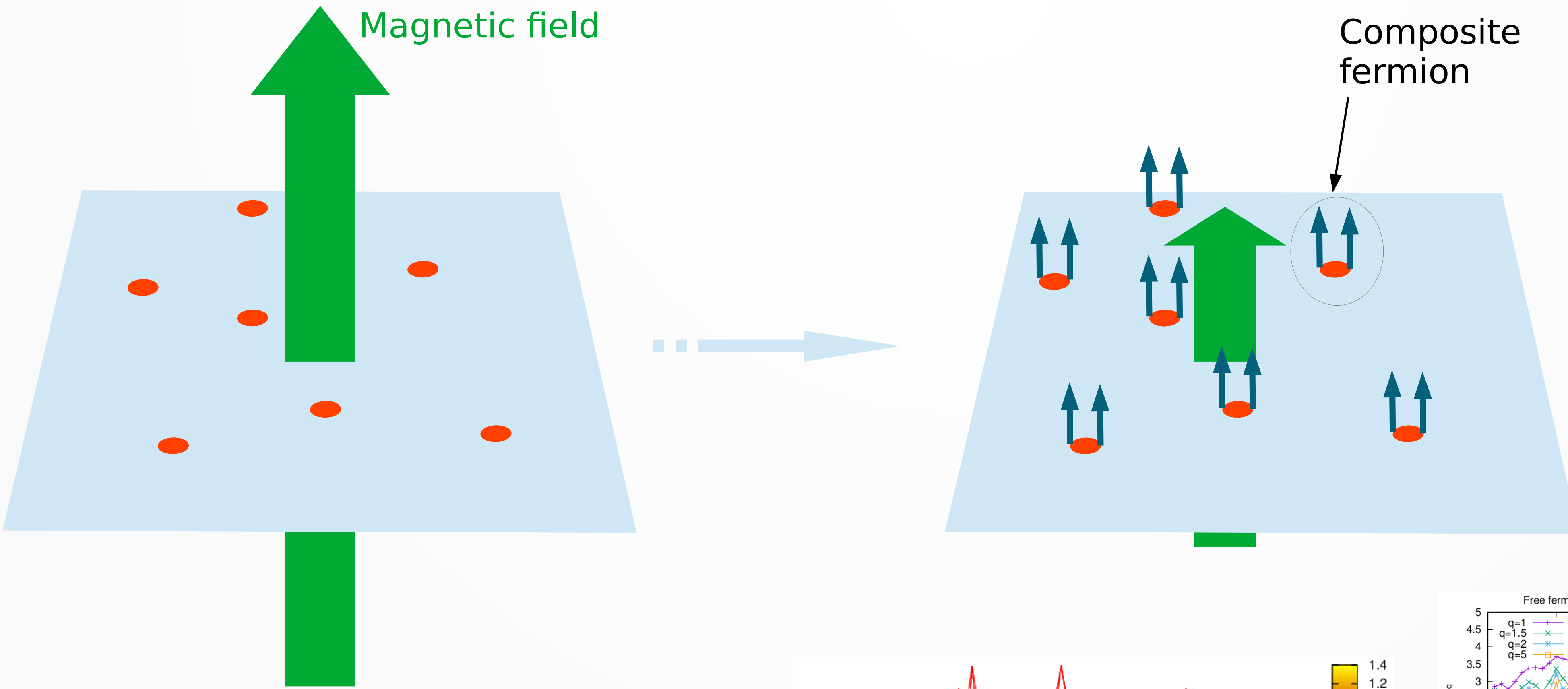
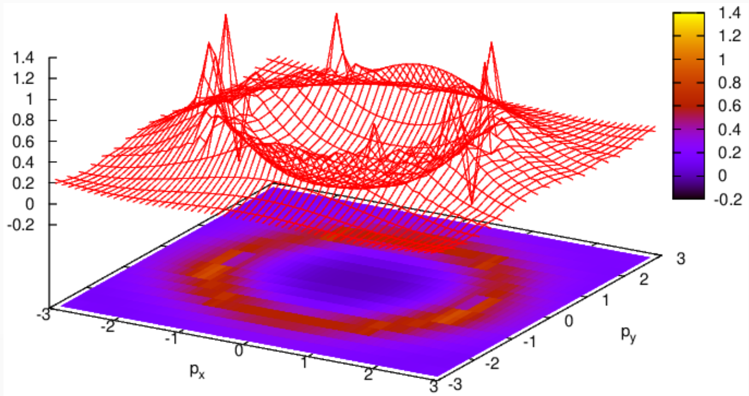
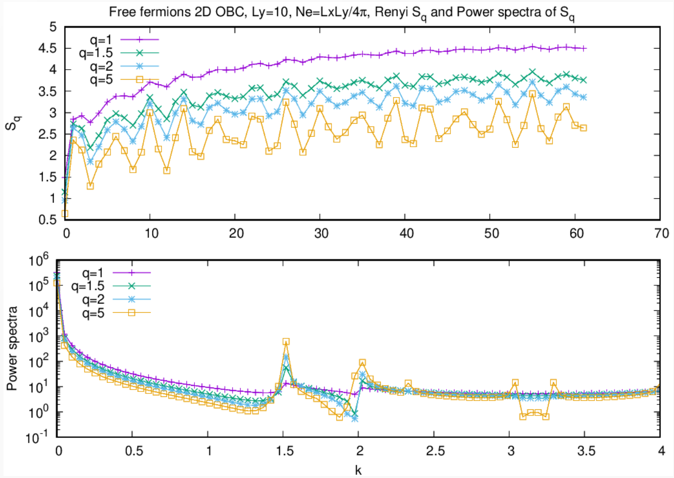
Half-filled quantum Hall problem on the cylinder, DMRG and composite fermions







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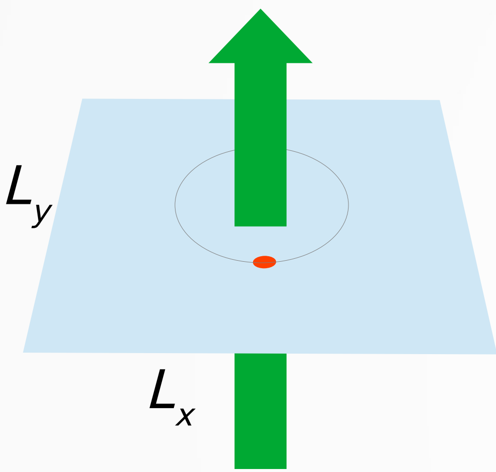
CEA-Grenoble, France.

Outline

1. Fractional quantum Hall effect
2. Effective 1D Hamiltonian for the LLL
3. Digression: DMRG0
4. Entanglement for free fermions
5. Results and conclusions

1. Fractional quantum Hall effect

Semi-classical



$$evB = mv^2/r$$
$$eBr^2 = mvr \sim \hbar$$
$$r^2 \sim \hbar/eB = l_B^2$$

Quantum mechanics[3]

$$H_e = \sum_j \frac{[\mathbf{p}_j + \frac{e}{c}\mathbf{A}(\mathbf{r}_j)]^2}{2m} + \sum_{i<j} V(\mathbf{r}_i - \mathbf{r}_j)$$

$$\mathbf{B} = B\hat{\mathbf{z}} = \nabla \times \mathbf{A}$$
$$(A_x, A_y) = (0, Bx)$$

Landau levels: $E_n = \left(n + \frac{1}{2}\right) eB/m, \quad n = 0, 1, 2...$

Macroscopic degeneracy: $N = L_x L_y / 2\pi l_B^2$ → Strongly interacting

Filling factor: $\nu = N_e/N$

LLL: $\varphi_j(\mathbf{r}) = \frac{e^{i\frac{2\pi}{L_y}jy}}{\sqrt{L_y}} \frac{e^{-\frac{(x-X_j)^2}{2\ell_B^2}}}{\pi^{1/4}\ell_B^{1/2}}$ $\qquad X_j \equiv \ell_B^2(2\pi/L_y)j$

At half-filling

HLR[1]

- electron+flux attachment
- Success:
  - Jain sequences: FQH (e) ↔ IQH (CF)
  - gapless Fermi surface of CFs at $\nu=1/2$
- N(CF)=Ne

Son[2]

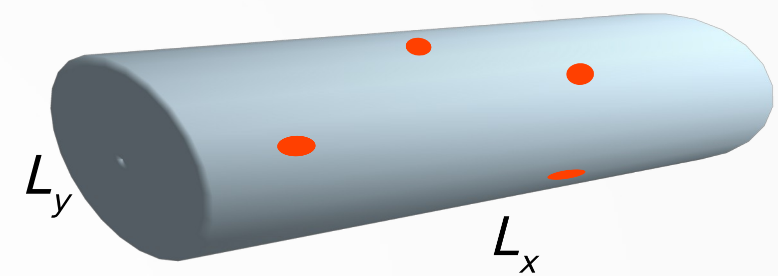
- Explicit PH symmetry
- Dirac composite fermion
- N(CF)=N/2

Solve using DMRG[3]
and measure!

[1] Halperin, B. I., Lee, P. A., & Read, N. (1993). Theory of the half-filled Landau level. Physical Review B, 47(12), 7312.
[2] Son, D. T. (2015). Is the composite fermion a dirac particle?. Physical Review X, 5(3), 031027.
[3] Geraedts, S. D., Zaletel, M. P., Mong, R. S., Metlitski, M. A., Vishwanath, A., & Motrunich, O. I. (2016). “The half-filled Landau level: The case for Dirac composite fermions”. Science, 352(6282), 197-201.

