IPAM, 4/23/2021

Chiral spin liquid phases in SU(N) quantum magnets

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Topological phases of matter

- Robustness of topological states

Topological quantum computation (no error correction needed !)

How to engineer topological phases ?

How to detect them: ground-state degeneracy
excitations
entanglement properties

Use topology !

Topological phases of matter

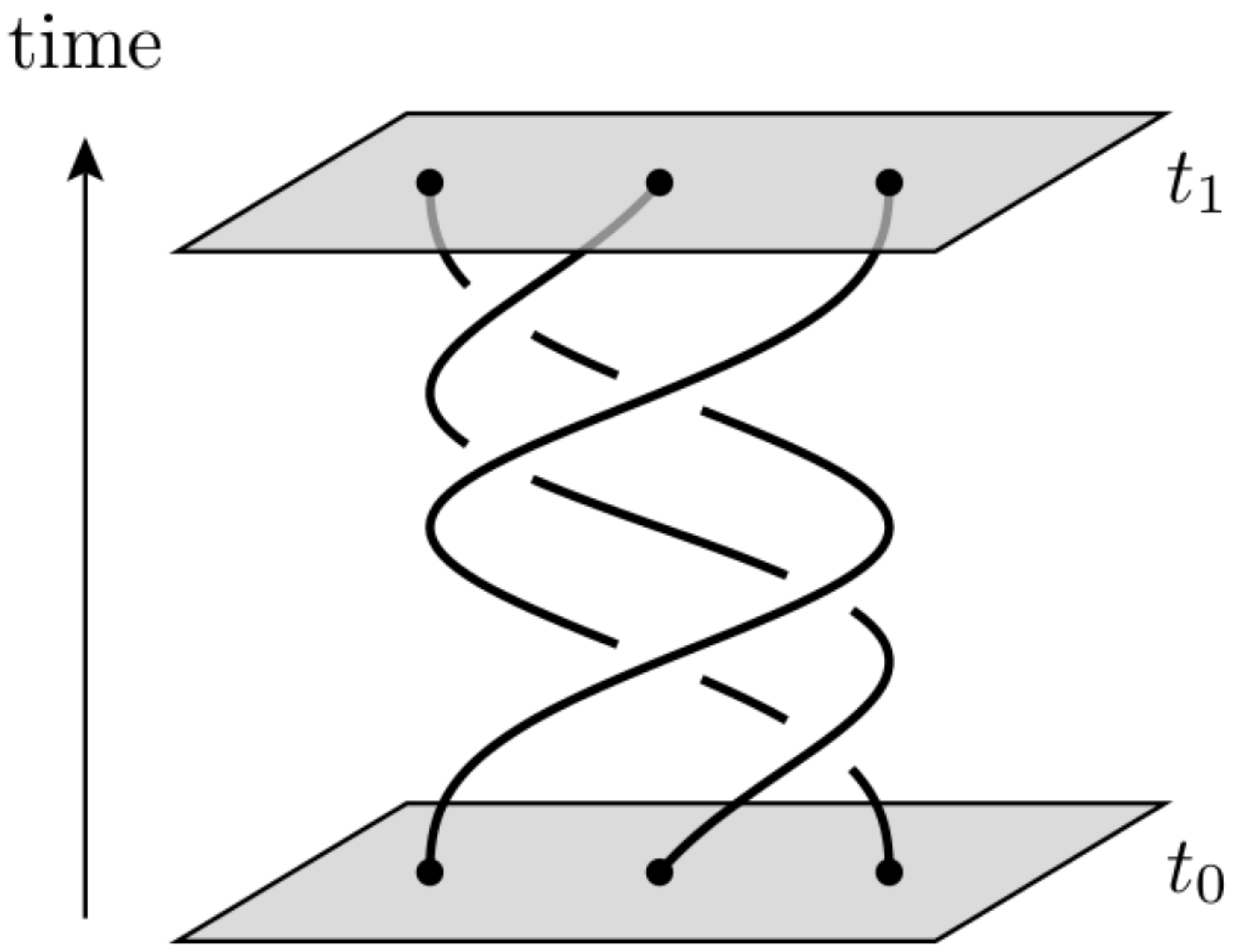
- Robustness of topological states

→ Topological quantum computation (no error correction needed !)

- Quasiparticles are anyons (fractional statistics) i.e. not necessarily bosons or fermions (spin statistics theorem breaks down in 2+1D)

- Excitations can be abelian or not

topological quantum field theory
(e.g. Chern-Simons),
braid group, fusion rules…



Chiral topological spin liquids

Chiral topological phase is found in the fractional quantum Hall (FQH) effect

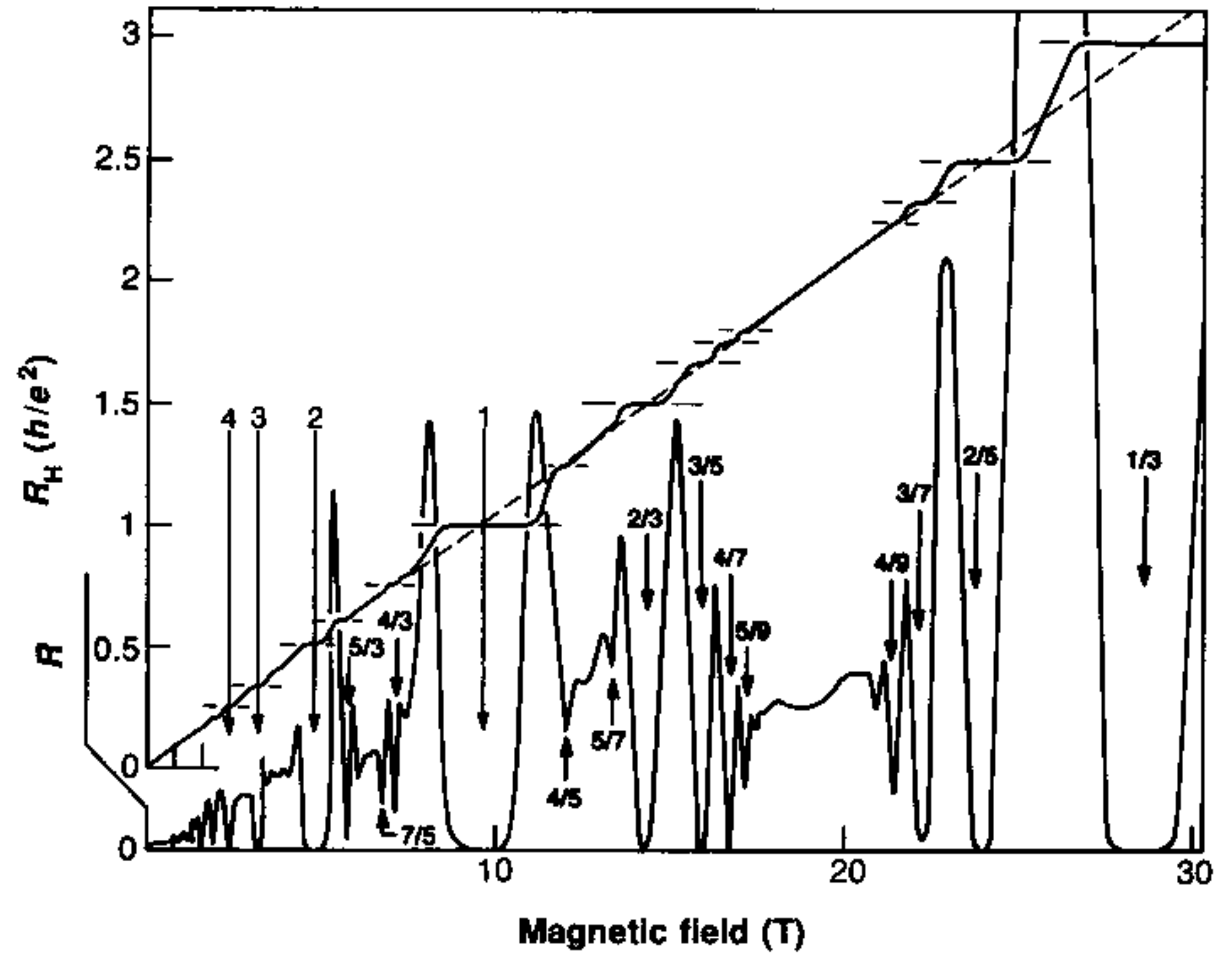


Fig. 2. Composite view showing the Hall resistance R_H and longitudinal

topological phases, exotic excitations
(**abelian or not**)
unconventional superconductor when doped

Is it possible to reach the same physics without Landau levels, on a lattice ?

See e.g. talk by Cécile Repellin

Mimic an effective magnetic field, flat bands etc.

Fractional Chern insulators

Look for lattice models with similar wavefunctions

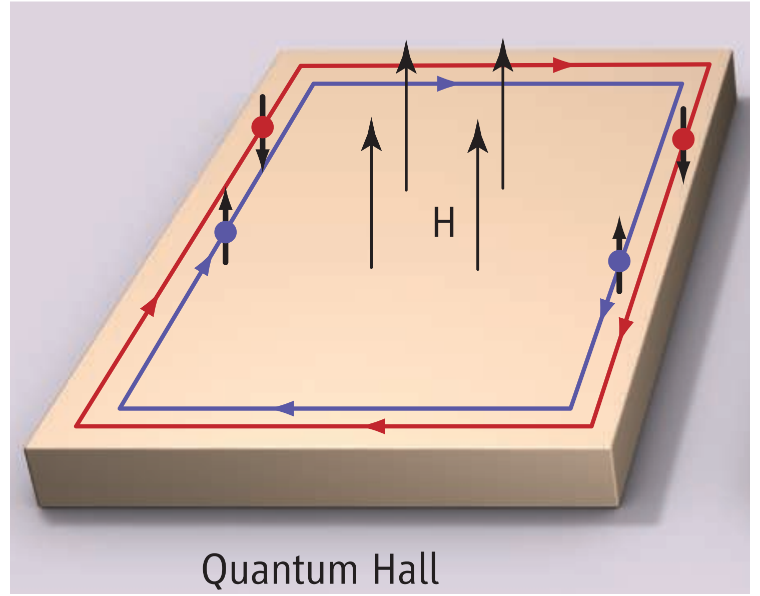
OUTLINE

- Introduction: chiral spin liquids in a nutshell
- Combined numerical methods of a family of SU(N) models
- Conclusions and outlook

Collaborators/Refs:

- Ji-Yao Chen, L. Vanderstraeten, S. Capponi, D. Poilblanc, Phys. Rev. B 98, 184409 (2018)
- Ji-Yao Chen, S. Capponi, A. Wietek, M. Mambrini, N. Schuch, D. Poilblanc, Phys. Rev. Lett. 125, 017201 (2020)
- **Ji-Yao Chen**, **Jheng-Wei Li**, **Pierre Nataf**, Sylvain Capponi, Matthieu Mambrini, Keisuke Totsuka, Hong-Hao Tu, Andreas Weichselbaum, Jan von Delft, Didier Poilblanc, in preparation

Chiral spin liquids (CSL)
= lattice analogue of FQH states



Low-energy physics described by 2+1 Chern-Simons theory

$\nu = \frac{1}{2}$ FQH state

lattice spin S=1/2 model

incompressible (gapped) in the bulk

same

charged e/2 fractional excitation

neutral s=1/2 fractional excitation

robust gapless chiral edge states

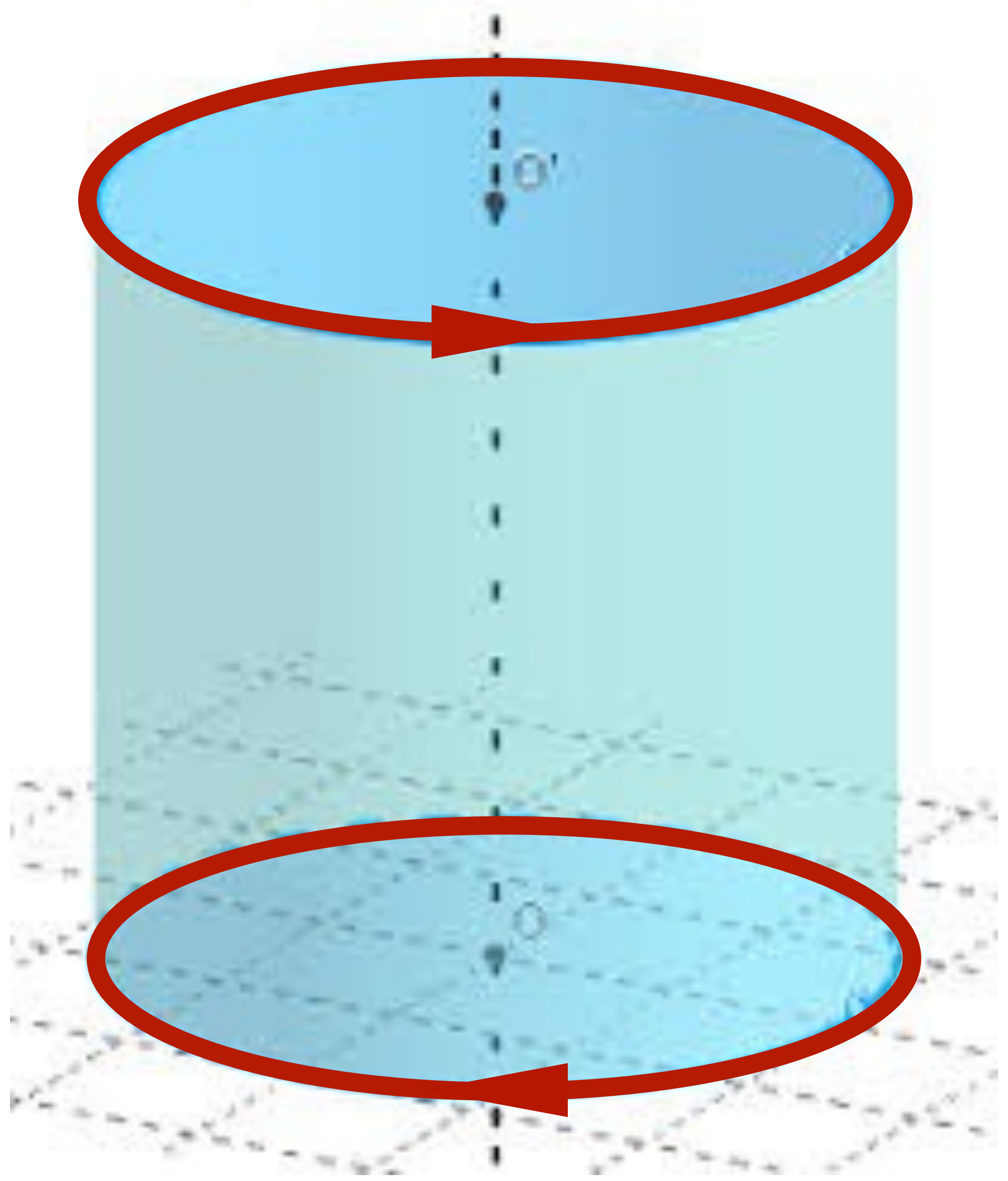
same

$SU(2)_1$ CFT

triangular lattice: Kalmeyer-Laughlin, 1987

chiral spin liquids = lattice analogs of FQH states

These states break time-reversal symmetry (T) and parity (P)



Protected edge modes
described by $SU(2)_1$ CFT

"Long range
entanglement"

Xiao-Gang Wen

Tensor networks formalism well suited

Dubail-Read 2015

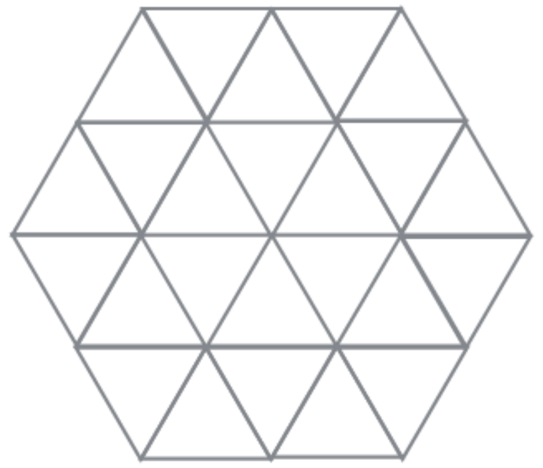
No-go theorem for a gaussian PEPS to have a bulk gap

Abelian CSL in spin-1/2 SU(2) models on frustrated lattices

breaks T

S=1/2 on triangular lattice

$$H = J_1 \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \vec{S}_i \cdot \vec{S}_j + J_\chi \sum_{\triangle/\triangledown} (\vec{S}_i \times \vec{S}_j) \cdot \vec{S}_k$$



PHYSICAL REVIEW B **96**, 075116 (2017)

Global phase diagram and quantum spin liquids in a spin-$\frac{1}{2}$ triangular antiferromagnet

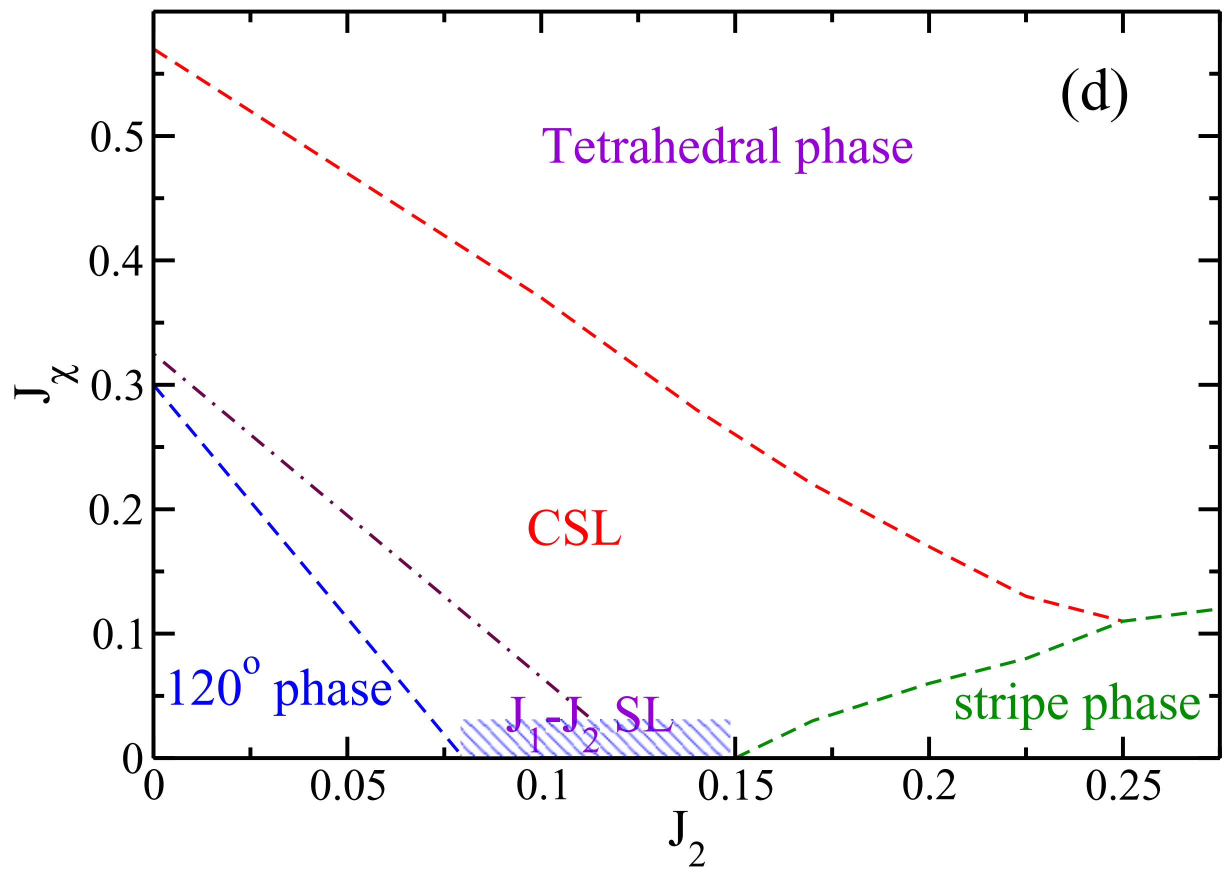
Shou-Shu Gong,[1] W. Zhu,[2] J.-X. Zhu,[2,3] D. N. Sheng,[4] and Kun Yang[5]

