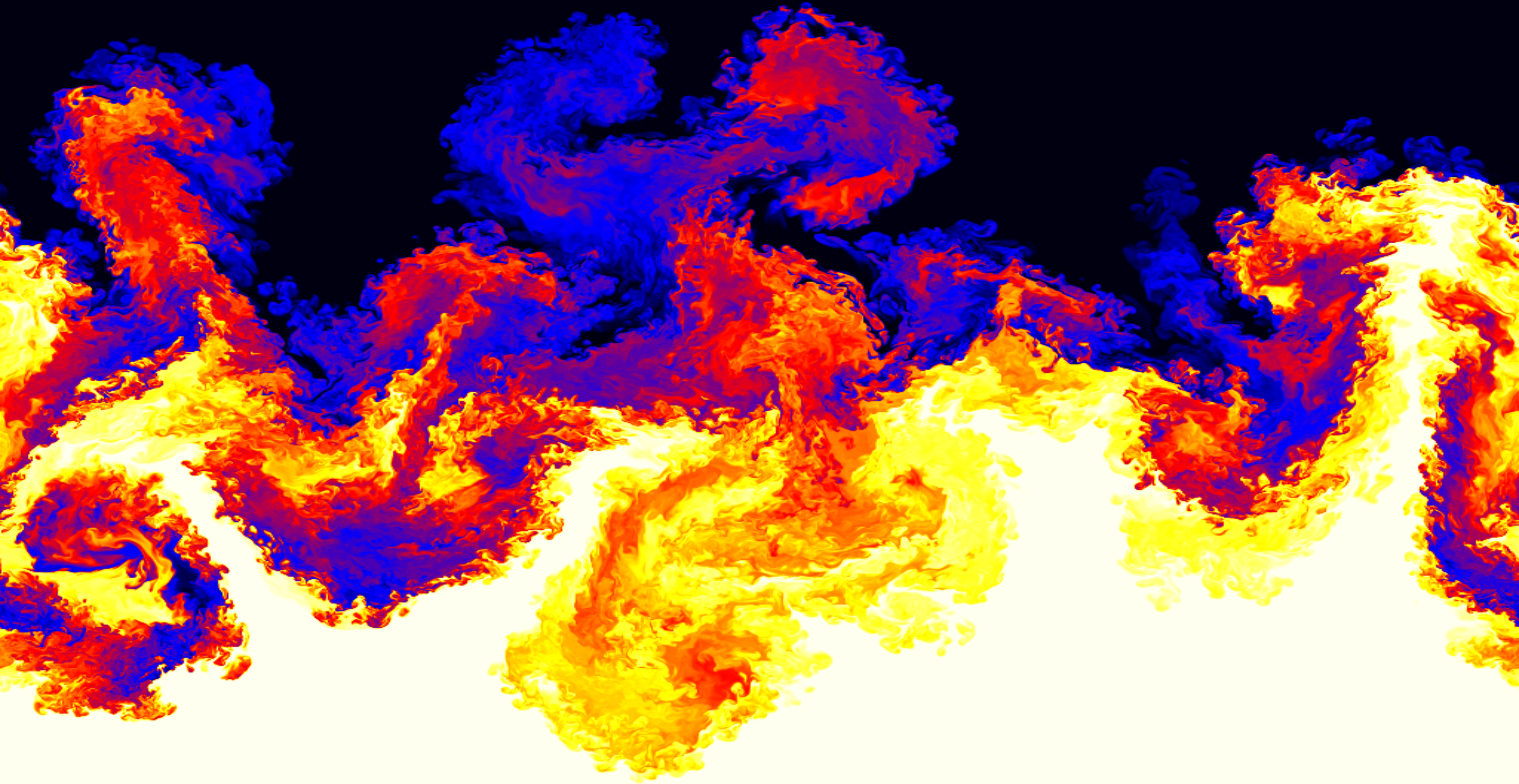
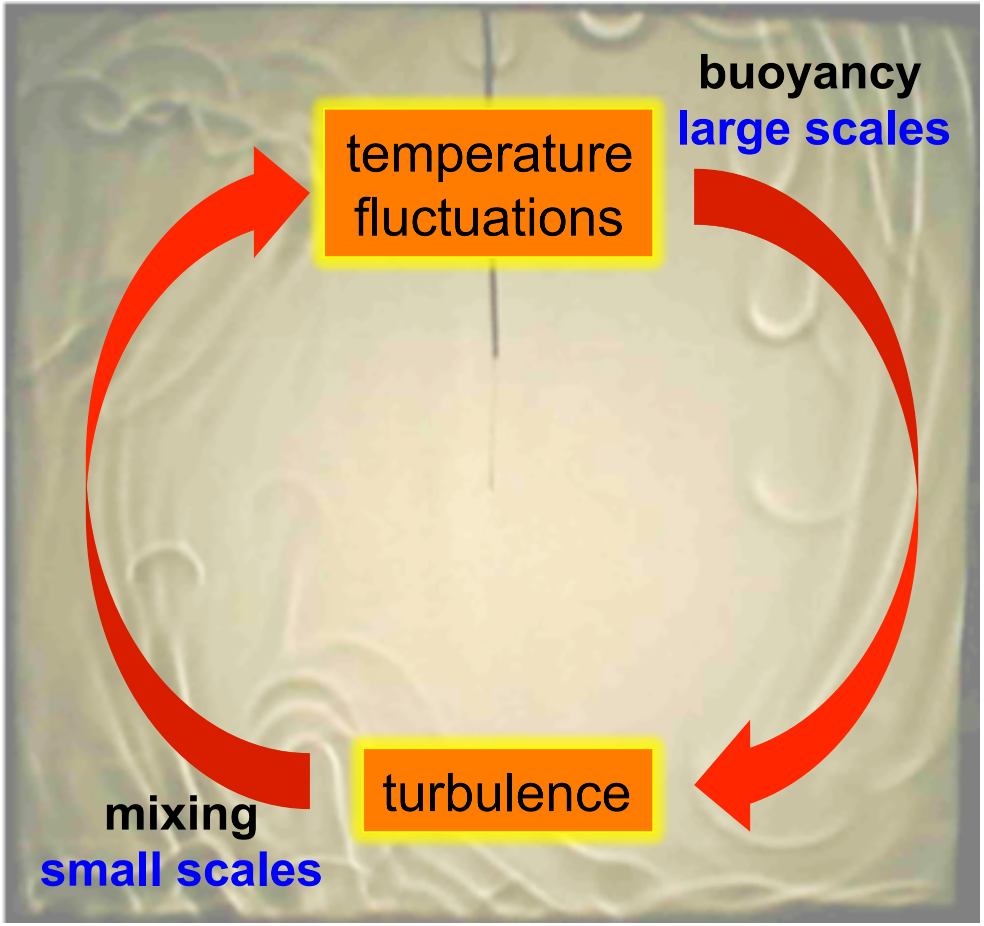
Geometrical confinement of Rayleigh-Taylor turbulent convection



Guido Boffetta (University of Torino)
Filippo De Lillo (Torino) Andrea Mazzino (Genova) Stefano Musacchio (Nice)

Turbulent convection



What is the effect of dimensionality on turbulent convection ?

