Spectral Signatures of Quasiparticle Interactions in Antiferromagnets

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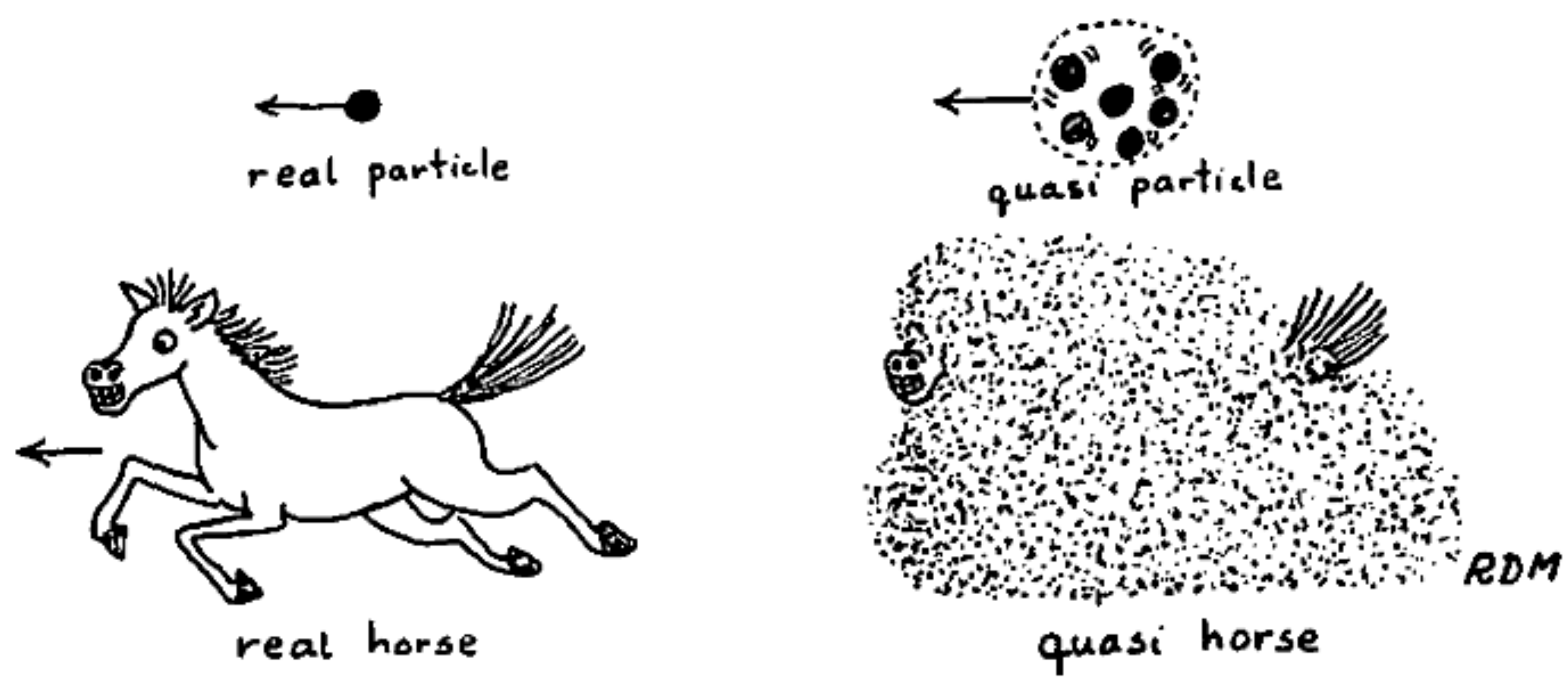
IPAM Workshop: Tensor Network States and Applications, April 20, 2021

Quasiparticles

Fundamental excitations of a many body ground state

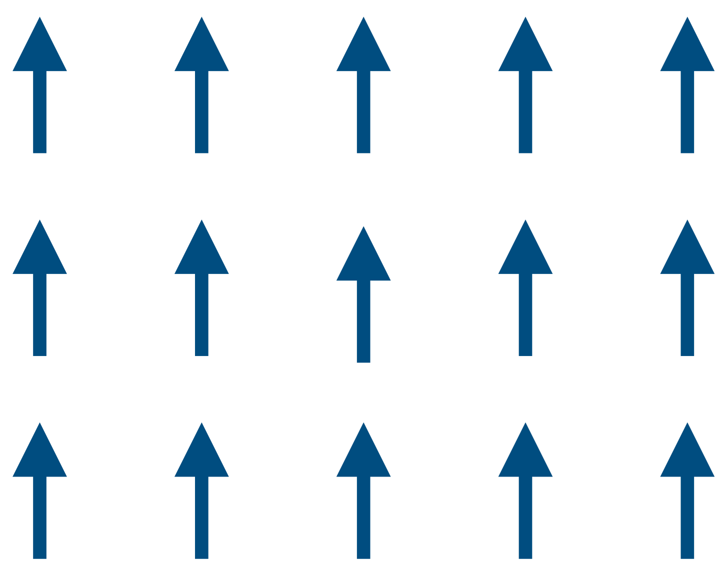
Behave like particles: single quasiparticle is long-lived



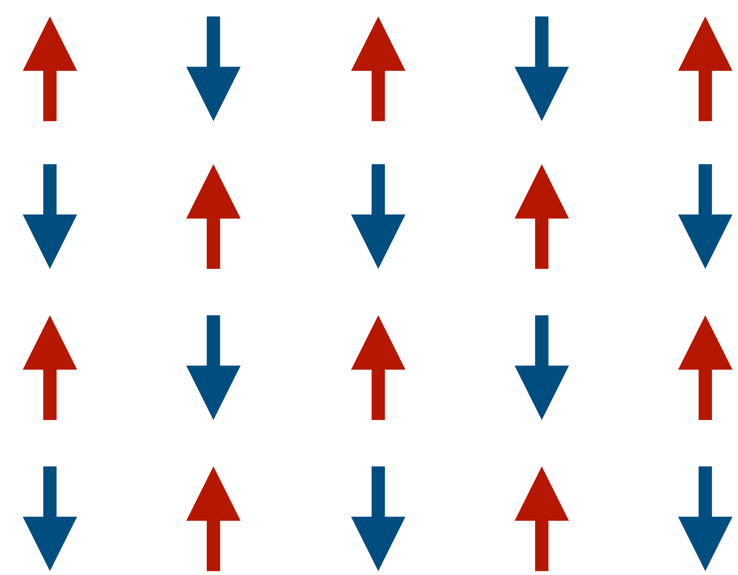
Magnetic systems

Electrons are immobile, pinned to the lattice $\Rightarrow$ System of localized spins
(i.e. electronic insulators)