PHYSICAL REVIEW B **95**, 035141 (2017)

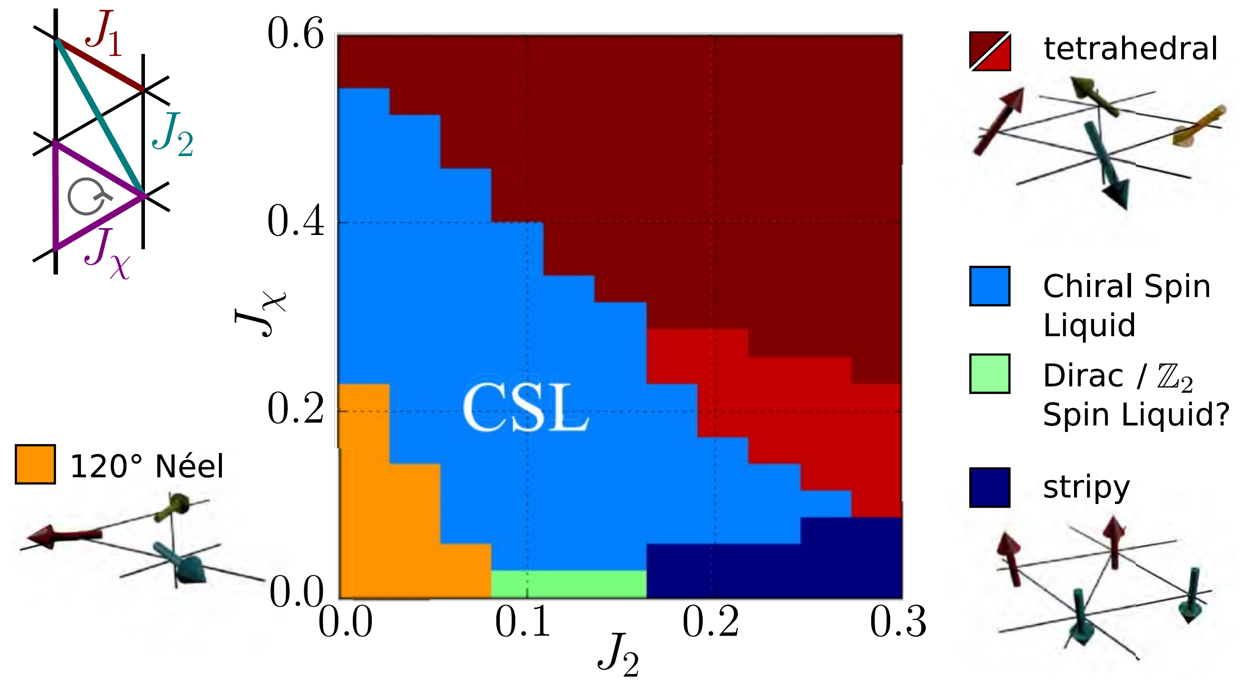
Chiral spin liquid and quantum criticality in extended $S = \frac{1}{2}$ Heisenberg models on the triangular lattice

Alexander Wietek* and Andreas M. Läuchli

DMRG



ED

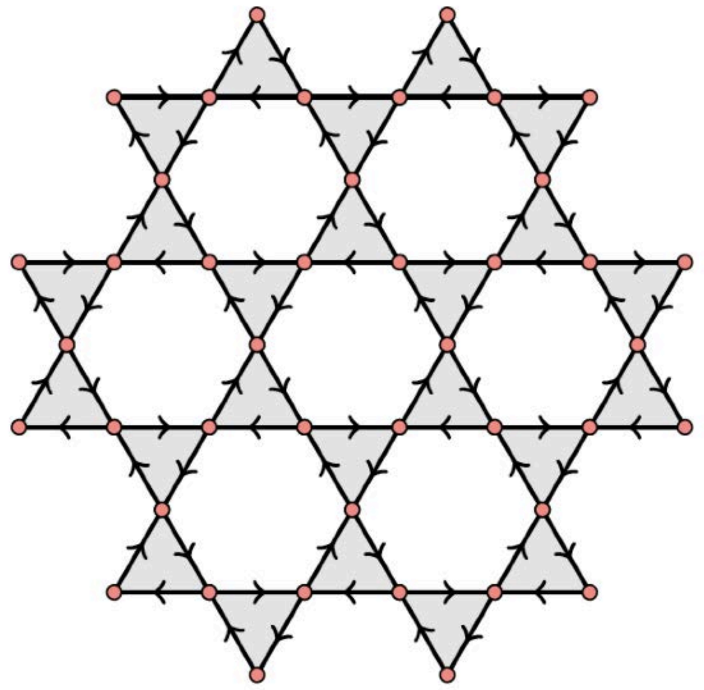


S=1/2 on kagome lattice

DMRG

Chiral spin liquid and emergent anyons in a Kagome lattice Mott insulator

B. Bauer[1], L. Cincio[2], B.P. Keller[3], M. Dolfi[4], G. Vidal[2], S. Trebst[5] & A.W.W. Ludwig[3]



S=1/2 on frustrated square lattice

PHYSICAL REVIEW B **96**, 121118(R) (2017)

PEPS

Investigation of the chiral antiferromagnetic Heisenberg model using projected entangled pair states

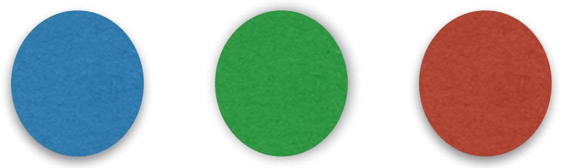
Didier Poilblanc

Abelian CSL in SU(N) models

Enlarging SU(2) to SU(N) is known to destabilize magnetic order

Several irreps (corresponding to Young's tableaux)

This talk: □

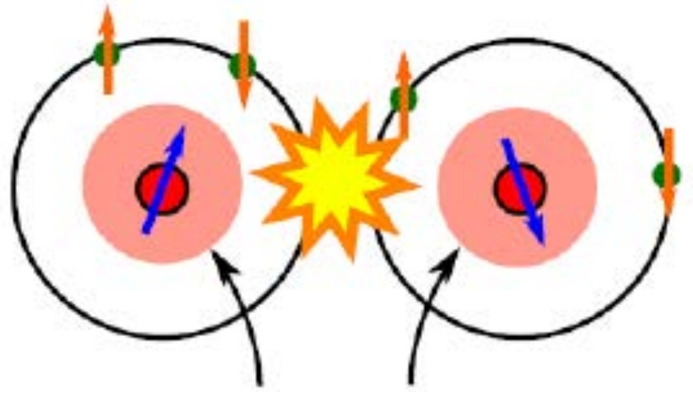


N "colors"

But SU(N) symmetry is also:

- approximately realized on condensed matter:
  SU(4) spin-orbital, SU(4) in graphene/TBG/moiré materials etc.

- exactly realized for alkaline-earth ultracold atoms since nuclear spin I is decoupled from electronic state: ^{173}Yb SU(6), ^{87}Sr SU(10)



PRL 105, 030402 (2010) PHYSICAL REVIEW LETTERS week ending 16 JULY 2010

Degenerate Fermi Gas of ^{87}Sr

B. J. DeSalvo, M. Yan, P. G. Mickelson, Y. N. Martinez de Escobar, and T. C. Killian

Selected for a Viewpoint in *Physics*
PRL 105, 190401 (2010) PHYSICAL REVIEW LETTERS week ending 5 NOVEMBER 2010

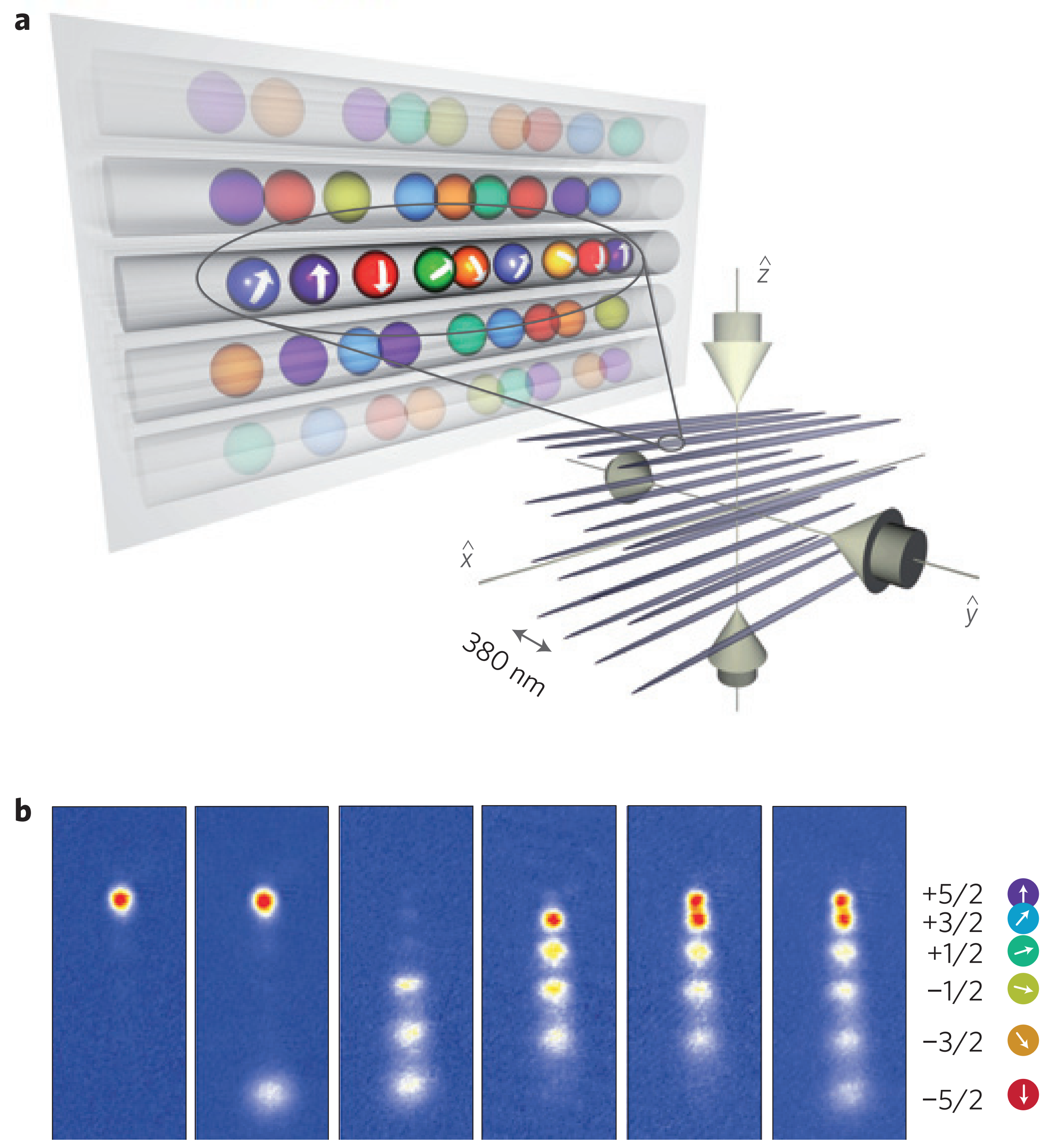
Realization of a SU(2) × SU(6) System of Fermions in a Cold Atomic Gas

Shintaro Taie,[1,*] Yosuke Takasu,[1] Seiji Sugawa,[1] Rekishu Yamazaki,[1,2] Takuya Tsujimoto,[1] Ryo Murakami,[1] and Yoshiro Takahashi[1,2]

LETTERS
PUBLISHED ONLINE: 2 FEBRUARY 2014 | DOI: 10.1038/NPHYS2878
nature physics

A one-dimensional liquid of fermions with tunable spin

Guido Pagano[1,2], Marco Mancini[1,3], Giacomo Cappellini[1], Pietro Lombardi[1,3], Florian Schäfer[1], Hui Hu[4], Xia-Ji Liu[4], Jacopo Catani[1,5], Carlo Sias[1,5], Massimo Inguscio[1,3,5] and Leonardo Fallani[1,3,5★]



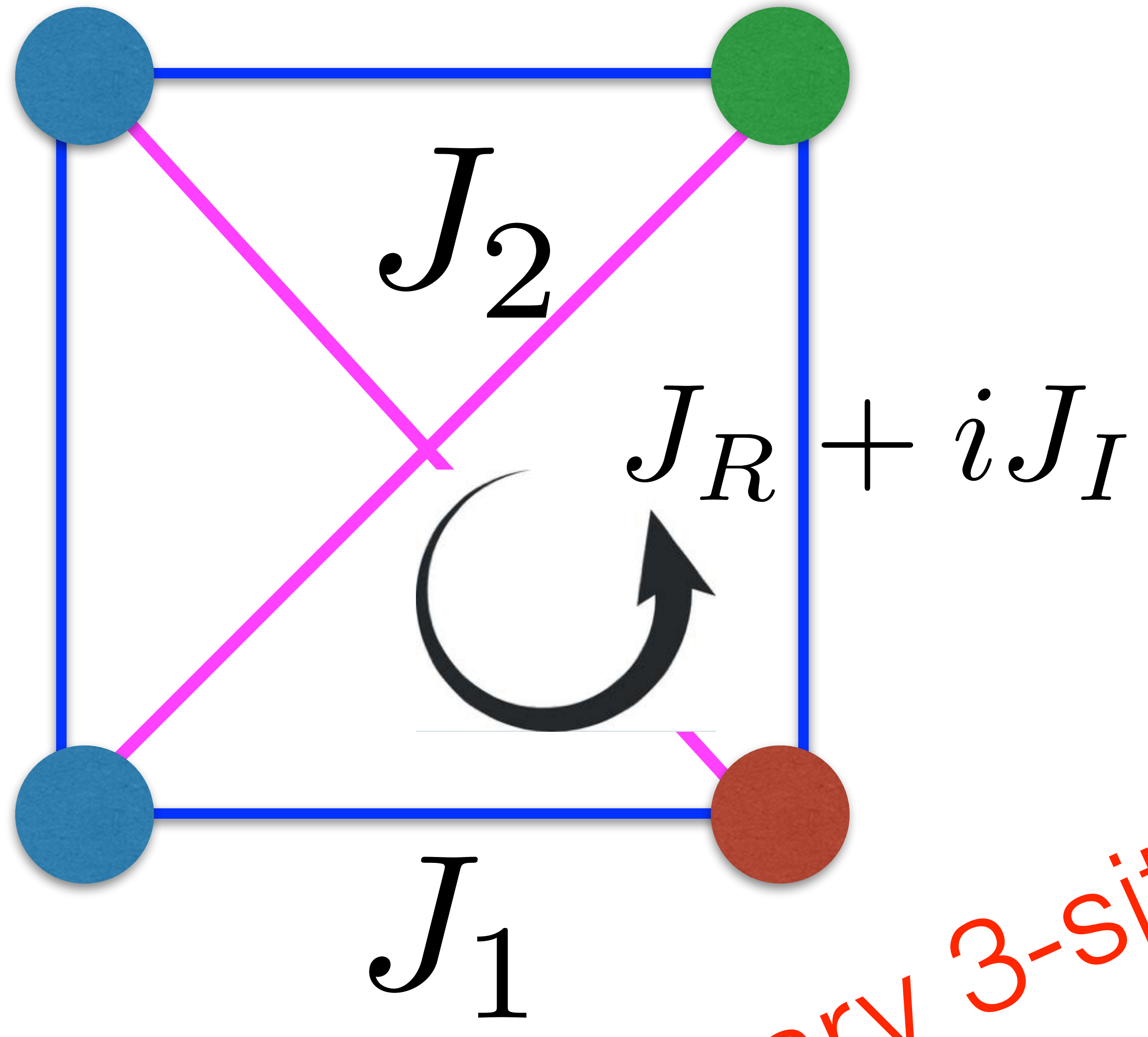
Models and definitions

Model defined in terms of permutation (SU(N) symmetry)

$$H = J_1 \sum_{\langle i,j \rangle} P_{ij} + J_2 \sum_{\langle\langle k,l \rangle\rangle} P_{kl} + J_{\mathrm{R}} \sum_{\triangle ijk} (P_{ijk} + P_{ijk}^{-1}) + iJ_{\mathrm{I}} \sum_{\triangle ijk} (P_{ijk} - P_{ijk}^{-1})$$

Square lattice, C4 symmetry

$$J_1 = 2J_2 = \frac{4}{3}\cos\theta \sin\phi,$$
$$J_{\mathrm{R}} = \cos\theta\cos\phi,$$
$$J_{\mathrm{I}} = \sin\theta.$$



imaginary 3-site permutation explicitly breaks T and P

PRL 117, 167202 (2016) PHYSICAL REVIEW LETTERS week ending 14 OCTOBER 2016

Chiral Spin Liquids in Triangular-Lattice SU(N) Fermionic Mott Insulators with Artificial Gauge Fields

Pierre Nataf,[1] Miklós Lajkó,[2] Alexander Wietek,[3] Karlo Penc,[4,5] Frédéric Mila,[1] and Andreas M. Läuchli[3]

Numerical methods for SU(N)

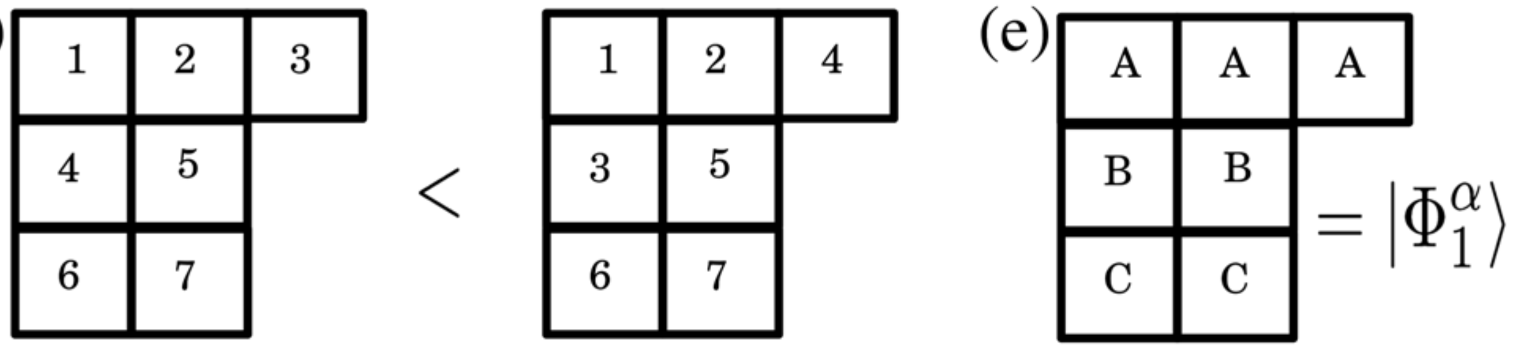
... an introduction

Analytics: large-N, mean-field, parton wavefunctions

- **Exact diagonalization** (U(1)+lattice symmetries or SU(N) symmetry)

using standard Young tableaux

(d) (e)



PRL **113**, 127204 (2014) PHYSICAL REVIEW LETTERS week ending 19 SEPTEMBER 2014

Exact Diagonalization of Heisenberg SU(N) Models

Pierre Nataf and Frédéric Mila

$SU(N)$	n	$f^{[k,\ldots,k]}$	$\frac{(n-1)!}{k!^N}$	$\mathcal{E}_{GS}$
$SU(5)$	25 (tilted)	701149020	2.5×10^{13}	-1.154324
$SU(5)$	25 (5×5)	701149020	2.5×10^{13}	-1.164712
$SU(5)$	20	1662804	1.5×10^{10}	-1.215377
$SU(8)$	16	1430	5.1×10^{9}	-1.572223
$SU(10)$	20	16796	1.2×10^{14}	-1.589218

using SU(N) U(1) (N-1 Cartan)

Numerical methods for SU(N)