FQHE: effective 1D Hamiltonian

$$H_e = \sum_j \frac{[\mathbf{p}_j + \frac{e}{c}\mathbf{A}(\mathbf{r}_j)]^2}{2m} + \sum_{i<j} V(\mathbf{r}_i - \mathbf{r}_j)$$



Projecting to the LLL:

$$\varphi_j(\mathbf{r}) = \frac{e^{i\frac{2\pi}{L_y}jy}}{\sqrt{L_y}} \frac{e^{-\frac{(x-X_j)^2}{2\ell_B^2}}}{\pi^{1/4}\ell_B^{1/2}}$$

$$X_j \equiv \ell_B^2(2\pi/L_y)j$$

$$H_{\text{el.int.}} = \sum_j \sum_{n\geq 0, m>0} \left[W_{mn} c_j^\dagger c_{j+n} c_{j+m+n} c_{j+m+2n}^\dagger + \text{H.c.}\right]$$

$$N = L_x L_y / 2\pi l_B^2,\ N_e = N/2$$

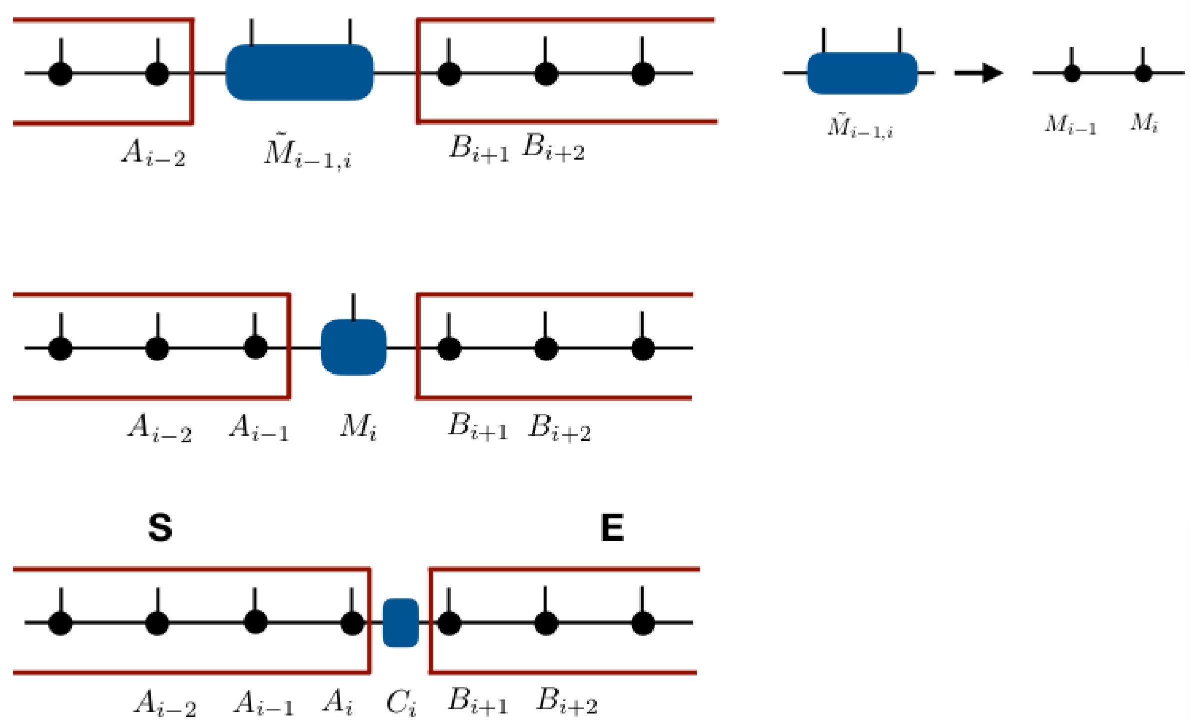
N sites, N_e electrons, only “local” two-particle interaction

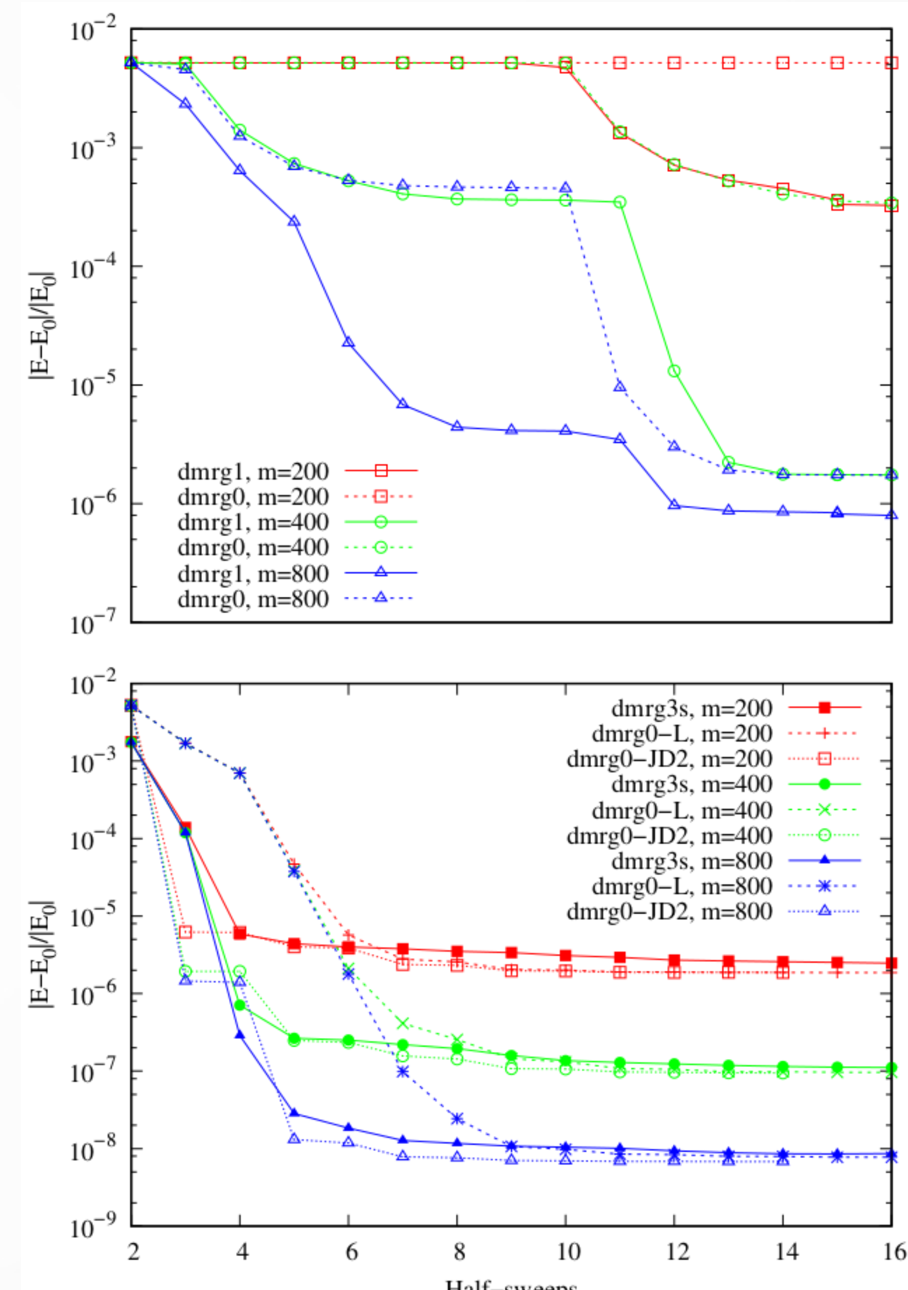
3. Digression: DMRG0

DMRG2

DMRG1

DMRG0



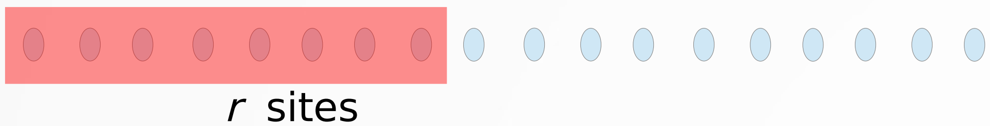


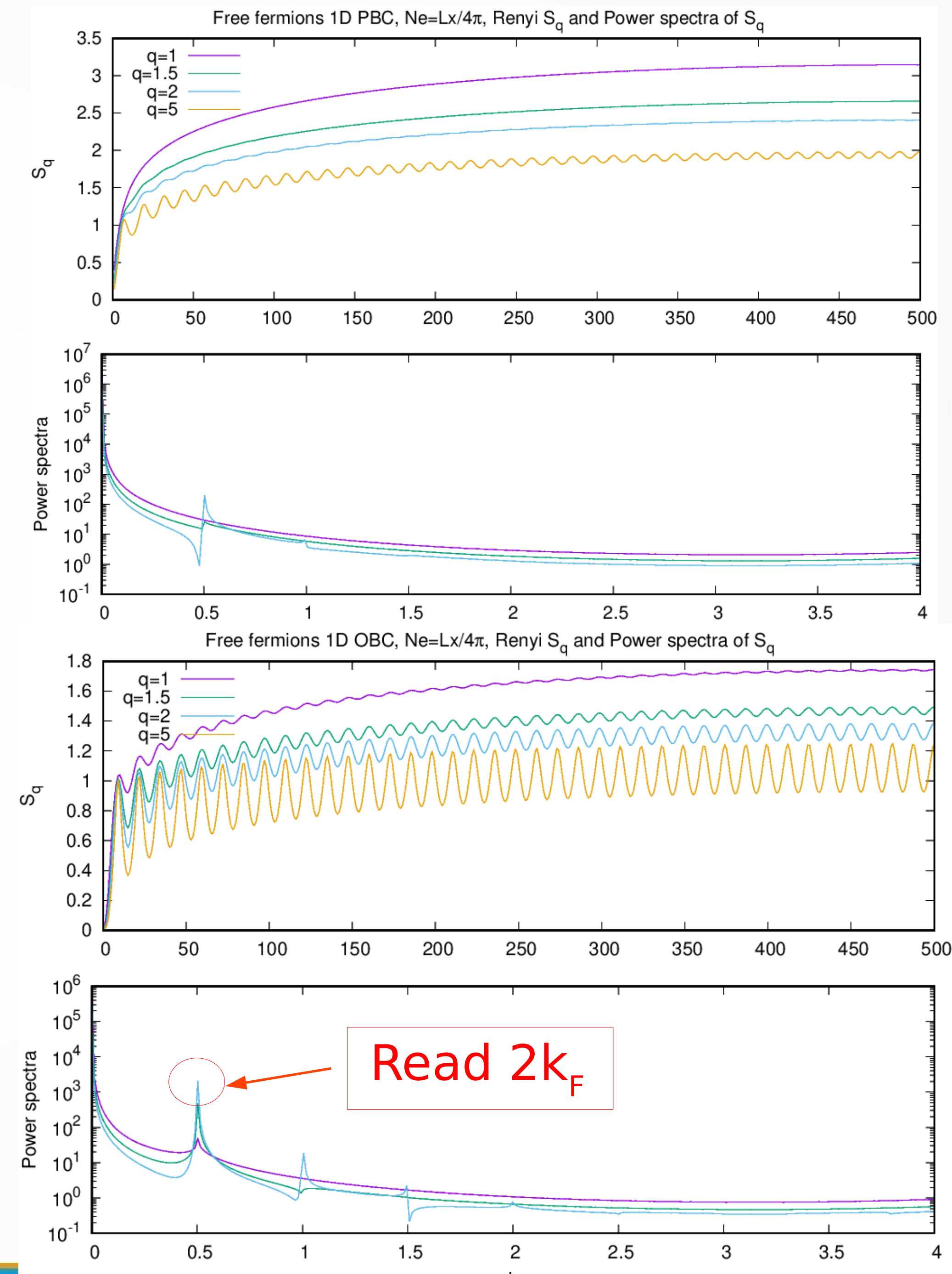
Optimal low-rank correction $\alpha\psi + \beta\tilde{\psi}$

$$|\tilde{\psi}\rangle \propto -\left[P(\hat{H}-\lambda)P\right]^{-1} P\hat{H}|\psi\rangle$$

[4] YNF & G. Torroba (2020). Zero-site density matrix renormalization group and the optimal low-rank correction. Physical Review B, 101(8), 085135.

