Scalable Bayesian Approaches for Freeway Traffic Systems

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Overview

I. Motivation for Vehicular Traffic Systems

II. Particle Methods

- Motivation and why is the approach needed
- Taxonomy of methods for estimation
- General Sequential Monte Carlo method
- Sampling Importance Resampling Particle Filter

III. Particle Filters for Traffic Flow Estimation, vs. UKFs

IV. Parallelised Particle Filters for Traffic Flow Estimation

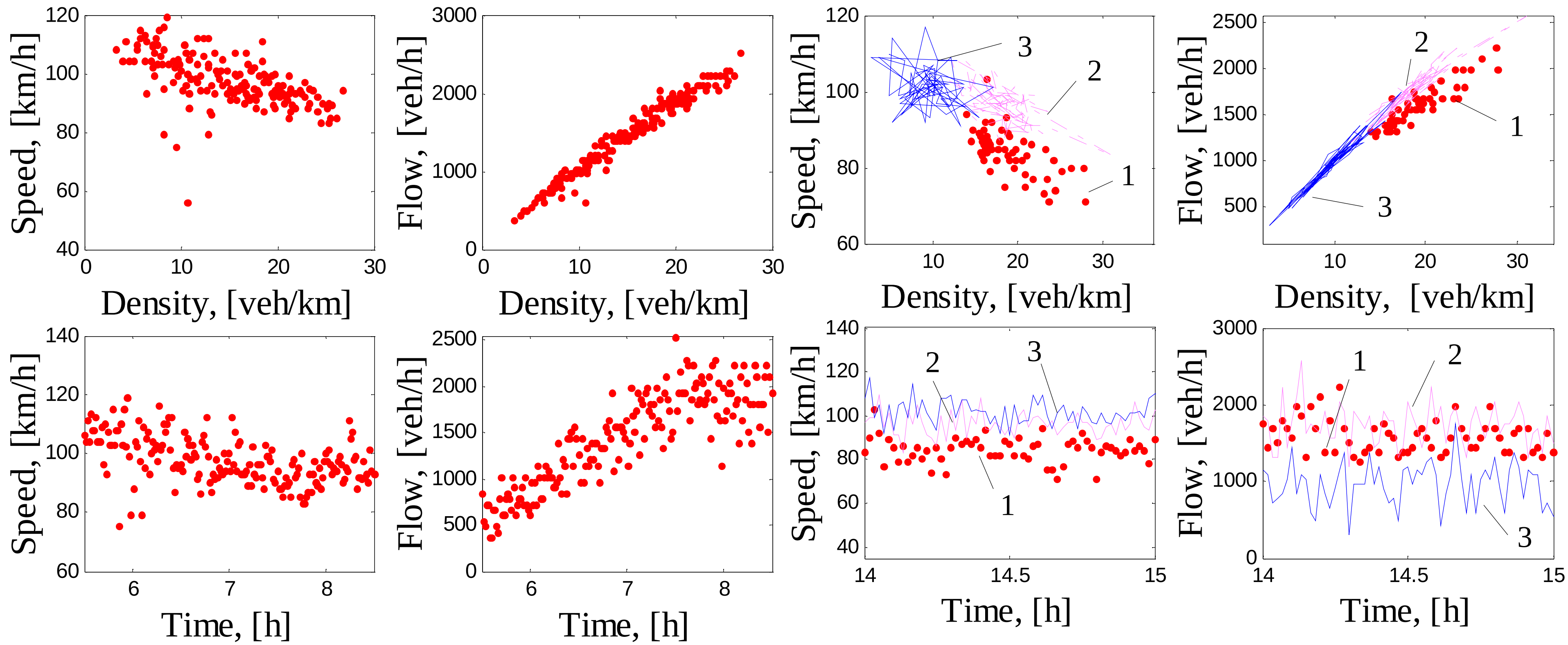
V. PFs for Weather Management in Traffic Systems

VI. Conclusions and Open Issues

Motivation

- Traffic flow on motorways: complex nonstationary, nonlinear phenomenon, with different modes such as: free flow motion, congestions, stop-and-go waves.
- Changes are due to **internal traffic dynamics**, and **external events** (e.g. accidents, road works, weather conditions).

Traffic Modes



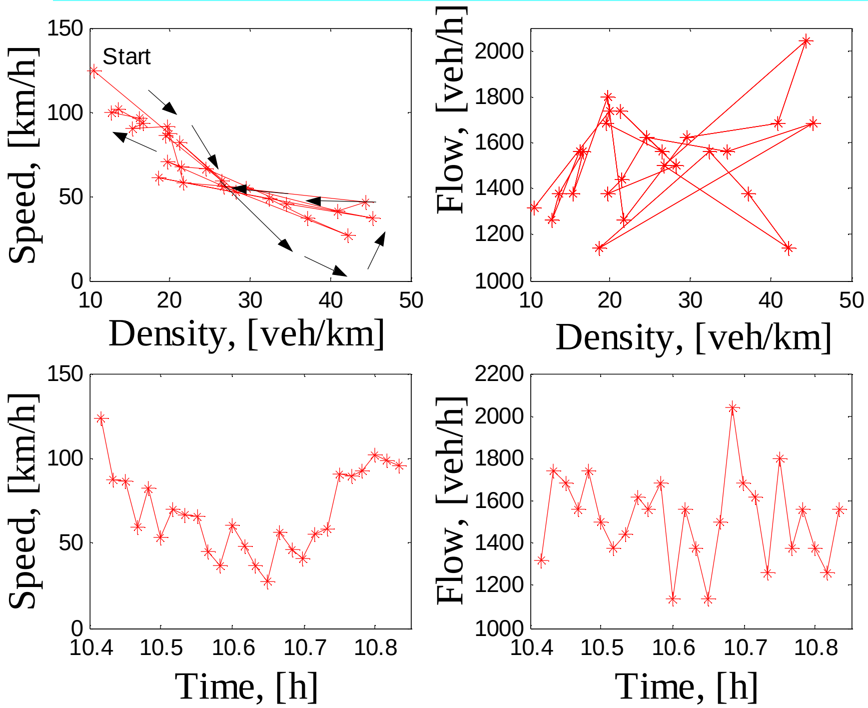
Free Flow Traffic

Synchronised Traffic

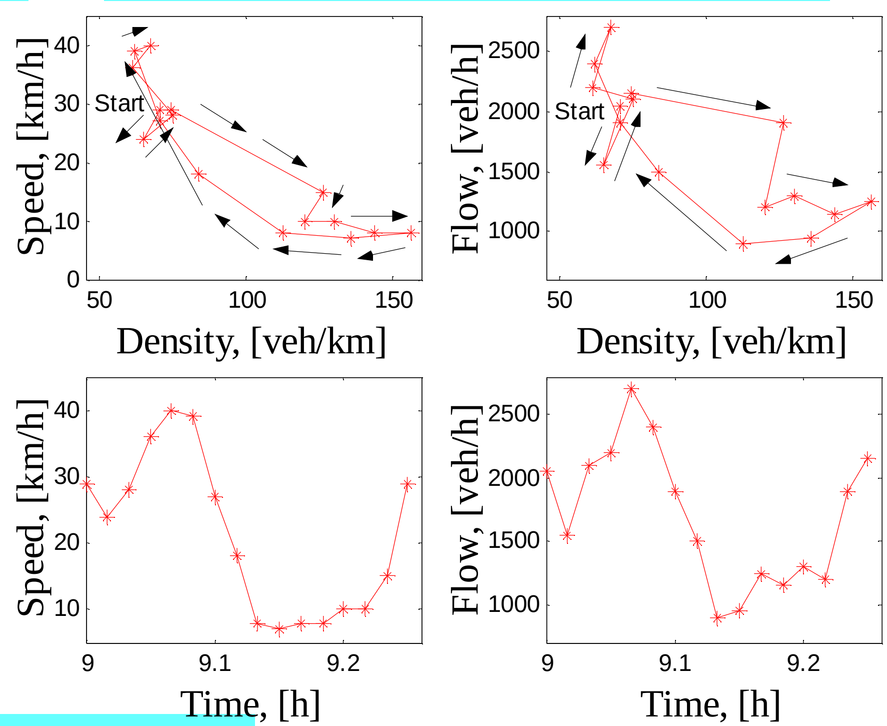
Data from Kennedy tunnel, Belgium, June 26, 2001

Traffic Modes & Transitions

Transition from a Synchronised to a Congested Mode



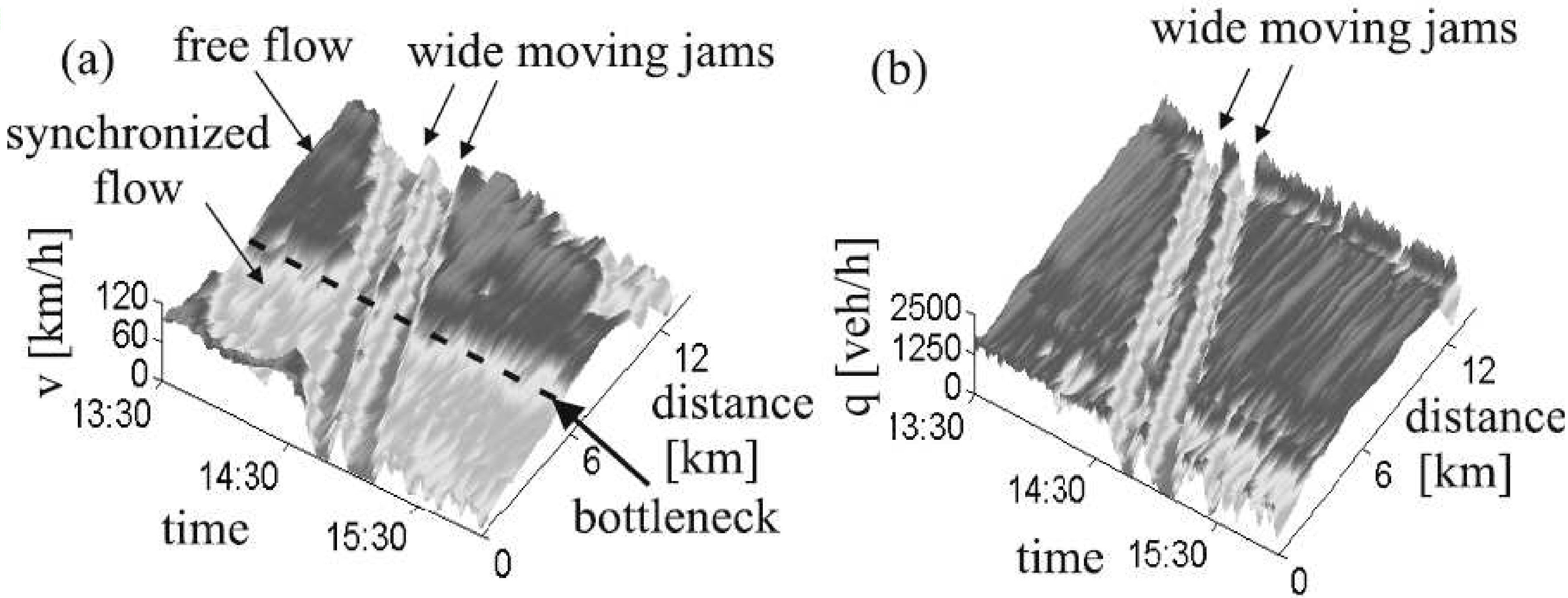
Transition from Congested to a Jammed Mode



Hysteresis loops

Data from Kennedy tunnel, Belgium, June 26, 2001

Traffic Modes



- Example of two wide moving traffic jams propagating in parallel with constant speed through free and congested traffic and across three freeway intersections I1, I2, and I3. From Kerner, 2000a, 2000b; see also Kerner and Rehborn, 1996a; Kerner, 1998b.

Traffic Flow Problems of Interest

* Analysis of traffic phenomena and modes, on-line detection

* Build up traffic and sensor models of traffic on motorways and in urban environment

  - for real-time applications and in block processing

* Develop models and filters reflecting different weather conditions

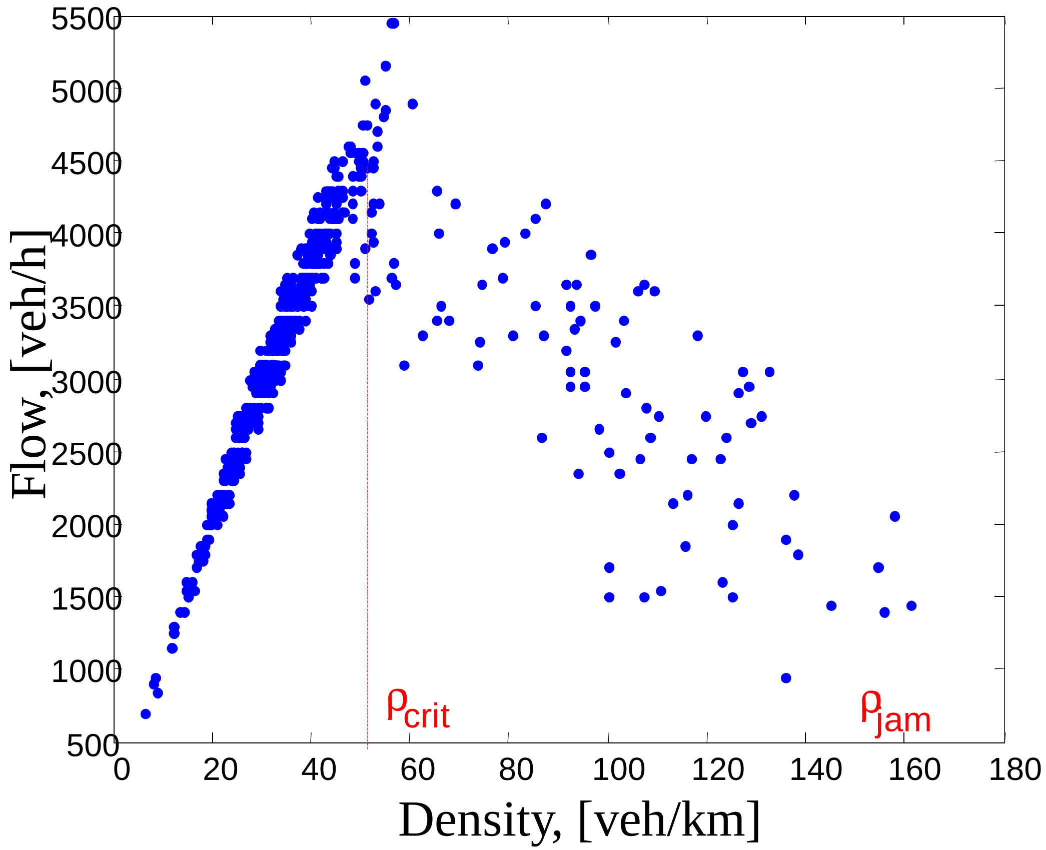
* Predict the traffic evolution over space and time

* Distributed methods for estimation

* Development of methods for traffic control, ameliorate traffic conditions, avoid congestions and jams

Vehicular Traffic Models

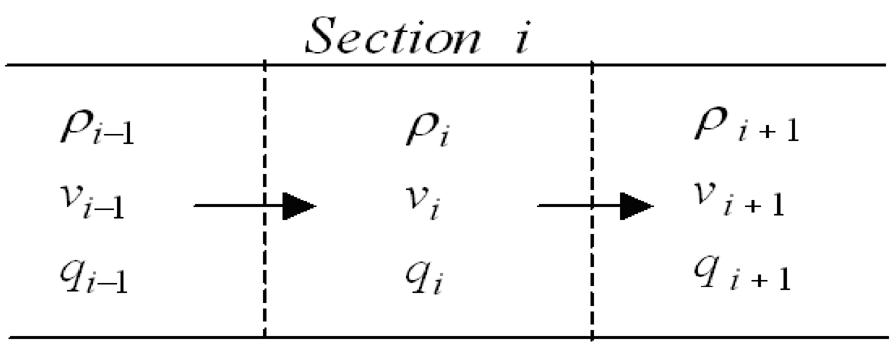
- Microscopic (particle based)
- Macroscopic (fluid-dynamic models)
- Mesoscopic (gas-kinetic)



The Fundamental Diagram

Macroscopic Models Suitable for Real-Time Estimation/ Prediction

- **First order** models:
  - Lighthill-Whitham (1955), Richards (1955), Daganzo (1984)
- **Second order models**, e.g., METANET, Papageorgiou et al. (1989, 2004)
- Other models: Boel and Mihaylova (2004, 2006)



The Cell Transmission Model of Daganzo

• S. Hoogendoorn, P. Bovy, State-of-the-art of Vehicular Traffic Flow Modeling, Journal of Systems Control Engineer – Proceedings of the Institution of Mechanical Engineers, Part I, Vol. 215, No. 14, pp. 283-303, 2001.
• D. Helbing, Traffic and Related Self-driven Many-particle Systems, Review Modern Physics, Vol. 73, pp. 1067-1141, 2002.