... an introduction

Analytics: large-N, mean-field, parton wavefunctions

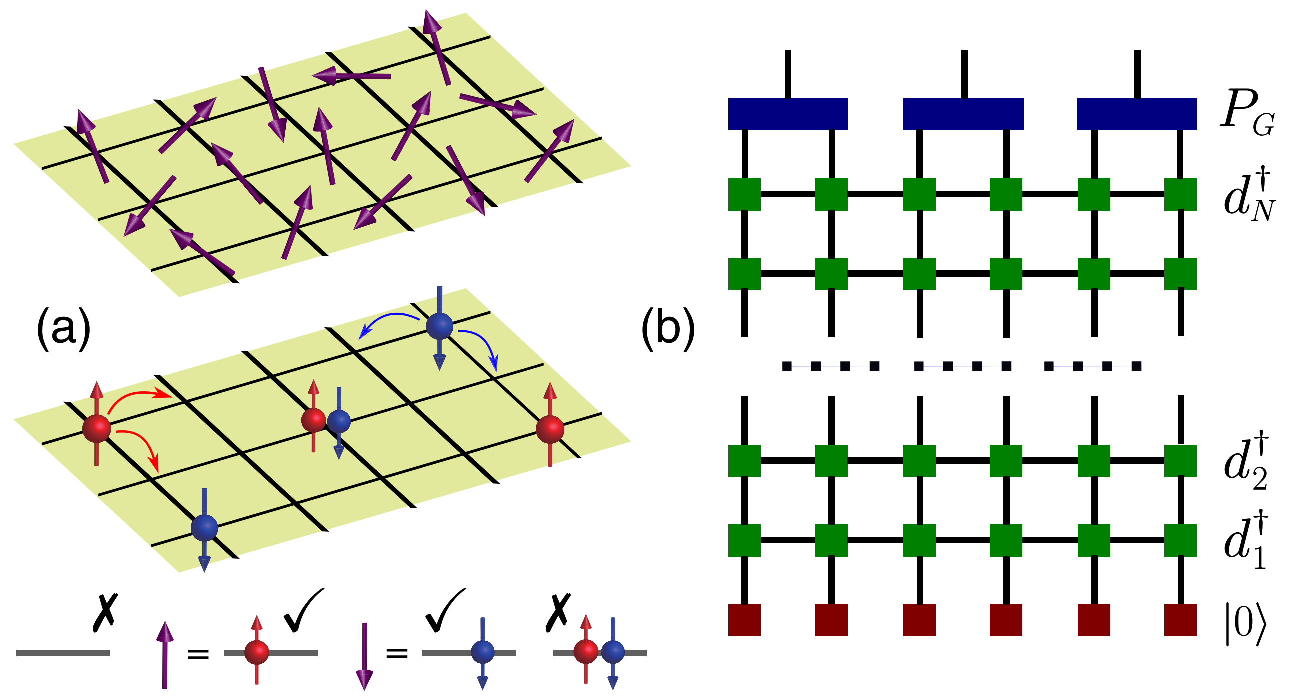
- Exact diagonalization (U(1)+lattice symmetries or SU(N) symmetry)

- **DMRG** (U(1) or SU(N) symmetry) + parton wavefunction

Projected Fermi sea

has a tensor network representation
MPO: D=2

using MPO-MPS compression -> MPS



PHYSICAL REVIEW LETTERS **124**, 246401 (2020)

Tensor Network Representations of Parton Wave Functions

Ying-Hai Wu,[1] Lei Wang,[2,3] and Hong-Hao Tu[4,*]

Numerical methods for SU(N)

... an introduction

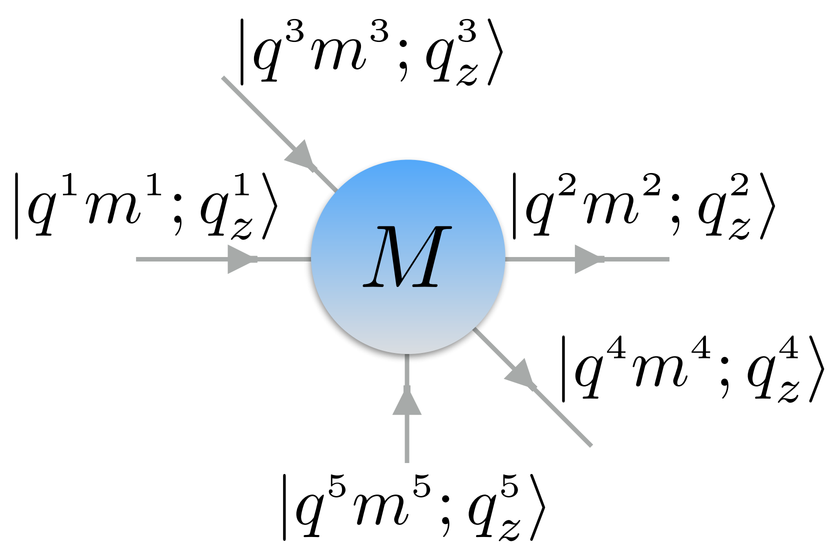
Analytics: large-N, mean-field, parton wavefunctions

- Exact diagonalization (U(1)+lattice symmetries or SU(N) symmetry)

- DMRG (U(1) or SU(N) symmetry) + parton wavefunction

- PEPS using SU(N) symmetric tensors + lattice point-group symmetry

Both DMRG and PEPS can use SU(N) symmetry, e.g. QSpace library



Andreas Weichselbaum

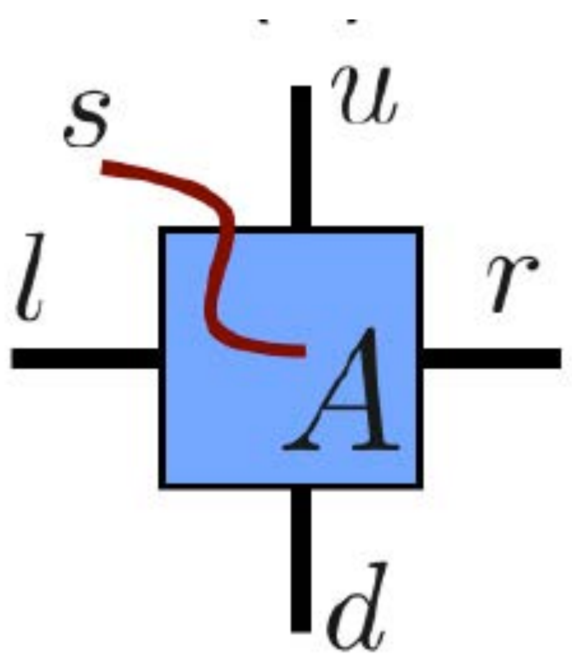
Chiral spin liquid with PEPS

Using a classification of SU(2)-invariant PEPS

PHYSICAL REVIEW B **94**, 205124 (2016)

Systematic construction of spin liquids on the square lattice from tensor networks with SU(2) symmetry

Matthieu Mambrini,[1] Román Orús,[2] and Didier Poilblanc[1]



* virtual space : $V = S_1 \oplus S_2 \oplus \cdots S_p$
* Irreps of point group
  (C4v for square lattice)

Chiral PEPS ansatz: $A = A_R + iA_I$

$$A_R = \sum_\alpha \lambda_\alpha A_\alpha^{(A_1)} \qquad A_I = \sum_\beta \gamma_\beta A_\beta^{(A_2)}$$

Different irreps !

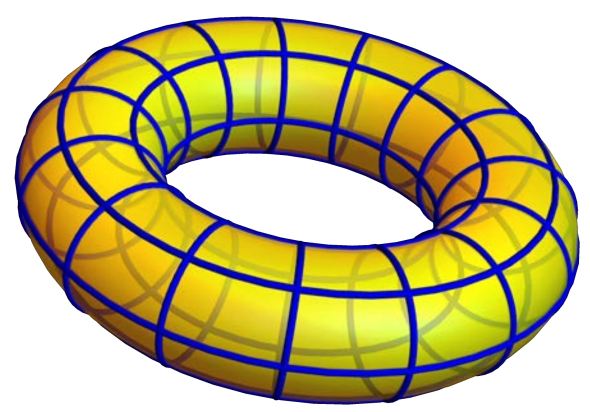
PHYSICAL REVIEW B **96**, 121118(R) (2017)

Investigation of the chiral antiferromagnetic Heisenberg model using projected entangled pair states

Didier Poilblanc

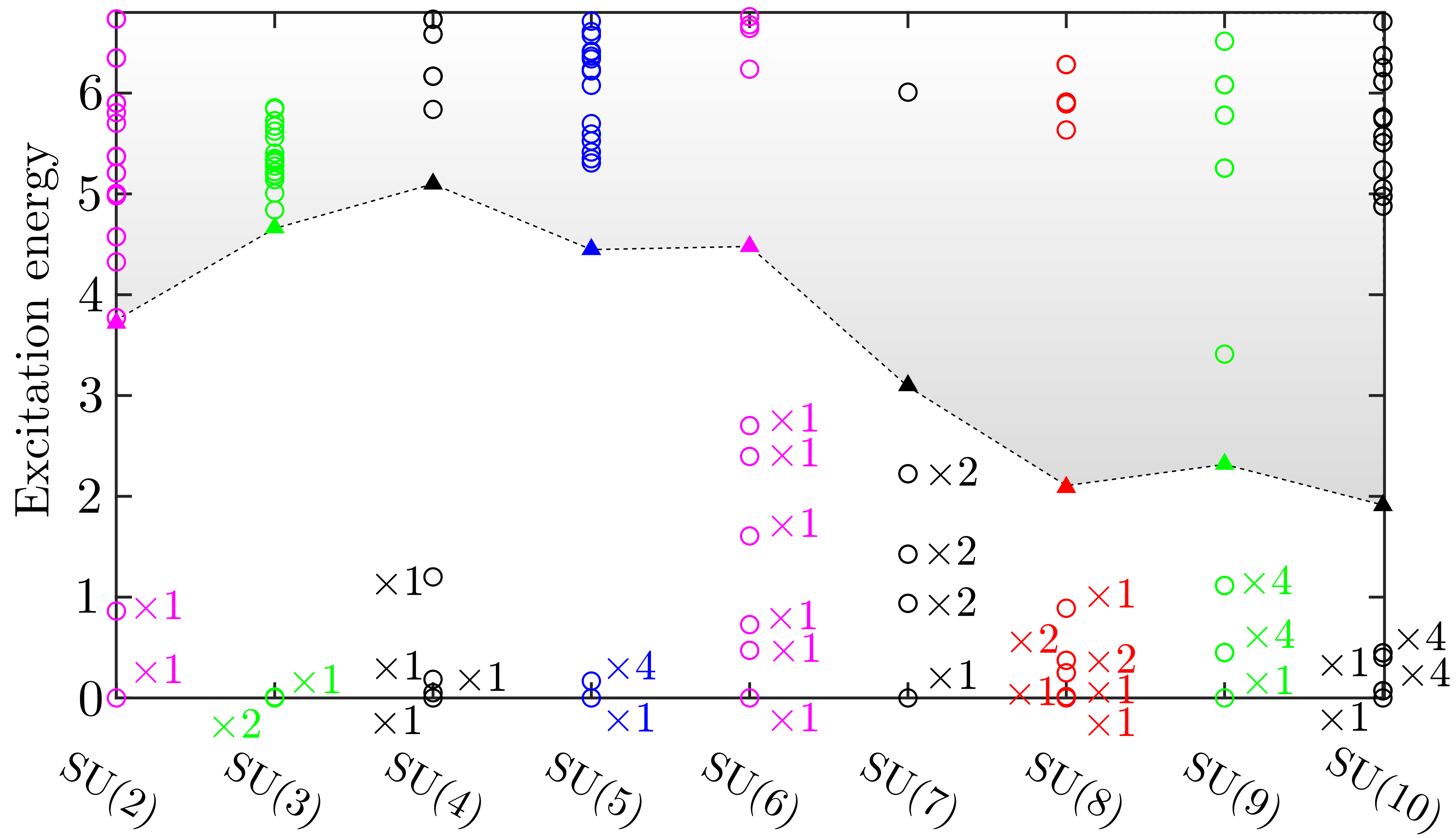
Can be generalized to SU(N)

Exact Diagonalization on torus

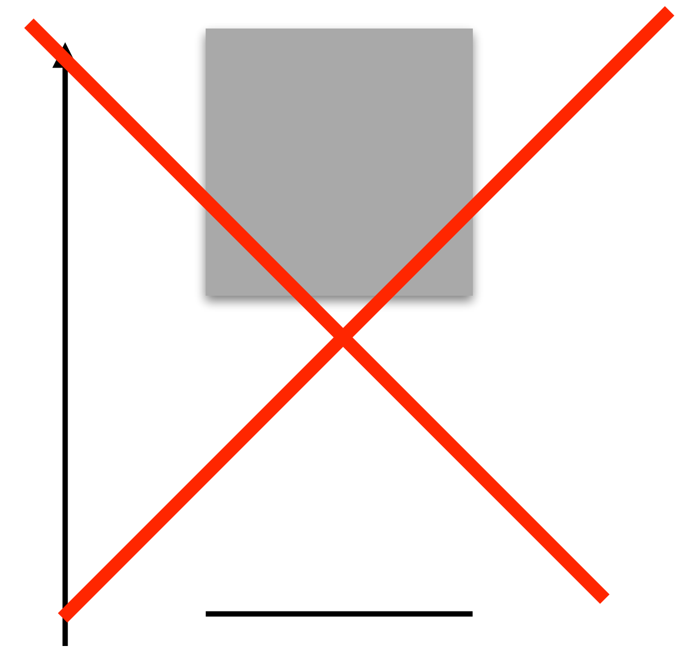


Predictions:

- If Ns=k*N: singlet ground-state degeneracy on a torus = N



gap above the N quasi-deg ground-states



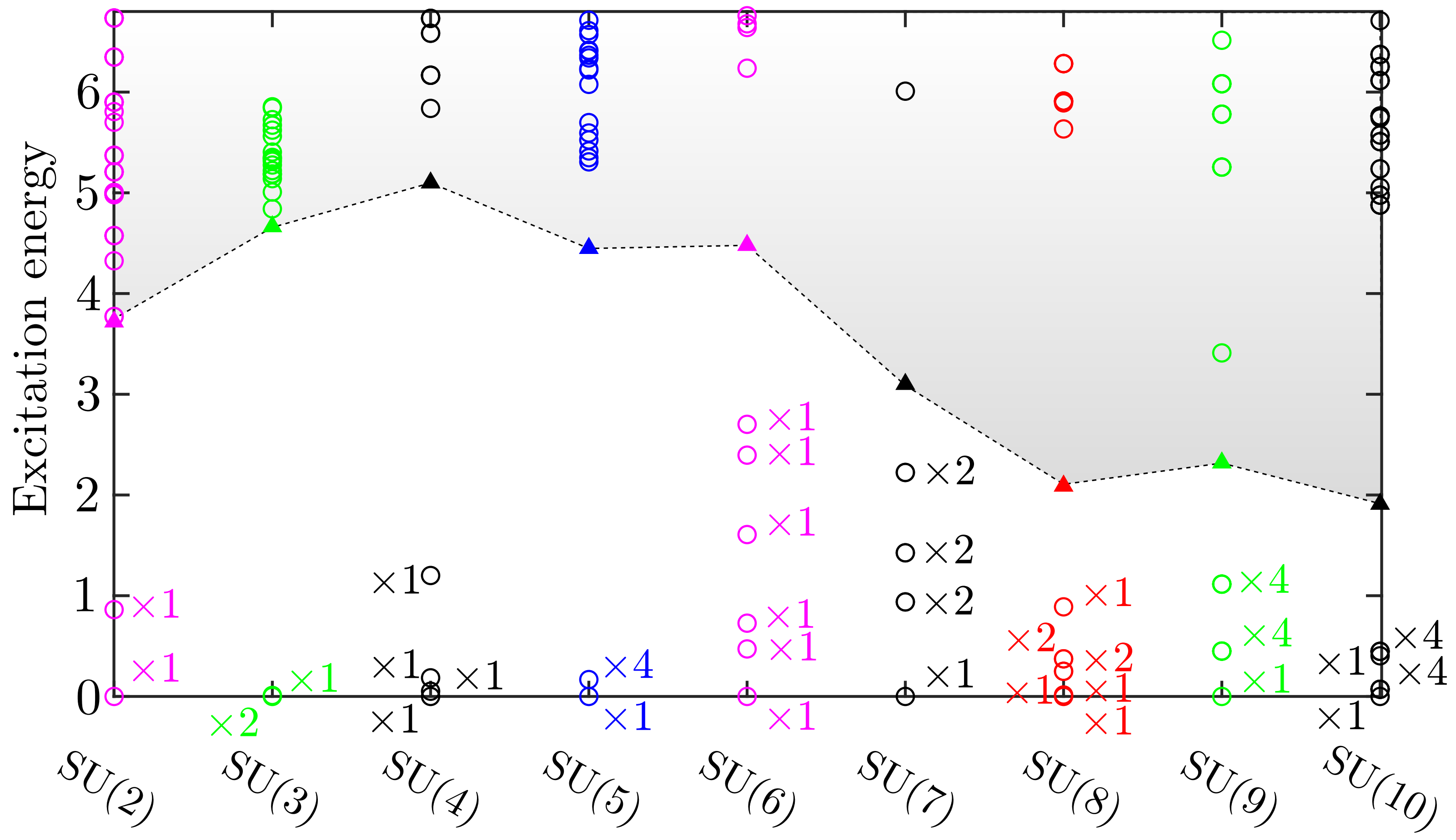
In 2d: generalization of Hastings-Oshikawa-Lieb-Mattis theorem
forbids a non-degenerate gapped state
gapless or discrete symmetry breaking or topological

