Quantum criticality and spin liquid phase in the Shastry-Sutherland model

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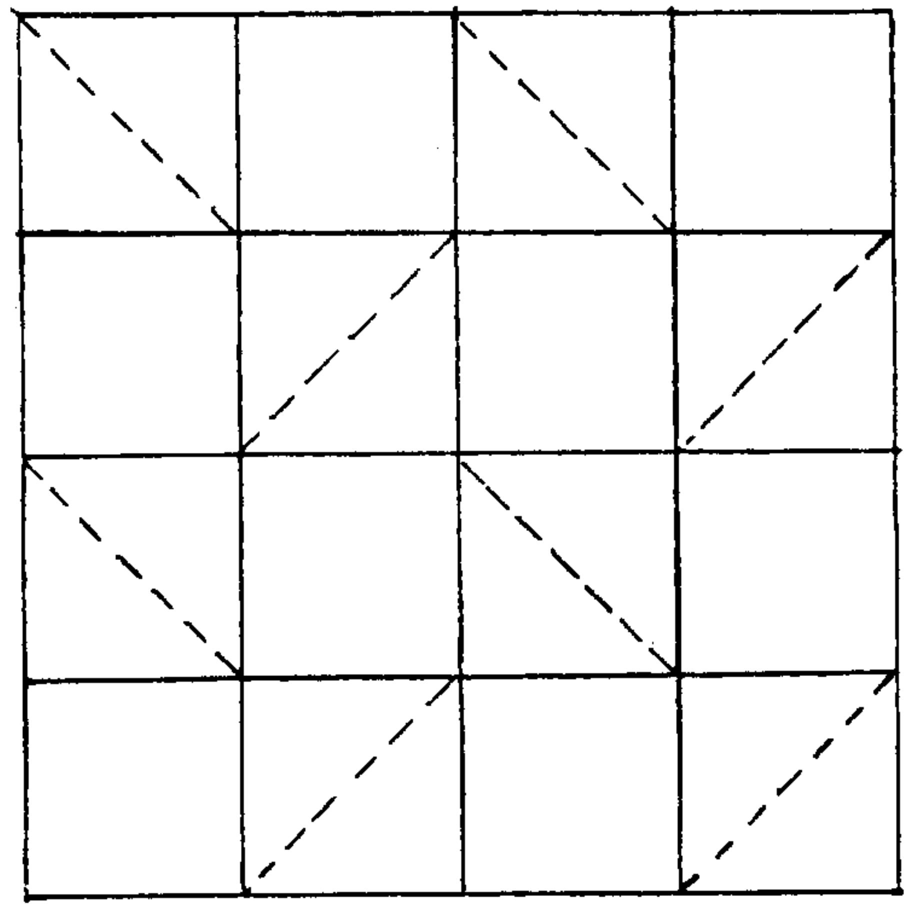
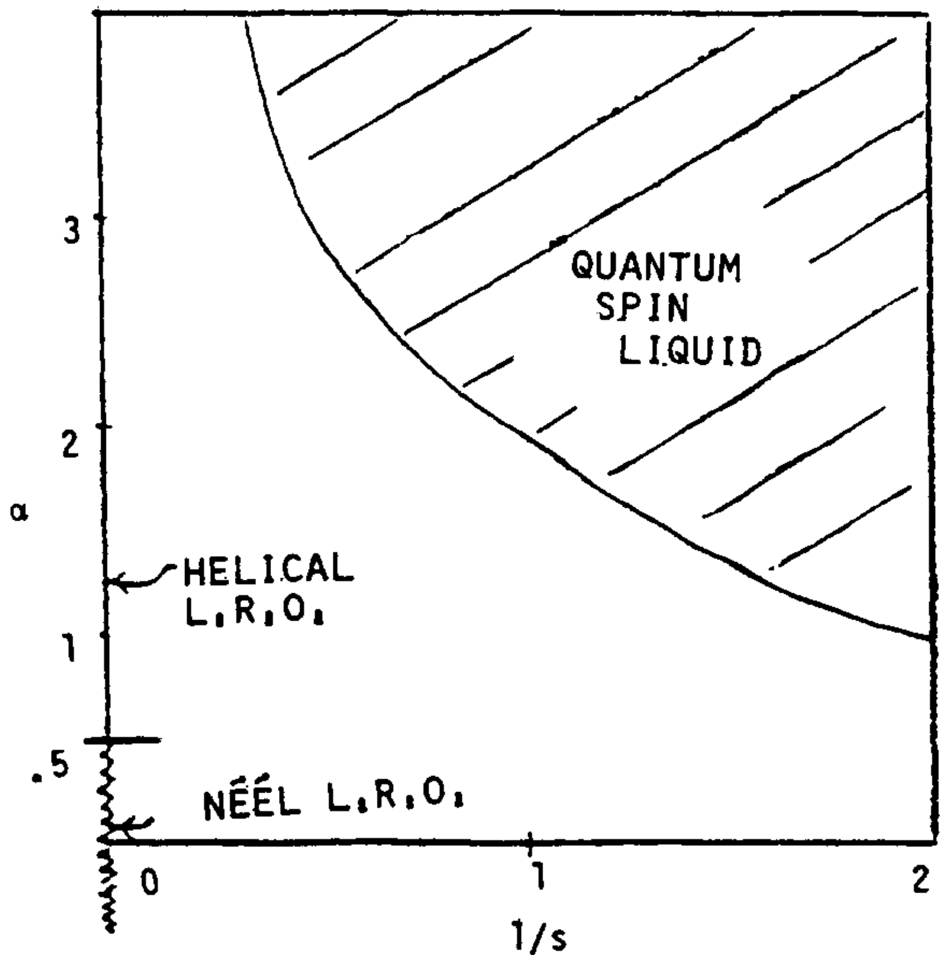
arXiv:2104.08887

Workshop II: Tensor Network States and Applications, IPAM, UCLA, April. 19, 2021

Exact ground state of Shastry-Sutherland model



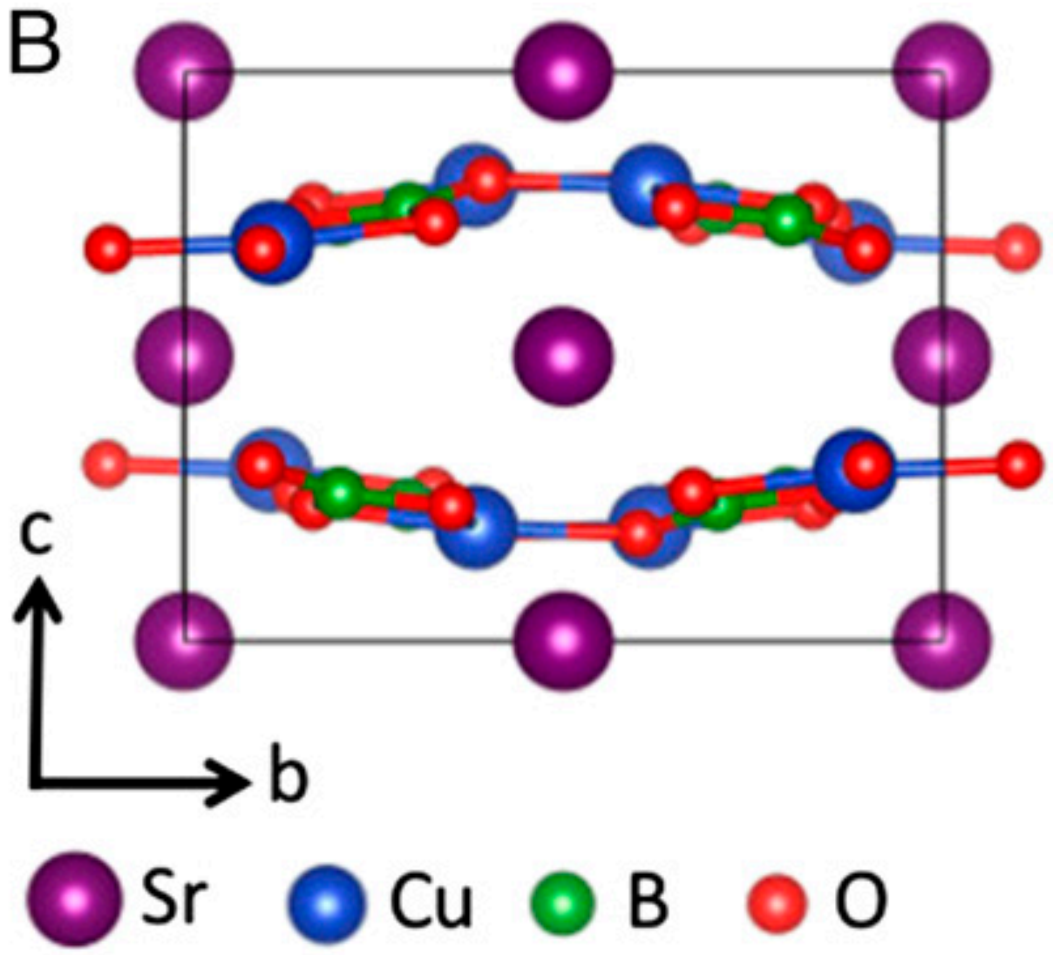


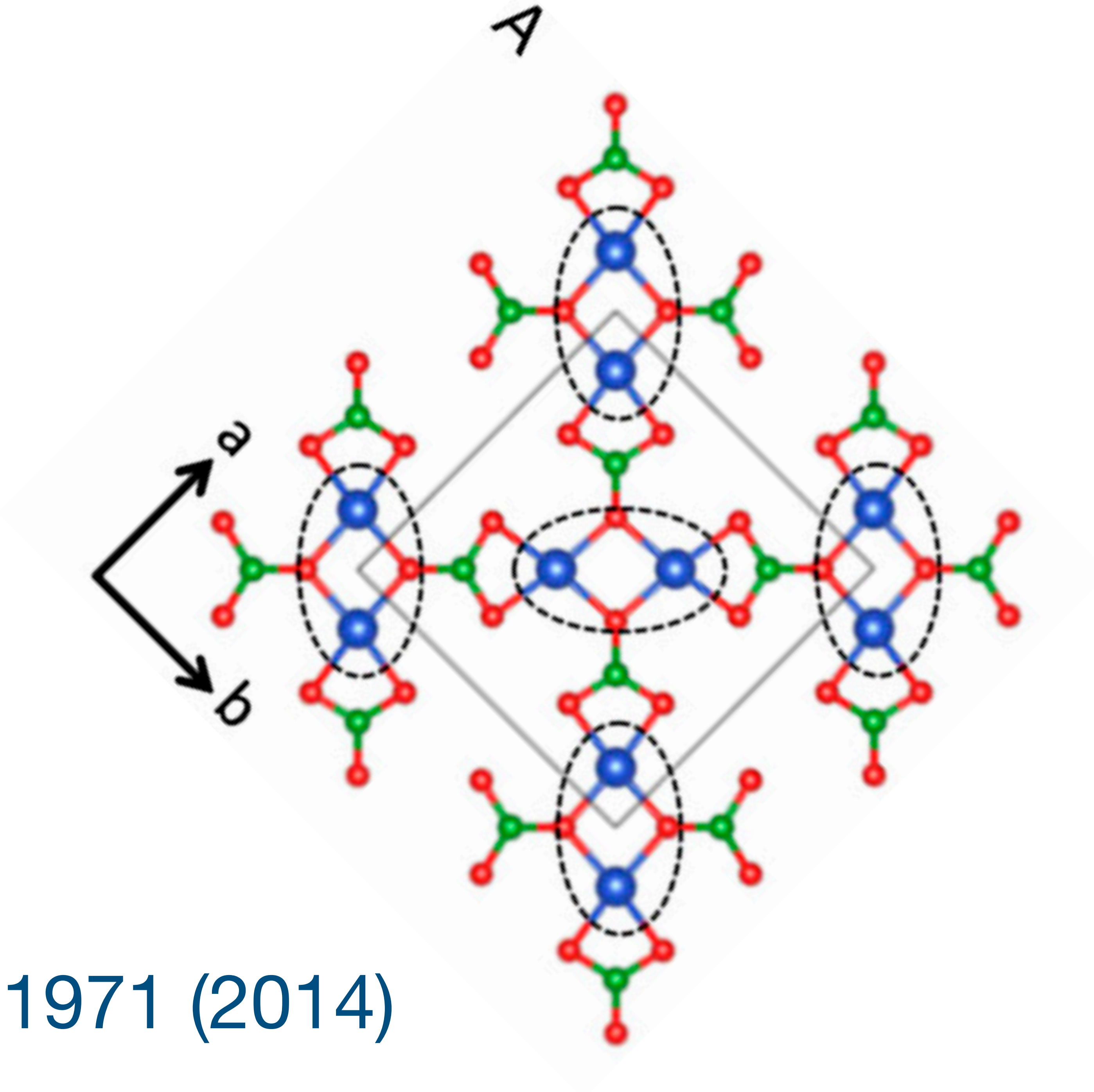
$$H = J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + J' \sum_{\langle\langle l,m \rangle\rangle} \vec{S}_l \cdot \vec{S}_m$$

Shastry and Sutherland, Physica 108B, 1069 (1981)

- Shastry-Sutherland proved that such defined model has an exact dimer singlet (DS) product ground state for $g = J/J' \leq 0.5$ at S=1/2
- At that time, they call it Quantum Spin Liquid (QSL) phase, because its form is of the type given by Anderson

SrCu2(BO3)2 realises Shastry-Sutherland model





PNAS 112, 1971 (2014)

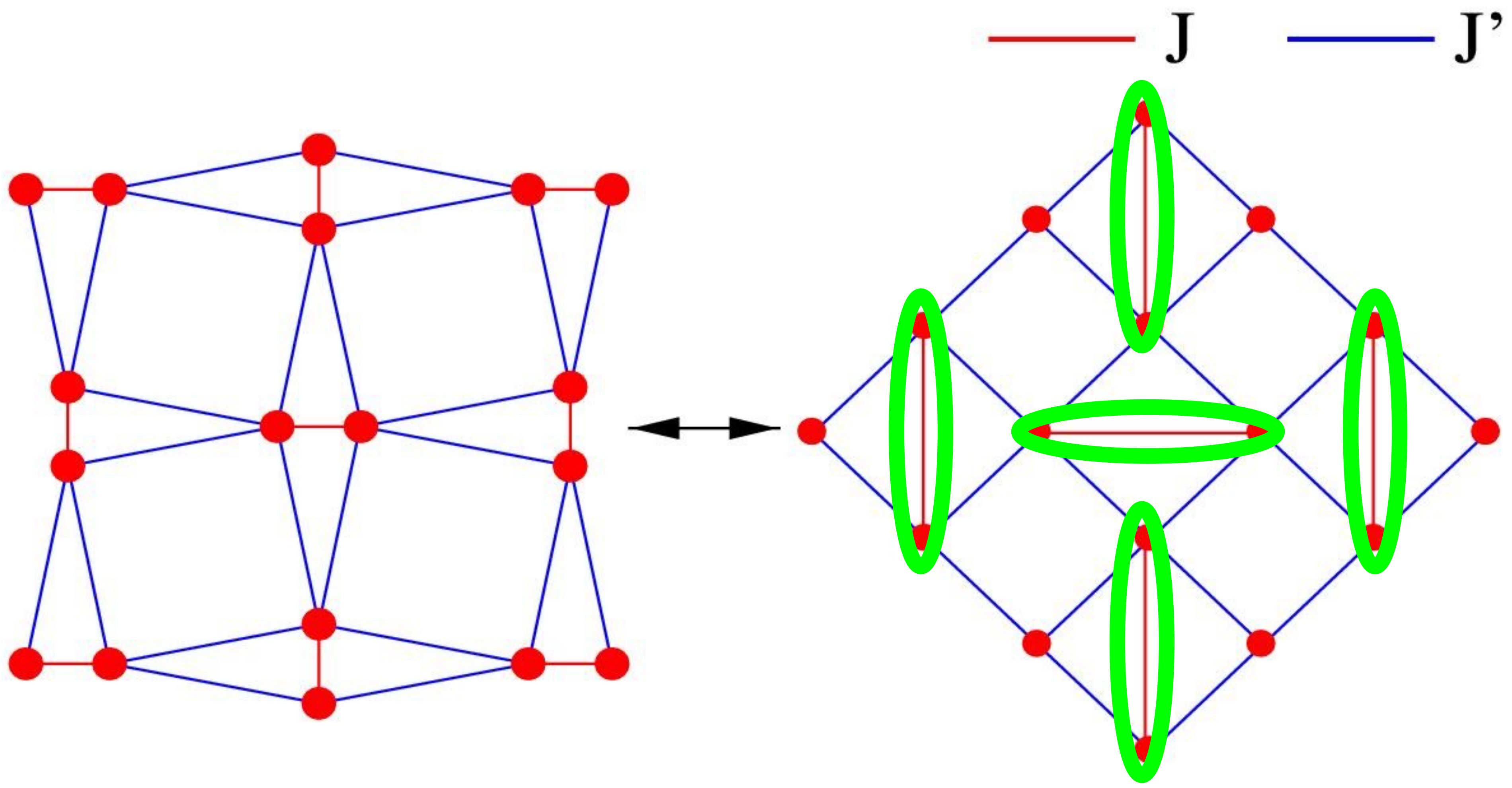
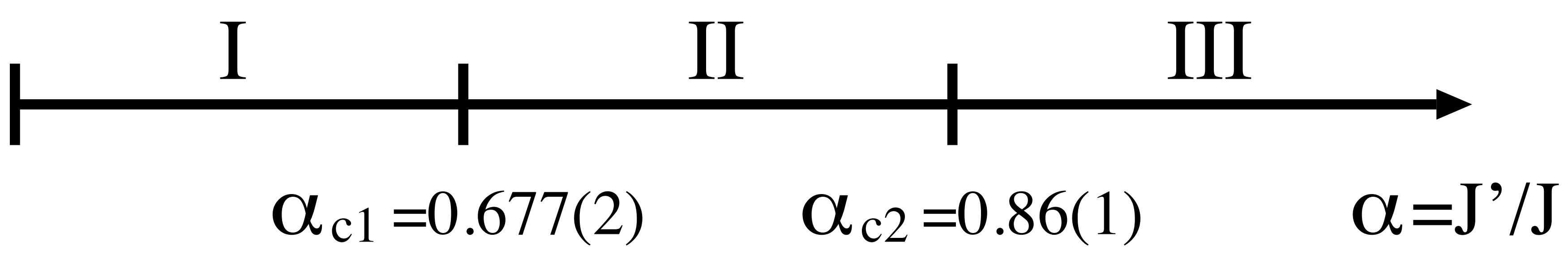


