Protected qubits using Josephson junctions and other superconducting elements.

Goal : all-electric protected* qubits

Qubits are not elementary devices (classical bits are not elementary either: an implementation consists of several transistors.)

Sub-goal : find a set of basic elements for quantum electric circuits

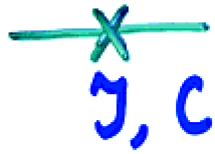
* Protected =

All unwanted interactions are exponentially suppressed.

"Simple" elements:

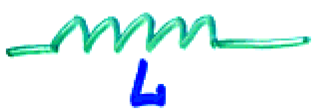
Capacitor :

Josephson junction :

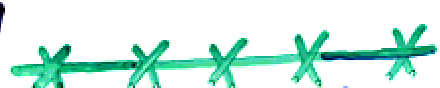


Standard

Inductor :



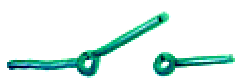
Usually implemented as a chain of Josephson junctions



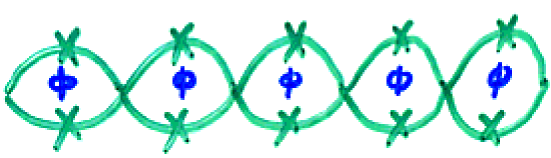
$$J \gg \frac{e^2}{C_2}$$

$$\left(L_{eff} = N \left(\frac{\hbar}{2e} \right)^2 J^{-1} \right)$$

Switch :



(Haviland at al 2000)

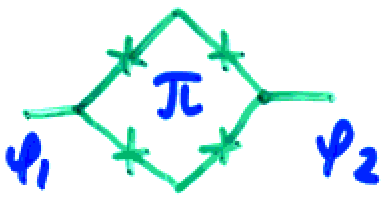


$\Phi \approx \pi$ $\Longrightarrow$ transition to an insulating state

(In dimensional units, $\Phi \approx \frac{\Phi_0}{2}$)

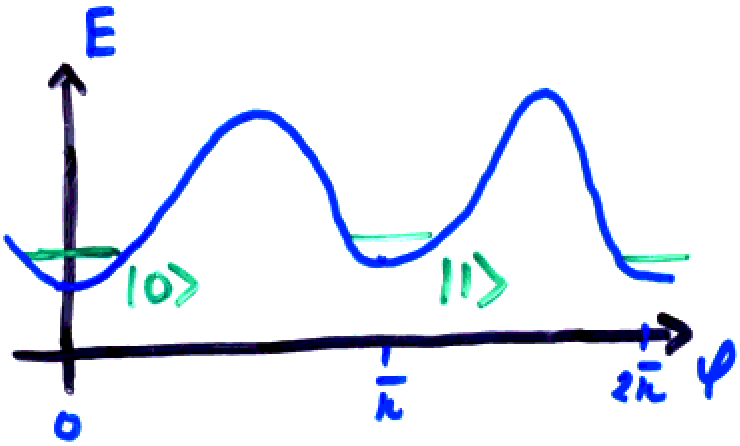
More exotic elements:

1) 0-π-contact (Douçot, Vidal 2002; Ioffe, Feigelman 2002)



$$E = -J_1 \cos 2\varphi - \underbrace{J_2 \cos\varphi}_{\text{error term}}$$

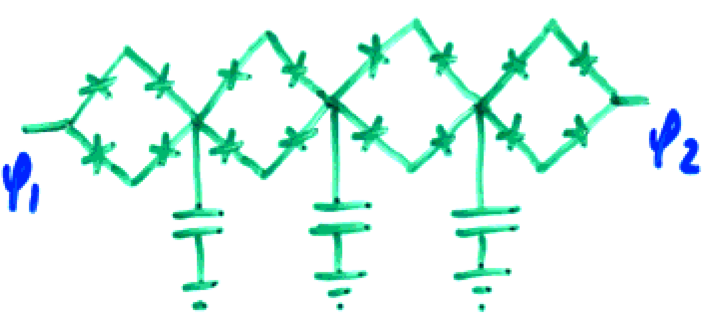
$$J_2 \ll J_1$$



$$\underline{\delta E = E_1 - E_0 = 2J_2}$$

May depend on the environment ⇒ decoherence

2) Protection (stabilization) $\delta E_{eff} \to 0$



$$J_{1,eff} \sim \frac{1}{N}$$

$$\boxed{\delta E_{eff} \sim \left(\frac{J_2}{t}\right)^{N-1}}$$

t: tunneling amplitude

$|0_L\rangle = |0000\rangle + |1100\rangle + \ldots$
(even number of 1s)

$|1_L\rangle = |1000\rangle + |0100\rangle + \ldots$
(odd number of 1s)

It seems that quantum protection requires a many-body system because a protected state is similar to a quantum code.