Ferromagnet

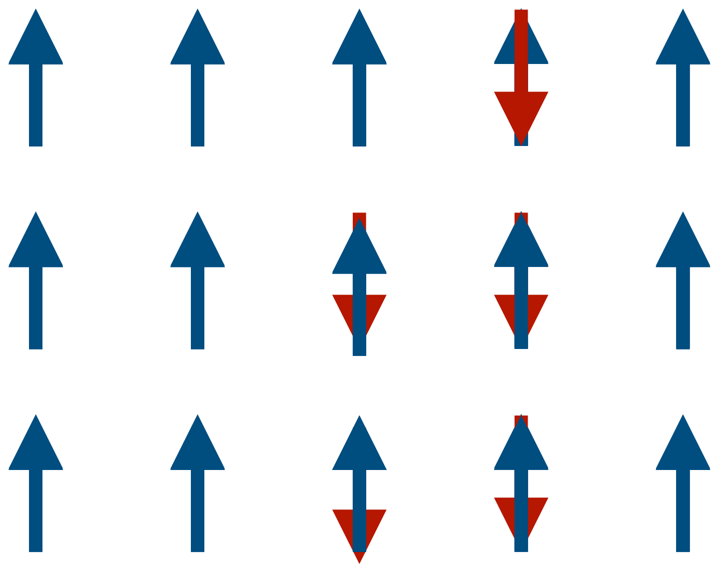


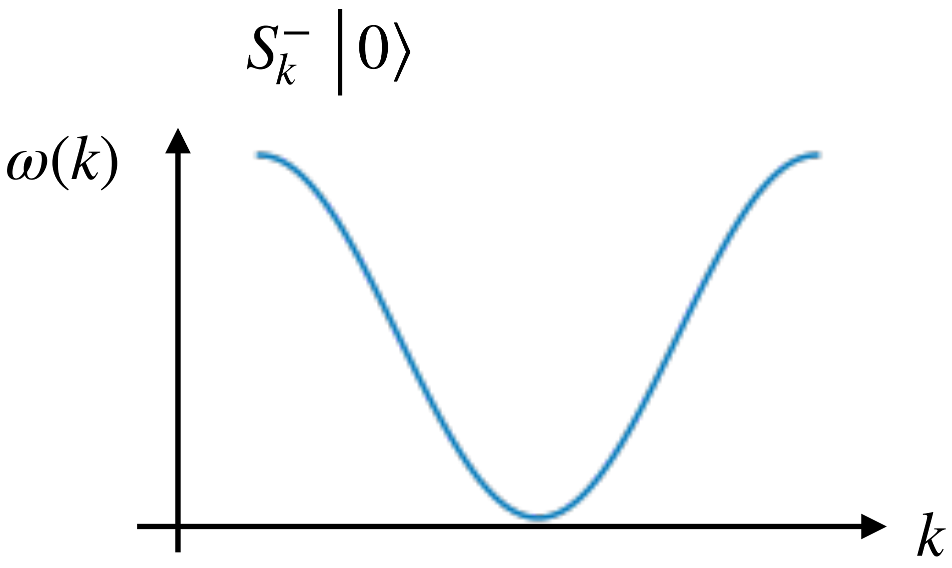
Antiferromagnet



Quasiparticles in magnetic systems

Spin waves (magnons) - bosonic quasiparticles



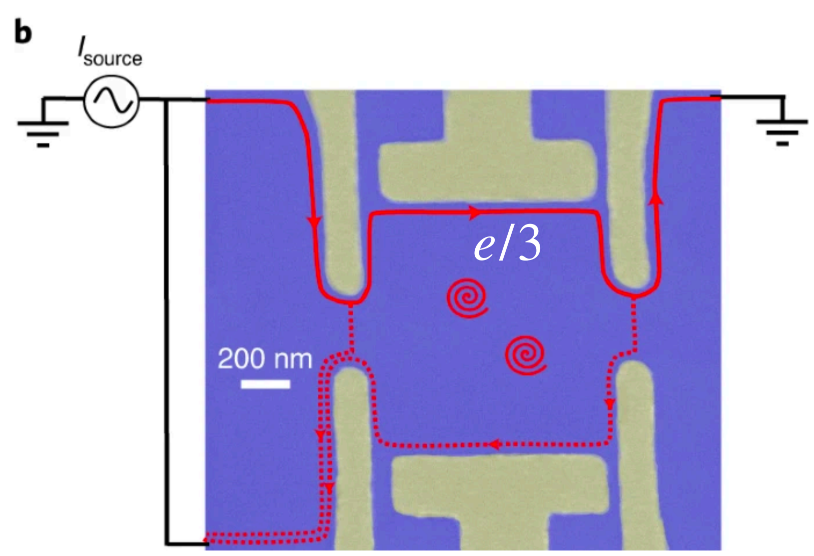


Beyond magnons - fractionalized quasiparticles

© R. E. Raspe, 1785

Carry fractional quantum numbers

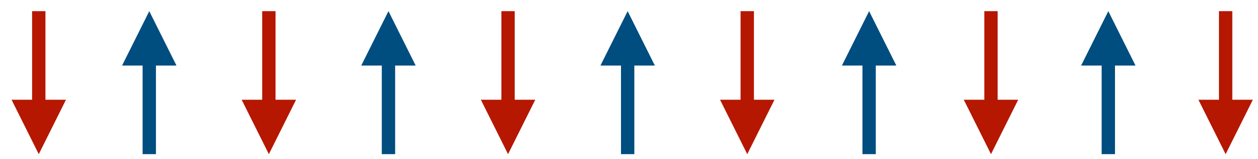
e.g. fractional charge in FQHE



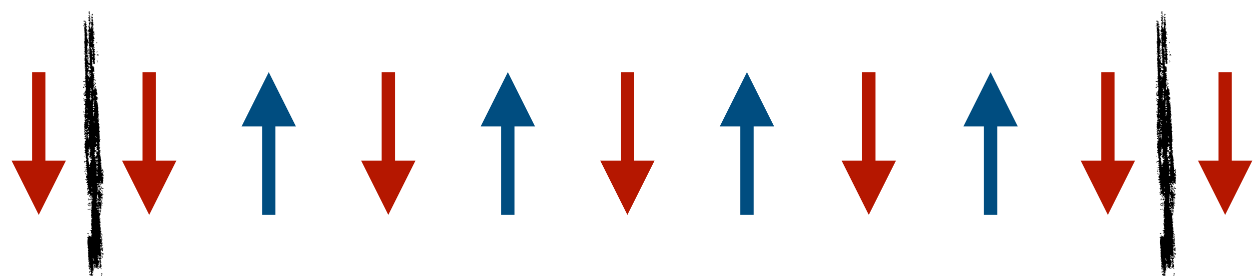
© Nakamura et al., Nature Phys. 2020

Beyond magnons - fractionalized quasiparticles

e.g. antiferromagnetic Ising chain



upon spin-flip domain walls are formed in pairs



$S = \frac{1}{2}$ excitation

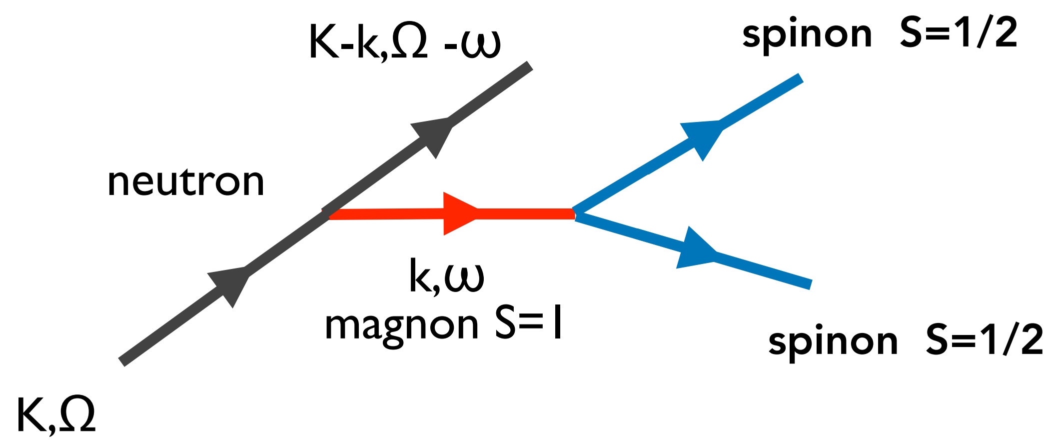
$S = \frac{1}{2}$ excitation

(spinon)

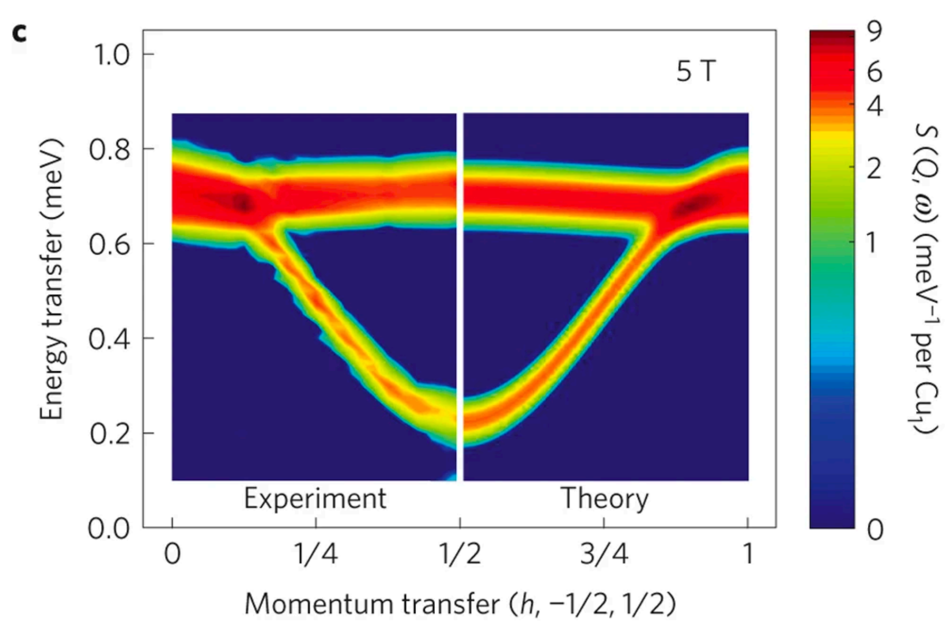
Dynamical susceptibility

$$S(\vec{k},\omega) = \int_{-\infty}^{\infty} dt e^{i\omega t} \int d\vec{r} e^{-i\vec{k}\cdot\vec{r}} \left\langle \vec{S}_r(t) \cdot \vec{S}_0(0) \right\rangle$$

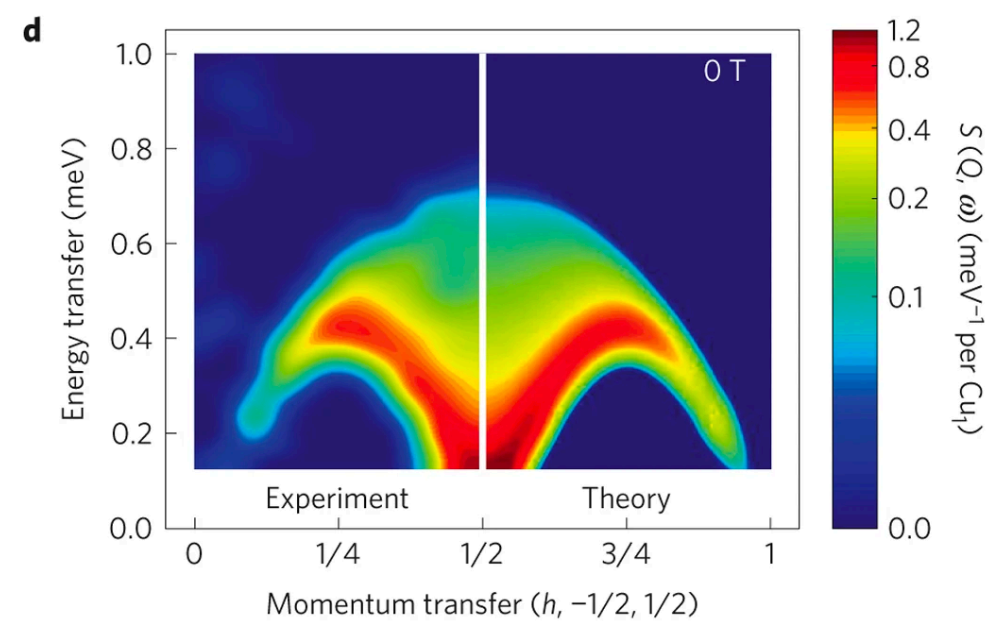
Measured in inelastic neutron scattering



Magnons - leave sharp peak



Spinons - show up as 2-particle continuum



Mourigal et al., Nature Physics 2013

Probing dynamical susceptibility numerically

$T = 0$

$$S^{+-}(\overrightarrow{k}, \omega) = \int_{-\infty}^{\infty} dt e^{i\omega t} \int d\vec{r} e^{-i\overrightarrow{k}\cdot\vec{r}} \langle 0 | S_r^+(t) S_0^-(0) | 0 \rangle$$

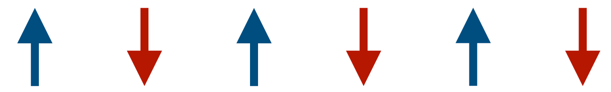
The ground state can be obtained using DMRG

$$\langle 0 | e^{iHt} S_r^+ e^{-iHt} S_0^- | 0 \rangle = e^{iE_0 t} \langle 0 | S_r^+ e^{-iHt} S_0^- | 0 \rangle$$

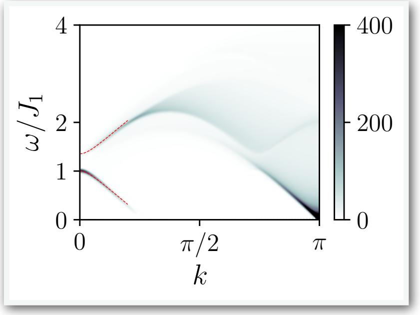
Time evolution of a quenched state

Outline

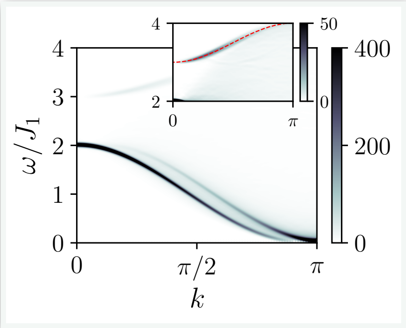
- Antiferromagnetic spin-1/2 chain in magnetic field

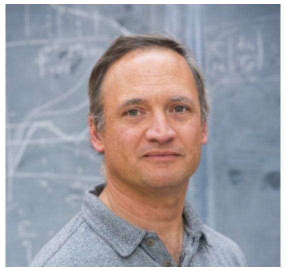


  - At low magnetization:
    Interactions between spinons induce
    a gap in the dynamical correlations



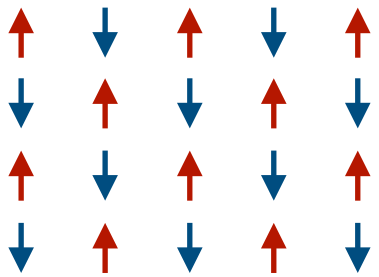
  - At high magnetization:
    Magnons (anti)-bound states

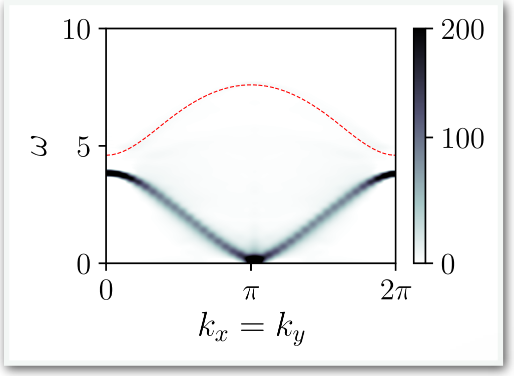




AK, Leon Balents, Oleg Starykh,
PRL 2020

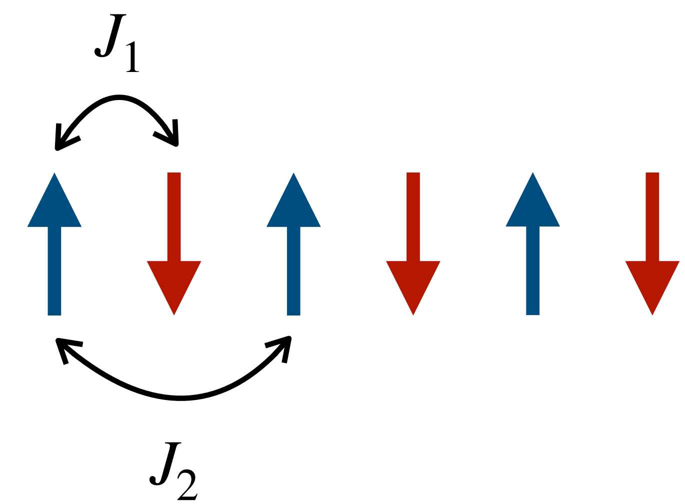
- (Anti)-bound states of
  magnons at higher dimensions

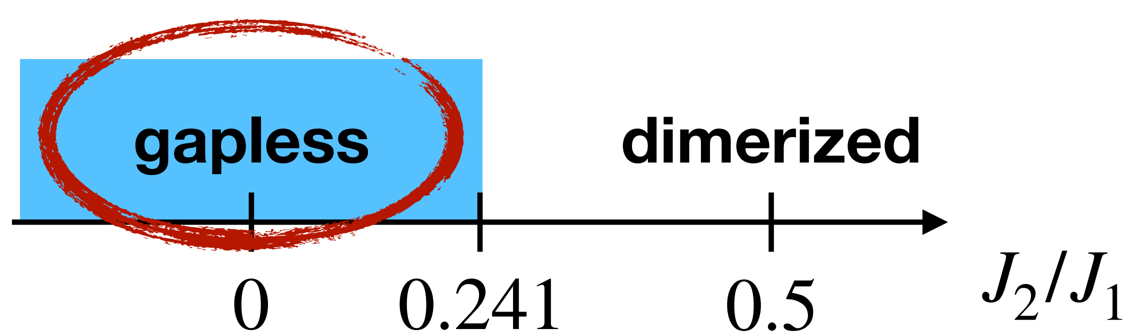




Spin-1/2 antiferromagnetic chain

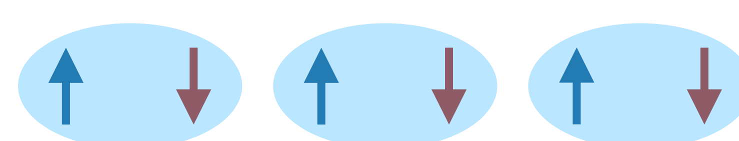
$$H = \sum_i J_1 \vec{S}_i \cdot \vec{S}_{i+1} + J_2 \vec{S}_i \cdot \vec{S}_{i+2}$$





