Pedestrian macroscopic models: game-theoretical versus mechanistic viewpoints

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'PEDIGREE' ANR Collaboration

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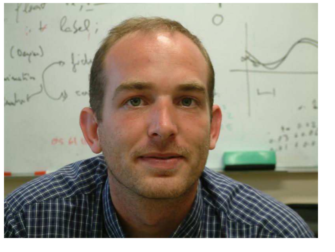

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Summary

1. Issues & context
2. The Heuristic-Based Model (HBM)
3. Mean-field models
4. Macroscopic model
5. Relation to game theory
6. Conclusion

1. Issues & context

Issues

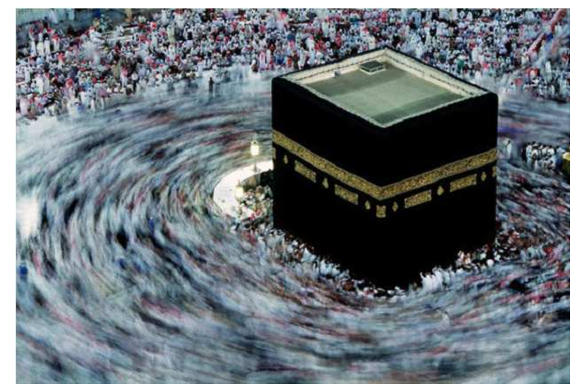
Safety

Avoid crowd disasters

e.g. Duisburg love parade

Cambodia water festival

Demonstration control



Design, comfort, efficiency

Terminals, shopping malls, etc.

Pedestrian models

Individual-Based Models (IBM)

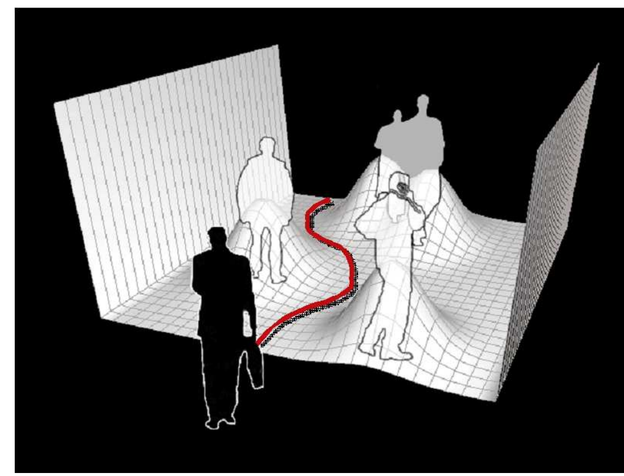
Each individual followed in time

Social force model [Helbing & Molnar, Phys. Rev. E51, 1995]

Analogy with physics:

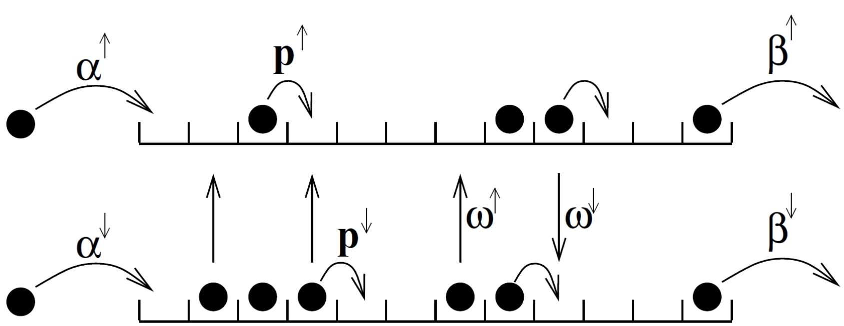
Attractive/repulsive forces

others ...