Not quite true: One can use a single continuous degree of freedom for quantum encoding (Gottesman Kitaev Preskill 2000)

Main conceptual result of the present work:

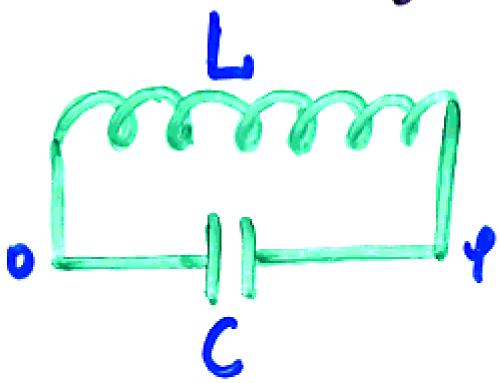
A sufficiently large superconducting inductor provides room for quantum encoding.

(Of course, Josephson junctions are also needed)

Conventions: $\hbar = 1$, $2e = 1$.

Resistance unit: $\frac{\hbar}{(2e)^2} \approx 1\,k\Omega$

In this units, the quantum resistance $\underline{R_q = \frac{h}{(2e)^2} \approx 6\,k\Omega}$ is simply $\underline{2\pi}$.



$$H = \frac{\varphi^2}{2L} + \frac{1}{2C}\underbrace{\left(\frac{\partial}{i\,\partial\varphi}\right)}_{q}{}^2$$

(charge operator)

$\langle \varphi^2 \rangle = \frac{1}{2}\sqrt{\frac{L}{C}}$ (characteristic impedance)

"Superinductor": $\langle \varphi^2 \rangle$ is large

$$\langle e^{i\varphi} \rangle = e^{-\frac{\langle \varphi^2 \rangle}{2}} = \exp\left(-\frac{1}{4}\sqrt{\frac{L}{C}}\right)$$

$$\boxed{\sqrt{\frac{L}{C}} \gg 4 \quad (4\,k\Omega \text{ in the usual units})}$$

in units of $\frac{\hbar}{(2e)^2}$

How to make a superinductor?

1) Coil of n loops : $\sqrt{\frac{L}{C_{geom}}} \sim R_{vac}\sqrt{\frac{\mu}{\varepsilon}}\, n$

$$R_{vac} = \frac{4\pi}{137} \qquad \left(\frac{4\pi}{c} \approx 377\,\Omega\right)$$

Too many loops are needed

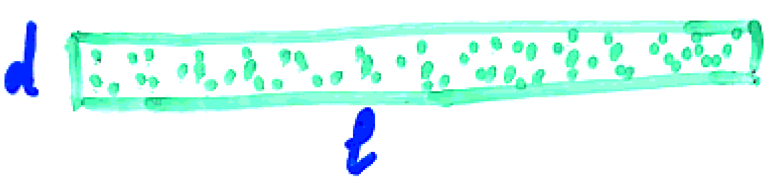
2) 

$$L = \frac{N}{J}$$

$\sqrt{\frac{L}{C_{chain}}} = \frac{N}{\sqrt{JC}} \gg 4$, but $\sqrt{JC} \gg 1$ is necessary to prevent phase slips.

Too many junctions...

3) Kinetic inductance



(Amorphous film)

$$L = \frac{\ell}{d\,\rho_s} \sim \frac{\ell}{d}\,\frac{R_\square}{\pi\Delta} = \frac{R}{\pi\Delta}$$

normal state resistance

(Assuming that quantum fluctuations are not too strong)

For kinetic inductors, $C_{geom.}$ is not a limiting factor because usually $\frac{1}{\sqrt{L C_{geom}}} > \Delta$.

$$\langle \varphi^2 \rangle \sim R = \frac{\ell}{d} R_\square$$

It is possible that $R_\square \sim 4\,K\Omega$

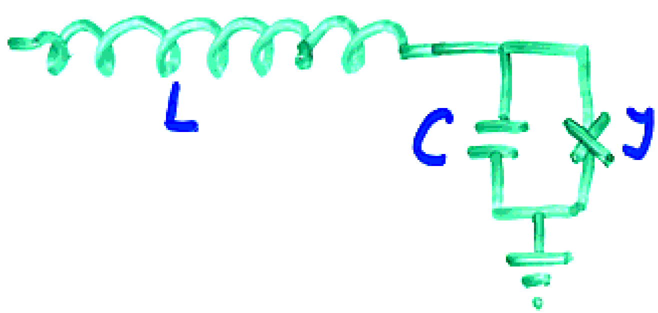
Superinductor requirements:

1) $\frac{\ell}{d} \gg \frac{4}{R_\square}$

2) $d \gg \xi$ to prevent phase slips

ξ ~ 30Å in MoGe

Typical application:



$\mu = \sqrt{JC} \sim 1$
(intermediate quantum regime)

