Wavelet-based definition of turbulent dissipation

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WHAT IS DISSIPATION ?

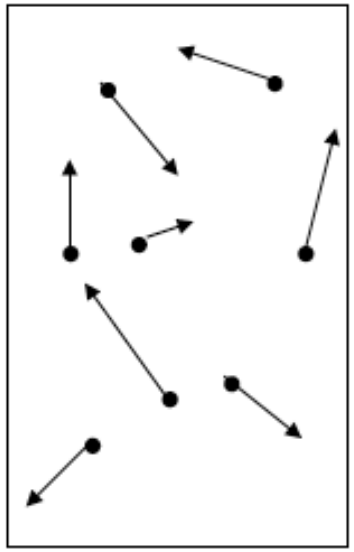
WHAT IS TURBULENCE ?

James Clerk Maxwell, 1877

'The notion of dissipated energy could not occur to a being who could trace the motion of every molecule and size it at the right moment. It is only to a being in the intermediate stage, who can hold of some forms of energy while others elude his grasp, that energy appears to be passing inevitably from the available to the dissipated state'

Maxwell, 'Diffusion', Encyclopedia Britannica, 1877

Molecular dissipation



- Lagrangian mechanics are insufficient to predict the behavior of the majority of many-particles systems.
- But equilibrium implies equivalence between a (very) large number of microscopic configurations.
- This principle of equivalence is sufficient to predict macroscopic properties at equilibrium!
- Statistical distributions compatible with this principle are entropy maxima.
- The phenomenon by which these equilibria are attained is called dissipation.

Dissipation is a matter of choice!

- The definition of dissipation depends on the definitions of equilibrium :
  - return to local equilibrium → collisional dissipation,
  - return to global equilibrium → fluid dissipation.
- Close to equilibrium, dissipation can be predicted by a linear theory (Onsager relations, …).
- Far from equilibrium, open questions remain:
  - does the standard dissipation still play a role?
  - is there another relevant dissipation?

Lewis Fry Richardson, 1930

'By an arbitrary choice we try to divide motions into two classes: those which we treat in detail, those which we smooth away by some process of averaging. Unfortunately these two classes are not always mutually exclusive. […] Diffusion is a compensation for neglect of details. The form of the law of diffusion depends entirely upon the arbitrary chosen method of averaging, which is always implied when diffusion or viscosity are mentionned. This calls attention to the desirability of making more explicit statements about smoothing operations than has hitherto been the custom;'

Richardson and Gaunt, 'Diffusion regarded as a compensation for smoothing', Memoirs Royal Met. Soc., 3, 30, 1930

G. I. Taylor, 1938

‘The fact that small quantities of very high frequency disturbances appear, and increase as the speed increases, seems to confirm the view frequently put forward by the author that the dissipation of energy is due chiefly to the formation of very small regions where the vorticity is very high’

Taylor, ‘The spectrum of turbulence’, Proc. Royal Soc. London A, 164, 1930

$\Longrightarrow$ We will focus on the vorticity field.

Hugh Dryden, 1948

‘The rapidly developing theory of random functions may possibly form the mathematical framework of an improved theory of turbulence. However, it is necessary to separate the random processes from the nonrandom processes. It is not yet fully clear what the random elements are in turbulent flows.’

Dryden, ‘Recent advances in the mechanics of boundary layer flow’, Adv. Applied Mech., ***1****, 1948*

$\Longrightarrow$ We propose to use the wavelet representation.

What is turbulence?

Turbulence is a state that **fluid flows** reaches when they become **unstable** and **highly fluctuating**.

Etymology of the word 'turbulence' *:*

turba-ae $\Longrightarrow$ *crowd, mob,*

turbo-inis $\Longrightarrow$ *vortex.*

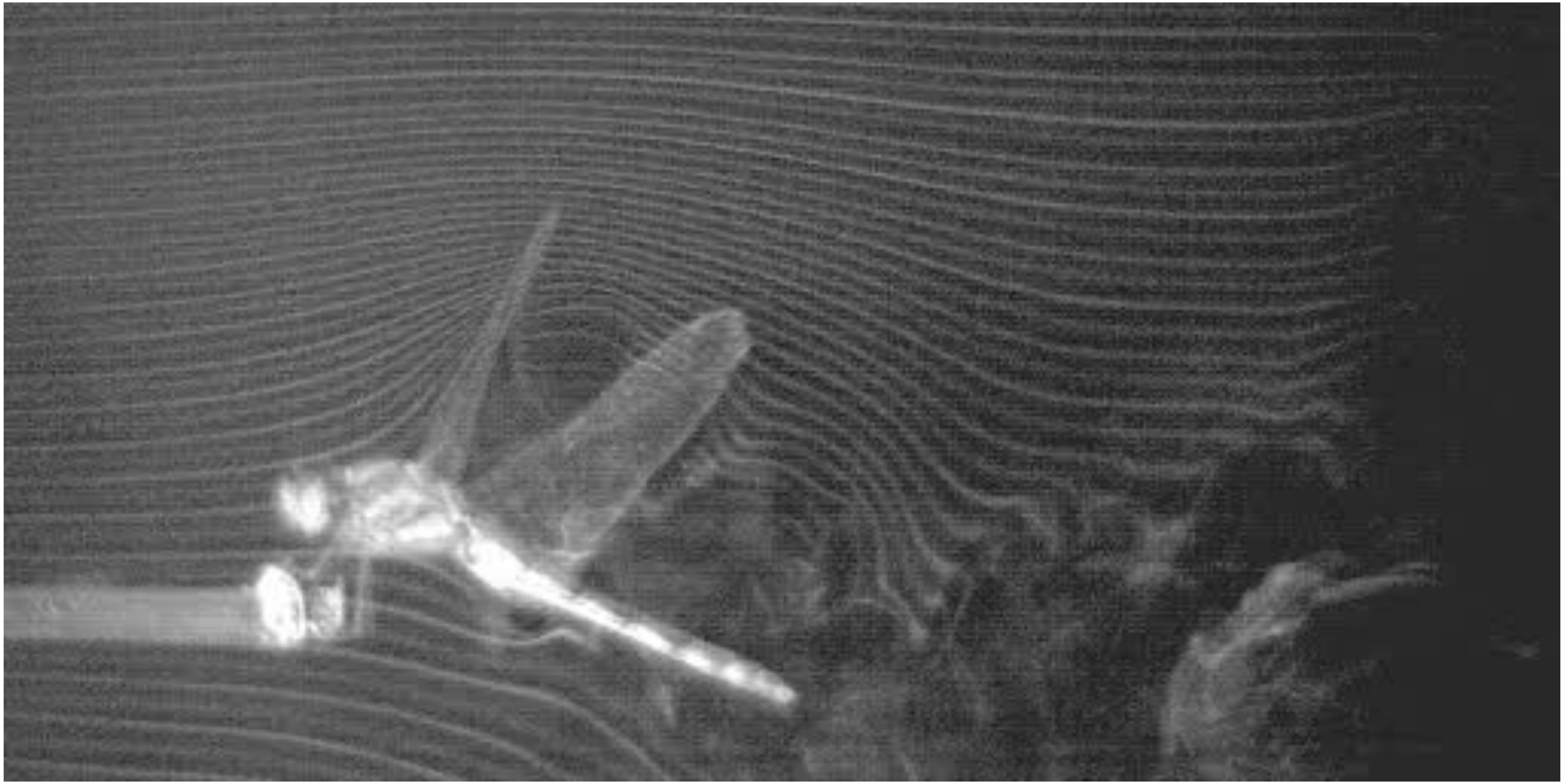
A turbulent flow is a **mob of vortices interacting together** on a wide range of temporal and spatial scales.

Hypotheses *:*

- The **fluid** is supposed to be a **continuous medium** since the scale at which one observes it is much larger than the mean free path of molecules,
- The **fluid flow** is supposed to be **incompressible**, *i.e.*, non-divergent.

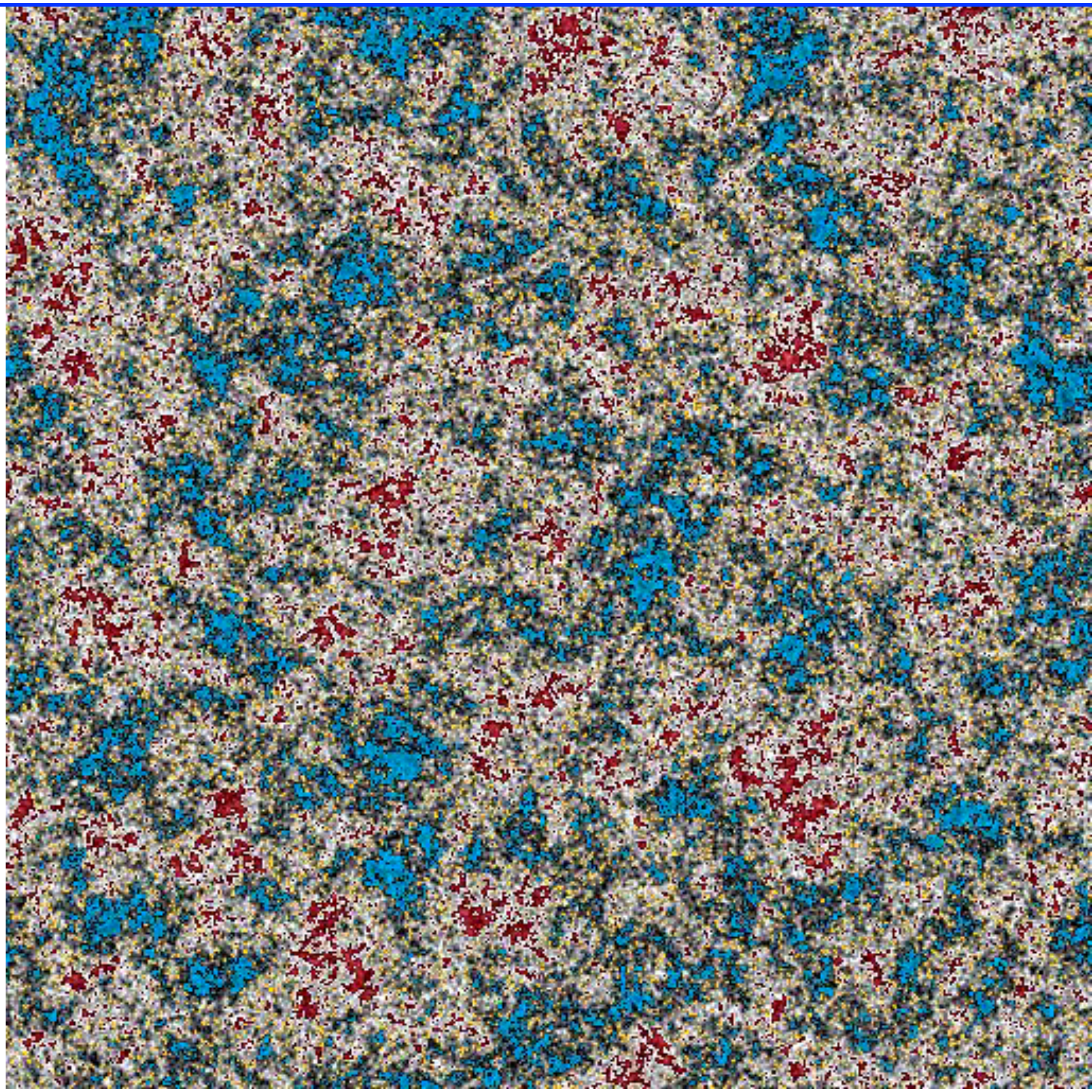
Fluid flows reach the **fully-developed regime** when they become **highly mixing**, which corresponds to **strong turbulence**.

Oberserved forced 3D turbulent flow



Simulated 2D decaying turbulent flow

Resolution N=512^2



Time evolution of the vorticity field from random initial conditions

ω *min* ω *max*

WHAT ARE WAVELETS ?

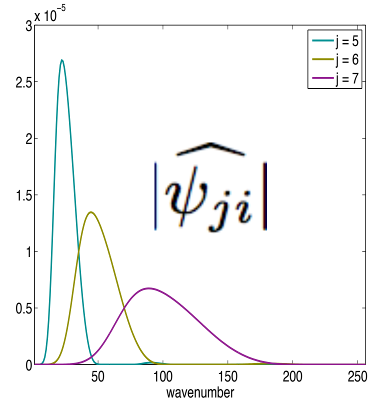
HOW TO DECOMPOSE TURBULENT FLOWS USING THEM ?