Cellular automata

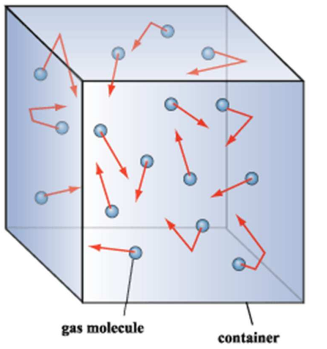
[Burstedde et al, Physica A 295, 2001]



Macroscopic models

Inspired by gas kinetics

[Henderson, Transp. Res. 8, 1974]



Static/dynamic field ($\sim$ chemotaxis)

[Hughes et al, Transp. Res. B36, 2002]

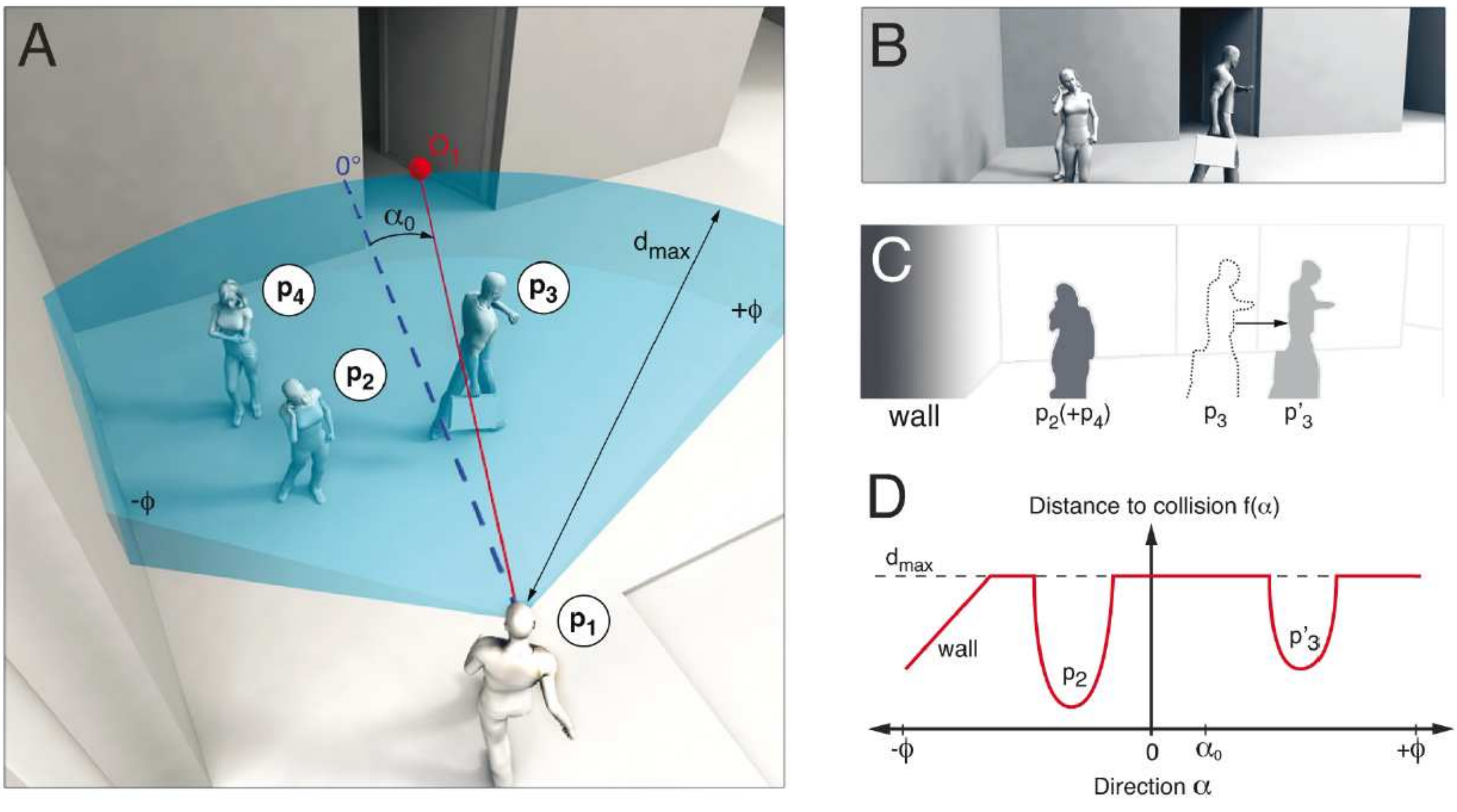
Inspired from road traffic

[Colombo et al, MMAS 28, 2005]

