Quantum computing with macroscopic fluorescence signals

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How to build a quantum computer

Requirements:

- well-defined qubits
- single qubit rotations
- read out of computational results
- one universal two-qubit gate
- communication between computers

Problems:

- vulnerability to parameter fluctuations
- spontaneous decay rates can lead to dissipation

Solutions:

- topological quantum computing
- quantum error correction
- measurement-based quantum computing
Where are now?

Ion trap experiments have succeeded in entangling up to eight ions with a relatively high fidelity.
Read out of atomic qubits
The read out of a single qubit

An ideal measurement:

According to the projection postulate, a quantum mechanical measurement of the qubit state $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$ requires the projection

$|0\rangle$ with probability $|\alpha|^2$

$|1\rangle$ with probability $|\beta|^2$

The result should be visible macroscopically on a measurement device.
Experimental setup

Level scheme:

\[ H_{\text{cond}} = \frac{1}{2}\hbar \Omega |1\rangle \langle 2| + \text{H.c.} - \frac{i}{2}\hbar \Gamma |2\rangle \langle 2| \]

No-photon time evolution: \(^1,^2\)

Basic idea

- The no-photon evolution damps away population in $|1\rangle$.
- The measurement result is indicated by no or many photons.

Advantages:
- simple and precise
- works even when using real photon detectors
- independent of concrete size of $\Omega$ and $\Gamma$
II

Macroscopic quantum jumps
Historical debate on quantum jumps

Schrödinger asserted that the application of QM to single systems would necessarily lead to nonsense such as quantum jumps. Bohr argued in response that the problem lay with the physics experiments of the time.¹,²

¹Bohr, Philos. Mag. 26, 476 (1913).
Macroscopic quantum jumps

The existence of a random fluorescence telegraph signal in the fluorescence of single ions, was predicted as early as 1975 by Dehmelt.\textsuperscript{1} About ten years later several groups verified them experimentally.\textsuperscript{2}


\textsuperscript{2}Nagourney et al., PRL 56, 2797 (1986); Sauter et al., PRL 57, 1696 (1986); Bergquist et al., PRL 57, 1699 (1986).
Quantum jump description

The no-photon evolution:  
\[ H_{\text{cond}} = \frac{1}{2} \hbar \Omega_b \left[ |b\rangle \langle e| + |g\rangle \langle b| + \text{H.c.} \right] - \frac{i}{2} \hbar \Gamma_d \left[ |b\rangle \langle b| + |d\rangle \langle d| + 2 |e\rangle \langle e| \right] - \frac{i}{2} \hbar \Gamma_b \left[ |b\rangle \langle b| + |e\rangle \langle e| \right] \]

Reset Operators:  
\[ R_d = |d\rangle \langle e| + |g\rangle \langle d| + |b\rangle \langle e| + |g\rangle \langle b| \]
\[ R_b = |b\rangle \langle e| + |g\rangle \langle b| \]

Characteristic time scales:  
\[ T_{\text{dark}} = \frac{1}{\Gamma_d}, \quad T_{\text{light}} = \frac{3 + 2 x^2 + x^4}{\Gamma_d}, \quad T_{\text{em}} = \frac{3 + 2 x^2 + x^4}{(2 + x^2) \Gamma_b} \]

with \( x \equiv \Gamma_b/\Omega_b \) and for \( \Gamma_d \ll \Gamma_b, \ \Omega_b \approx \Gamma_b \).
Transition into a dark period

Possible trajectory of the four-level toy model for $\Omega_b = \Gamma_b$ and $\Gamma_d = 10^{-2} \Gamma_b$. The upper figure shows the population in the dark state $|b\rangle$; the vertical lines mark photon emissions. The population in $|b\rangle$ eventually reaches one.
Again, the spontaneous emission of a photon results in the build up of population in $|b\rangle$. This time, another photon is emitted before the dark state population reaches one. The system remains in a macroscopic light period.
III

Entangled pair generation $^1$

$^1$Metz, Trupke, and Beige, PRL 97, 040503 (2006).

Experimental setup to entangle two atoms

The successful generation of a maximally entangled atom pair is triggered on a macroscopic dark period. The laser should be turned off once the cavity emission stops.
An adiabatic elimination of the excited states due to a large detuning $\Delta$ shows that the atoms remain mainly in their ground states.
Comparison with toy model