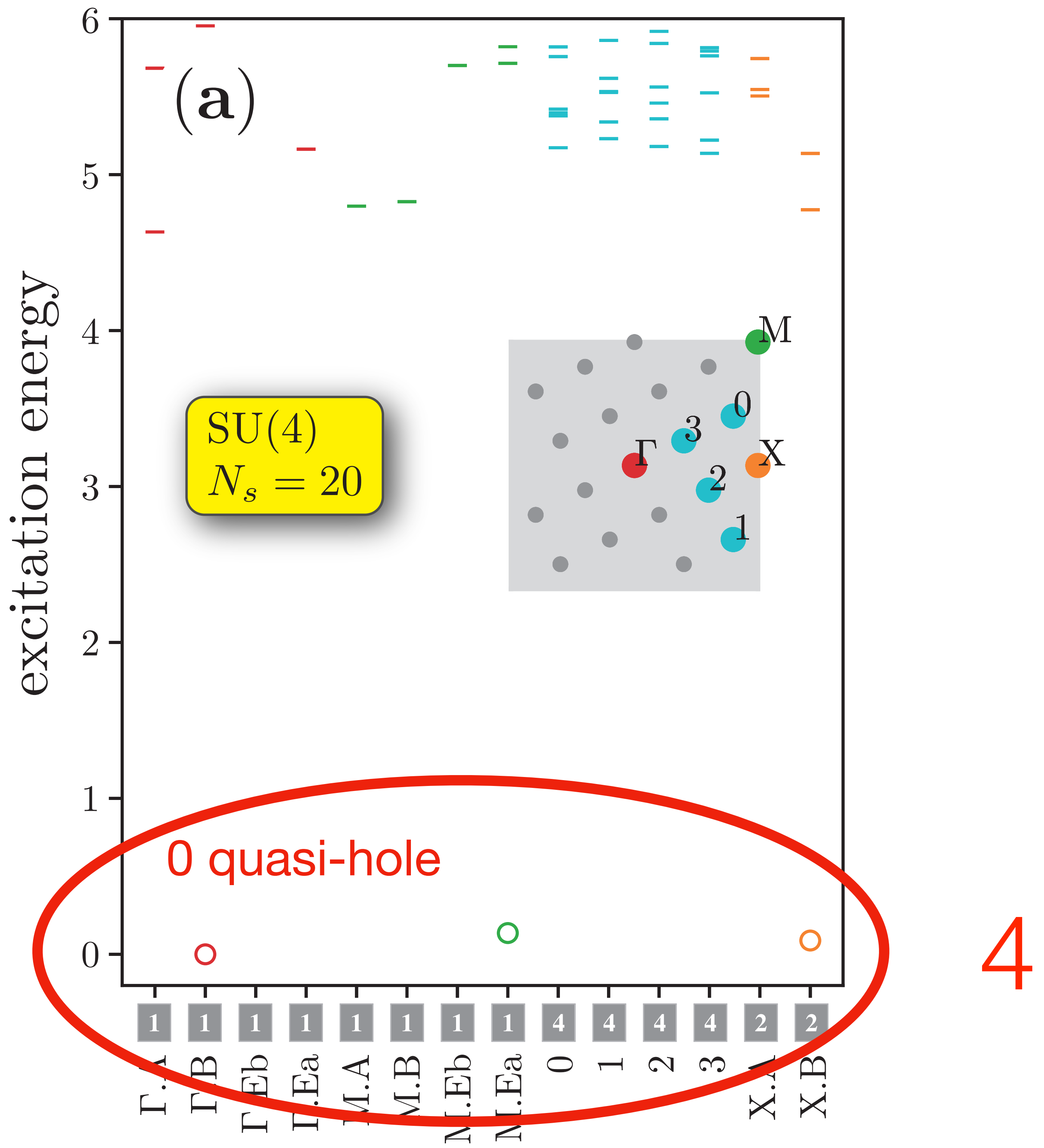
Exact Diagonalization on torus

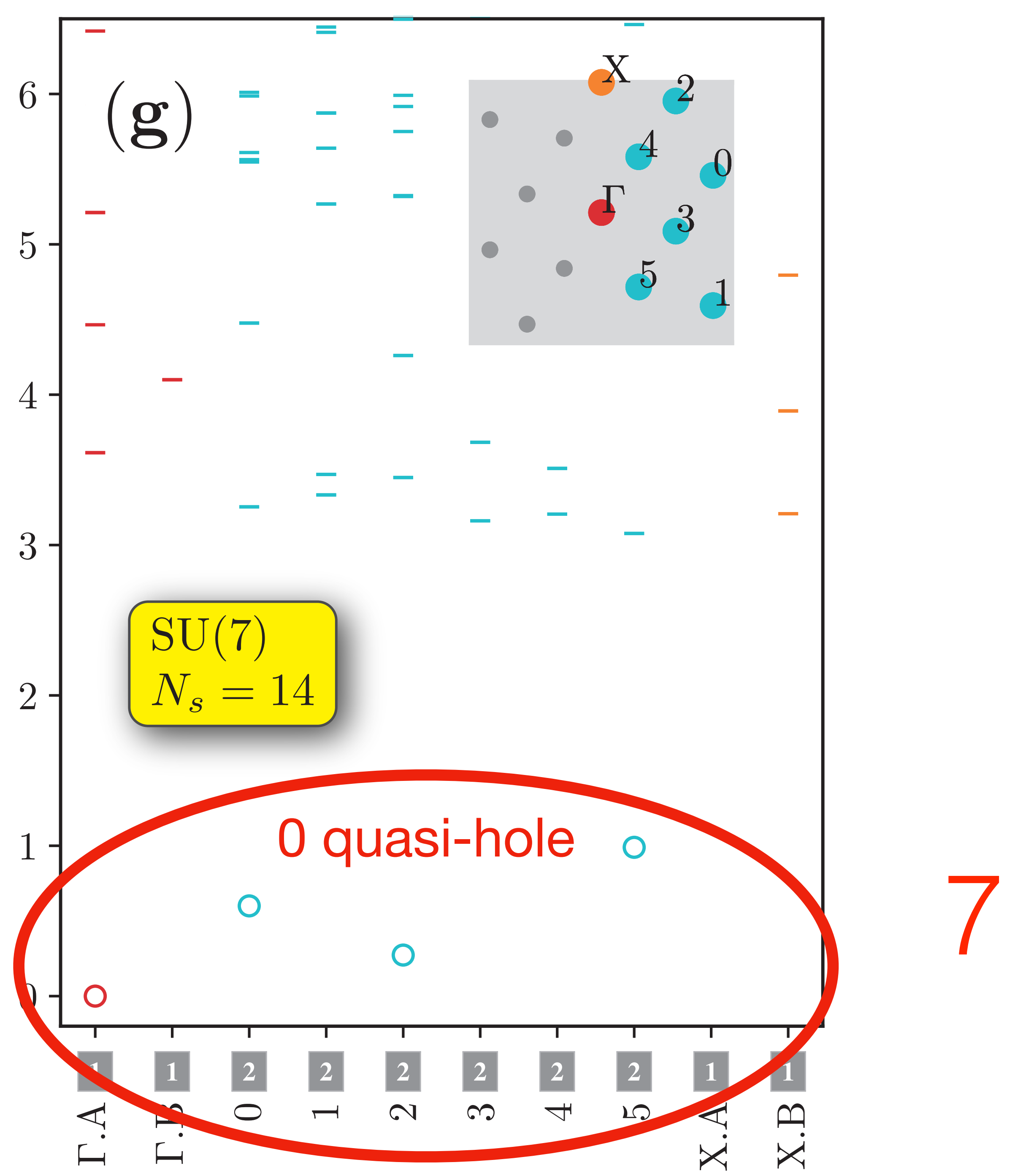
Predictions:

- If Ns=k*N: singlet ground-state degeneracy on a torus = N
- Lattice momenta can be obtained from a generalized Pauli principle

Haldane, Bernevig, Regnault,….







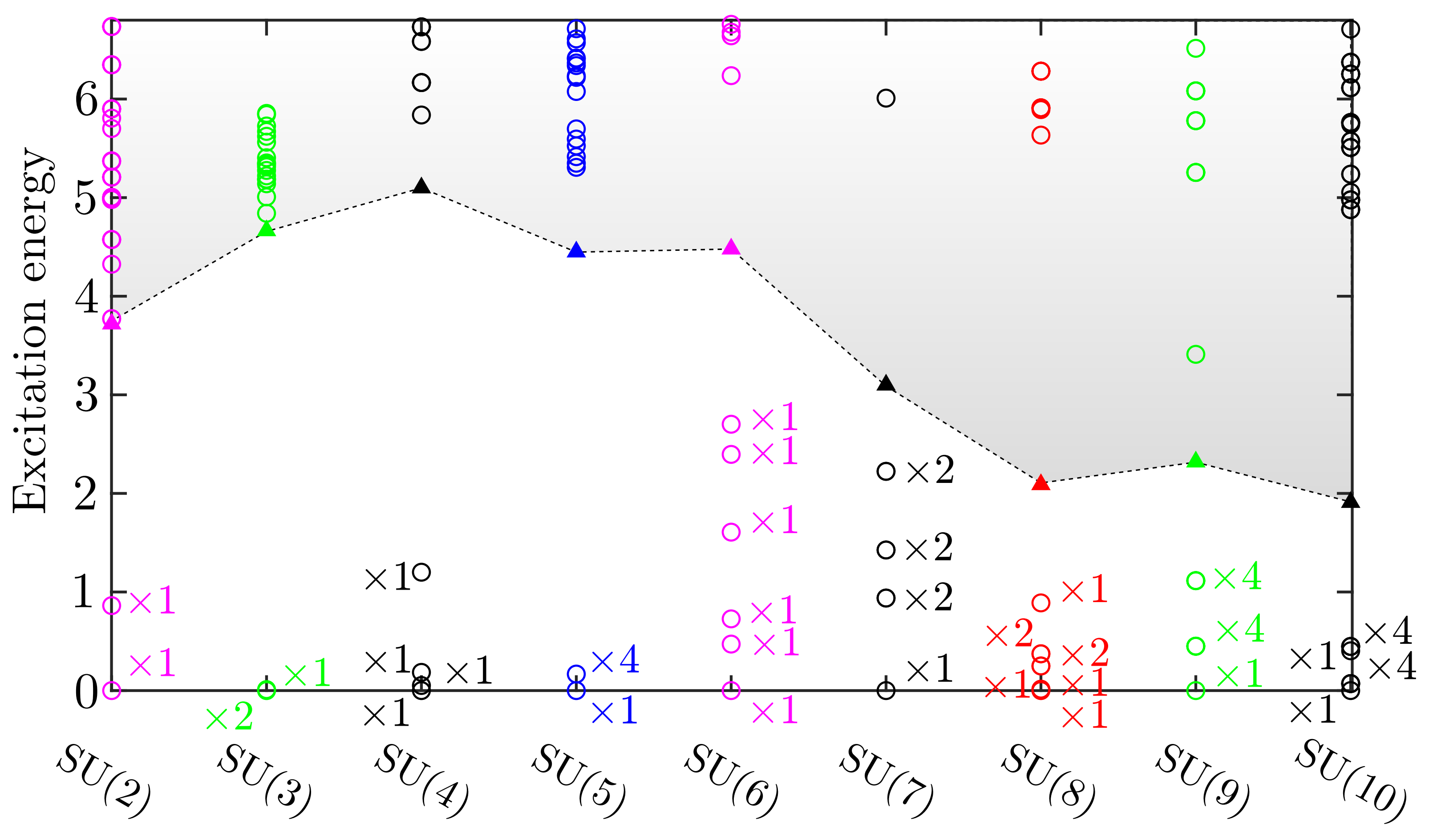
Exact Diagonalization on torus

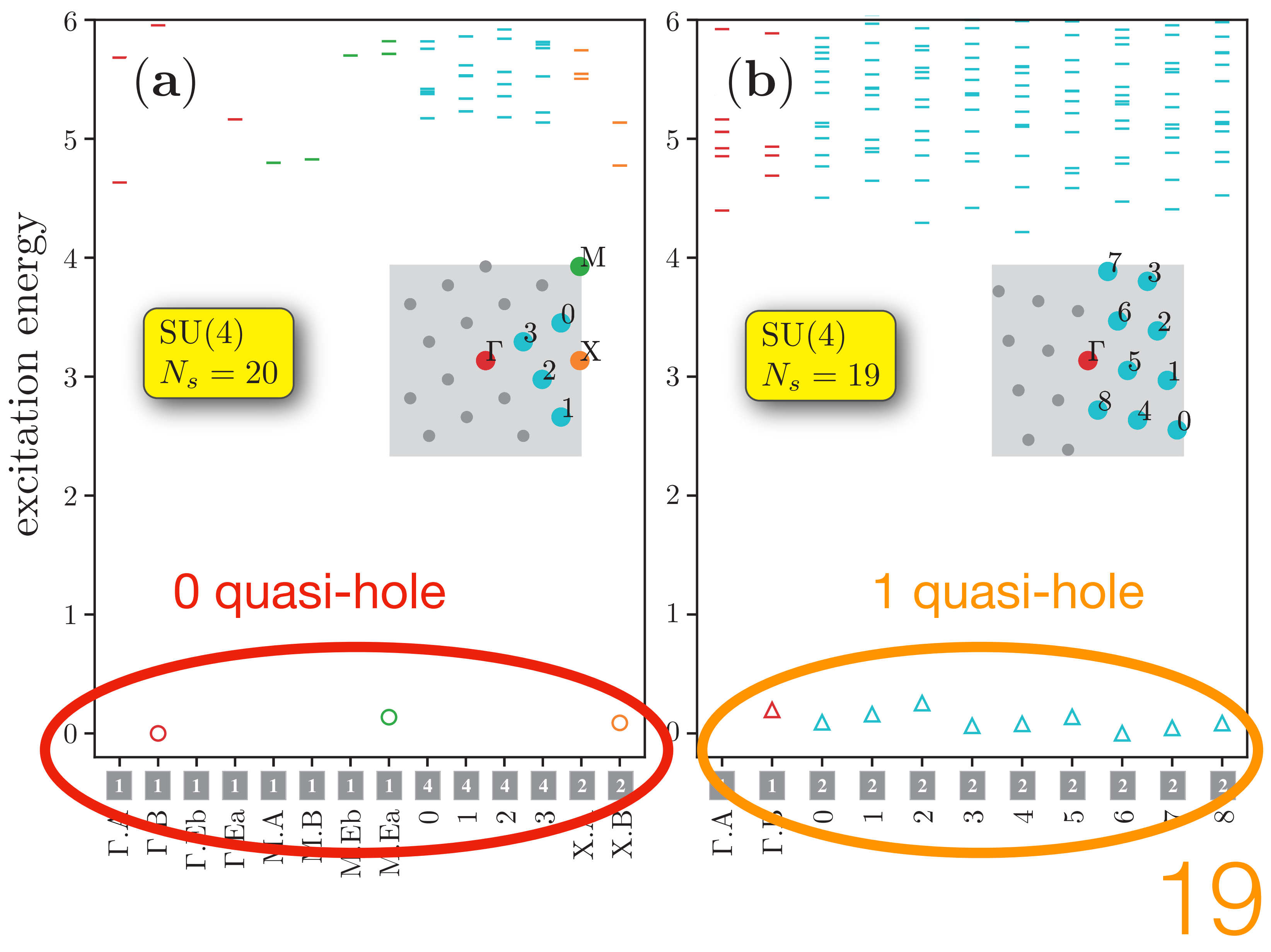
Predictions:

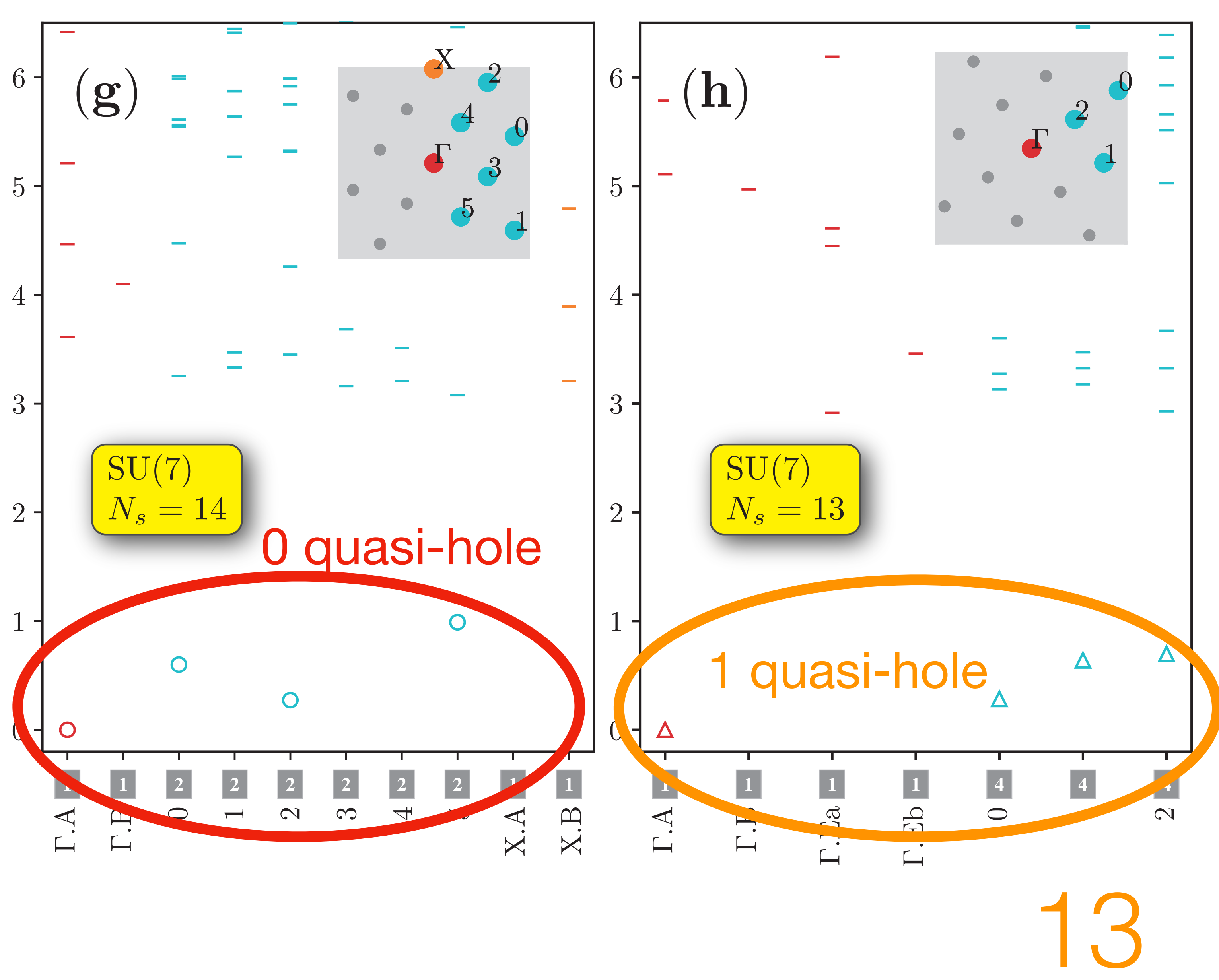
- If Ns=k*N: singlet ground-state degeneracy on a torus = N
- Lattice momenta can be obtained from a generalized Pauli principle

Haldane, Bernevig, Regnault,….

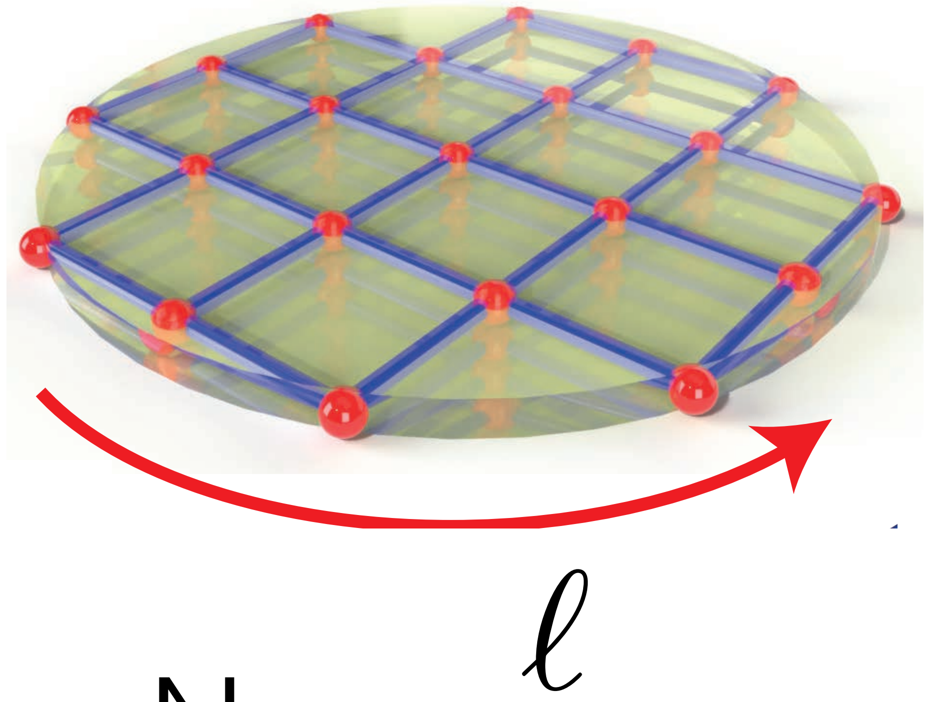
- Quasi-hole counting: deg=Ns, 1 per momentum sector