The Problem of Interest

- We are mainly interested in **estimating** the state x at time k from the measurements up to time

  $k'=k$ (**filtering**),

  and

  prediction $k' > k$

  opposite to

  smoothing $k' < k$

- No restrictions to linear processes or Gaussian noises

The Dynamic System Model

- **State transition (motion) equation**

$x_k = f(x_{k-1}, u_{k-1}, v_{k-1})$

$f(.)$: evolution function (possibly nonlinear)

$x_{k,}\, x_{k-1} \in \Re^{n_x}$: current and previous state

$v_{k-1} \in \Re^{n_v}$: system noise (usually non-Gaussian)

u_{k-1}: $\Re^{u}$ known input (control process, e.g.)

The state depends only on the previous step: i.e. first order Markov process

- **Measurement equation**

$z_k = \mathrm{h}(x_k, n_k)$

$z_k \in \Re^{n_z}$: measurement

$h(.)$: measurement function (possibly nonlinear)

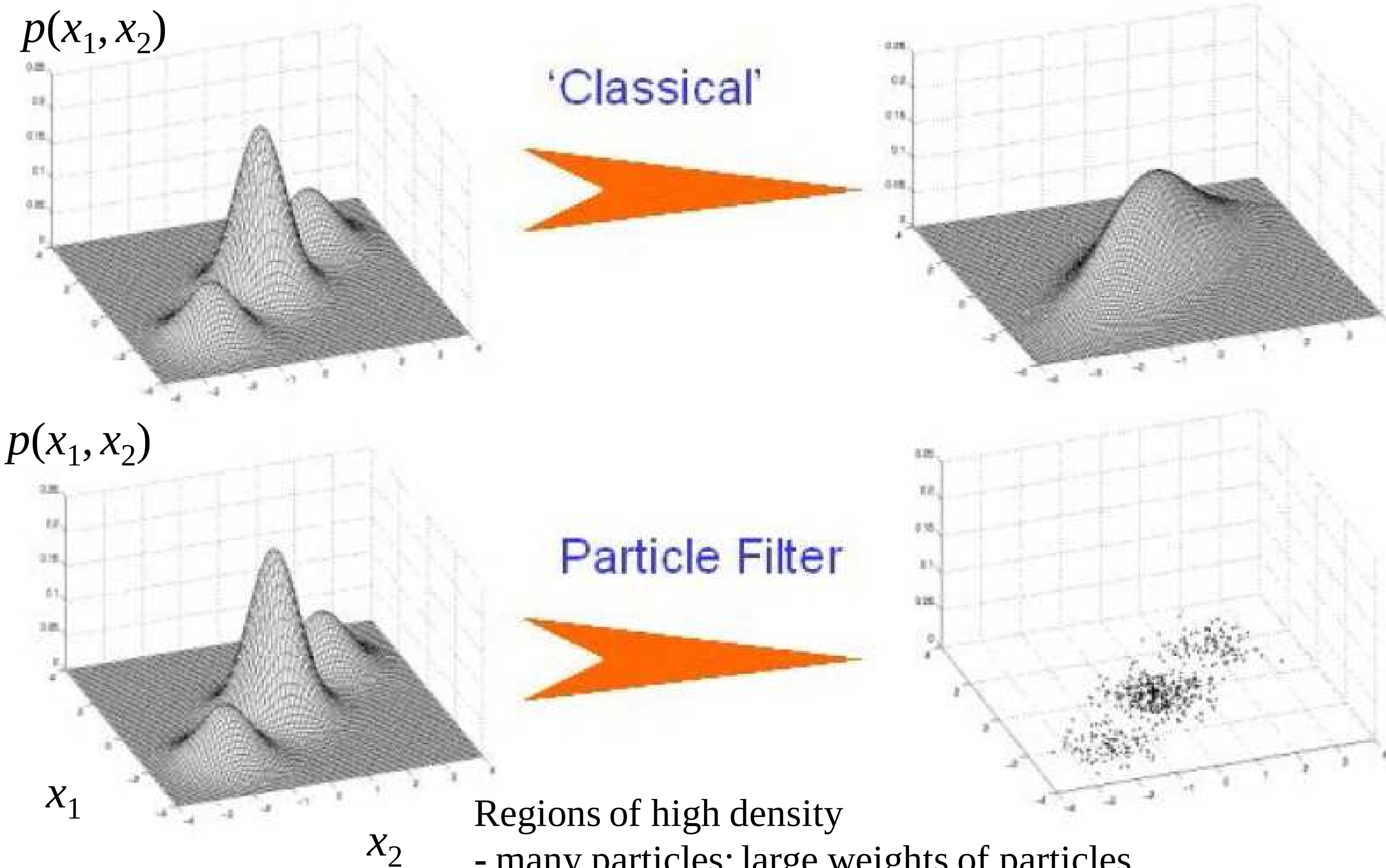
n_k : measurement noise (usually non-Gaussian)

Main Methods for Estimation

- Methods with **linear models** and **Gaussian noises**
  - the Kalman filter
  - the Extended KF: requires linearisation (difficult for the traffic models with interconnected components)
  - derivative free filters: unscented KF, central differences filter, others (can work with nonlinear models)

- **Mixtures of Gaussian models**
  - the Gaussian sum filter
- **Nonparametric methods**
  - Particle filtering methods
- Models with partial linear structure (work with KF), nonlinear part solve with PFs: Rao-Blackwellisation (Karlsson et al, 2005, Mihaylova et al 2007)

Why Sequential Monte Carlo Methods



Regions of high density
- many particles; large weights of particles
Discrete approximations of continuous pdf

Suitable for:

- **nonlinear models** (for traffic dynamics, measurement equation)
- **non-Gaussian** noises, **multimodalities**, affords incorporation of **constraints**
- able to cope with different **uncertainties** (in the data, models)
- allows for **fusing information** from different measurement sources
- applicable to **real-time** problems, can be **parallelised**

Bayesian Methodology

$$\Pr(X \mid D) = \frac{\Pr(D \mid X)\Pr(X)}{\Pr(D)}$$

Bayes Rule

- X: state, D: data
- Prediction
- Correction step

Posterior PDF = likelihood * prior / evidence

Traffic State Estimation Within Bayesian Framework

- The posterior state probability density function (PDF) is estimated given a data set

$$z_{1:k} = \{z_1, z_2, \ldots, z_k\}$$

- The sensor information updates recursively the state distribution.

Prediction : $p(x_k \mid z_{1:k-1}) = \int p(x_k \mid x_{k-1}) p(x_{k-1} \mid z_{1:k-1}) dx_{k-1}$

Update : $p(x_k \mid z_{1:k}) = \dfrac{p(z_k \mid x_k) p(x_k \mid z_{1:k-1})}{p(z_k \mid z_{1:k-1})}$

The conditional state PDF $p(x_k / z_{1:k})$ is represented as a set of random samples which are updated and propagated by a particle filter.

The General Monte Carlo Method

- Method to solve intractable integrals, e.g., with complex PDF

$$E[x] = \int g(x)p(x)dx$$

- g(x): some function, *p*(*x*): a PDF with complex form
- Since this integral cannot be solved analytically, random samples are generated from p(*x*) by representing the PDF with random samples

$$p(x) = \frac{1}{N}\sum_{i=1}^{N}\delta(x - x^{(i)})$$

- and the Monte Carlo approximation follows

$$E[x] = \int g(x)p(x)dx \approx \int g(x)\frac{1}{N}\sum_{i=1}^{N}\delta(x - x^{(i)})dx = \frac{1}{N}\sum_{i=1}^{N}g(x^{(i)})$$

Doucet et al (2001), Liu (2001)

Update Step

$$p(x_k \mid z_{1:k}) = \frac{p(z_k \mid x_k)\,p(x_k \mid z_{1:k-1})}{p(z_k \mid z_{1:k-1})}$$

Posterior state PDF = likelihood * prior / evidence

- **Prediction**: $p(x_k|x_{k-1})$, from the transition prior, e.g., METANET or another traffic model
- **Update**: from the likelihood, based on the observation model
  - Usually concentrates the state PDF by combining the likelihood of current measurement with the predicted state.

- **Evidence**: the normalising term

$$p(z_k \mid z_{1:k-1}) = \int p(z_k \mid x_k)\,p(x_k \mid z_{1:k-1})dx_k$$

A Particle Filter for Traffic Flow Estimation

I. Initialisation : $k = 0$.

Generate samples $\{x_0^{(l)}\} \sim p(x_0), \quad l = 1,2,\ldots,N$ and initial weights $w^{(i)} = 1/N$

II. For $k = 1,2,\ldots,$

(1) Prediction: sample according to the traffic model $x_k^{(l)} \sim p(x_k \mid x_{k-1}^{(l)})$ for segments between two boundaries where measurements arrive

(2) Measurement processing step (only for $t_k \equiv t_s$ on boundaries between segments where measurements are available

For $l = 1,2,\ldots, N,\ w_s^{(l)} = w_{s-1}^{(l)} p(z_s \mid x_s^{(l)})$

Normalise the weights: $\hat{w}_s^{(l)} = w_s^{(l)} / \sum_{l=1}^{N} w_s^{(l)}$

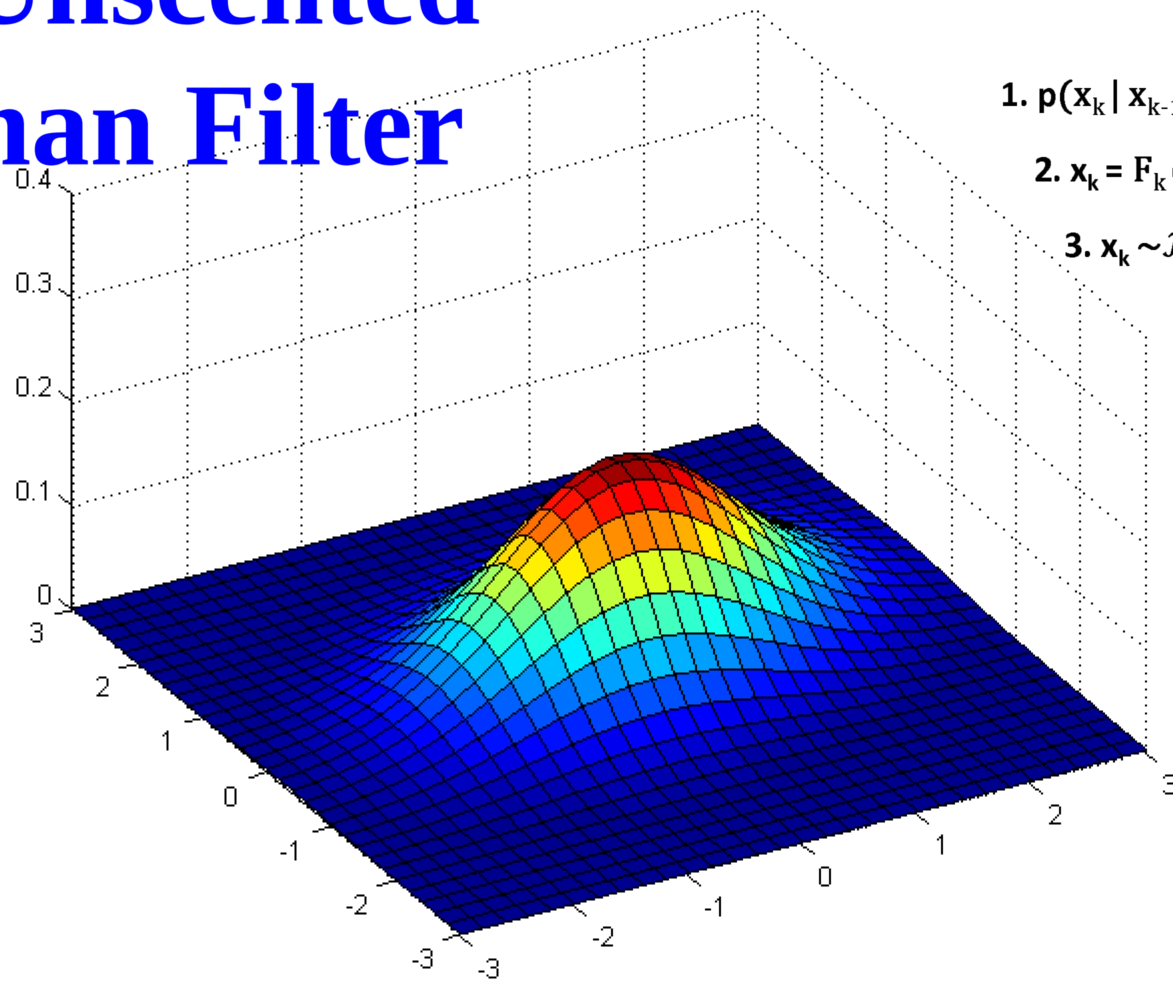
(3) Output $\hat{x}_s = \sum_{l=1}^{N} w_s^{(l)} x_s^{(l)}$,

(4) Selection (resampling) step

(5) Increase k and iterate to step 2.