Fig borrowed from F. Mila

- SrCu2(BO3)2 (SCBO) is a layered spin-1/2 magnetic material with magnetic degrees of freedom made of 3d electrons of Cu2+
- SS model can well describe the Antiferromagnetic interactions among Cu atoms within one layer. At ambient pressure, SCBO is in the dimer product phase

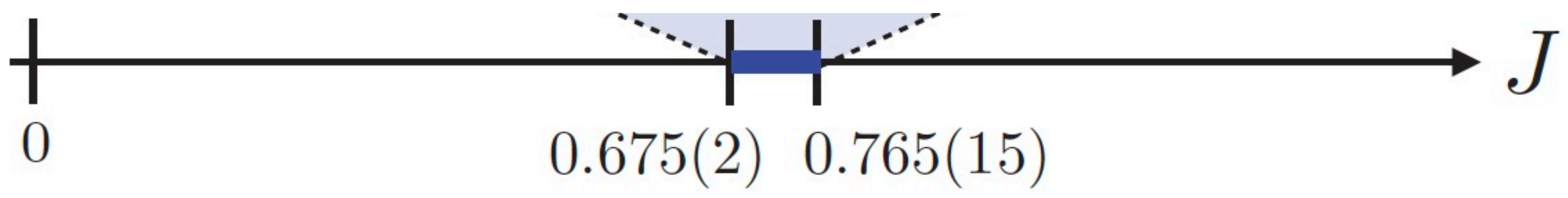
Theoretical results of phase diagram for SS model

- Through Series expansion, Koga and Kawakami (PRL 2000) discovered the plaquette singlet (PS) phase in between dimer singlet and AFM phases



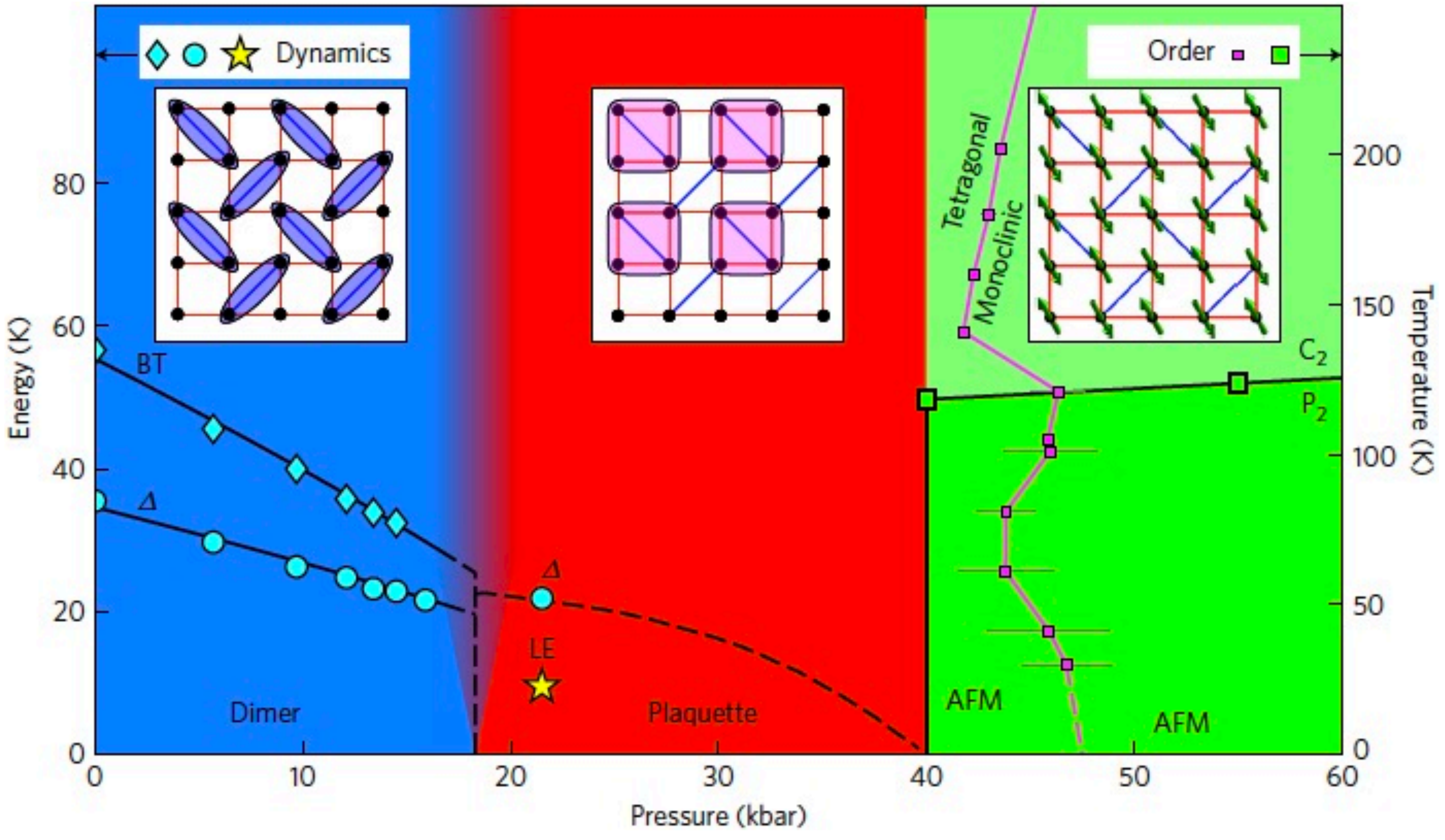
- Using ED, Lauchili, Wessel and Sigrist (PRB 2002) found two-fold degeneracy of PS phase

- Using iPEPS, Corboz and Mila (PRB 2013) improves the accuracy of phase boundary and finds two first order phase transitions

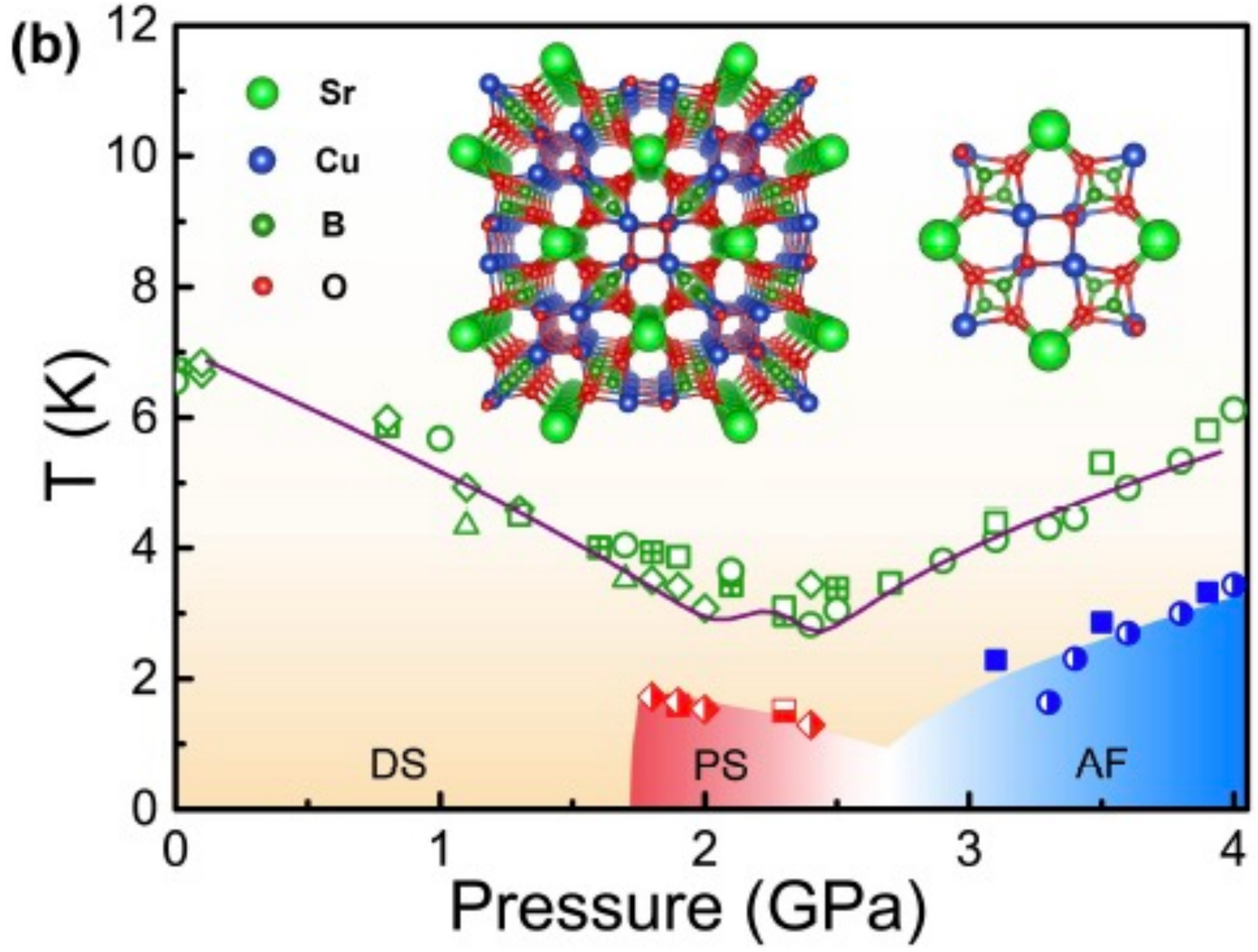


- Using iDMRG, Lee et al (PRX 2019) , claim a Deconfined Quantum Criticality Point (DQCP) between PS and AFM phases

SCBO under pressure



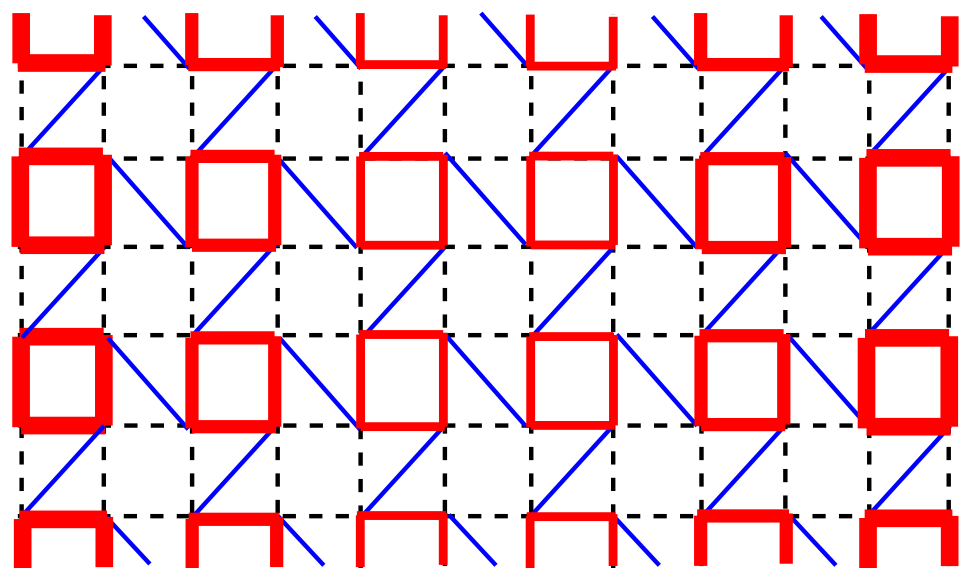
Zayed et al, NP 13, 962 (2017)



Guo et al, PRL 124, 206602 (2020)

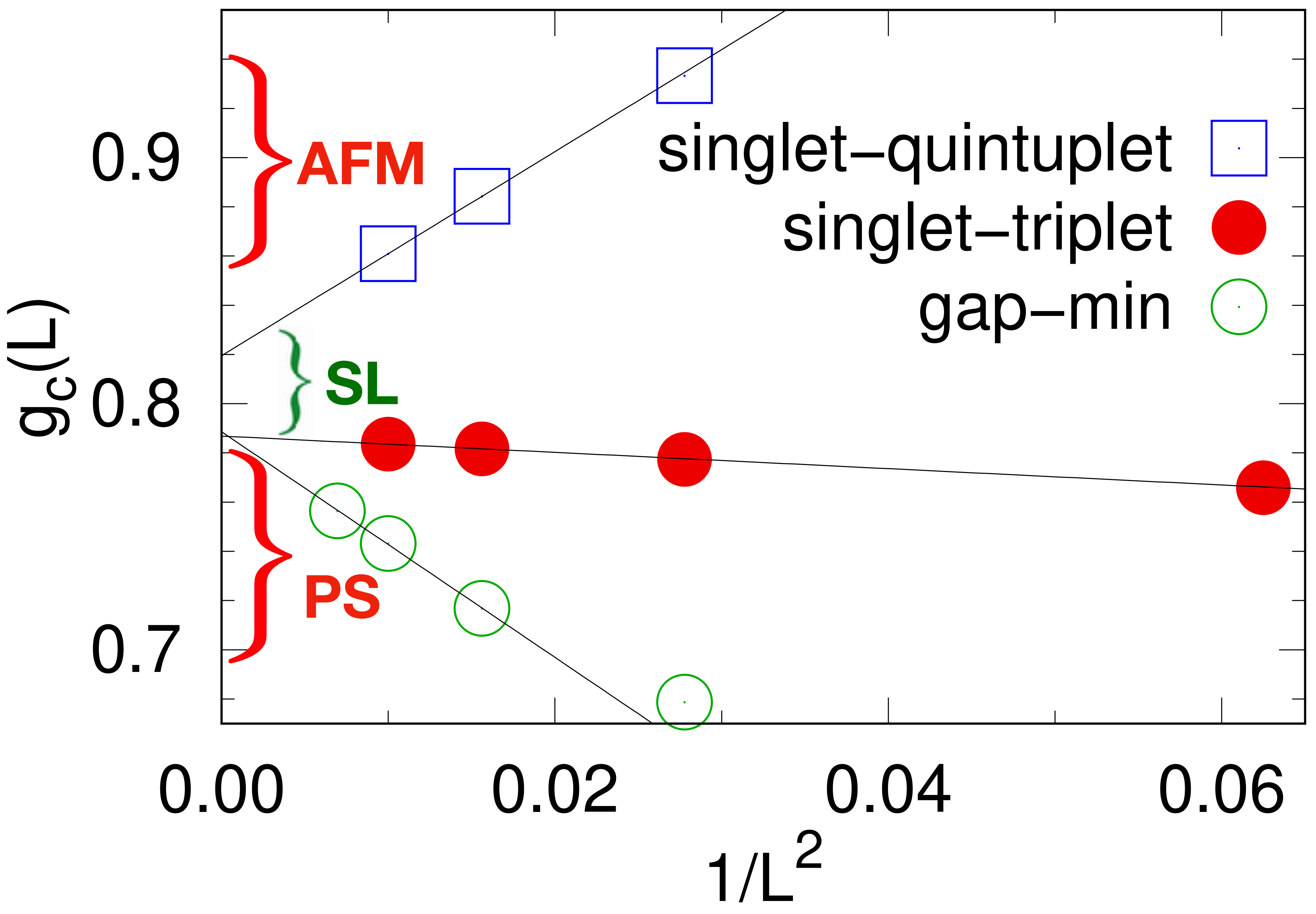
- Turn on pressure play the role of increasing g=J/J′, produces plaquette singlet and Antiferromagnetic phases