Exactly solvable by Bethe ansatz

Majumdar-Gosh

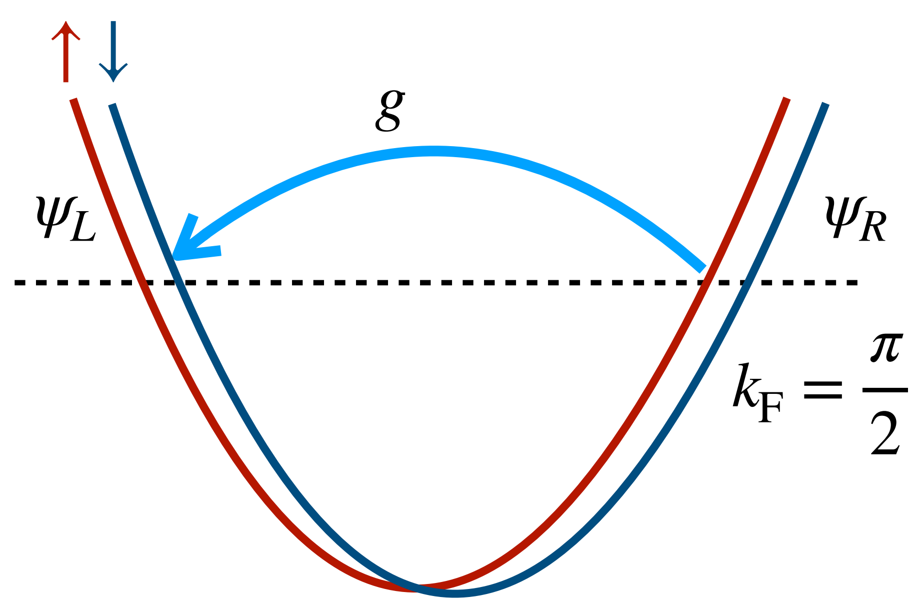


Low energy description of the gapless phase

Half-filled band of spinons

$$\vec{S}_i \sim \vec{J}_R(x_i) + \vec{J}_L(x_i) + (-1)^i \vec{N}(x_i)$$

$$\vec{J}_{R/L} = \frac{1}{2}\psi^\dagger_{R/L}\vec{\sigma}\psi_{R/L}, \quad \psi_{R/L} = \begin{pmatrix} \psi_{R/L,\uparrow} \\ \psi_{R/L,\downarrow} \end{pmatrix}$$



Effective Hamiltonian

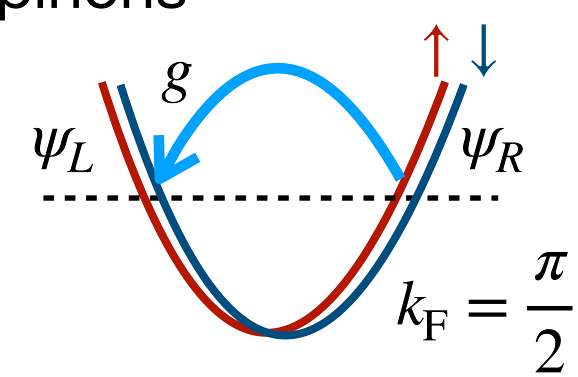
$$H = \underbrace{\int dx \left(\psi^\dagger_R(-iv_F\partial_x)\psi_R + \psi^\dagger_L(iv_F\partial_x)\psi_L\right)}_{H_0} \underbrace{-g\int dx\, \vec{J}_R \cdot \vec{J}_L}_{V}$$

free left/right moving spinons

density-density interactions between left and right moving spinons

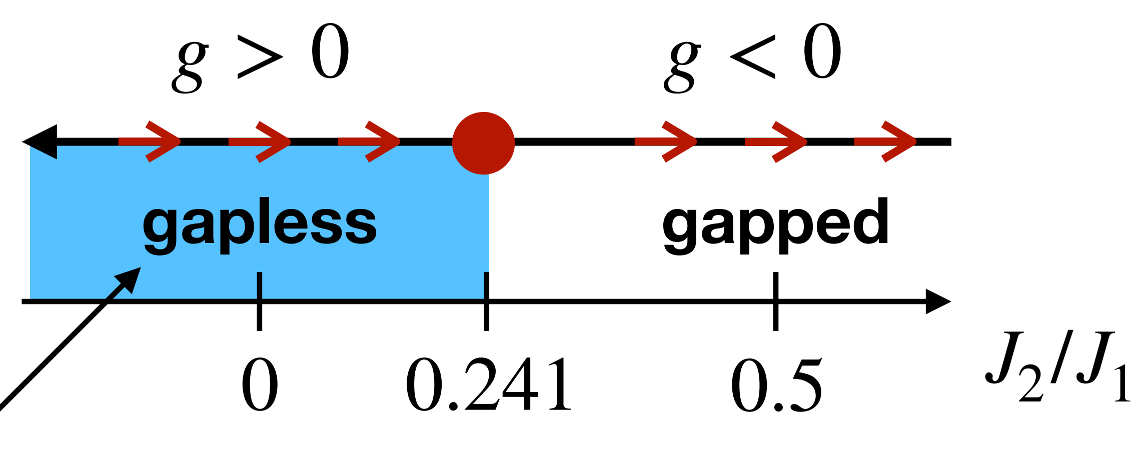
Spin-1/2 antiferromagnetic chain

half-filled band
of spinons



$$H = H_0 - g \int dx\, \vec{J}_R \cdot \vec{J}_L$$

$$\vec{J}_{R/L} = \frac{1}{2} \psi^{\dagger}_{R/L} \vec{\sigma} \psi_{R/L}$$



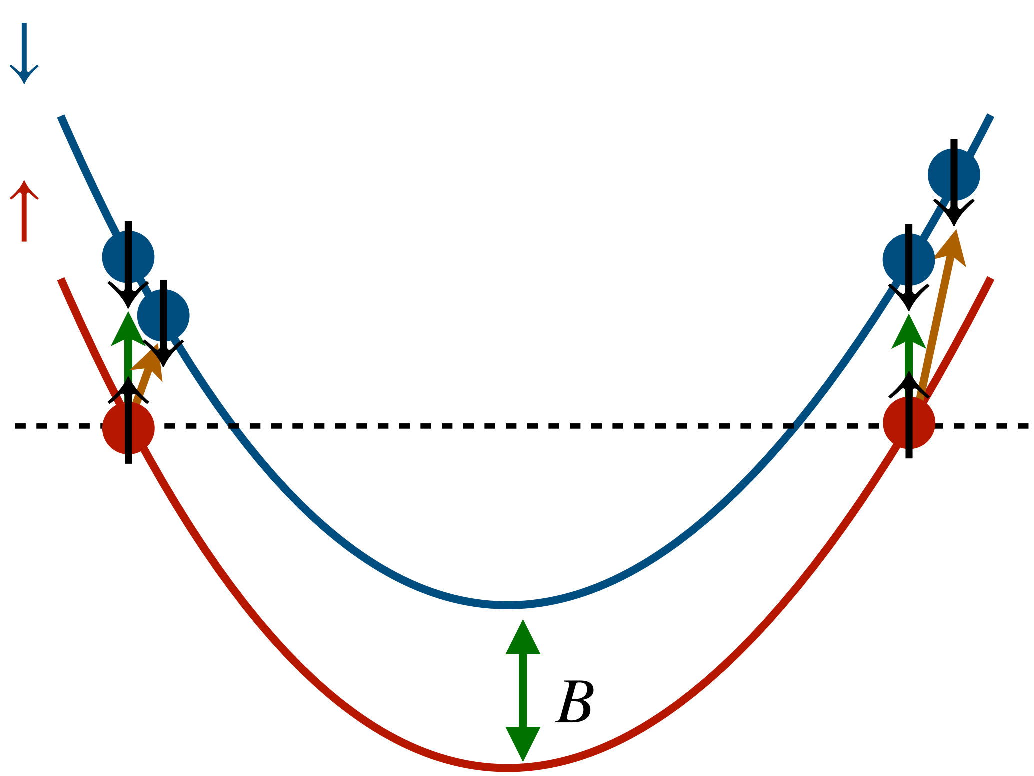
backscattering interaction
is *marginally irrelevant!*

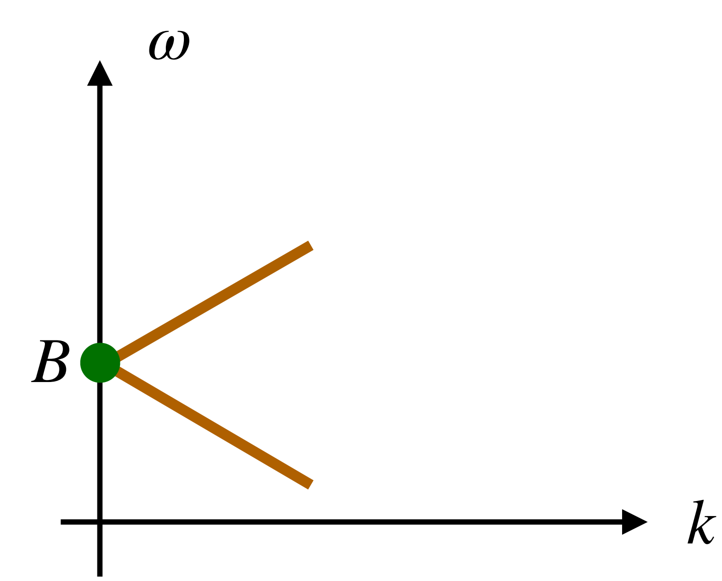
Dynamical susceptibility of spin-1/2 AFM chain in magnetic field

Non-interacting limit - small Zeeman field splits the up/down bands

$$H = H_0 - B\int dx \left[\underbrace{J^z_R(x) + J^z_L(x)}_{\text{Magnetization}}\right]$$

$$S^{+-}(k,\omega) = \sum_m \left|\langle m|\, S^-_k\, |0\rangle\right|^2 \delta(\omega - E_m)$$



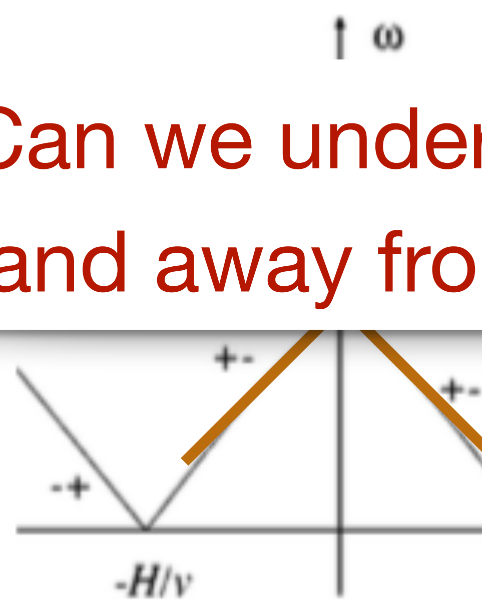


Spin-1/2 antiferromagnetic chain in magnetic field

PHYSICAL REVIEW B, VOLUME 65, 134410

Electron spin resonance in $S=\frac{1}{2}$ antiferromagnetic chains

Masaki Oshikawa[1] and Ian Affleck[2,*]

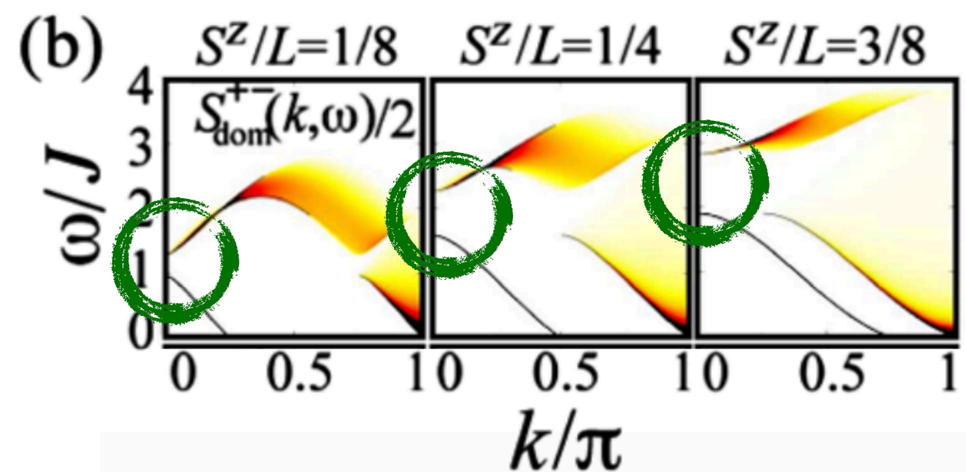


week ending
23 JANUARY 2009

romagnetic

Masanori Kohno

Can we understand the effect of g systematically (and away from the integrable limit with $J_2 = 0$)?