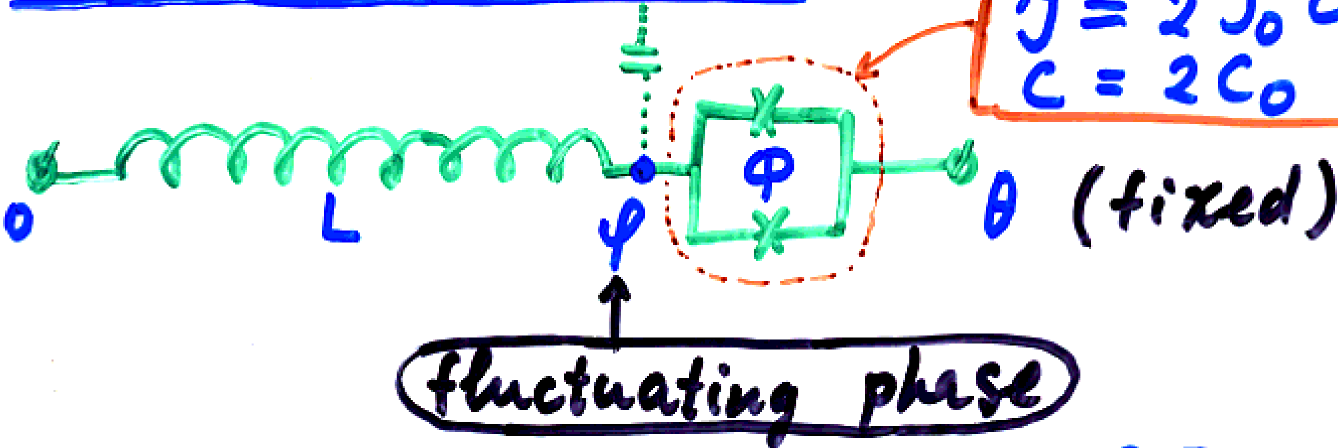
For $Al - Al_2O_3$ junctions $\omega_p = \sqrt{\frac{J}{C}} \sim 20\,GHz$
$\sim 1\,K$

$J \sim C^{-1} \sim \omega_p \Rightarrow C \sim 5 \cdot 10^{-15} F$
in the junction

The geometric capacitance is small if $\ell \ll 50\mu m$

Adiabatic switch

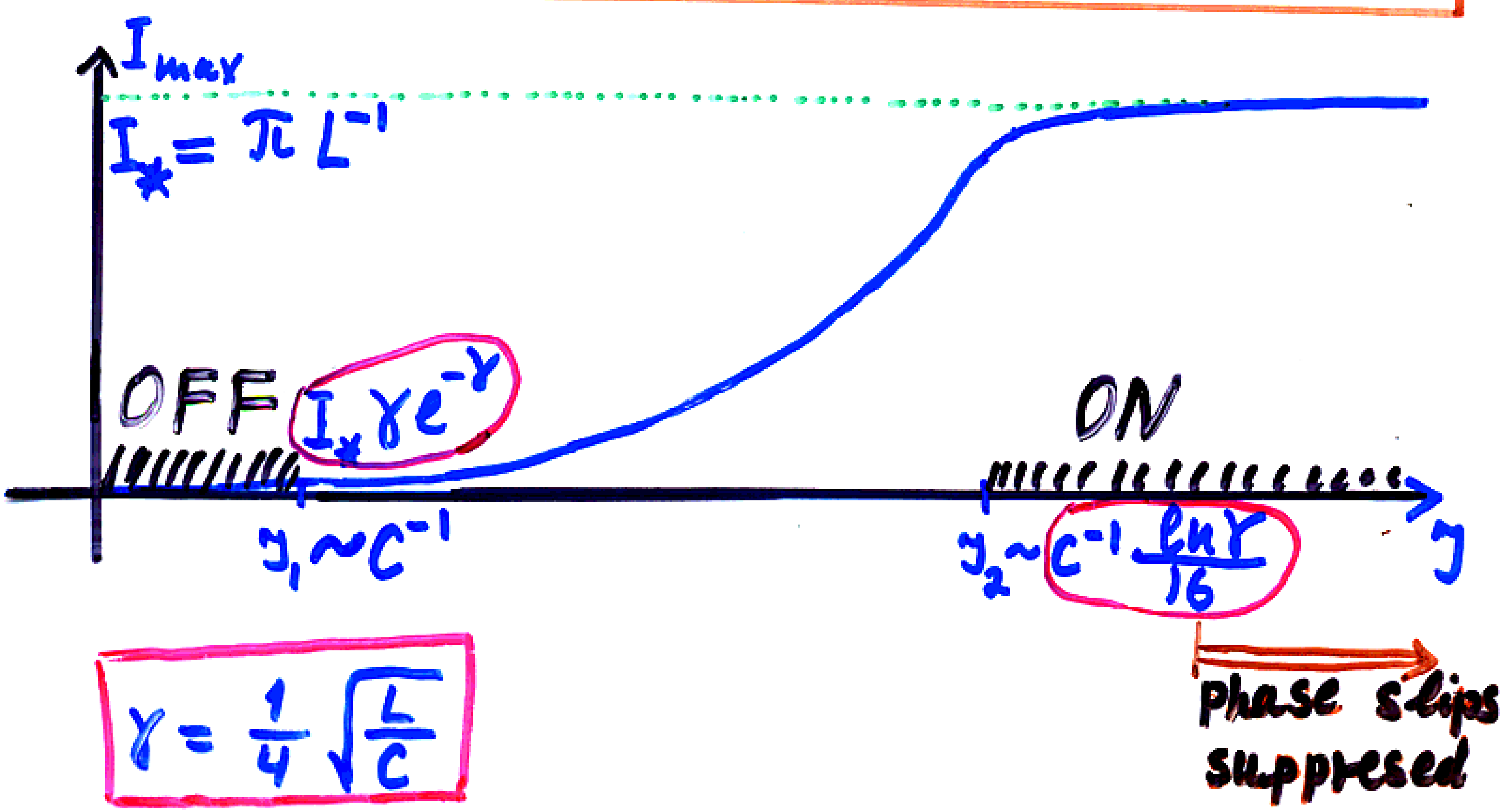
$$J = 2J_0 \cos\frac{\varphi}{2}$$
$$C = 2C_0$$