3D-2D: statistics of small-scale fluctuations & the appearance of the Bolgiano scale

3D-1D: subdiffusive growth of large scale quantities

Small scale statistics of turbulent convection

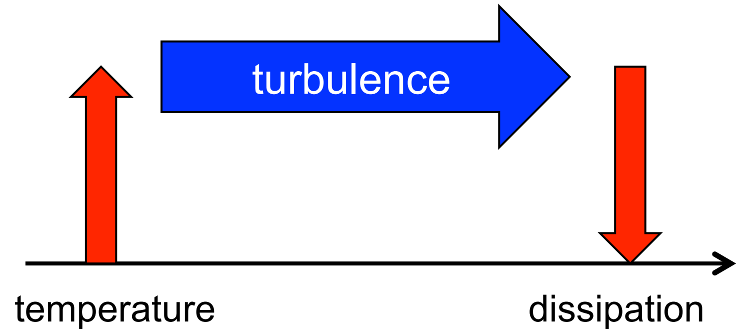
see: D. Lohse, K.Q. Xia, Ann. Rev. Fluid Mech. **42** (2010)

Kolmogorov-Obukhov-Corrsin (1949-51)

temperature is passive at small scales

$$\delta u(r) \simeq \varepsilon^{1/3} r^{1/3}$$

$$\delta T(r) \simeq \varepsilon_\theta^{1/2} \varepsilon^{-1/6} r^{1/3}$$

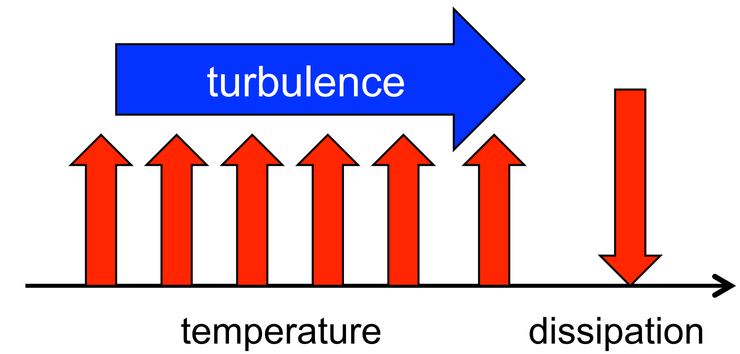


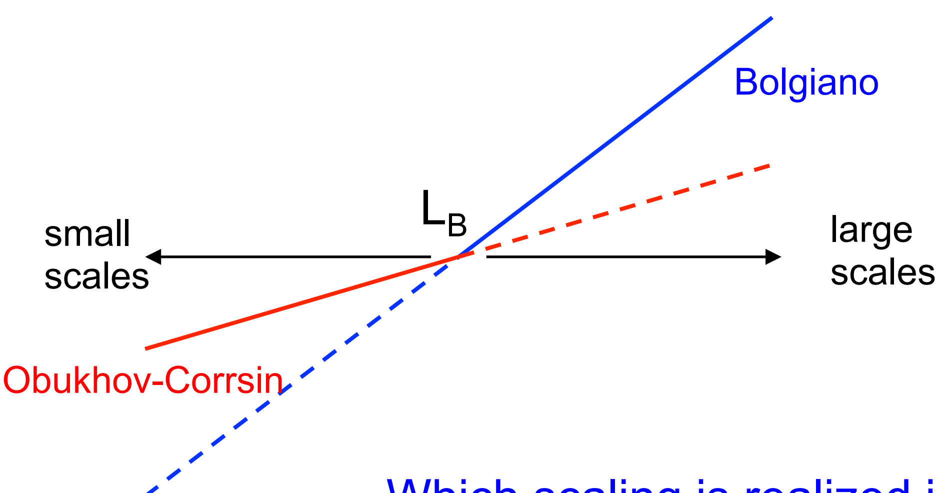
Bolgiano (1959)

temperature is active at all scales

$$\delta u(r) \simeq (\beta g)^{2/5} \varepsilon_\theta^{1/5} r^{3/5}$$

$$\delta T(r) \simeq (\beta g)^{-1/5} \varepsilon_\theta^{2/5} r^{1/5}$$





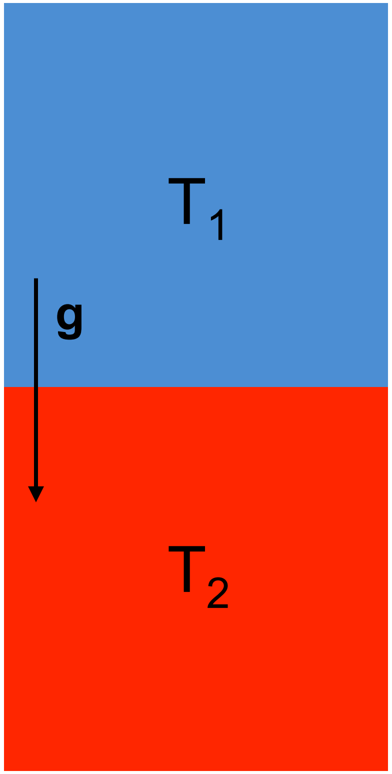
Crossover at Bolgiano scale:

$$L_B \simeq \varepsilon^{5/4} \varepsilon_\theta^{-3/4} (\beta g)^{-3/2}$$

Which scaling is realized in turbulent convection ?

Rayleigh-Taylor turbulence

Instability at the interface of two fluids of different densities with relative acceleration.
Single fluid at two temperatures (densities) : $\theta_0 = T_2 - T_1$



Atwood: $A \equiv \dfrac{\rho_1 - \rho_2}{\rho_1 + \rho_2} \simeq \dfrac{1}{2}\beta\theta_0$ (β: thermal expansion coef.)

For small A the Boussinesq approximation for an incompressible fluid holds:

$$\begin{cases} \partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p + \nu\Delta\mathbf{u} - \beta\mathbf{g}T \\ \partial_t T + \mathbf{u}\cdot\nabla T = \kappa\Delta T \end{cases}$$

with initial condition: $\begin{cases} \mathbf{u}(\mathbf{x},0) = 0 \\ T(\mathbf{x},0) = -(1/2)\theta_0 \,\mathrm{sgn}(z) \end{cases}$

Simple setup for turbulent convection
(no boundaries, no large scale circulation)

Phenomenology of RT turbulence

Energy balance:

$$\frac{dE}{dt} = -\frac{dP}{dt} - \varepsilon = \beta g \langle wT \rangle - \varepsilon$$

turbulent kinetic energy produced from potential energy
and dissipated at a rate $\varepsilon = \nu \langle (\nabla u)^2 \rangle$

Dimensional balance: $\frac{du_{rms}^2}{dt} = \beta g \theta_0 u_{rms}$ implies

Large scale velocity fluctuations $u_{rms} \simeq Agt$

Turbulent mixing layer of width $h(t) \simeq Agt^2$

Kinetic energy pumped at a rate $\varepsilon_I \simeq \frac{u_{rms}^3}{h} \simeq (Ag)^2 t$

time evolving turbulence

M. Chertkov, Phys. Rev. Lett. **91** (2003)

Small scale theory of RT turbulence

Ansatz: buoyancy negligible at small scales

$$\begin{cases} \partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p + \nu\Delta\mathbf{u} - \beta\mathbf{g}T \\ \partial_t T + \mathbf{u}\cdot\nabla T = \kappa\Delta T \end{cases}$$

$$\beta g \delta_r T \ll \frac{\delta_r u^2}{r}$$ (small Richardson number)

passive temperature in turbulent flow with time dependent flux

$$\varepsilon(t) \simeq (Ag)^2 t$$

Small scale velocity/temperature fluctuations follow Kolmogorov-Obukhov scaling

$$\delta_r u(t) \simeq u_L(t)\left(\frac{r}{h(t)}\right)^{1/3} \simeq (\beta g\theta_0)^{2/3} t^{1/3} r^{1/3}$$