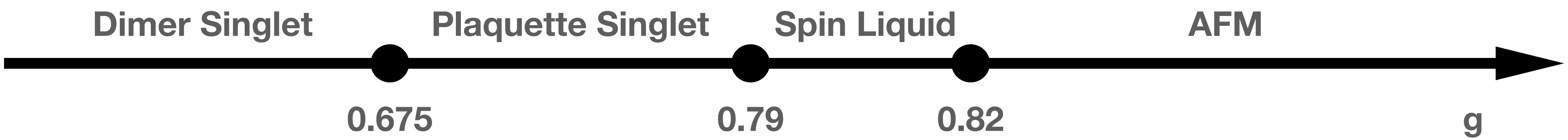
Quantum Criticality and Spin Liquid phase in SS model



J first neighbor
J′ second neighbor
g=J/J′

$$H = J\sum_{\langle ij\rangle} \mathbf{S}_i\cdot\mathbf{S}_j + J'\sum_{\langle ij\rangle'} \mathbf{S}_i\cdot\mathbf{S}_j$$





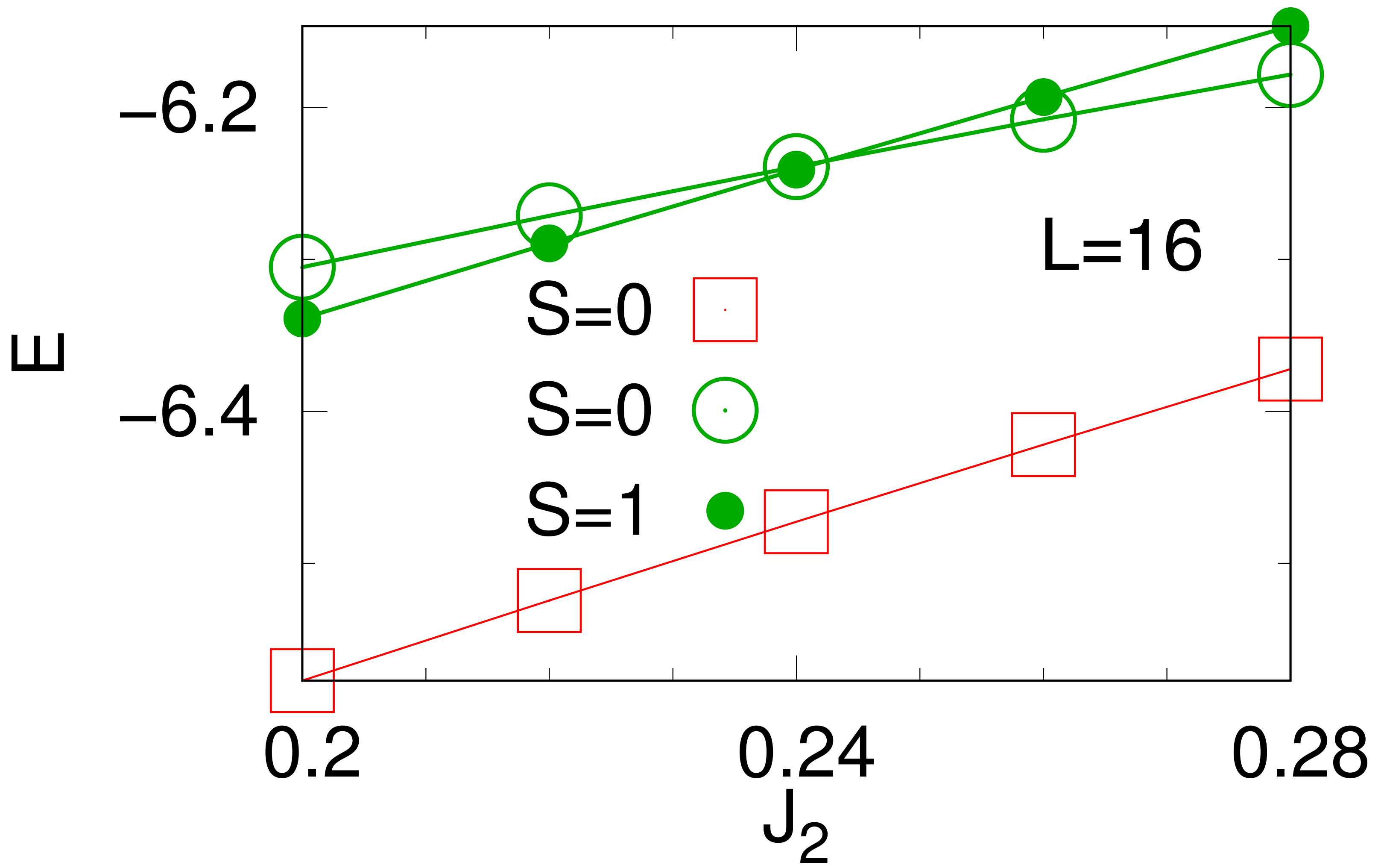
Critical level crossing and the ground state order parameters suggest a gapless spin liquid within a small region g∈[0.79:0.82]

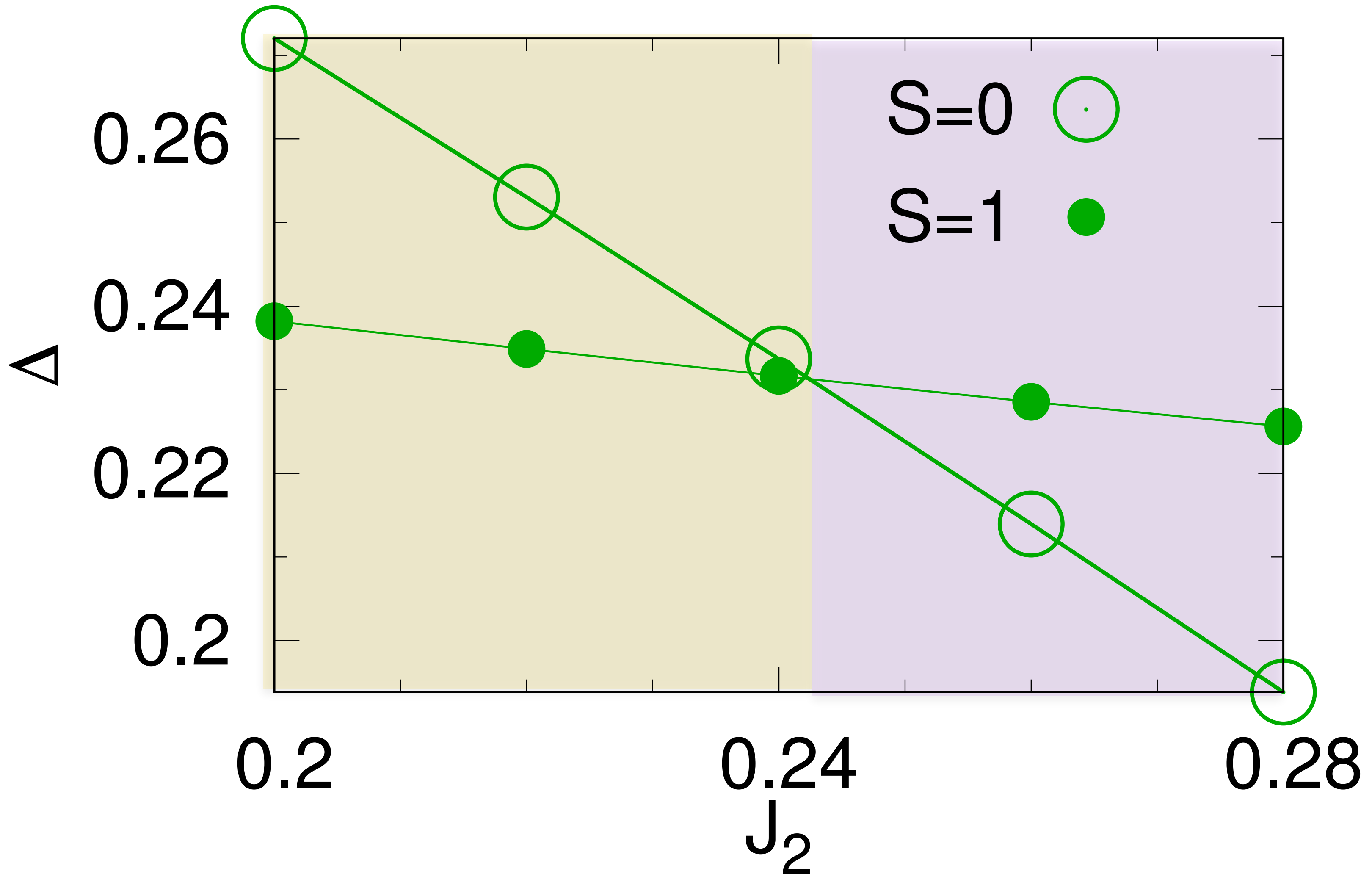
Yang, Sandvik and Wang, arXiv:2104.08887

Rest of the talk

- Excited state critical level crossings in one and two dimensions to detect quantum phase transitions
- Targeting bulk excited eigenstates in finite size DMRG algorithm for quasi-two dimensional cylinder
- Critical level crossing analysis for the Shastry-Sutherland model, existence of a narrow SL phase straddles between PS and AFM states
- Other evidence of the SL phase, order parameters and gap scalings
- Discussions and conclusions

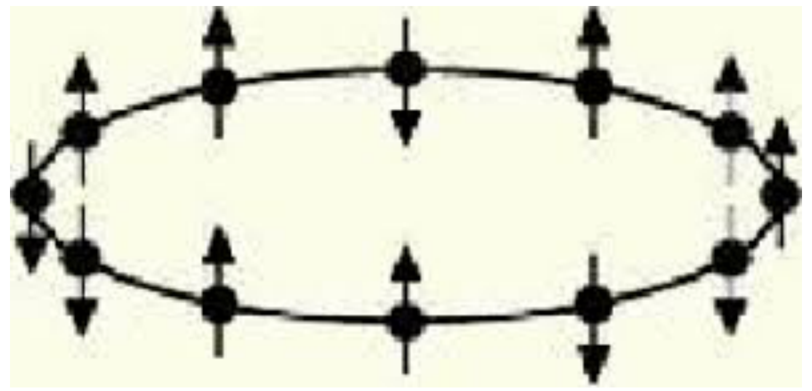
Critical level crossings in one dimension



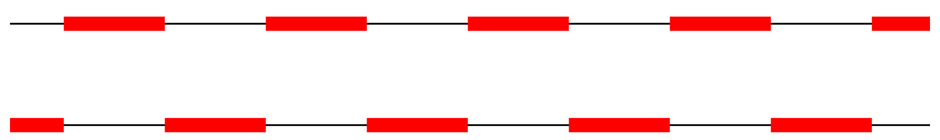


Spin-1/2 chain J1-J2 model

$$H_{J_1-J_2} = J_1 \sum_{\langle i,j \rangle} \mathbf{S}_i \mathbf{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \mathbf{S}_i \mathbf{S}_j$$