Continuous / orthogonal wavelets

<table>
<tr><td colspan="2">Analyzing functions are
translates and dilates
of an oscillating function (of zero mean) </td></tr>
<tr><td colspan="2">Well localized in both space and wavenumber</td></tr>
<tr><td colspan="2">$$\tilde{f}(l, \vec{x}) = \langle \psi_{l,\vec{x}} | f \rangle$$</td></tr>
<tr><td>Continuous wavelets
$$\psi_{l,\vec{x}}(x') = \frac{1}{l^{n/2}} \psi(\frac{\vec{x}' - \vec{x}}{l})$$</td><td>Orthogonal wavelets
$$\psi_{j,i}(x') = 2^{j/2} \psi(2^j x' - i)$$</td></tr>
<tr><td>• Translates and dilates
vary continuously
• Redundant representation</td><td>• Translates and dilates are
on a discrete dyadic grid
• Orthogonal basis</td></tr>
<tr><td>• Coefficients are easy to read
• Unfold in both space and scale
• For analysis</td><td>• Coefficients not easy to read
• sampled on a dyadic grid
• For filtering and compression</td></tr>
</table>

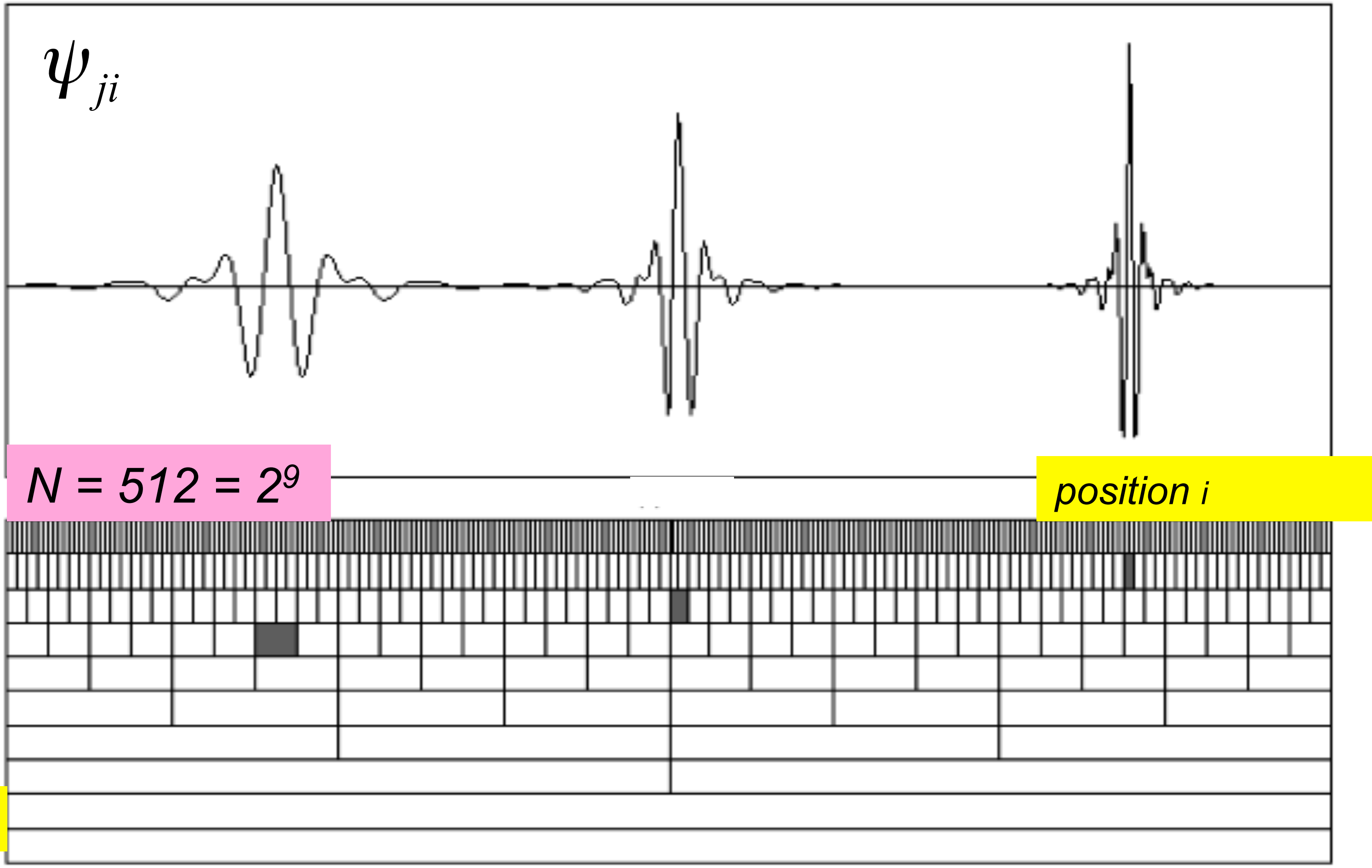
Orthogonal wavelet representation

Wavelets



wavelet coefficients

$$\tilde{f}_{ji} = \langle \psi_{ji} | f \rangle$$



Mallat, 2008
A wavelet tour of signal processing, 3rd edition, Academic Press

3D orthogonal wavelets

- **fast algorithm** with **linear complexity**
- **no redundancy** between the coefficients

A 3D vector field v(x) sampled on $N = 2^{3J}$ *equidistant grid points*

$\psi_\lambda(\boldsymbol{x})$ *3D wavelet* $\longrightarrow$ *orthogonal wavelet series*

$$\boldsymbol{v}(\boldsymbol{x}) = \sum \widetilde{\boldsymbol{v}}_\lambda \psi_\lambda(\boldsymbol{x}), \quad \widetilde{\boldsymbol{v}}_\lambda = \langle \boldsymbol{v}, \psi_\lambda \rangle$$

$$\Lambda = \{\lambda = (j, i_n, \mu),\ j = 0, ..., J-1,\ i_n = 0, ..., 2^j - 1, n = 1, 2, 3,\ \text{and}\ \mu = 1, ..., 7\}$$

$N_j = 7 \times 2^{3j}$, *wavelet coefficients at a scale indexed by j*

We use Coifman 12 wavelet, which are
compactly supported with four vanishing moments.

Wavelet-based diagnostics

- Local intermittency measure

$$I(l,x) = \frac{\left|\tilde{f}(l,x)\right|^2}{\left\langle \left|\tilde{f}(l,x)\right|^2 \right\rangle_x}$$

- Local Reynolds number

M. F., 1992,
Ann.Rev.Fluid Mech.,
***24**, 395-457*

$$R_e(l,x) = \frac{|\overline{v}(l,x)|l}{\nu}$$

- Local Rossby number

$$R_o(l,x) = \frac{|\overline{v}(l,x)|l}{2\Omega l}$$

OR

$$R_o(l,x) = \frac{\overline{\omega}(l,x)}{2\Omega}$$

How to decompose turbulent flows?

'In 1938 Tollmien and Prandtl suggested that turbulent fluctuations might consist of two components, a diffusive and a non-diffusive. Their ideas that fluctuations include both random and non random elements are correct, but as yet there is no known procedure for separating them.'

Hugh Dryden, Adv. Appl. Mech., 1, 1948

mean + turbulent fluctuations
= mean + non random + random
= mean + coherent structures + incoherent noise

⟹ *Coherent Vorticity Extraction (CVE)*

turbulent dynamics
= chaotic non diffusive + stochastic diffusive
= inviscid nonlinear dynamics + turbulent dissipation

⟹ *Coherent Vorticity Simulation (CVS)*

M. F., 1992
Ann. Rev. Fluid Mech., ***24***

M. F., Schneider, Kevlahan, 1999,
Phys. Fluids, ***11*** *(8)*

M. F., Pellegrino, Schneider, 2001
Phys. Rev. Lett., ***87*** *(5)*

How to extract coherent structures?

Since there is not yet a universal definition of coherent structures
which emerge out of turbulent fluctuations,
we adopt an apophetic method :
instead of defining what they are, we define what they are not.

For this we propose the minimal statement :
'Coherent structures are not noise'

$\Longrightarrow$

Extracting coherent structures becomes a denoising problem,
not requiring any hypotheses on the structures themselves
but only on the noise to be eliminated.

Choosing the simplest hypothesis as a first guess,
we suppose we want to eliminate an additive Gaussian white noise,
and for this we use a nonlinear wavelet filtering.

M.F., Schneider et al., 2003
*Phys. Fluids, **15** (10)*

Azzalini, M. F., Schneider, 2005
*ACHA, **18** (2)*

Wavelet-based denoising algorithm

Apophatic method :

- *no hypothesis on the structures,*
- *only hypothesis on the noise,*
- *simplest hypothesis as our first choice.*

Hypothesis on the noise :

$$f_n = f_d + n$$

n Gaussian white noise,
$<f_n^2>$ variance of the noisy signal,
N number of coefficients of f_n.

Wavelet decomposition :

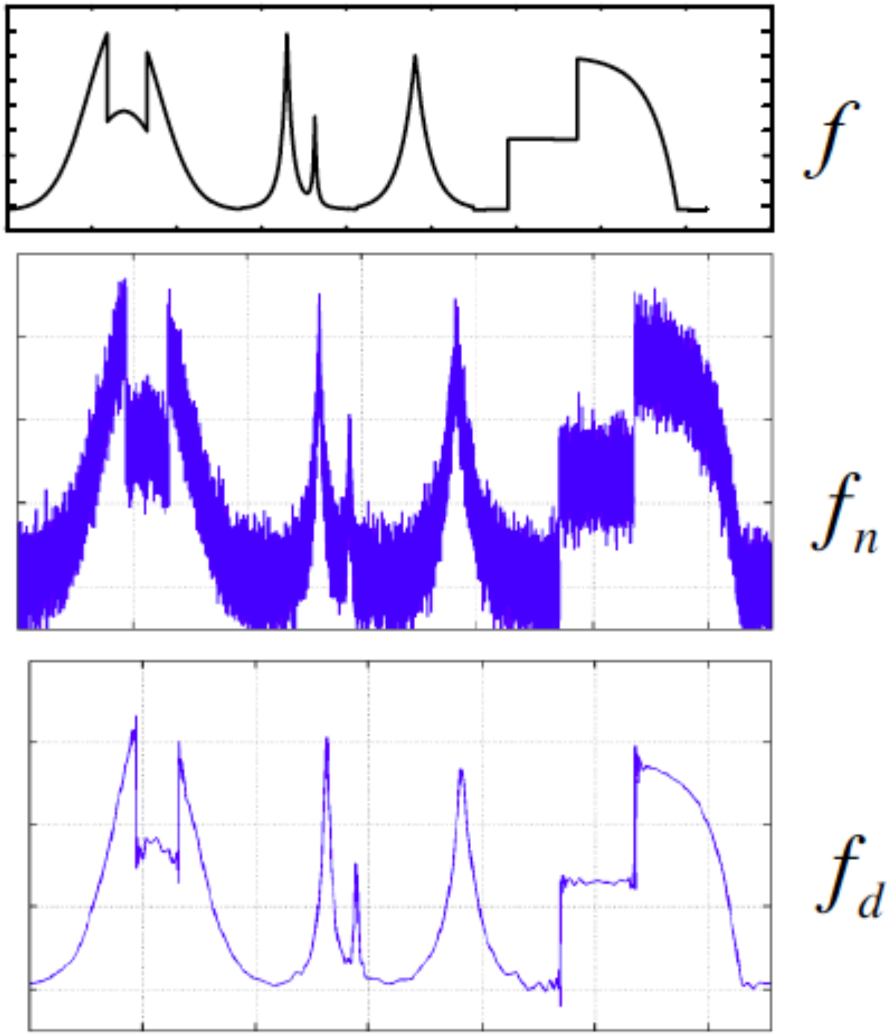
$$\tilde{f}_{ji} = < f \mid \psi_{ji} >$$ *j scale, i position*

Estimation of the threshold :

$$\varepsilon_n = \sqrt{2 < f_n^{\ 2} > \ \ln(N)}$$

Wavelet reconstruction :

$$f_d = \sum_{ji : \left| \tilde{f}_{ji} \right| > \varepsilon_n} \tilde{f}_{ji} \psi_{ji}$$



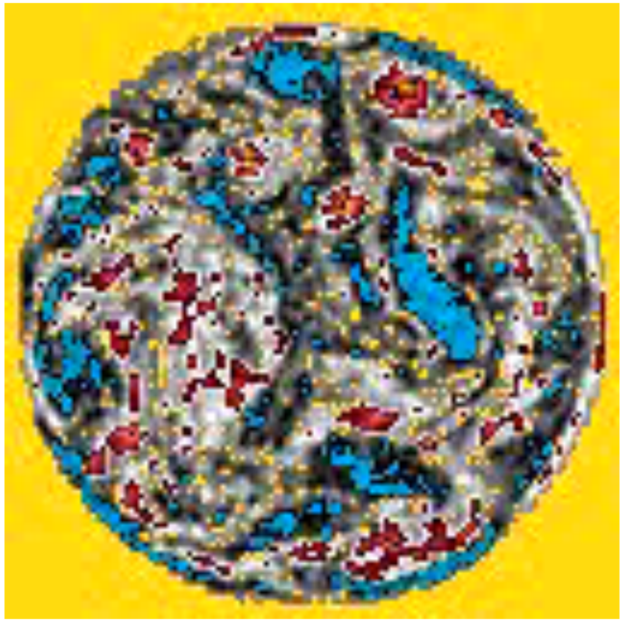
*Donoho, Johnstone, Biometrika, **81**, 1994*

*Azzalini, M. F., Schneider, ACHA, **18** (2), 2005*

WAVELET DECOMPOSITION OF 2D AND 3D TURBULENT FLOWS ?