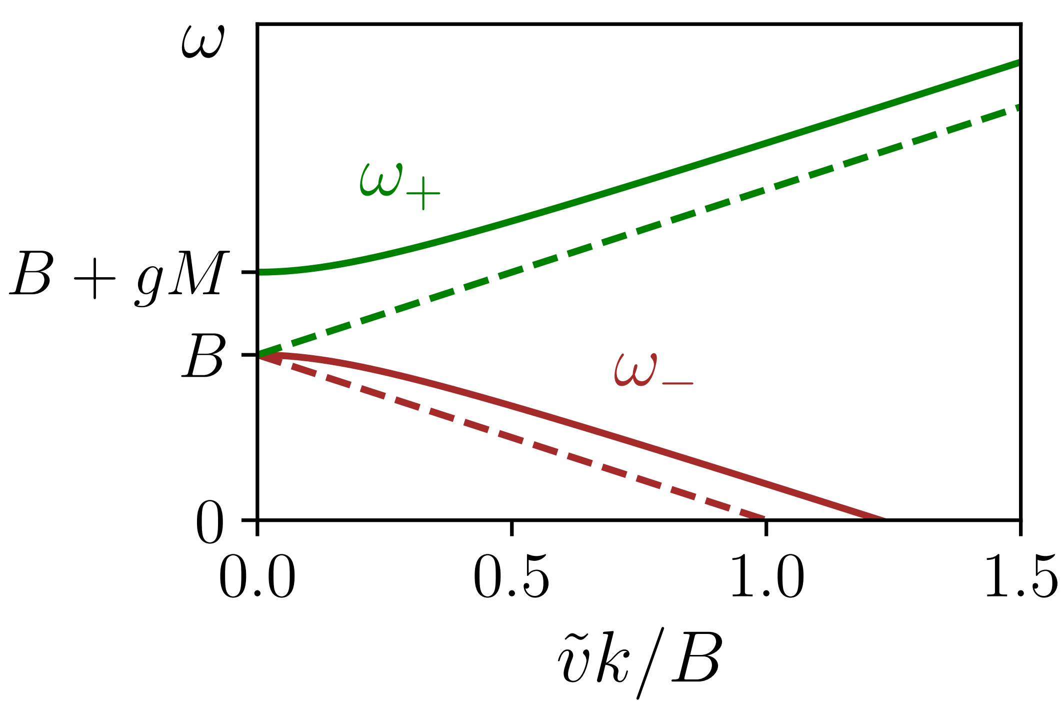
Dynamical susceptibility - analytical results

RPA-like treatment in the vicinity of $k = 0$

$$\chi^{\pm}(k,\omega) = M\left(\frac{A_+(k)}{\omega - \omega_+(k)} + \frac{A_-(k)}{\omega - \omega_-(k)}\right) \qquad \tilde{v} = v\sqrt{1 - g^2\chi_0^2/4}$$

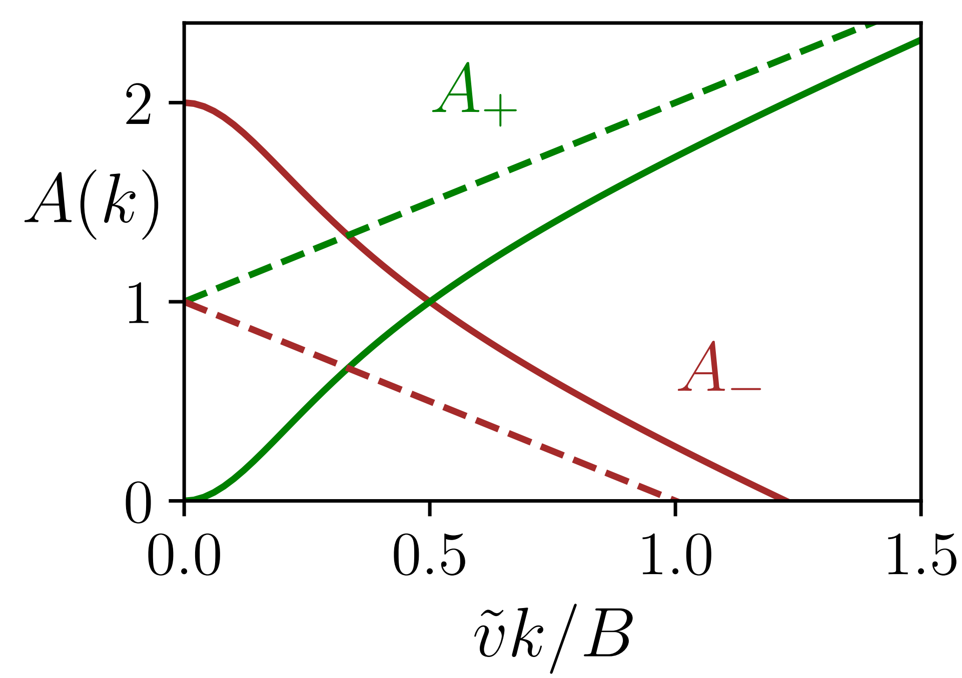
Dispersion

$$\omega_{\pm}(k) = B + \Delta \pm \sqrt{\Delta^2 + \tilde{v}^2 k^2}$$



Spectral weight

$$A_{\pm}(k) = 1 \pm \frac{\tilde{v}^2 k^2 - B\Delta}{B\sqrt{\Delta^2 + \tilde{v}^2 k^2}}$$

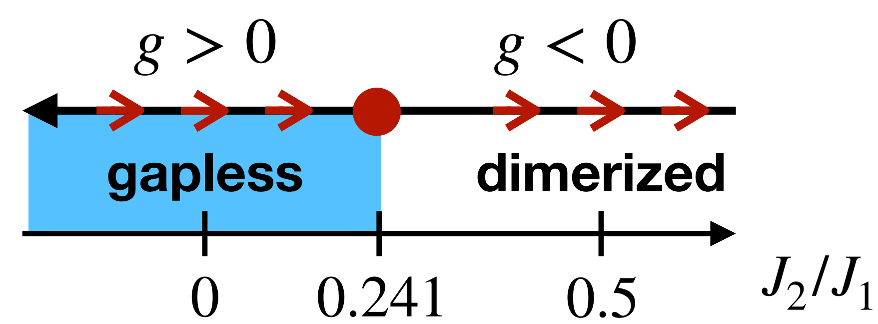


dashed lines - non-interacting limit ($g = 0$)

Numerical results

Transverse dynamical spin susceptibility:

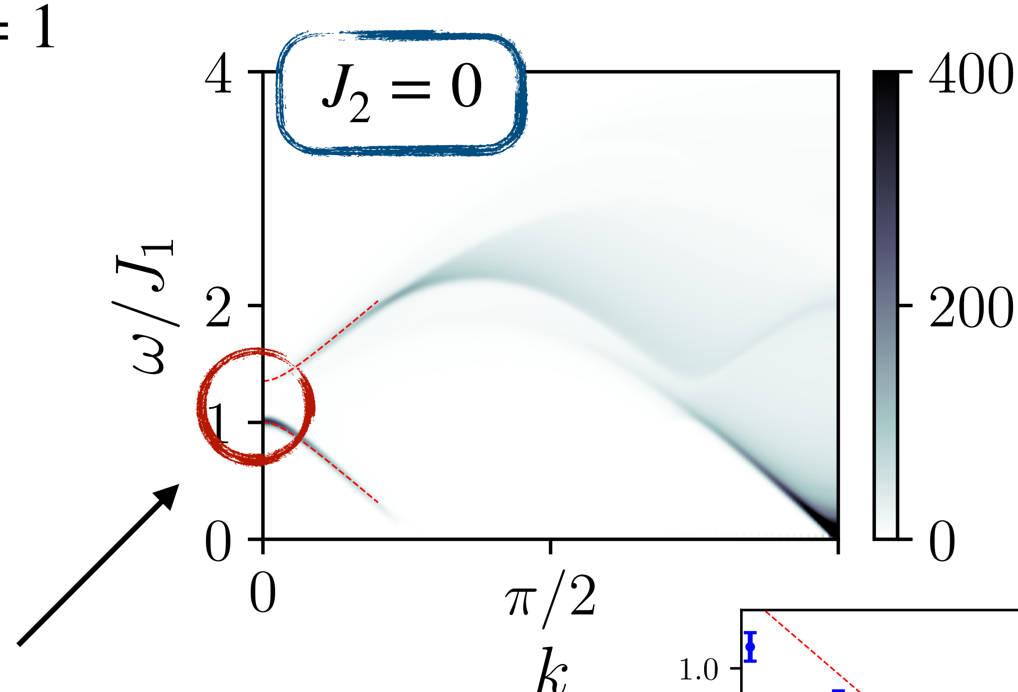
$$S^{+-}(\overrightarrow{k},\omega) = \int_{-\infty}^{\infty} dt e^{i\omega t} \int d\vec{r} e^{-i\overrightarrow{k}\cdot\vec{r}} \langle 0 \big| S_r^+(t) S_0^-(0) \big| 0 \rangle$$