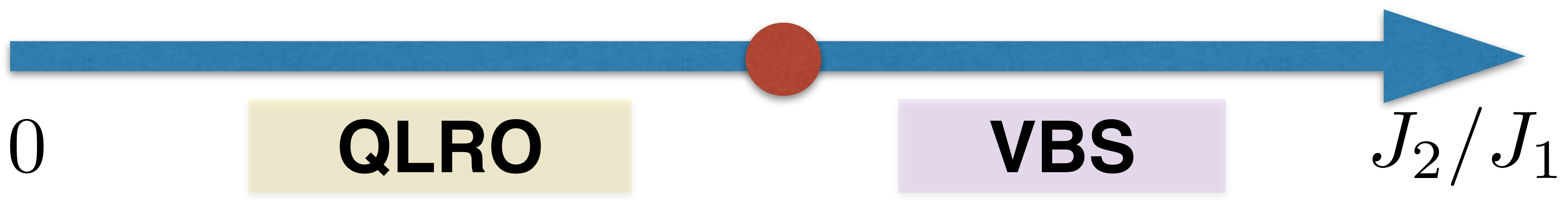


J2=0.5J1 exact VBS

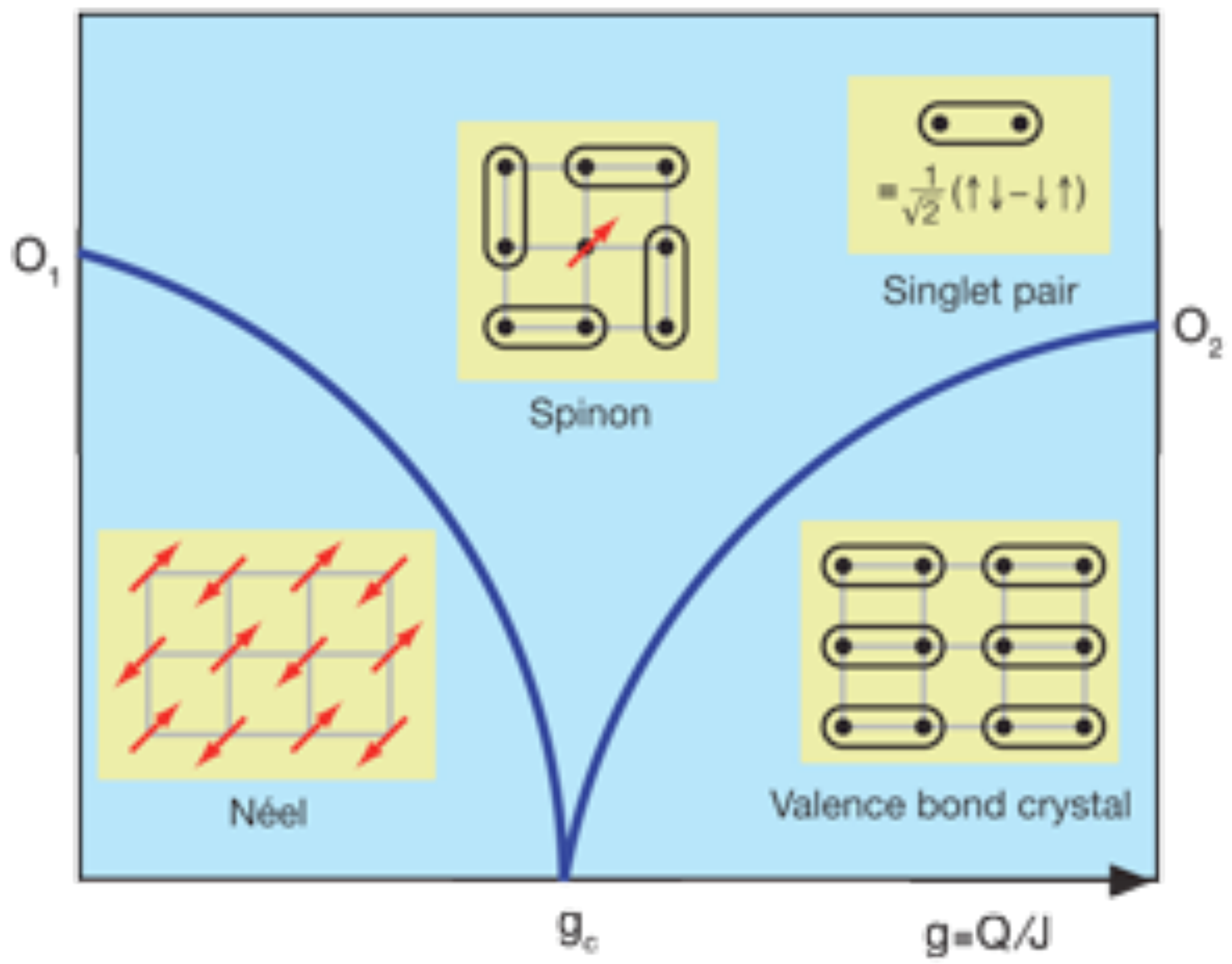


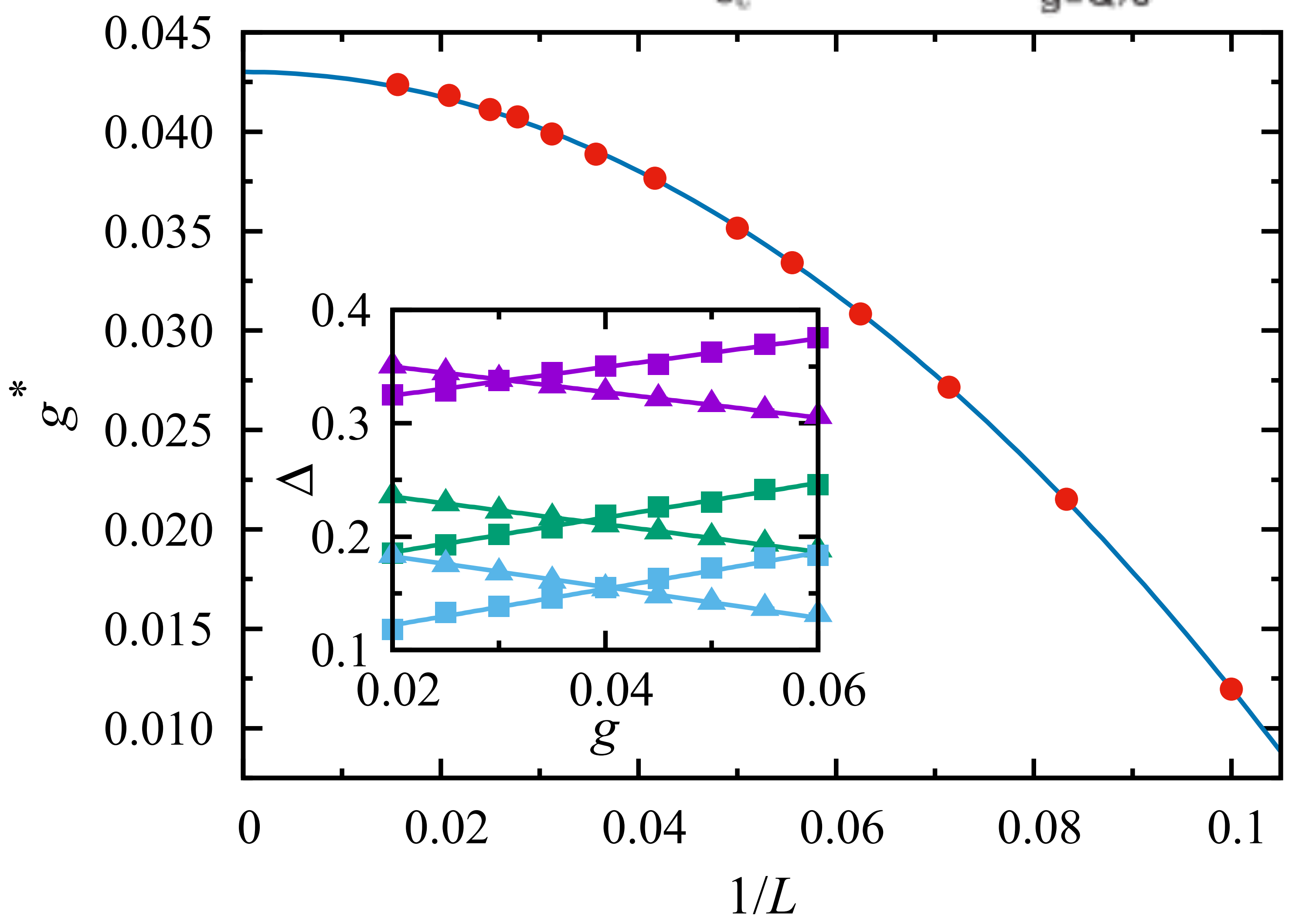
Field theory predicts a drift of $g_c(L) = g_c(\infty) + aL^{-2}$

Eggert PRB 54, R9612 (1996)



Critical level crossings in two dimension



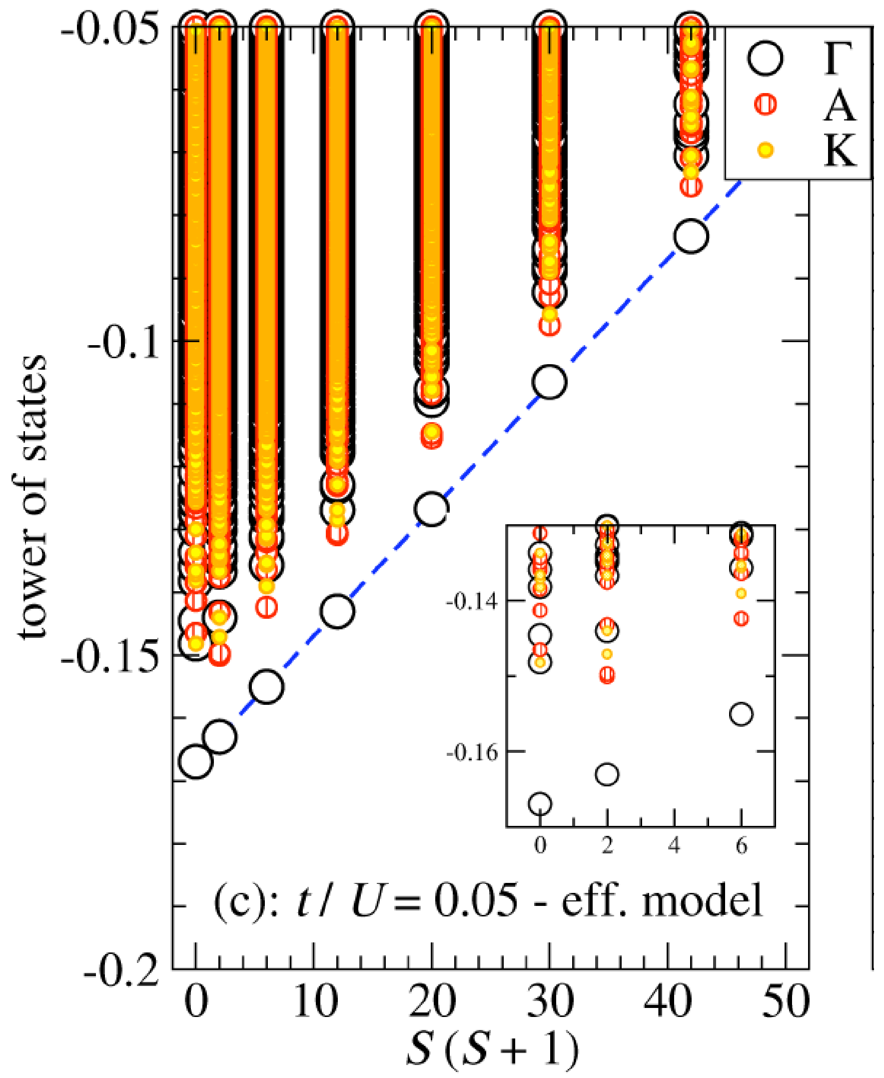


$$g_c(L) = g_c(\infty) + aL^{-2}$$

Suwa, Sen and Sandvik, PRB 94, 144416 (2010)

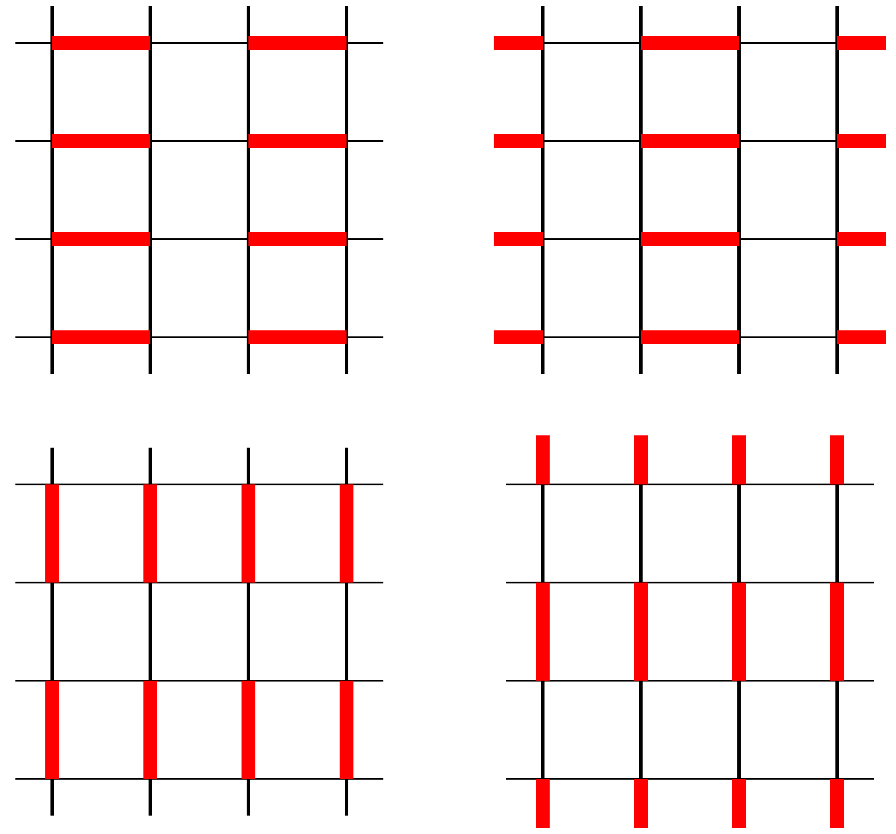
$$H = -J\sum_{\langle ij \rangle} P_{ij} - Q\sum_{\langle ijkl \rangle} P_{ij}P_{kl}$$

$$\Delta_S = J_{\text{eff}}\frac{S(S+1)}{N}$$

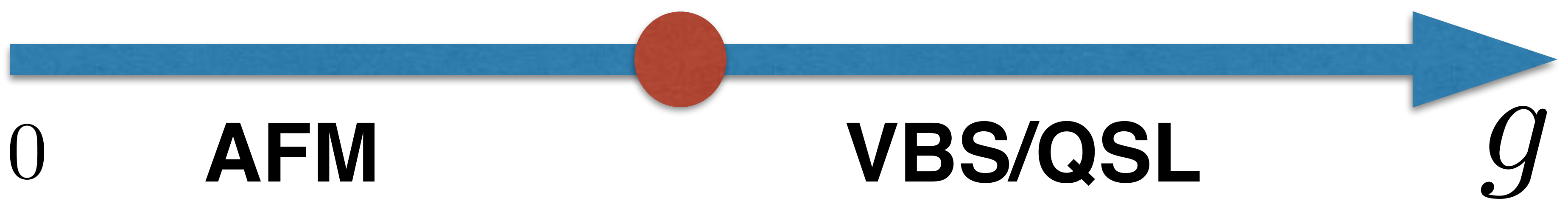


Anderson Tower excitation

$$\Delta_0 \approx e^{-\alpha L}$$



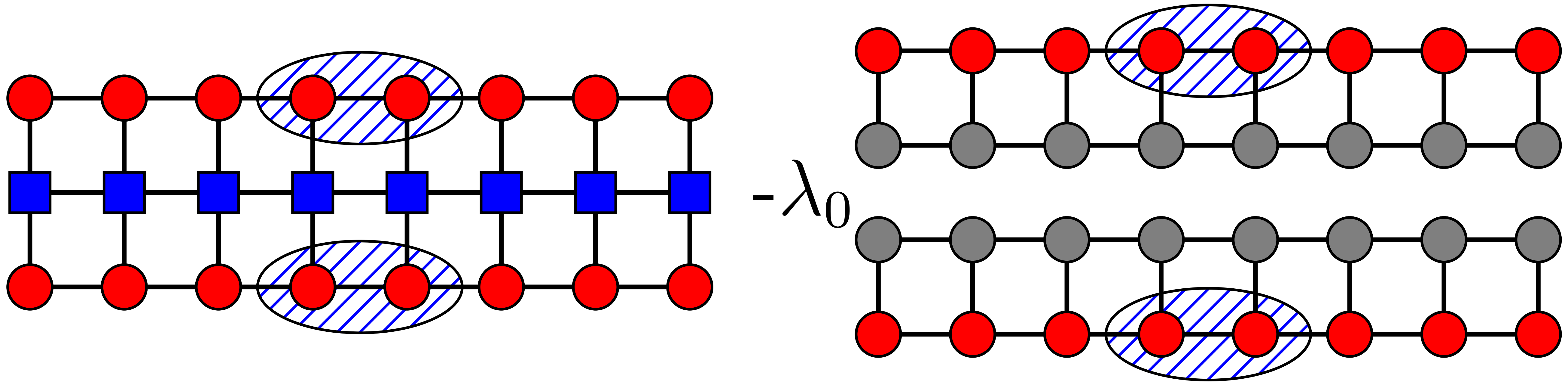
Ground state degeneracy



Targeting excited states with finite size DMRG

$$H_i = H - \sum_{j=1}^{i} e_j |\psi_j\rangle\langle\psi_j| \quad H|\psi_j\rangle = e_j|\psi_j\rangle$$

$$H_i = \sum_{j=1}^{N} e_j |\psi_j\rangle\langle\psi_j| - \sum_{j=1}^{i} e_j |\psi_j\rangle\langle\psi_j|$$



Wang and Sandvik, PRL 121, 107202 (2018)

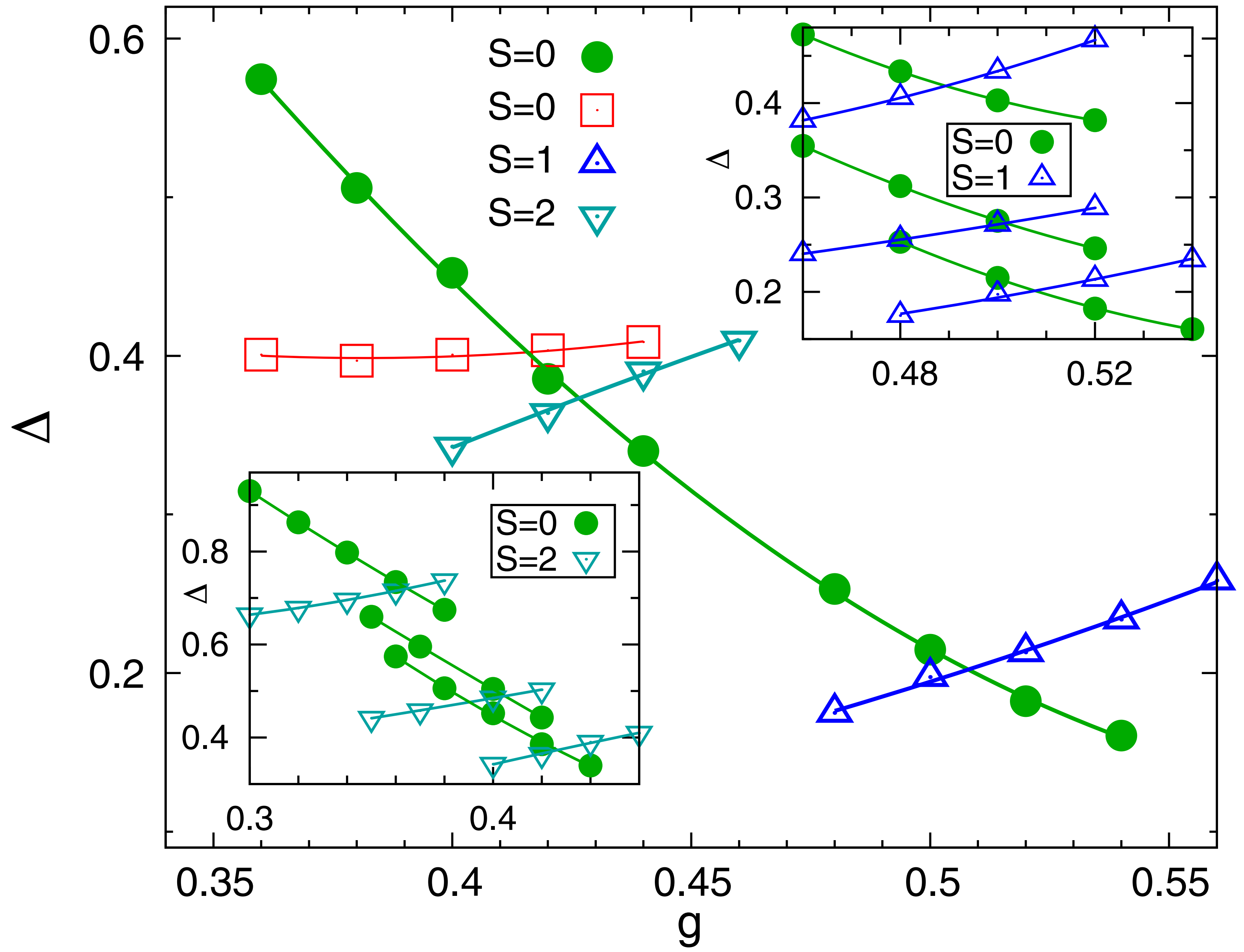
Schollwock, Ann. Phys. 326, 96 (2011)