4. Entanglement for free fermions in 1D





Rényi entropy:

$$S_n(V) = \frac{1}{1-n} \log(\mathrm{Tr}(\rho_V^n))$$

Friedel oscillations!

$$S_n(r) = \frac{n+1}{6n} \log\left(\frac{r}{\epsilon}\right) + A\, f_n \frac{\cos(2k_F r)}{(2k_F r)^{\frac{2}{n}}} + \dots$$

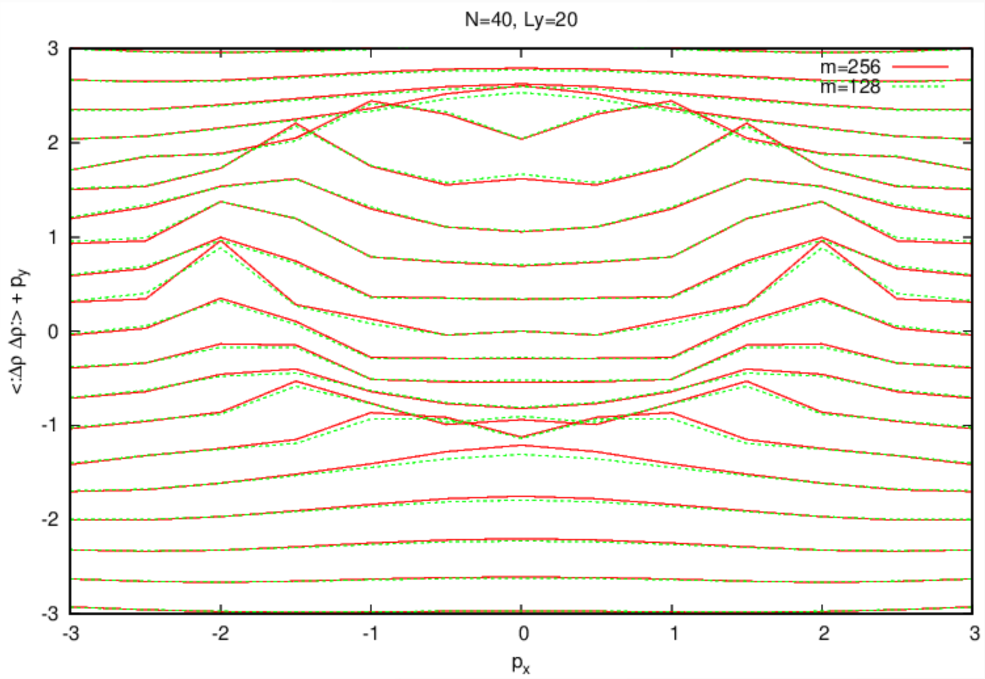
$$f_n = \frac{2}{1-n} \left(\frac{\Gamma((1+n^{-1})/2)}{\Gamma((1-n^{-1})/2)}\right)^2 .$$

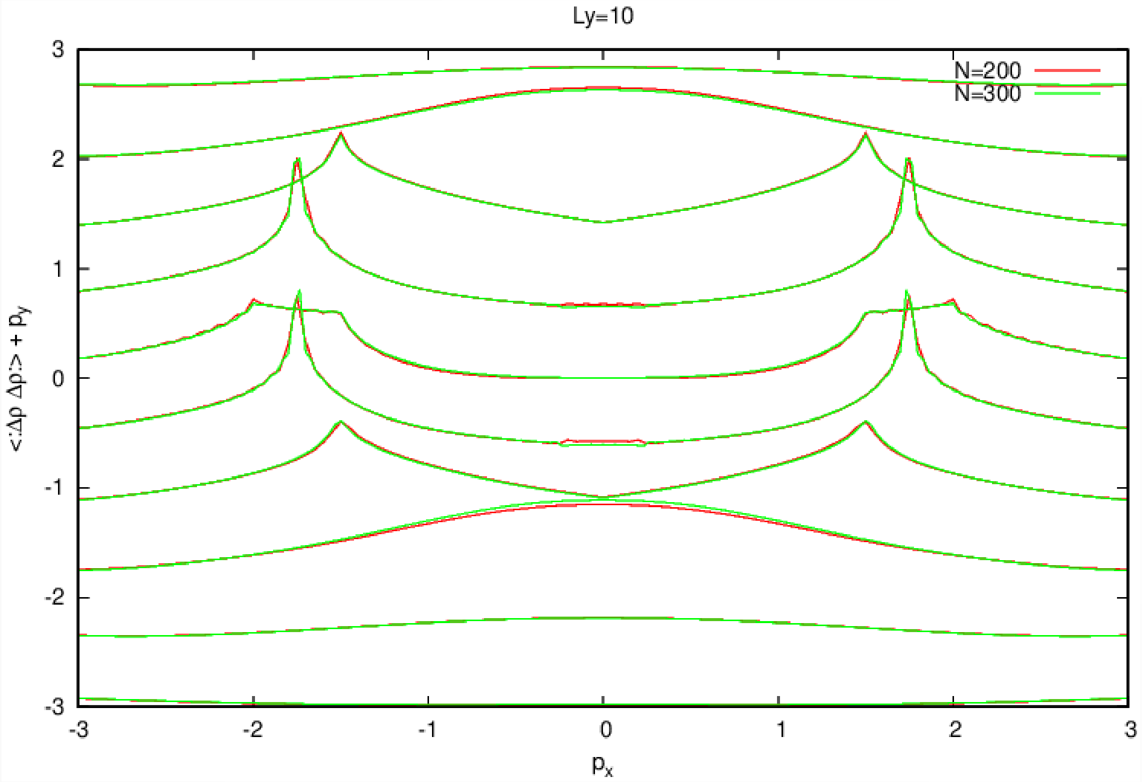
[5] Calabrese, P., Campostrini, M., Essler, F., & Nienhuis, B. (2010). Parity effects in the scaling of block entanglement in gapless spin chains. Physical Review Letters, 104(9), 095701.
[6] Daguerre, L., Medina, R., Solís, M., Torroba, G. (2021) "Aspects of quantum information in finite density field theory". J. High Energ. Phys. 2021, 79.

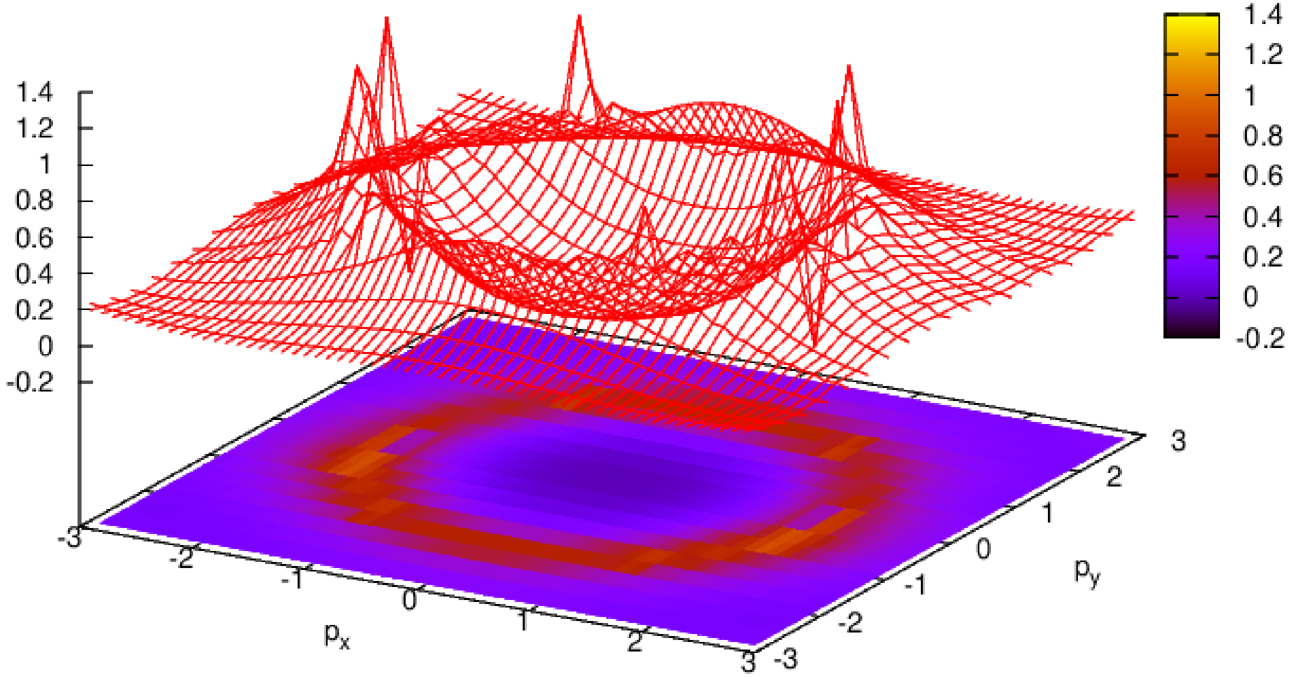
5. Results

Reproduce the density-density correlation physics[3]