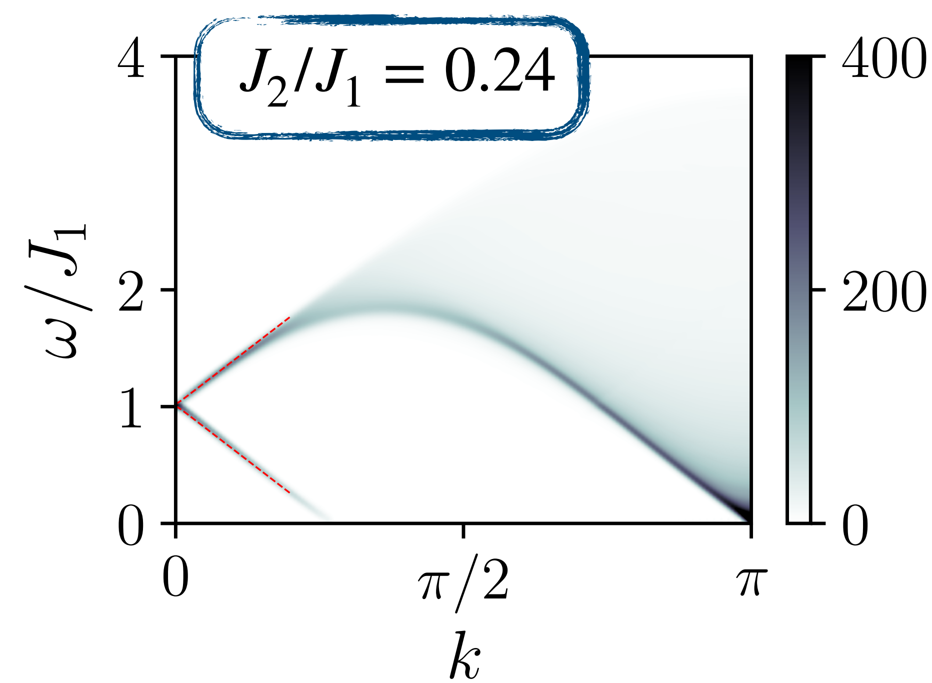


Finite interactions between spinons

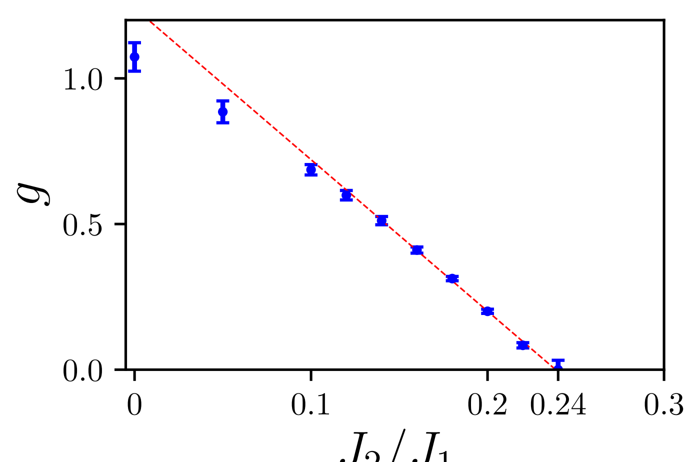
Non-interacting limit $g = 0$

$B/J_1 = 1$

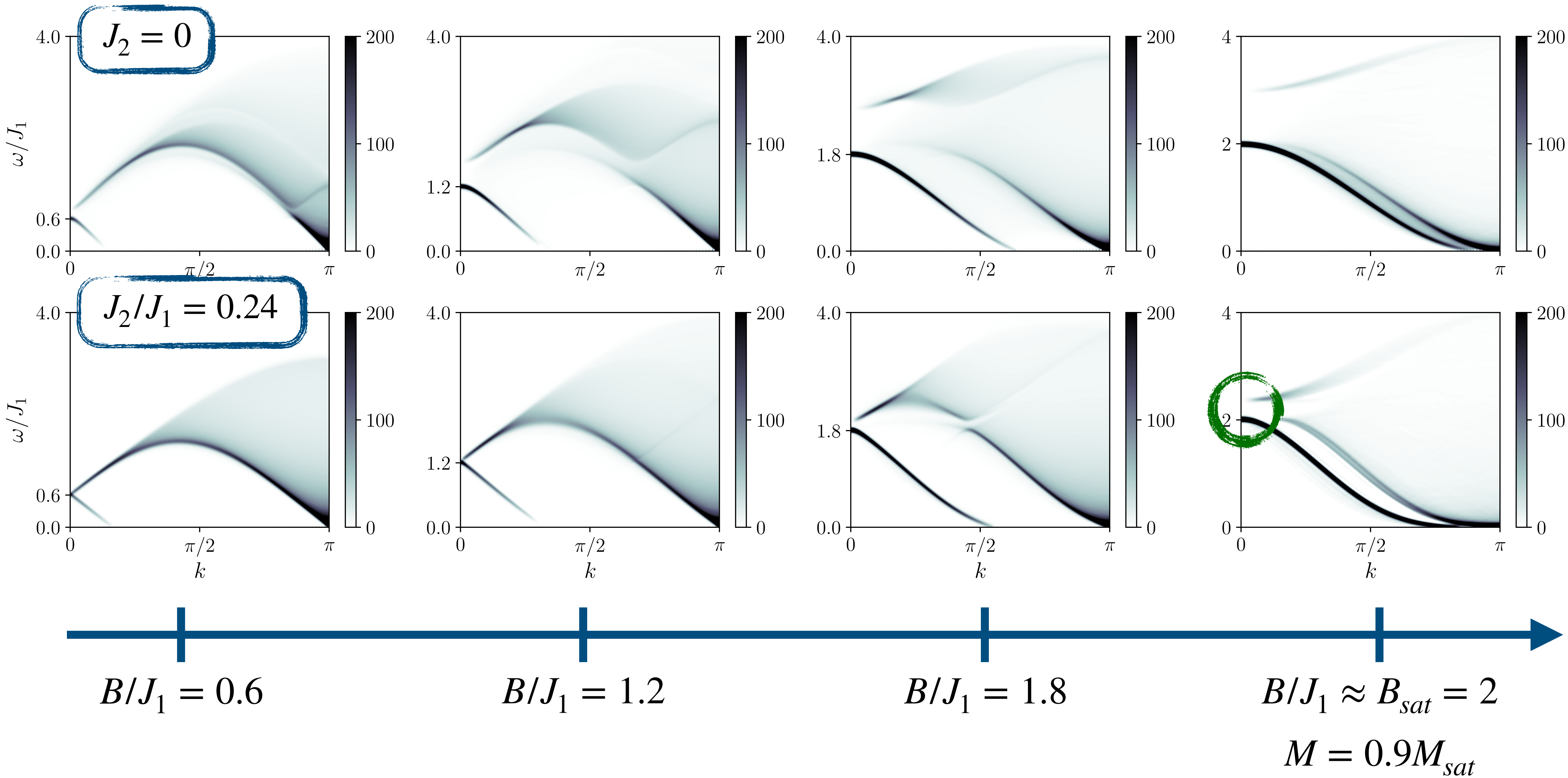




Magnitude of the gap allows us to extract interaction strength

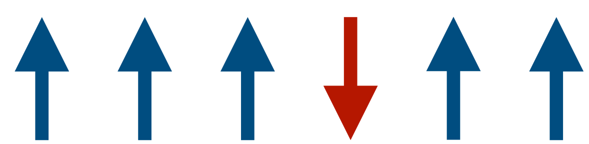


Increasing the magnetic field



Large magnetization limit

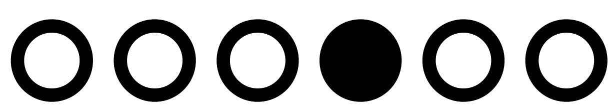
Mapping to spinless fermions



$$H = \sum_i 2J\left(S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+\right) + J^z S_i^z S_{i+1}^z - B S_i^z$$

$$S_i^- \sim c_i^\dagger$$

$$S_i^z = \frac{1}{2} - n_i$$



$$H = \sum_i \frac{J}{2}\left(c_i^\dagger c_{i+1} + h.c.\right) - J^z n_i n_{i+1} + (B - J_z) n_i$$

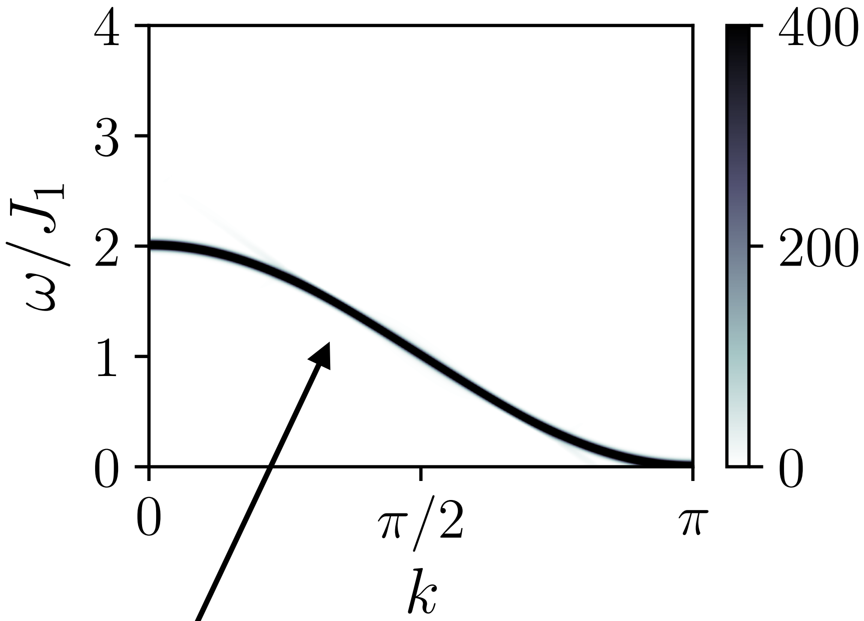
Single magnon dispersion

$$\omega(k) = J\cos k + B - J_z$$

Interaction strength

Dynamical susceptibility in the large magnetization limit

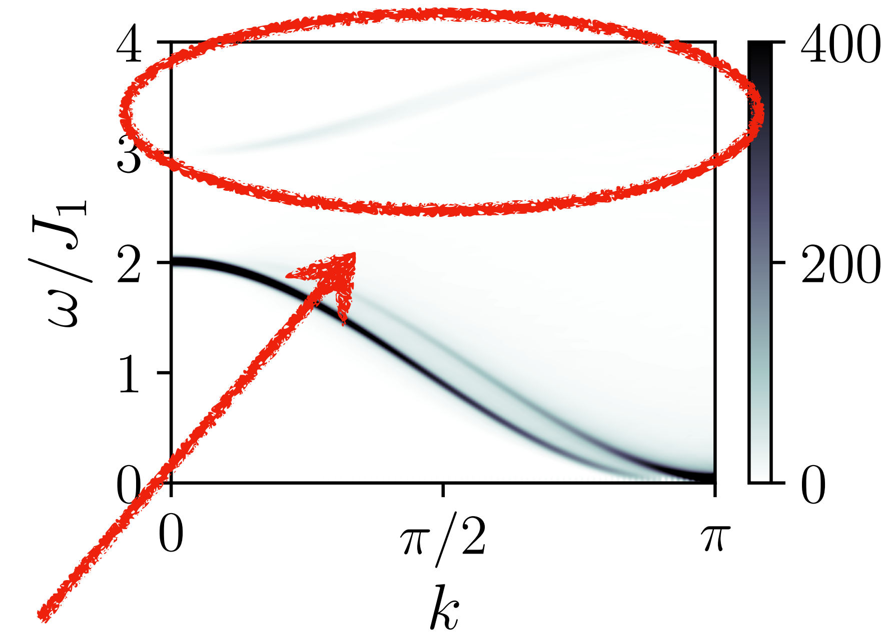
$J^z = J,\, M = M_{sat}$



Single magnon dispersion

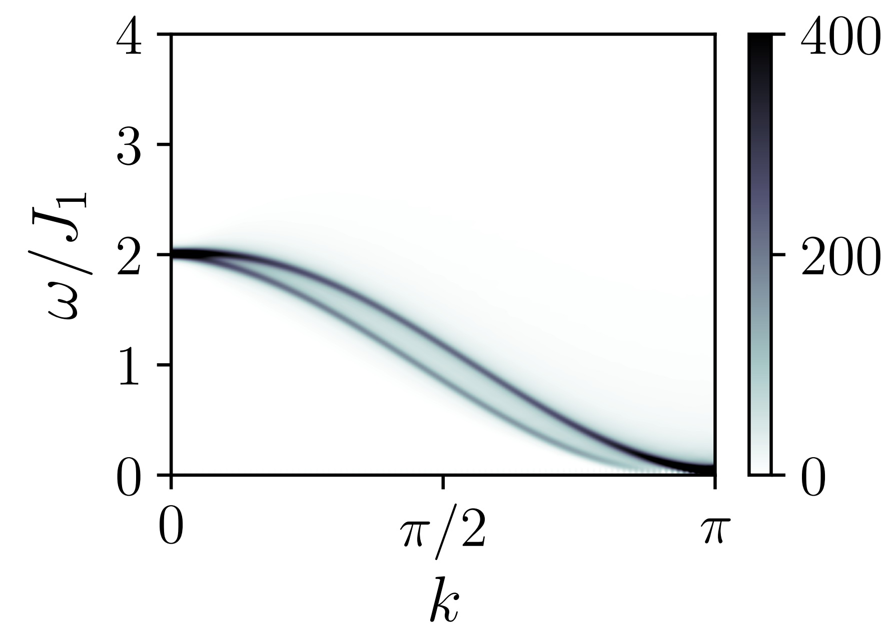
$$\omega(k) = J \cos k + B - J_z$$

$J^z = J,\, M = 0.9M_{sat}$



Interactions effect!