2. The Heuristic-Based Model (HBM)

Heuristic-Based Model (HBM)

[Moussaïd, Helbing, Theraulaz, PNAS 2011]



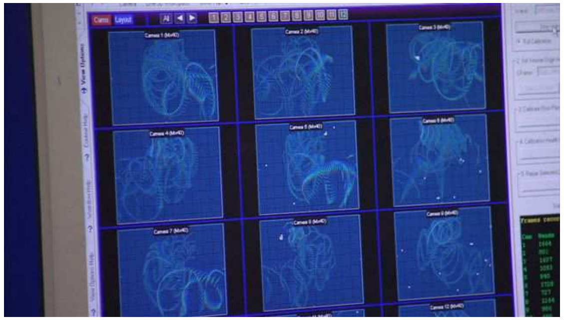
Experiments

Motion capture system

- Sensors reflect infra-red light
- Reflection point camera recorded
- Triangulation $\longrightarrow$ coordinates

Circular arena

- Avoids boundary effects

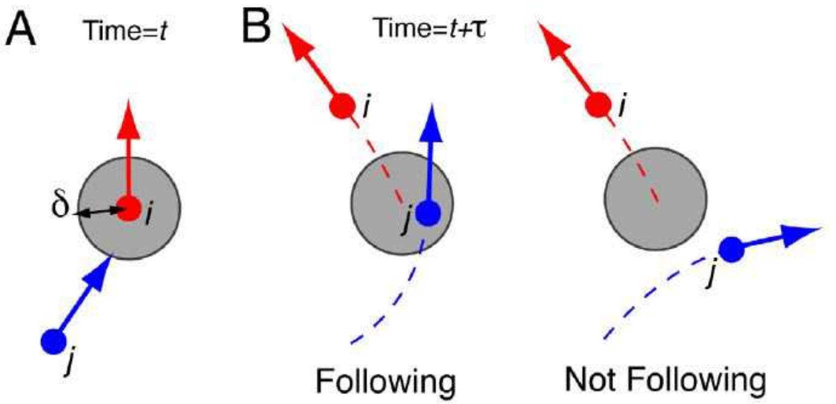


Experiments vs model

Lane formation

Lane definition

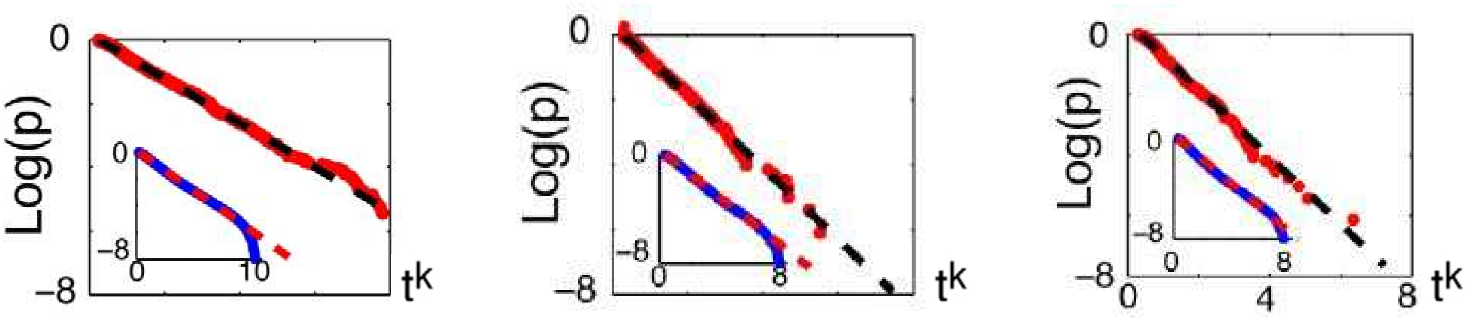