Wavelet filtering of an experimental rotating tank flow

PIV
$N=128^2$

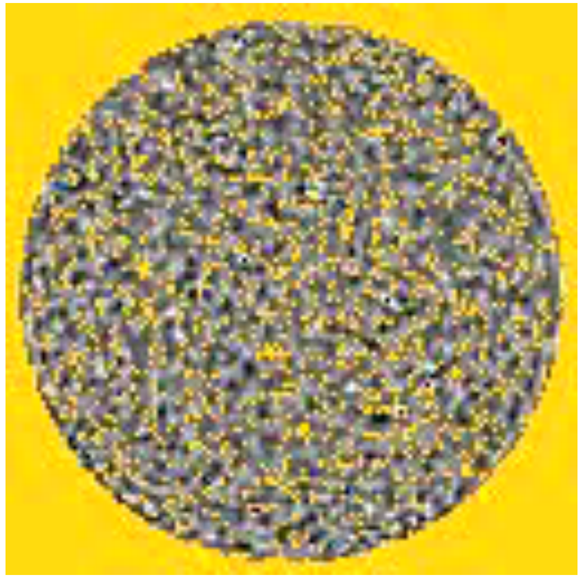


Total vorticity
100% E
100% Z

2% N

Coherent vorticity
99% E
80% Z

98% N



Incoherent vorticity
1% E
3% Z

ω_{imin} ω_{max}

A posteriori proof of coherence

PIV
N=128^2

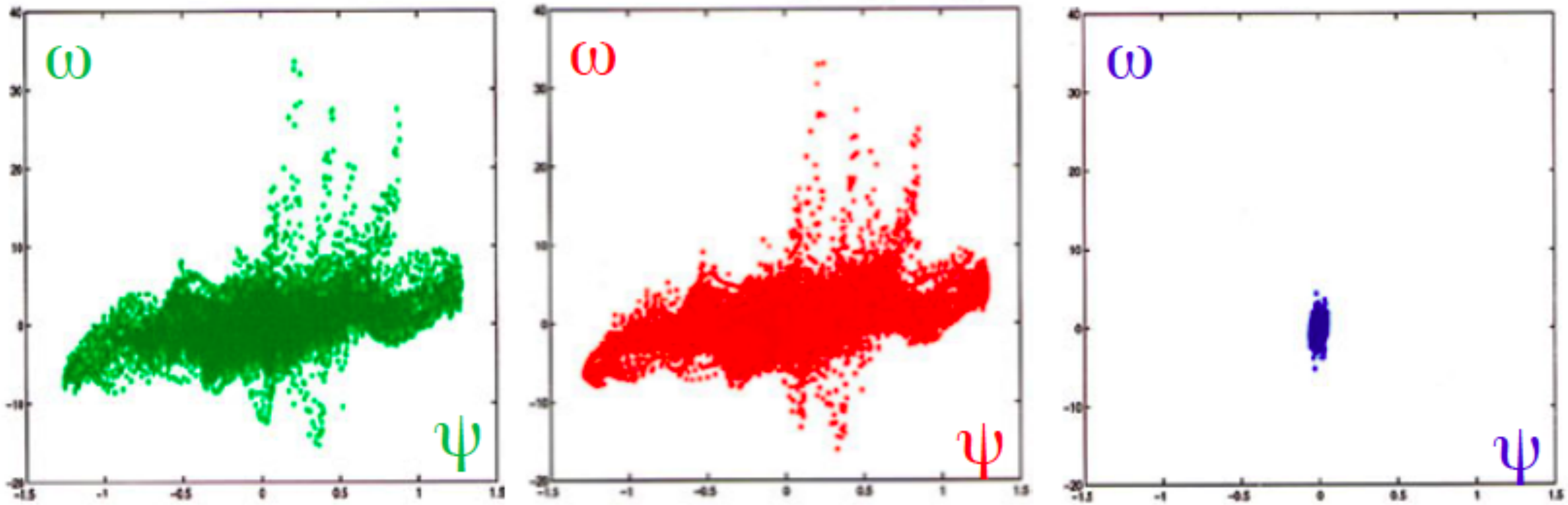
Coherent structures are locally (in space and time) steady solutions of Euler equation, thus, for 2D flows:

$$\omega = f(\psi)$$

Depletion of nonlinearity in coherent structures

Arnold, 1965,
Joyce & Montgomery, 1973
Robert & Sommeria, 1991

Total **Coherent** **Incoherent**

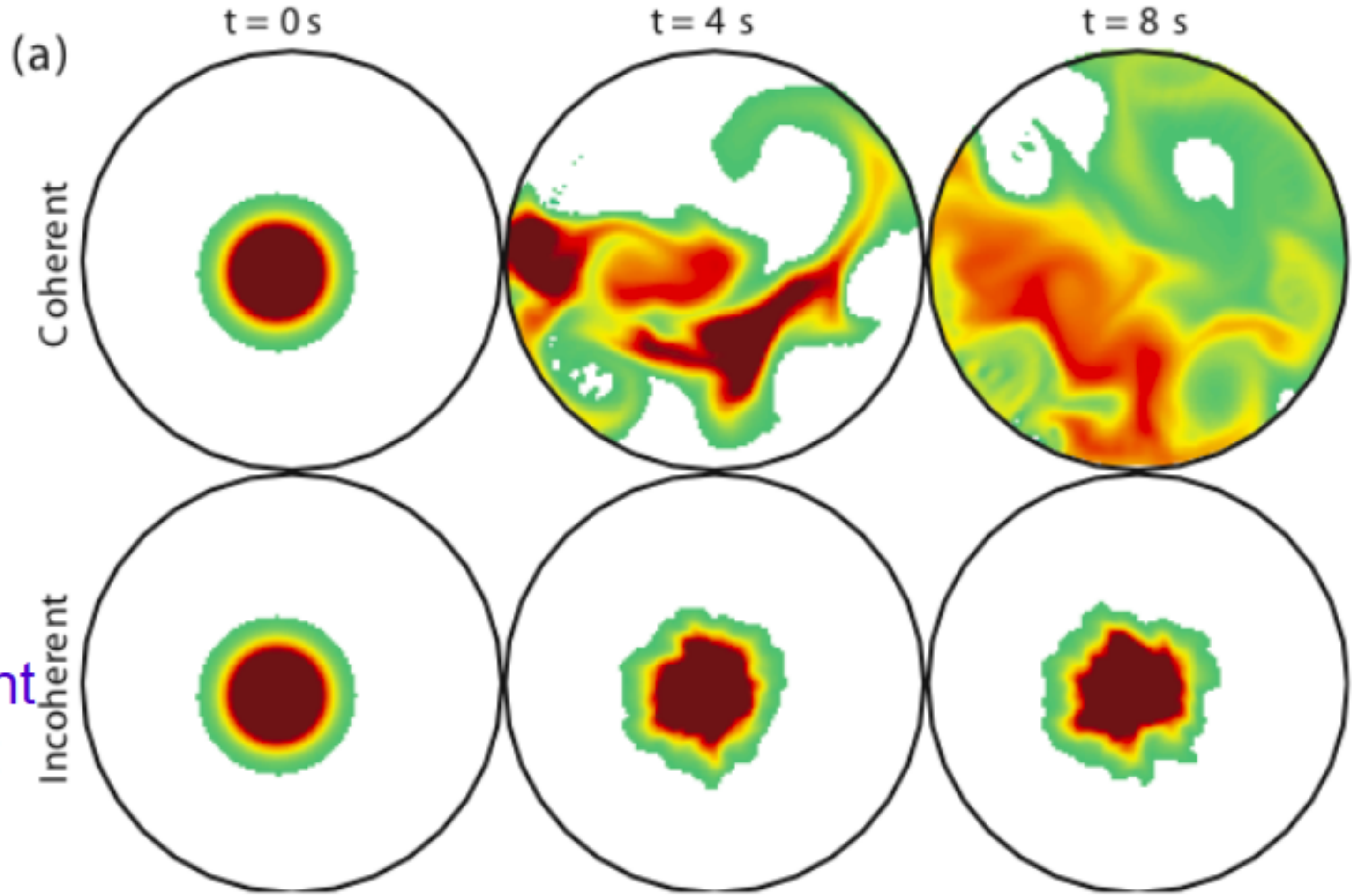


Passive scalar advection
in laboratory experiment

PIV
N=128^2

Transport by the coherent vortices :

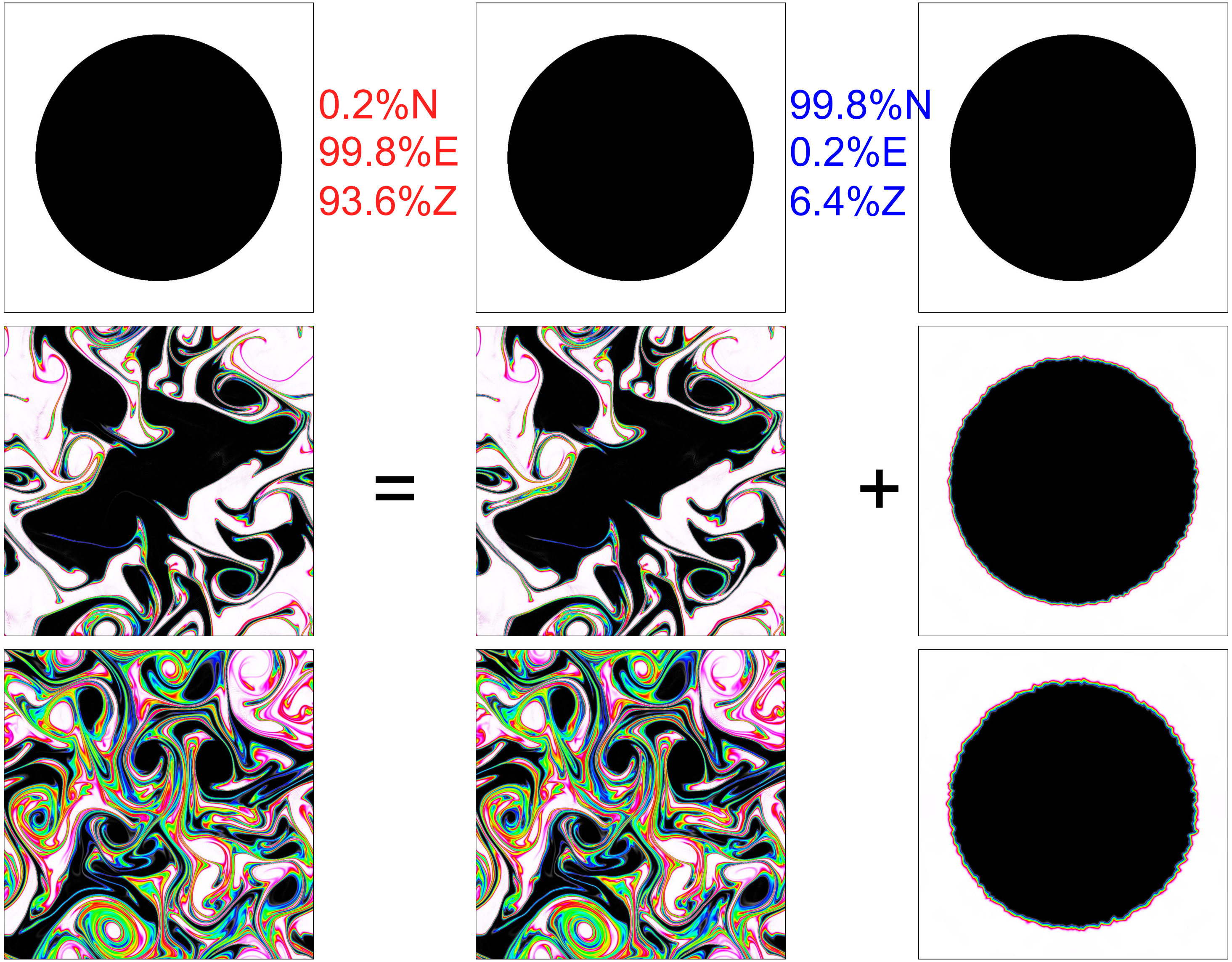
Diffusion by the incoherent background :



Transport and diffusion of a passive scalar

DNS
$N=512^2$

Transport by the coherent flow and turbulent dissipation by the incoherent flow



Total flow | Coherent flow | Incoherent flow

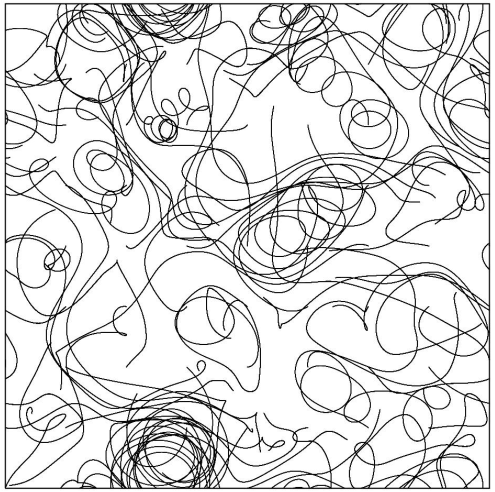
Beta, Schneider, M.F., 2003, Nonlinear Sci. Num. Simul., 8

