Entanglement entropy in quantum Hall states

Kareljan Schoutens
Institute for Theoretical Physics
University of Amsterdam

FOM

UNIVERSITEIT VAN AMSTERDAM

IFA

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References

with M. Haque (Dresden), O. Zozulya (UvA),
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Outline

1. Interpreting entanglement
2. Bipartite entanglement entropy and area law
3. Many-body entanglement
4. Laughlin on the sphere
5. Orbital partitioning — topological order
6. Particle partitioning — exclusion statistics

Interpreting entanglement

- 1935 — Schrödinger introduces \`Verschränkung' as key notion in quantum mechanics.

> *\`\`... I would not call that <u>one</u> but rather <u>the</u> characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought. By the interaction the two representatives [the quantum states] have become entangled.''*

Interpreting entanglement

- 1935 — Schrödinger introduces `Verschränkung’ as key notion in quantum mechanics
- since 1980s — entanglement recognized as key resource for quantum communication and quantum computation
- since 1990s — entanglement measures considered for characterizing nature of quantum states of matter

Bipartite entanglement entropy

- measure of mutual entanglement between parts of quantum system

  - system partitioned into A and B blocks
  - B degrees of freedom traced out: $\rho_A = tr_B \rho$
  - entropy defined as $S_A = -tr\left[\rho_A \ln \rho_A\right] = S_B$

- example: two spins 1/2

$$|\psi\rangle = |\uparrow\downarrow\rangle \qquad\qquad |\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle\right)$$

$$S_A = 0 \qquad\qquad S_A = \ln 2$$

Area law for bipartite entanglement entropy

- for spatial partitioning: expect in general that entanglement entropy will be proportional to area separating the A and B blocks, leading to

$$S_A \propto L_A^{d-1}$$

- exceptions to the area law, as well as sub-leading corrections, are tell-tale of the quantum many-body state

Many-body entanglement

Bipartite entanglement entropies of a quantum many-body state can shed light on

- quantum criticality
- topological order
- correlations
- …

Entanglement and criticality

1D critical systems

$$S_A = \frac{c}{3}\ln(L_A)$$

Holzhey-Larsen-Wilczek 1994
Vidal-Latorre-Rico-Kitaev 2003
Calabrese-Cardy 2004

2D critical systems: subleading logarithm in area law

$$S_A = 2f_s(L/a) + \alpha c\ln(L/a)$$

Fradkin-Moore 2006

Topological entanglement entropy

Topological order in a 2D system is captured by a sub-leading term in the dependence of spatial entanglement entropy on the radius L_A of the A block

$$S_A = \alpha L_A - \gamma + O(L_A^{-1})$$

The topological entanglement entropy is related o the total quantum dimension according to

$$\gamma = \ln D \qquad D = \sqrt{\sum_i d_i^2}$$

Kitaev-Preskill 2006, Levin-Wen 2006

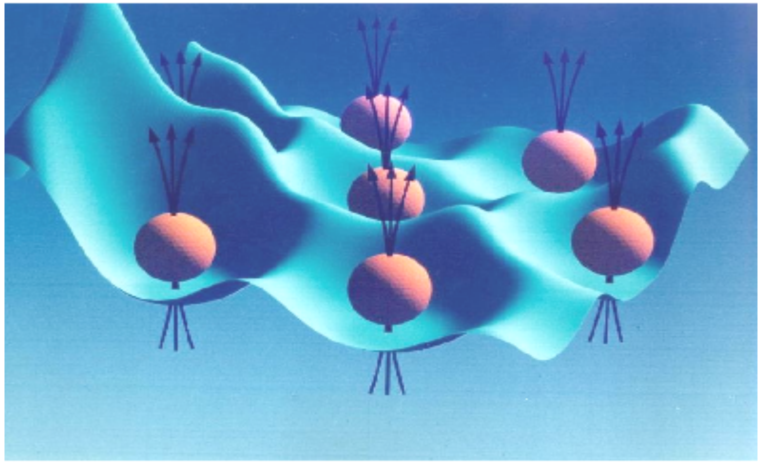
Entanglement and correlations

Subleading terms in entanglement entropy for particle partitioning $n_A, n_B = N - n_A$ hold clue to correlation effects and exclusion statistics.