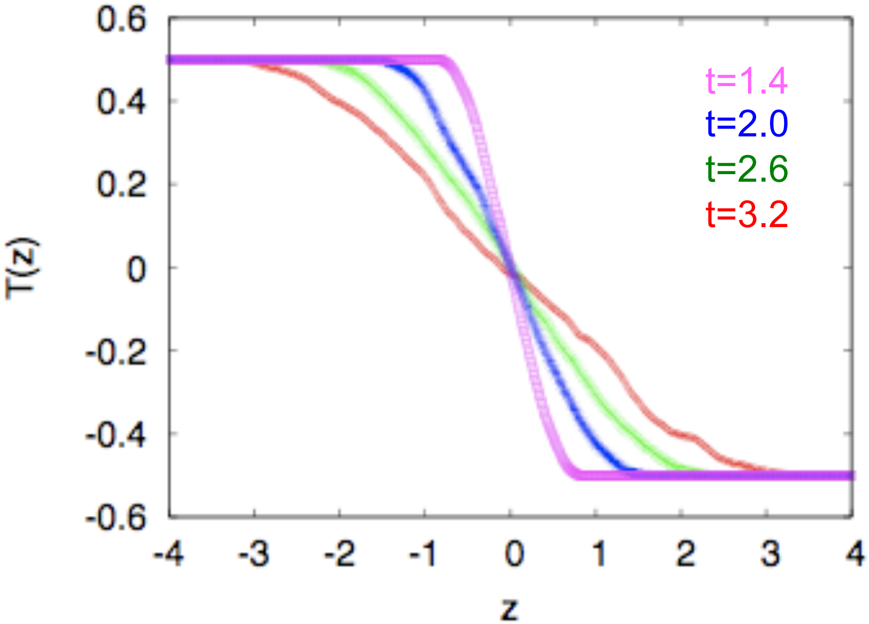
$$\delta_r T(t) \simeq \theta_0\left(\frac{r}{h(t)}\right)^{1/3} \simeq \frac{\theta_0}{(\beta g\theta_0)^{1/3}} t^{-2/3} r^{1/3}$$

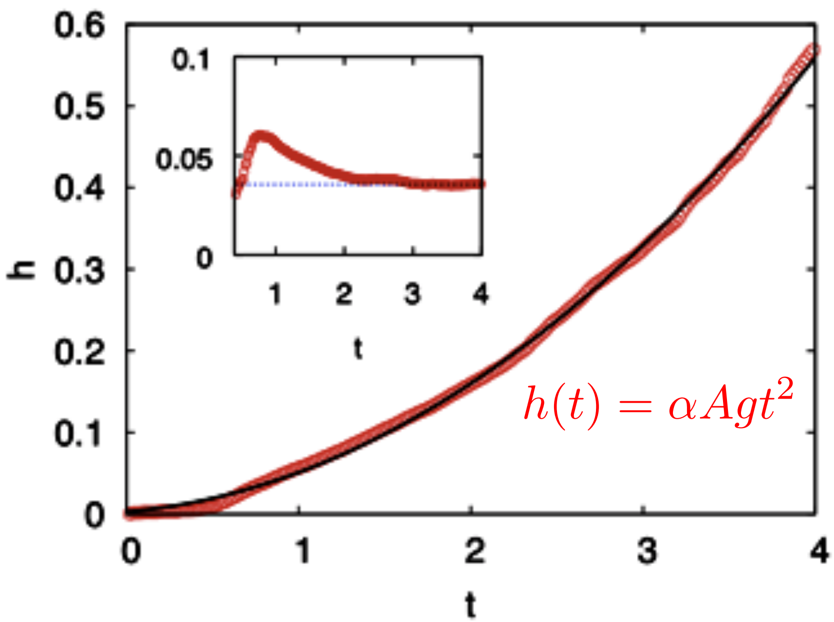
consistency:

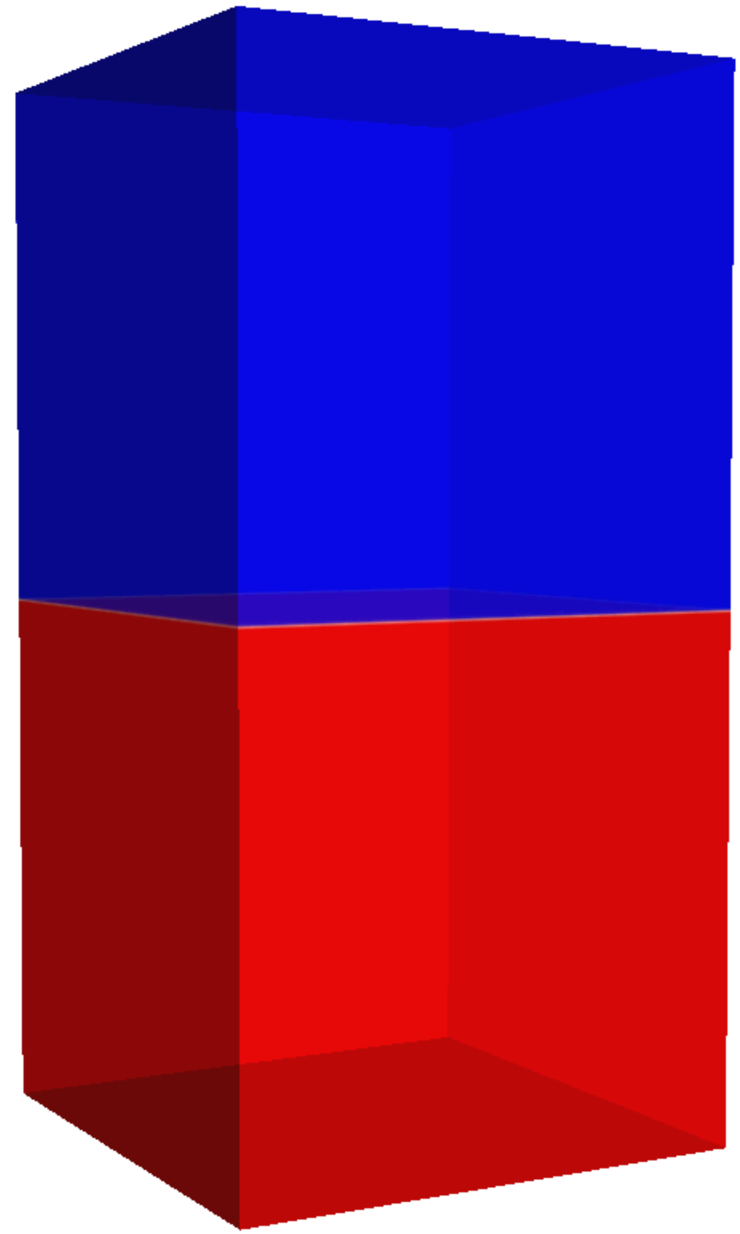
$$Ri = \frac{\beta g\delta_r T(t)}{\delta_r u^2(r)/r} \simeq \left(\frac{r}{h(t)}\right)^{2/3} \to_{r\to 0} 0$$

Inconsistent in 2D where the energy flows to large scale (buoyancy dominated)

Numerical simulation of RT turbulence







Self-similar evolution of spectra

Collapse of kinetic energy and temperature variance spectra at different times.