Advection of tracer particles

DNS
N=128^2

0.2 %	*of coefficients*	*99.8 %*	*of coefficients*
99.8 %	*of kinetic energy*	*0.2 %*	*of kinetic energy*
93.6 %	*of enstrophy*	*6.4 %*	*of enstrophy*

by the total flow = **by the coherent flow** + **by the incoherent flow**



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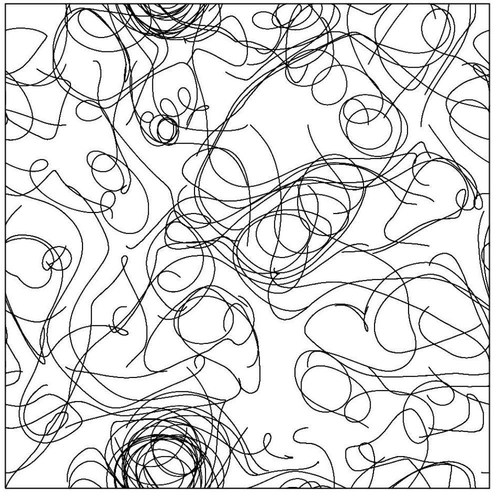

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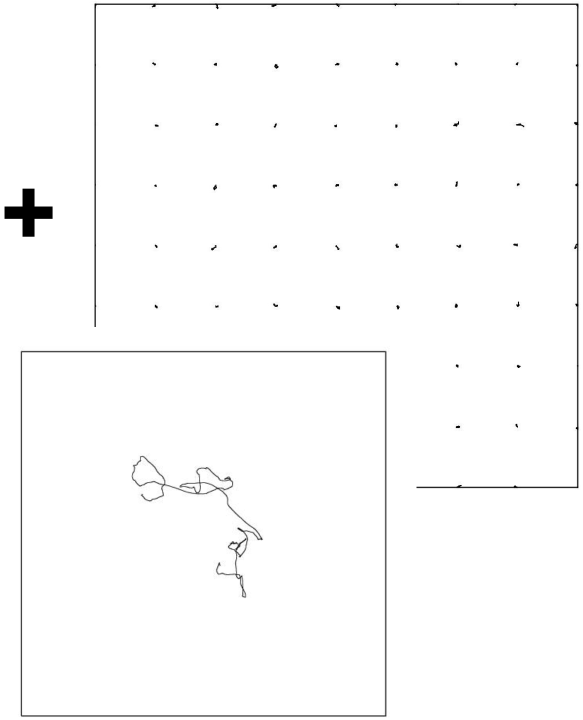


Transport by vortices

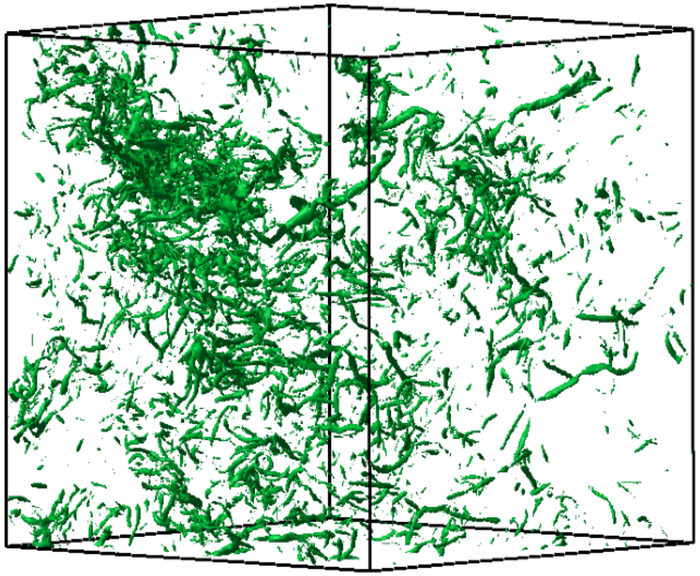
Beta,Schneider, M.F. 2003, Nonlinear Sci. Num. Simul., ***8***