Mihaylova, Boel, Hegyi (2004, 2007, 2011)

The Unscented Kalman Filter



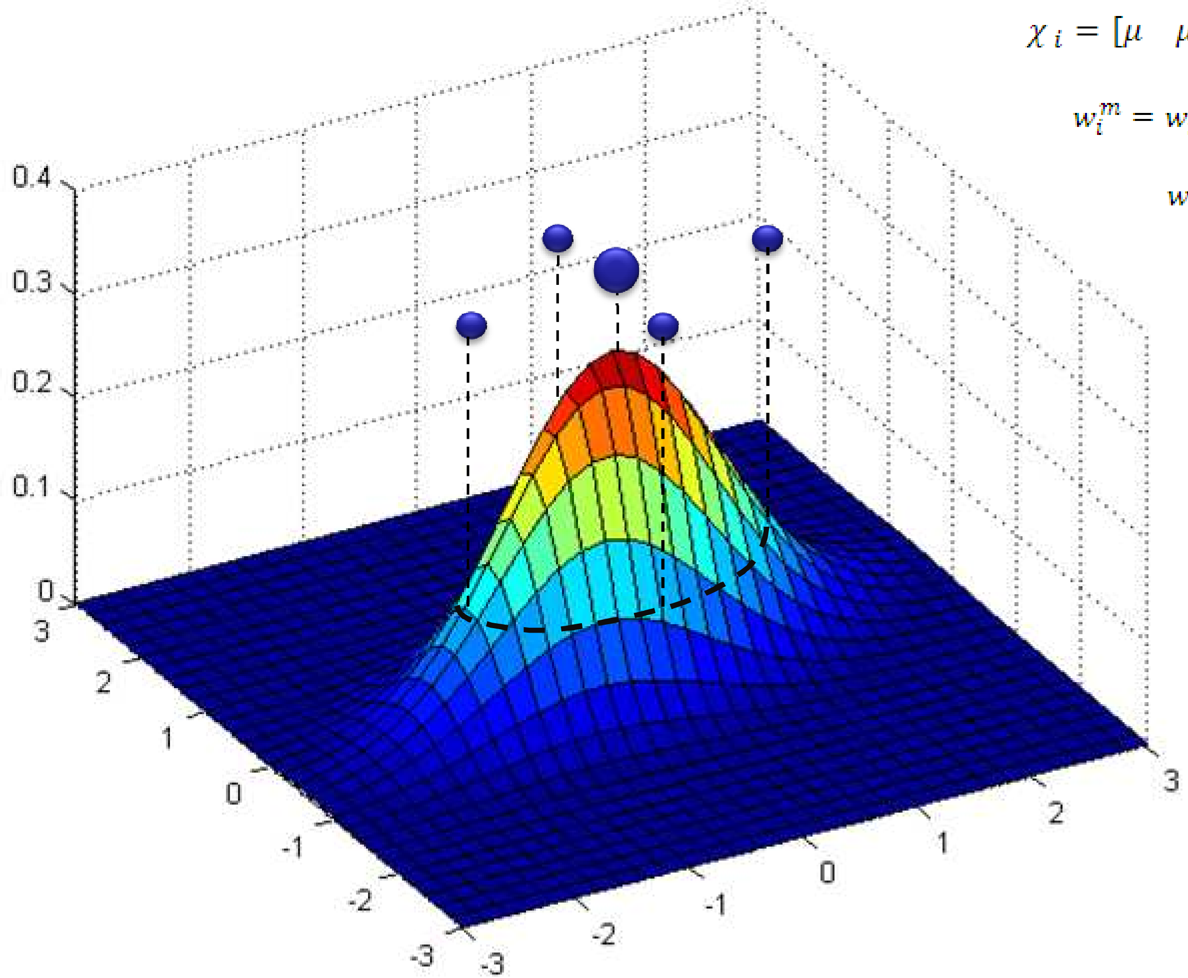
1. $p(x_k \mid x_{k-1}, z_{k-1}) \sim \mathcal{N}(\mu_k, \Sigma_k)$

2. $x_k = F_k(x_{k-1}, \nu_k) \qquad \nu_k \sim \mathcal{N}(0, Q_k)$

3. $x_k \sim \mathcal{N}(F_k(\mu_{k-1}), F_k \Sigma_{k-1} F_k^T + Q_k)$

1. The system state can be represented by a probability distribution

2. The system state equations describe how the dynamical system evolves with the passage of time and relate the states to the measurements.
3. Propagation typically diffuses and translates the prior distribution yielding the *a priori* distribution.



$$\chi_i = [\mu \quad \mu - \gamma(\sqrt{P})_i \quad \mu + \gamma(\sqrt{P})_i]$$

$$w_i^m = w_i^c = \frac{1}{2[N_x + \lambda]} \quad i = 1, \ldots, 2N_x$$

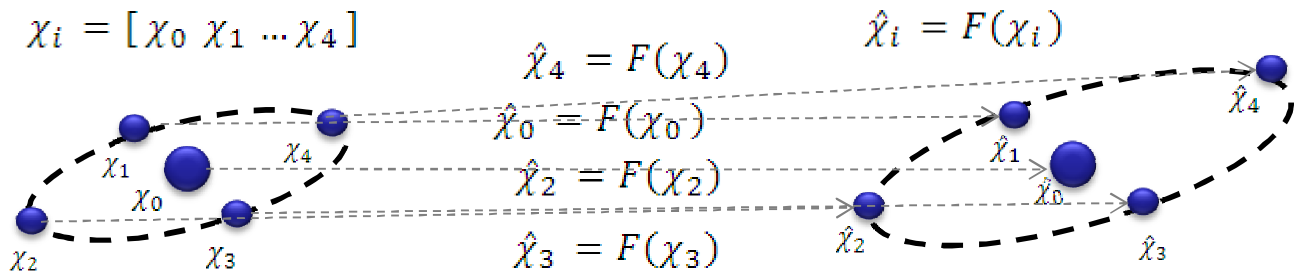
$$w_0^m = \frac{\lambda}{N_x + \lambda}, \lambda = \sqrt{\alpha^2(N_x + \kappa)}$$

$$w_0^c + (1 - \alpha^2 + \beta)$$

1. The UKF uses a set of weighted deterministically selected sample points to represent the prior distribution with mean and covariance P.

2. Weights for each point to incorporate prior knowledge of the distribution.

3. 2N+1 sigma-points can be utilised to completely describe the second order statistics of a probability distribution. These statistics can be recovered by a weighted sum of the σ-points.



$$\mu = \sum_{i=0}^{n} w_i^m \chi_i$$

$$Cov = \sum_{i=0}^{n} w_i^c (\chi_i - \mu)(\chi_i - \mu)^T$$

$$\hat{\mu} = \sum_{i=0}^{n} w_i^m \hat{\chi}_i$$

$$Cov = \sum_{i=0}^{n} w_i^c (\hat{\chi}_i - \mu)(\hat{\chi}_i - \mu)^T$$

1. The set of sigma points describe the sufficient statistics of the prior distribution

2. The sigma points are propagated through the system state equations.

3. The mean and covariance of the transformed distribution (a priori state estimate) can be calculated by a weighted sum of the transformed sample points.

Results from the PF and UKF with Synthetic Data

METANET

Discretised in space and time stochastic system model (METANET)

$$\rho_i(k+1) = \rho_i(k) + \frac{\Delta t}{L_i l_i}[q_{i-1}(k) - q_i(k)] + \eta_\rho(k),$$

Law of conservation of vehicles

$$q_i(k) = \rho_i(k) v_i(k) l_i,$$

$$v_i(k+1) = v_i(k) + \underbrace{\frac{\Delta t}{L_i} v_i(k)[v_{i-1}(k) - v_i(k)]}_{convection\quad term} + \underbrace{\frac{\Delta t}{T}\{v^e[\rho_i(k)] - v_i(k)\}}_{relaxation\quad term} - \underbrace{\frac{\nu\,\Delta t[\rho_{i+1}(k) - \rho_i(k)]}{T\,L_i[\rho_i(k) + \kappa]}}_{anticipation} + \eta_v(k),$$

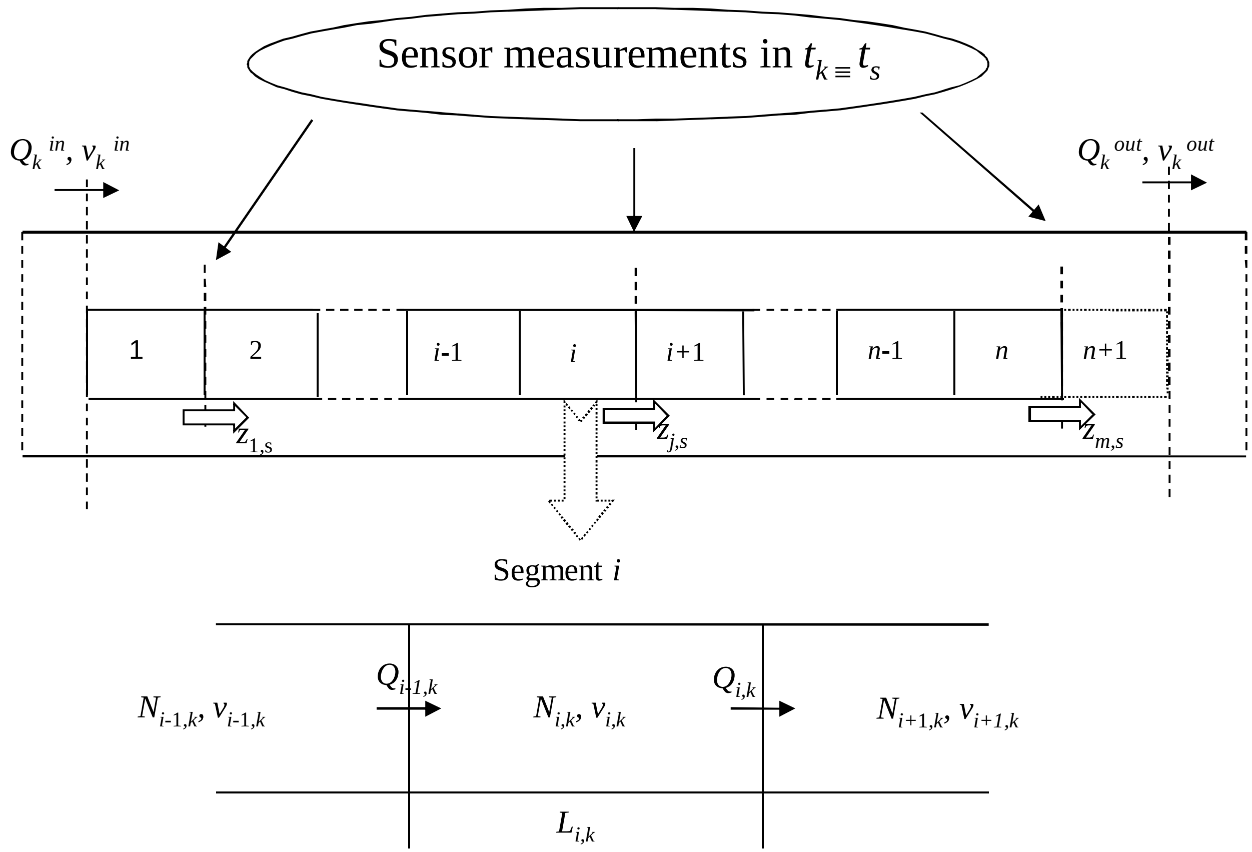
$$v^e[\rho_i(k)] = v_f \exp\left\{ -\frac{1}{a_m}\left(\frac{\rho_i(k)}{\rho_{crit}}\right)^{a_m} \right\}$$

$\rho_i(k)$ traffic density [*veh/km/lane*], number of vehicles per length unit per lane

$v_i(k)$ average speed [*km/h*], $q_i(k)$ traffic flow [*veh/h/lane*]; l_i - the number of lanes.

A Compositional Traffic Model

- Stochastic
- Dynamic sending and receiving functions
- Comprises an equation for the speed



$$L \geq v_{max} \Delta t$$

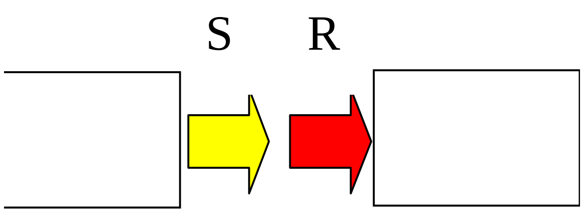
Boel and Mihaylova (2004 and 2006)

Sending and Receiving Functions

- **Sending flow S:** the volume of traffic that can leave a cell
- **Receiving flow R**: the volume of traffic that a cell can receive
- **Capacity q_capacity**: the theoretical maximal flow associated to the critical density

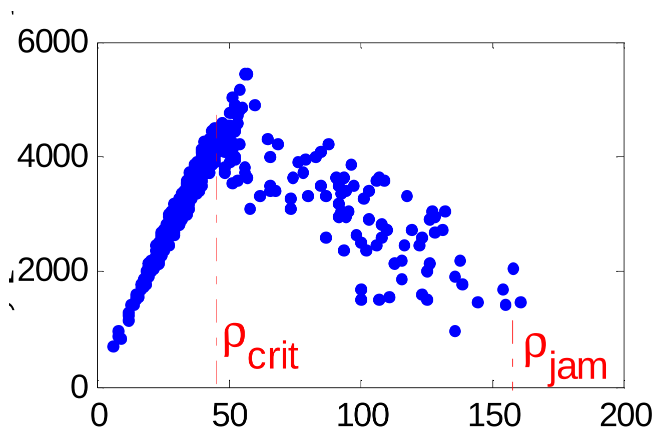
$$S_k = \begin{cases} q_{free\ flow}, & for\ \rho < \rho_{crit} \\ q_{capacity}, & for\ \rho \geq \rho_{crit} \end{cases} \qquad R_k = \begin{cases} q_{free\ flow}, & for\ \rho \geq \rho_{crit} \\ q_{capacity}, & for\ \rho < \rho_{crit} \end{cases}$$

$$q_k = \min\{S_k, R_k, q_{capacity}\}$$

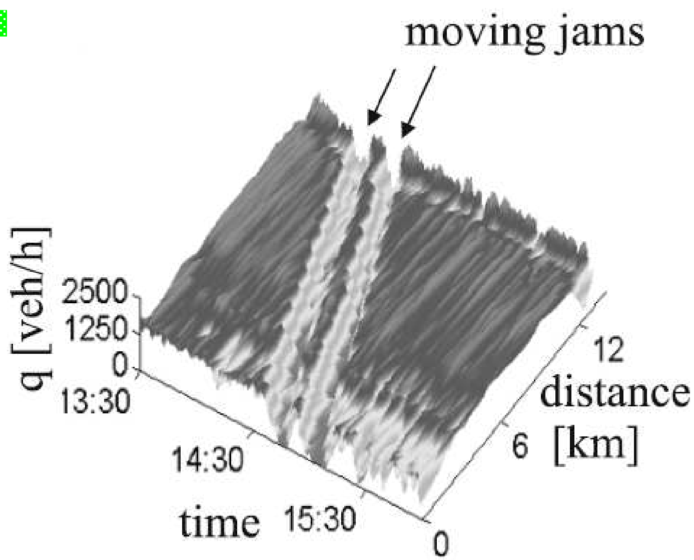


- deterministic, static,
- no speed relationships

Daganzo, 1994



Compositional Traffic Model



$$x_{1,k+1} = f_1(Q_k^{in}, v_k^{in}, x_{1,k}, x_{2,k}, \eta_{1,k}),$$
$$x_{i,k+1} = f_i(x_{i-1,k}, x_{i,k}, x_{i+1,k}, \eta_{i,k}),$$
$$x_{n,k+1} = f_n(x_{n-1,k}, x_{n,k}, Q_k^{out}, v_k^{out}, \eta_{n,k}),$$

$$x_k = (x_{1,k}^T, x_{2,k}^T, \ldots, x_{n,k}^T)^T$$

$$x_{i,k} = (N_{i,k}, v_{i,k})^T$$

Boel & Mihaylova (2006, TRB)