$J^z = 0,\, M = 0.9M_{sat}$



2-magnon (anti-)bound states

Solving an effective Schrodinger equation for 2-magnon states

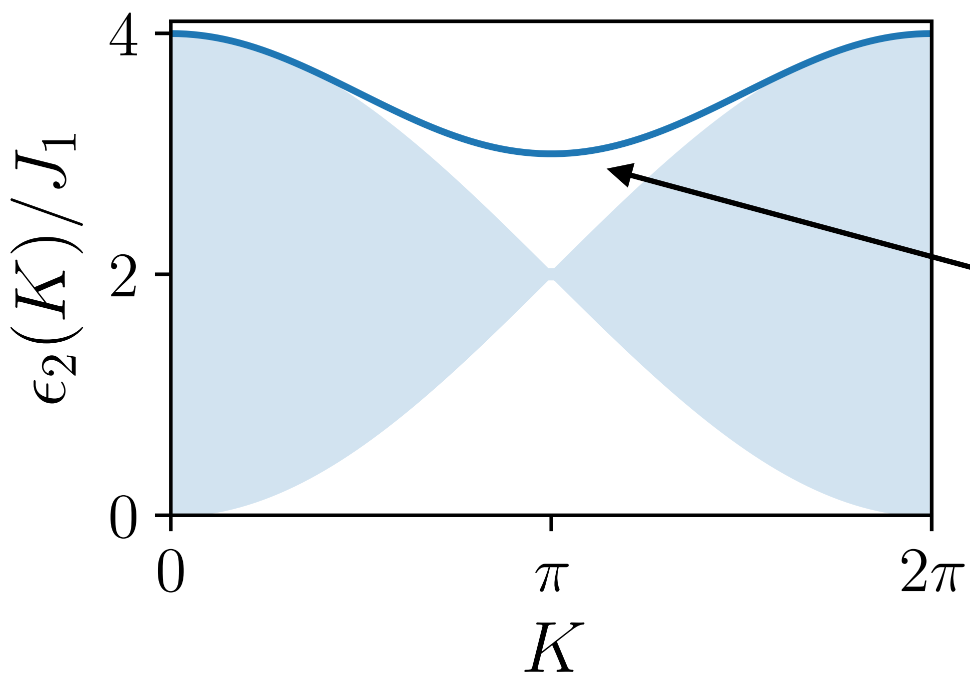
$$|2\rangle = \sum_{n,m} \Psi_{n,m} S_n^- S_m^- |0\rangle$$

fully polarized state

$$\Psi_{n,m} = e^{iK(n+m)/2} f(|n-m|)$$

center of mass momentum

2-magnon spectrum:

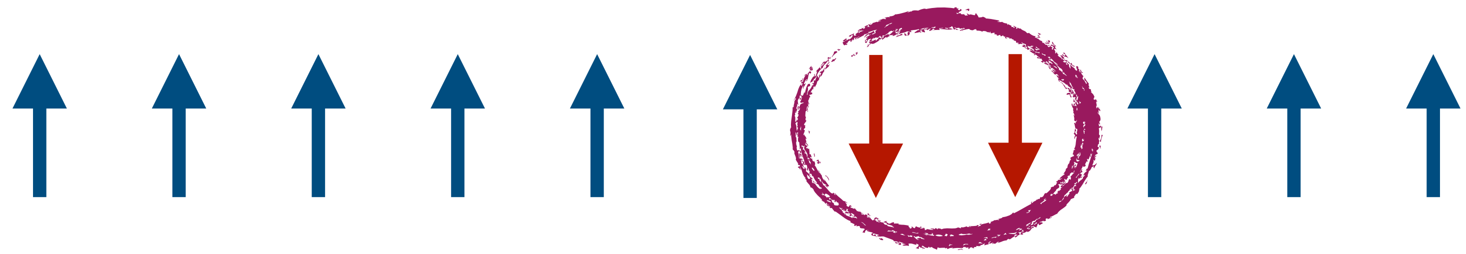


bound state above the 2-magnon continuum!

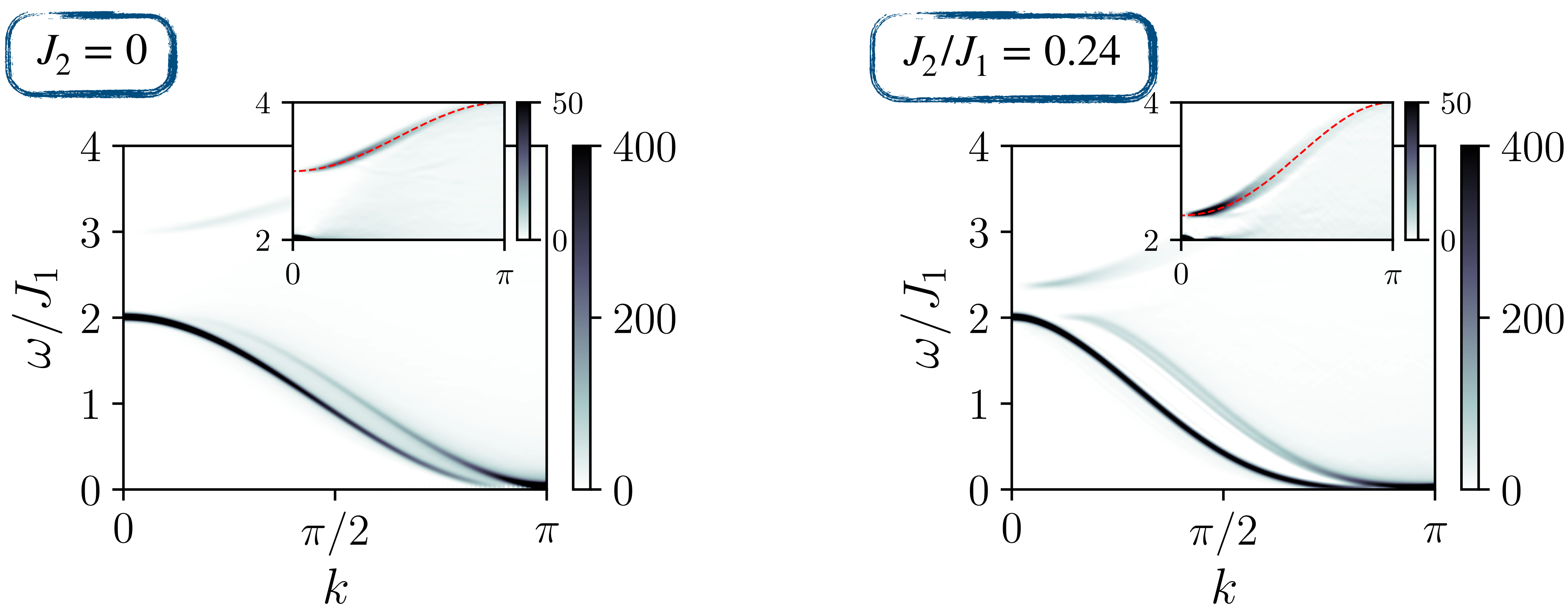
* works also for $J_2 \neq 0$

Dynamical susceptibility at large magnetization

$$\left\langle 2_{\pi+k} \right| \; S_k^- \; \left| 1_\pi \right\rangle$$



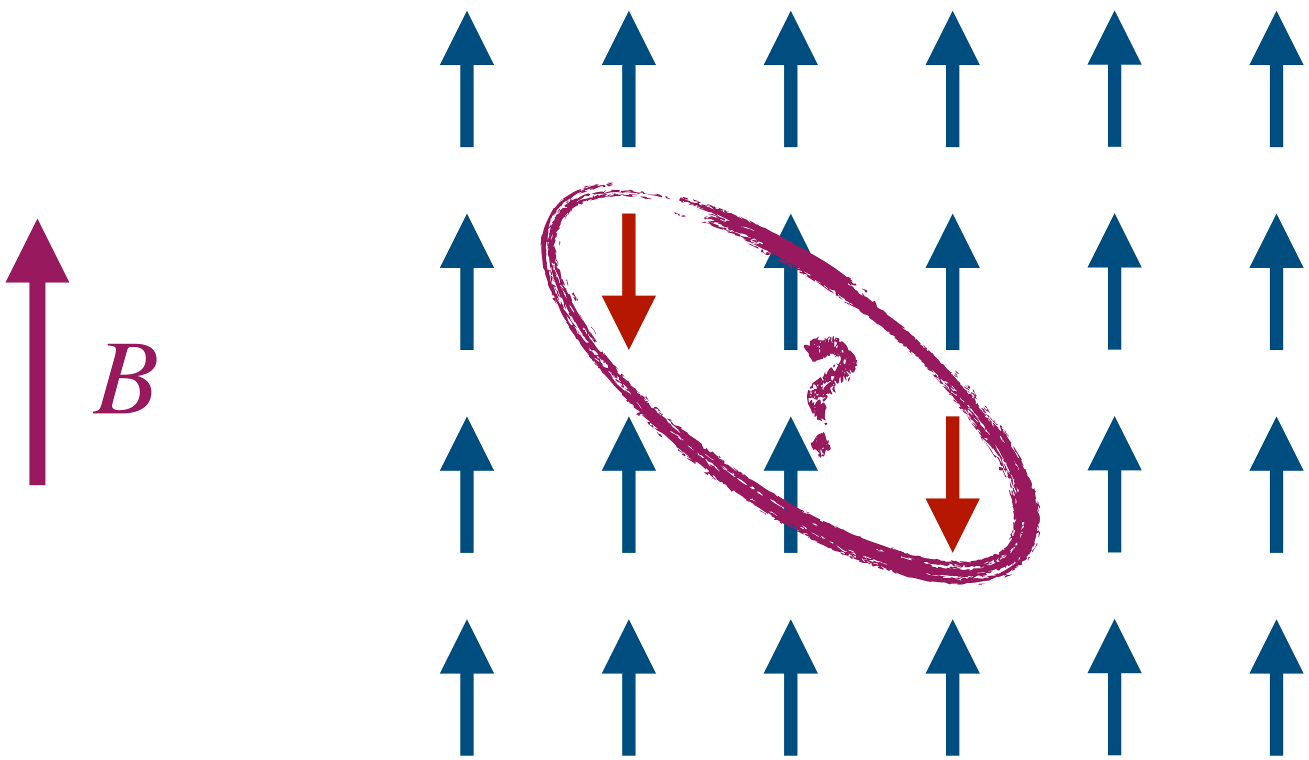
$$S^{+-}(k,\omega) = \left| \left\langle 2_{\pi+k} \right| S_k^- \left| 1_\pi \right\rangle \right|^2 \delta(\omega - \epsilon_2(\pi + k)) + \dots$$



In the Heisenberg limit these are Bethe string solutions!

see also Kohno PRL 2009, Yang et al. PRB 2019

Higher-dimensional antiferromagnets
in the large magnetization regime



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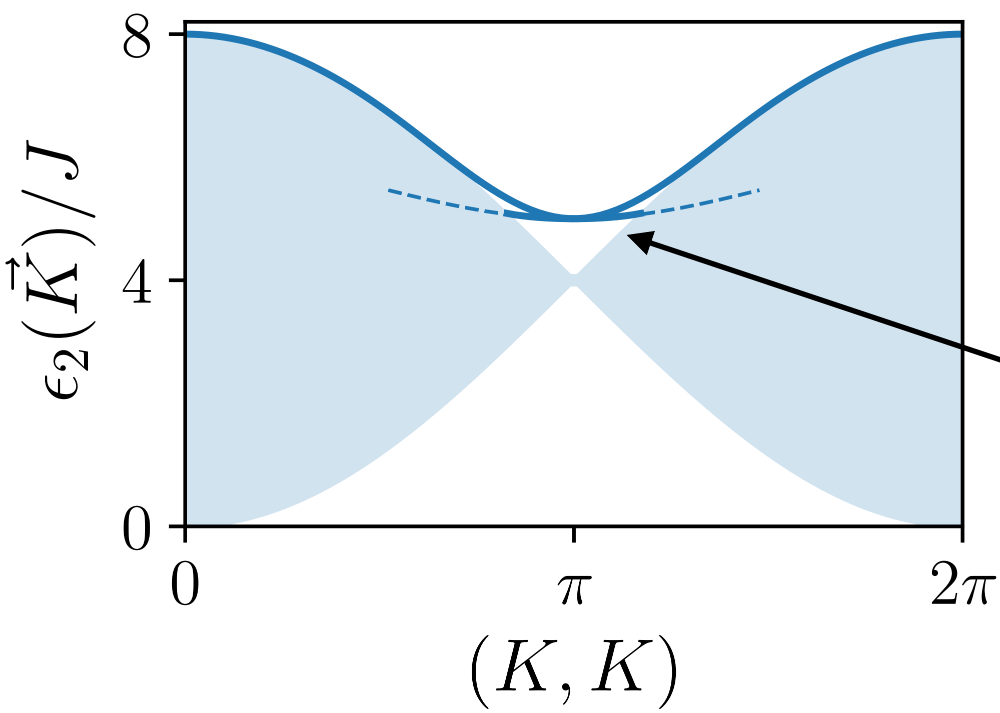
Oleg Starykh