- Mapping the Fermi surface via the structure factor
- Singularities at $2k_F$
- We use open boundary conditions

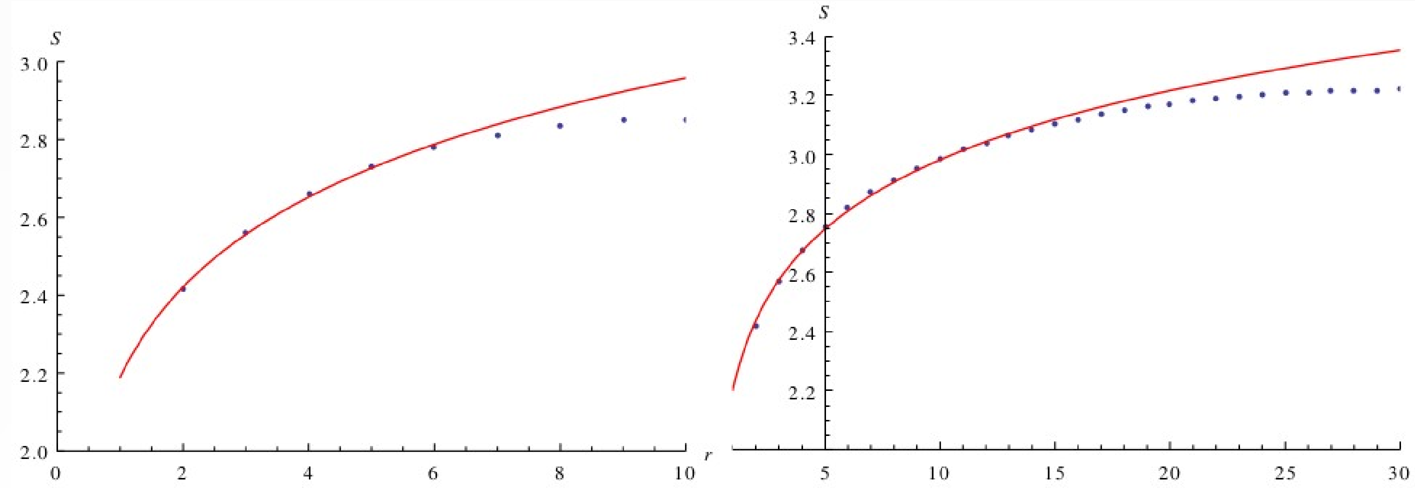


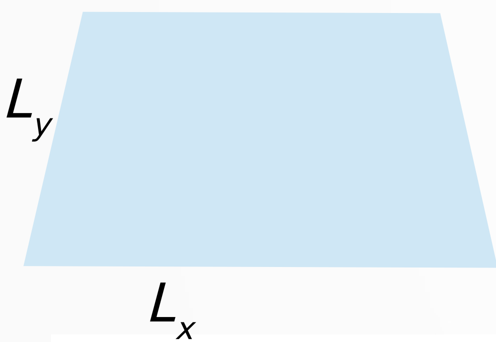


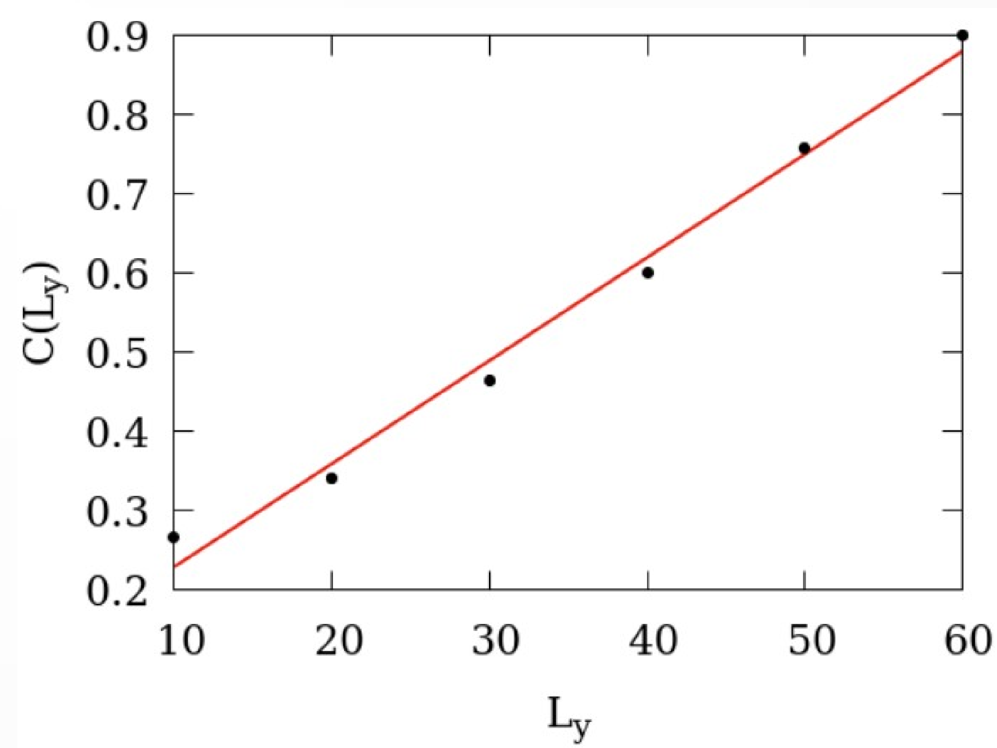


S~C log(r)

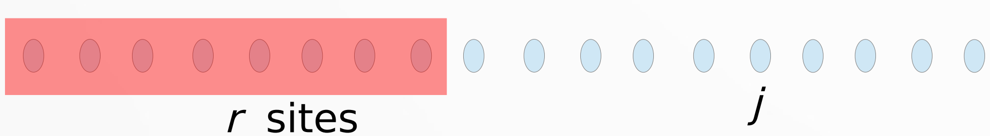
- hard to see log(r)
- periodic system or oscillations
- essentially limits the system size