1. *Forward wave*: for $i = 1, 2, \ldots, n$

$$S_{i,k} = max\left(N_{i,k}\frac{v_{i,k}.\Delta t_k}{L_i} + \eta_{S_i,k},\ N_{i,k}\frac{v_{min}.\Delta t_k}{L_i}\right) \quad (4)$$

and set $Q_{i,k} = S_{i,k}$. (5)

2. *Backward wave*: for $i = n, n-1, \ldots, 1$

$$R_{i,k} = N_{i+1,k}^{max} - N_{i+1,k} + Q_{i+1,k}, \quad (6)$$

where $N_{i+1,k}^{max} = (L_{i+1}\ell_{i+1,k})/(A_\ell + v_{i+1,k}t_d)$.

$$if \quad S_{i,k} < R_{i,k}, \quad Q_{i,k} = S_{i,k}, \quad (7)$$

$$else \quad Q_{i,k} = R_{i,k}, \quad v_{i,k} = Q_{i,k}L_i/(N_{i,k}\Delta t_k), \quad (8)$$

3. Update the number of vehicles inside segments,

for $i = 1, 2, \ldots, n$

$$N_{i,k+1} = N_{i,k} + Q_{i-1,k} - Q_{i,k}, \quad (9)$$

4. Update the density, for $i = 1, 2, \ldots, n$

$$\rho_{i,k+1} = N_{i,k+1}/(L_i\ell_{i,k+1}), \quad (10)$$

$$\rho_{i,k+1}^{antic} = \alpha\rho_{i,k+1} + (1-\alpha)\rho_{i+1,k+1}. \quad (11)$$

5. Update of the speed, for $i = 1, 2, \ldots, n$

$$v_{i,k+1}^{interm} = \begin{cases} \frac{v_{i-1,k}Q_{i-1,k} + v_{i,k}(N_{i,k} - Q_{i,k})}{N_{i,k+1}}, & \text{for } N_{i,k+1} \neq 0, \\ v_f, & \text{otherwise}, \end{cases}$$

$$v_{i,k+1}^{interm} = max(v_{i,k+1}^{interm}, v_{min}),$$

$$v_{i,k+1} = \beta_{k+1}v_{i,k+1}^{interm} + (1-\beta_{k+1})v^e(\rho_{i,k+1}^{antic}) + \eta_{v_i,k+1},$$

where

$$\beta_{k+1} = \begin{cases} \beta^I, & \text{if } |\rho_{i+1,k+1}^{antic} - \rho_{i,k+1}^{antic}| \geq \rho_{threshold}, \\ \beta^{II} & \text{otherwise.} \end{cases}$$

Measurement Equations

Consider m sensors along the freeway stretch. Traffic states are measured at discrete time instants t_1, t_2, …, t_s, …,

Overall measurement vector:

$$z_s = \left(z_{1,s}^T, z_{2,s}^T \ldots, z_{m,s}^T\right)^T$$

$$z_{j,s} = \left(Q_{j,s}, v_{j,s}\right)^T, \quad j \in J = \{1,2,\ldots,m\}$$

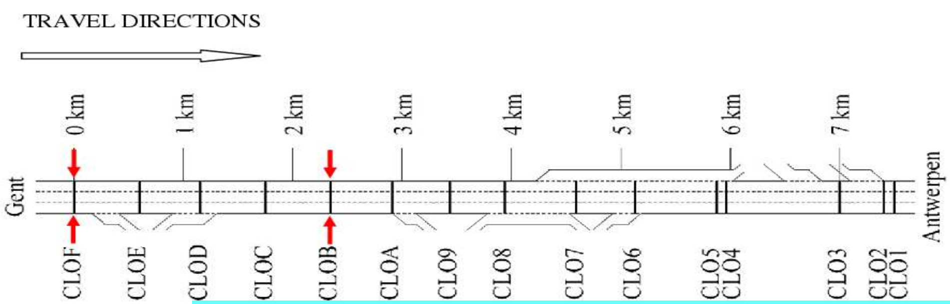
The measurement intervals $\Delta t_s = t_{s+1} - t_s$ is typically several times longer than the time update interval $\Delta t_k = t_{k+1} - t_k$

$$z_s = h(x_s, \xi_s),$$

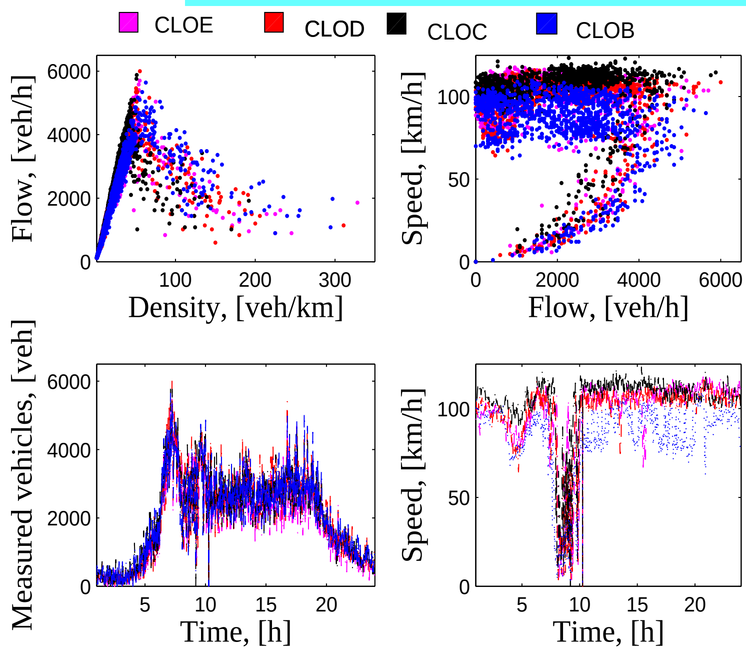
where

$$z_{j,s} = \begin{pmatrix} \overline{Q}_{j,s} \\ \overline{v}_{j,s} \end{pmatrix} + \xi_{j,s}$$

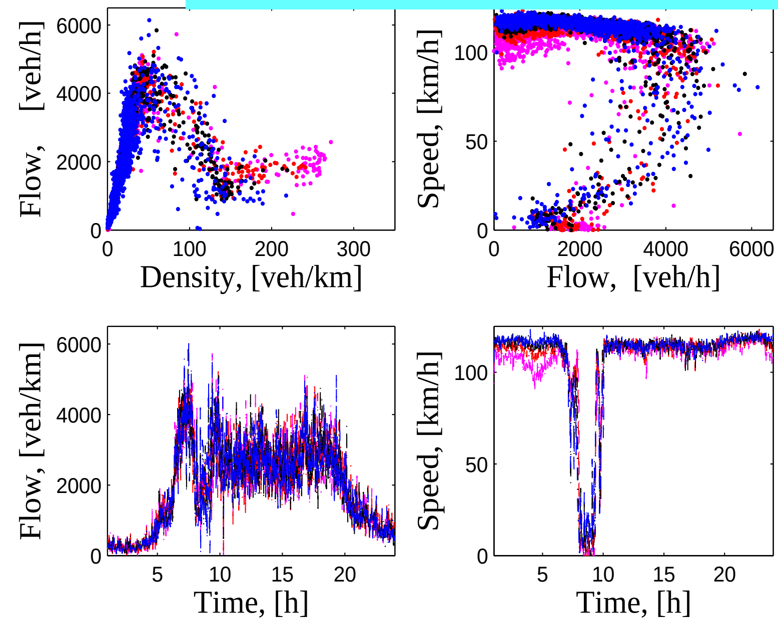
Results from Modelling. Comparison with Real Data from Belgium (Gent-Antwerp)



Real data



Results from the developed compositional model



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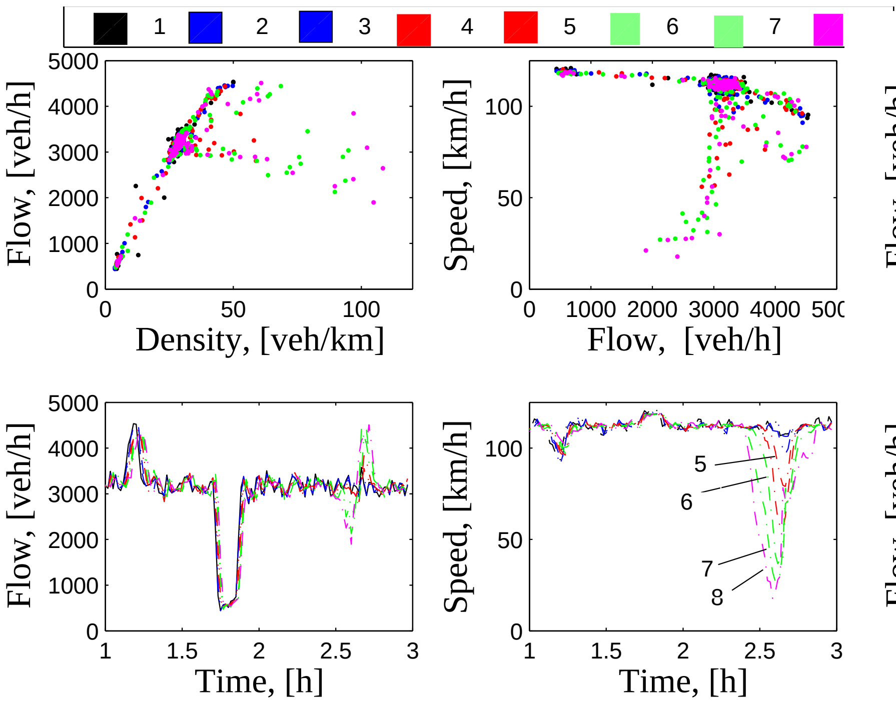
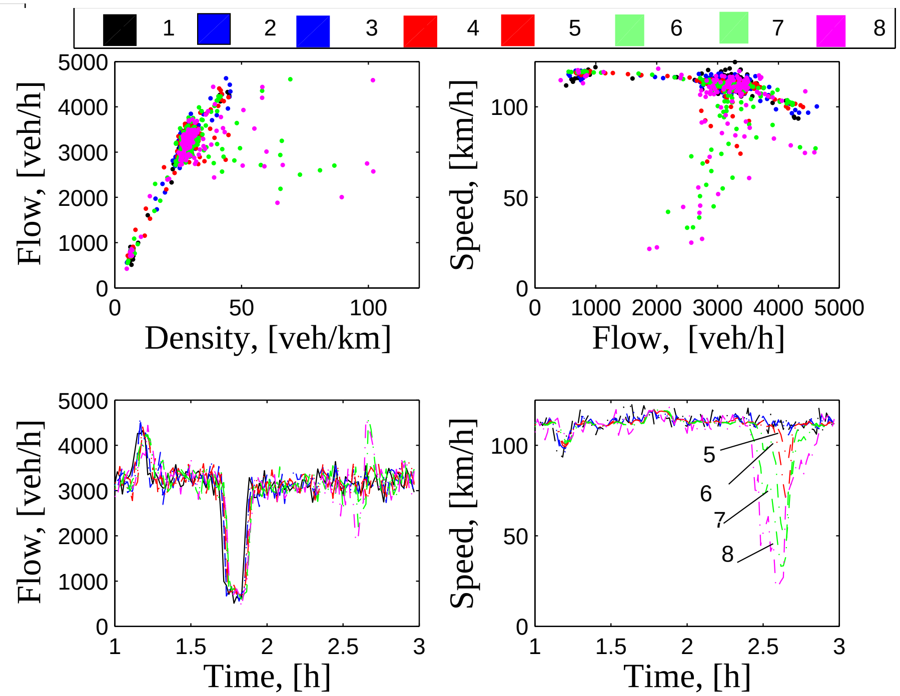
Results with Synthetic Data