For the ν=1/m Laughlin states

$$S_A - \ln\binom{N_\phi + 1}{n_A} \approx -\frac{1}{N}\frac{m-1}{m} n_A(n_A - 1) + O(\frac{1}{N^2})$$

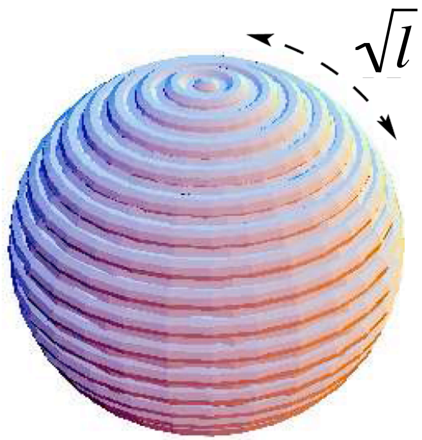
How entangled is a fqH state?



$$\Psi(z_1,\ldots,z_n) = \prod_{i<j}(z_i - z_j)^m e^{-\sum_i \frac{|z_i|^2}{4}}$$

- entanglement among spatial regions?
- entanglement among constituent particles?

Laughlin wave functions



- N fermions in spherical geometry, filling factor $\nu=1/m$
- monopole at center provides magnetic field; total flux

$$N_\phi = m(N-1)$$

- eigenstates of orbital angular momentum localized on latitude lines → Lowest Landau Level orbitals
- l -th orbital, $l = 0,1,\ldots,N_\phi$ localized at distance $\propto \sqrt{l}$ from north pole
- spherical projection onto plane gives standard Laughlin wavefunction

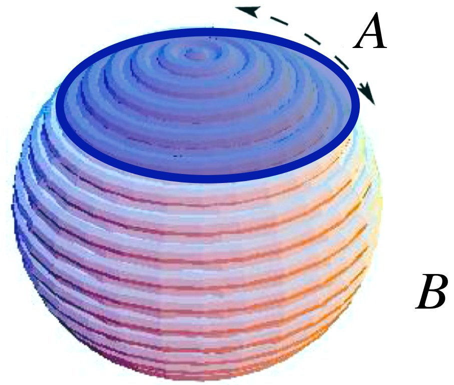
$$\Psi(z_1,\ldots,z_n) = \prod_{i<j}(z_i - z_j)^m e^{-\sum_i \frac{|z_i|^2}{4}}$$