by clustering method



Cluster lifetime statistics

$p(t)dt =$ probability that lifetime $\in [t, t + dt]$

Streched exponential $p(t) = p_0 \, e^{at^k}, \quad k = 0.4$

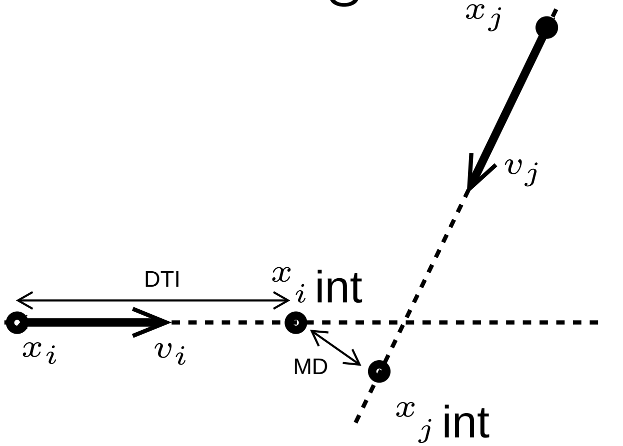


In insert: results of model (See [Moussaid et al, PlosCB 2012])

Perception phase

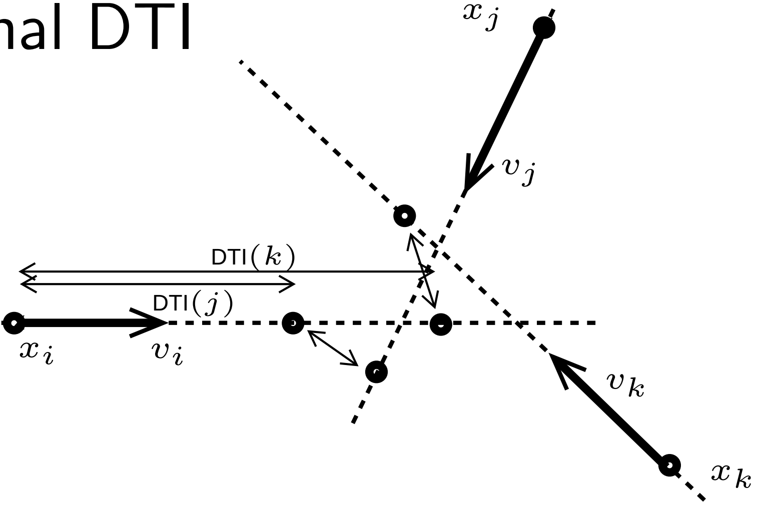
Pedestrians have constant speed

- Evaluation assumes pedestrians move on straight lines
- Distance to Interaction (DTI)
- Minimal Distance (MD)