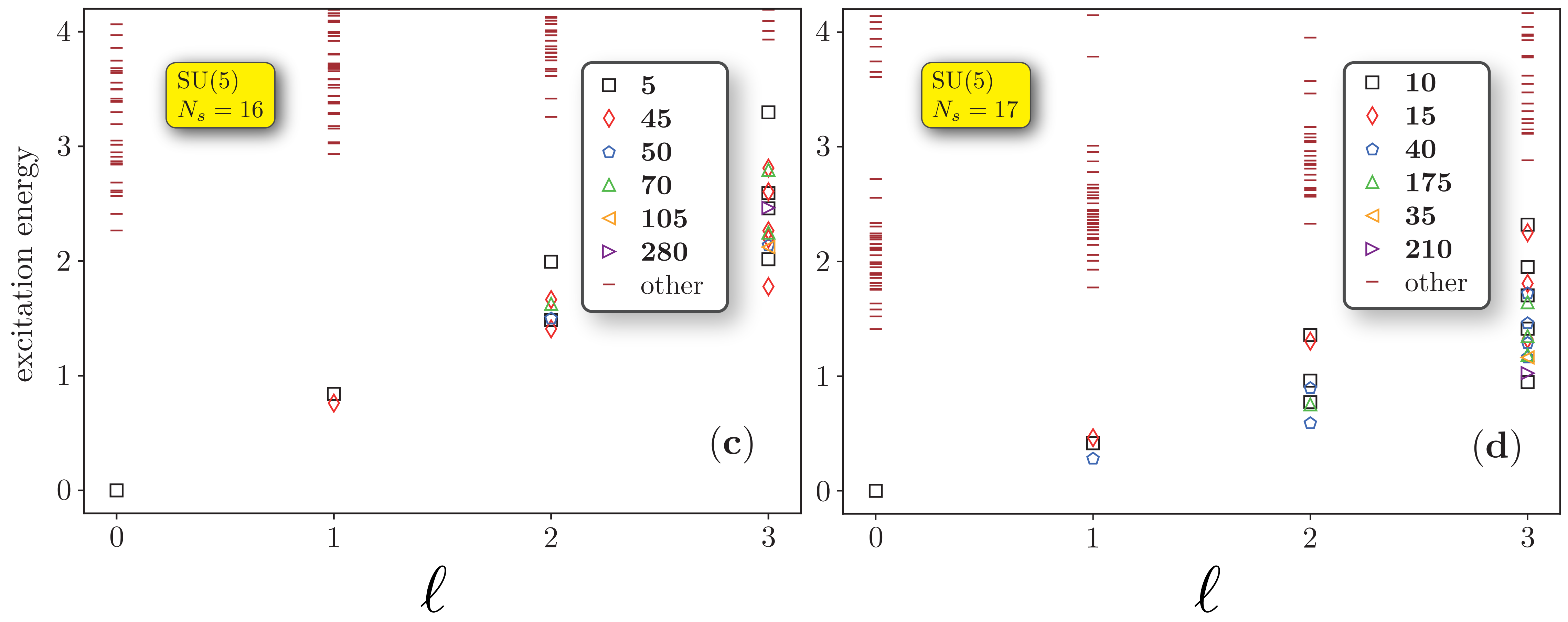


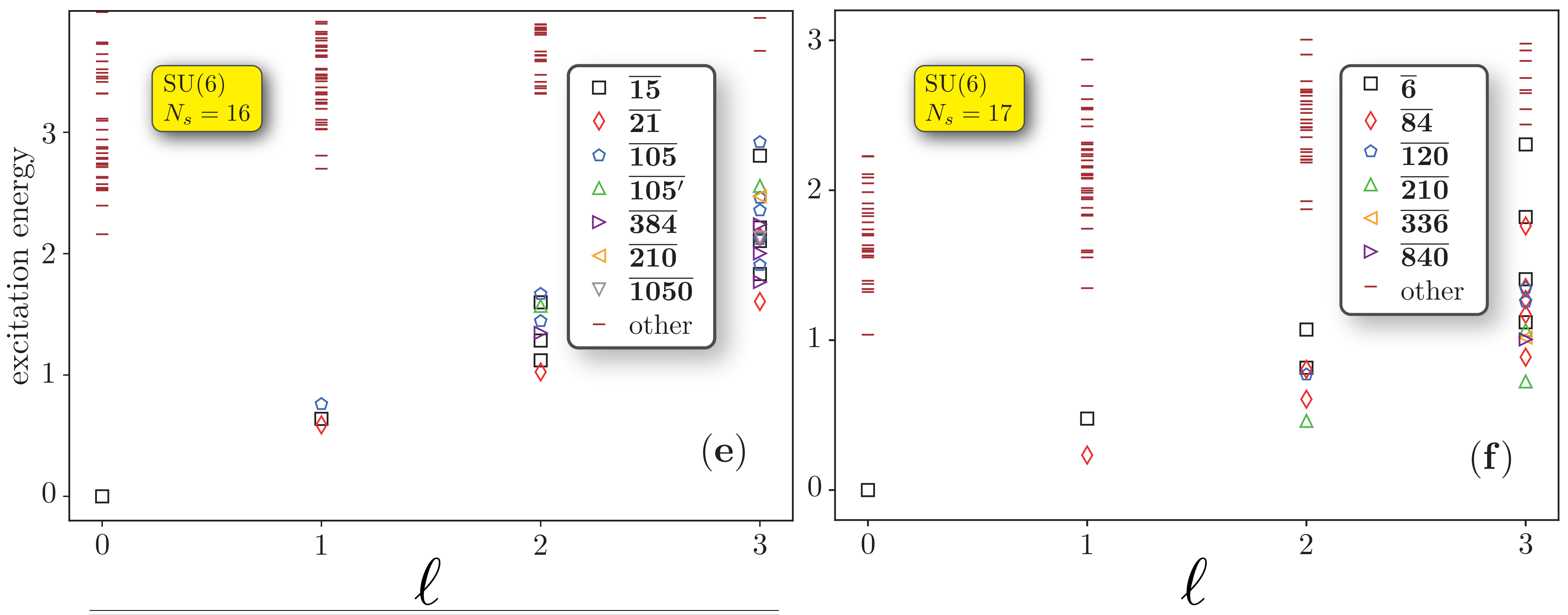


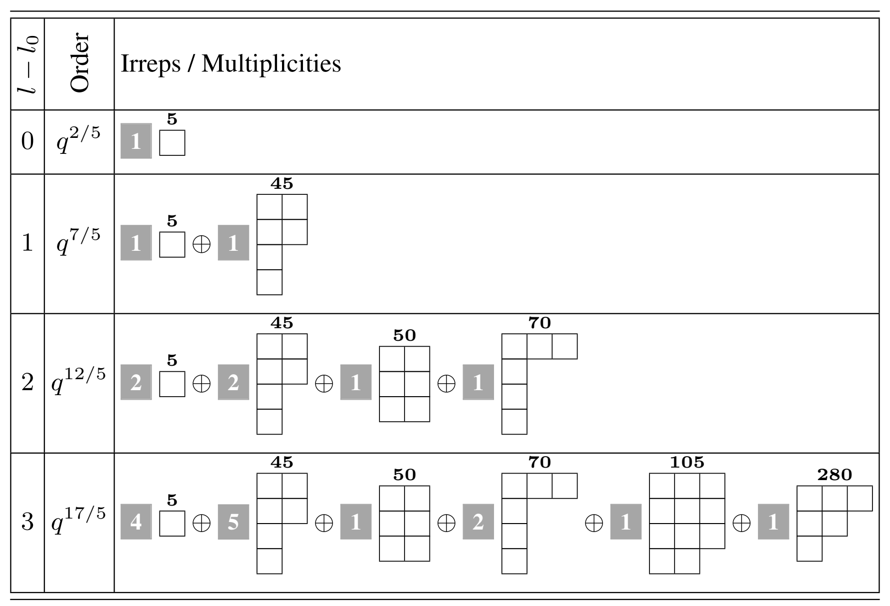
Exact Diagonalization on **open** cluster

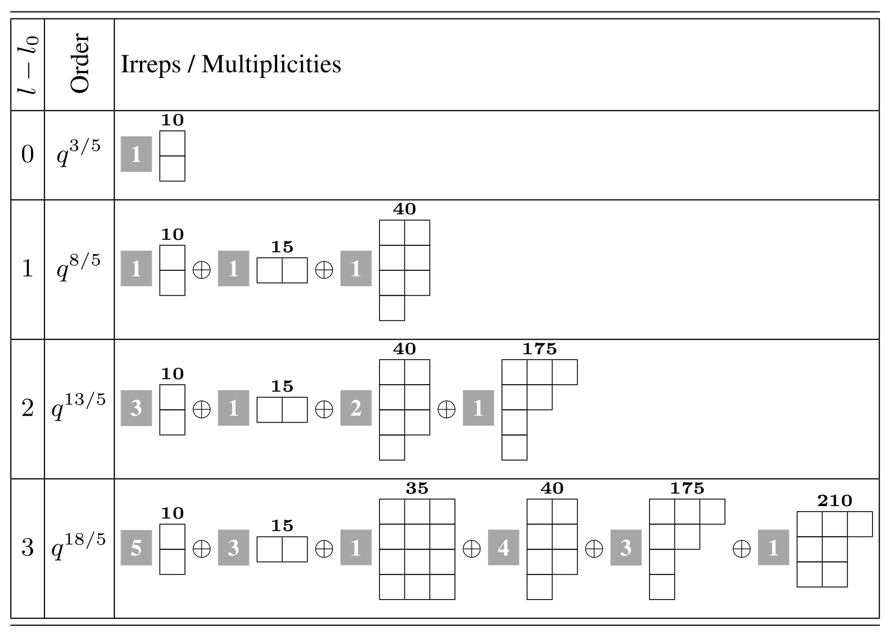


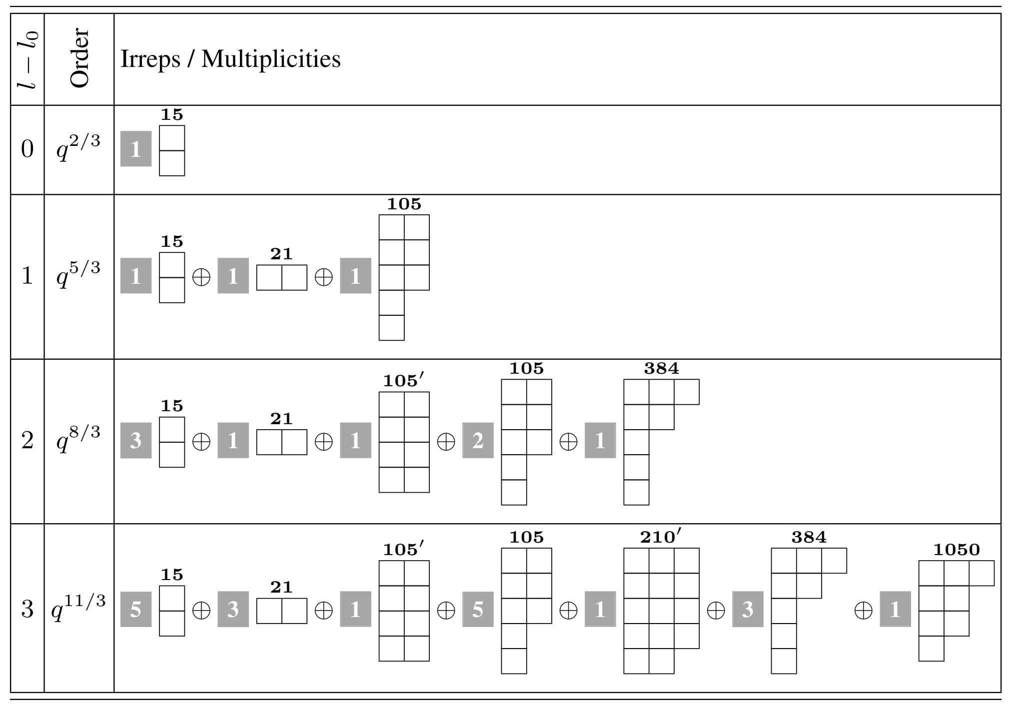
- SU(N)$_1$ chiral CFT counting depending on the number of sites Ns vs N

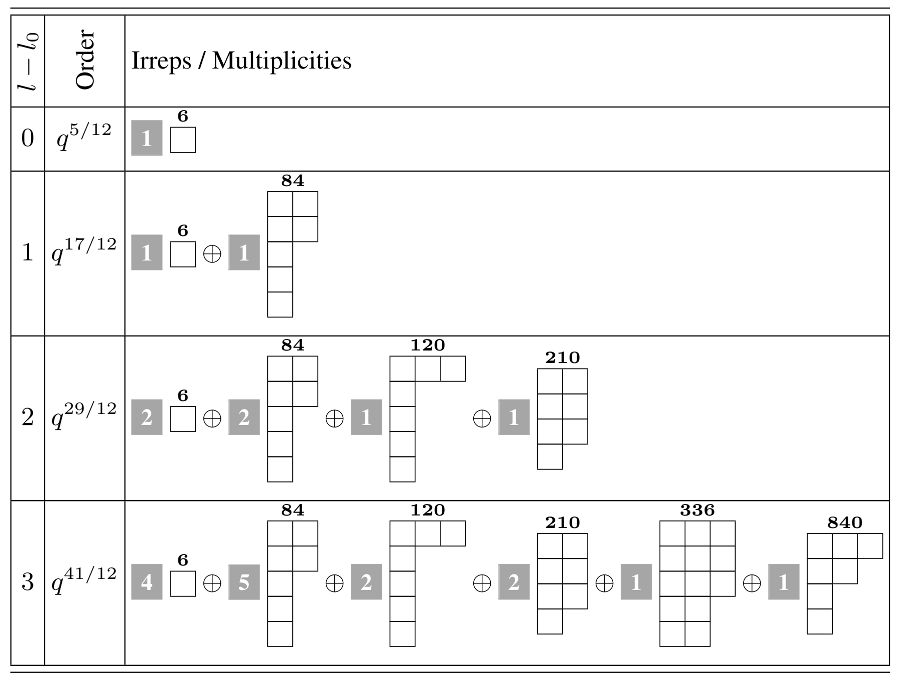










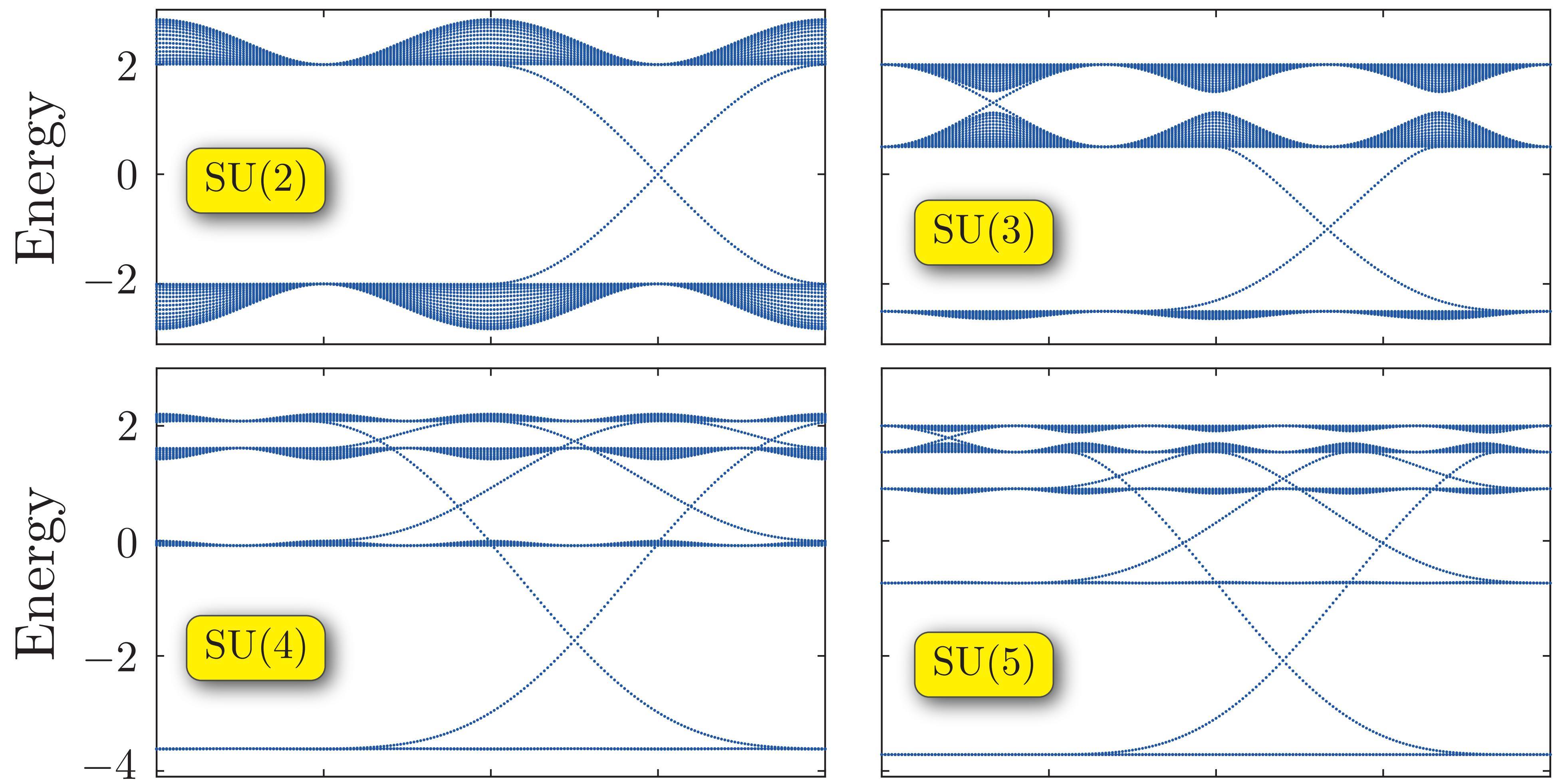


perfect agreement !

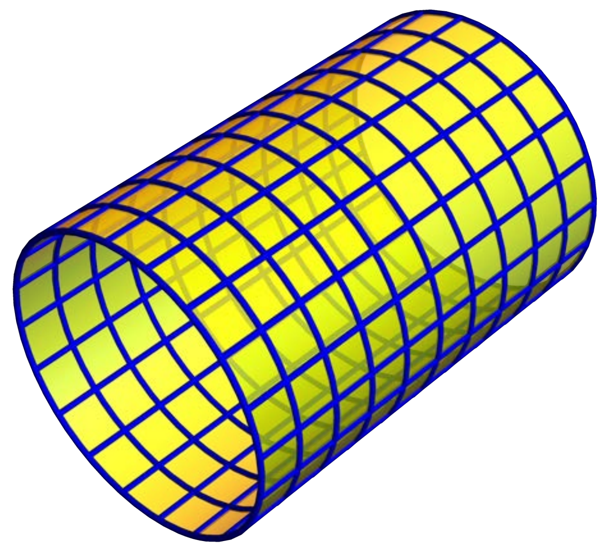
DMRG

Wu, Wang, Tu, PRL 124, 246401 (2020)