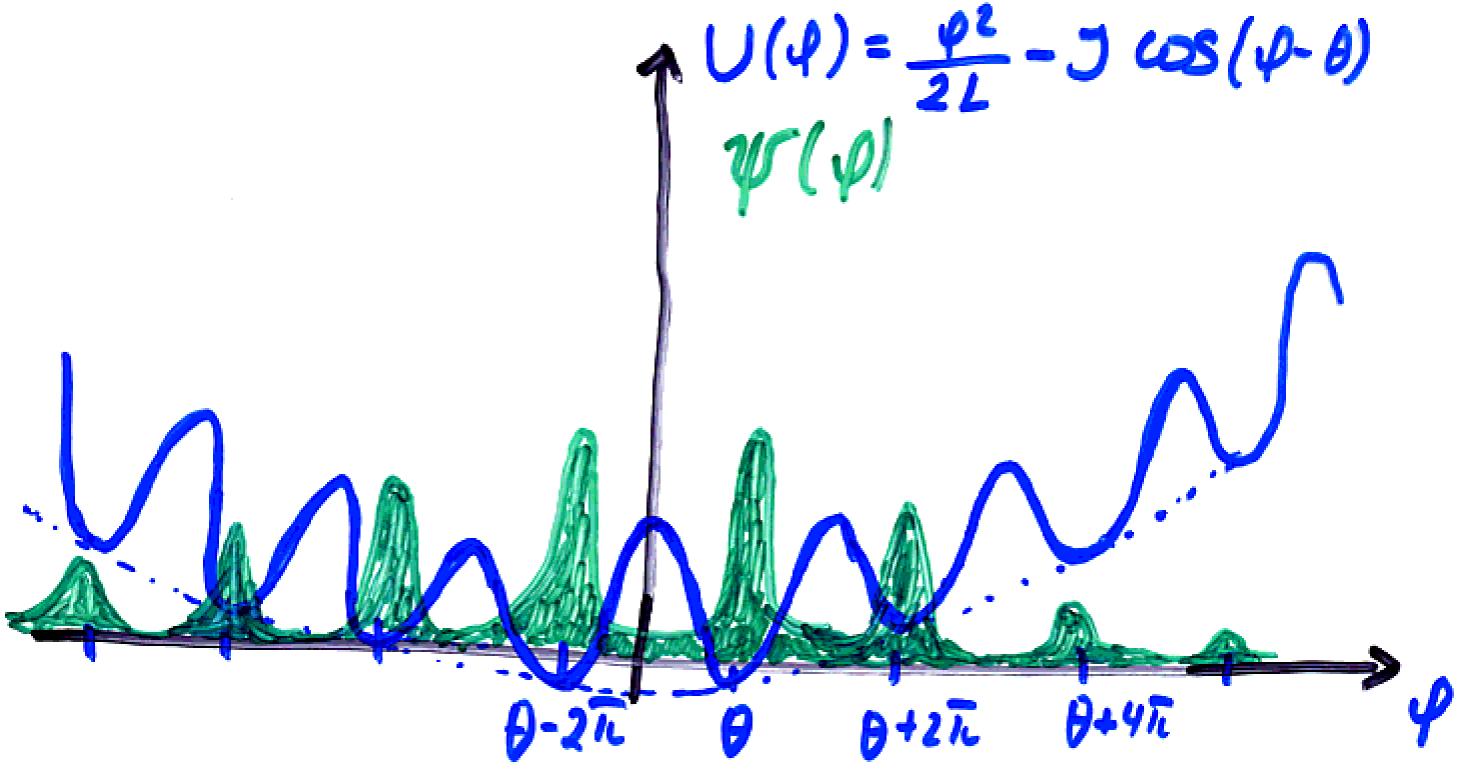
Superconducting current $I(\theta) = \dfrac{\partial E}{\partial \theta}$