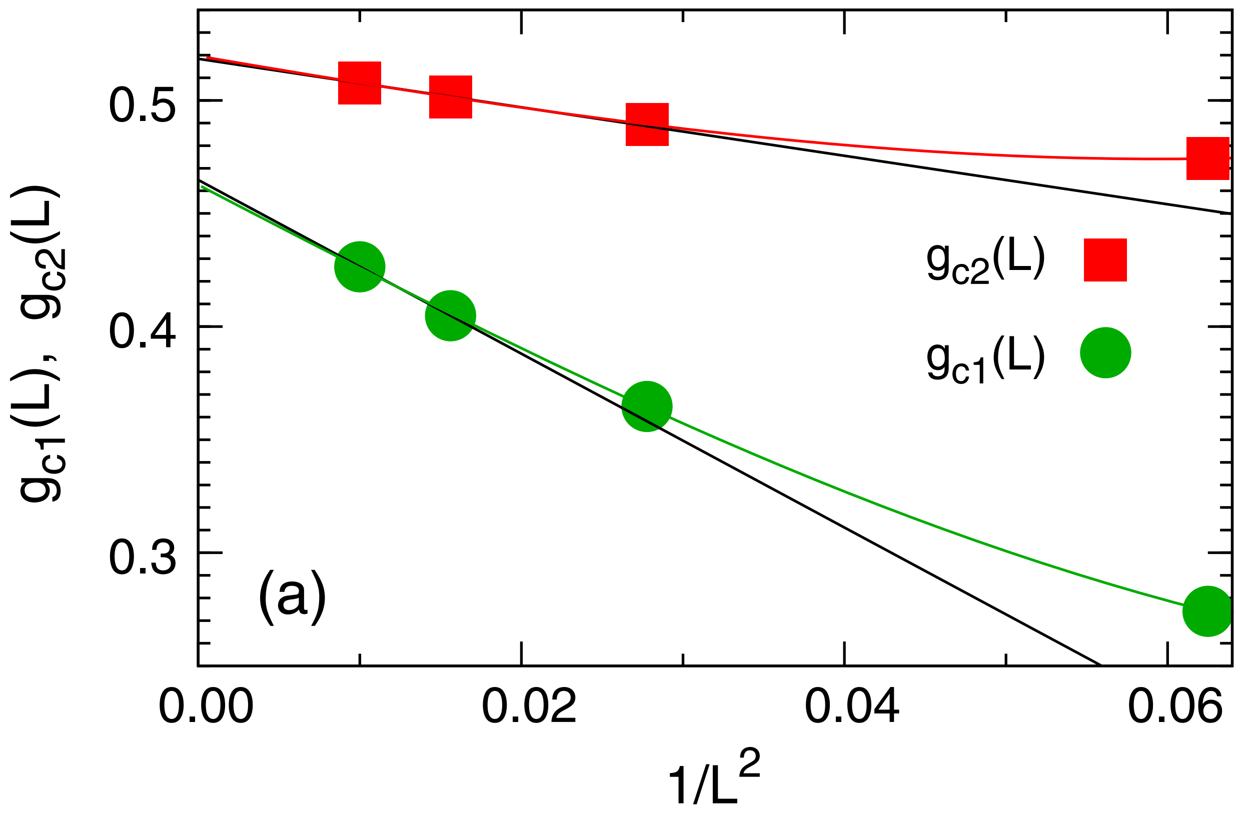
- DMRG/MPS is very successful in targeting the ground state
- Lowest excited state can be targeted “like a ground state” for the Hamiltonian H_i where the pre-calculated ground state space is projected out

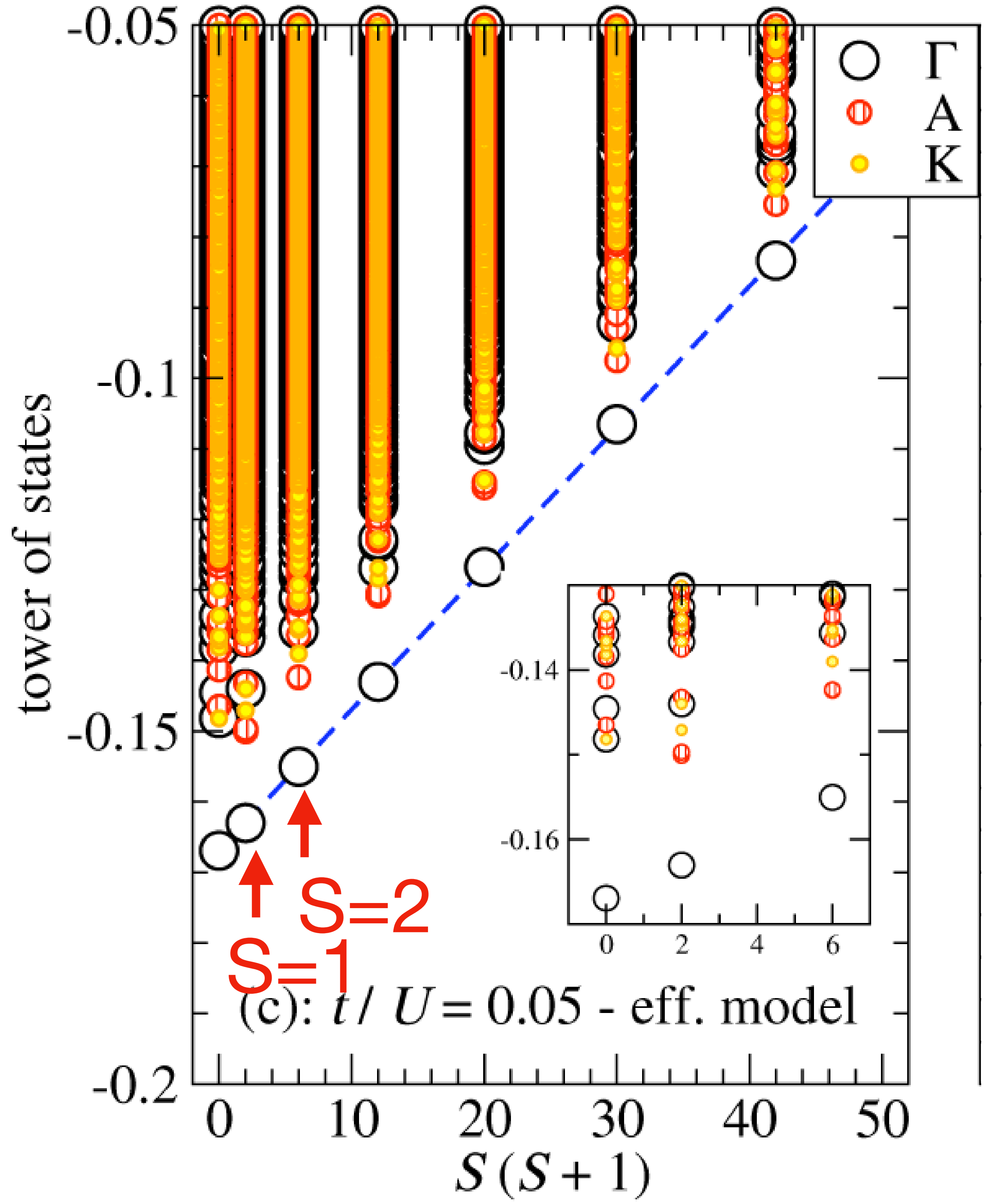
Critical level crossings in square lattice J1-J2 model

$$H_{J_1-J_2} = J_1 \sum_{\langle i,j \rangle} \mathbf{S}_i \mathbf{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \mathbf{S}_i \mathbf{S}_j$$

Anderson Tower States in AFM

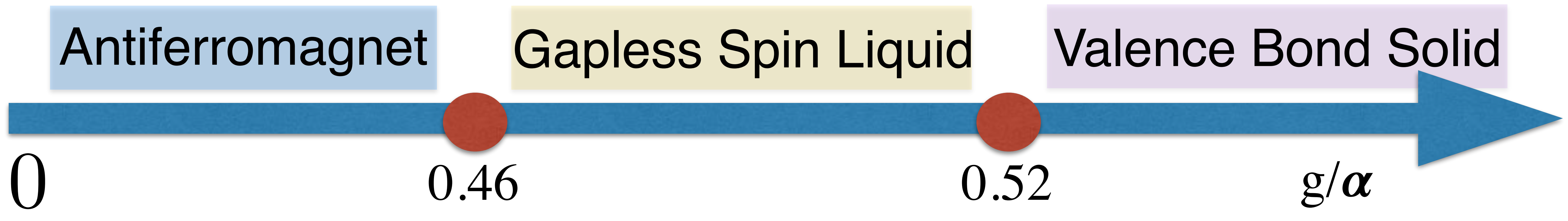




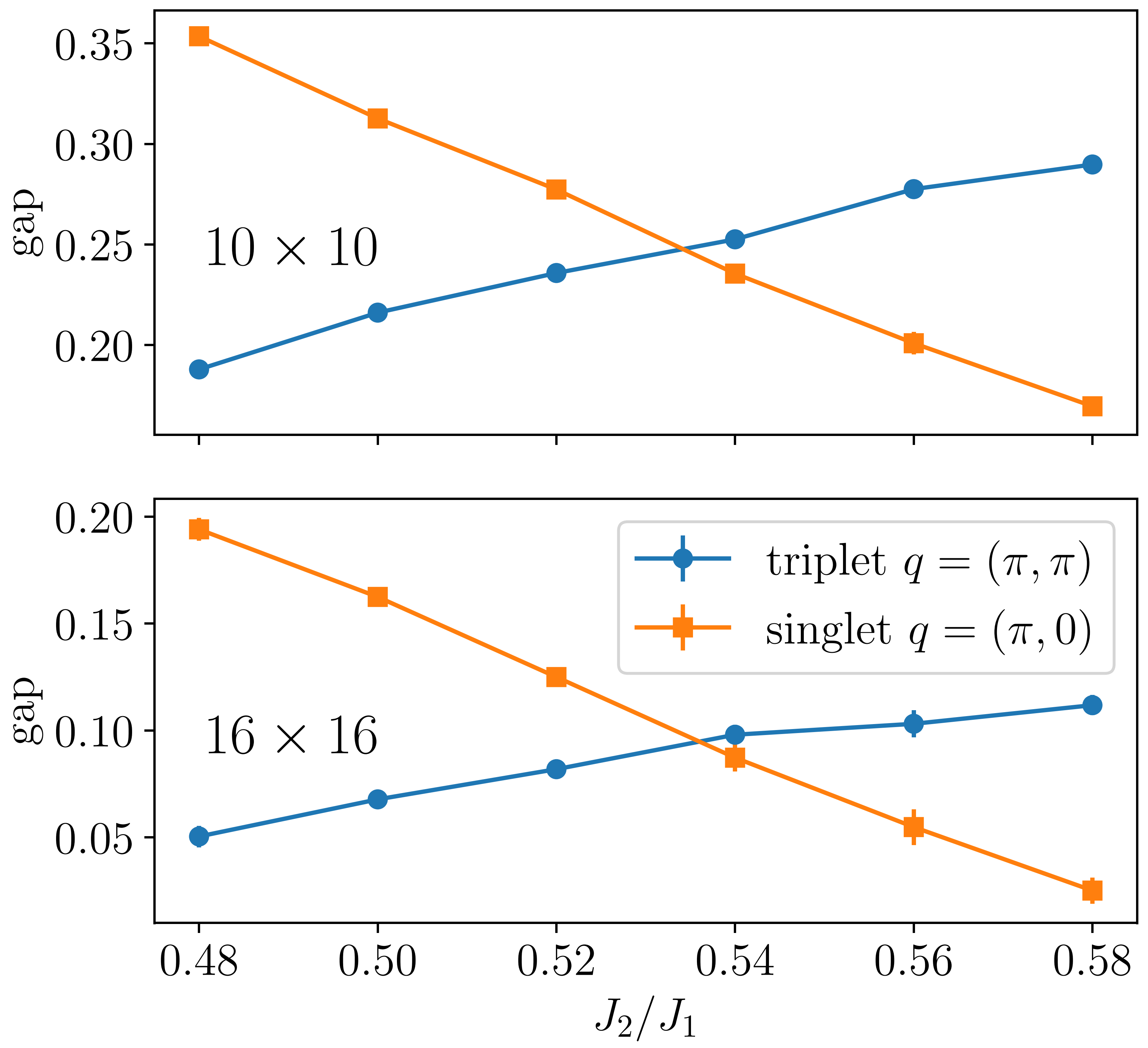


Wang and Sandvik, PRL 121, 107202 (2018)

Yang et al. New J. Phys. **14** 115027 (2012)

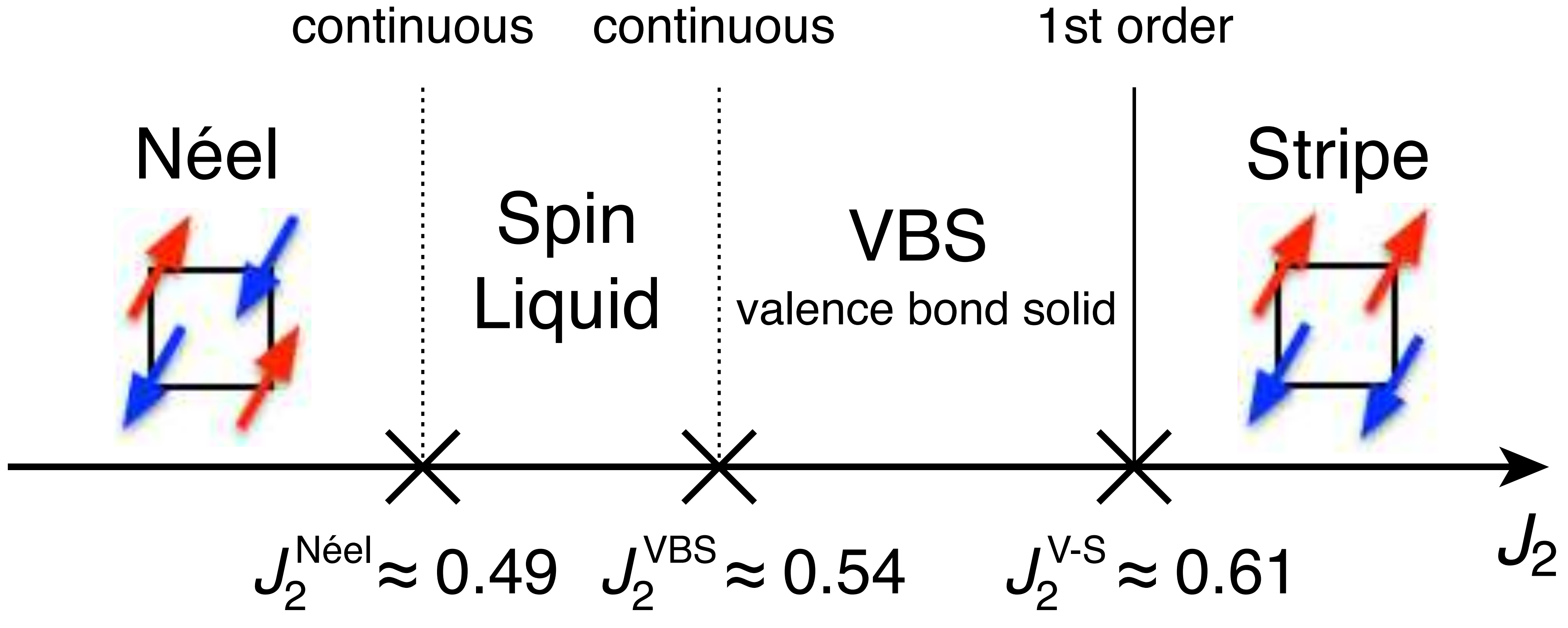


Critical level crossings in square lattice J1-J2 model



Square lattice J1-J2 model

$$H_{J_1-J_2} = J_1 \sum_{\langle i,j \rangle} \mathbf{S}_i \mathbf{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \mathbf{S}_i \mathbf{S}_j$$