Orbital partitioning

- orbital partitioning

  A-block — orbitals $l = 0, \ldots l_A - 1$

  B-block — orbitals $l = l_A, \ldots N_\phi$

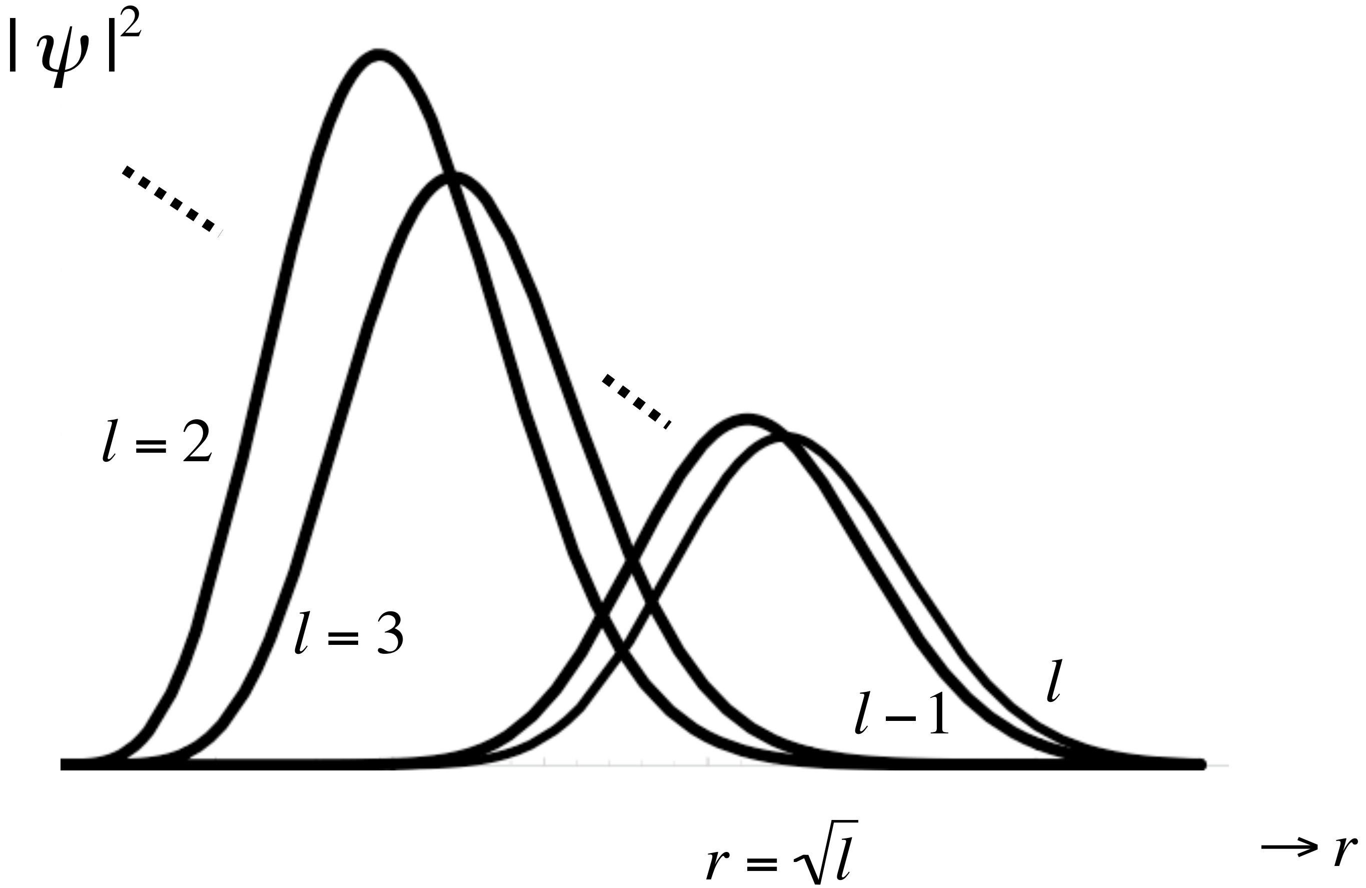