The level scheme of the combined atom-cavity system resembles the previously considered four-level toy model.
Quantum jump simulation of the light and dark periods of the atom-cavity system for \( \Delta = 50 \kappa, \Gamma = 0.05 \kappa, g = \Omega_L = \kappa, \Omega_M = 0.05 \kappa \) and \( \eta = 1 \).
The characteristic time scales

\[ T_{\text{cav}} = (3 + 4x^2) \cdot \frac{\kappa \Delta^2}{4g^2\Omega_L^2} \approx \frac{3\kappa \Delta^2}{4g^2\Omega_L^2} \]

\[ T_{\text{light}} = \frac{3 + 16x^2 + 16x^4}{(3 + 8x^2)\Gamma} \cdot \frac{16\Delta^2}{\Omega_L^2} \approx \frac{16\Delta^2}{\Gamma \Omega_L^2} = \frac{64g^2}{3\kappa\Gamma} T_{\text{cav}} \]

\[ T_{\text{dark}} = \frac{16\Delta^2}{3\Gamma \Omega_L^2} = \frac{64g^2}{9\kappa\Gamma} T_{\text{cav}} \]

for \( \Omega_M < g, \kappa, \Gamma, \Omega_L \ll \Delta, \Gamma_0 = \Gamma_1 = \frac{1}{2}\Gamma \) and \( x \equiv \Omega_L^2/4\Delta\Omega_M \ll 1 \)

Crucial for the ability to distinguish a light from a dark period is that \( T_{\text{dark}} \) is sufficiently longer than \( T_{\text{cav}} \). Turning off the laser field \( \Omega_L \) within a dark period is enough to complete the state preparation.
Achieving fidelities above 0.9 is possible even when using a relatively modest cavity with $C \equiv g^2/\kappa \Gamma$ is as low as 10 and when using a real-life single photon detector with an efficiency as low as $\eta = 0.2$. 

Fidelity of the prepared state

![Graph showing Fidelity vs. t/T_{dark} with data points and curves for Numerics and analytical F limits.](image-url)
Finite detector efficiency

\[
\lim_{t \to \infty} F(t) = \frac{3}{2} \left[ 3 + \sqrt{9 - 48\eta C + 256(\eta C)^2 - 16\eta C} \right]^{-1}
\]
Advantages and disadvantages

**Advantages:**
- no coherent control required
- robust against parameter fluctuations
- relatively large decay rates are allowed
- finite detector efficiencies are allowed
- deterministic

**Disadvantages:**
- symmetry is important
- scaling by placing more atoms into cavity is feasible but complicated
IV

Cluster state growth with parity measurements

Fusion of clusters via parity measurements

Fusion of entangled pairs into linear clusters:

\[ P_{01} + P_{10} = |01\rangle\langle 01| + |10\rangle\langle 10| \]

combined with a Hadamard rotation can be used to fuse two shorter cluster states into a larger one.
Fusion of linear cluster states into 2D clusters for one-way quantum computing:

Once a 2D cluster state has been build, a one-way quantum computation can be performed without having to create additional entanglement, using only single-qubit rotations and measurements.
V

Parity measurements with classical fluorescence signals

\[1\] Metz, Schön, and Beige, *Atomic cluster state build up with classical fluorescence signals* (in preparation).
Experimental setup to realise a parity measurement

Basic idea:

Atomic levels:

The successful completion of the projection $P_{01} + P_{10}$ is indicated by the emission of photons as if there is only one emitting atom inside the resonator.
Atom chip experiments in London

Miniaturised quantum optics experiments:

For example, Ed Hinds’ group in London helped to develop a new atom chip technology and mounted optical cavities on a chip.
Relatively low emission rate

Possible trajectory for $\Gamma = 0.1\kappa$, $g = \kappa$, $\eta = 0.5$, $\Delta = 50\kappa$, $\Omega = \kappa$ and given the initial state

$$|\psi(0)\rangle = \frac{1}{2}(|01\rangle - |10\rangle)(|01\rangle - |10\rangle)$$

$$= \frac{1}{2}(|0101\rangle - |0110\rangle - |1001\rangle + |1010\rangle).$$
Possible trajectory for \( \Gamma = 0.1\kappa,\ g = \kappa,\ \eta = 0.5,\ \Delta = 50\kappa,\ \Omega = \kappa \) and given the initial state

\[
|\psi(0)\rangle = \frac{1}{2} (|0101\rangle - |0110\rangle - |1001\rangle + |1010\rangle).
\]
Fidelity of the desired final state

The parity measurement does not destroy the correlations with the atoms outside the cavity and can be used to build cluster states.
Finite detector efficiencies

Fidelity of the final state for $C = 20$ for different detector efficiencies for an optimised version of the proposed protocol.
Advantages and disadvantages

Advantages:

• no coherent control required
• robust against parameter fluctuations
• relatively large decay rates are allowed
• finite detector efficiencies are allowed

Disadvantages:

• 50% success rate
• symmetry is important
• projection not on 1D but 2D subspace
VI

Final remarks
Measurement-based quantum computing

Examples: • quantum computing using dissipation (Beige; Pachos; Franson)
• one-way quantum computing (Raussendorf & Briegel)
• linear optics quantum computing (KLM; Leung; Nielsen)

1Lapaire, Kok, Dowling, and Sipe, PRA 68, 042314 (2003).
Final remarks 1,2,3

- Easily available measurements can be used to perform entangling operations with a very high fidelity.
- Such measurements do not require ideal photon detectors and are feasible with current technology.
- We employed symmetry properties, hybrid character and dissipation of the system.

1 Metz, Trupke, and Beige, PRL 97, 040503 (2006).