Ferrari and Becca, PRB 102, 014417 (2020)

Poilblanc et al, SciPost Phys. 7, 041 (2019)

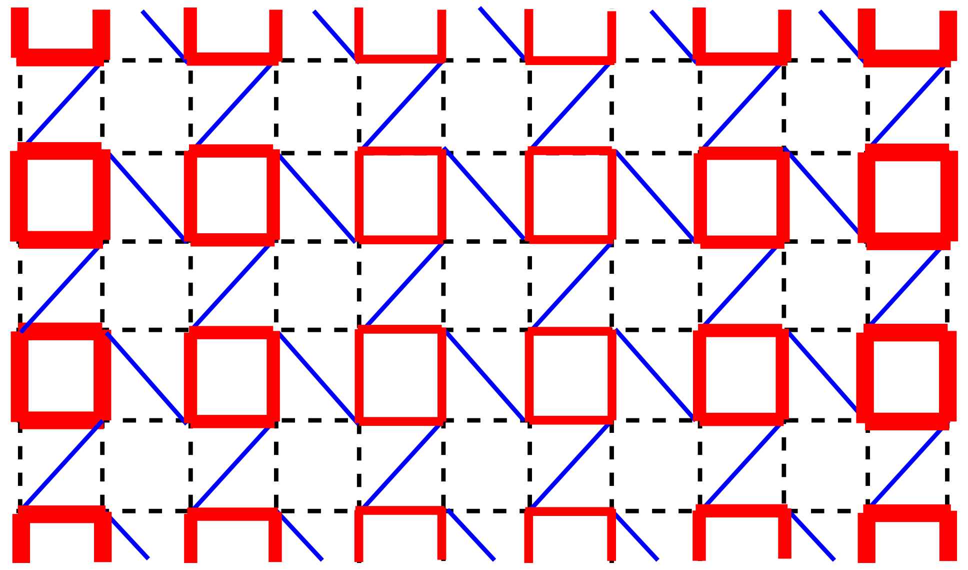
Nomura and Imada, arXiv:2005.14142

Liu et al, arXiv:2009.01821

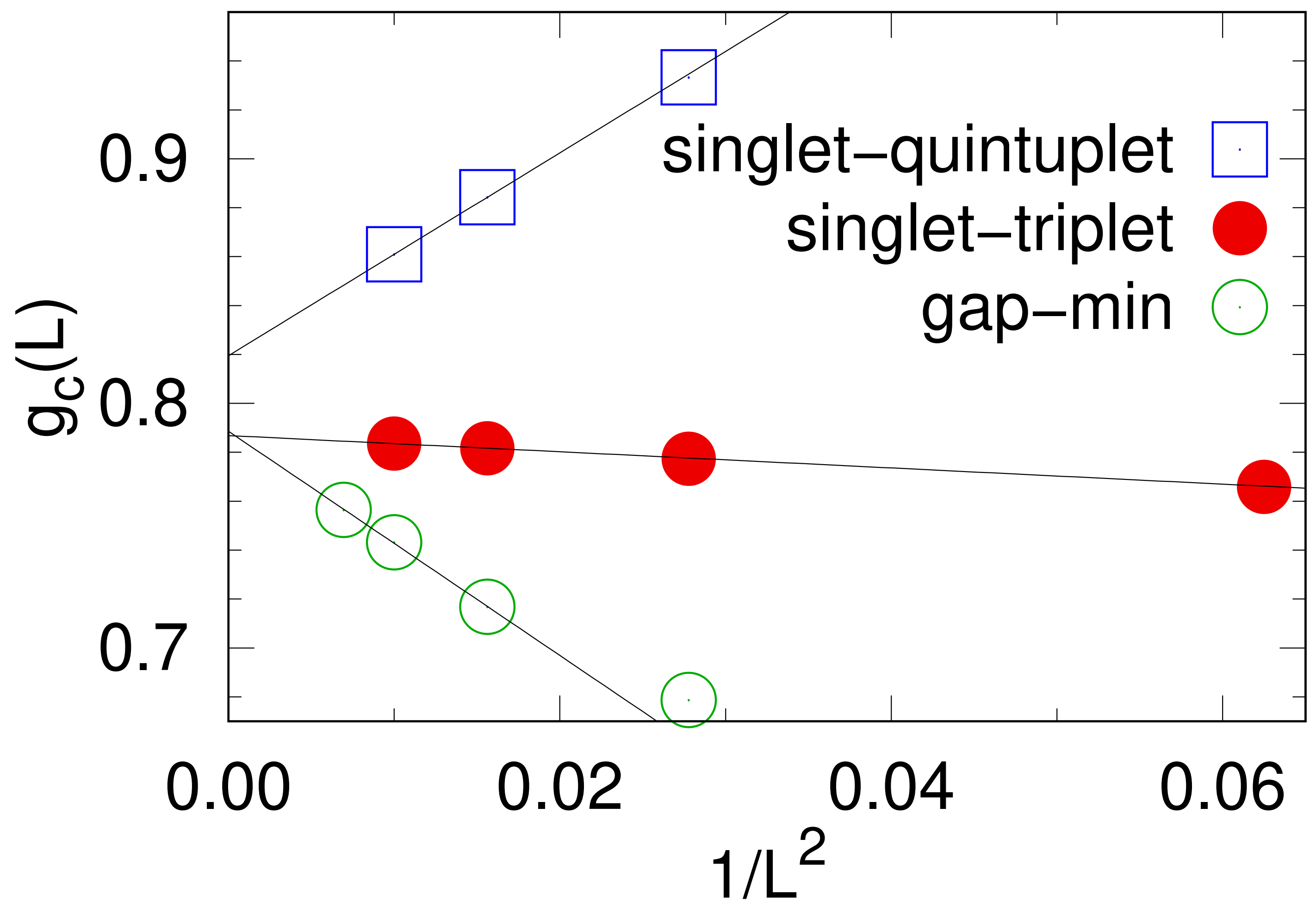
A consensus has been reached for a gapless SL phase in square lattice J1-J2 model

Shackleton et al (will appear tonight) proposed a field theory to look into this scenario

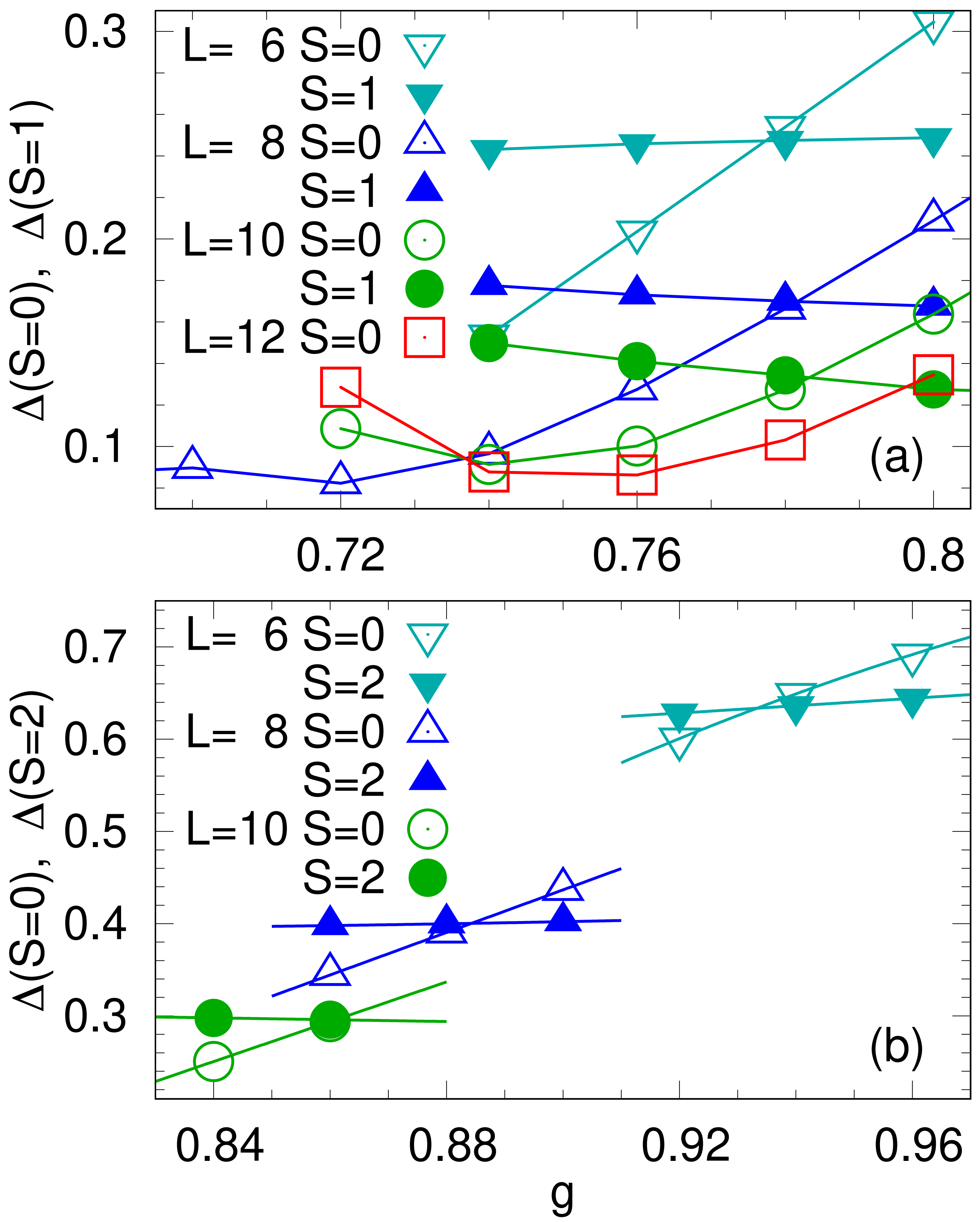
Critical level crossings in SS model



4n×2n cylinder



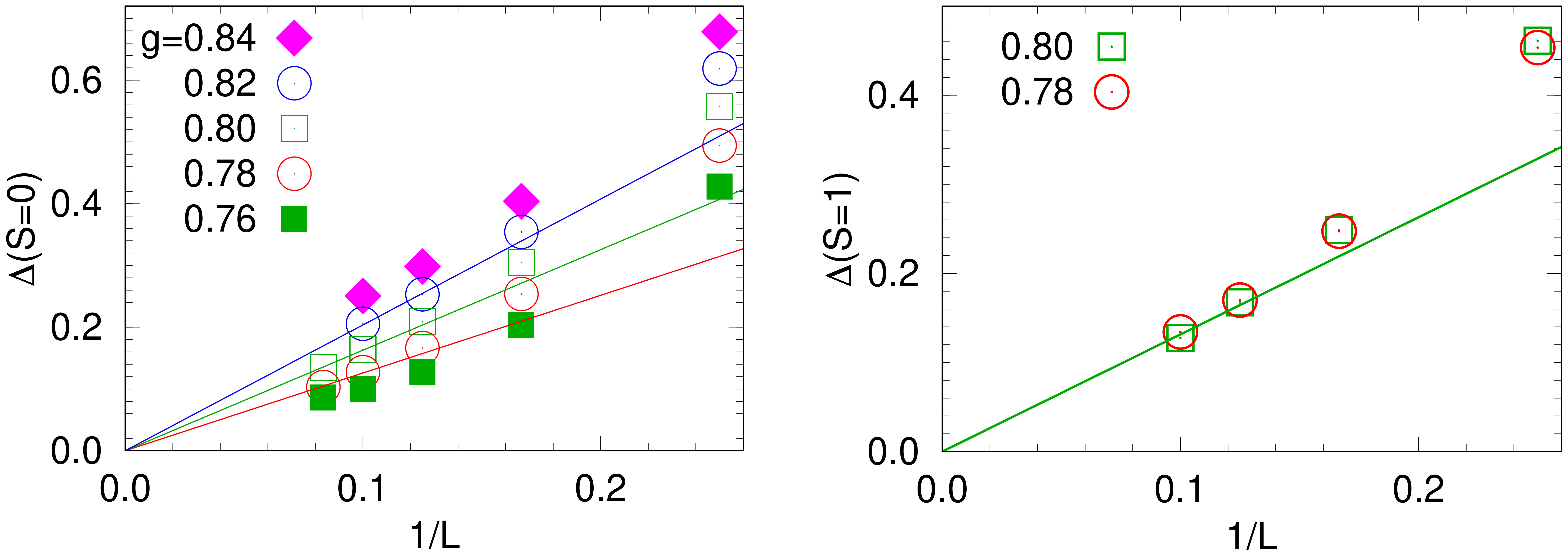
$$g_c(L) = g_c(\infty) + aL^{-2}$$



Yang, Sandvik and Wang, arXiv:2104.08887

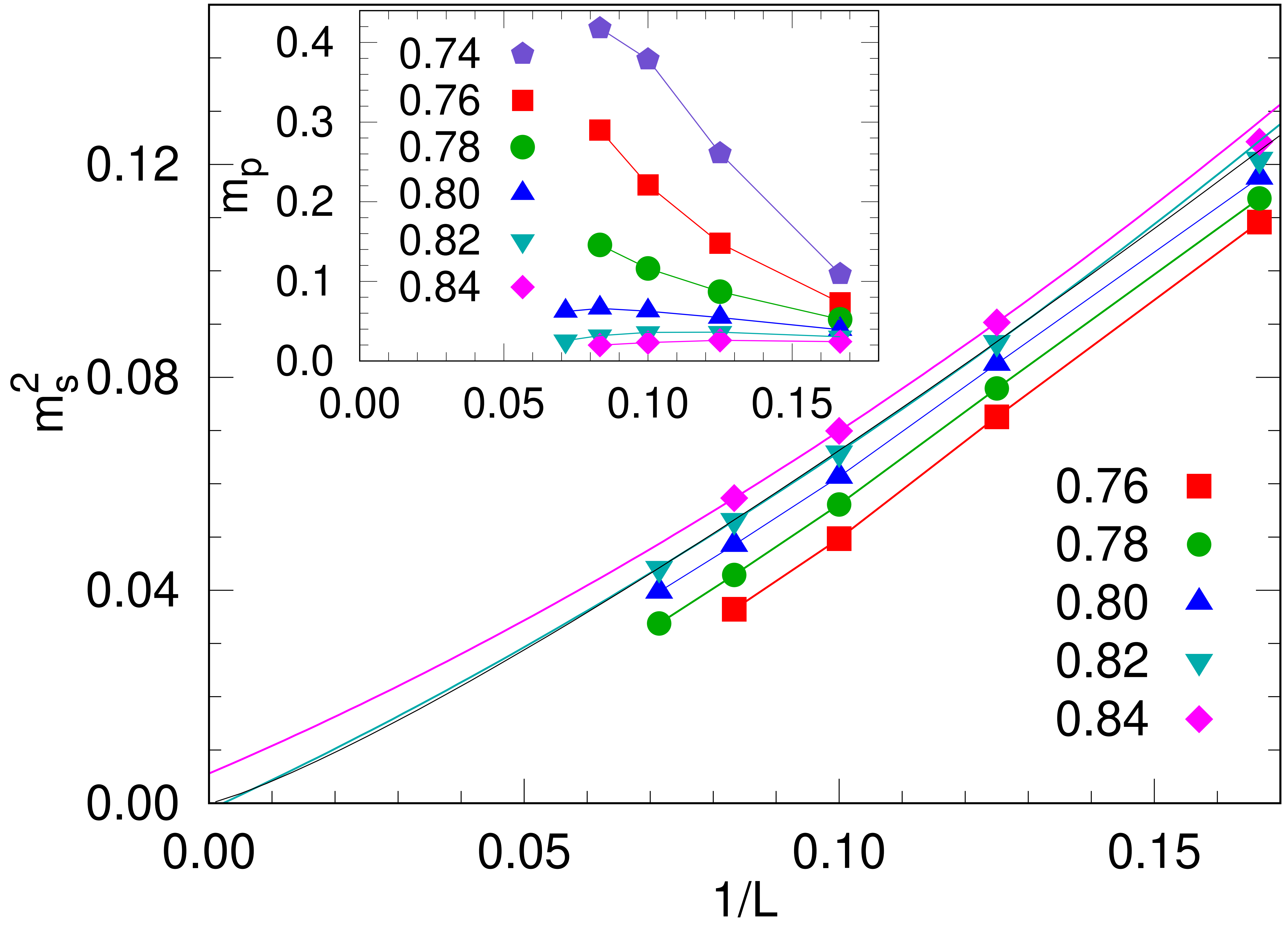
- Open boundary serves as pinning field and breaks Z2 symmetry of the Hamiltonian, the ground state is unique in PS phase
- In addition to singlet-triplet and singlet-quintuplet level crossings, there is singlet gap minimum, signaling vanishing domain wall energy (the end of PS phase)
- The drifting of critical value follows perfectly a $1/L^2$ correction. The spin liquid region, although small, is unlikely to shrink to zero