Particle Filter

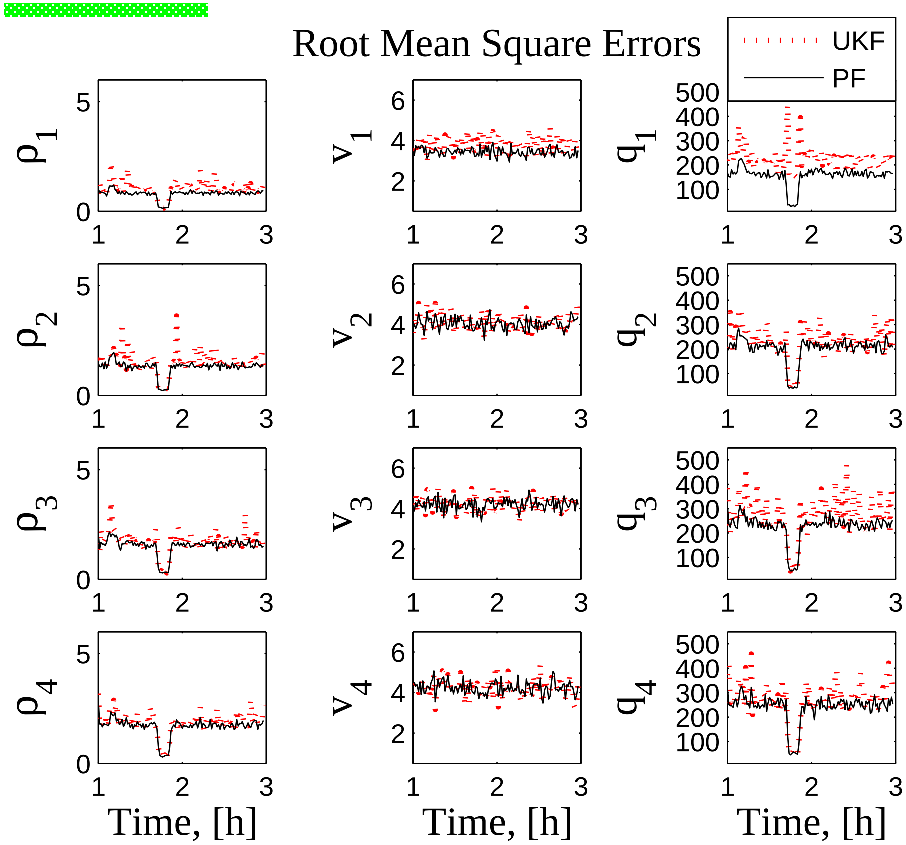


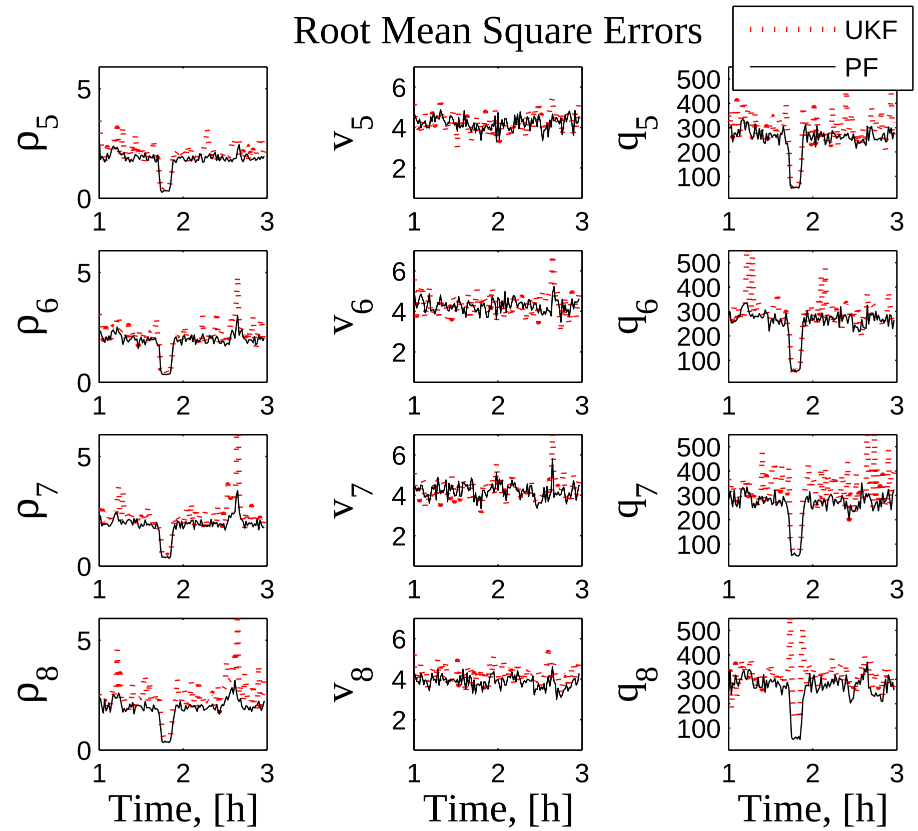
Unscented Kalman Filter



Measurements: in segments 1 and 8

Results with Synthetic Data





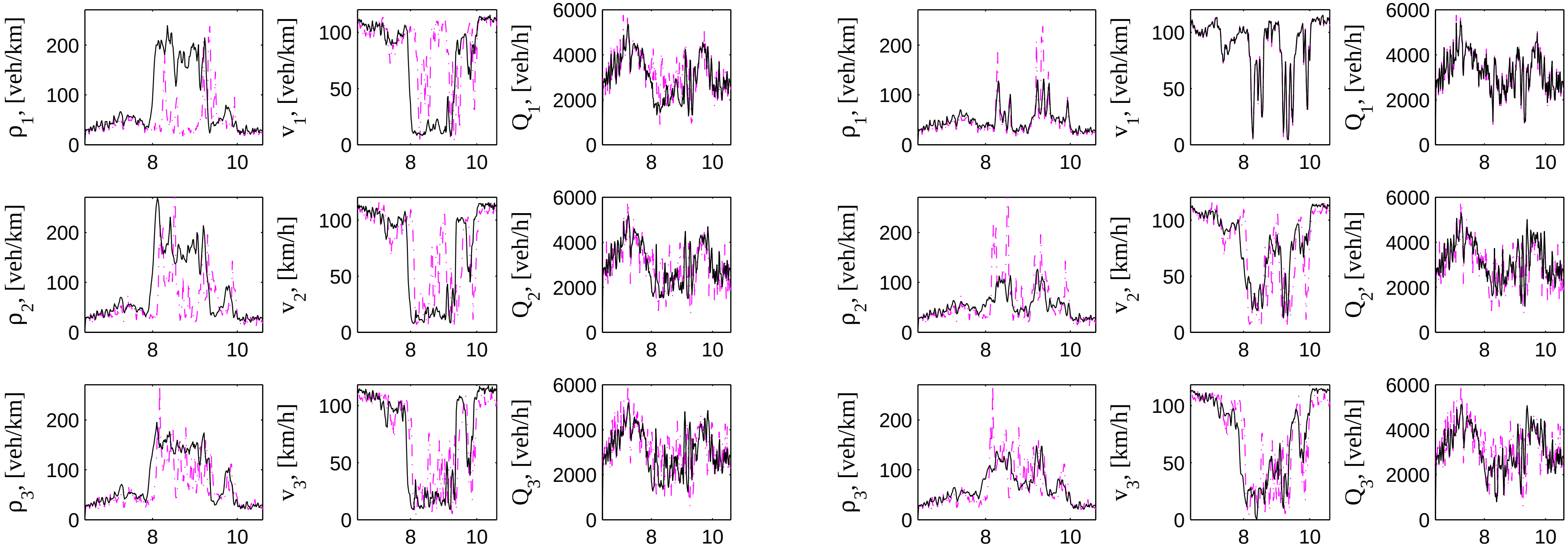
Results with Real Data from the Particle Filter

PF

PF estimates: solid line
Measured: dashed line

UKF

UKF estimates: solid line
Measured: dashed line



Segments 1-3

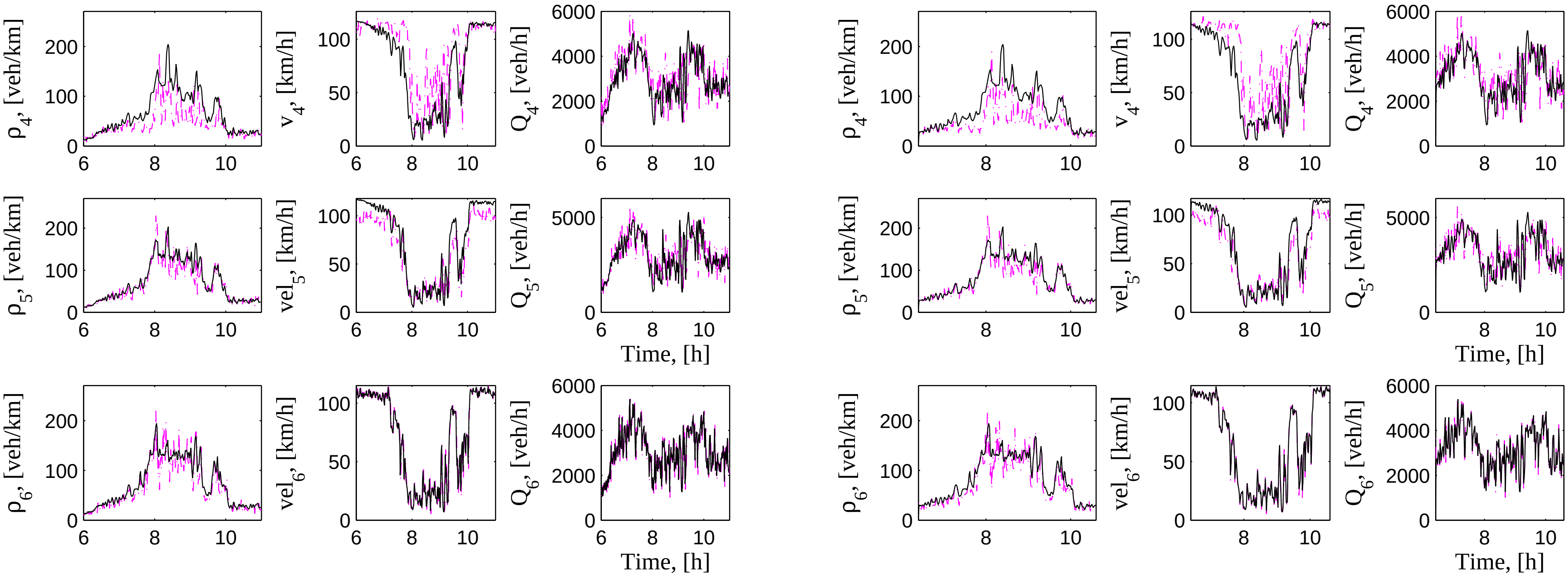
Results with Real Data from the Particle Filter

PF

PF estimates: solid line
Measured: dashed line

UKF

UKF estimates: solid line
Measured: dashed line



Segments 4-8

Parallelised Particle Filters for Freeway Traffic State Estimation

L. Mihaylova, A. Hegyi, A. Gning and R. Boel, Parallelized Particle and Gaussian Sum Particle Filters for Large Scale Traffic Systems, I*EEE Transactions on Intelligent Transportation Systems*, 2011, in press

A. Hegyi, L. Mihaylova, R. Boel and Z. Lendek, Parallelized Particle Filtering for Freeway Traffic State Tracking, *Proc. of the European Control Conf.*, Greece, 2007, TuD15.3, pp. 2442-2449

L. Mihaylova, R. Boel, A. Hegyi, Freeway Traffic Estimation within Recursive Bayesian Framework, *Automatica*, 2007, Vol. 43, No. 2, pp. 290-300, February.

Parallelised Particle Filters for Freeway Traffic State Estimation

<u>Aims</u>:

- Cope with the high computational demands.
- For traffic state estimation the required number of particles grows exponentially with network size.
- Achieve:
  - high accuracy
  - deal with nonlinearities and non-Gaussian processes

<u>Approach:</u> Parallelise the traffic network

- Why parallelisation is possible:
  - A traffic network can be simulated in parallel (limited interaction at subnetwork boundaries),
- Measurements are related to local states.

Related Works

Algorithms transmitting:

- particles and their weights between processing units (PUs)
- communicating statistical characteristics

- G. Hendeby, R. Karlsson, F. Gustafsson, Particle Filtering: The Need for Speed, *EURASIP Journal on Advances in Signal Processing*, 2010.
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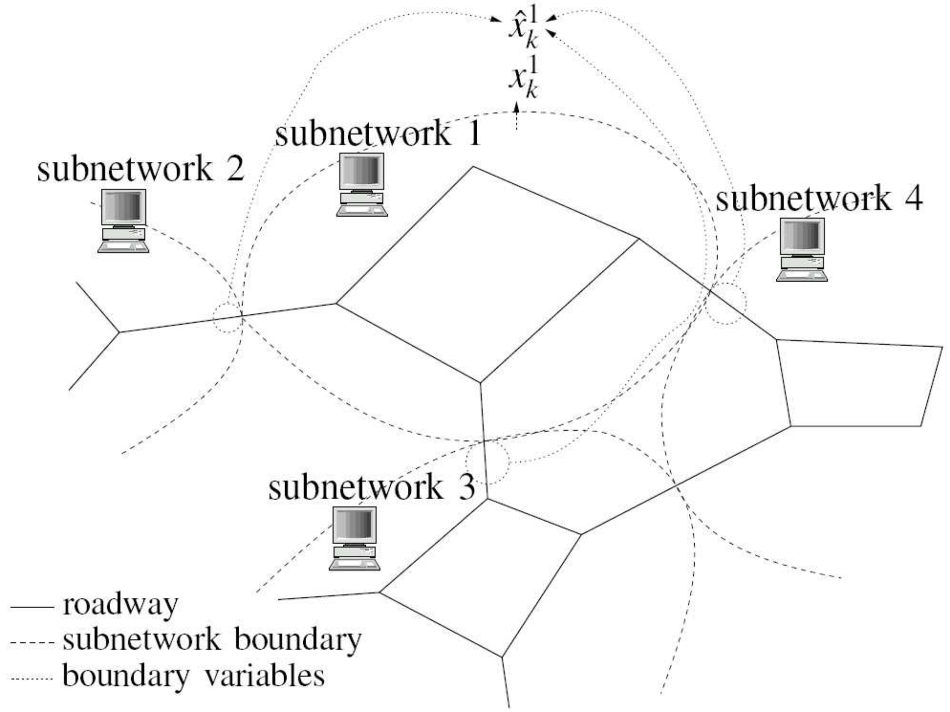
Main Idea: Partition the Traffic Network into Subnetworks



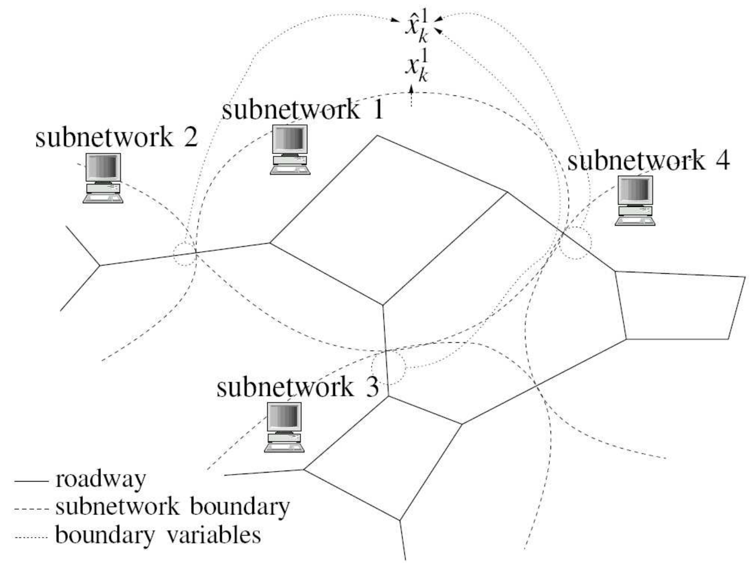
- Applicable: when the whole traffic state vector can be partitioned into subsets of states and most interactions are within the subsets
- A traffic network can be **simulated** in parallel
- Divide the traffic network into sub-networks where each PU is responsible for one sub-network.
- Variables of neighbouring segments are communicated