I_{max} change as a function of J by many orders of magnitude

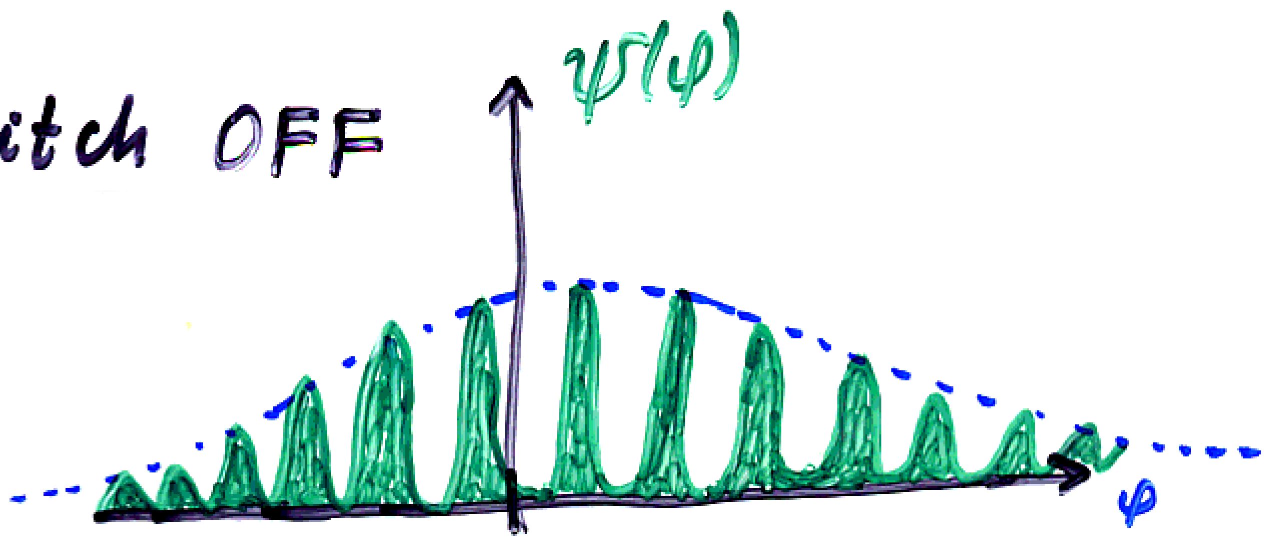
$$\boxed{H = \frac{\varphi^2}{2L} - J\cos(\varphi - \theta) + \frac{1}{2C}\left(\frac{\partial}{i\partial\varphi} - n_g\right)^2}$$



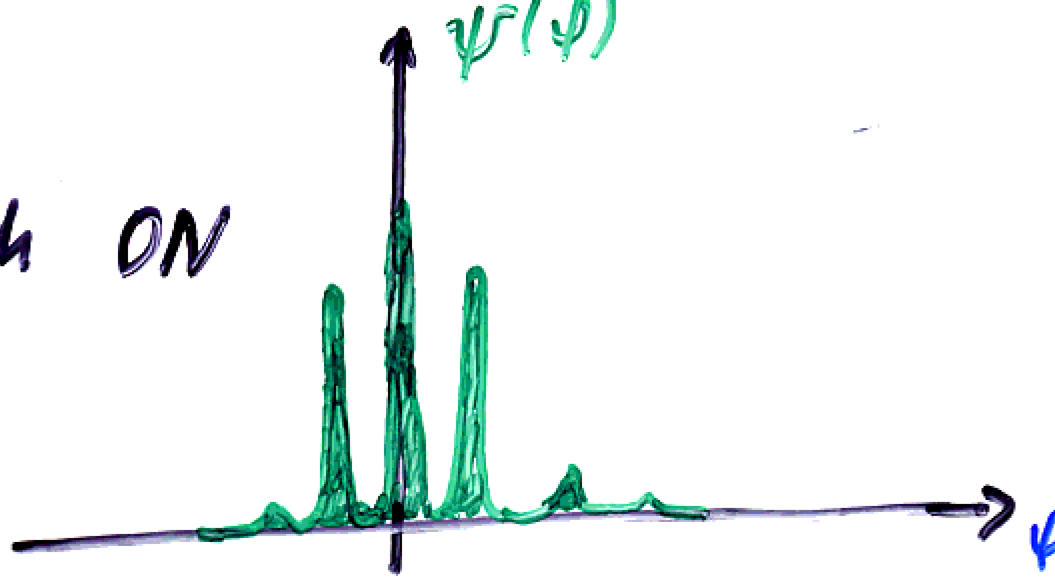
$$\boxed{\gamma = \frac{1}{4}\sqrt{\frac{L}{C}}}$$



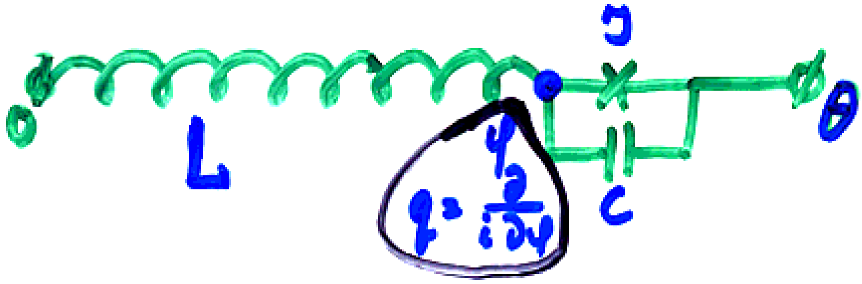
switch OFF



switch ON



The physics is very simple:



