and v_0^{ARZ} sufficiently large is a well–posed boundary value problem of the given system and exhibits a weak self–similar solution.

Discussion of Riemann solver at the junction

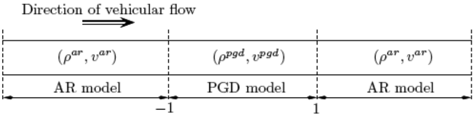


FIG. 2. Modeling road accidents using different traffic regimes.

$$
\begin{array}{ll}
(ARZ) & \rho_t + (\rho v)_x = 0,\ (\rho v)_t + (\rho v w)_x = 0,\ w = v + p(\rho) \\
(PGD) & \rho_t + (\rho v)_x = 0,\ (\rho u)_t + (\rho u^2)_x = 0
\end{array}
$$

- $x_0 = -1$: From the proof a condition for the existence of $\delta-$shock waves arise, namely, there exists a $\delta-$solution to (PGD), if and only if
$$w(\rho_0^{ARZ}, v_0^{ARZ}) > v_0^{PGD}$$
- $x_0 = +1$: For v_0^{ARZ} small there is a lack of uniqueness. Additional conditions need to be imposed to resolve this. The situation amounts to the case when the flow in the ARZ is smaller than in the PGD part.

Discussion of the "accident condition"

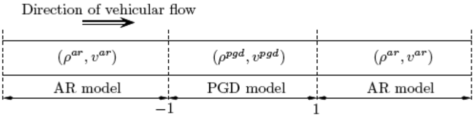


FIG. 2. Modeling road accidents using different traffic regimes.

$$
\begin{array}{ll}
(ARZ) & \rho_t + (\rho v)_x = 0,\ (\rho v)_t + (\rho v w)_x = 0,\ w = v + p(\rho) \\
(PGD) & \qquad\qquad \rho_t + (\rho v)_x = 0,\ (\rho u)_t + (\rho u^2)_x = 0
\end{array}
$$

- $x_0 = -1$:

  $$w(\rho_0^{ARZ}, v_0^{ARZ}) > v_0^{PGD}$$

  implies $\delta-$solution
- Claim: The condition is stable under perturbations of the pressure.

Stability result: Setting

$$
\begin{array}{rll}
(ARZ) & x \le x_0 & \rho_t + (\rho u)_x = 0,\ (\rho v)_t + (\rho v w)_x = 0,\ w = v + p(\rho) \\
(ARZ-\lambda) & x \ge x_0 & \rho_t + (\rho u)_x = 0,\ (\rho v)_t + (\rho v w)_x = 0,\ w = v + \lambda p(\rho) \\
CPL & x = x_0 & (\rho v)(x_0-, t) = (\rho v)(x_0+, t) \\
 & & (v + p(\rho))(x_0-, t) = (v + \lambda p(\rho))(x_0+, t)
\end{array}
$$

- Consider two coupled ARZ systems at $x_0 = -1$ with pressure $\lambda\ p(\rho)$ for $x > x_0$
- Formal limit for $\lambda = 0$ is pressureless gas dynamics
- Coupling conditions: conservation of mass and equality of traffic pressure
- Riemann initial data fulfilling accident condition
- $\implies$ well–posed boundary value problem

Stability result: Limit for $\lambda \to 0$

$$
\begin{array}{rll}
(ARZ) & x \le x_0 & \rho_t + (\rho u)_x = 0,\ (\rho v)_t + (\rho v w)_x = 0,\ w = v + p(\rho) \\
(ARZ - \lambda) & x \ge x_0 & \rho_t + (\rho u)_x = 0,\ (\rho v)_t + (\rho v w)_x = 0,\ w = v + \lambda p(\rho) \\
CPL & x = x_0 & (\rho v)(x_0-, t) = (\rho v)(x_0+, t) \\
 & & (v + p(\rho))(x_0-, t) = (v + \lambda p(\rho))(x_0+, t)
\end{array}
$$

- In the proof of the well–posedness result the construction of an intermediate state $U^* = (\rho^*, v^*)$ appears
- This state fulfills the coupling condition and a Riemann problem give rise to non–positive waves for $x \le x_0$ and non–negative waves for $x > x_0$
- The pressure of U^* is

$$
p(\rho^*) = \frac{w_0 - v_0^\lambda}{\lambda}
$$

  and tends to infinity for $\lambda \to 0$
- p is a regular function of ρ and therefore $\rho^* \to \infty \approx \delta$

Results obtained from data (Preliminary)

- Current statistics of accidents do not include density and velocity of traffic condition close–by
- Camera observations are not fully evaluated yet
- So far, **one** accident could be retrieved which is clearly insufficient to give a fair assesment of the model
- Accident happened at time 12:30:10 on kilometer 328.6 in direction Dortmund
- Data retrieved from cameras at kilometers 383.9 (prior), 328.6 (accident), 381.4, 381.2 (after)
- Data available for 120 sec before and after the accident

Measurements

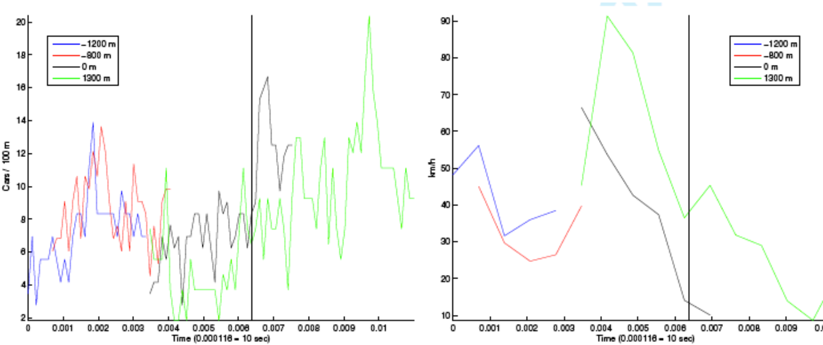
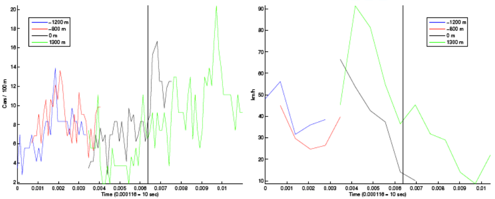


FIG. 10. Density and velocity on the A1 highway at different points on the highway over time. The black vertical line the time of the accident ,i.e., at time 12:30:10. Green data are taken at a point 1300m prior to the accident, black data at of the accident, blue (1200 m) and red data (800 m) after the accident.

- Left: density, right: velocity over time at different locations
- Location of data is green: prior, black: at, blue/red: after accident

Possible interpretation



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- Black data as initial data for PGD, green data as ARZ data clearly not constant
- Observation: $v^{ARZ} > v^{PGD}$ for some time prior to the accident
- Observation: Accidents occurs when density ρ^{ARZ} becomes significantly larger
- Indicates a dependence of accident situations like $w(\rho^{ARZ}, v^{ARZ}) > v^{PGD}$

Thank you for your attention.

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