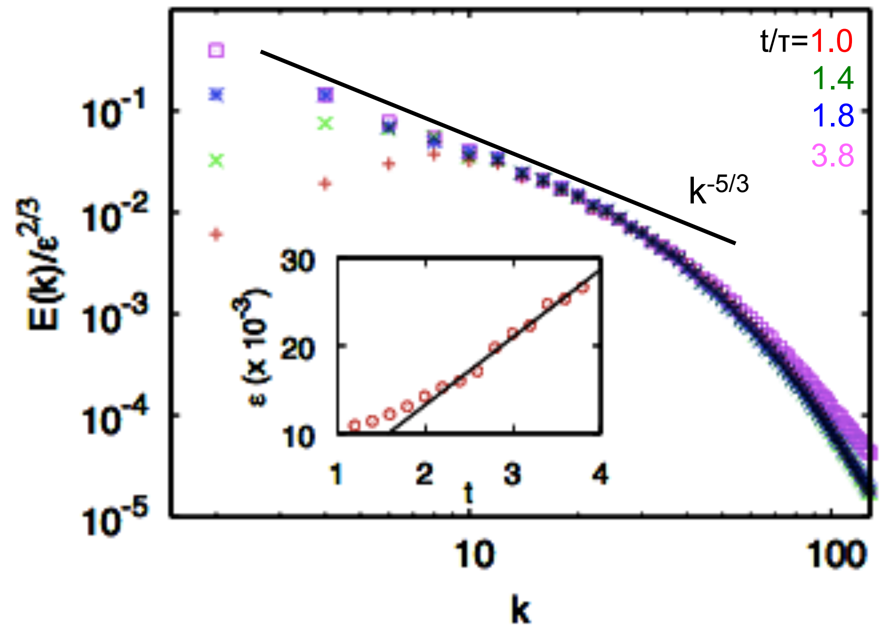
Insets: time evolution of kinetic energy dissipation $\varepsilon \approx t$ and temperature variance dissipation $\varepsilon_T \approx t^{-1}$

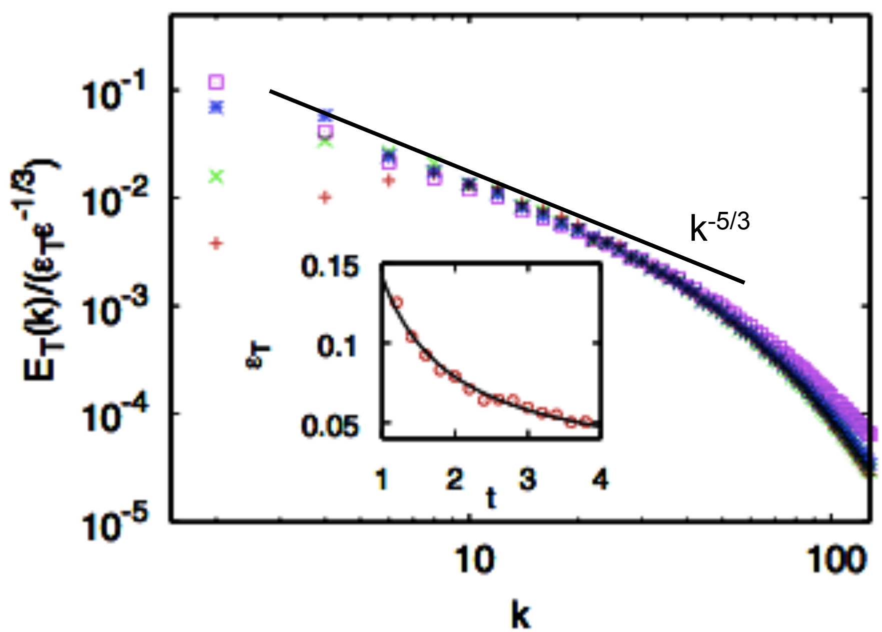
Spatial-temporal scaling in agreement with dimensional theory

$$E(k,t) \simeq (\beta g \theta_0)^{4/3} t^{2/3} k^{-5/3}$$

$$E_T(k,t) \simeq \theta_0^2 (\beta g \theta_0)^{-2/3} t^{-4/3} k^{-5/3}$$

+ intermittency corrections for higher order statistics





G.Boffetta, A.Mazzino, S.Musacchio, L.Vozella, *PRE* **79** (2009)
G.Boffetta, F.De Lillo, S.Musacchio, *PRL* **104** (2010)
G.Boffetta, A.Mazzino, S.Musacchio, L.Vozella, *Phys. Fluids* **22** (2010)

+ intermittency corrections

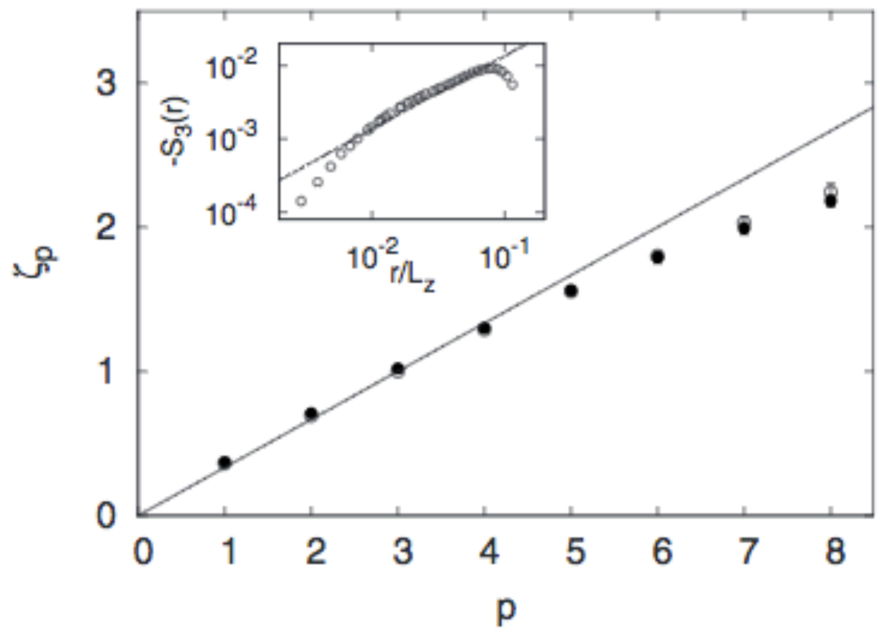


FIG. 4. Scaling exponents of isotropic longitudinal velocity structure functions $S_p(r)=\langle(\delta_r\mathbf{v}\cdot\hat{\mathbf{r}})^p\rangle(\hat{\mathbf{r}}=\mathbf{r}/r)$ for the late stage of RT turbulence (open circle). Exponents are computed by compensation of $S_p(r)$ with $S_3(r)$, according to the extended self-similarity procedure [21] averaging inside the mixing layer and on all directions. Filled circles: scaling exponents from simulations of homogeneous isotropic turbulence at $R_\lambda=381$ [22]. Line represents dimensional prediction $\zeta_p=p/3$. Inset: third-order isotropic longitudinal structure function $S_3(r)$. The line represents Kolmogorov's four-fifth law $S_3(r)=-4/5\epsilon r$.