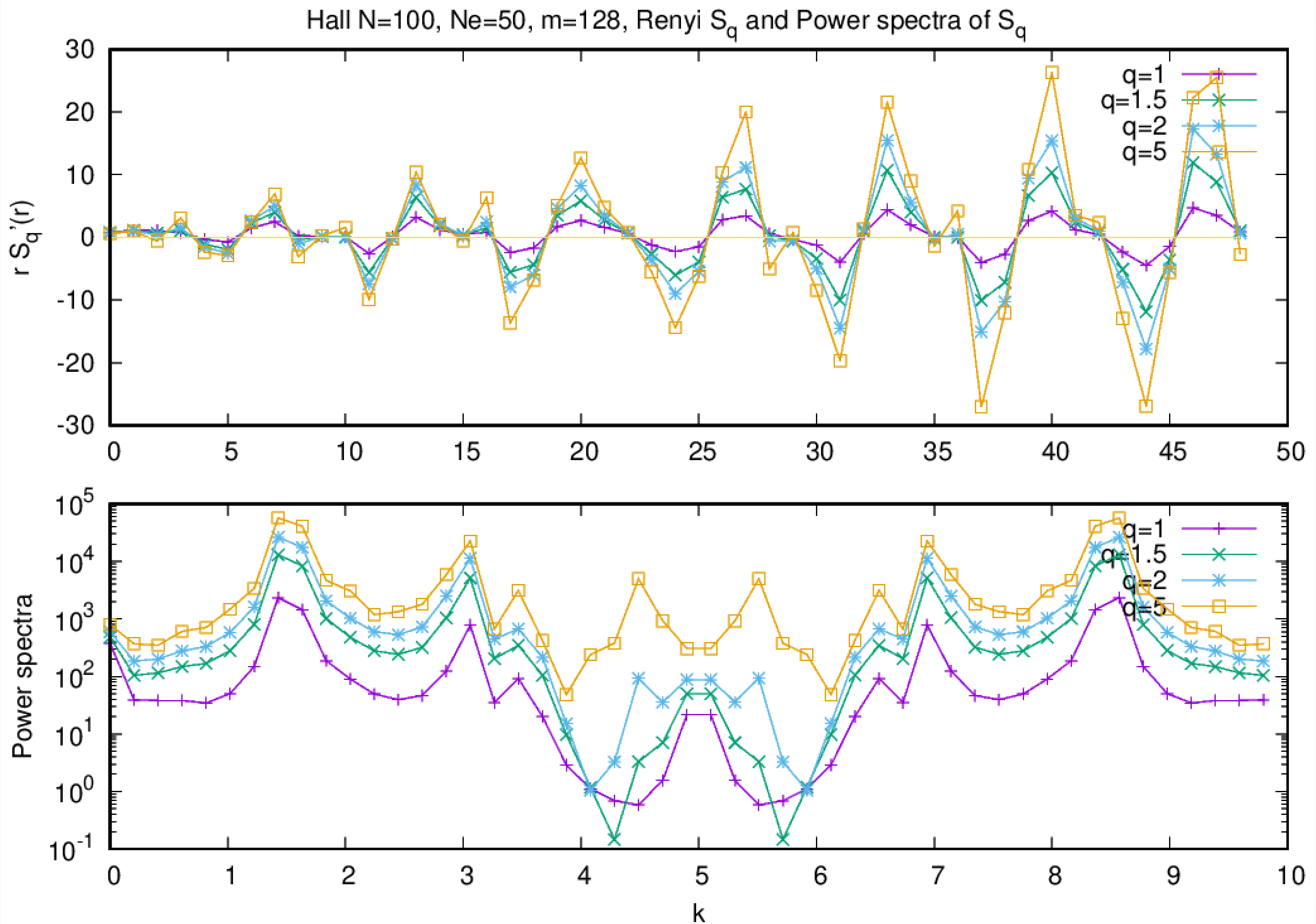


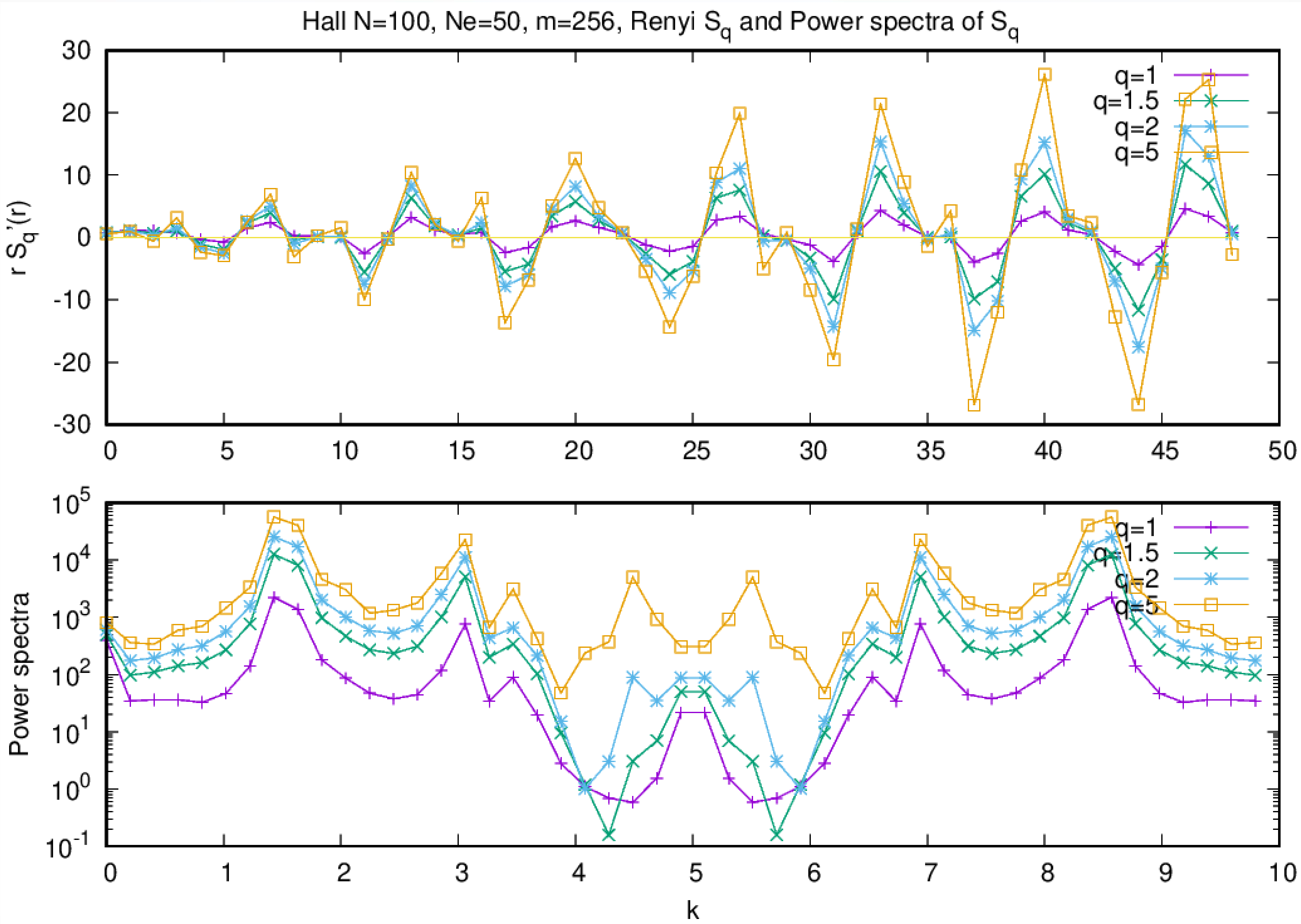


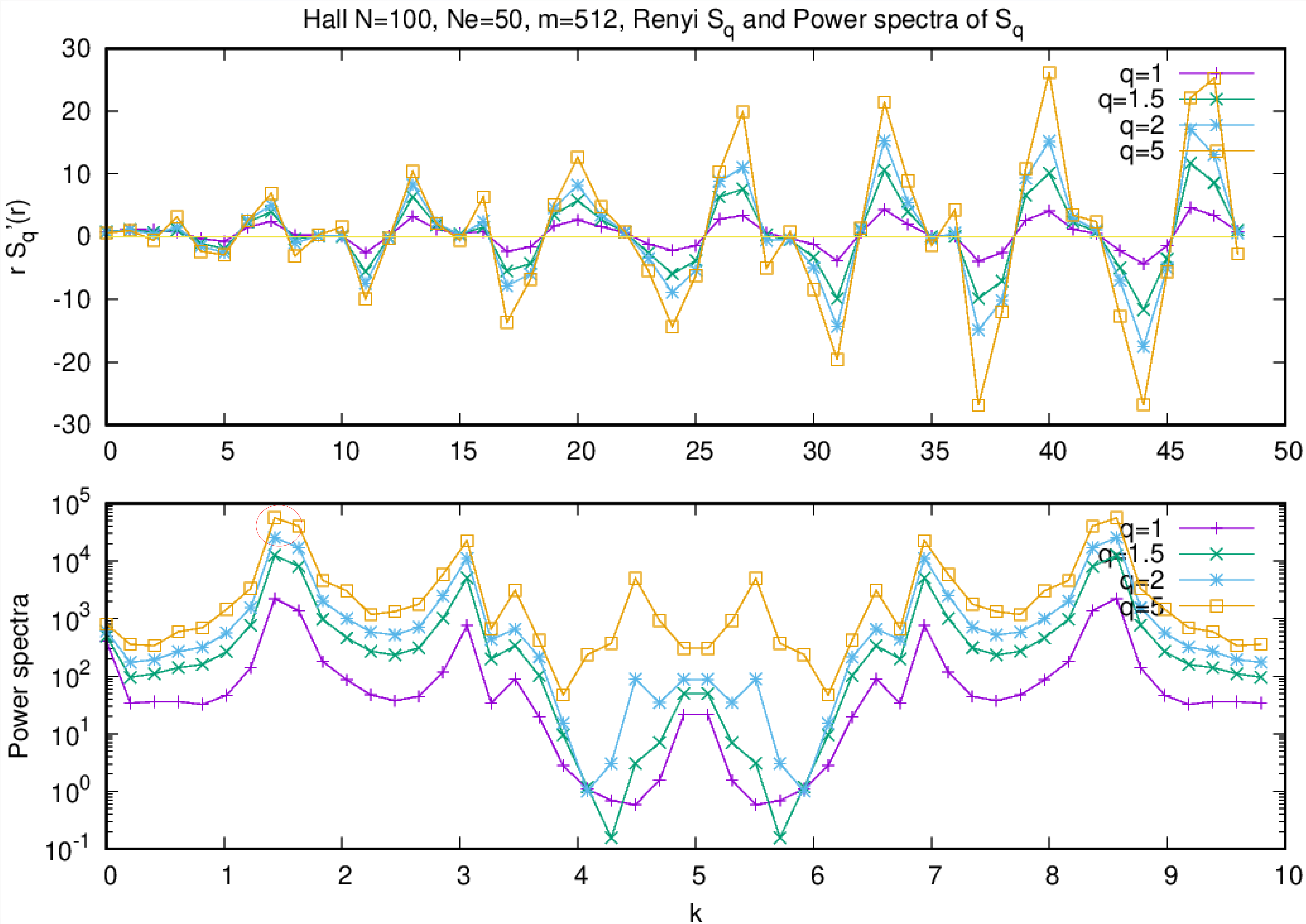
$$\varphi_j(\mathbf{r}) = \frac{e^{i\frac{2\pi}{L_y} j y}}{\sqrt{L_y}} \; \frac{e^{-\frac{(x-X_j)^2}{2\ell_B^2}}}{\pi^{1/4}\ell_B^{1/2}} \qquad X_j \equiv \ell_B^2 (2\pi/L_y) j$$



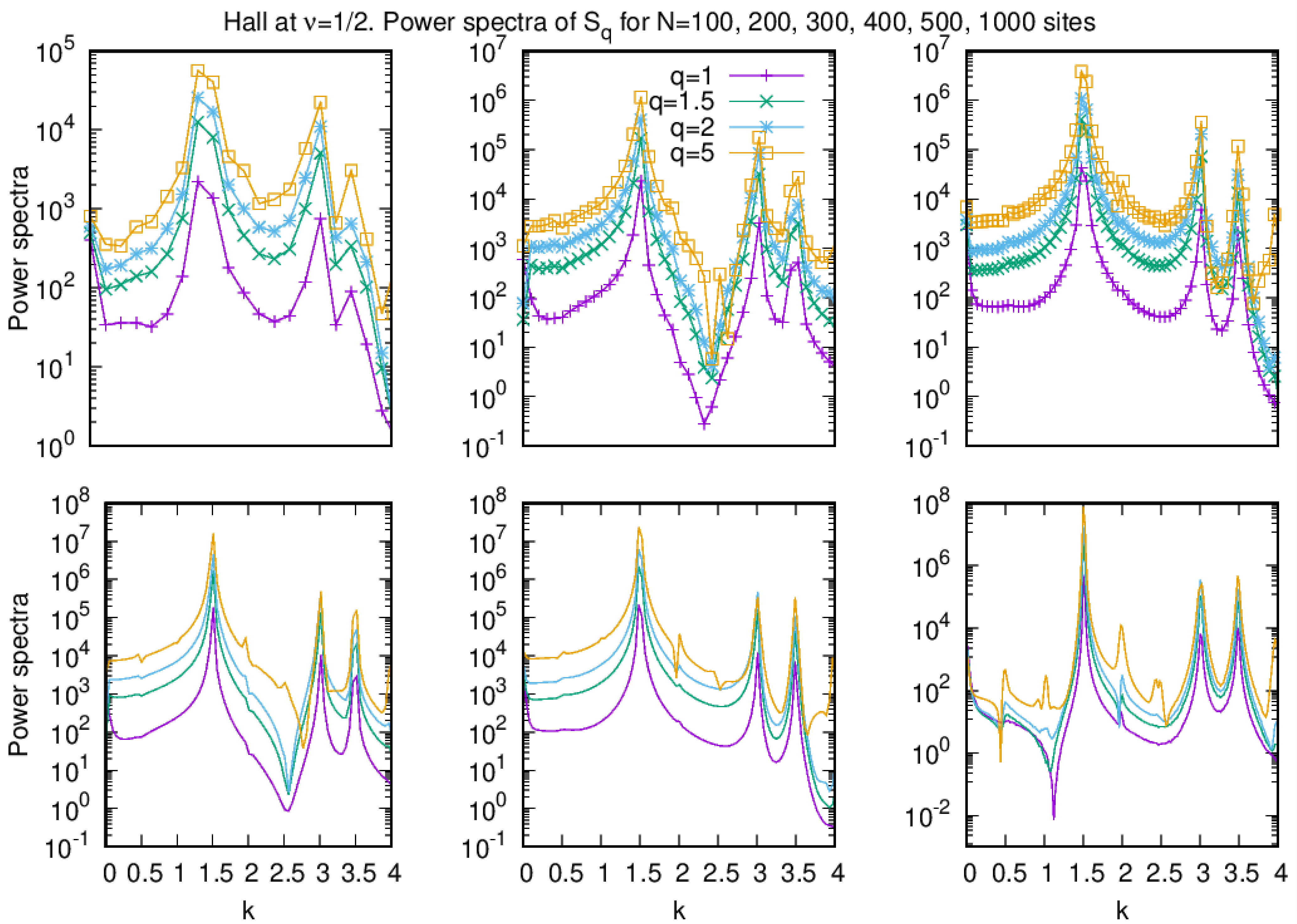
However the oscillations are robust!