- boundary between orbitals $l_A - 1$ and l_A located

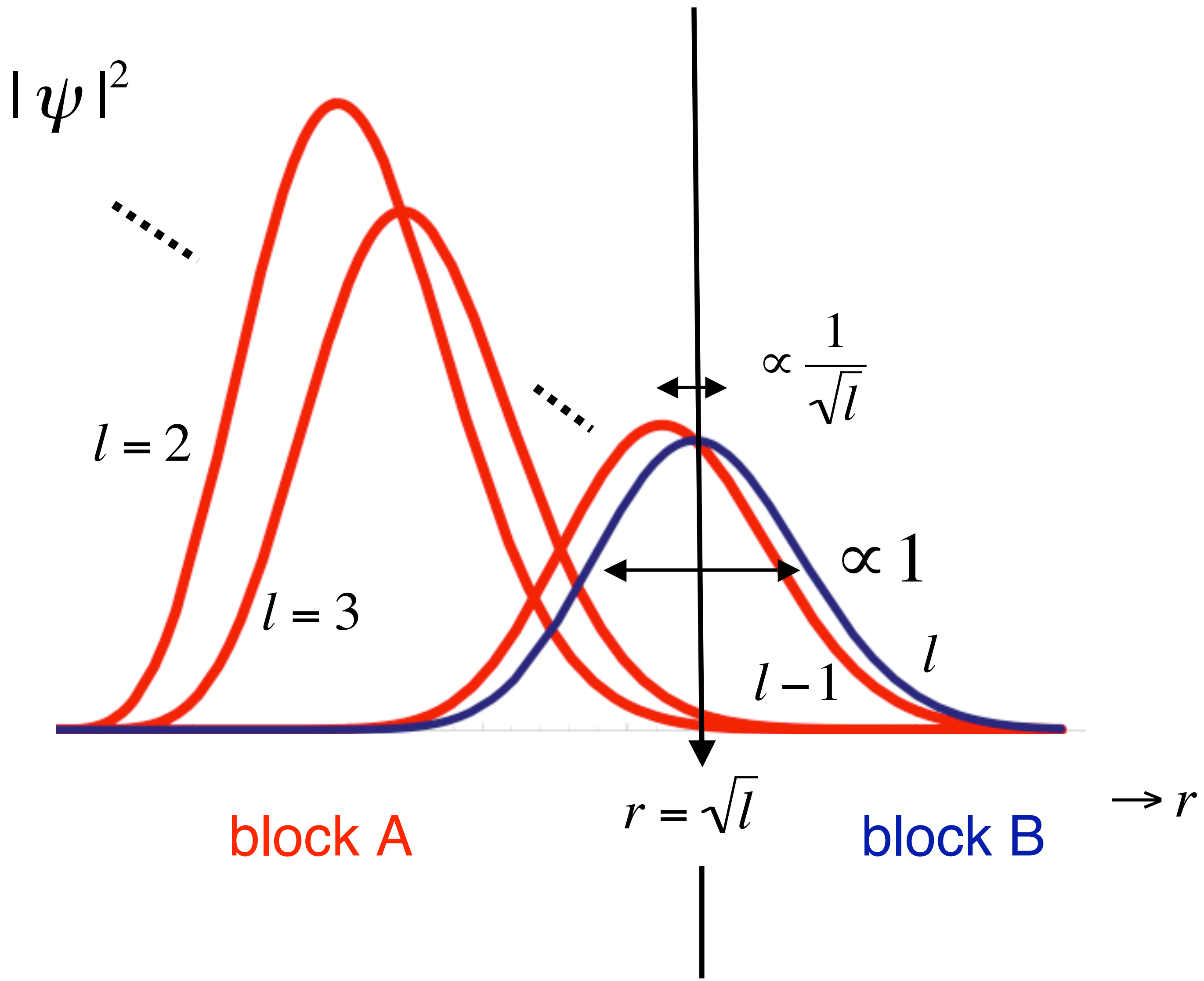
  at $r = \sqrt{l_A}$; expect asymptotic behavior