RT turbulence in 2D

M. Chertkov, Phys. Rev. Lett. **91** (2003)

Buoyancy balances inertia at all scales: $\beta g \delta_r T \simeq \frac{\delta_r u^2}{r}$ (Ri=O(1))

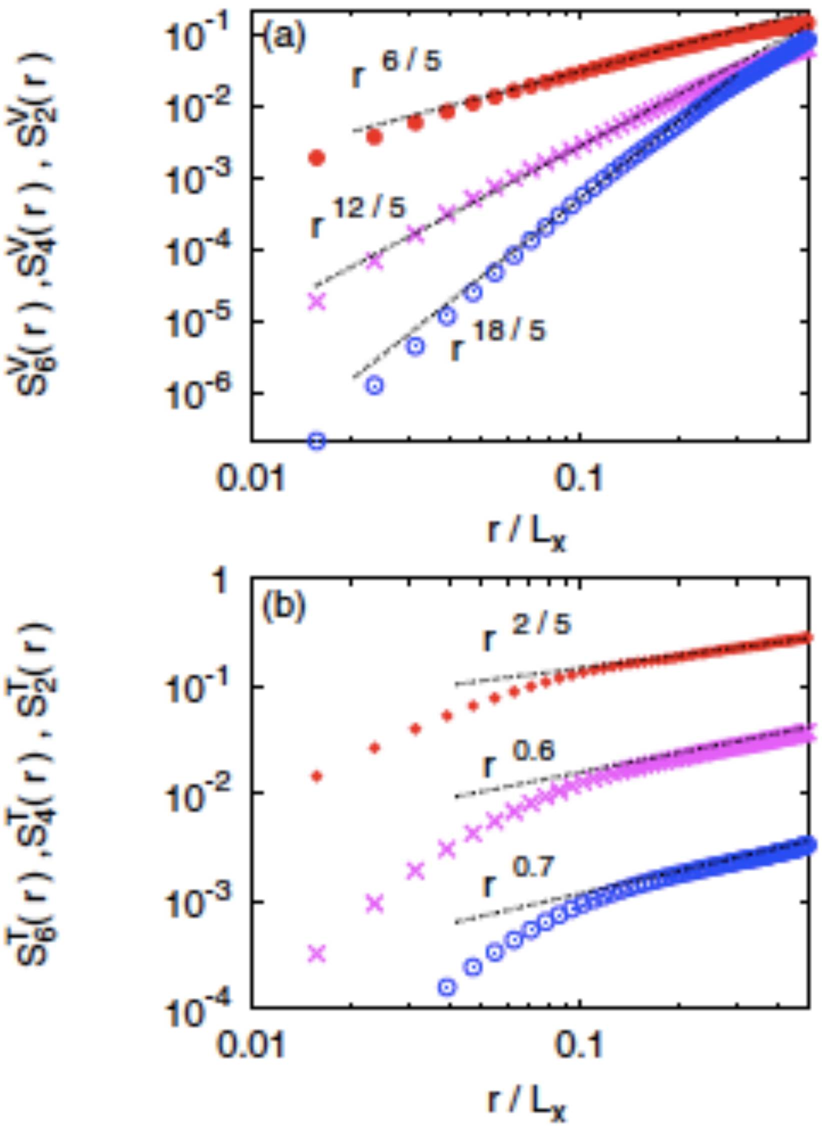
Direct cascade of temperature fluctuations

$$\varepsilon_T(t) \simeq \frac{\delta_r u \delta_r T^2}{r} \simeq \frac{\delta_r u^5}{r^3(\beta g)^2} \simeq \frac{u_L^5}{h^3(\beta g)^2}$$

Inverse cascade of velocity fluctuations follows (time evolving) Bolgiano scaling (+ intermittency correction for temperature fluctuations)

$$\delta_r u(t) \simeq u_L(t)\left(\frac{r}{h(t)}\right)^{3/5} \simeq (\beta g \theta_0)^{2/5} t^{-1/5} r^{3/5}$$

$$\delta_r T(t) \simeq \theta_0 \left(\frac{r}{h(t)}\right)^{1/5} \simeq \frac{\theta_0}{(\beta g \theta_0)^{1/5}} t^{-2/5} r^{1/5}$$

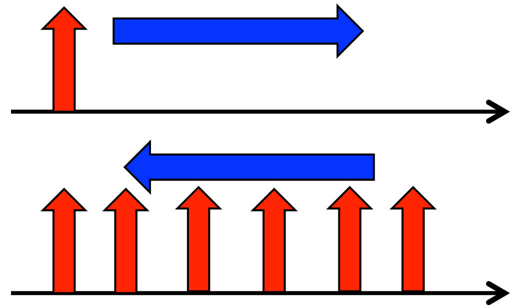


A.Celani, A.Mazzino, L.Vozella, Phys. Rev. Lett. **96** (2006)

Where is the Bolgiano scale L_B ?

3D direct cascade

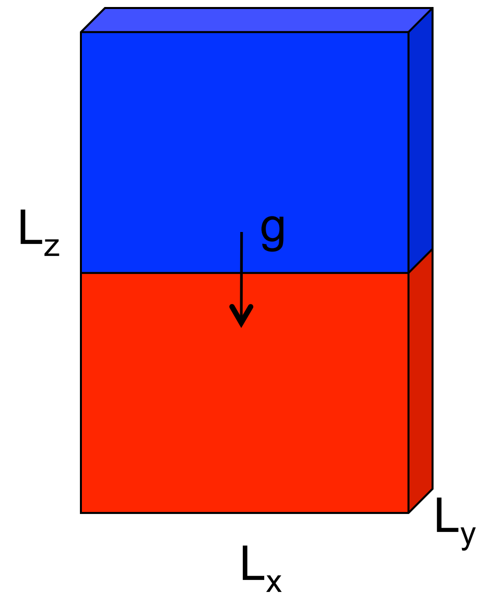
2D inverse cascade



$L_B \approx L$ (integral scale)

$L_B \approx 0$

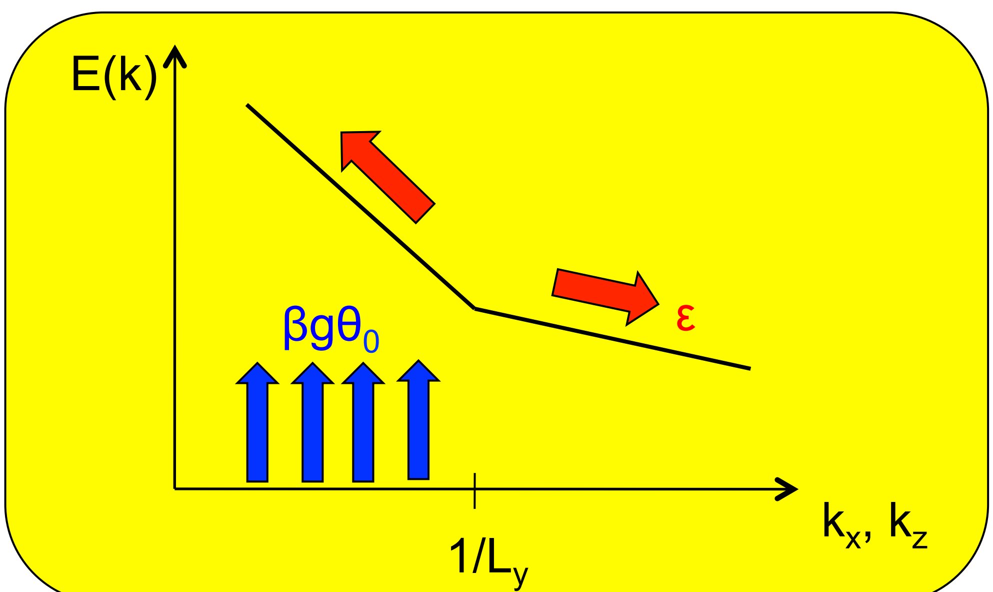
Idea: L_B is determined by the smallest size of the box