In case of multiple encounters

- Take the minimal DTI



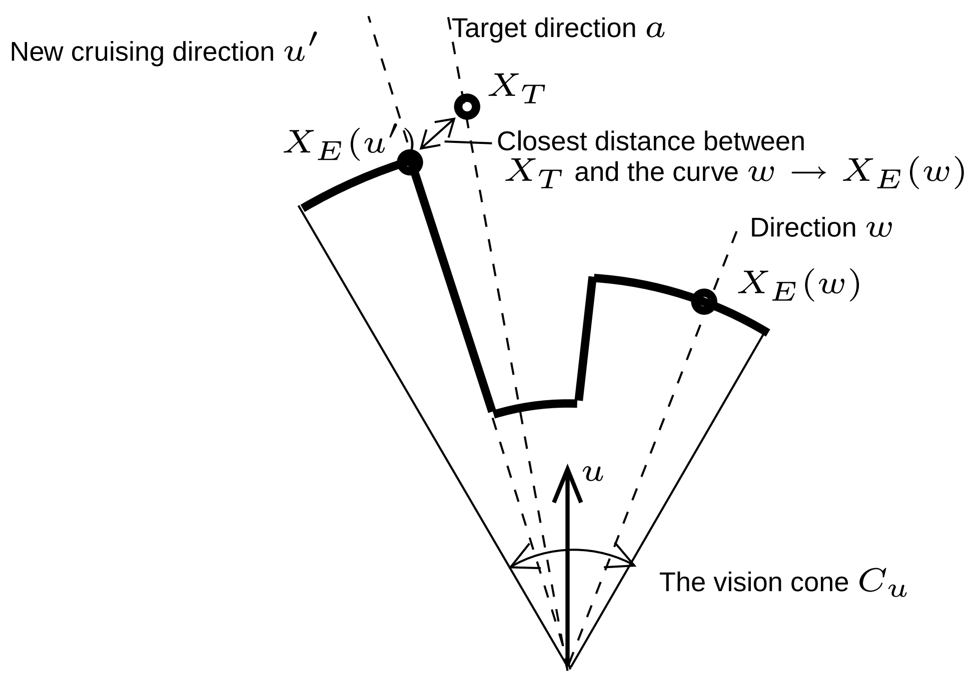
Decision phase

Optimisation: Discrete time step

New cruising direction u' chosen such that

Estimation $X_E(u')$ minimizes distance to target X_T

$\|X_E(w) - X_T\|^2$ among test directions w



Time continuous model

N Particles (pedestrians) $i = 1, \ldots, N$

Position $x_i(t)$, velocity $u_i(t)$, Target direction $a_i(t)$

with $|u_i(t)| = 1$, $|a_i(t)| = 1$, i.e. u_i, $a_i \in \mathbb{S}^1$

$$\dot{x}_i = c u_i,$$

$$du_i = F_i \, dt + P_{u_i^\perp}(\sqrt{2d} \circ dB_i(t))$$

Speed c, noise intensity d, Stratonowich sense $\circ$

Force $F_i \perp u_i$, $P_{u_i^\perp}$ maintains $|u_i| = 1$

Force

Test velocity directions $w \in \mathbb{S}^1 \quad \longrightarrow \quad$ Potential $\Phi_i(w,t)$

$$\Phi_i(w,t) = \frac{k}{2} \, |D_i(w) w - L a_i|^2$$

Reaction rate k, horizon L

$D_i(w)$ maximal walkable distance in direction w

Force $F_i(t)$ defined by steepest descent of Φ_i

$$F_i(t) = -\nabla_w \Phi_i(u_i(t),t)$$

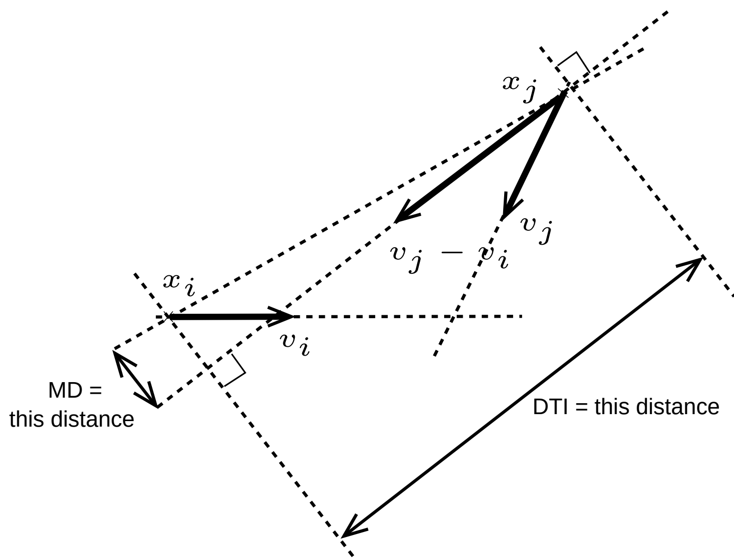
Maximal walkable distance $D_i(w)$

DTI of 'i' against 'j' when 'i' walks in direction w: $D_{ij}(w)$

$$D_i(w) = \text{"}\min_j\text{"}\, D_{ij}(w)$$

For continuum model, replace 'min' by average

e.g. harmonic average in some interaction region



3. The Mean-Field Model

Mean-Field Model

Distribution function $f(x, u, a, t) \quad x \in \mathbb{R}^2, \quad u, \; a \in \mathbb{S}^1$