Hall N=100, Ne=50, m=128, Renyi S_q and Power spectra of S_q
r S_q'(r)
q=1
q=1.5
q=2
q=5
Power spectra
k


Hall N=100, Ne=50, m=256, Renyi S_q and Power spectra of S_q
r S_q'(r)
q=1
q=1.5
q=2
q=5
Power spectra
k

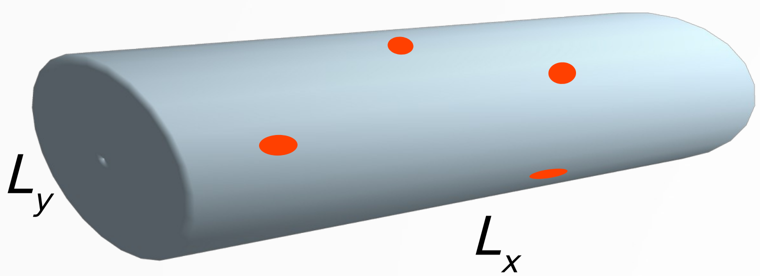


Hall N=100, Ne=50, m=512, Renyi S_q and Power spectra of S_q
$r S_q'(r)$
q=1
q=1.5
q=2
q=5
Power spectra
k



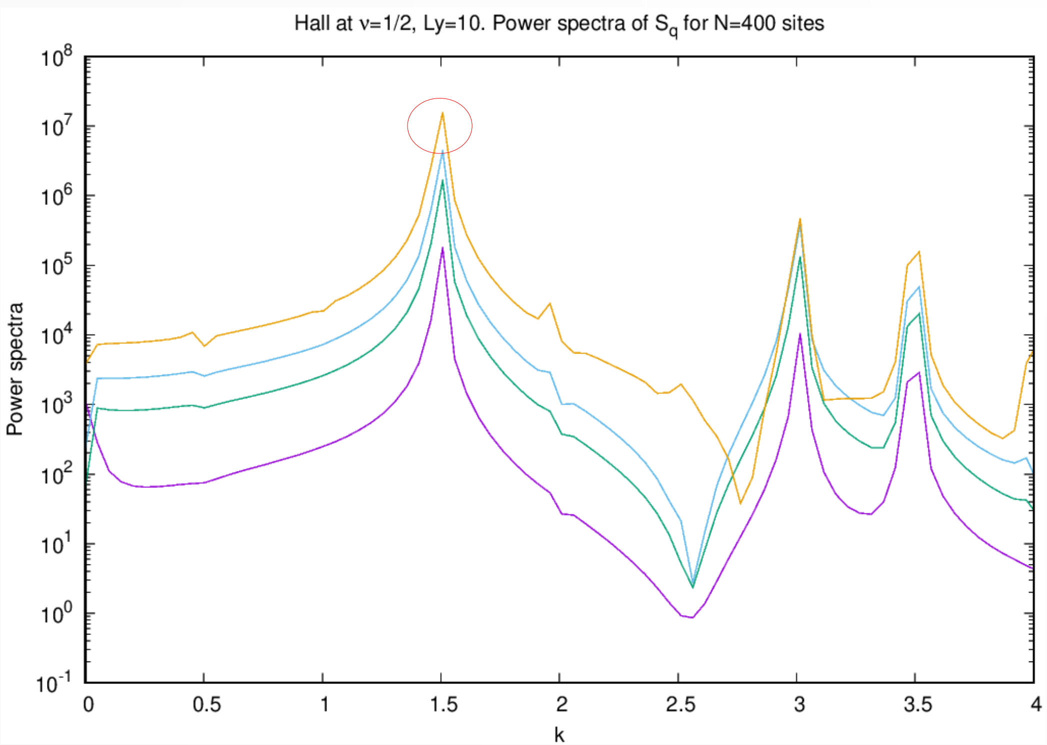
Hall at ν=1/2. Power spectra of S_q for N=100, 200, 300, 400, 500, 1000 sites
Power spectra
k
q=1
q=1.5
q=2
q=5