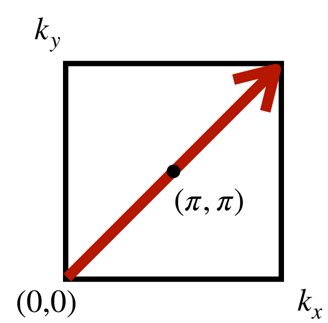
High field regime

Square lattice antiferromagnet

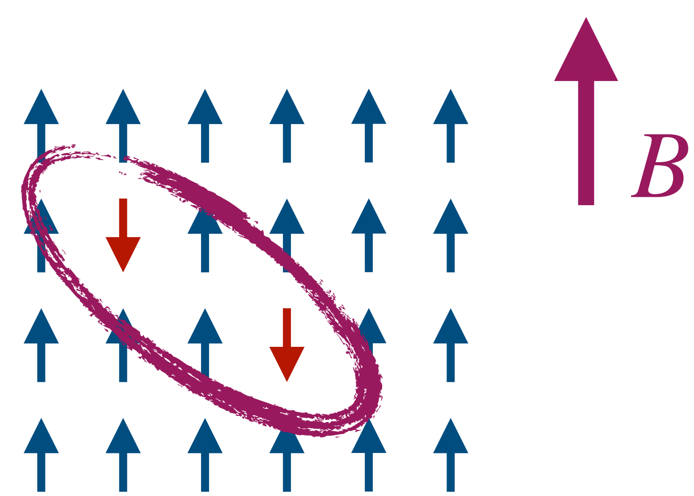
$$H = \sum_{<i,j>} J \overrightarrow{S}_i \cdot \overrightarrow{S}_j - B \sum_i S_i^z$$

$B > B_{\rm sat}$ - solving 2-magnon Schrodinger equation





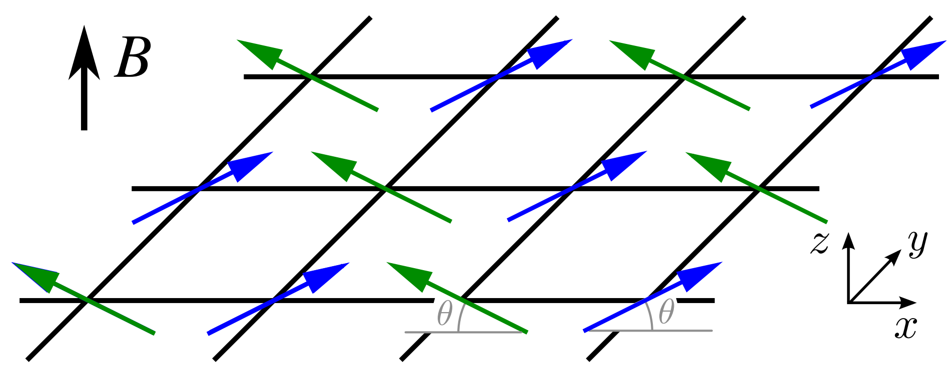
2-magnon bound states



High field regime

Below saturation $B < B_{\rm sat}$ magnons condense (long range order)

$\sin\theta = B/B_{sat}$



2D square lattice

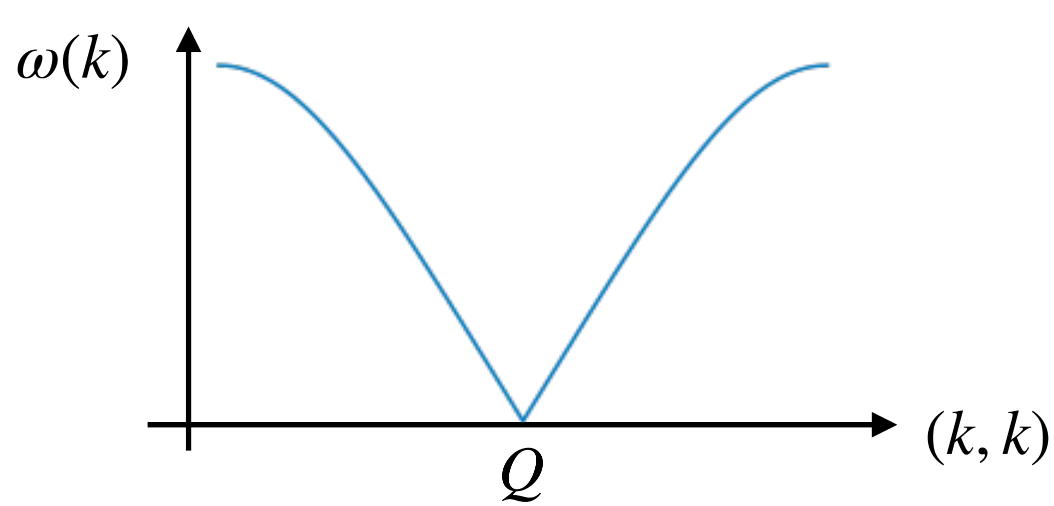
$\vec{Q} = (\pi, \pi)$

Mourigal, Zhitomirsky, Chernyshev, PRB 2010

Magnons dispersion within linear spin wave theory

$$H_{\rm LSWT} = \sum_k \omega(k) b_k^\dagger b_k \qquad \omega(k) = 2J\sqrt{(1+\gamma_k)(1-\cos 2\theta \gamma_k)}$$

$$\gamma_k = \frac{1}{2}\left(\cos k_x + \cos k_y\right)$$

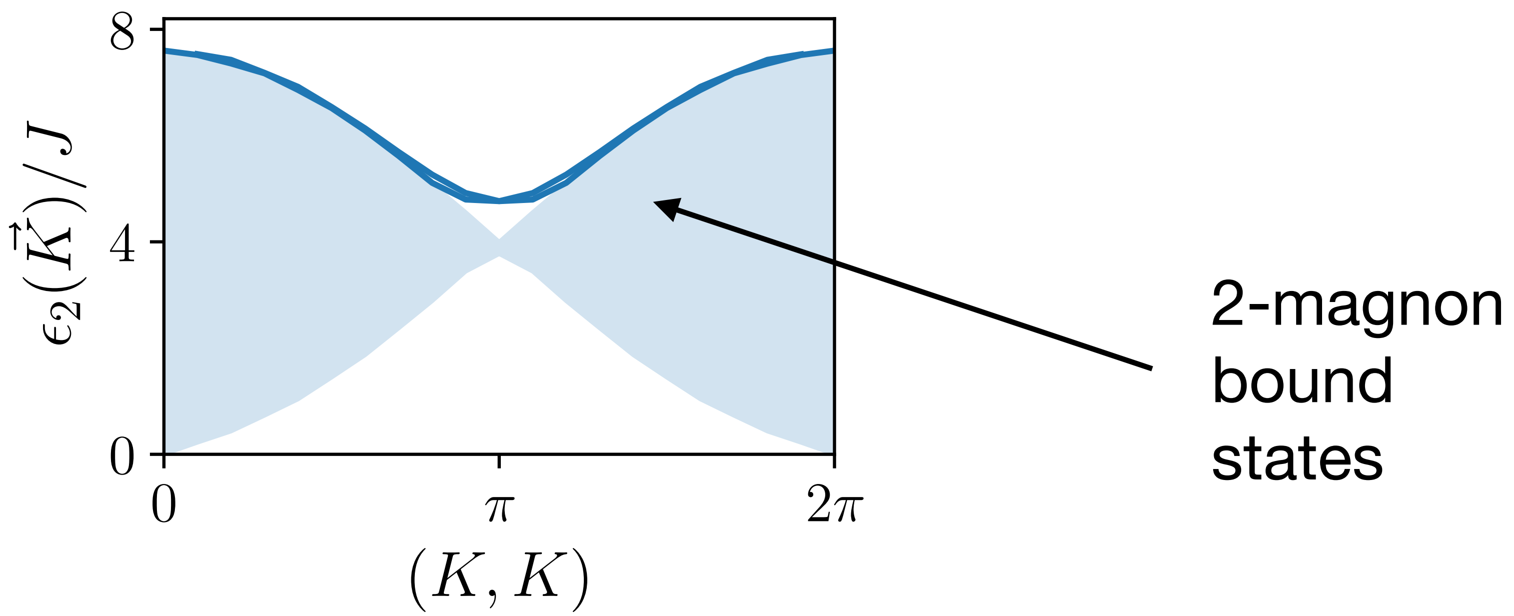


Beyond linear spin wave theory

Projecting interactions onto the 2-magnon subspace $\left|k_1, k_2\right\rangle = b^\dagger_{k_1} b^\dagger_{k_2} \left|GS\right\rangle$

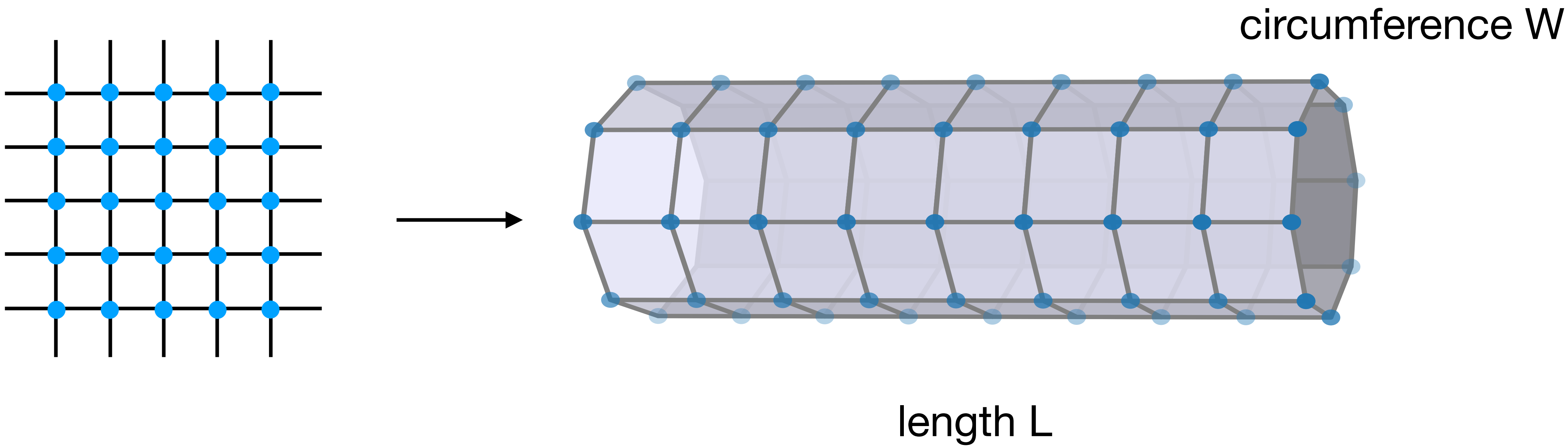
$$H_{\text{eff}} = \left\langle k_1', k_2' \right| H_{\text{int}} \left| k_1, k_2 \right\rangle \qquad H_{\text{int}} = \sum_{\tilde{k}_{1,..,4}} F(\tilde{k}_1, .., \tilde{k}_4)\delta(\tilde{k}_1 + \tilde{k}_2 - \tilde{k}_3 - \tilde{k}_4) b^\dagger_{\tilde{k}_1} b^\dagger_{\tilde{k}_2} b_{\tilde{k}_3} b_{\tilde{k}_4}$$