$$S_{l_A} \cong -\gamma + \alpha\sqrt{l_A}$$

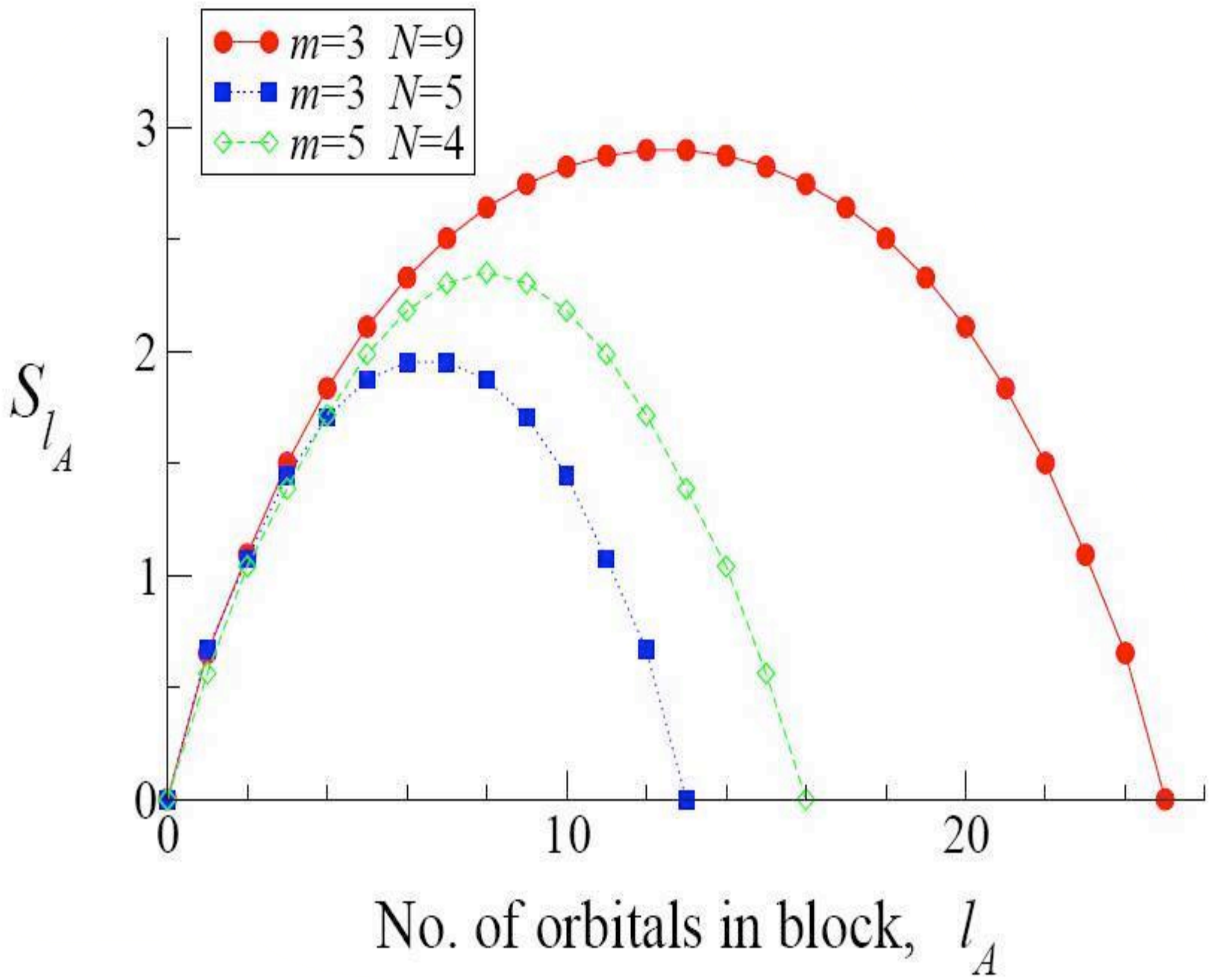
LLL orbitals



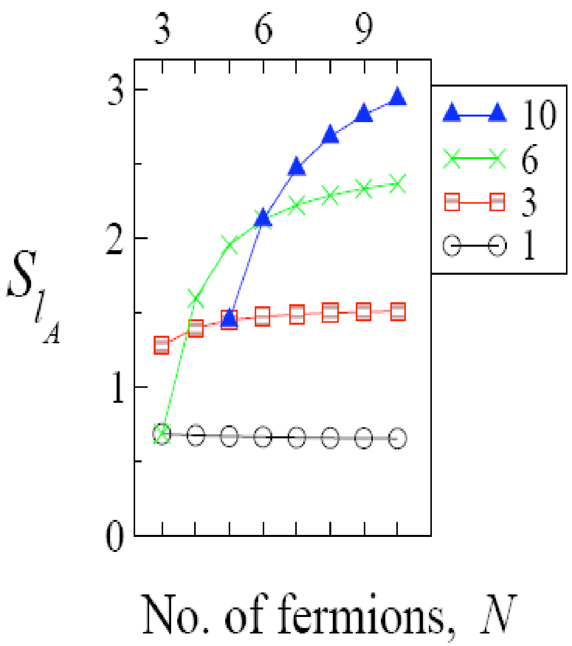
LLL orbitals

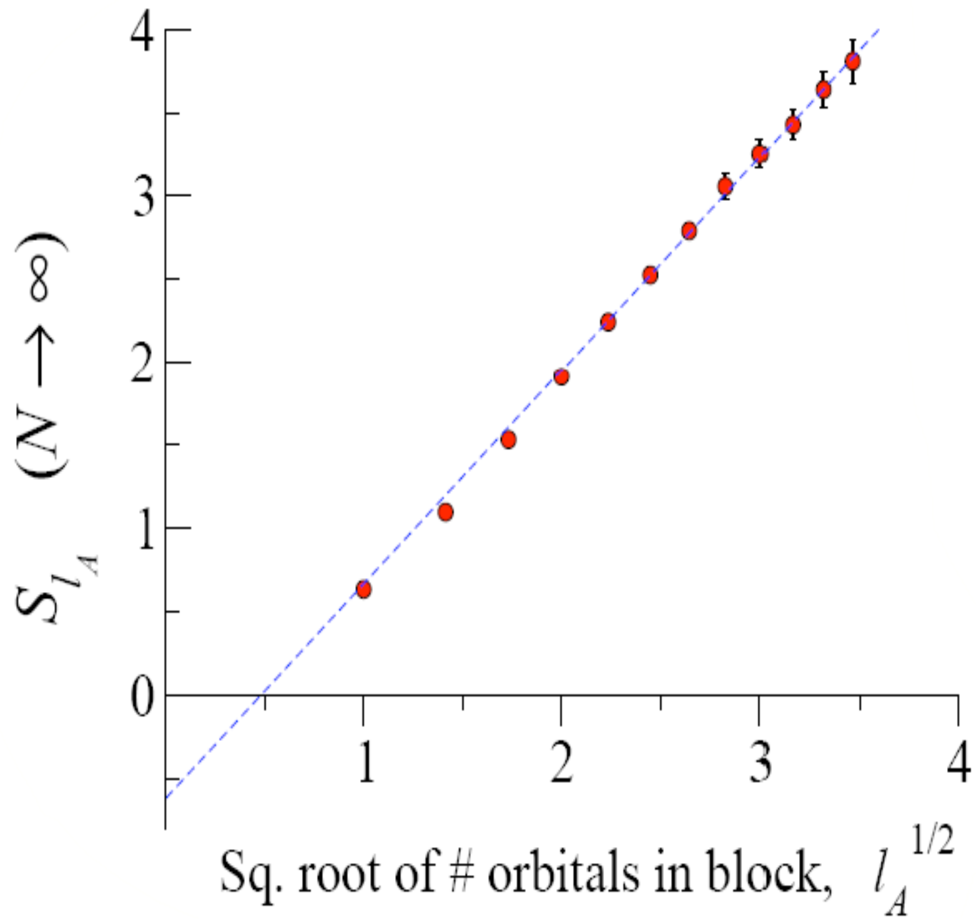


Orbital partitioning



Orbital partitioning -- extrapolating N → ∞

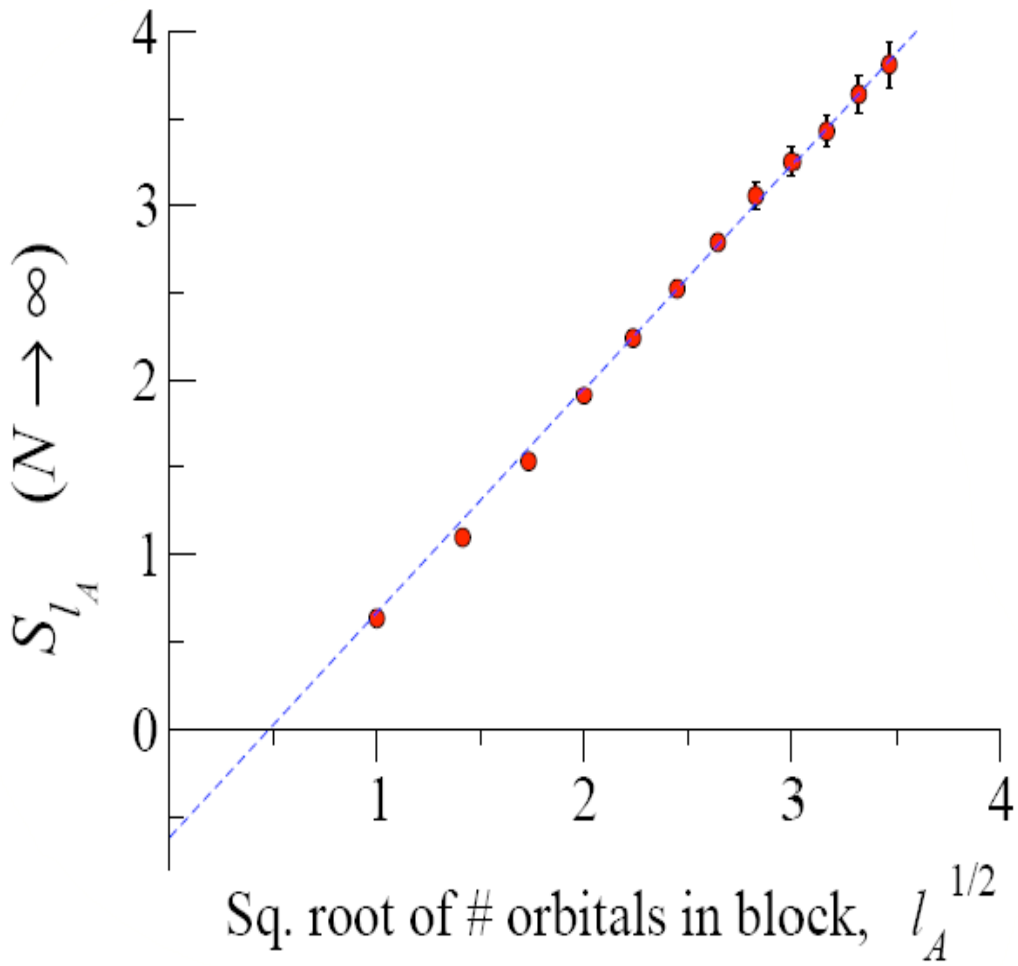




extrapolating $N \rightarrow \infty$

for fixed l_A at *m=3*

Orbital partitioning -- extracting γ



best fit to

$$S_{l_A} \cong -\gamma + \alpha\sqrt{l_A}$$

$$\rightarrow \quad \gamma = 0.60 \pm 0.10$$

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from TQFT

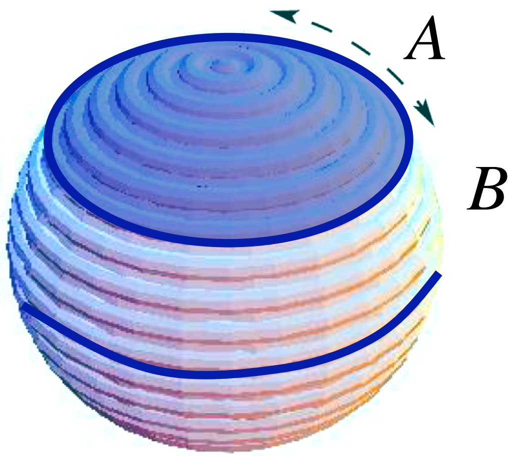
$$\gamma_3 = \log\sqrt{3} = 0.55$$

Orbital partitioning, II

- orbital partitioning

  A-block — orbitals $l = 0, \ldots l_A - 1$