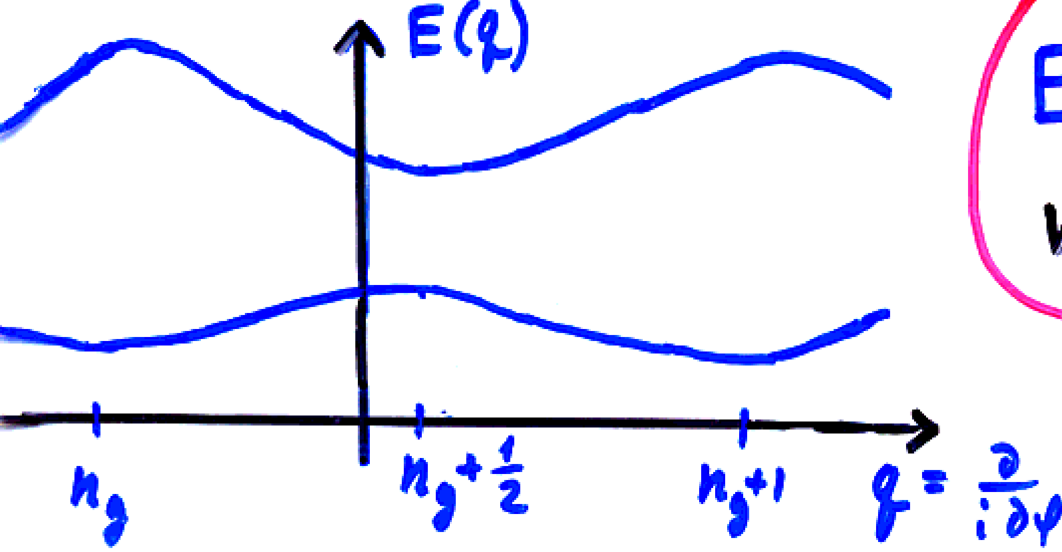
1) If $L \to \infty$ then

$\varphi \approx \theta + 2\pi n$

$$H \approx -J \cos(\varphi - \theta) + \frac{1}{2C}\left(\frac{\partial}{i\partial\varphi} - n_g\right)^2$$

φ varies from $-\infty$ to $+\infty$

Band structure:



$$E = \frac{1}{C} f(\mu, q - n_g)$$

where $\mu = JC$

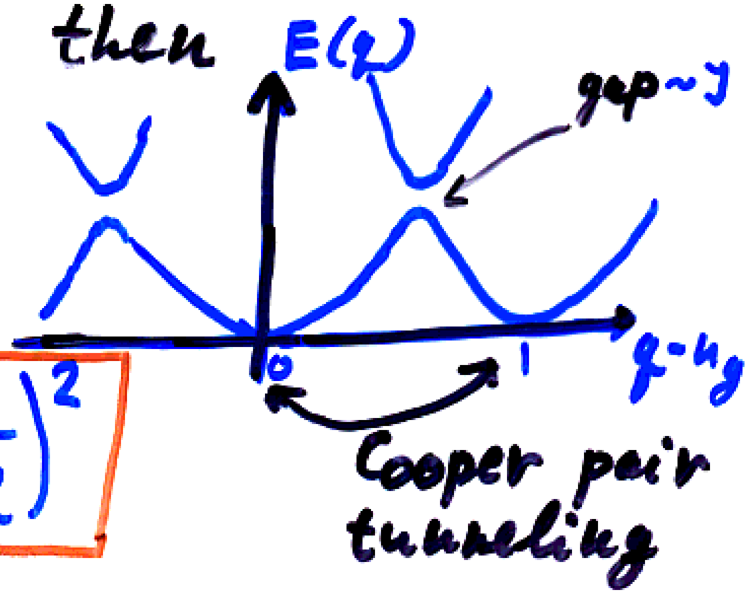
If $\mu \gg 1$ then $f(\mu, q) \approx -t\cos(2\pi q)$

where $\boxed{t \sim \sqrt{\mu}\, e^{-8\sqrt{\mu}}}$ ← phase slip amplitude $(\times C)$

Switch is "closed" if $\boxed{C^{-1} t \ll L^{-1}}$

2) If $\mu = JC \ll 1$ then

$$E(q) \approx \frac{(q-n_g)^2}{2C}$$



$$\boxed{H_{eff} = E(q) + \frac{1}{2L}\left(\frac{\partial}{i\,\partial q}\right)^2}$$

Critical current $I_{max} \sim$ tunneling amplitude between $q = n_g$ and $q = n_g + 1$

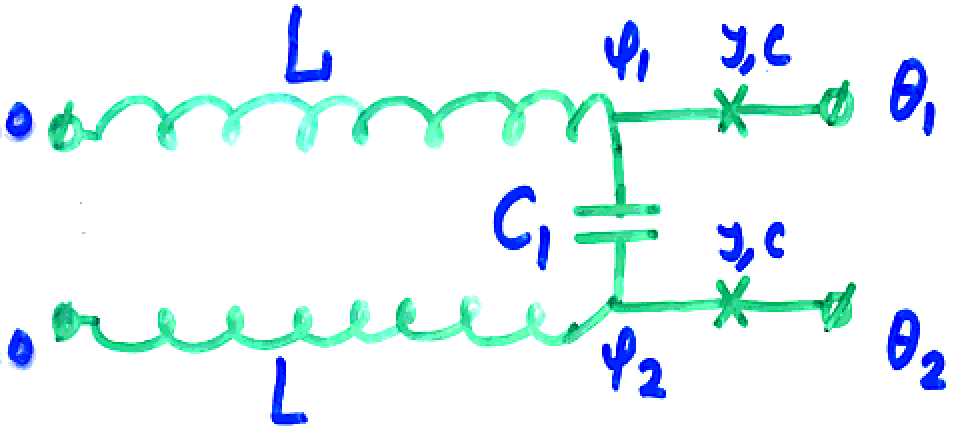
$$I_{max} \sim L^{-1}\gamma e^{-\gamma},$$

where $\gamma = \frac{1}{4}\sqrt{\frac{L}{C}}$

Caveat : Large numeric factors may be involved.