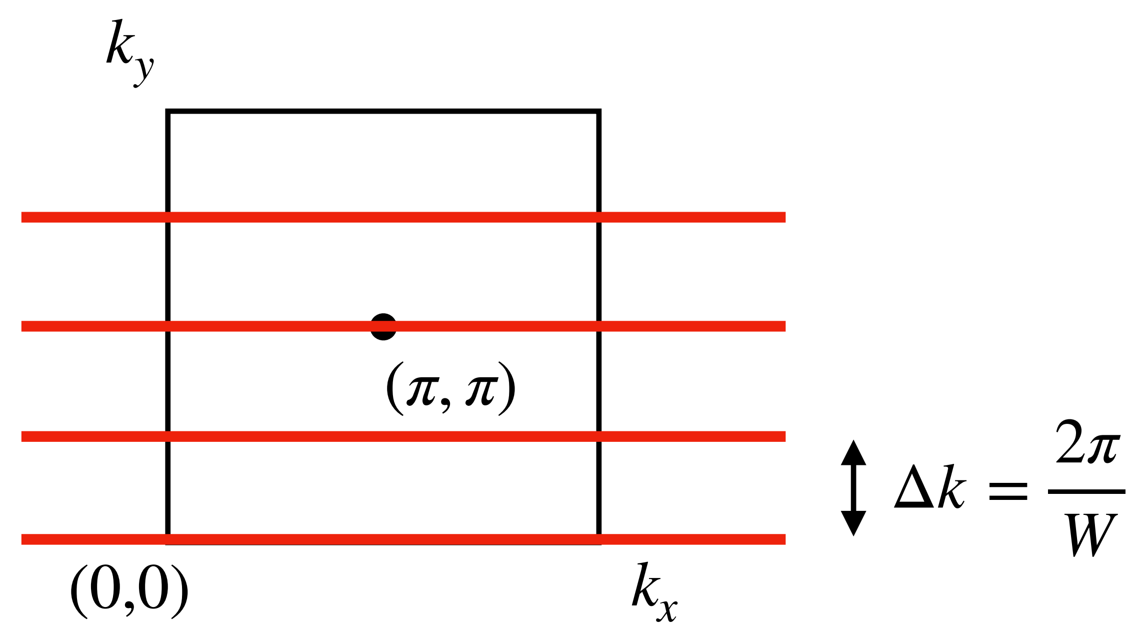
$B = 0.95 B_{sat}$



* possible decay channels of the 2-magnon state not taken into account

Probing the appearance of bound-states numerically



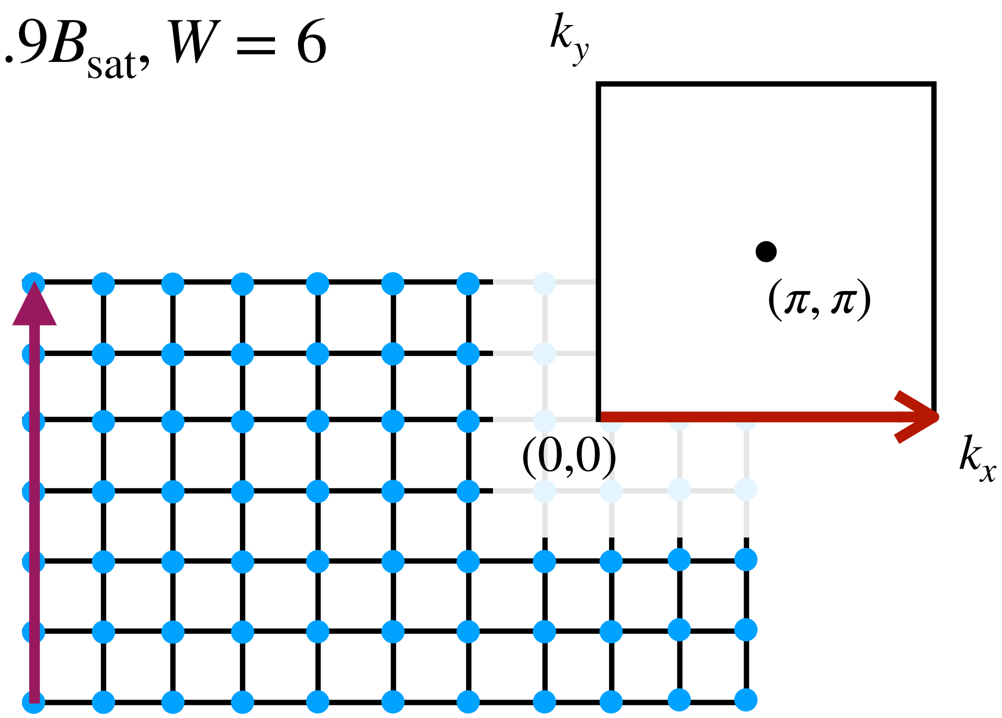


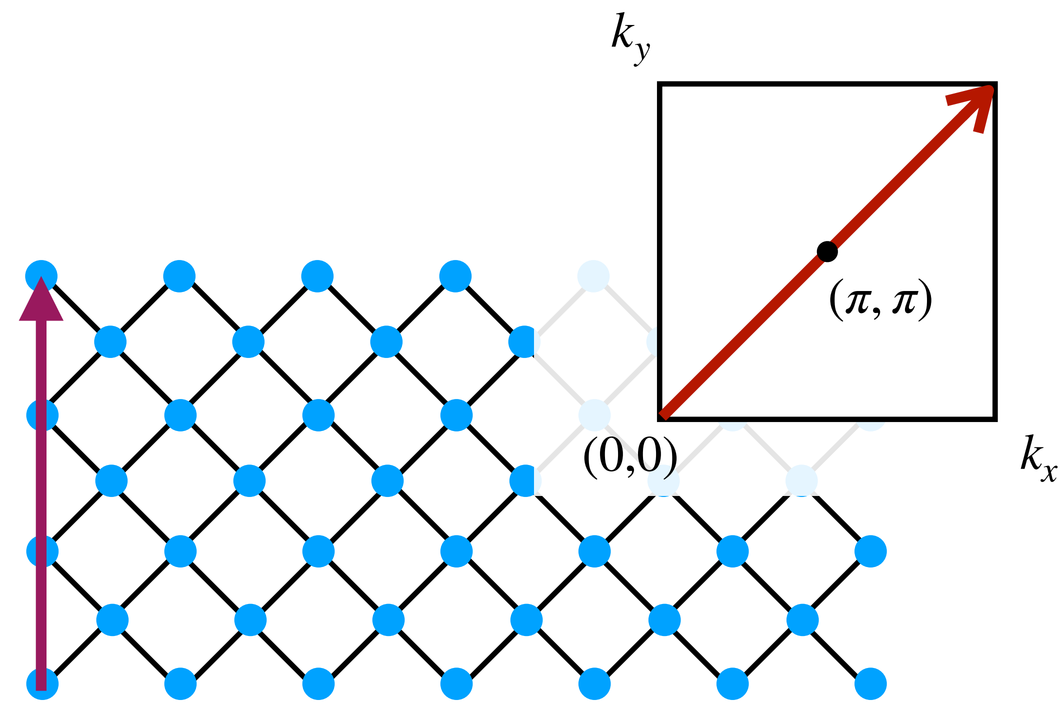
*using MPO representation of the time-evolution operator

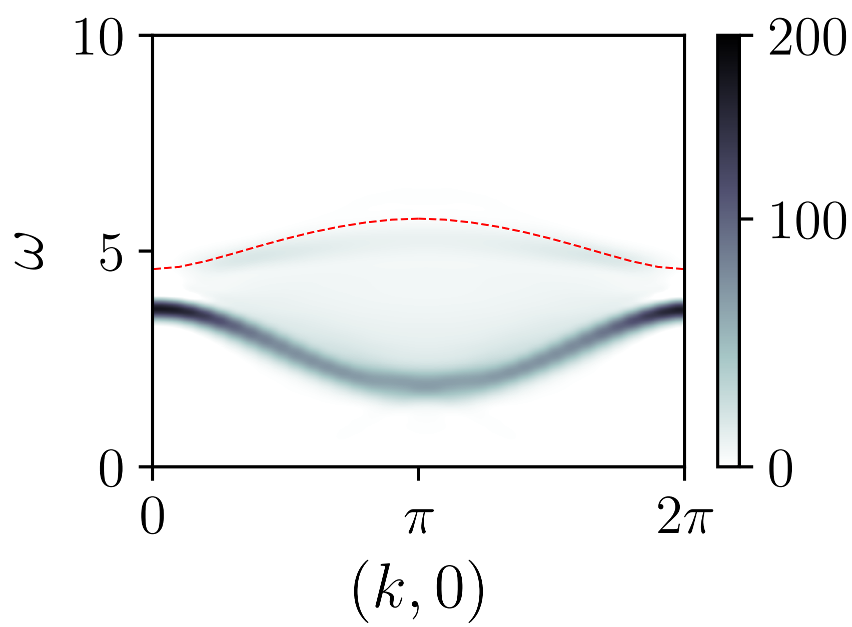
Zaletel et al. PRB 2015

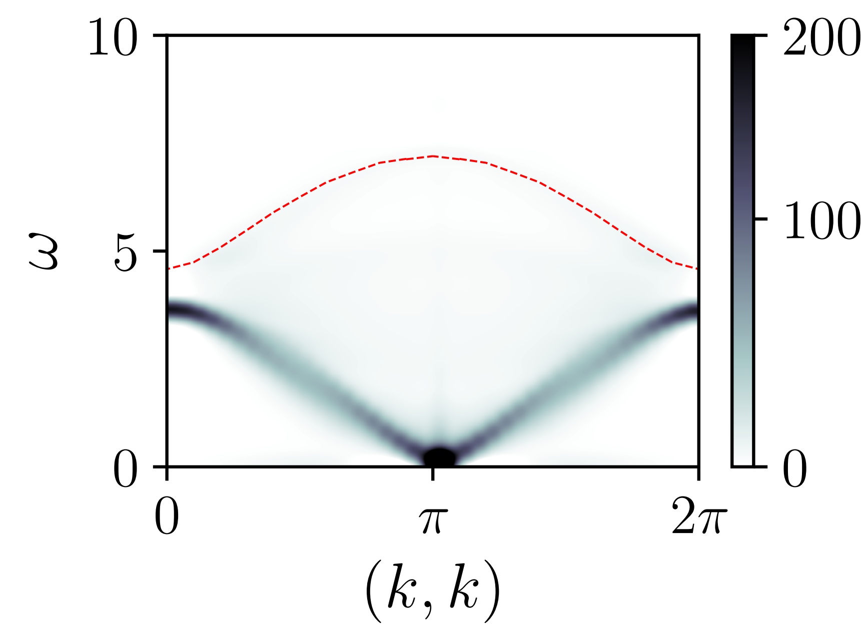
Dynamical susceptibility of square lattice AFM

$B = 0.9B_{\mathrm{sat}}, W = 6$

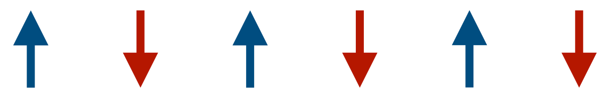


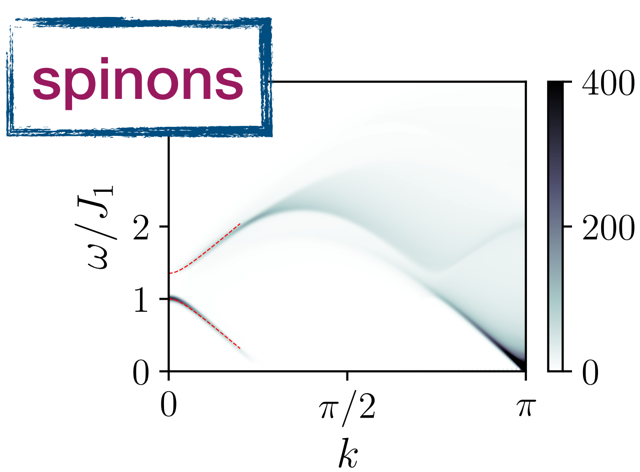


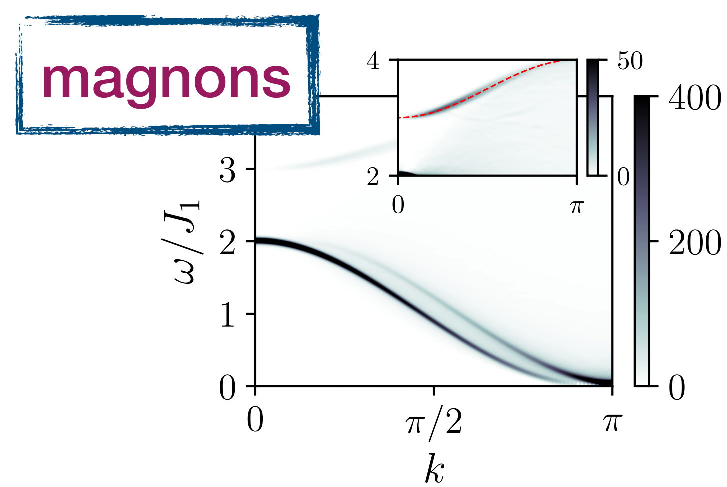




Summary

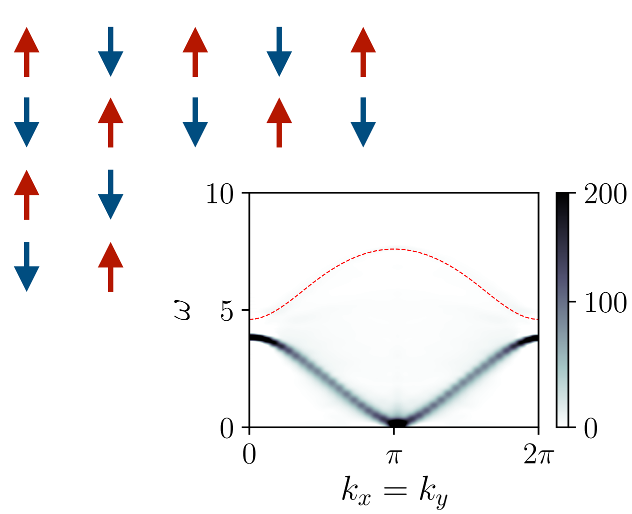
- 1D 
  - We have identified clear signatures of quasiparticle interactions in the transverse dynamical susceptibility





AK, Leon Balents,
Oleg Starykh,
PRL 2020

- Interactions between magnons give rise to bound states in higher-dimensional AFMs in high fields



- Scaling of intensity with system size
- Evolution of the bound state with magnetization
- Other lattices