Partitioning the Traffic Network into Subnetworks

The state and measurement vectors are partitioned into S subvectors

$$x_k = [(x_k^1)^T, (x_k^2)^T, \ldots, (x_k^S)^T]^T$$

$$z_k = [(z_k^1)^T, (z_k^2)^T, \ldots, (z_k^S)^T]^T$$



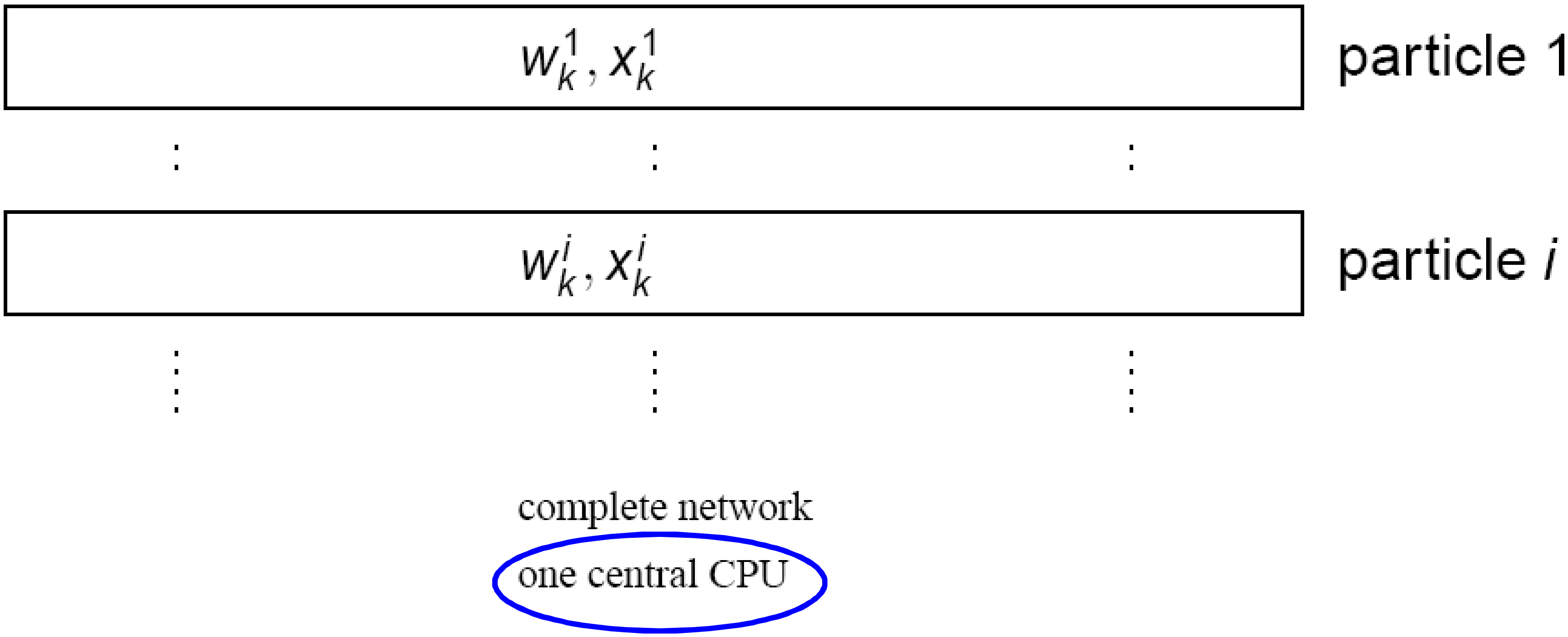
$$x_k^s = f_k^s(x_{k-1}^s, \hat{x}_{k-1}^s, v_{k-1}^s),$$

$$z_k^s = h_k^s(x_k^s, n_k^s),$$

Two types of states:
Internal and boundary states

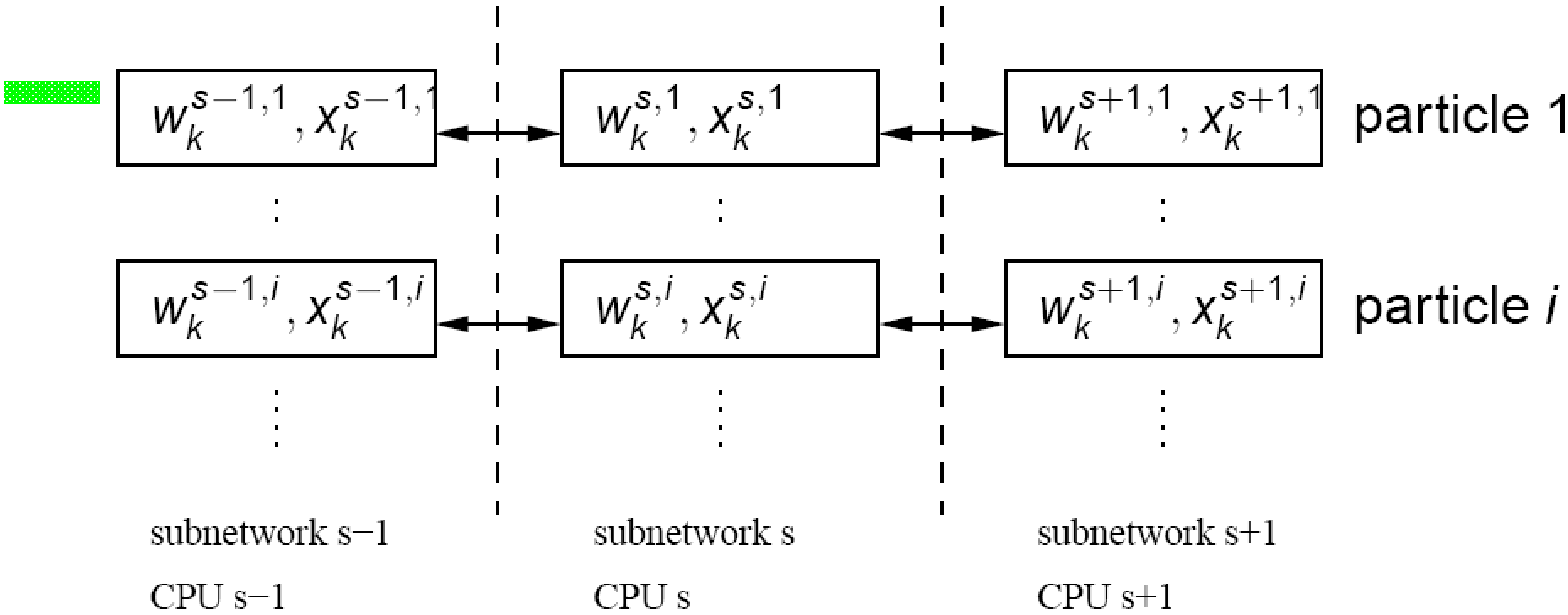
The vector $\hat{x}_{k-1}^s$ collects all neighbouring state variables that act as an input to subnetwork s.

Centralised Approach



- Global states and weights
- Communications only for measurements

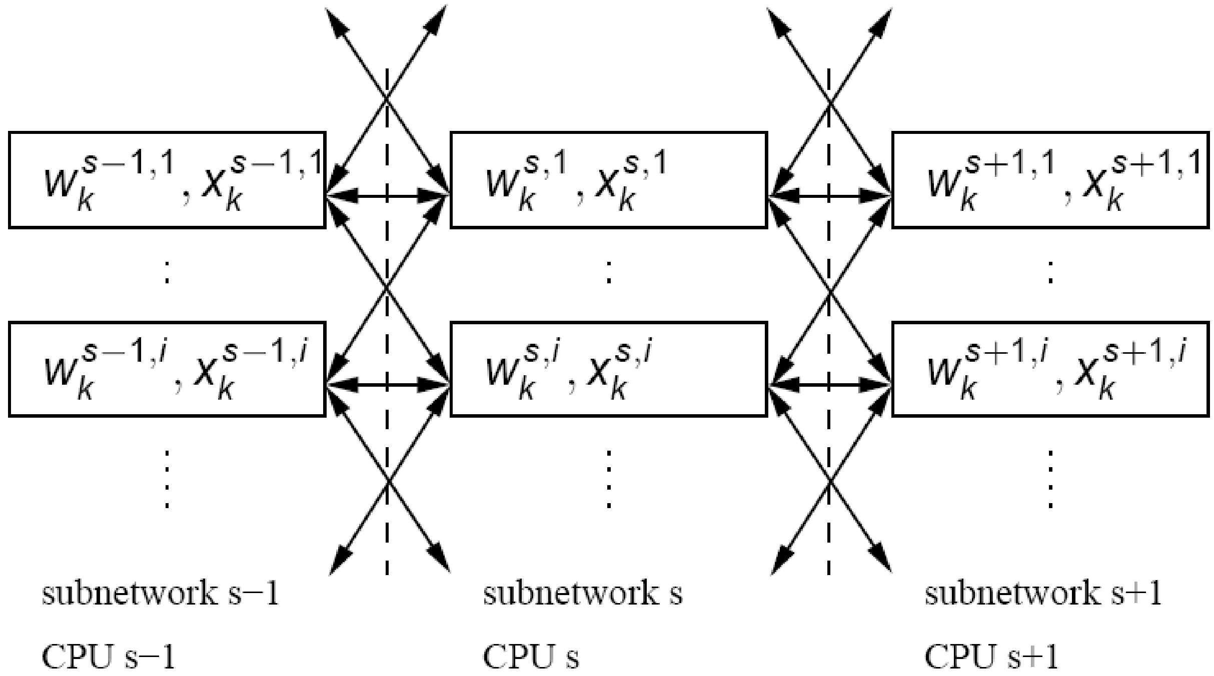
Approach I: Shared Particles



- The same as the centralised particle filter, but calculations are distributed over several processing units.
- Communication of states over boundaries
- Communication of weights to a central unit when resampling is necessary.

$$w_k^i \propto w_{k-1}^i \prod_{s=1}^{S} w_{k-1}^{s,i}$$

Approach II: Separate Particles



- Neighbour combination: based on weights
- Communicate **neighbouring** states over the boundaries,
- No need of central unit for resampling.
- Assuming independence of state and measurement noise.

Centralised Particle Filter

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$$x_k = f(x_{k-1}, v_{k-1}),$$

$$z_k = h(x_k, n_k),$$

$$p(x_k|z_{1:k}) = \frac{p(z_k|x_k)p(x_k|z_{1:k-1})}{p(z_k|z_{1:k-1})}$$

$$\{x^i_{0:k}, w^i_k\}^I_{i=1}, \quad \{x^i_{0:k}, i = 0, \ldots, I\} \quad \{w^i_k, i = 0, \ldots, I\}$$

The posterior density at *k* is approximated as:

$$p(x_{0:k}|z_{1:k}) \approx \sum_{i=1}^{I} w^i_k \delta(x_{0:k} - x^i_{0:k})$$

$$w^i_k \propto w^i_{k-1} \frac{p(z_k|x^i_k)p(x^i_k|x^i_{k-1})}{q(x^i_k|x^i_{k-1}, z_k)}$$

- Typically $q(x_k|x^i_{k-1}, z_k) = p(x_k|x^i_{k-1})$ $\quad w^i_k \propto w^i_{k-1} p(z_k|x^i_k)$

Partitioning the Traffic Network into Subnetworks

Assumptions:

- Communicate only the variables that serve as an input to subnetwork *s*, not all states of neighbouring networks.
- Measurements in a subnetwork depend only on the state in that subnetwork.
- **Independent state noises** between the subnetworks
- **Independent measurement noises** between the networks

Boundary states

$$p(x_k|x_{k-1}) = \prod_{s=1}^{S} p(x_k^s|x_{k-1}^s, \hat{x}_{k-1}^s)$$

$$p(z_k|x_k) = \prod_{s=1}^{S} p(z_k^s|x_k^s)$$

Approach II

$$p(x_k^{s,i}|x_{k-1}^{s,i}) = \int_{\hat{x}_{k-1}^{s,j}} \{p(x_k^{s,i}|x_{k-1}^{s,i},\hat{x}_{k-1}^{s,j})p(\hat{x}_{k-1}^{s,j}|x_{k-1}^{s,i})\}\, d\hat{x}_{k-1}^{s,j}$$

Applying Monte Carlo sampling to the product

$$p(x_k^{s,i}|x_{k-1}^{s,i},\hat{x}_{k-1}^{s})p(\hat{x}_{k-1}^{s}|x_{k-1}^{s,i})$$

with a proposal distribution $q(\hat{x}_{k-1}^{s}|x_{k-1}^{s,i})$ results in the approximation

$$p(x_k^{s,i}|x_{k-1}^{s,i}) \approx \sum_j \frac{p(x_k^{s,i}|x_{k-1}^{s,i},\hat{x}_{k-1}^{s,j})p(\hat{x}_{k-1}^{s,j}|x_{k-1}^{s,i})}{q(\hat{x}_{k-1}^{s,j}|x_{k-1}^{s,i})}$$

$\hat{x}_{k-1}^{s,i}$: state variables at the boundaries

Approach II

- By assumption the pdf of the communicated state variables is independent on $x_{k-1}^{s,i}$ and then

$$p(\hat{x}_{k-1}^{s,j}|x_{k-1}^{s,i}) = p(\hat{x}_{k-1}^{s,j})$$

- Taking one sample from $\hat{x}_{k-1}^{s,j_i} \sim p\left(\hat{x}_{k-1}^{s}\right)$ for each *i* and choosing

$$q(\hat{x}_{k-1}^{s,j}|x_k^{s,i}) = p(\hat{x}_{k-1}^{s,j})$$

$$p(x_k^{s,i}|x_{k-1}^{s,i}) \approx p(x_k^{s,i}|x_{k-1}^{s,i},\hat{x}_{k-1}^{s,j_i})$$

$$w_k^{s,i} = w_{k-1}^{s,i} \frac{p(z_k^{s,i}|x_k^{s,i})p(x_k^{s,i}|x_{k-1}^{s,i},\hat{x}_{k-1}^{s,j_i})}{q(x_k^{s,i}|x_{k-1}^{s,i},\hat{x}_{k-1}^{s,j_i},z_k^{s})}$$

Approach II

- There is no central PU
- Communications only between the neighbouring PUs: statistics of neighbouring states is exchanged

Advantages of Approach II over Approach I

- Requires less particles: the dimension of the state space is reduced by a factor *S* (if all subnetworks have the same number of states).
- For each subnetwork a different number of particles can be used

Disadvantage of Approach II

- An approximation is introduced in the interaction (joint pdf) of the local states with the states in neighbouring subnetworks.

Gaussian Sum Particle Filters for Traffic Flow Estimation

- Approximate the state filtering and state prediction probability density functions (pdfs) with sums of Gaussian pdfs

$$p(\mathbf{x}_k|\mathbf{z}_{1:k}) = \sum_{g=1}^{G} w_{k,g}\mathcal{N}(\mathbf{x}_k; \boldsymbol{\mu}_{\mathbf{x}_k,g}, \boldsymbol{\Sigma}_{\mathbf{x}_k,g})$$

$$p(\mathbf{x}_k|\mathbf{z}_{1:k-1}) = \int p(\mathbf{x}_k|\mathbf{x}_{k-1})p(\mathbf{x}_{k-1}|\mathbf{z}_{1:k-1})d\mathbf{x}_{k-1}$$

$$\approx \int \sum_{g=1}^{G} \bar{w}_{k-1,g}\mathcal{N}(\mathbf{x}_{k-1}; \bar{\boldsymbol{\mu}}_{k-1,g}, \bar{\boldsymbol{\Sigma}}_{\mathbf{x}_{k-1,g}})p(\mathbf{x}_k|\mathbf{x}_{k-1})d\mathbf{x}_{k-1}$$

$$= \sum_{g=1}^{G} \bar{w}_{k-1,g} \int \mathcal{N}(\mathbf{x}_{k-1}; \bar{\boldsymbol{\mu}}_{k-1,g}, \bar{\boldsymbol{\Sigma}}_{\mathbf{x}_{k-1,g}})p(\mathbf{x}_k|\mathbf{x}_{k-1})d\mathbf{x}_{k-1}$$

- Output estimate $\hat{\mathbf{x}}_k = \sum^{G} w_{k,g}\boldsymbol{\mu}_{k,g}$

$$\hat{\boldsymbol{\Sigma}}_{\mathbf{x}_k} = \sum_{g=1}^{G} w_{k,g}[\boldsymbol{\Sigma}_{\mathbf{x}_{k,g}} + (\hat{\mathbf{x}}_k - \boldsymbol{\mu}_{k,g})(\hat{\mathbf{x}}_k - \boldsymbol{\mu}_{k,g})^T]$$

A Parallelised Gaussian Sum PF for Traffic Flow Estimation

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Initialize: $\mu_{\mathbf{x}_{0,g}}, \Sigma_{\mathbf{x}_{0,g}}, w_{0,g} = \frac{1}{G}, g = 1, \ldots, G$, at $k = 0$.

for $k = 1, 2, \ldots, K$,

for $s = 1 : S$ (for each subnetwork)

Prediction

for $g = 1, \ldots, G$, draw

$$\{\breve{\mathbf{x}}_{k-1}^{s,j}\}_{j=1}^{N} \sim p(\breve{\mathbf{x}}_{k-1}^{s}) \quad (31)$$

$$\{\mathbf{x}_{k-1,g}^{j}\}_{j=1}^{N} \sim \mathcal{N}(\mathbf{x}_{k-1}; \mu_{\mathbf{x}_{k-1,g}}, \Sigma_{\mathbf{x}_{k-1,g}}) \quad (32)$$

end

for $g = 1, \ldots, G$, sample
$\{\mathbf{x}_{k,g}^{j}\}_{j=1}^{N} \sim p(\mathbf{x}_{k,g} | \mathbf{x}_{k-1,g}^{j}, \breve{\mathbf{x}}_{k-1}^{s,j})$, from (1).
end

for $g = 1, \ldots, G$, update weights $\bar{w}_{k,g} = w_{k-1,g}$. **end**
for $g = 1, \ldots, G$, from $\{\mathbf{x}_{k,g}^{j}\}_{j=1}^{N}$ obtain sample mean $\bar{\mu}_{\mathbf{x}_{k,g}}$ and sample covariance $\bar{\Sigma}_{\mathbf{x}_{k,g}}$.
end

- Communicates only the statistics (mean and covariance) of the states on the boundaries
- Similar to the parallelised PF with separate particles
- Transmit estimates of the boundary conditions between the boundaries of each subnetwork.