Setup with large aspect ratio $L_y << L_x , L_z$

* scales $r << L_y$: 3D Kolmogorov-Obukhov

* scales $r >> L_y$: 2D Bolgiano



transverse geometrical scale becomes the Bolgiano scale

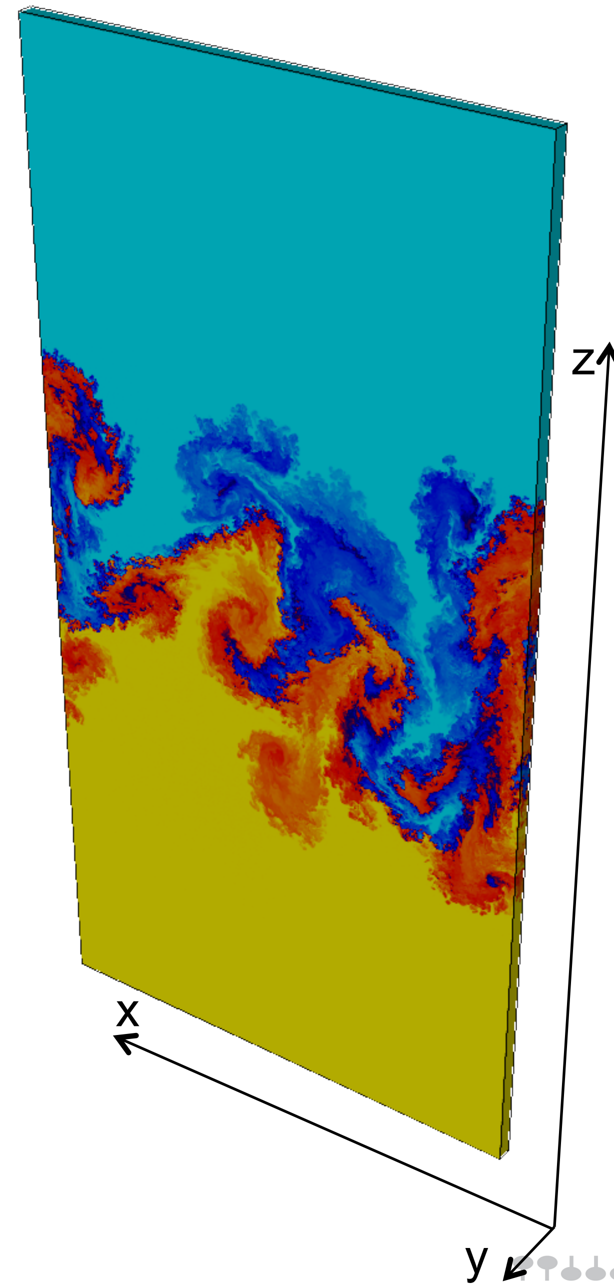
Quasi-2D Rayleigh-Taylor turbulence

RT system confined in a thin convective cell

Aspect ratio $L_y / L_x = 1 / 32$, $L_z / L_x = 2$
Periodic b.c.: no material walls

* $h(t) < L_y$: 3D phenomenology
- Kolmogorov scaling
- passive temperature

* $h(t) > L_y$: 2D phenomenology
- Bolgiano scaling
- active temperature



temperature field from simulations at 4096x128x8192 - HPC grant

G.Boffetta, F.De Lillo, A.Mazzino, S.Musacchio, JFM **690**, 426 (2012)

A first signature of 3D – 2D transition: energy balance

$$\frac{dE}{dt} = -\frac{dP}{dt} - \varepsilon$$

h(t) < L_y 3D
$$\begin{cases} \dfrac{dE}{dt} \simeq t \\ \varepsilon \simeq t \end{cases}$$

h(t) > L_y 2D
$$\begin{cases} \dfrac{dE}{dt} \simeq t \\ \varepsilon \simeq 0 \end{cases}$$

In quasi-2d there is a residual direct energy flux (given by matching the velocity at r=L_y)

$$\delta_r u(t) \simeq \varepsilon(t)^{1/3} r^{1/3} \qquad (r \ll L_y)$$

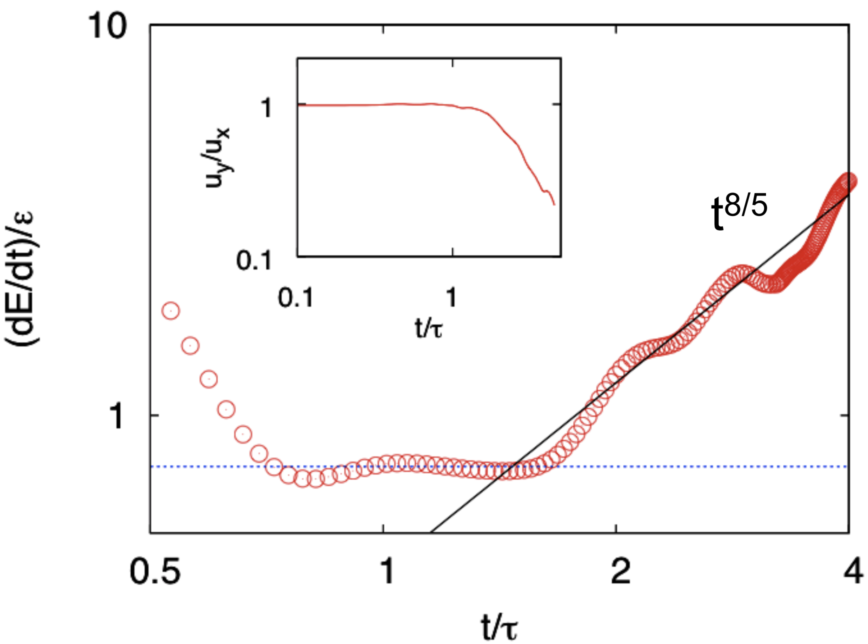
$$\delta_r u(t) \simeq (\beta g \theta_0)^{2/5} t^{-1/5} r^{3/5} \qquad (r \gg L_y)$$