Diffusion by Brownian motion

Wavelet filetring of an homogeneous 3D turbulent flow

DNS
N=2048^3

$|\omega|=5\sigma$ with $\sigma=(2Z)^{1/2}$



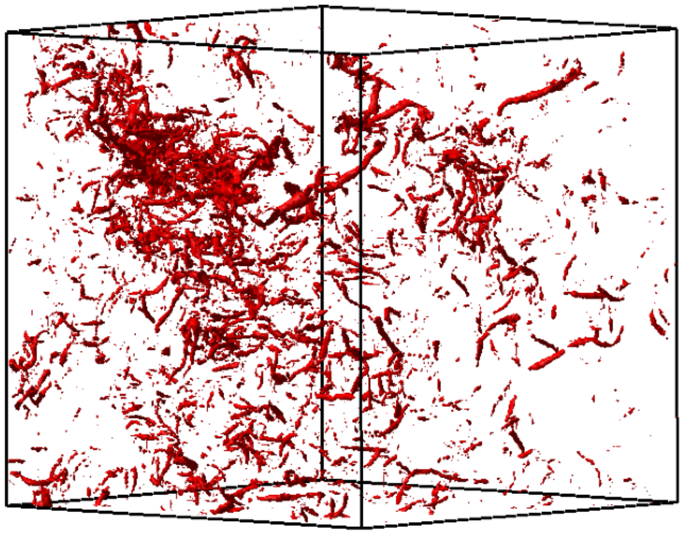
Coherent vorticity

2.6 % N coefficients
80% enstrophy
99% energy

Incoherent vorticity

97.4 % N coefficients
20 % enstrophy
1% energy

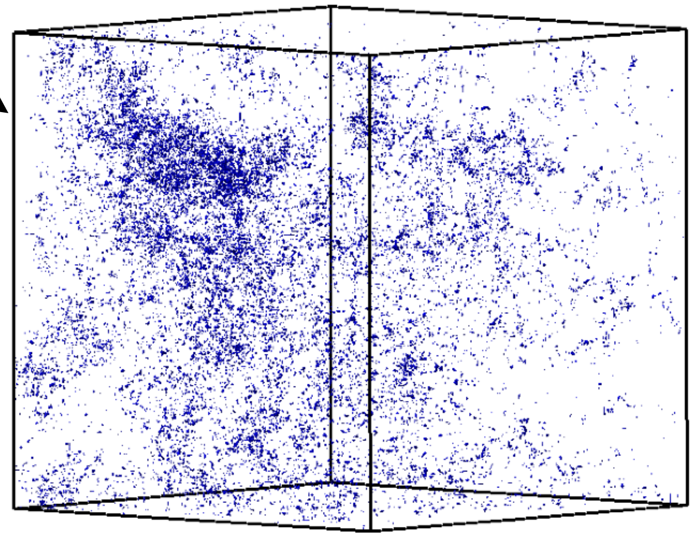
Total vorticity



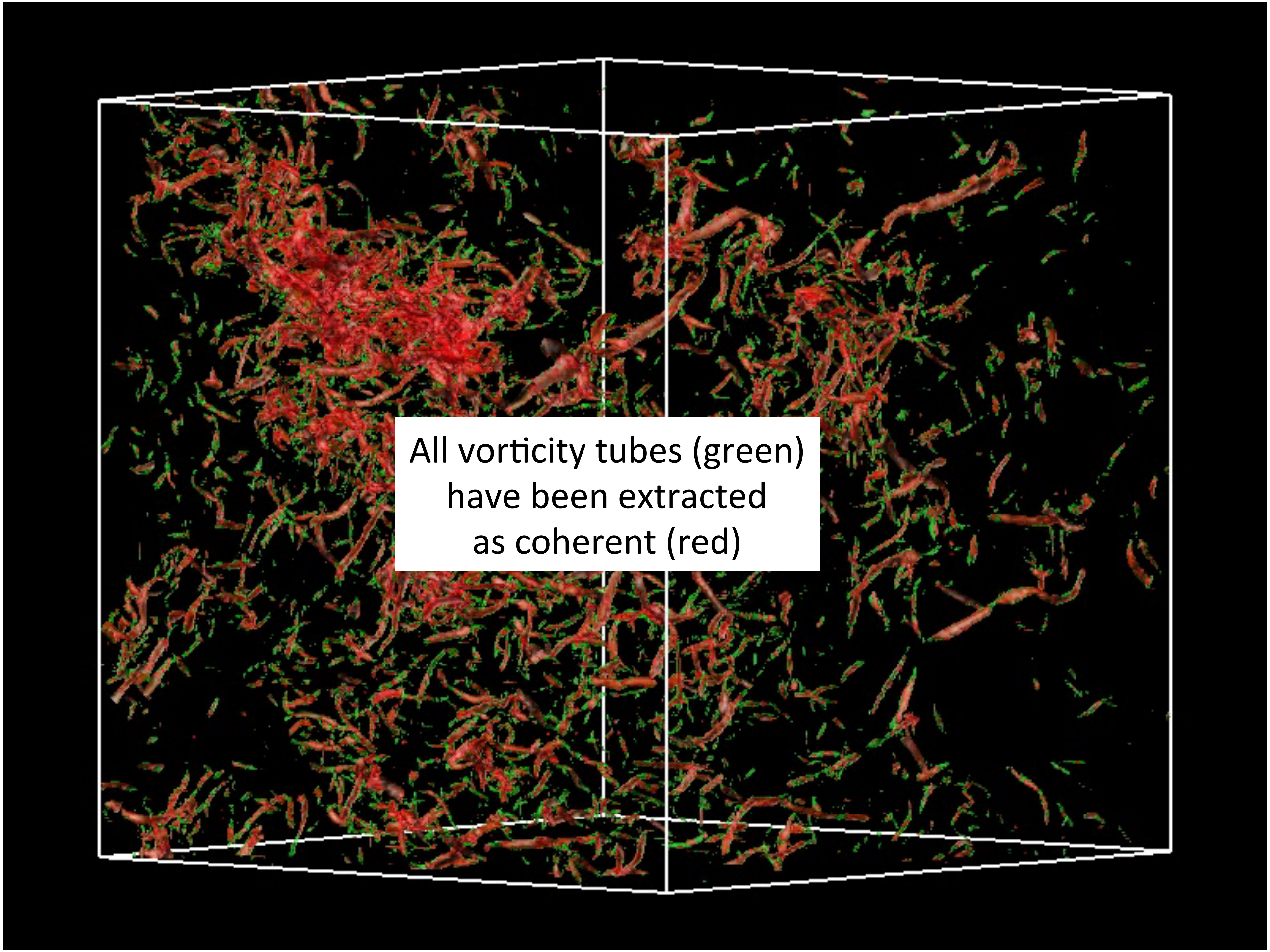
$|\omega|=5\sigma$

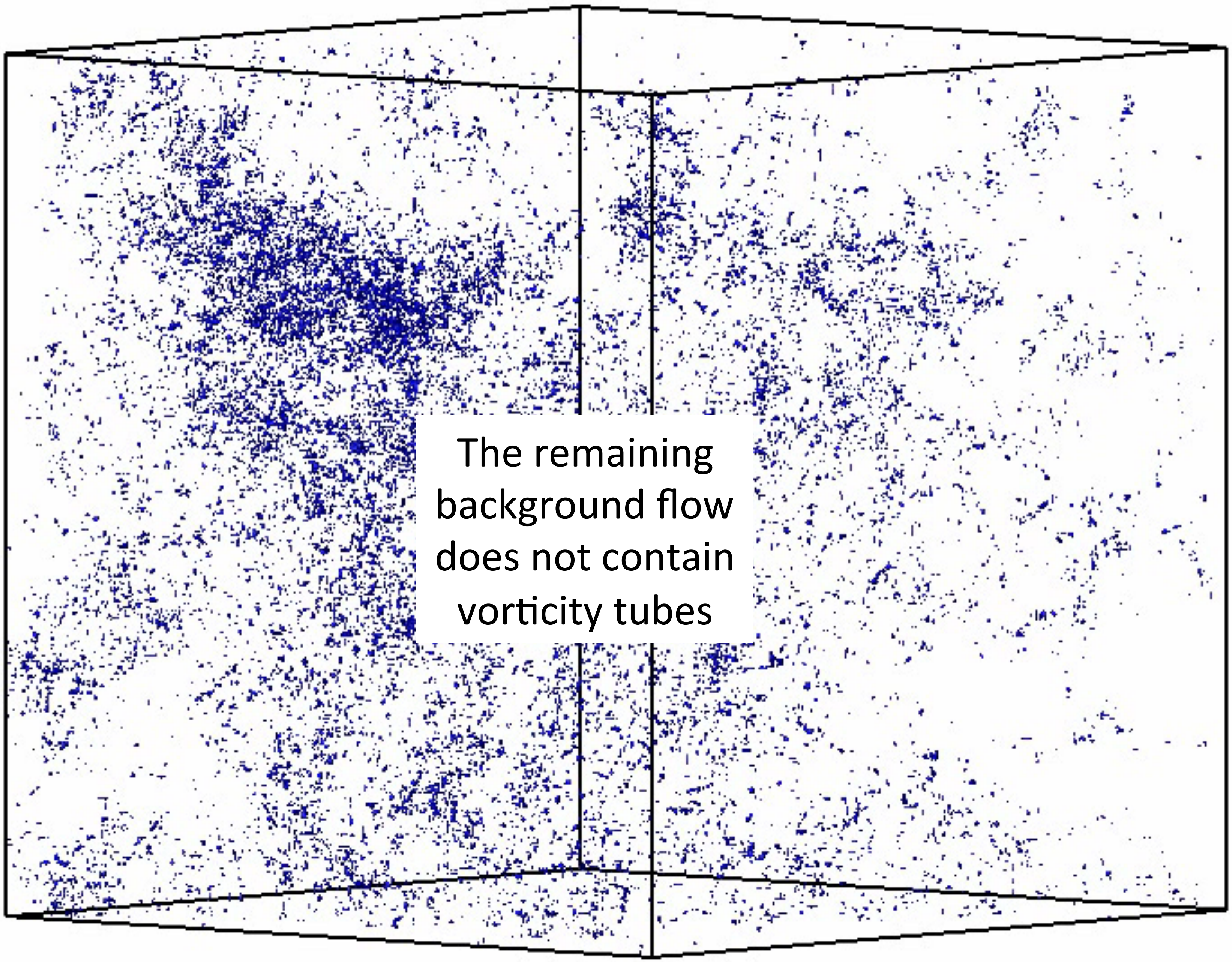
+

*Okamoto, Yoshimatsu, Schneider, M.F., Kaneda, 2007, Phys. Fluids, **19**, 1159*



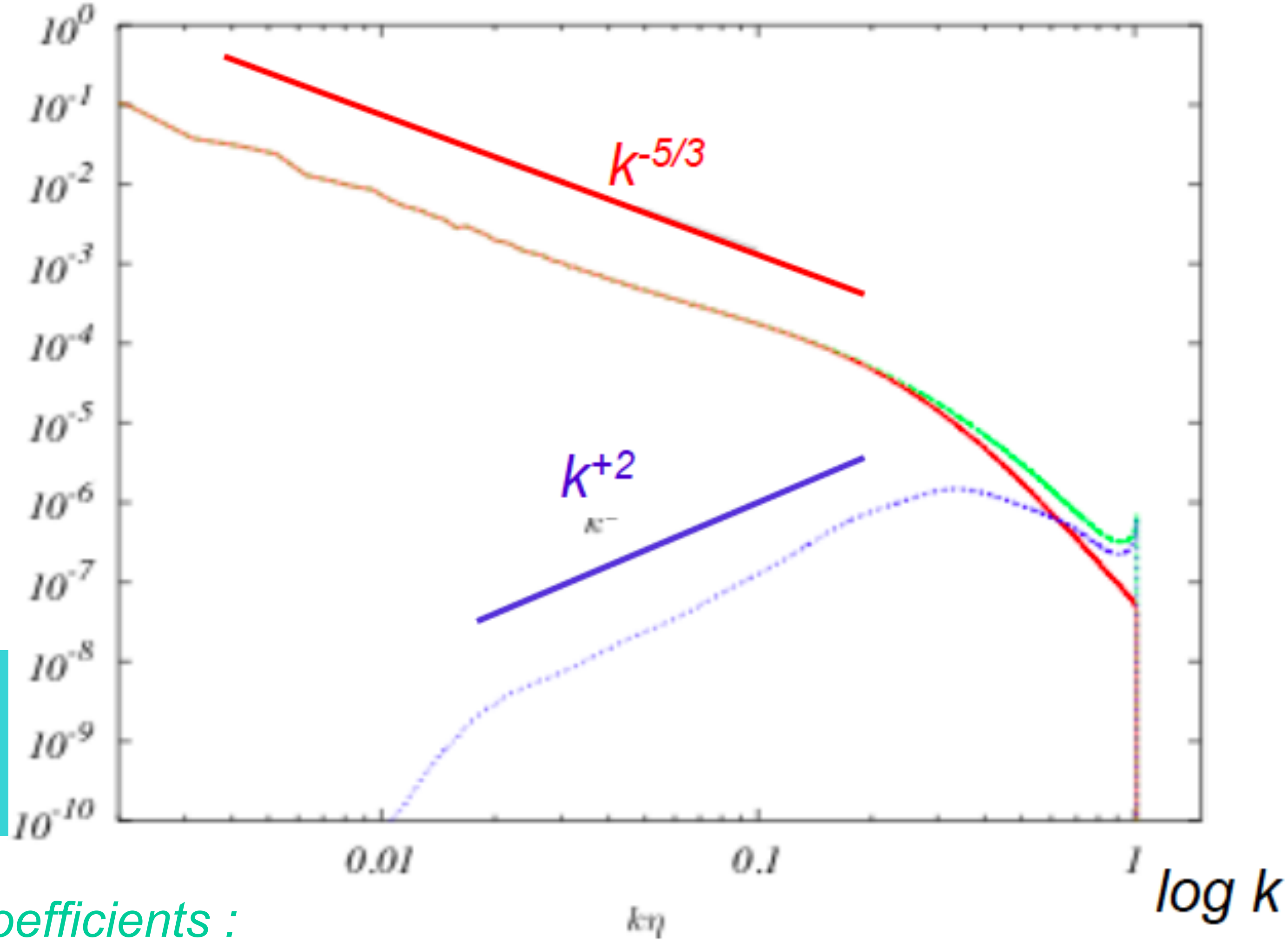
$|\omega|=5/3\sigma$


All vorticity tubes (green)
have been extracted
as coherent (red)


The remaining background flow does not contain vorticity tubes

Energy spectrum

DNS
$N=2048^3$



Okamoto, Yoshimatsu, Schneider, M.F., Kaneda, 2007,
Phys. Fluids, ***19****, 1159*

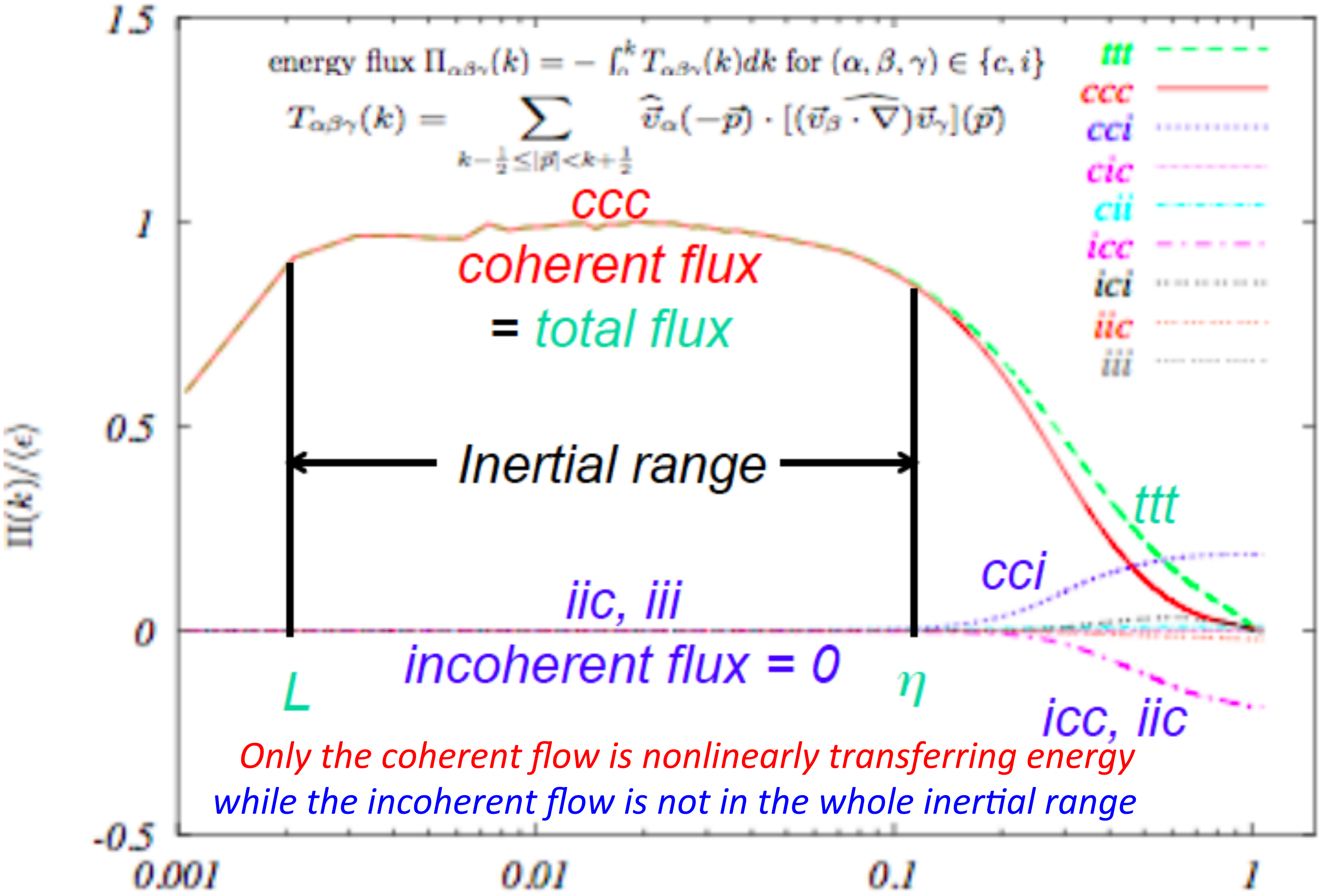
Retained wavelet coefficients :

2.6 % N coefficients
80% enstrophy
99% energy

Multiscale Coherent
$k^{-5/3}$ scaling, i.e.
long-range correlation

Multiscale Incoherent
k^{+2} scaling, i.e.
energy equipartition

Nonlinear transfers and energy fluxes

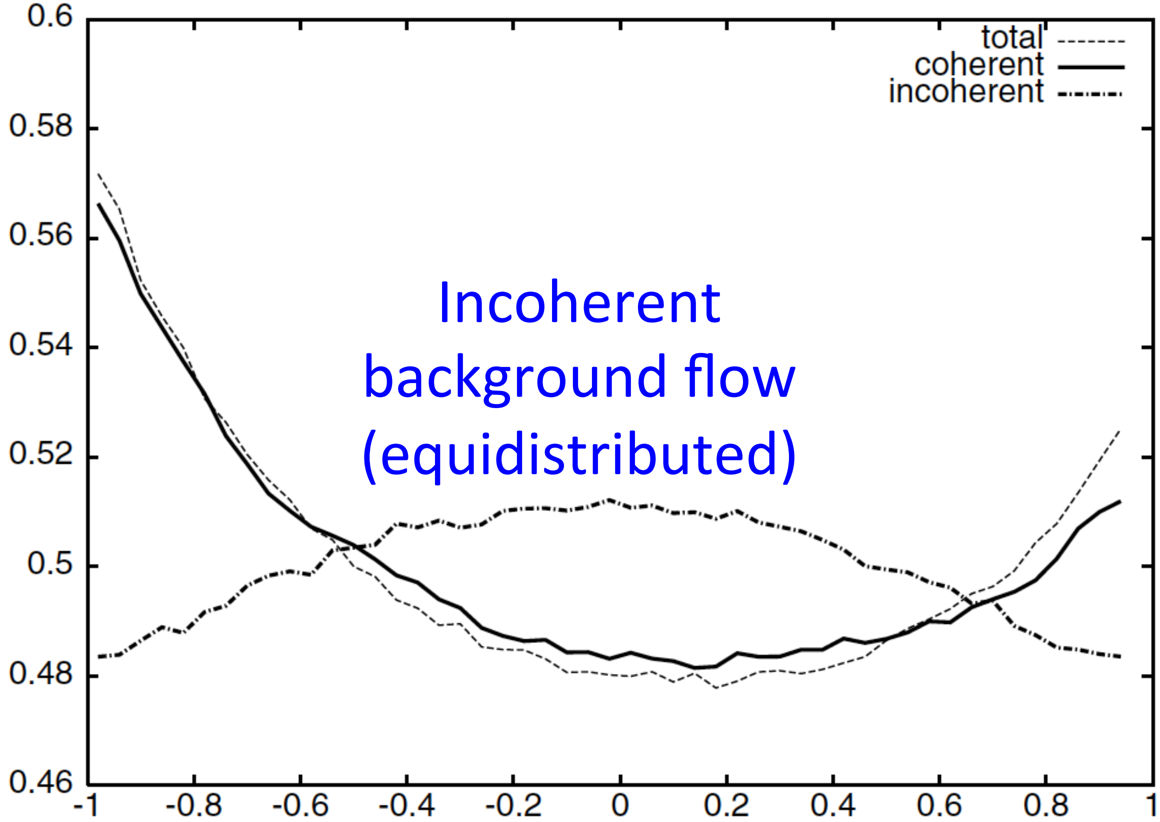


Okamoto, Yoshimatsu, Schneider, M.F., Kaneda, 2007, Phys. Fluids, **19**, 1159

PDF of relative helicity

Coherent
vortex tubes
with depletion
of nonlinearity
(peaked at |h|=1)