DC transformer with 1:1 current ratio

(current mirror)



$$4\mathcal{J}C \sim 1$$

$$\gamma = \frac{1}{8}\sqrt{\frac{L}{C}} \gg 1$$

$$C_1 \gtrsim C \ln \gamma$$

$$H = \frac{\varphi_+^2}{L} + \frac{\varphi_-^2}{4L} + \frac{q_+^2}{4C} + \frac{q_-^2}{2C_1 + C} - 2\mathcal{J} \cos\frac{\varphi_- - \theta_1 + \theta_2}{2} \times \cos\left(\varphi_+ - \frac{\theta_1 - \theta_2}{2}\right)$$

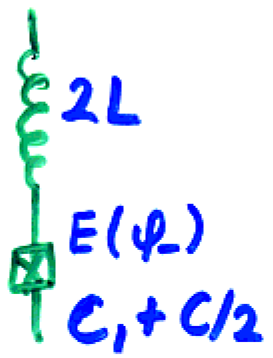
where $\underbrace{\varphi_+ = \frac{\varphi_1 + \varphi_2}{2}}_{\text{fast}}$, $\underbrace{\varphi_- = \varphi_1 - \varphi_2}_{\text{slow}}$

Equivalent circuits:

For φ_+:

$I_{max} \sim L^{-1} \gamma e^{-\gamma}$ (very small)

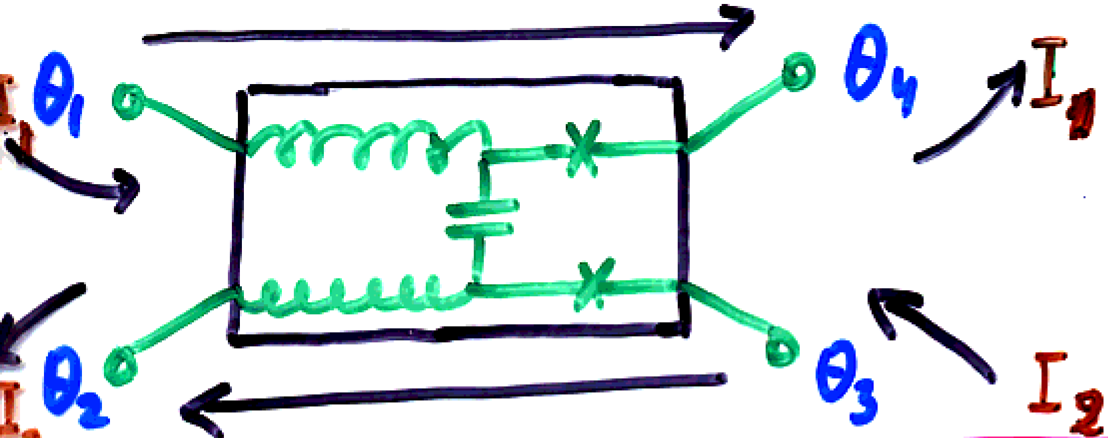
For φ_-:



$I_{max} \sim L^{-1}$ (sufficiently large)

$$E(\theta_1,\theta_2) = \underbrace{F(\theta_1-\theta_2)}_{\min\limits_n \frac{1}{4L}(\theta_1-\theta_2-2\pi n)^2} + \underbrace{g(\theta_1,\theta_2)}_{\sim L^{-1}\gamma e^{-\gamma}}$$

More general situation:

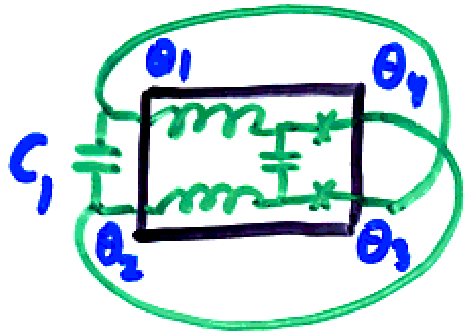


$$\boxed{E = F(\theta_1-\theta_2+\theta_3-\theta_4) + g(\theta_4-\theta_1,\ \theta_3-\theta_2)}$$

$$I_1 \approx I_2 : \quad \begin{cases} I_1 - I_2 = \dfrac{\partial g}{\partial \theta_1} + \dfrac{\partial g}{\partial \theta_2} \sim L^{-1}\gamma e \\ I_1 \approx I_2 < \boxed{I_{max} \approx \dfrac{\pi}{2L}} \end{cases}$$