$$\varepsilon(t) \simeq (\beta g \theta_0)^{6/5} L_y^{4/5} t^{-3/5}$$

thus

$$\left(\frac{dE}{dt}\right) / \varepsilon \sim t^{8/5}$$

transition from direct to inverse flux when h(t) reaches L_y



Simultaneous presence of a direct and an inverse cascade of energy

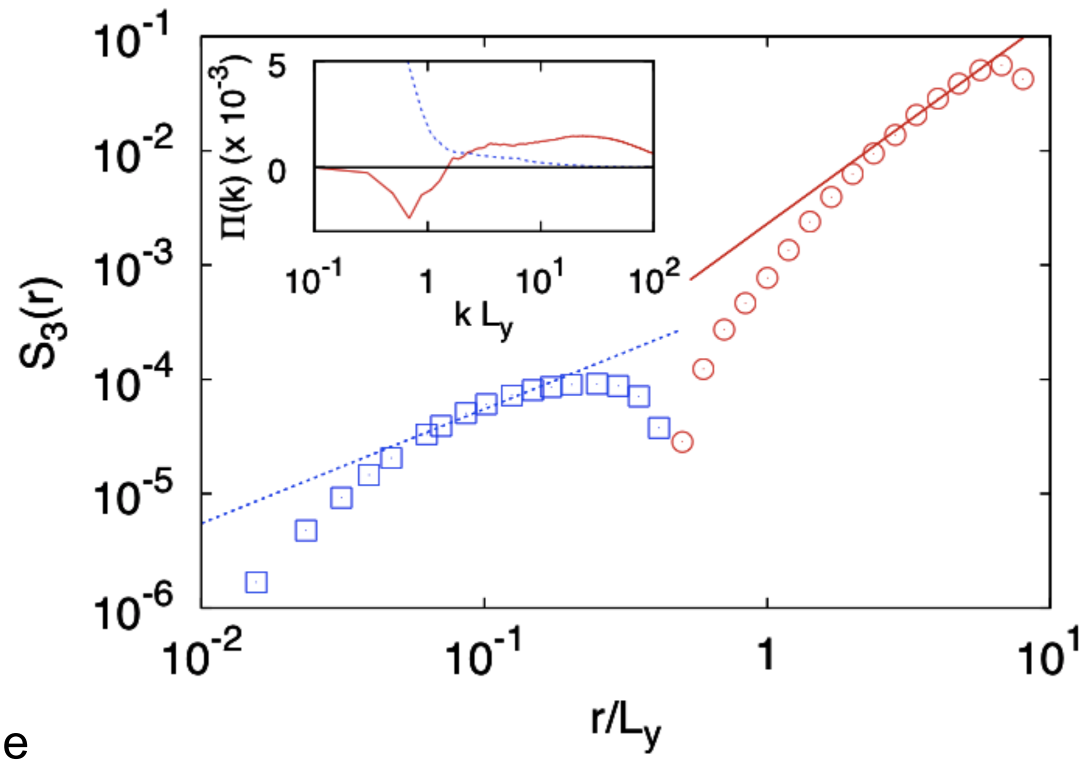
r < L$_y$

$$-S_3(r) = -\left\langle \left(\delta_r u\right)^3 \right\rangle = \frac{4}{5}\varepsilon r$$

r > L$_y$

$$S_3(r) \simeq +r^{9/5}$$

Third-order velocity SF at late times



The sign of $S_3(r)$ indicates a change in the direction of the turbulent cascade

3D turbulent cascade at small scales

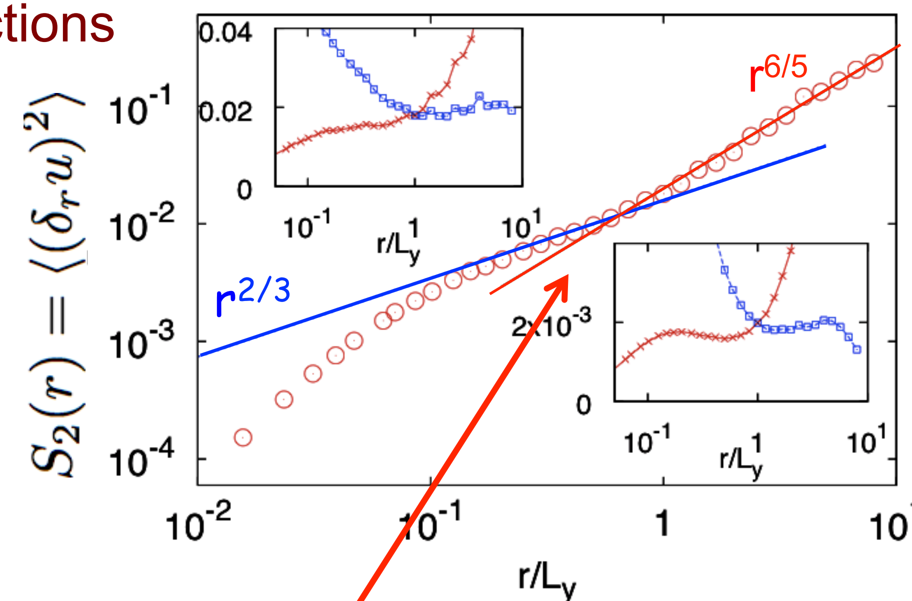
2D inverse cascade at large scales

Inset: contributions to energy flux in Fourier space by the nonlinear term and by the buoyancy term

Velocity and temperature structure functions

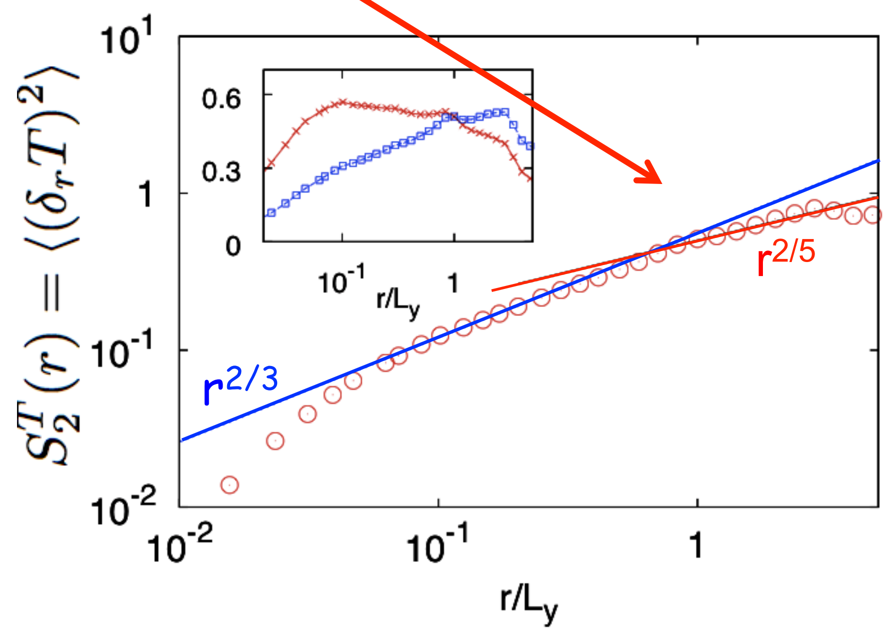
Kolmogorov-Obukhov scaling
at small scales (passive temperature)

Bolgiano scaling
at large scales (active temperature)



L_y is the Bolgiano scale

First evidence of a Bolgiano scale
(i.e. two scalings) in turbulent convection



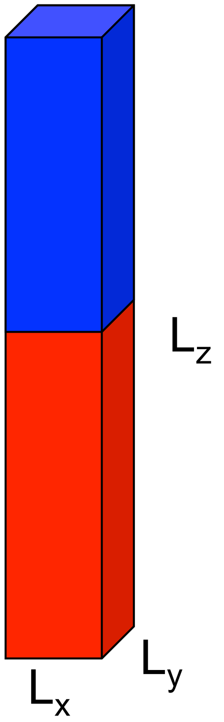
(see D. Lohse, K.Q. Xia,
Ann. Rev. Fluid Mech. **42** (2010))

Quasi-1D Rayleigh-Taylor turbulence

Two-regimes:

* $h(t) < L_x$: 3D RT turbulence

* $h(t) > L_x$: ?