Singlet and triplet gaps close to the SL phase



- Singlet gap and triplet gap within the putative SL phase are consistent with 1/L scaling, system sizes are too small to check possibility of a slightly higher z

Order parameters in SS model

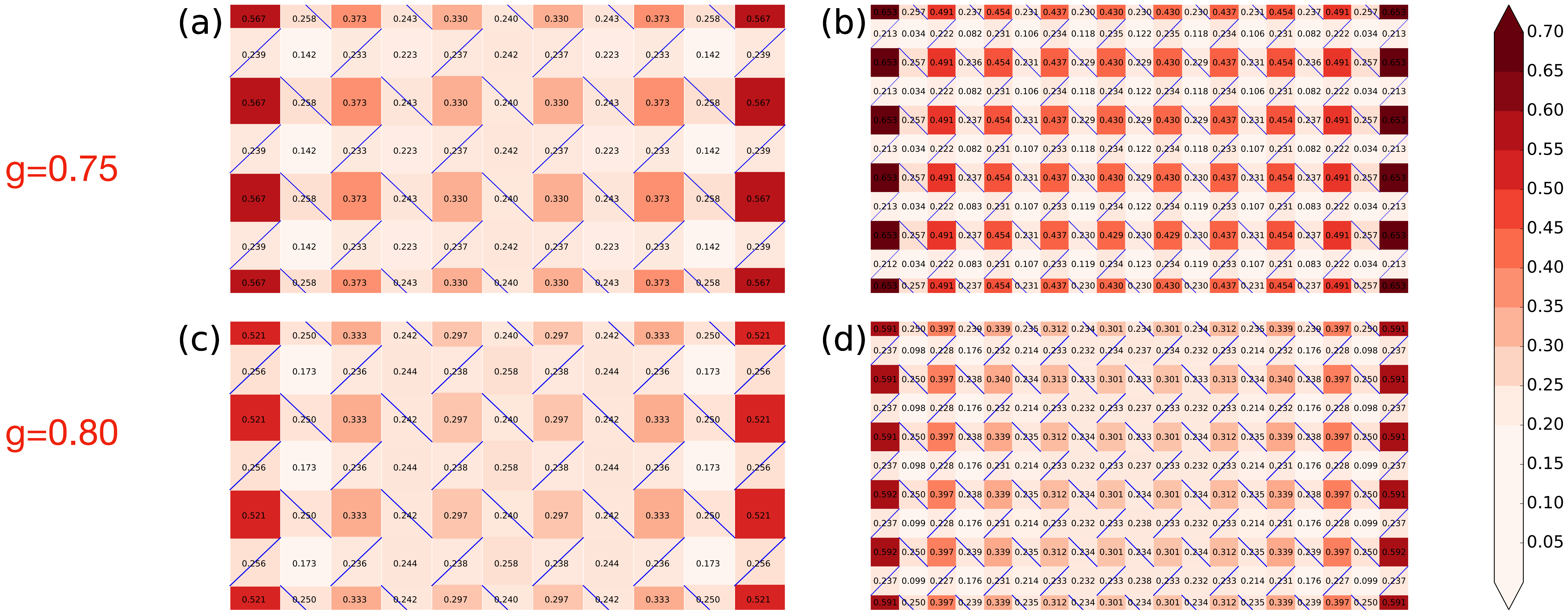


$$m_s^2 = L^{-4} \sum_{ij} \phi_{ij} \langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle \quad \phi_{ij} = \pm 1$$

$$m_p = \langle \mathbf{Q}_{\mathbf{R}} - \mathbf{Q}_{\mathbf{R}'} \rangle \quad \mathbf{Q}_{\mathbf{r}} \equiv \frac{1}{2}(\mathbf{P}_{\mathbf{r}} + \mathbf{P}_{\mathbf{r}}^{-1})$$

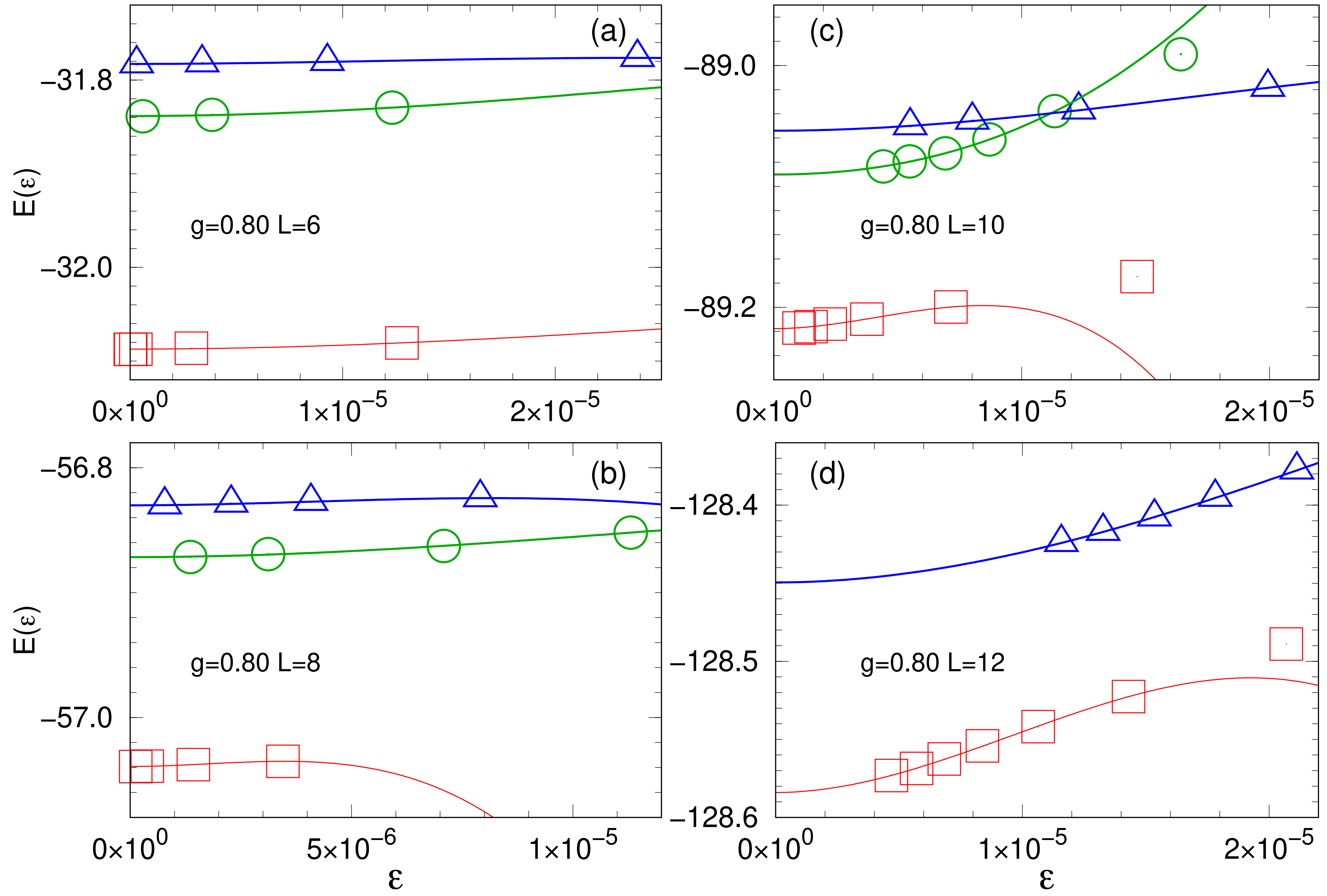
- A second order polynomial fit to m_s^2 at g=0.82 gives a slightly negative value, means no AFM order at g=0.82, a power law fit $m_s^2 \propto 1/L^{1+\eta}$ produce η=0.2
- Within PS phase g=0.74 to 0.78, m_p increase quickly with L, reflecting large fluctuations and no stable order in small systems
- Outside PS phase at g=0.82, 0.84, m_p increase first then decrease with L, reflecting a disordered bulk
- At g=0.80, m_p starts to turn downwards at L=14, indicating vanishing plaquette order, thus within the SL

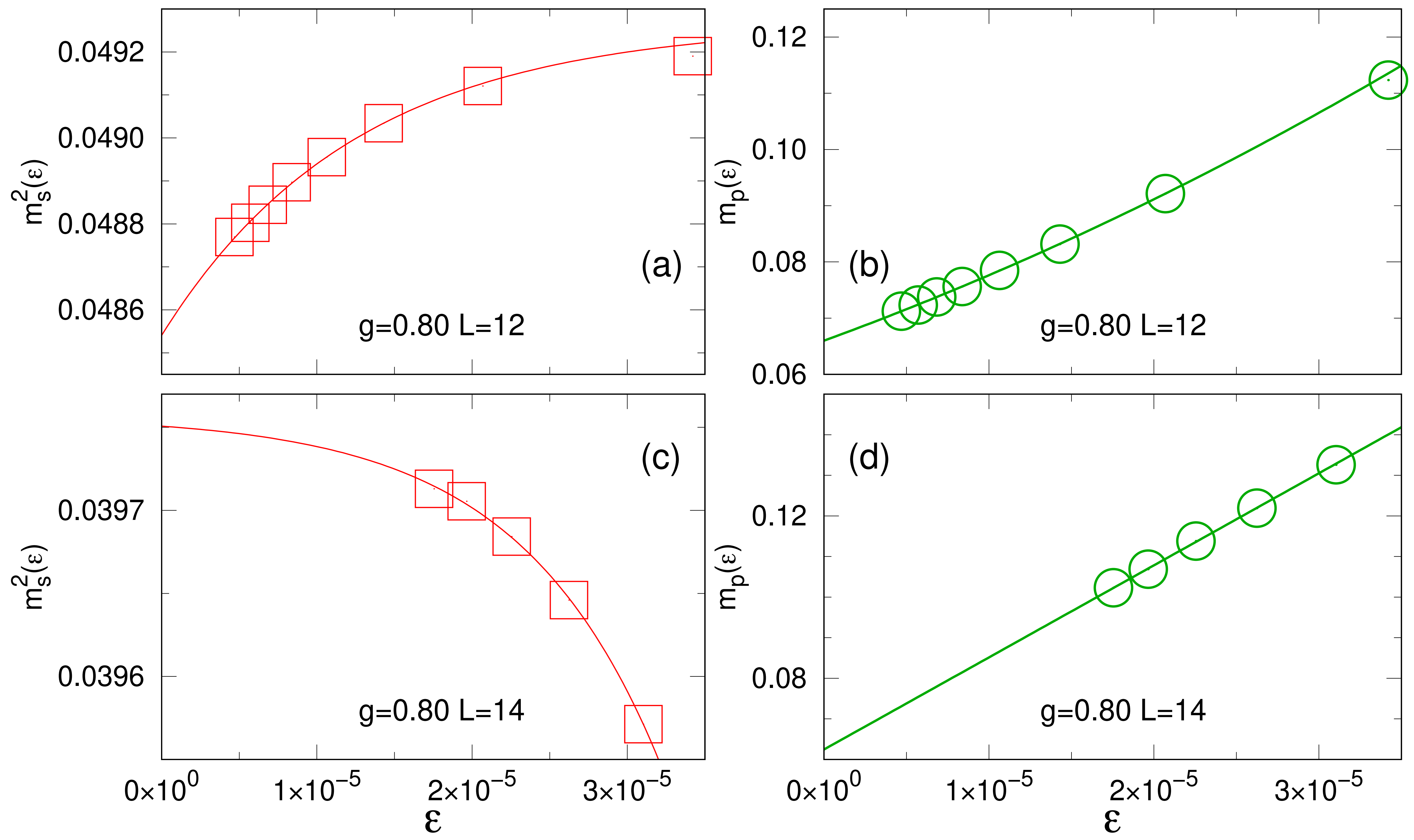
Plaquette order landscape in SS model



- Plaquette landscape at g=0.75 and g=0.80 for size L=6 and 10, where enhanced boundary plaquette order with increasing L is clearly visible

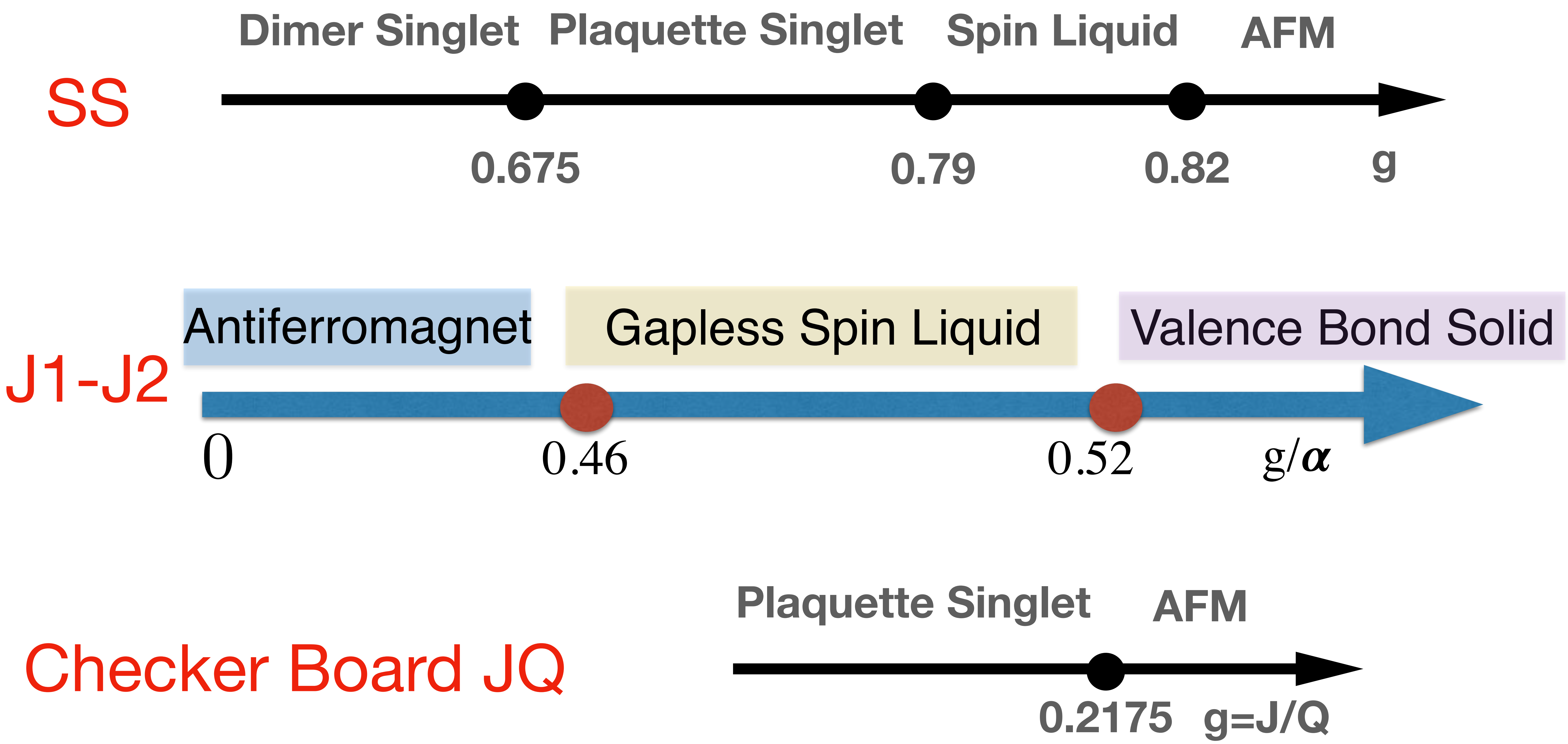
Convergence of eigen energies and order parameters