- Parton construction is useful to boost DMRG convergence

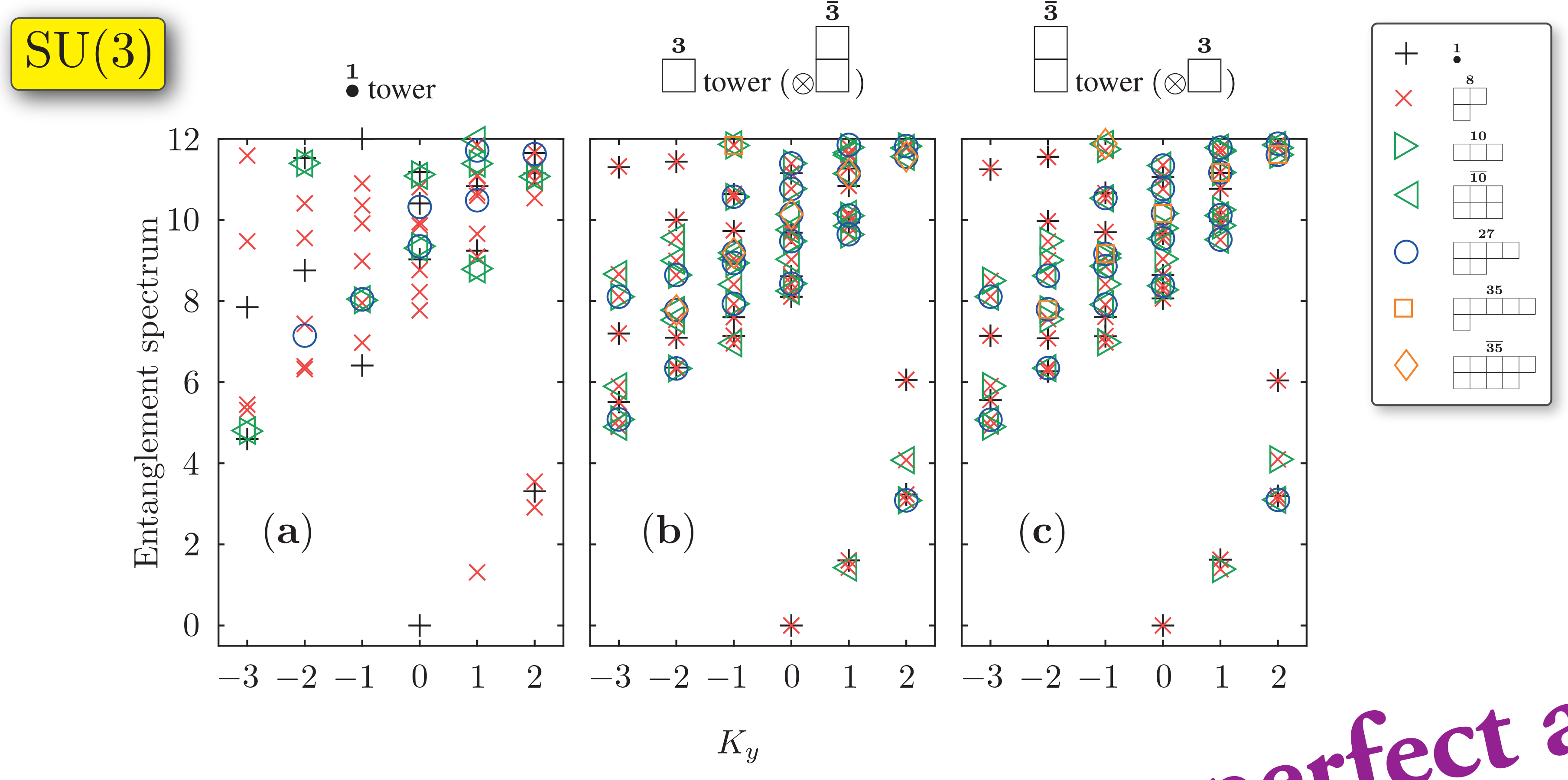


Spectrum on cylinder vs ky
Exact zero-mode edge states

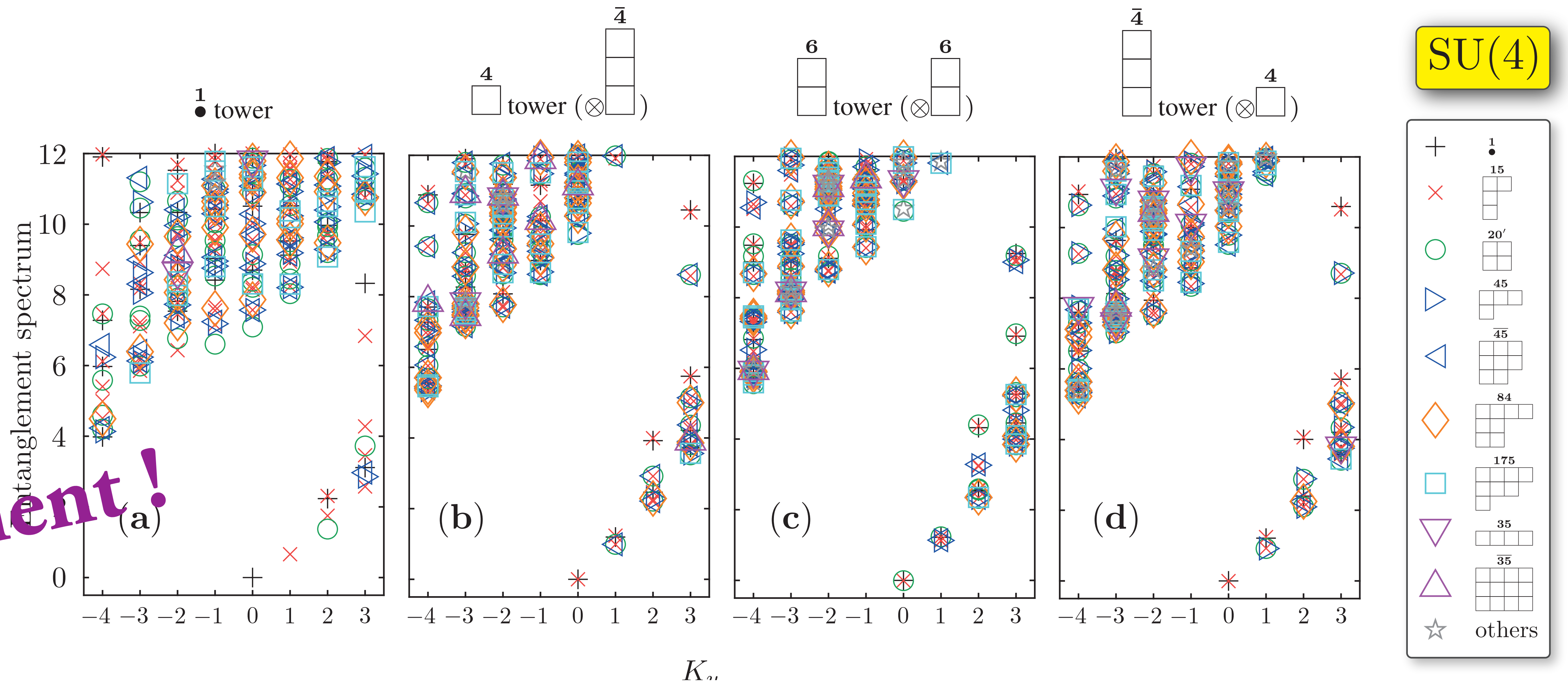


Construct N different minimally entangled states to target different excitations

- Probe entanglement spectrum as fingerprint of topological order



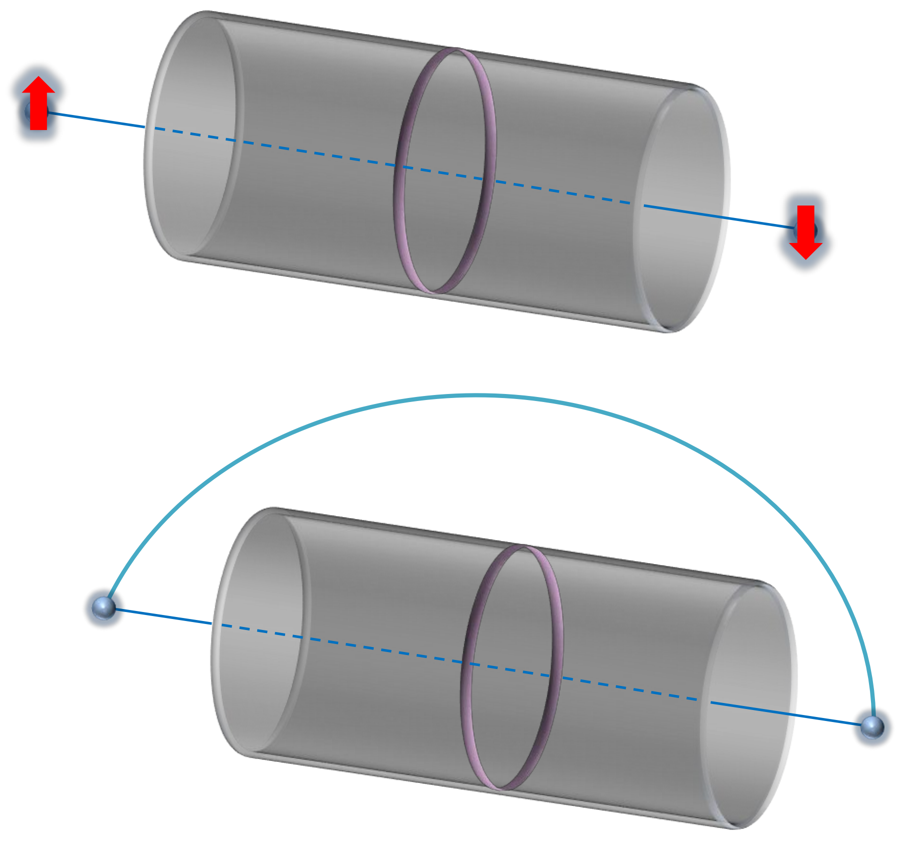
perfect agreement !



DMRG SU(N) subtleties

Hong-Hao Tu et al.

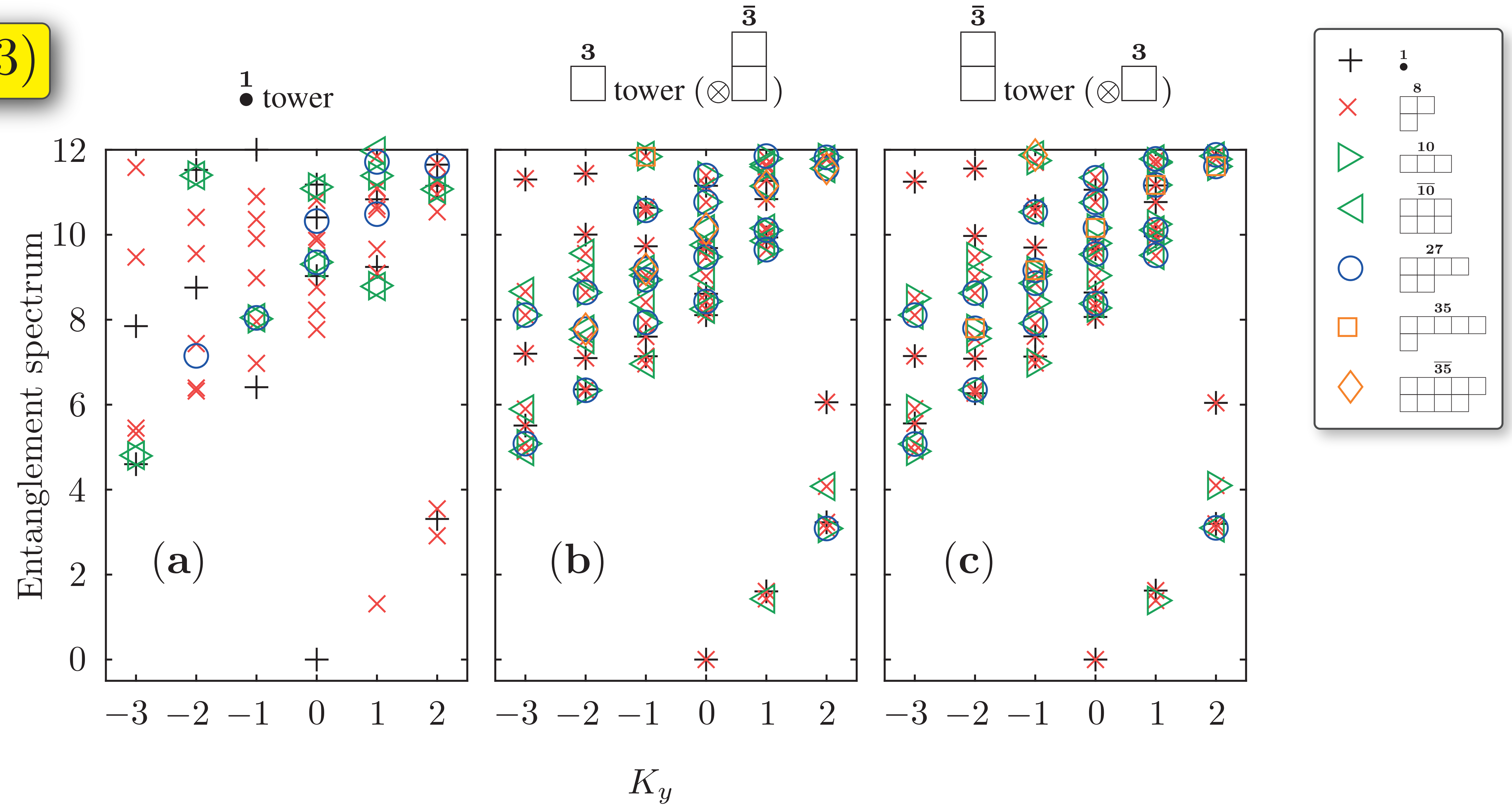
Semion sector
SU(2) case



$$|\psi_s\rangle = P_G \gamma^\dagger_{L\uparrow}\gamma^\dagger_{R\downarrow}|\mathrm{FS}\rangle$$ *not* a singlet

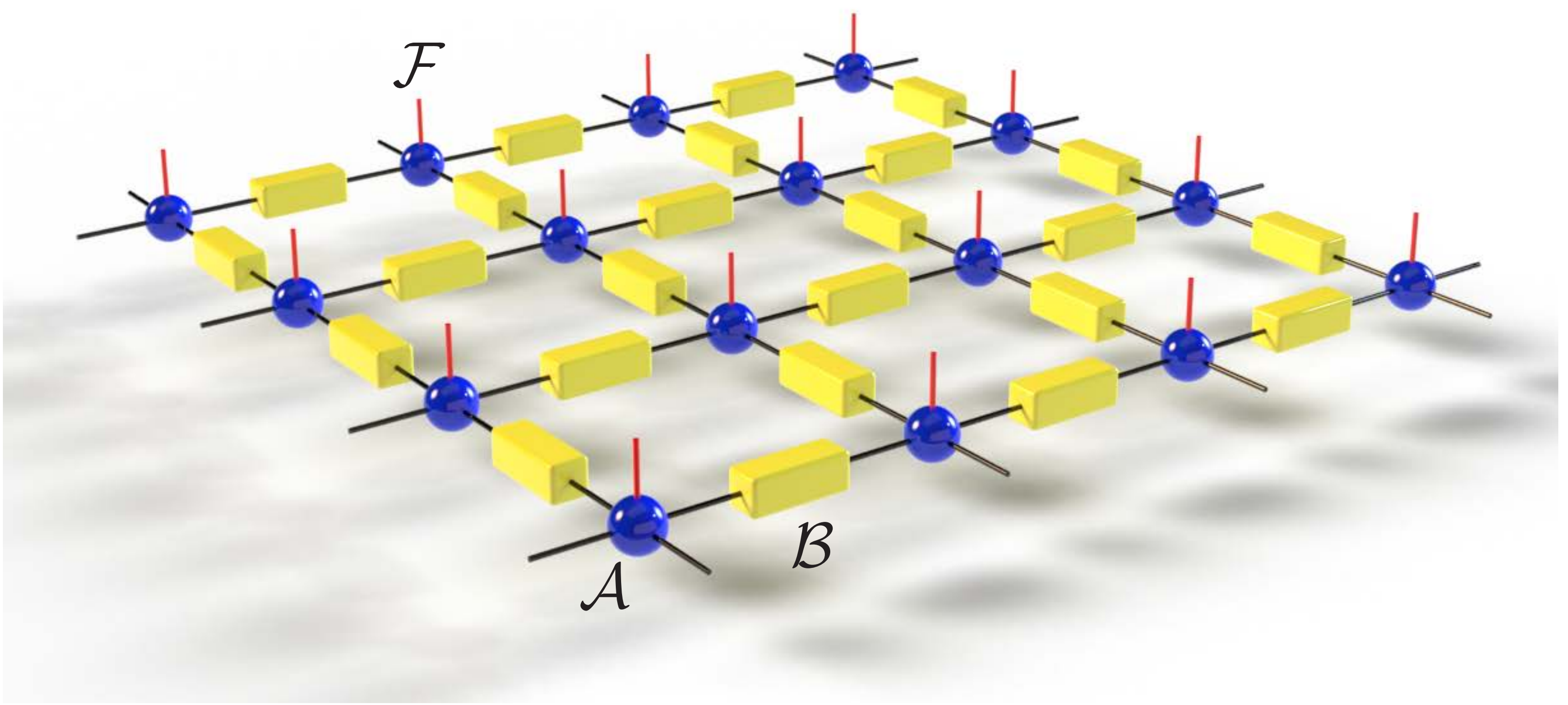
$$P_G(\gamma^\dagger_{L\uparrow}\gamma^\dagger_{R\downarrow} - \gamma^\dagger_{L\downarrow}\gamma^\dagger_{R\uparrow})|\mathrm{FS}\rangle$$ singlet

SU(3)



Entanglement spectrum=
two copies of “semion” conformal towers

PEPS



•Symmetric PEPS construction

virtual space: $\mathcal{V}_N = \bullet \oplus \square \oplus \cdots \oplus \left.\begin{array}{|c|}\hline \\ \hline \\ \hline \\ \hline \\ \hline\end{array}\right\} N-1$

$\mathcal{A} : (\mathcal{V}_N)^{\otimes z} \to \mathcal{F}$ $\mathcal{B} : (\mathcal{V}_N)^{\otimes 2} \to \bullet$

CSL breaks P and T but not PT

$$\mathcal{A} = \mathcal{A}_R + i\mathcal{A}_I = \sum_{a=1}^{N_R} \lambda_a^R \mathcal{A}_R^a + i \sum_{b=1}^{N_I} \lambda_b^I \mathcal{A}_I^b$$

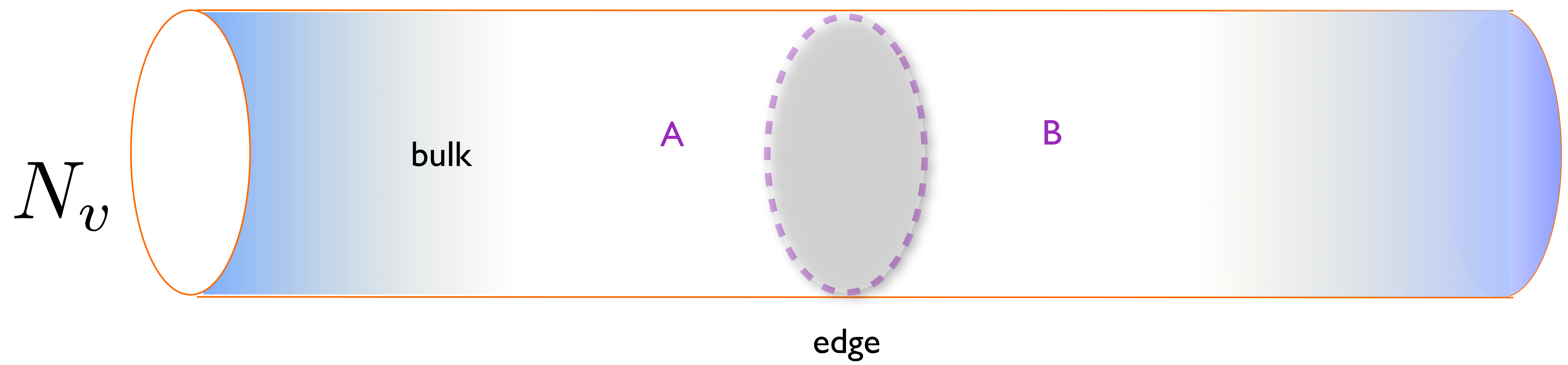
Tensor is a linear combination of point-group SU(N) symmetric ones