The state predictive density is approximated as

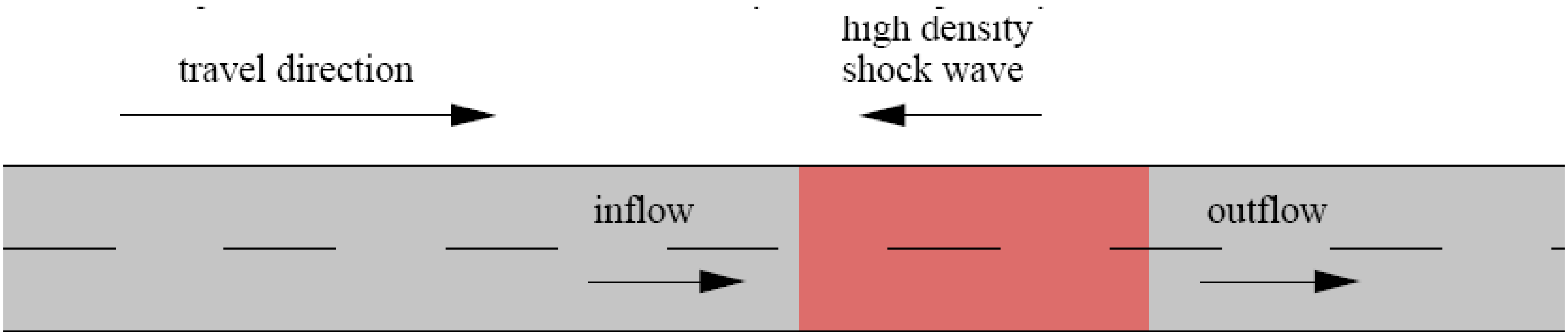
$$p(\mathbf{x}_k | \mathbf{z}_{1:k-1}) = \sum_{g=1}^{G} \bar{w}_{k,g} \mathcal{N}(\mathbf{x}_k; \bar{\mu}_{\mathbf{x}_{k,g}}, \bar{\Sigma}_{\mathbf{x}_{k,g}}).$$

Measurement Update
for $g = 1, \ldots, G$, draw samples
$\{\mathbf{x}_{k,g}^{(j)}\}_{j=1}^{N} \sim \mathcal{N}(\mathbf{x}_k; \bar{\mu}_{\mathbf{x}_{k,g}}, \bar{\Sigma}_{\mathbf{x}_{k,g}})$.
end
for $g = 1, \ldots, G$, $j = 1, \ldots, N$ compute the weights $W_{k,g}^{j} \propto p(\mathbf{z}_k | \mathbf{x}_{k,g}^{j})$ where the likelihood $p(\mathbf{z}_k | \mathbf{x}_{k,g}^{j})$ is calculated from (2).
end
for $g = 1, \ldots, G$, $j = 1, \ldots, N$ estimate the mean and covariance from (27) and (28)
end

- Normalize the weights
Output: the state estimate and its covariance are calculated respectively from (24) and (25).
Resampling
- Resample the weights $w_{k,g}$

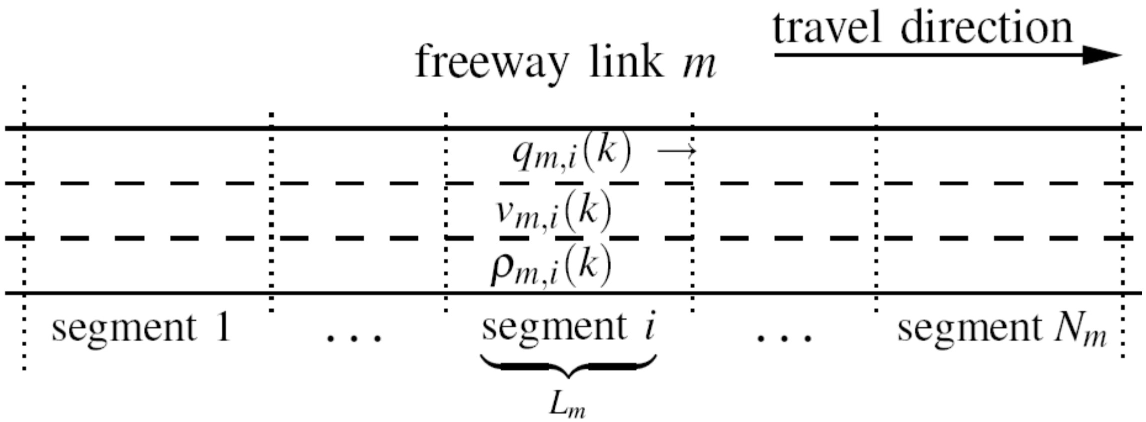
- Set $w_{k,g} = \frac{1}{G}$. **end**
end // of S loop (for each subnetwork)
- Set $k - 1 \longrightarrow k$

Experimental Setup



- Motorway with a traffic jam
- Research questions:
- Compare the centralised filter and approaches 1 and 2 for several numbers of particles
  - Tracking accuracy
  - Computational complexity (CPU time)
  - Communication
- Each test executed 100 times.

Experimental Setup



- Two links, two lanes, 10 segments in each link;
- Measurements: at segments 1 and 10 every minute
- State update step: 10 seconds
- Boundary conditions estimated as part of the state vector
- Gaussian noises
- State vector = [states, boundary states]

$$x_k = [\rho_1(k), \ldots, \rho_N(k), v_1(k), \ldots, \; v_N(k), v_0(k), q_0(k), \rho_{N+1}]^{\mathrm{T}}$$

- METANET model for state update

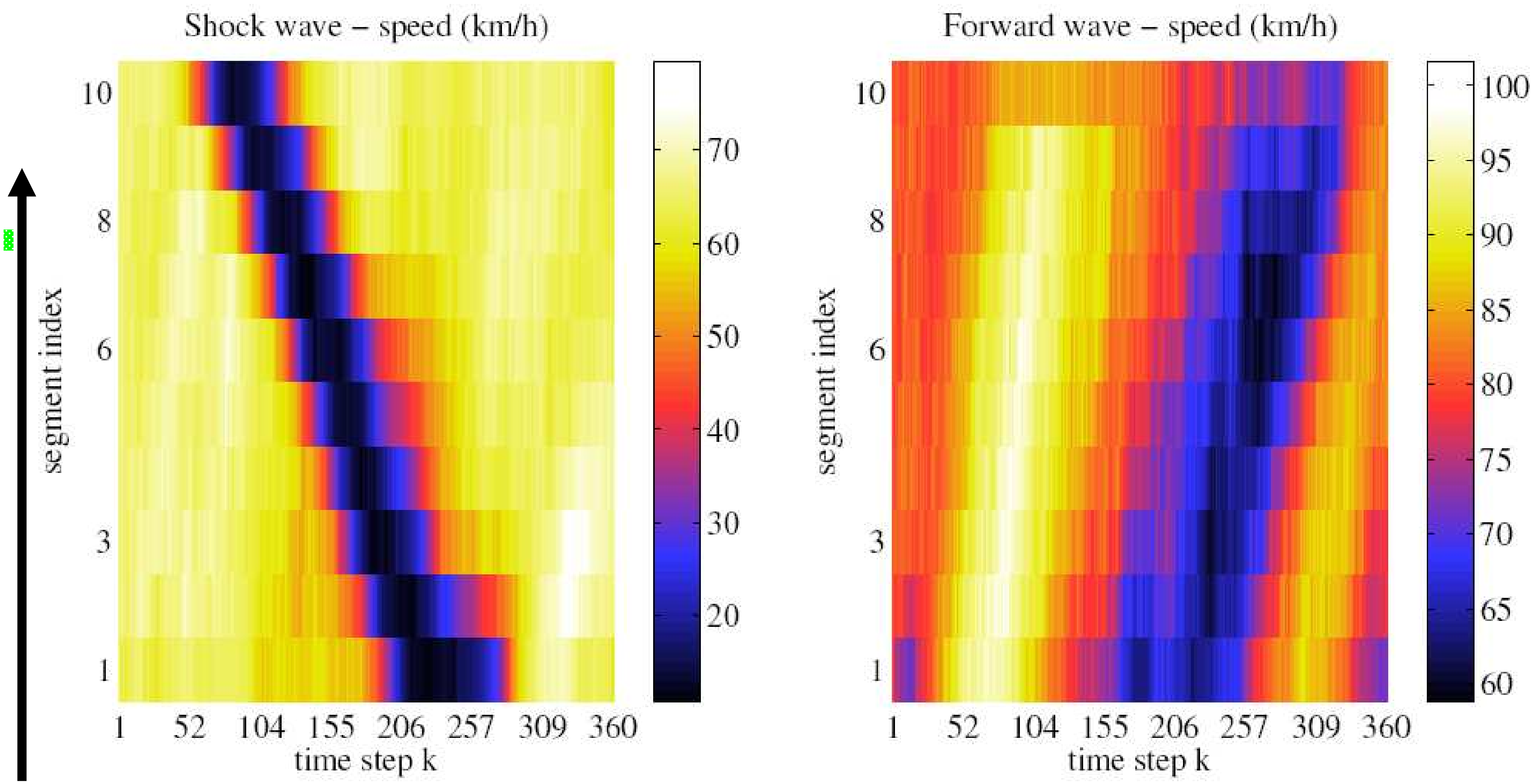
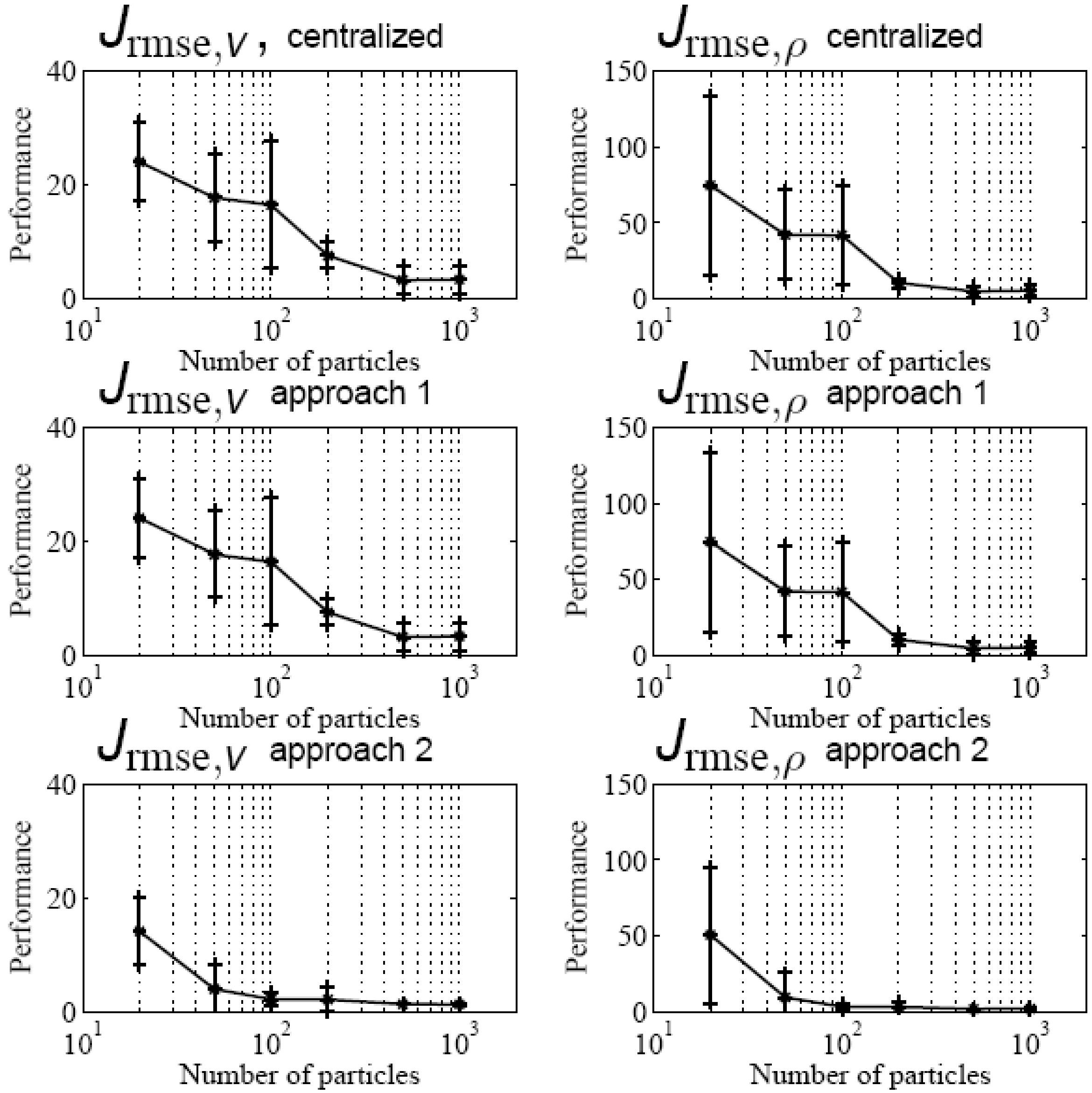


Fig. 3. The shock wave (left) and the forward wave (right) scenario, used for the evaluation of the filters. The travel direction is from segment 1 to 10. The colors indicate the speed. Please note the difference in color bar scales: the shock wave scenario includes a wider range of speed since it also contains congested traffic.