Probability to find pedestrians at x

with velocity u and target velocity a at time t

$$\partial_t f + \nabla_x \cdot (cuf) + \nabla_u \cdot (Ff) = d\Delta_u f$$

$$F = -\nabla_w \Phi_{(x,a,t)}(u)$$

$$\Phi_{(x,a,t)}(w) = \frac{k}{2} |D_{(x,t)}(w)\, w - La|^2$$

$D_{(x,t)}(w)$ walkable distance of subject at x

in direction w: functional of f

Case: local interactions / no blind zone

Supposes interaction region "very small"

$$D^{-1}_{(x,t)}(w) = \frac{\int_{(v,b)\in\mathbb{T}^2} K(|v-w|)\, f(x,v,b,t)\, dv\, db}{\int_{(v,b)\in\mathbb{T}^2} f(x,v,b,t)\, dv\, db}$$

where K is analytically known (related to the DTI)

If blind zone, $K = K(u, |v-w|)$

Then $D = D_{(x,u,t)}(x)$ and $\Phi = \Phi_{(x,u,a,t)}(w)$

Dependence of Φ on u problematic

Subsequent macroscopic theory cannot be developed

Other closures can be done

4. Macroscopic model

Hydrodynamic scaling

Let $D(u)$ be arbitrary and define

$$Q_D(f) = -\nabla_u \cdot (F_D f) + d\Delta_u f$$

$$F_D(u,a) = -\nabla_u \Phi_D(u,a), \quad \Phi_D(u,a) = \frac{k}{2}|D(u)\, u - La|^2$$

For $f(u,a)$ arbitrary, define

$$D_f^{-1}(u) = \frac{\int_{(v,b)\in\mathbb{T}^2} K(|v-u|)\, f(v,b)\, dv\, db}{\int_{(v,b)\in\mathbb{T}^2} f(v,b)\, dv\, db}$$

Then mean-field model can be written

$$\partial_t f + \nabla_x \cdot (cuf) = \frac{1}{\varepsilon} Q_{D_f}(f)$$

'Generalized' Von-Mises (GVM) distributions

For given $D(u)$, solutions f of $Q_D(f) = 0$ are of the form

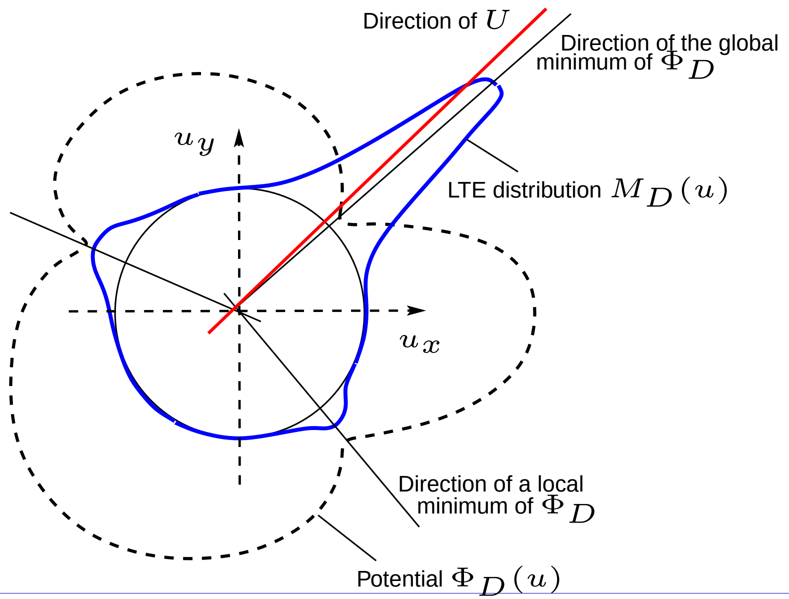
$$f(u,a) = \rho(a)\, M_D(u,a)$$

with $\rho(a)$ arbitrary and

$$M_D(u,a) = \frac{1}{Z_D(a)} \exp\big(- \frac{\Phi_D(u,a)}{d} \big)$$

where $Z_D(a)$ is s.t.