Free fermions on the cylinder

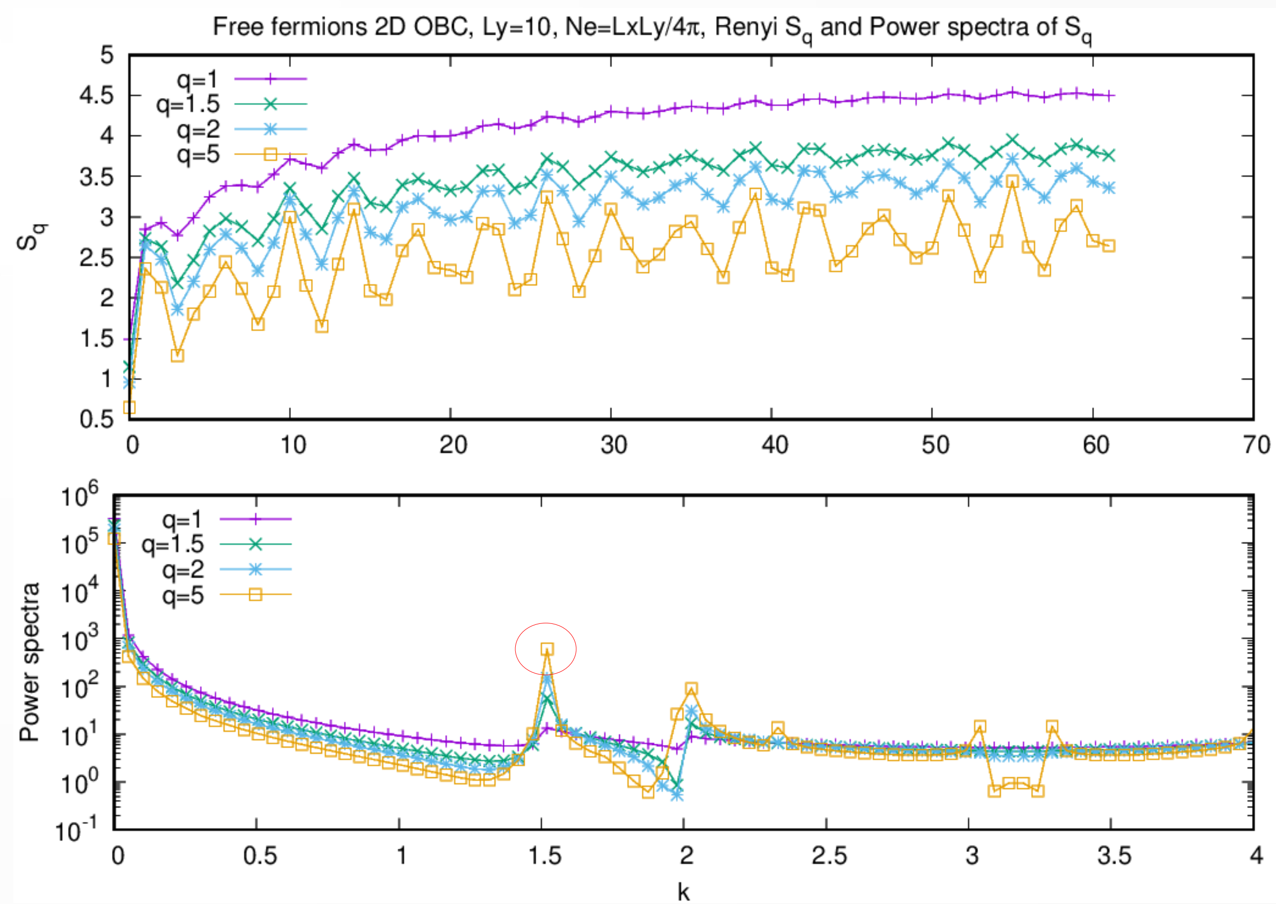


$$N_e = \frac{L_x L_y}{4\pi l_B^2} \Longrightarrow n_e = \frac{1}{4\pi}$$

Hall problem

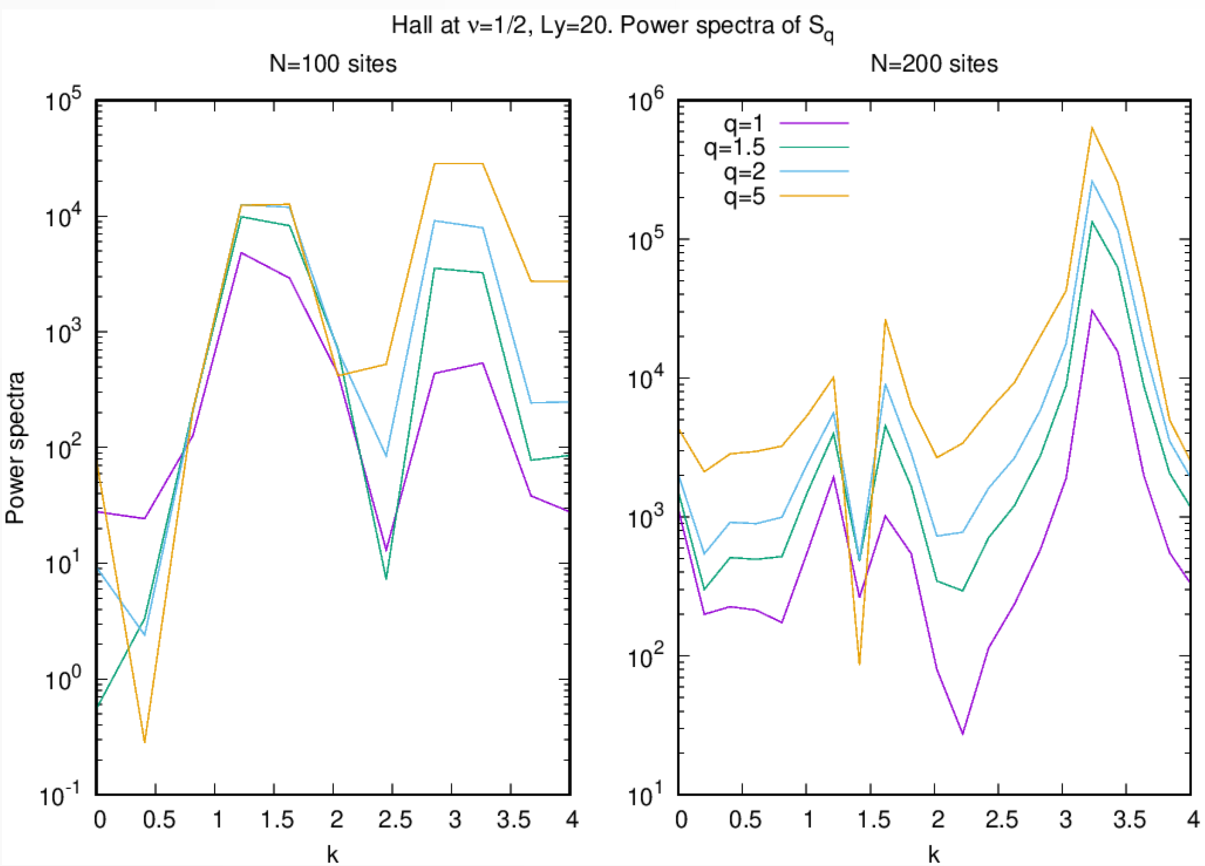


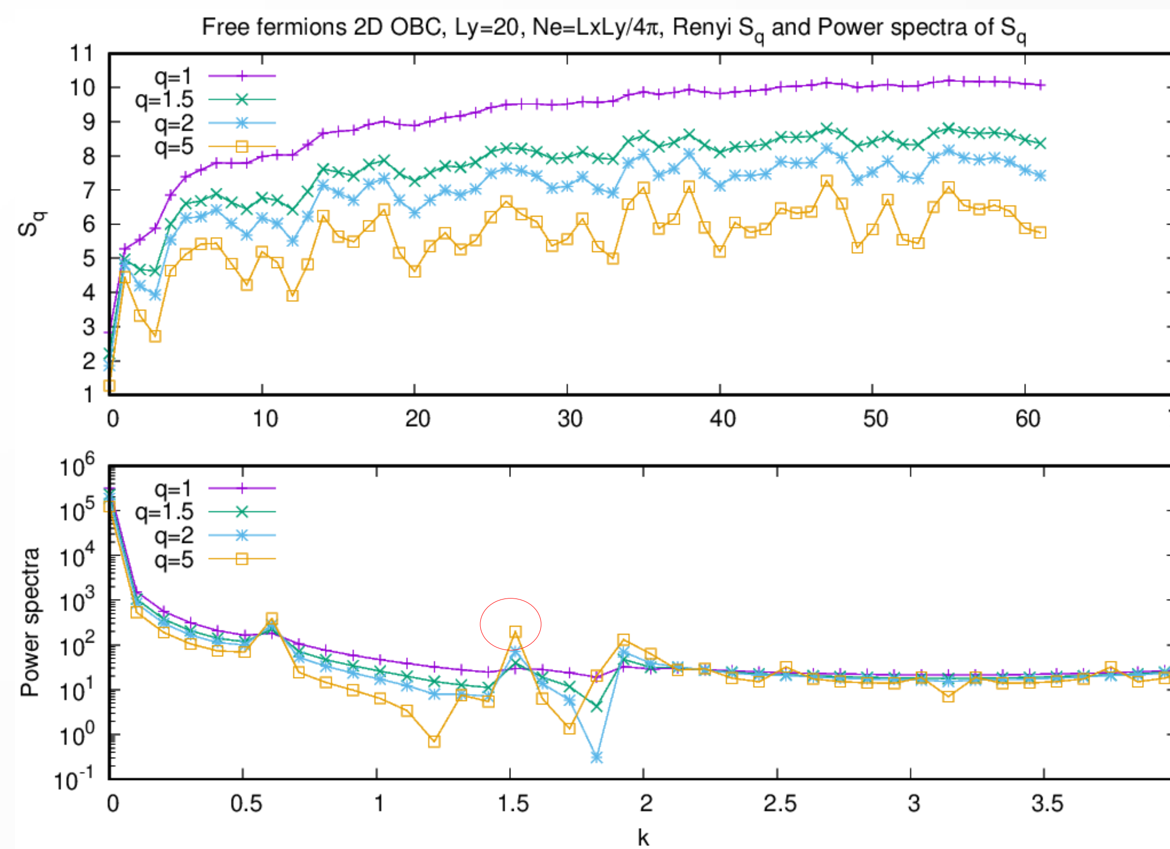
Free fermions



Work in progress …

$L_y=20$

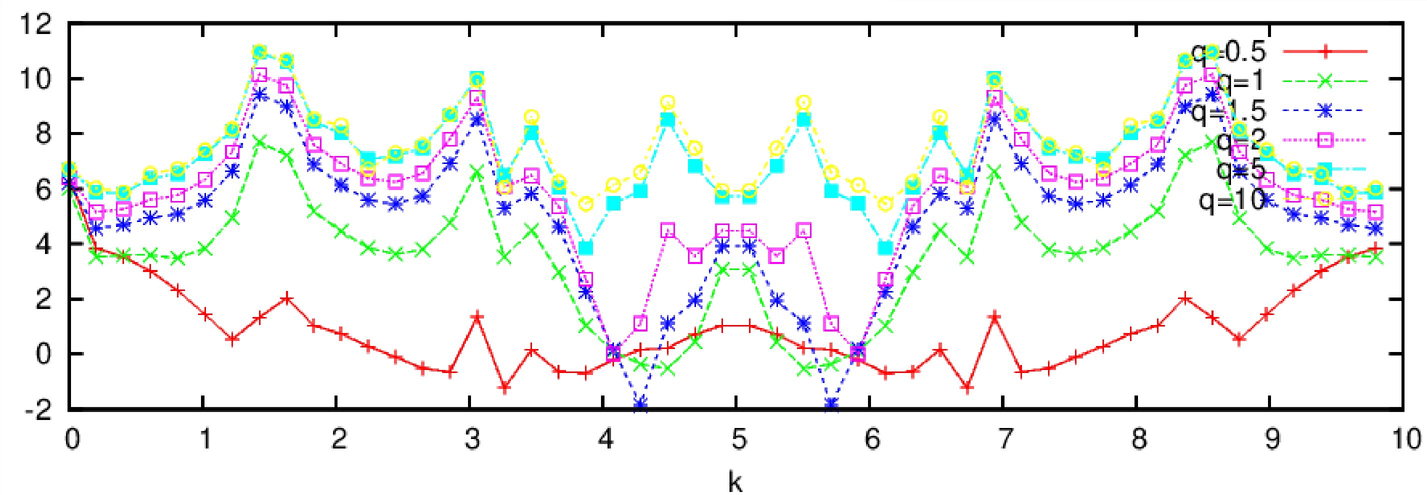




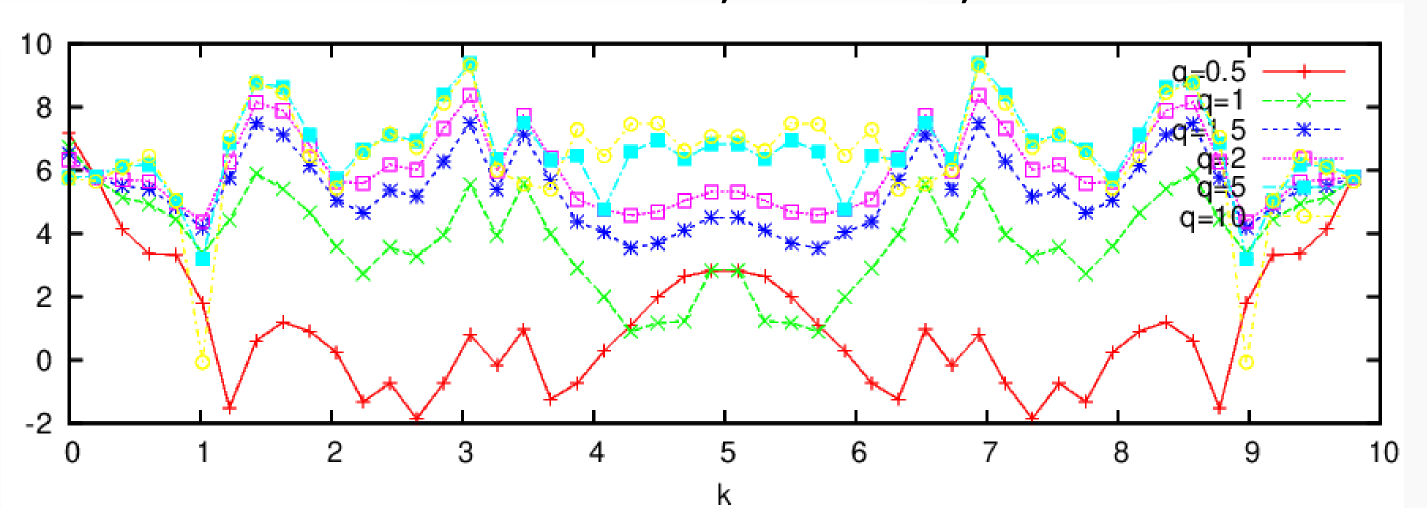
Doped system

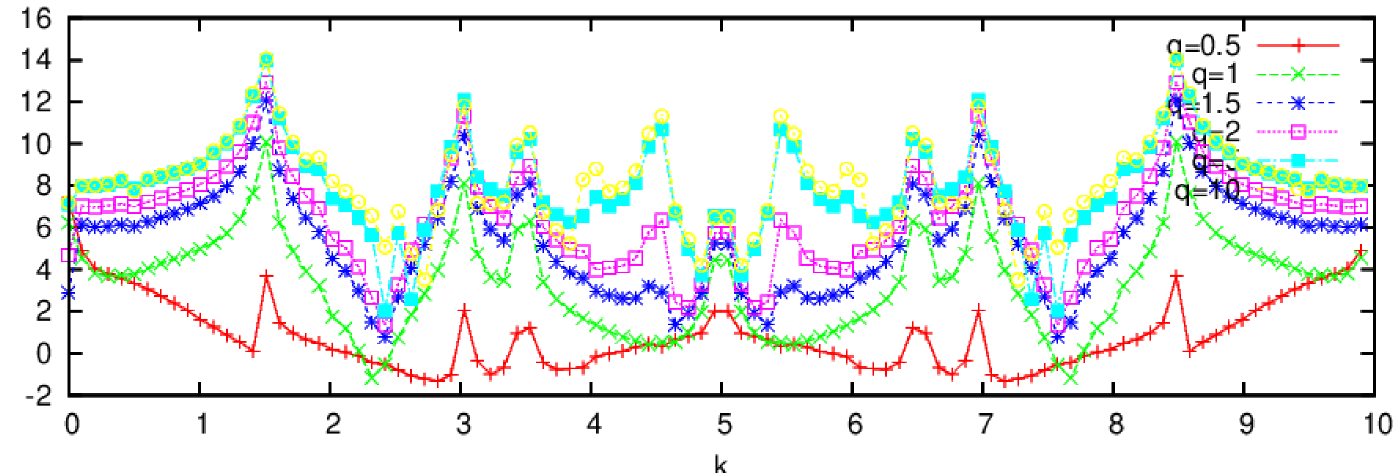
- Many interesting dynamics:
  - “magic” filling factors
  - composite bosons
- If the “k_F” varies then N(CF) depends on Ne →HLR,
- otherwise → Son

N=100, Ne=50

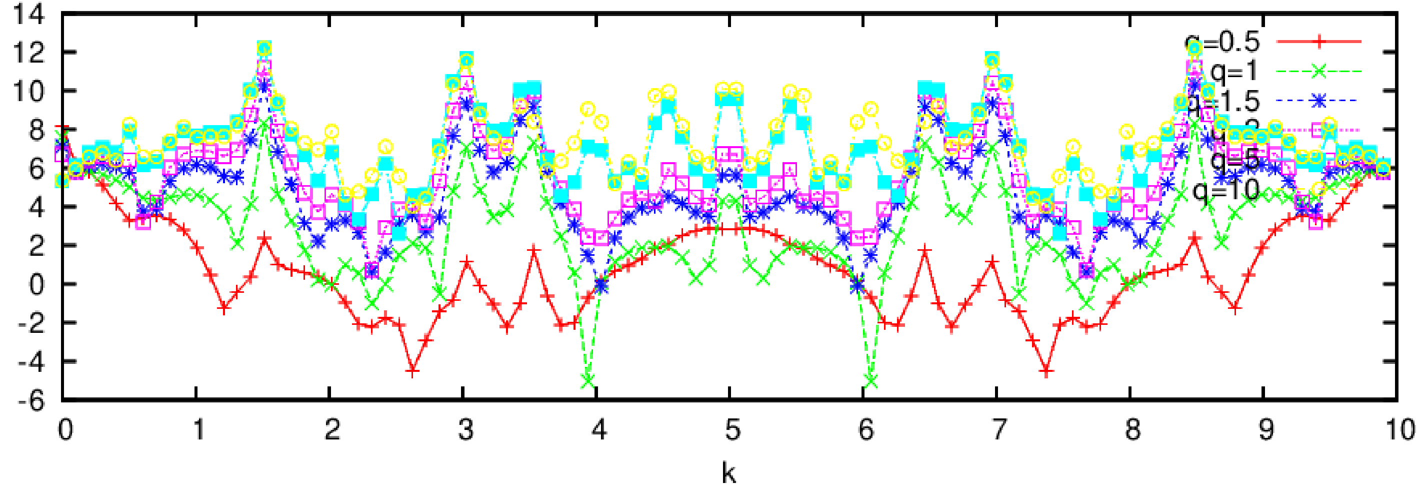


N=100, Ne=49,51

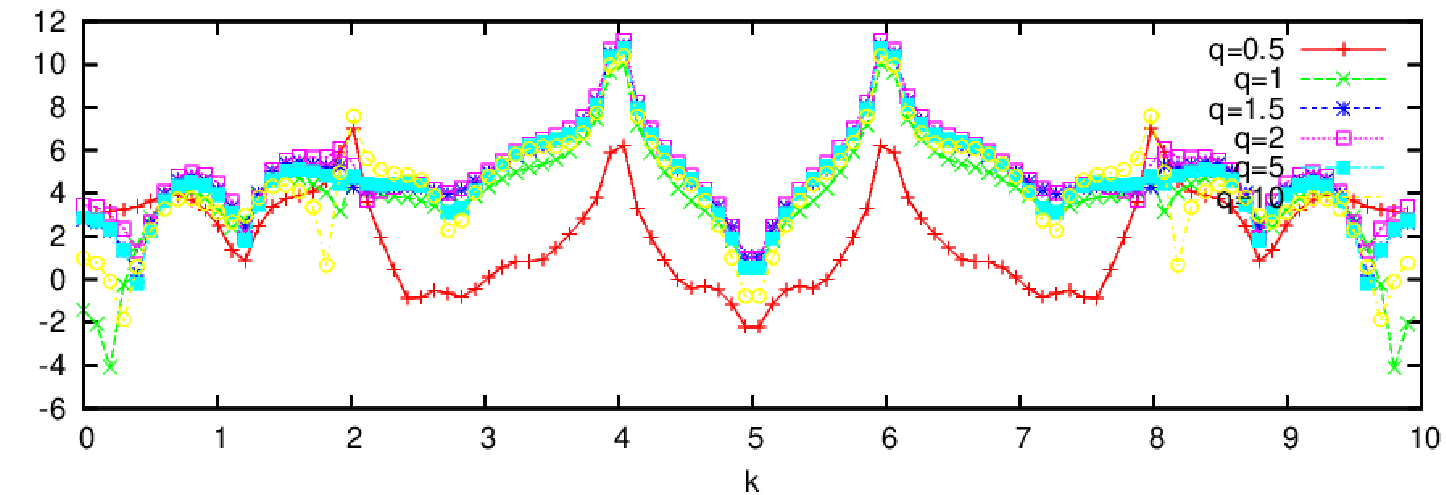




N=200, Ne=100