•Optimization is performed using CTMRG

few coefficients

PEPS: entanglement spectrum

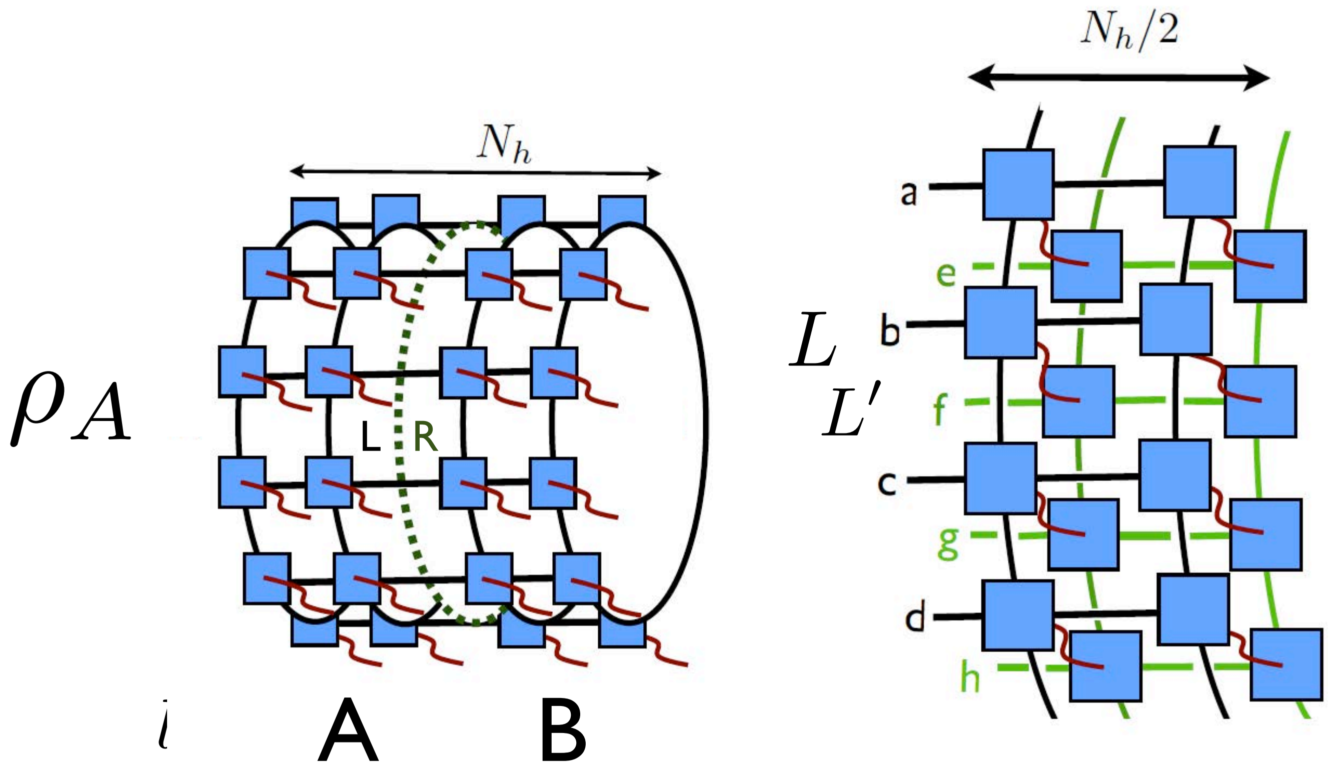
infinite PEPS cylinder



$$\rho_A = \mathrm{Tr}_B |\Psi\rangle\langle\Psi|$$ reduced density matrix

Li & Haldane

Entanglement spectrum is identical to a CFT boundary spectrum



σ_b^2

lives" on the boundary

Basic formula: $\rho_A = U \sigma_b^2 U^\dagger$

isometry: maps 2D onto 1D

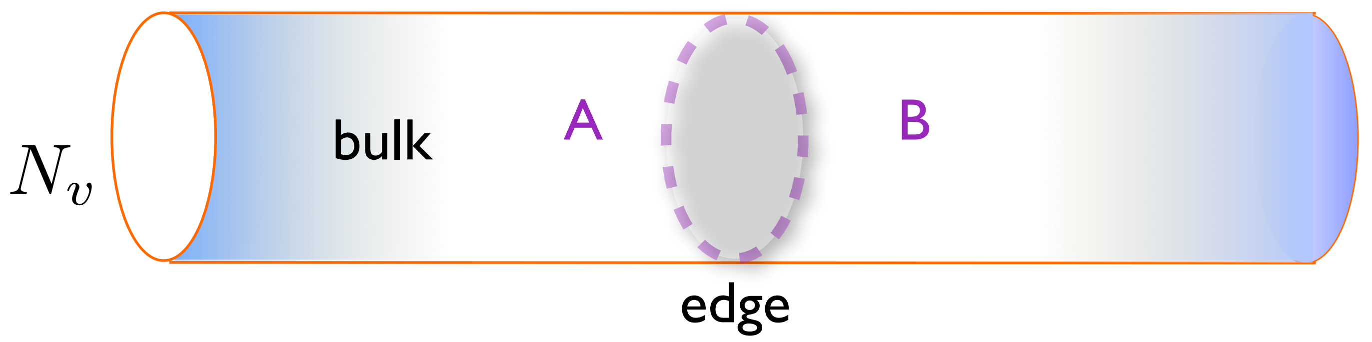
PHYSICAL REVIEW B **83**, 245134 (2011)

Entanglement spectrum and boundary theories with projected entangled-pair states

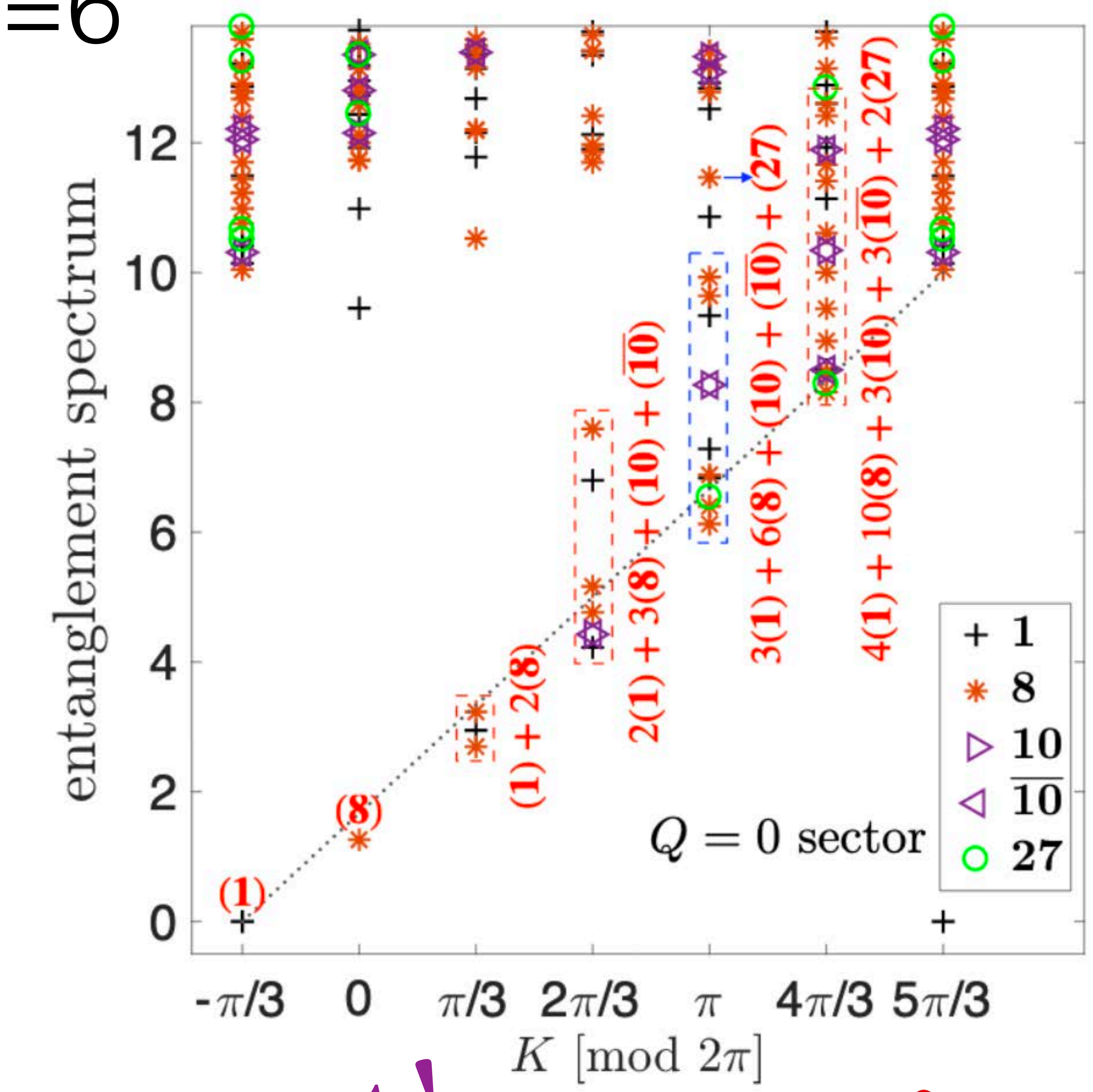
J. Ignacio Cirac,[1] Didier Poilblanc,[2] Norbert Schuch,[3] and Frank Verstraete[4]

PEPS: entanglement spectrum

infinite PEPS cylinder



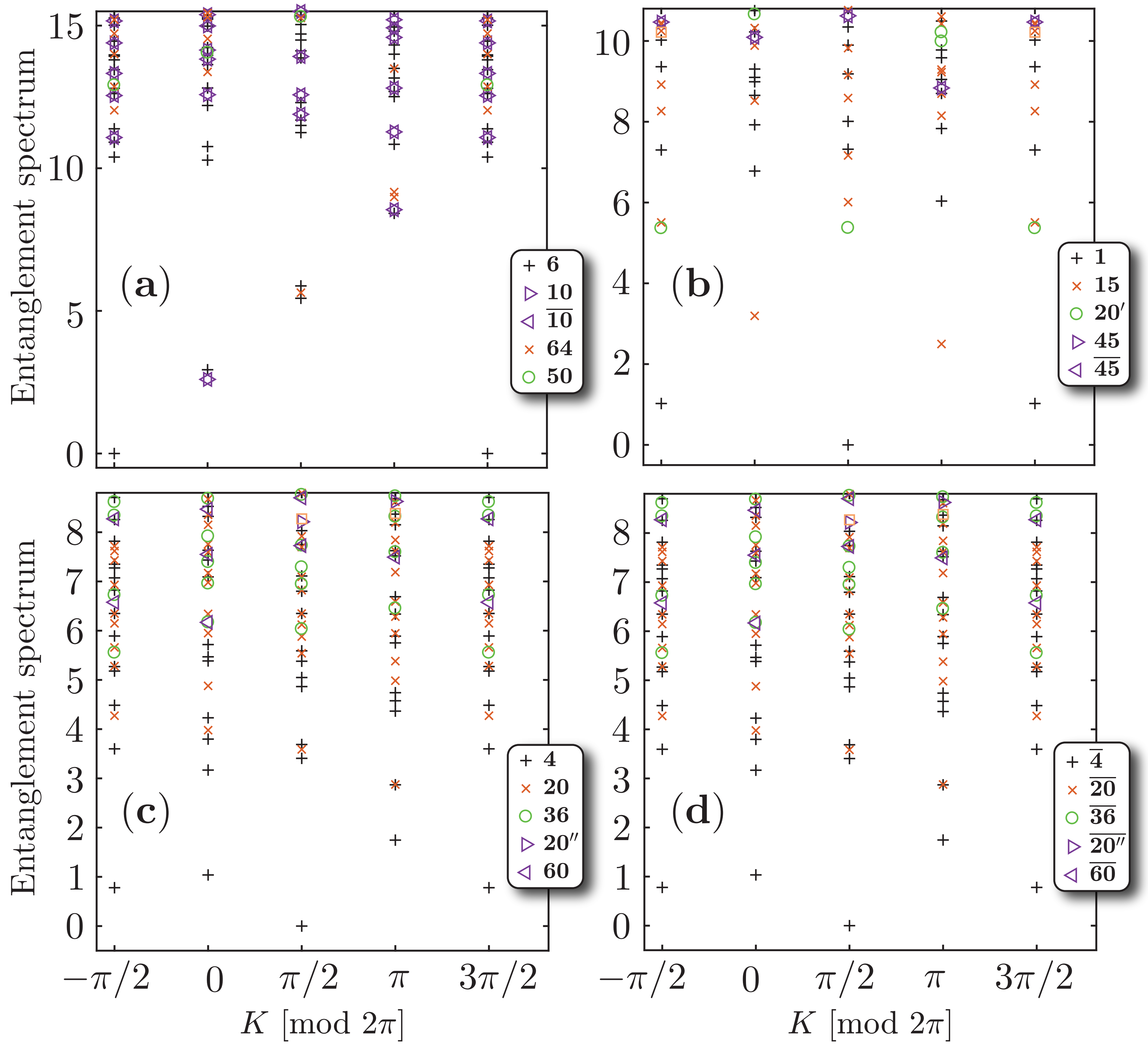
SU(3) Nv=6



SU(4), Nv=4, full SU(N) symmetry

D=15

$\chi = 1350$

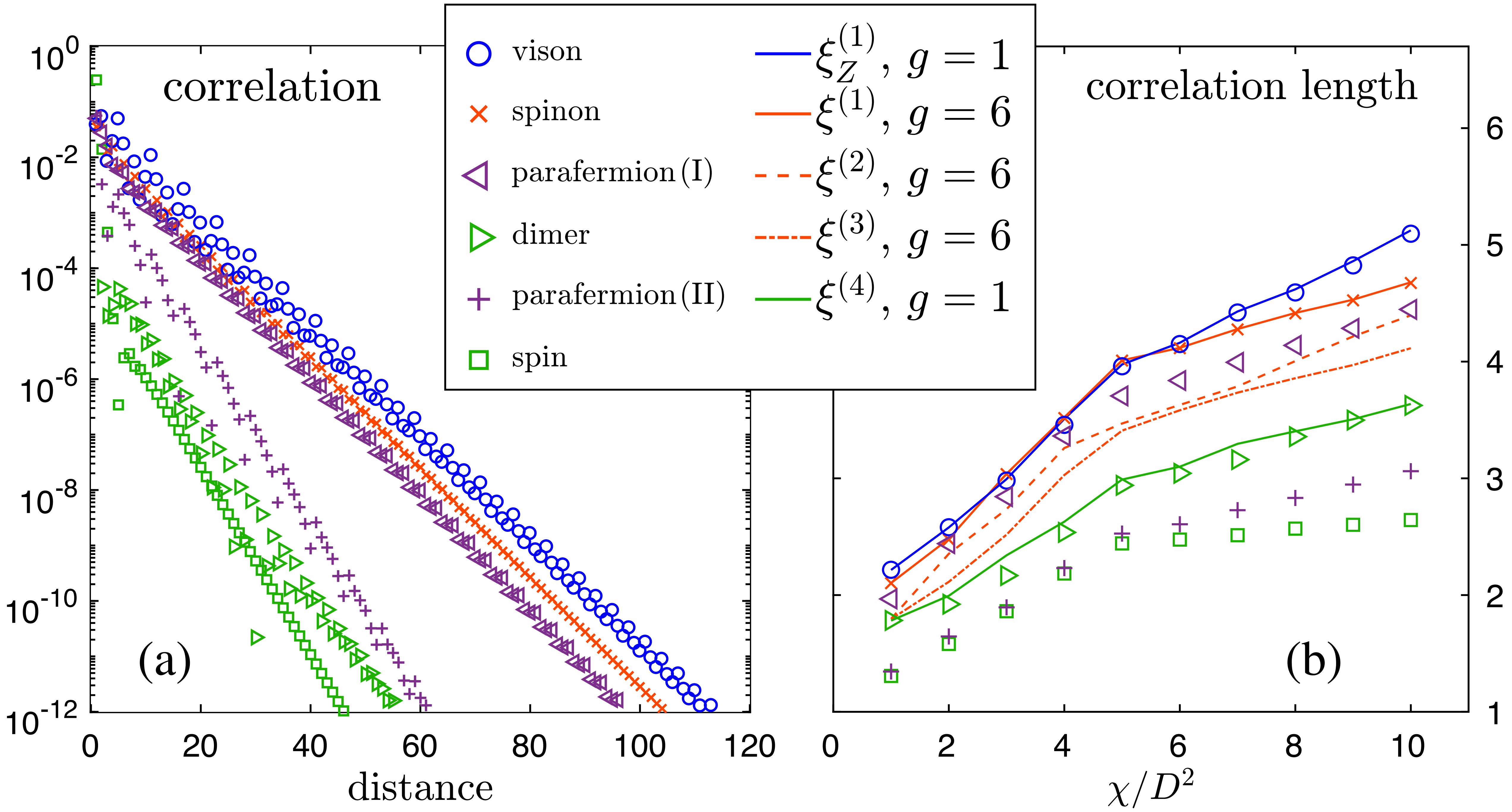


perfect agreement !

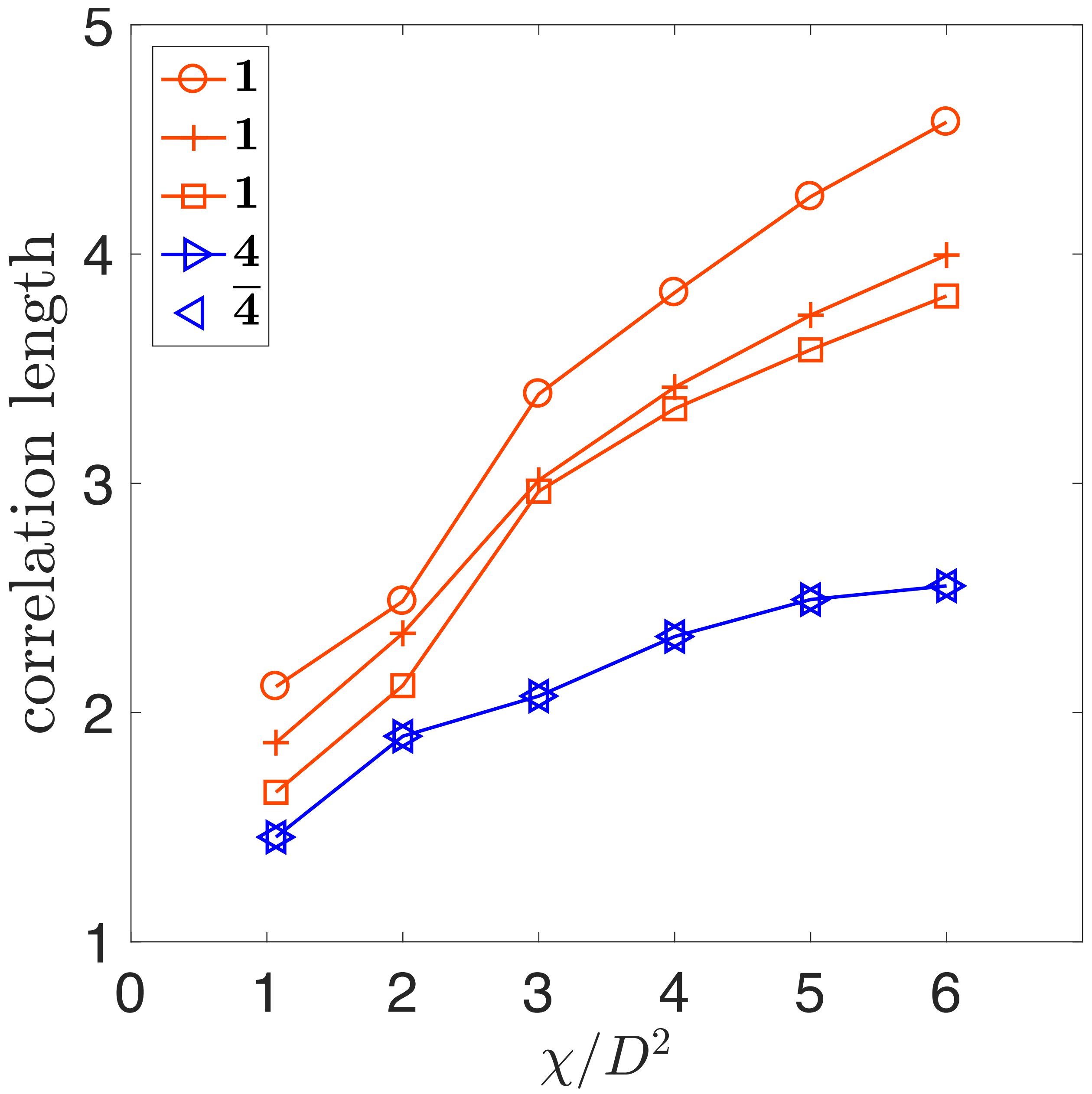
duplication of chiral branches in some sectors

PEPS: correlation lengths

SU(3)



SU(4)



Correlations in the bulk

Correlation length directly from transfer matrix

No saturation so presumably gapless state…

Dubail-Read 2015

No-go theorem for a free-fermion PEPS to have a bulk gap
Is it also true in the interacting case ?