$\int M_D(u,a)\, du = 1$



Solutions f of $Q_{D_f}(f) = 0$:

are GVM $\quad f = \rho(a)\, M_D(u,a)$

such that $\quad D = D_{\rho M_D}$

Leads to a fixed point equation

$$D^{-1}(u) = \frac{\int_{(v,b)\in\mathbb{T}^2} K(|v-u|)\,\rho(b)\,M_D(v,b)\,dv\,db}{\int_{b\in\mathbb{S}^1} \rho(b)\,db}$$

Mathematical theory open

Here we assume that for any function $\rho(a)$:

there exists a 'distinguished' solution D_ρ

Hydrodynamic limit

When $\varepsilon \to 0$, formally we have

$$f^\varepsilon \longrightarrow \rho_{(x,t)}(a)\, M_{D_{\rho_{(x,t)}}}(u,a)$$

where $\rho_{(x,t)}(a)$ satisfies the continuity eq.

$$\partial_t \rho_{(x,t)}(a) + \nabla_x \cdot (c\rho_{(x,t)}(a) U_{\rho_{(x,t)}}(a)) = 0$$

and $U_{\rho_{(x,t)}}(a)$ is the mean equilibrium velocity

$$U_\rho(a) = \int_{u \in \mathbb{S}^1} M_{D_\rho}(u,a)\, u\, du$$

5. Relation to game theory

Game definition

Spatially homogeneous case:

For probability $f(u,a)$, introduce the 'cost function'

$$\mu_f(u,a) = \Phi_{D_f}(u,a) + d \ln f(u,a)$$

Non-cooperative anonymous game with a continuum of players (aka 'Mean-Field Game [Lasry & Lions])

each pedestrian (player) tries to minimize its cost

by acting on its own decision variable u

Nash equilibrium

f_{NE} is a Nash Equilibrium if

No player can reduce its cost by acting on its control variable u

f_{NE} is a Nash Equilibrium iff $\exists K$ s.t.

$$\mu_{f_{\text{NE}}}(u,a) = K, \quad \forall (u,a) \in \text{Supp}(f_{\text{NE}})$$
$$\mu_{f_{\text{NE}}}(u,a) \geq K, \quad \forall (u,a) \in \mathbb{T}^2$$

The following statements are equivalent:

f is an equilibrium of the kinetic model

and is therefore a GVM distribution

f is a Nash Equilibrium for the Mean-Field Game

defined by cost function μ_f

Hydrodynamic model

Spatially inhomogeneous case

Hydrodynamic model is obtained by

- Taking the continuity equation (i.e. taking the first moment of kinetic eq. wrt u)
- Closing the model by taking the local Nash Equilibrium

See a general framework for

- Kinetic models coupled with Mean-Field Games in
  - [D., Liu, Ringhofer, MFG driven by local Nash equilibria, JNLS 2013]
- and applications to economics in
  - [D., Liu, Ringhofer, JSP 2014 and Phil. Trans.. A (to appear)]

6. Conclusion

Heuristic-Based model of Moussaid, Helbing Theraulaz

Takes into account following behavior

Derivation of

Time continuous IBM

instantenous minimization replaced by steepest descent

Mean-Field Model

approximation of local interactions, no blind zone

Hydrodynamic Model

Equilibria can be interpreted as

Nash equilibria of a Mean-Field Game

View of pedestrians as fully rational agents

$\neq$ mechanistic view of most models