  B-block — orbitals $l = l_A, \ldots l_B - 1$



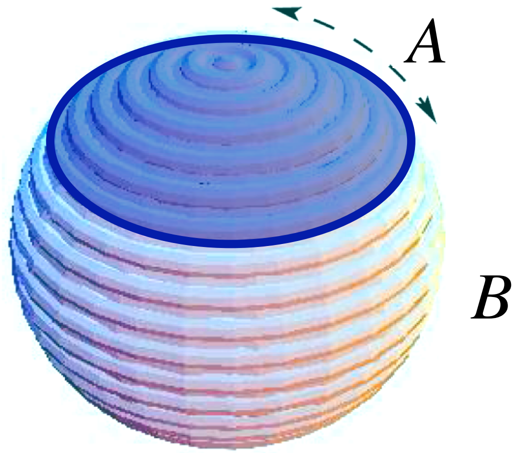
- combination

$$S_B - S_A - S_{AB} \cong \gamma$$

gives topological entanglement entropy if

$$0 << l_A << l_B << N_\phi$$

Perspective



- if sufficient accuracy can be achieved, the method allows the extraction of the total quantum dimension from finite-size wave-functions

- particularly interesting for wavefunctions obtained from exact diagonalization of realistic potentials:
  - direct probe of (universal) topological order
  - alternative to overlap with model wavefns

- work in progress [Haque, Zozulya, KjS]

Particle partitioning, I

- particle partitioning

  A-block — n_A particles

  B-block — $(N - n_A)$ particles

- maximal entropy for n_A fermions in $N_\phi + 1$ orbitals

$$S_A \leq \ln \begin{pmatrix} N_\phi + 1 \\ n_A \end{pmatrix}$$

- correlations in the many-body state lead to reduced value of S_A

Particle partitioning, II

- example:

  m=3, *N* particles, *3N-2* orbitals, 1-particle states carry orbital angular momentum $L = 3(N-1)/2$

  → possible states at of $n_A = 2$ fermions:

$$L = (3N-4), (3N-6), \ldots, 1(0)$$

  → total number of states

$$\frac{1}{2}(3N-2)(3N-3)$$

  → of these the multiplet at $L = 3N-4$ is absent, reducing the total number of states to

$$\frac{1}{2}(3N-2)(3N-3) - [2(3N-4)+1] = \frac{1}{2}(3N-4)(3N-5)$$

Particle partitioning, III

- general:
  using quasi-hole counting results [Read-Rezayi,1996]
  we improved the upper bound