Abelian CSL: spontaneous T-breaking

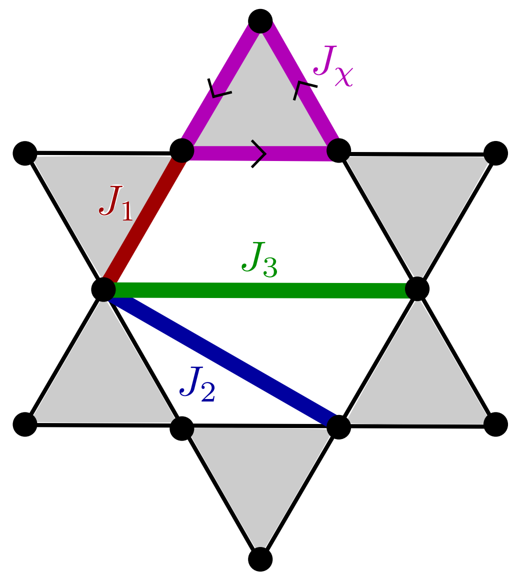
Topological CSL can also be found in the **absence** of explicit T-breaking

SU(2) Heisenberg

PRL **112,** 137202 (2014) PHYSICAL REVIEW LETTERS week ending 4 APRIL 2014

Chiral Spin Liquid in a Frustrated Anisotropic Kagome Heisenberg Model

Yin-Chen He,[1] D. N. Sheng,[2] and Yan Chen[1,3]



PHYSICAL REVIEW B **92**, 125122 (2015)

Nature of chiral spin liquids on the kagome lattice

Alexander Wietek,* Antoine Sterdyniak, and Andreas M. Läuchli

SU(2) Hubbard

Quantum Spin Liquid with Emergent Chiral Order in the Triangular-lattice Hubbard Model

Bin-Bin Chen,[1,2] Ziyu Chen,[1] Shou-Shu Gong,[1,*] D. N. Sheng,[3] Wei Li,[1,4,†] and Andreas Weichselbaum[5,2,‡]

PHYSICAL REVIEW X **10,** 021042 (2020)

Chiral Spin Liquid Phase of the Triangular Lattice Hubbard Model: A Density Matrix Renormalization Group Study

Aaron Szasz,[1,2,3,*] Johannes Motruk,[1,2] Michael P. Zaletel,[1,2,4] and Joel E. Moore[1,2]

SU(4)

An $SU(4)$ chiral spin liquid and quantized dipole Hall effect in moiré bilayers

Ya-Hui Zhang[1], D. N. Sheng[2], and Ashvin Vishwanath[1]

Non-abelian case: SU(2)

non abelian FQHS

Moore-Read

Read-Rezayi

incompressible (gapped) in the bulk

non abelian fractional excitation

gapless chiral edge states

$SU(2)_k$ CFT

Spin analogue ?

Parent Hamiltonian approach

Coupled wire construction

This topological phase hosts $SU(2)_2$ non-abelian Ising anyons

Moore-Read state corresponds to **spin-1** lattice model

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IOP Institute of Physics

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FAST TRACK COMMUNICATION

Exact parent Hamiltonians of bosonic and fermionic Moore–Read states on lattices and local models

Ivan Glasser[1], J Ignacio Cirac[1], Germán Sierra[2,3] and Anne E B Nielsen[1]

Proposed parent Hamiltonian is rather complicated, long-range

$$
\begin{aligned}
H = & J_1 \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J_2 \sum_{\langle\langle k,l \rangle\rangle} \mathbf{S}_k \cdot \mathbf{S}_l \\
& + K_1 \sum_{\langle i,j \rangle} (\mathbf{S}_i \cdot \mathbf{S}_j)^2 + K_2 \sum_{\langle\langle k,l \rangle\rangle} (\mathbf{S}_k \cdot \mathbf{S}_l)^2 \\
& + K_c \sum_{\square} [\mathbf{S}_i \cdot (\mathbf{S}_j \times \mathbf{S}_k) + \mathbf{S}_j \cdot (\mathbf{S}_k \times \mathbf{S}_m) \\
& + \mathbf{S}_i \cdot (\mathbf{S}_j \times \mathbf{S}_m) + \mathbf{S}_i \cdot (\mathbf{S}_k \times \mathbf{S}_m)],
\end{aligned}
$$

→ truncated approximate spin-1 model on the square lattice

→ ***numerics needed***

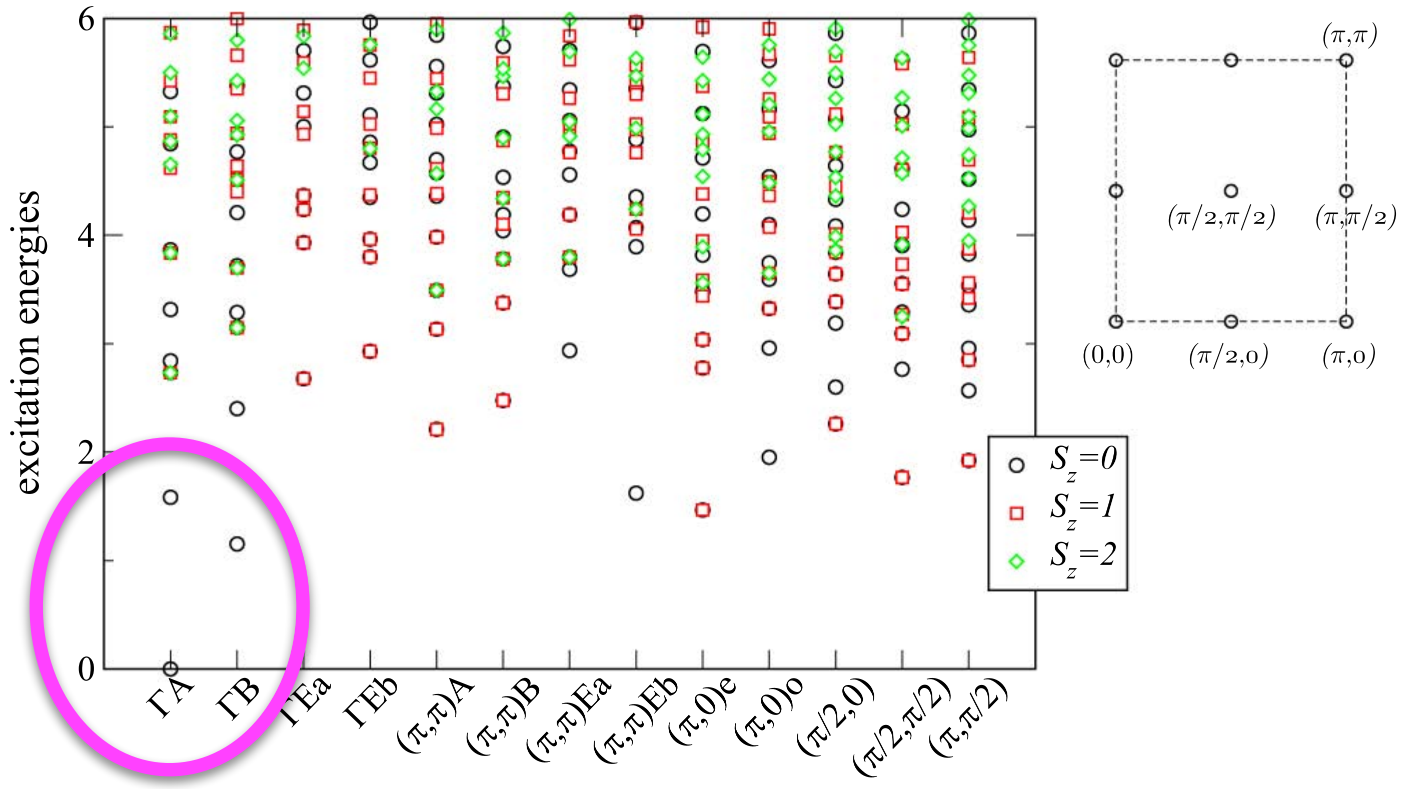
Combined ED/DMRG/PEPS study

PHYSICAL REVIEW B **98**, 184409 (2018)

Non-Abelian chiral spin liquid in a quantum antiferromagnet revealed by an iPEPS study

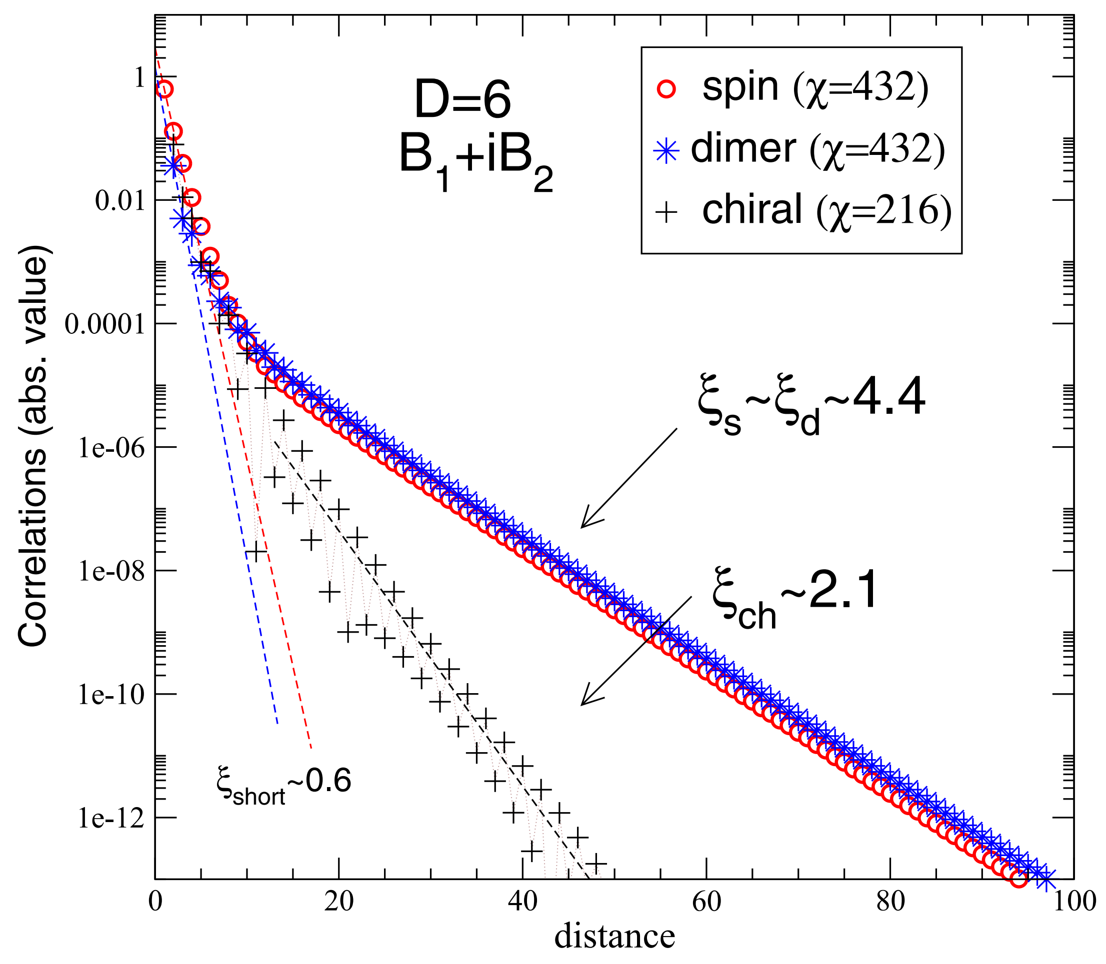
Ji-Yao Chen,[1] Laurens Vanderstraeten,[2] Sylvain Capponi,[1] and Didier Poilblanc[1,3]

ED on a torus



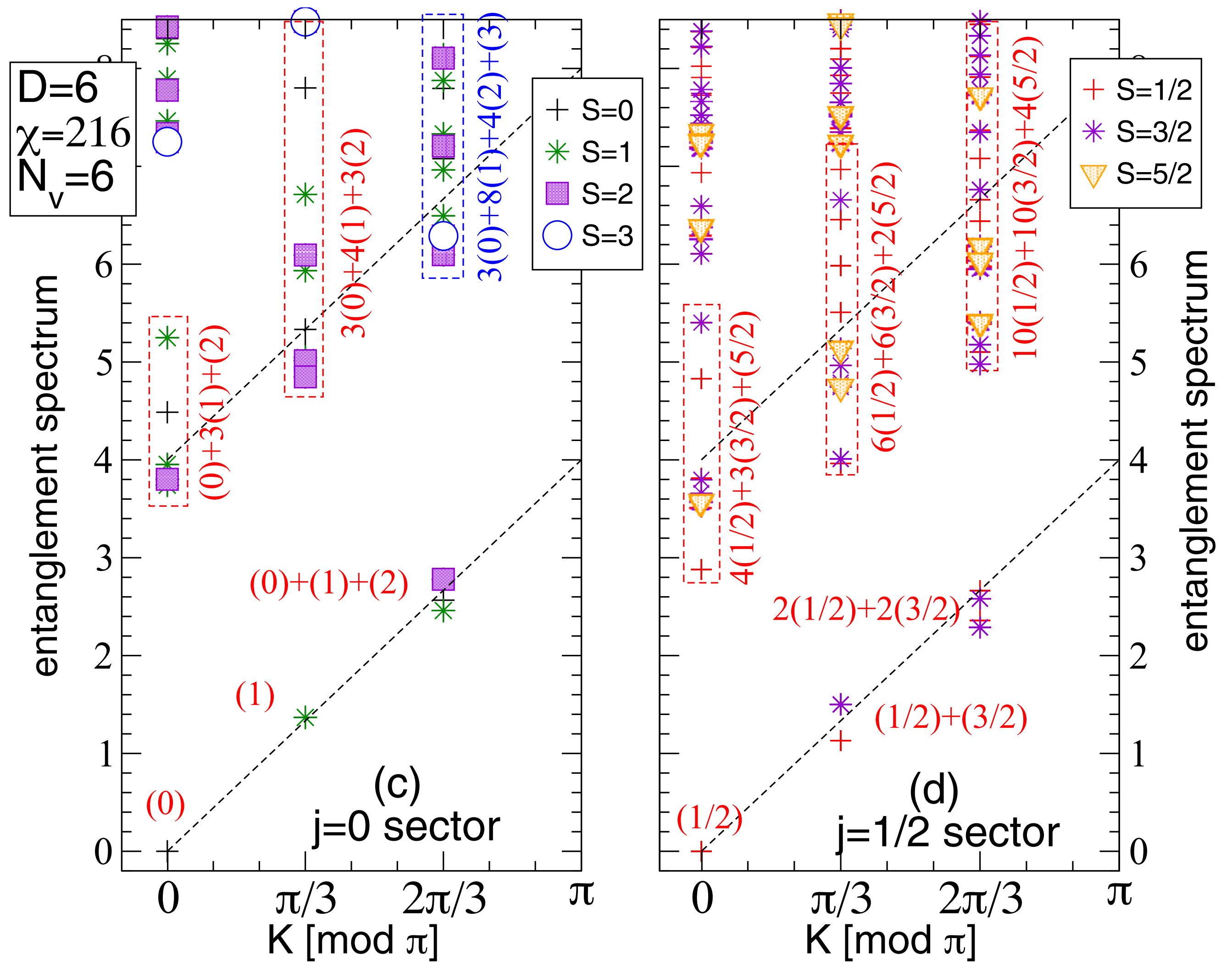
3 states

PEPS bulk correlation



gossamer critical tail

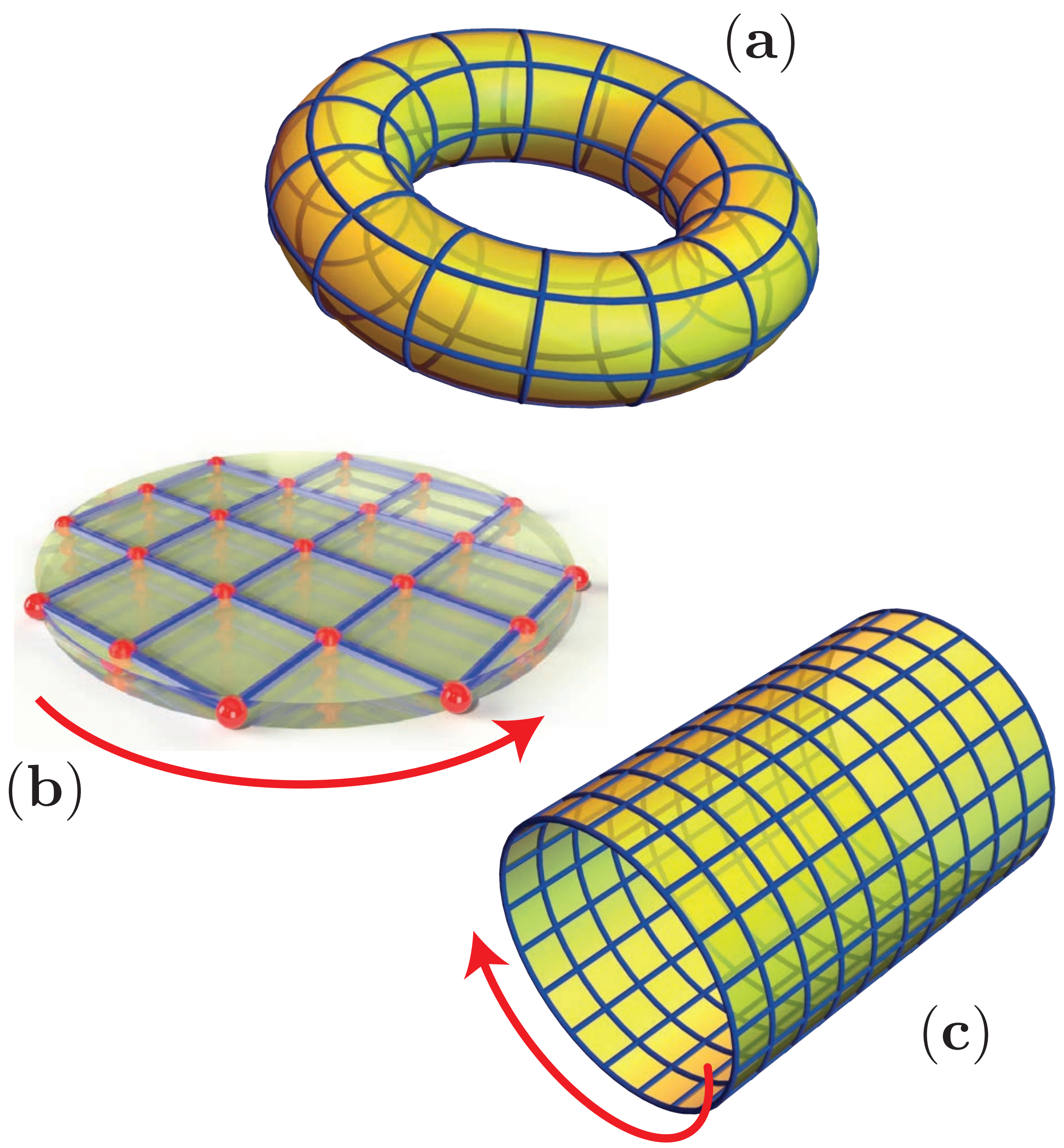
PEPS edge entanglement



agrees with **SU(2)$_2$** WZW

c=3/2

Conclusion and outlook



- Simple SU(N) spin models hosting topological chiral spin liquids
- Important to combine different numerical techniques to validate all properties
- Characterization of edge states and entanglement properties
- Also non-Abelian CSL with $SU(2)_2$, $SU(2)_3$, $SU(3)_2$, etc… edges physics

Ising anyons

Fibonacci anyons

allows universal computation

Collaborators/Refs:

- Ji-Yao Chen, L. Vanderstraeten, S. Capponi, D. Poilblanc, Phys. Rev. B 98, 184409 (2018)
- Ji-Yao Chen, S. Capponi, A. Wietek, M. Mambrini, N. Schuch, D. Poilblanc, Phys. Rev. Lett. 125, 017201 (2020)
- Ji-Yao Chen, Jheng-Wei Li, Pierre Nataf, Sylvain Capponi, Matthieu Mambrini, Keisuke Totsuka, Hong-Hao Tu, Andreas Weichselbaum, Jan von Delft, Didier Poilblanc, in preparation