$$h = \frac{\vec{V} \cdot \vec{\omega}}{|\vec{V}||\vec{\omega}|}$$

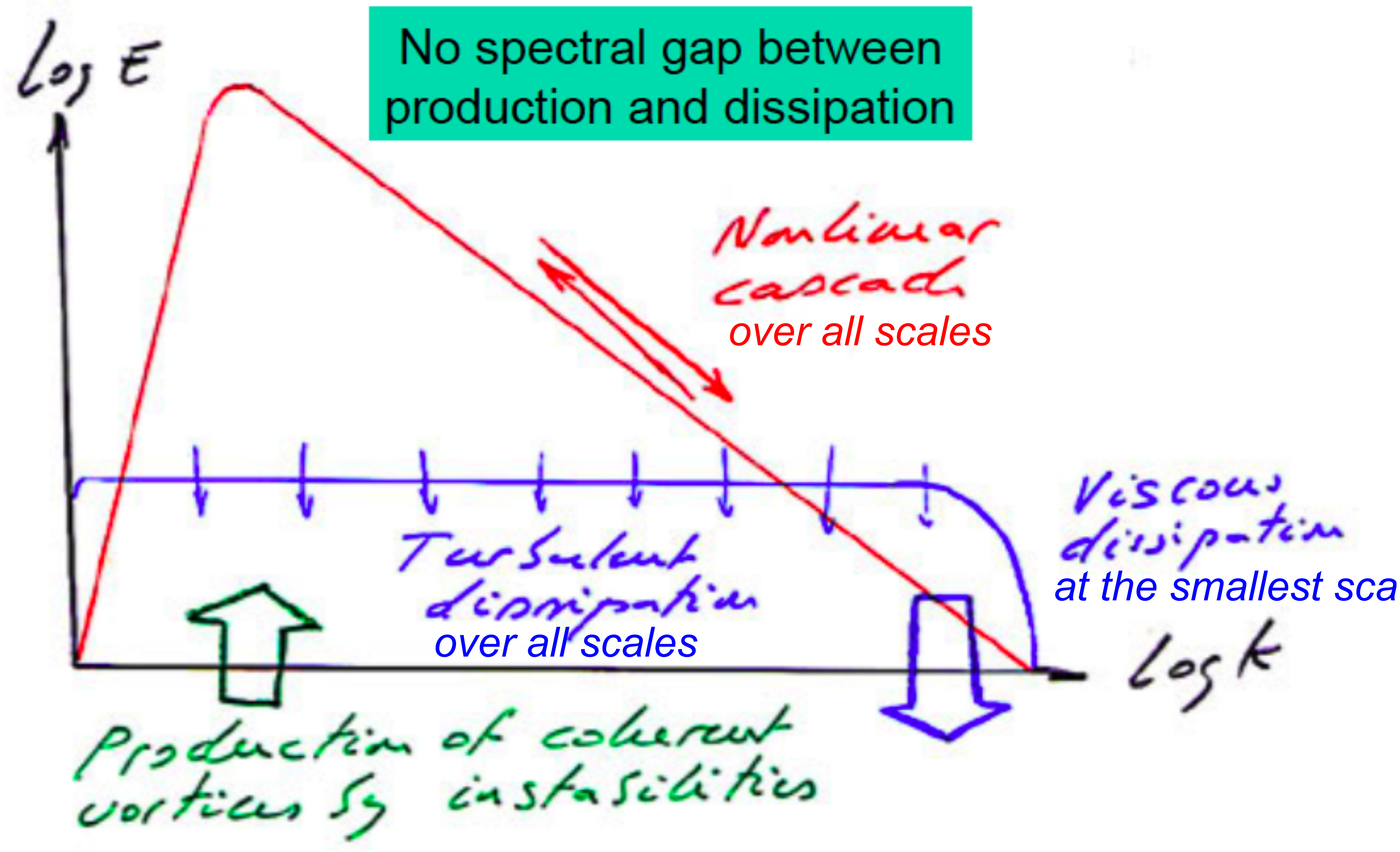


M. F., Pellegrino and Schneider, 2001,
'Coherent vortex extraction in 3D turbulent flows using orthogonal wavelets',
*Phys. Rev. Lett.., **87**(5), 2001*

CONCLUSION AND PERSPECTIVES

New interpretation of the turbulence cascade

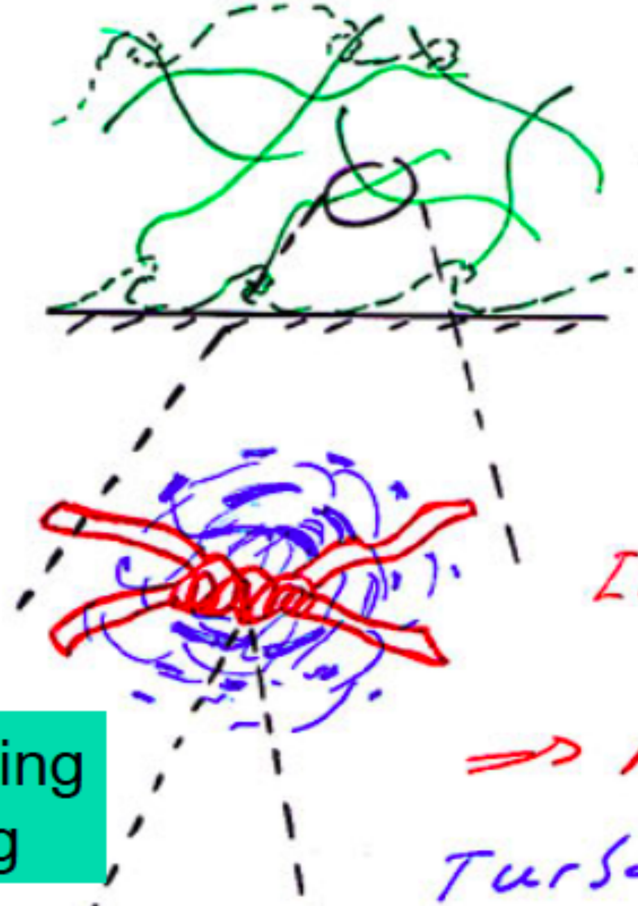
Fourier space viewpoint



New interpretation of the turbulence cascade

Physical space viewpoint

No vortex fission 'a la Richardson'