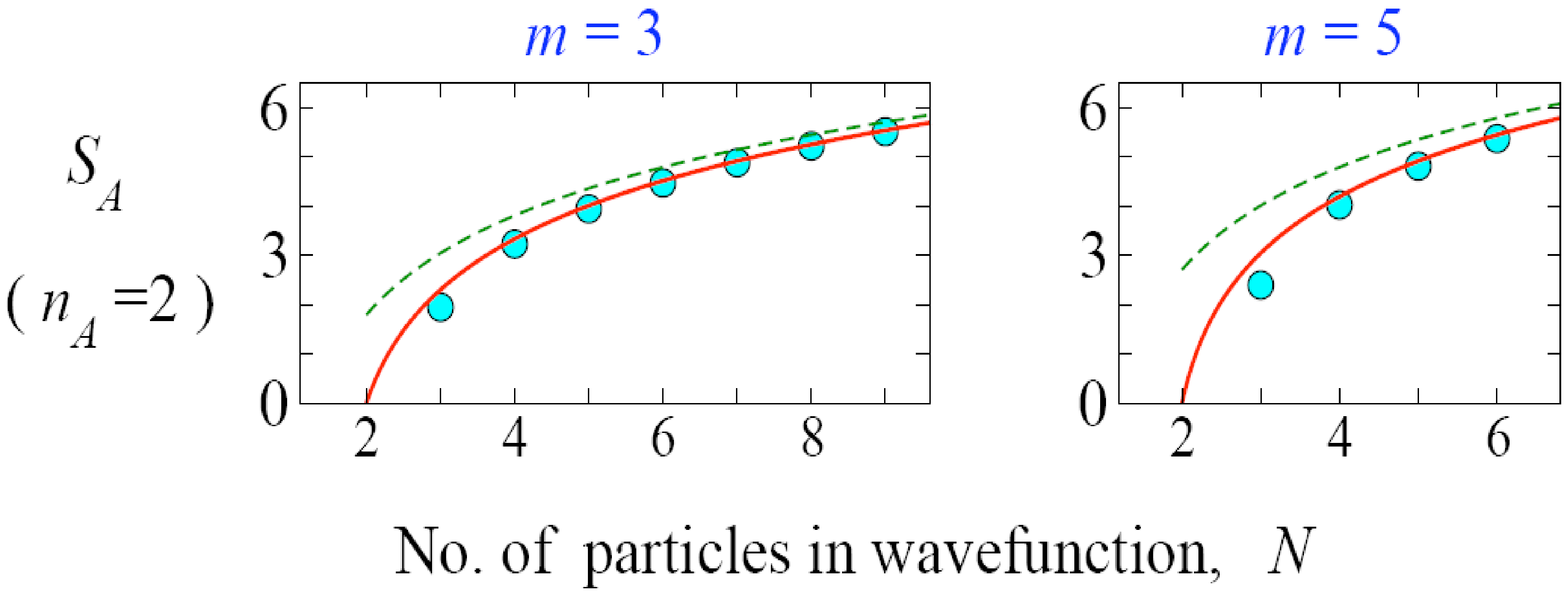
$$S_A \leq \ln \binom{N_\phi + 1}{n_A} \longrightarrow S_A \leq \ln \binom{N_\phi + 1 - (m-1)(n_A - 1)}{n_A}$$

- interpretation in terms of exclusion statistics:
  the reduced number of states is precisely equal to the number of ways n_A particles can be put into $N_\phi + 1$ orbitals, observing a minimal distance of *m* between adjacent occupied orbitals

Particle partitioning, IV

particle partitioning for $n_A = 2$ particles

[- - - - free fermion bound, ——— improved bound]



Particle partitioning, V

- further interpretation:
  comparing with the free fermion bound we have a $1/N$ expansion

$$S_A - \ln\binom{N_\phi + 1}{n_A} \leq -\frac{1}{N}\frac{m-1}{m} n_A(n_A - 1) + O(\frac{1}{N^2})$$

- in this concrete case, the leading correction term can be traced to the short distance behavior of the 2-body correlator $g_2(r)$

- more generally, the leading $1/N$ term is a global measure for the leading correlations in a state of interacting fermions

Particle partitioning, VI

- for the Moore-Read state at ν=1/m, m even

rigorous upper bounds

$$S_A - \ln\binom{N_\phi + 1}{n_A} \leq -\frac{1}{N^2}\frac{3}{4}n_A(n_A-1)(n_A-2) + O(\frac{1}{N^3}) \quad \text{for} \quad m=2$$

$$-\frac{1}{N}\frac{m-2}{m}n_A(n_A-1) + O(\frac{1}{N^2}) \quad \text{for} \quad m>2$$

conclusions

Bipartite entanglement entropy with orbital partitioning reveals topological order in a given LLL wave function. Practical use of this hinges on the accuracy that can be achieved with finite size wavefunctions.

Bipartite entanglement entropy with particle partitioning reveals correlations and exclusion statistics properties satisfied by fqH wave functions.