Results: Accuracy



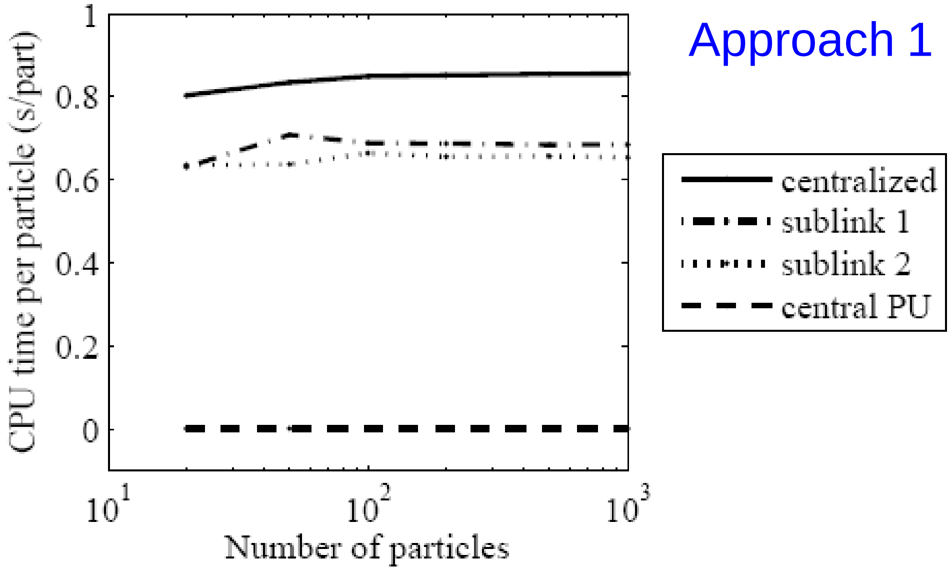
$$J_{\mathrm{rmse},\rho} = \sqrt{\frac{(\hat{\rho}_{i,k} - \tilde{\rho}_{i,k})^2}{KN_m}}$$

Scenario with the shock wave,
500 particles in the PFs

CPU Time vs Number of Particles

Communications

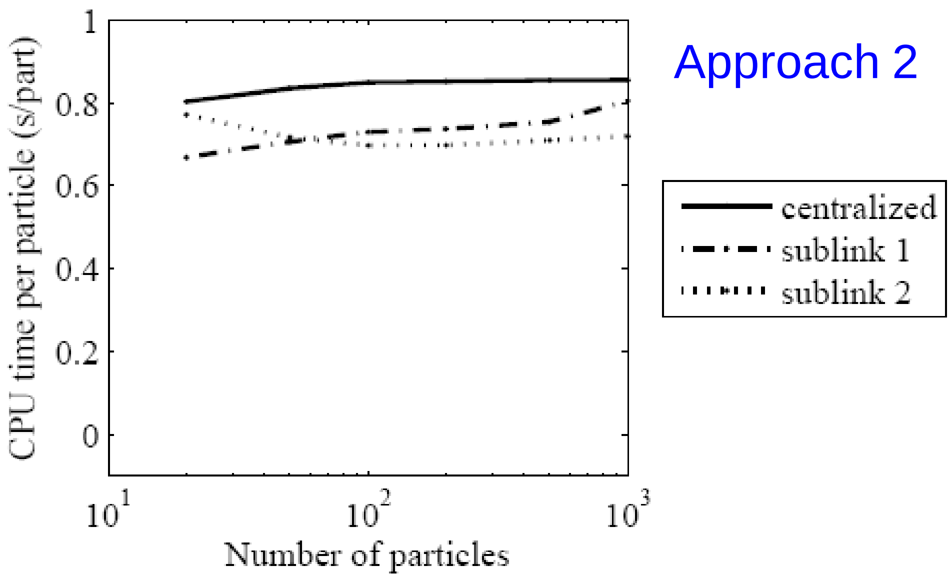
Approach 1



N	Centralized PF	PPF1	PPF2
20	4	124	44
50	4	304	104
100	4	604	204
200	4	1204	404
500	4	3004	1004
1000	4	6004	2004

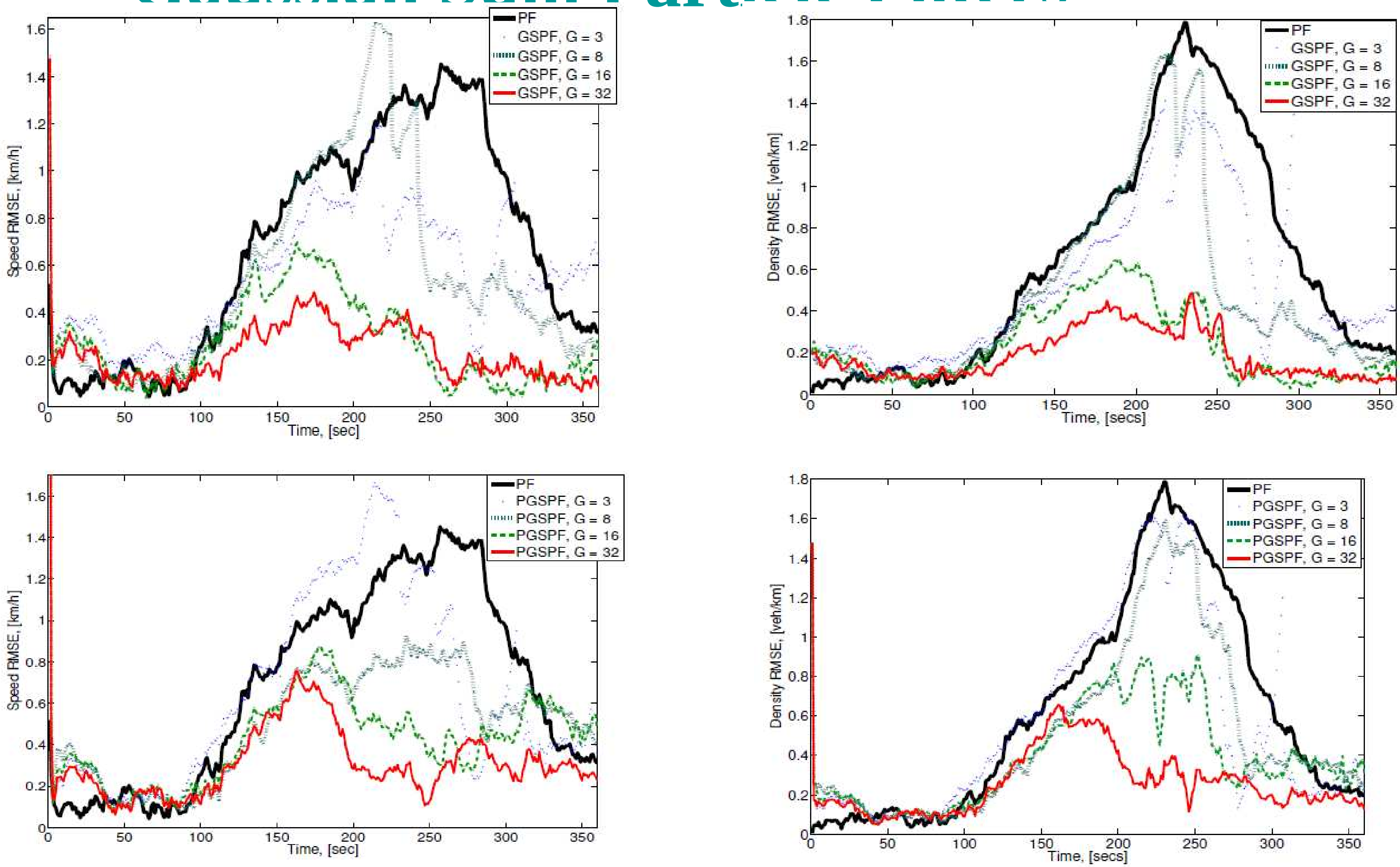
Number of communicated real numbers as a function of the number of particles N

Approach 2



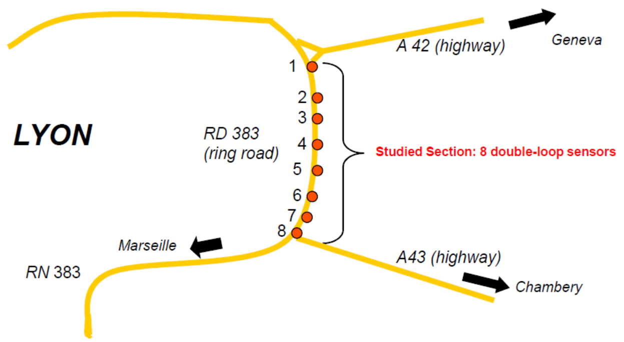
G	Centralized GSPF: as the Centralized PF	PGSPF
1	4	12
3	4	28
8	4	68
16	4	132
24	4	196
32	4	260

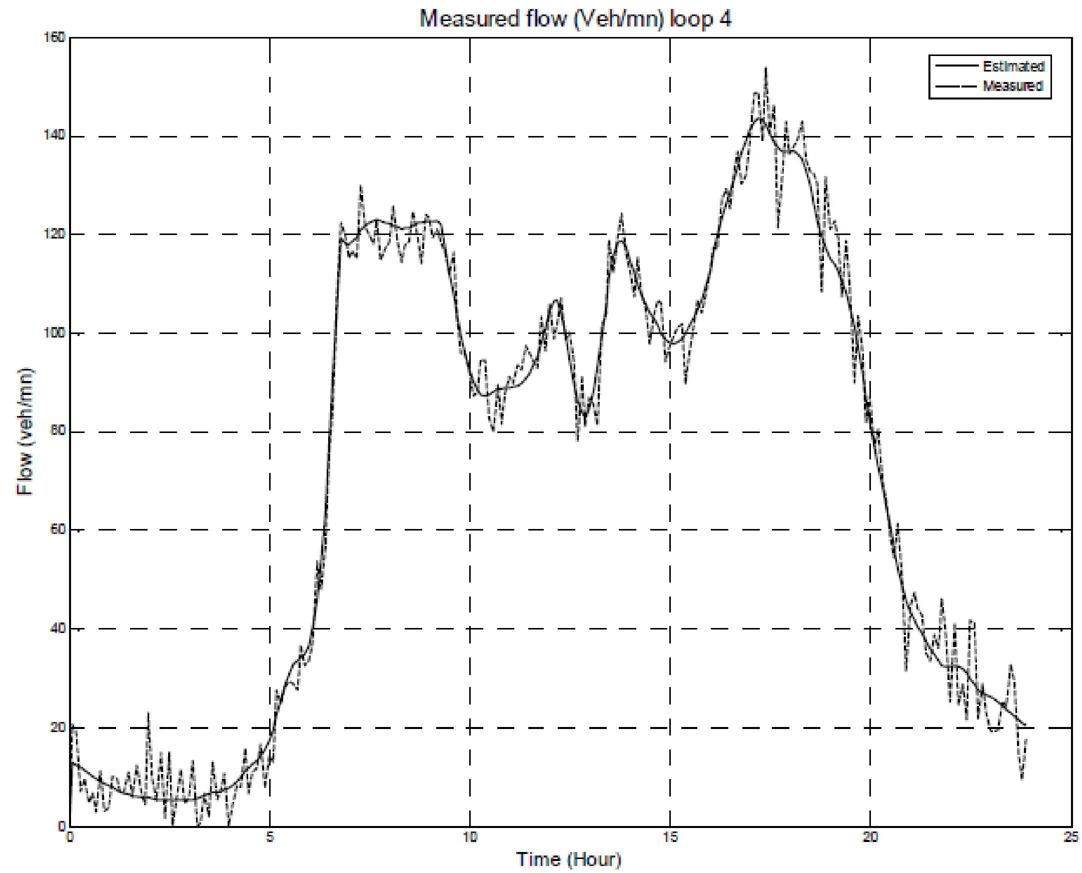
Centralised PF vs Centralised Gaussian Sum Particle Filters

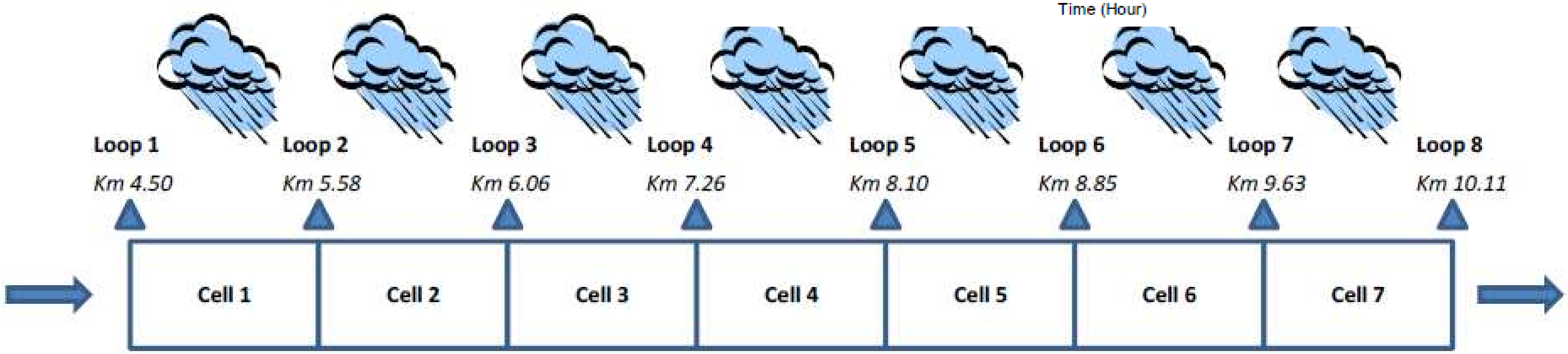


- **Parallelised PFs vs Parallelised GSPFs**

Integrating the Impact of Rain Into Traffic Management







- Based on the first order model of Lighthill and Whitham, 1955

Billot, El Faouzi, Sau, De Vuyst (2010)

- Precipitation data and traffic data

Summary

- Centralised and parallelised PFs and Gaussian Sum PFs are can give efficient traffic flow estimation
- The estimation accuracy of the parallelised PF 1 with shared particles: compared with the centralised PF
- The estimation accuracy of the parallelised PF 2 (separate particles) is higher than the accuracy of the parallelised PF 1 with shared particles
- The accuracy of the Gaussian Sum PFs (centralised and parallelised) - higher than the accuracy of the PFs (centralised and parallelised)
- The Gaussian Sum PFs are more computationally efficient than the PFs because they require transmission of estimated boundary states and their covariances
- The proposed approach can be extended to other applications

Conclusions and Open Issues

- **Open issues**:
  - distributed estimation, other techniques
  - algorithms robust to missing data and sensor failures
  - what is the optimal configuration of the detectors (optimal sensor placement)
- Fusion of sensor data from different types of sensors (e.g., from radars and video cameras)
- Modelling traffic to reflect different weather conditions
- Prediction/ filtering of traffic behaviour, e.g., based on Markov linear jump models (for control purposes)
- Group object tracking: track the behaviour of a group as a whole

Thank you for your attention ☺ !

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