PRODUCTION

Webs of vortex tubes produced by instabilities in shear layers

INERTIAL RANGE

Intermittent vortex interactions

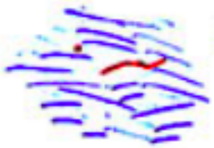
Vortex stretching and bursting

⇒ Nonlinear cascade

+

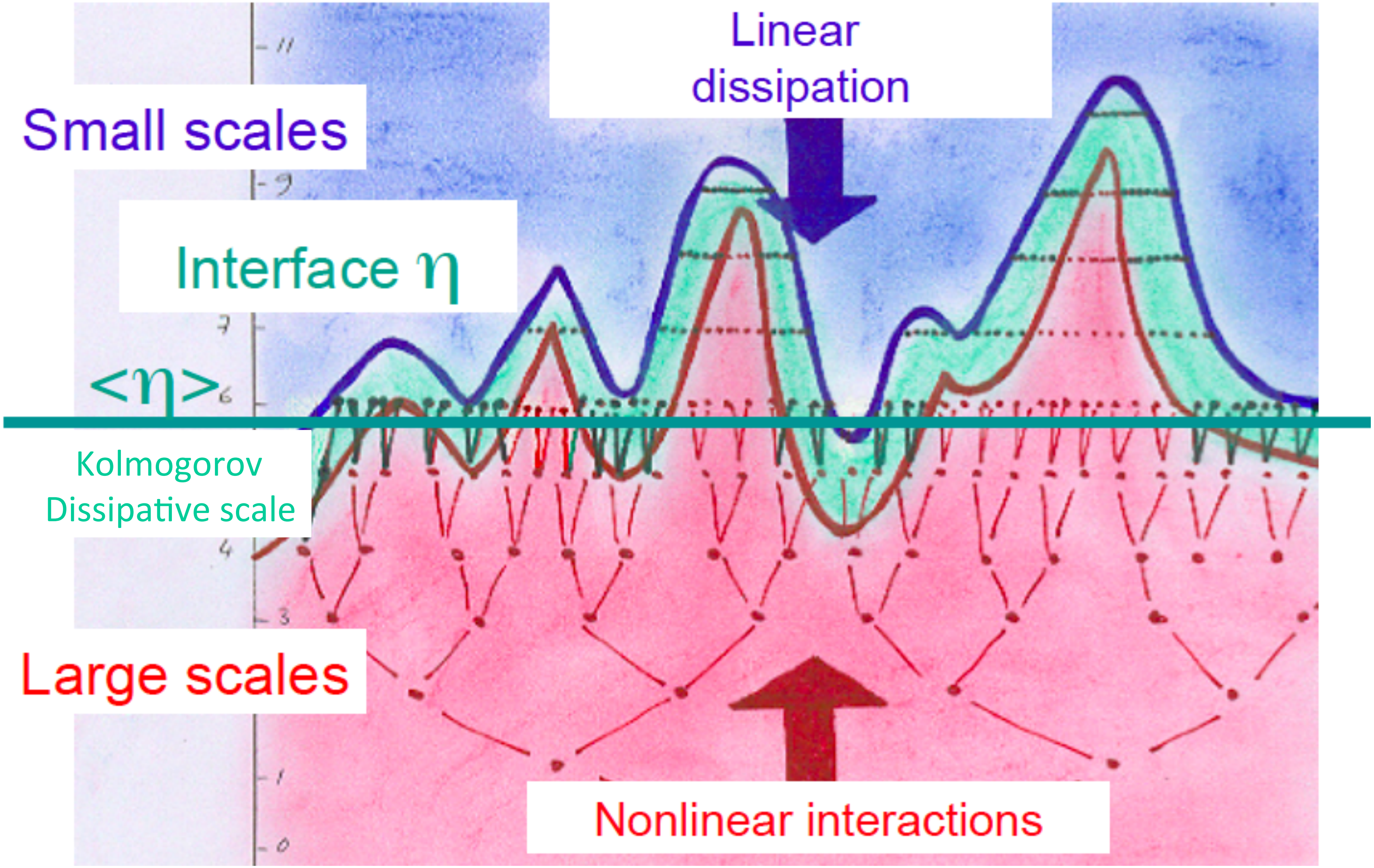
Turbulent dissipation

production of background noise which is damped in the viscous range

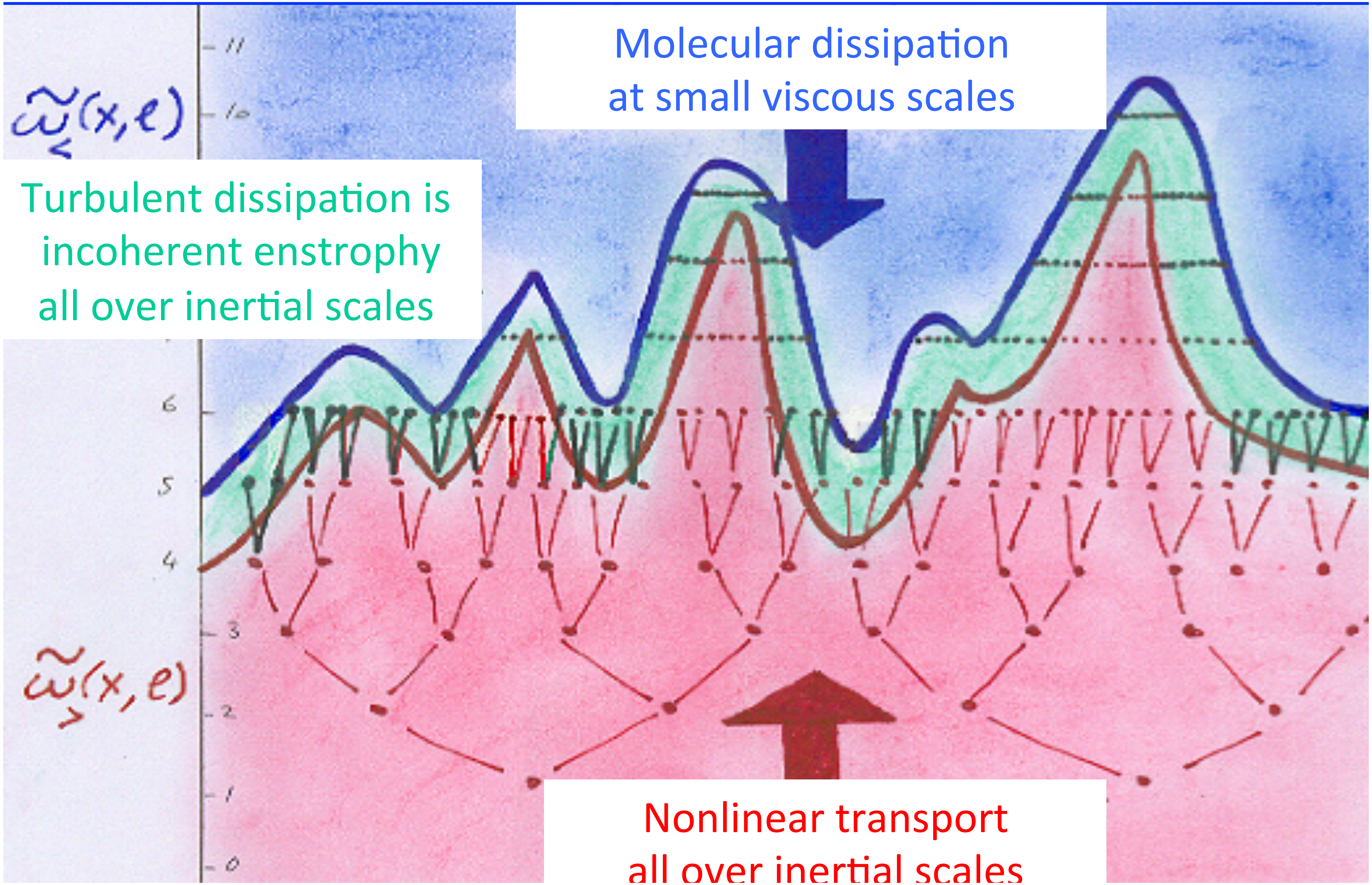


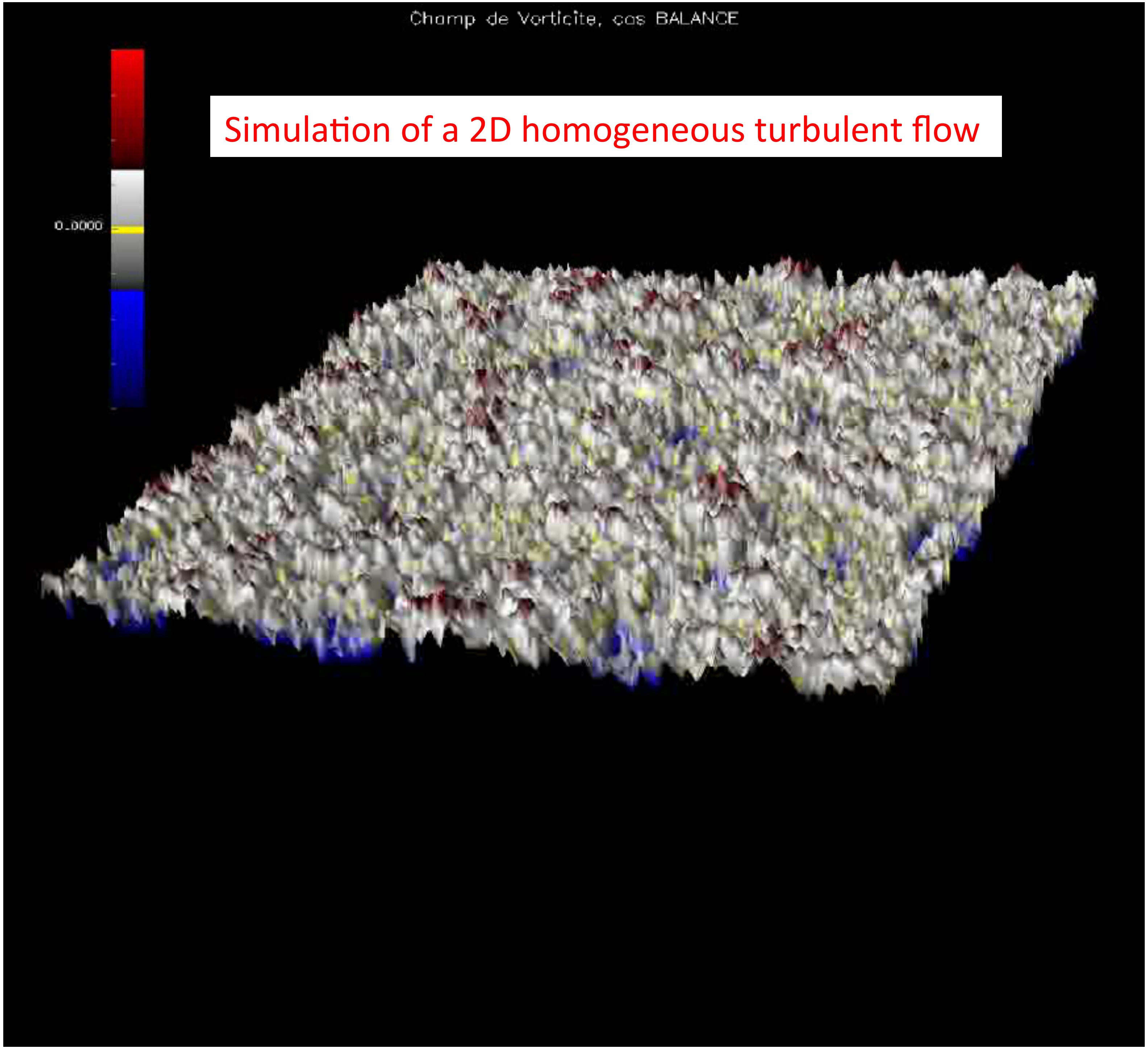
New interpretation of turbulence cascade

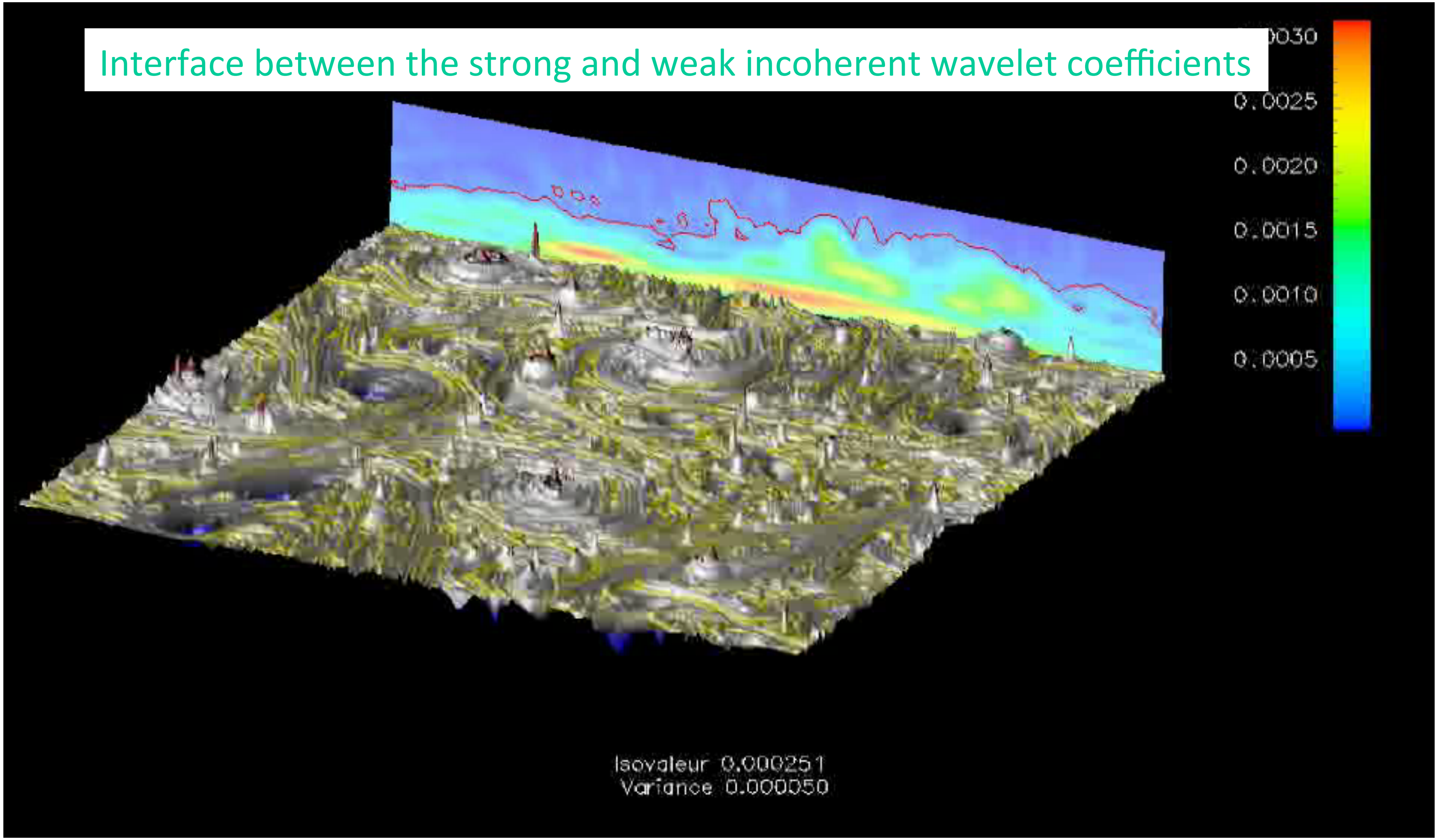
Wavelet space viewpoint



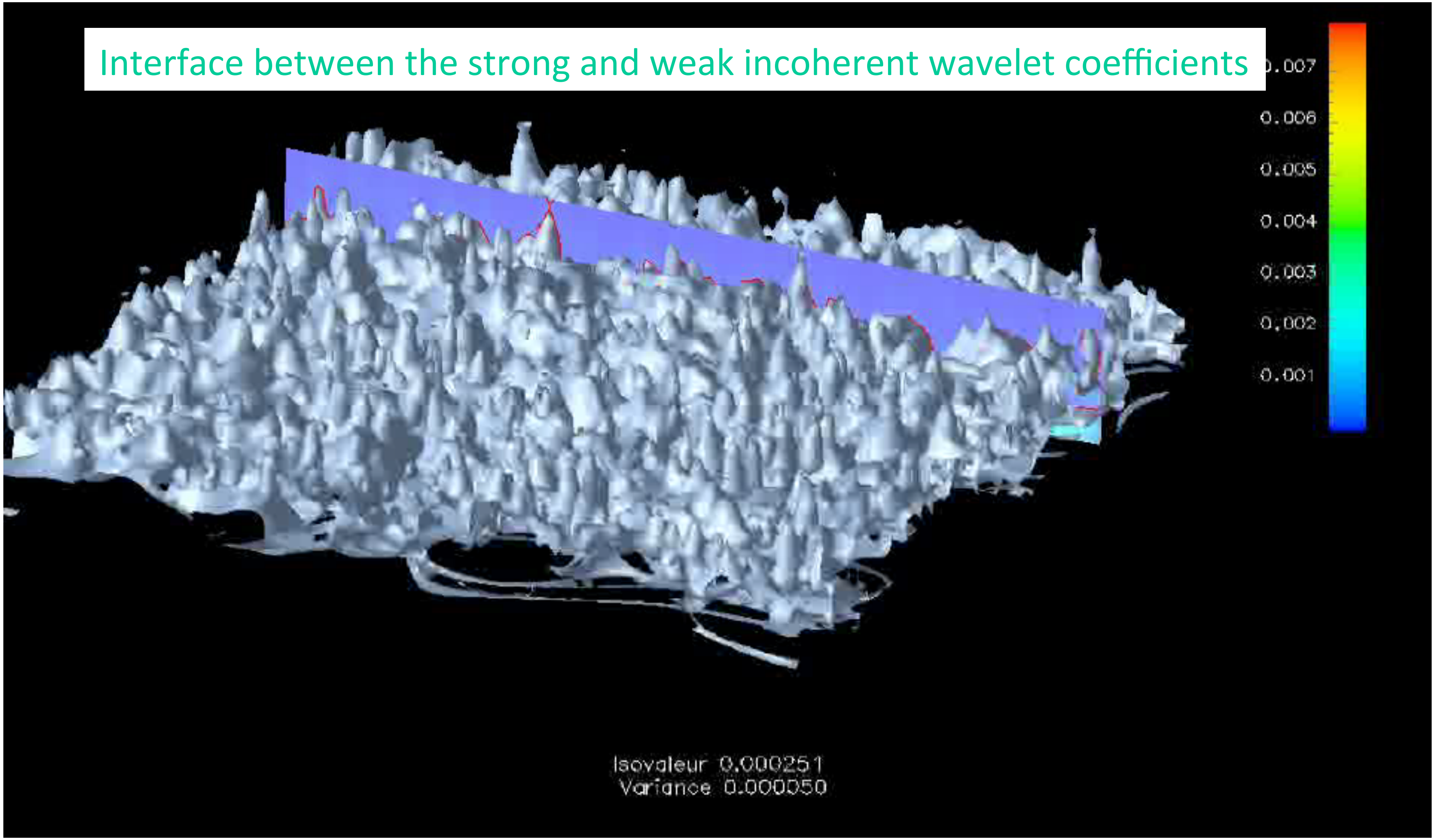
Wavelet-based definition of turbulent dissipation




Champ de Vorticite, cas BALANCE
Simulation of a 2D homogeneous turbulent flow
0.0000



The strong wavelet coefficients are below the interface
and correspond to the flow generated by the coherent vortices
The weak wavelet coefficients are above the interface
and correspond to the incoherent dissipative background flow



The strong wavelet coefficients are below the interface
and correspond to the flow generated by the coherent vortices
The weak wavelet coefficients are above the interface
and correspond to the incoherent dissipative background flow

‘We conjecture that turbulent flows can be described as a superposition of metastable coherent vortices that are not in statistical equilibrium. Their nonlinear interactions are responsible for the chaotic behaviour of turbulent flows and generate a random incoherent flow, which then relaxes towards statistical equilibrium and is dissipated at the smallest scales.’