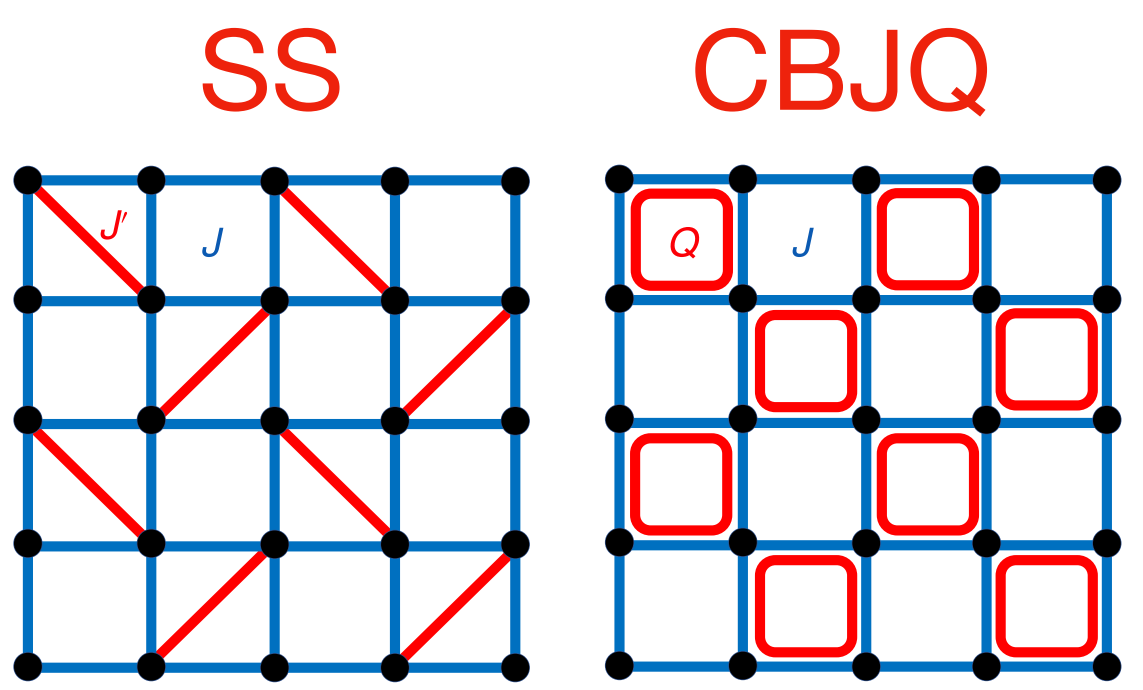


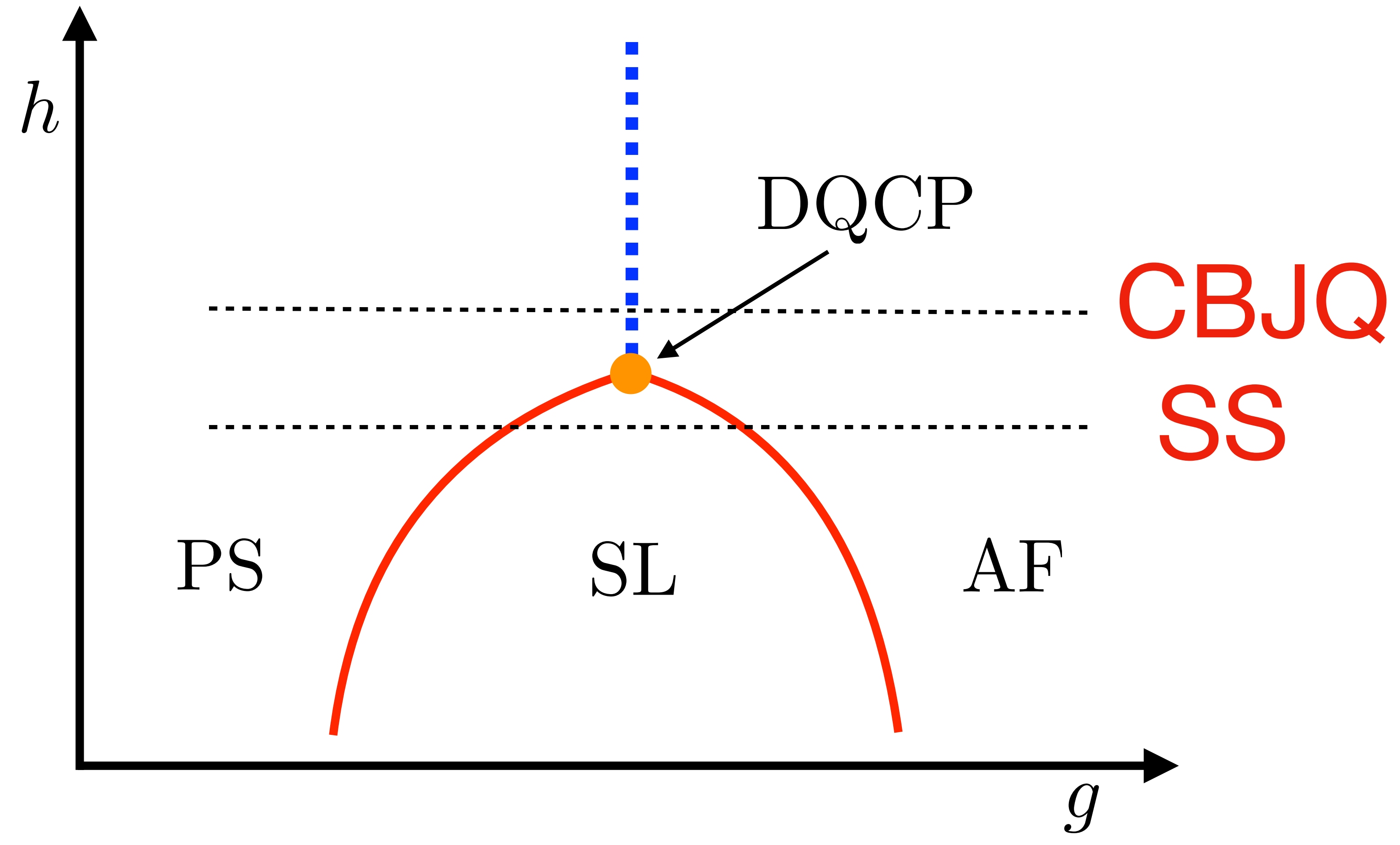
- For energies third order polynomial fits without linear term are employed, to ensure monatomic behavior. For order parameters, fits are polynomial without constraint

Discussions and conclusions



Zhao, Weinberg and Sandvik, NP 15, 678 (2019)

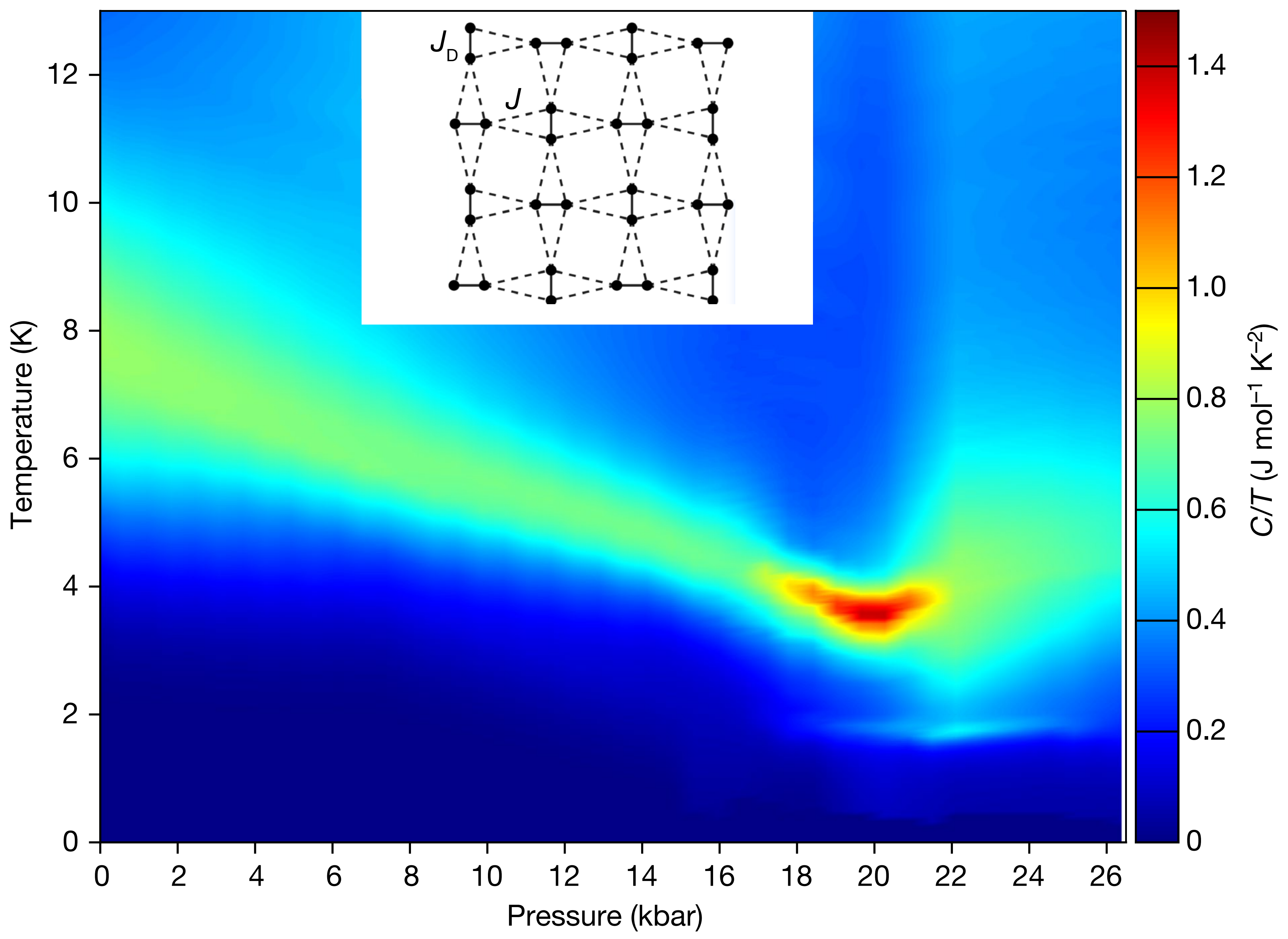




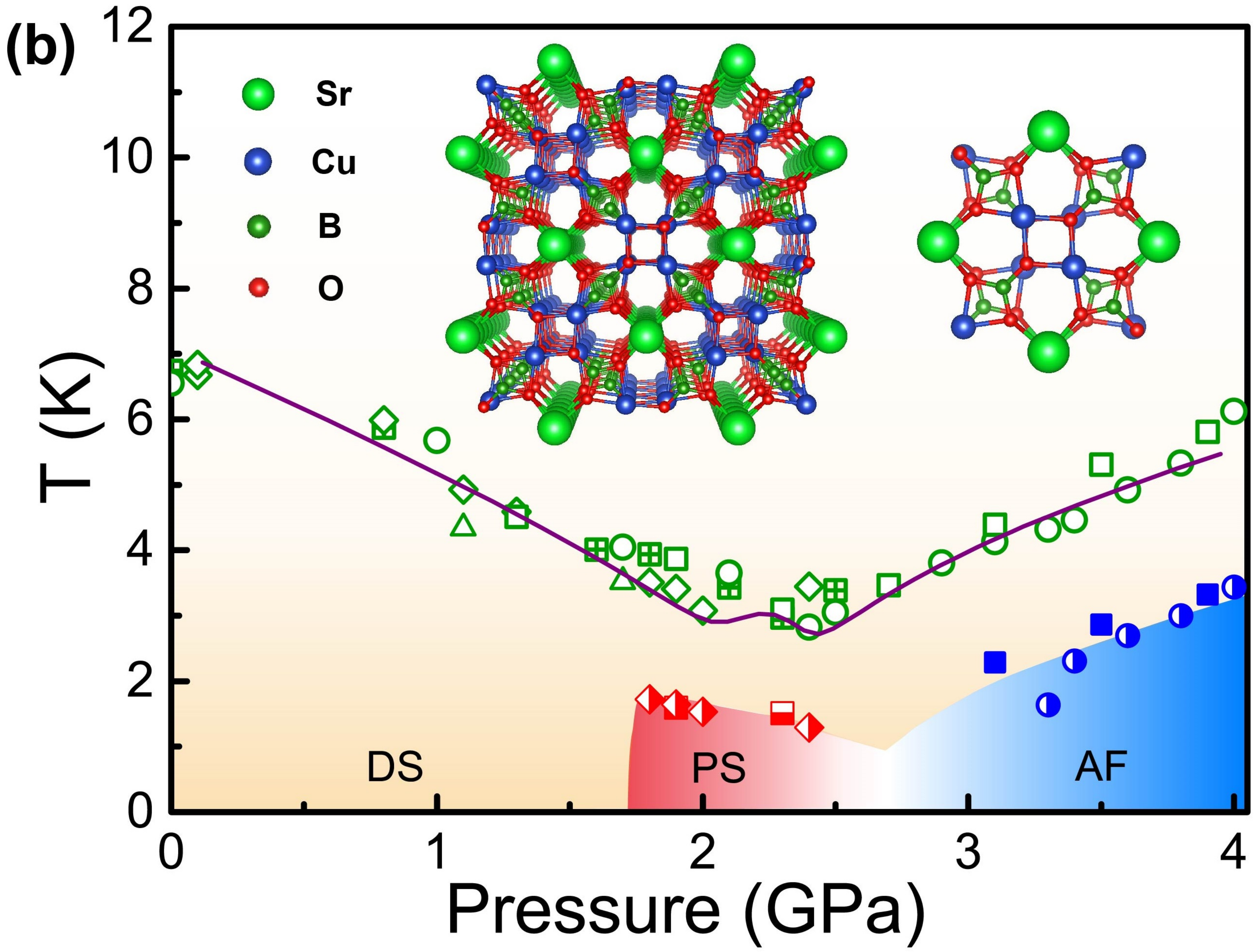
- The two models (SS and J1-J2) have different symmetries (translation), both SL phases have gapless singlet and triplet excitations, and continuous transitions to neighboring AFM and VBS phases

- Turn on another relevant field operator of the same symmetry can tune the transition in the two-parameter plane and across the Deconfined Quantum Critical Point

Specific heat measurement of SCBO



Jimenez et al, Nature 592, 370 (2021)



Guo et al, PRL 124, 206602 (2020)

- In low-T specific heat measurement, there is absence of finite temperature Ising type phase transition at pressures 2.6 to 3 Gpa, which is right between PS and AFM phases, supporting the scenario of a SL