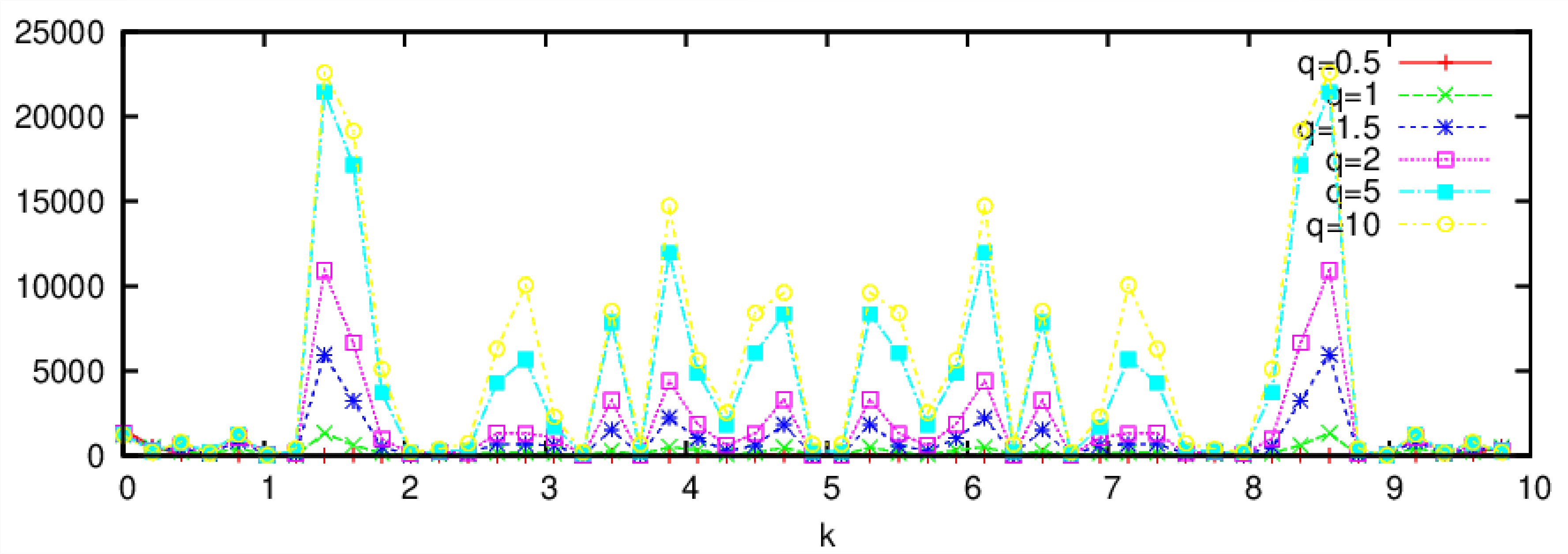
N=200, Ne=99,101



N=200, Ne=80,120

Medium doped system

- If the "k_F" varies then N(CF) depends on Ne →HLR, otherwise → Son



N=100, Ne=46,54

Not enough precision yet!

Conclusions

- Open system instead of iDMRG or periodic DMRG as a different approximation to the same physics
- Renyi entropy (Sq) as a good proxy to study the physics of FQHE
- Friedel oscillations of Sq are robust wrt to both the MPS bond dimension and the MPS length.
- They seem to be consistent with the picture of free fermions in the same geometry and filling.

Perspectives

- Explore L_y to check free-fermion conjecture
- Systematic study of doped systems could discard HLR or Son proposal!!

Thank you very much!