M. F., Pellegrino and Schneider, 2001,
‘Coherent vortex extraction in 3D turbulent flows using orthogonal wavelets’,
*Phys. Rev. Lett.., **87**(5), 2001*

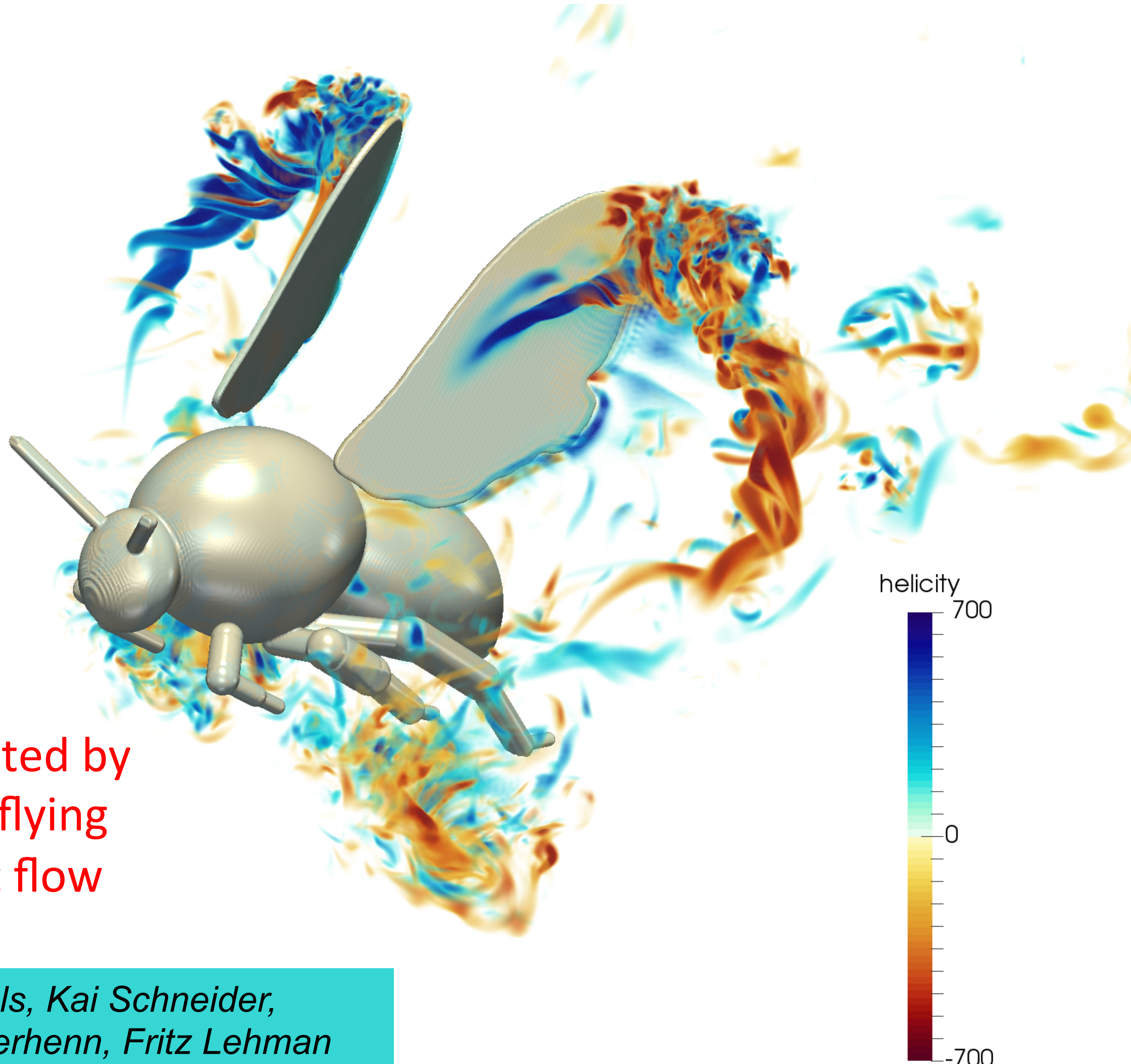
‘We conjecture that the wavelet representation, formulated in terms of both space and scale, allows such a decoupling between organized motions out of statistical equilibrium and random motions in statistical equilibrium. Both components are multiscale but have different probability distributions and correlations.’

M. F., Pellegrino and Schneider, 2001,
‘Coherent vortex extraction in 3D turbulent flows using orthogonal wavelets’,
*Phys. Rev. Lett.., **87**(5), 2001*

‘This gives us incentives to extend the CVS method to compute three-dimensional Navier-Stokes equations in an adaptive wavelet basis, remapped at each time step to track the nonlinear vortex dynamics in both space and scale, as we have done for two-dimensional turbulent flows. The advantage of the CVS method is to combine an Eulerian representation of the solution in a wavelet basis with a Lagrangian strategy to adapt the basis in space and scale, to track the formation, advection, and dissipation of vortex tubes whatever their scales.’

M. F., Pellegrino and Schneider, 2001,
‘Coherent vortex extraction in 3D turbulent flows using orthogonal wavelets’,
*Phys. Rev. Lett.., **87**(5), 2001*

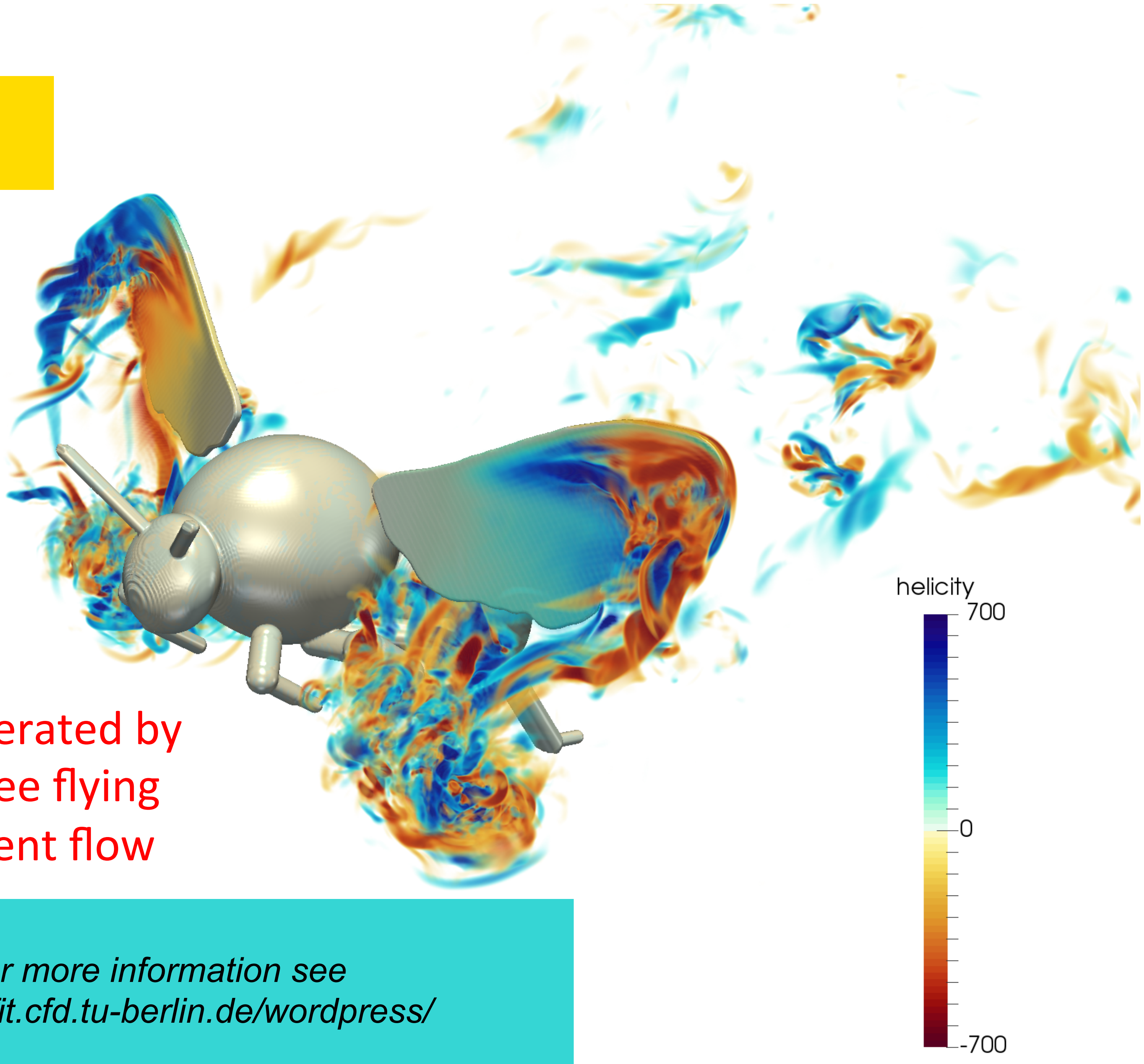
DNS
N=680/10^6



Helicity generated by
a bumblebee flying
in a turbulent flow

Thomas Engels, Kai Schneider,
M. F., Joern Sesterhenn, Fritz Lehman
Collaboration ANR-AIFIT

DNS
$N=680.10^6$

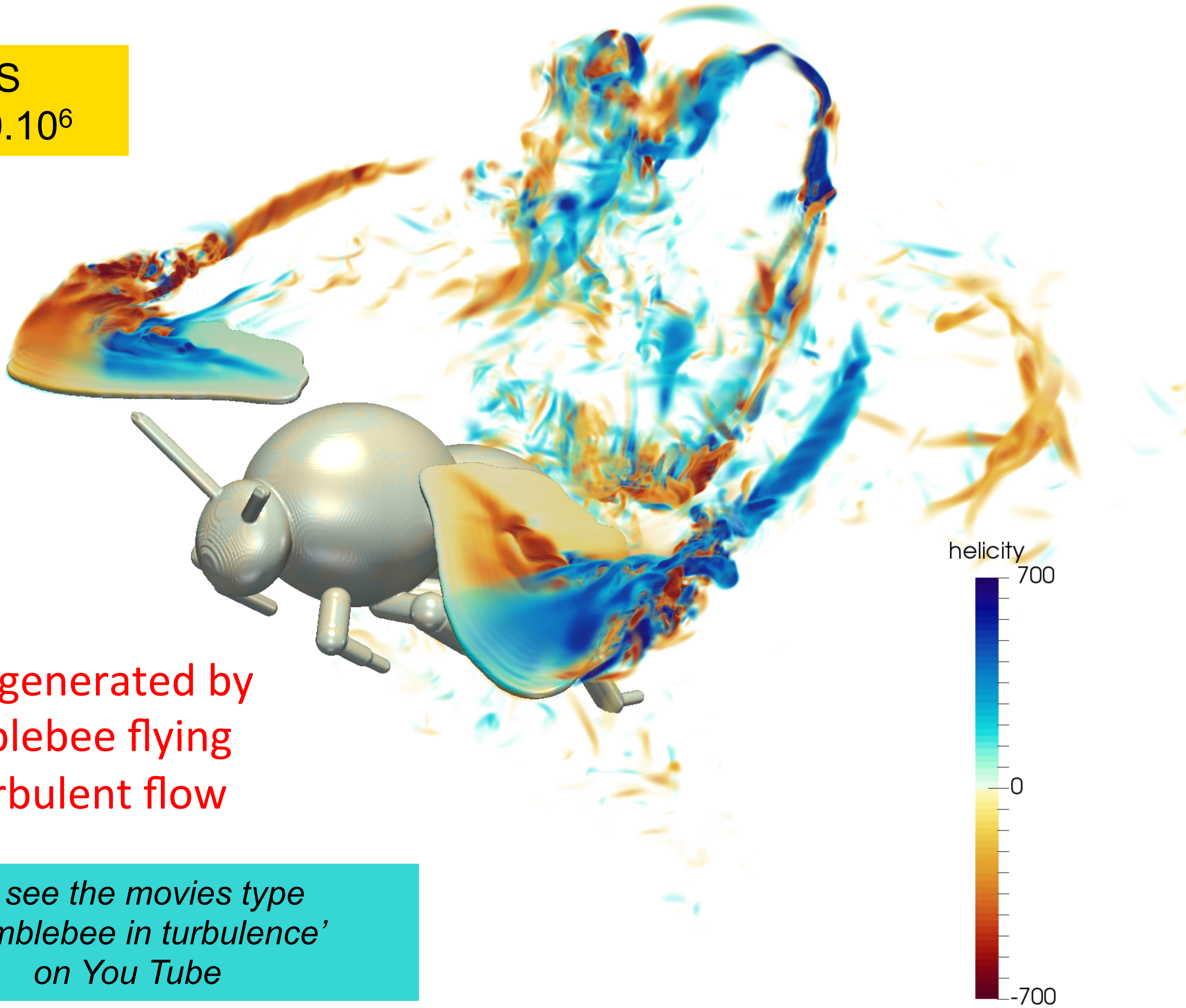


Helicity generated by
a bumblebee flying
in a turbulent flow

For more information see
http://aifit.cfd.tu-berlin.de/wordpress/

DNS

N=680.10^6



Helicity generated by a bumblebee flying in a turbulent flow

To see the movies type
'Bumblebee in turbulence'
on You Tube