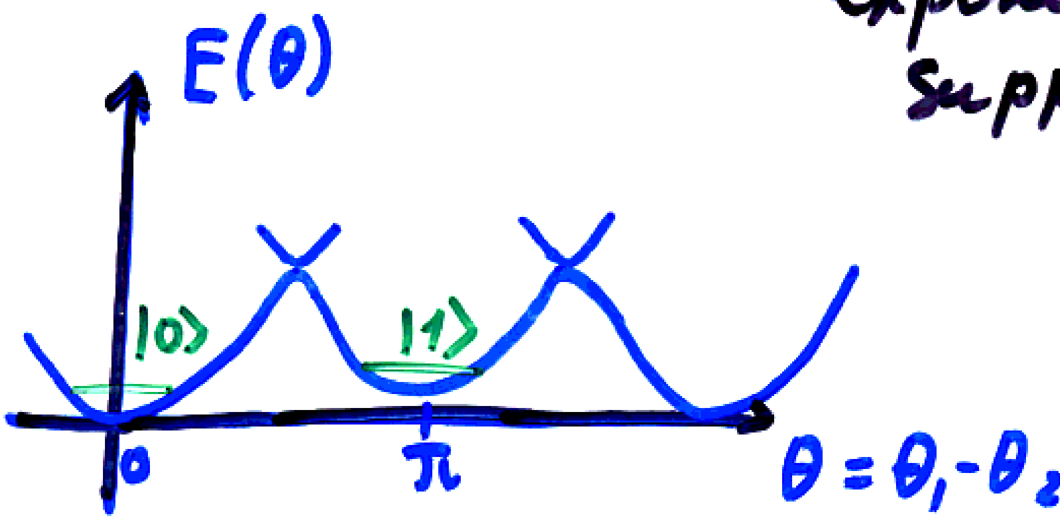
No dissipation $\Rightarrow$ $V_{14} \approx V_{23}$

Turning a current mirror into a qubit



$$\theta_1 = \theta_3$$
$$\theta_2 = \theta_4$$

$$E = F(2(\theta_1 - \theta_2)) + \underbrace{h(\theta_1 - \theta_2)}_{\text{exponentially suppressed}}$$

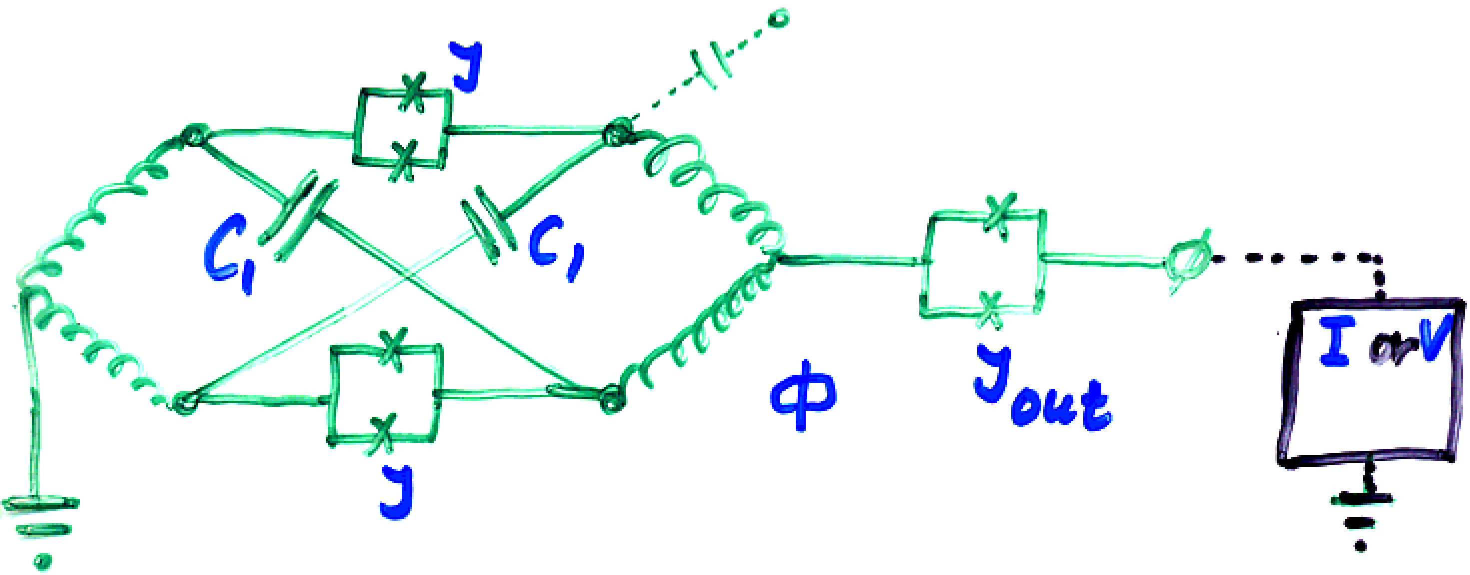


To prevent tunneling $|0\rangle \longleftrightarrow |1\rangle$,

we need $\boxed{JC_1 \gg 1}$

No phase slips in the inductors

Complete qubit



1) Quiet state :

J ON

J_{out} OFF

2) Phase measurement :

$|0\rangle$ vs $|1\rangle$

J ON

J_{out} ON

$$I_{out} \sim \frac{1}{L}$$

3) Dual measurement

$$\frac{|0\rangle + |1\rangle}{\sqrt{2}} \text{ vs } \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

J OFF

J_{out} OFF

$$V_{out} \sim \frac{1}{C_1}$$