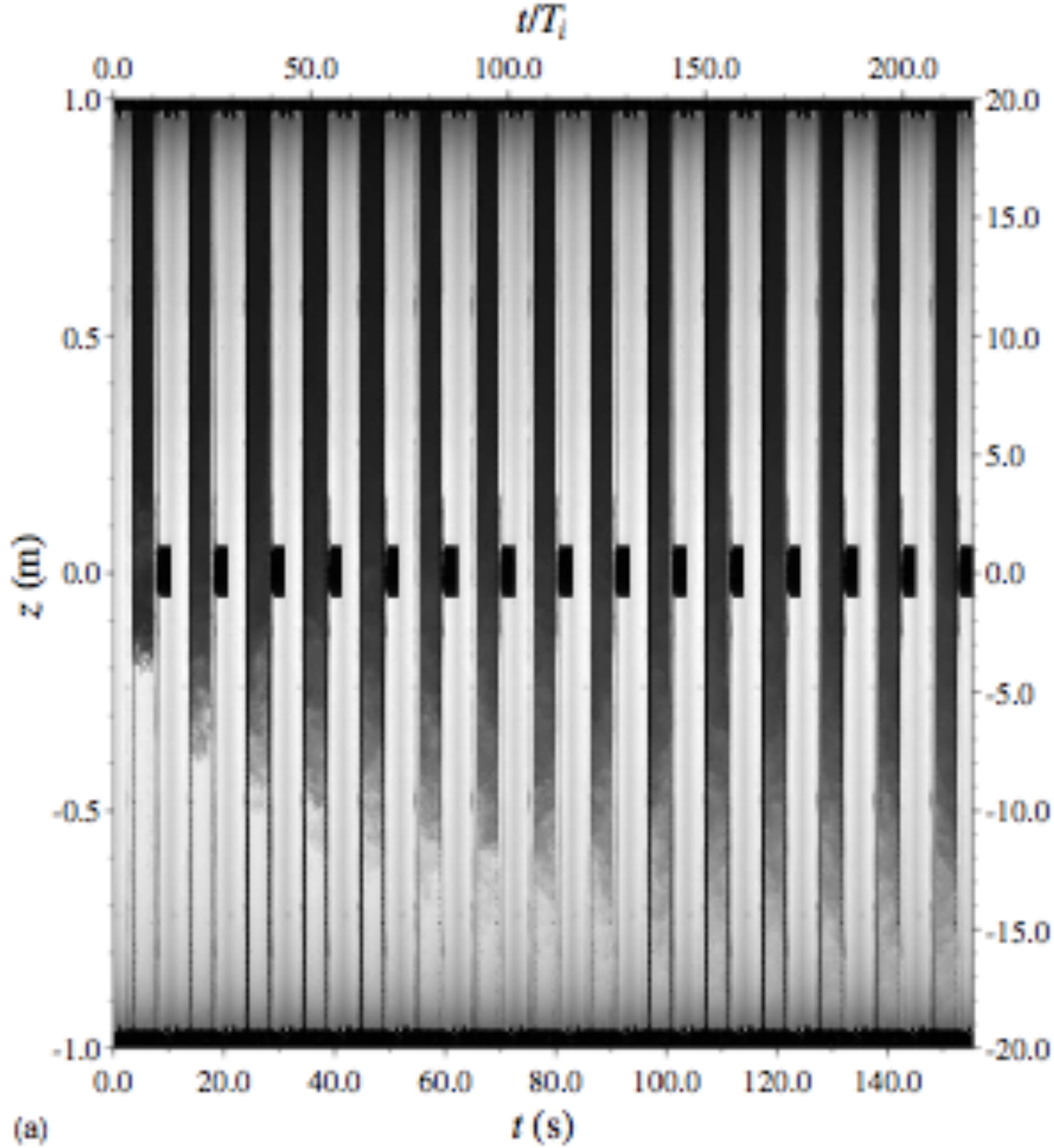
L_x , L_y << L_z

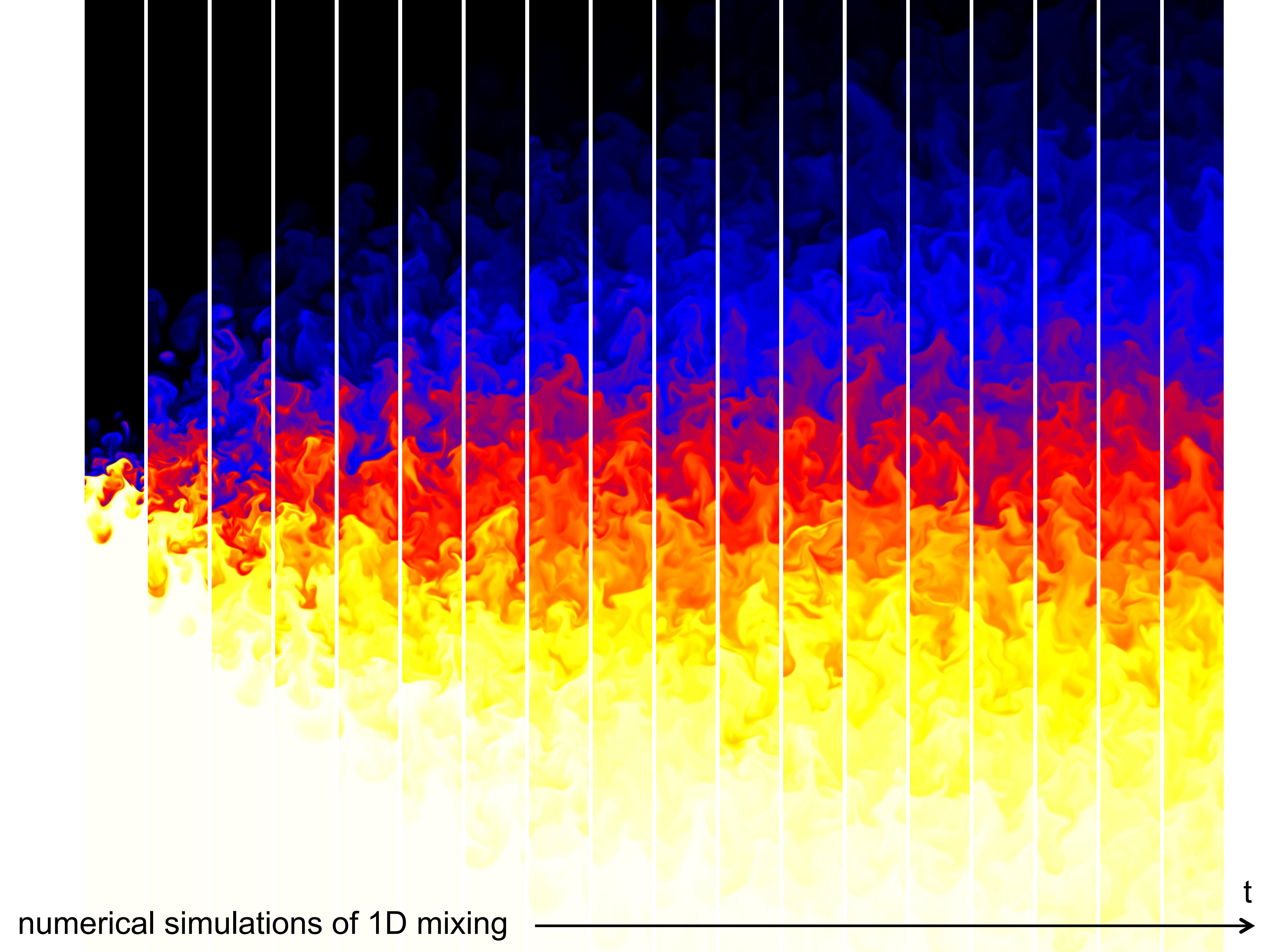
Physical motivation: mixing efficiency in stratified fluids

Experiments in quasi-1D mixing

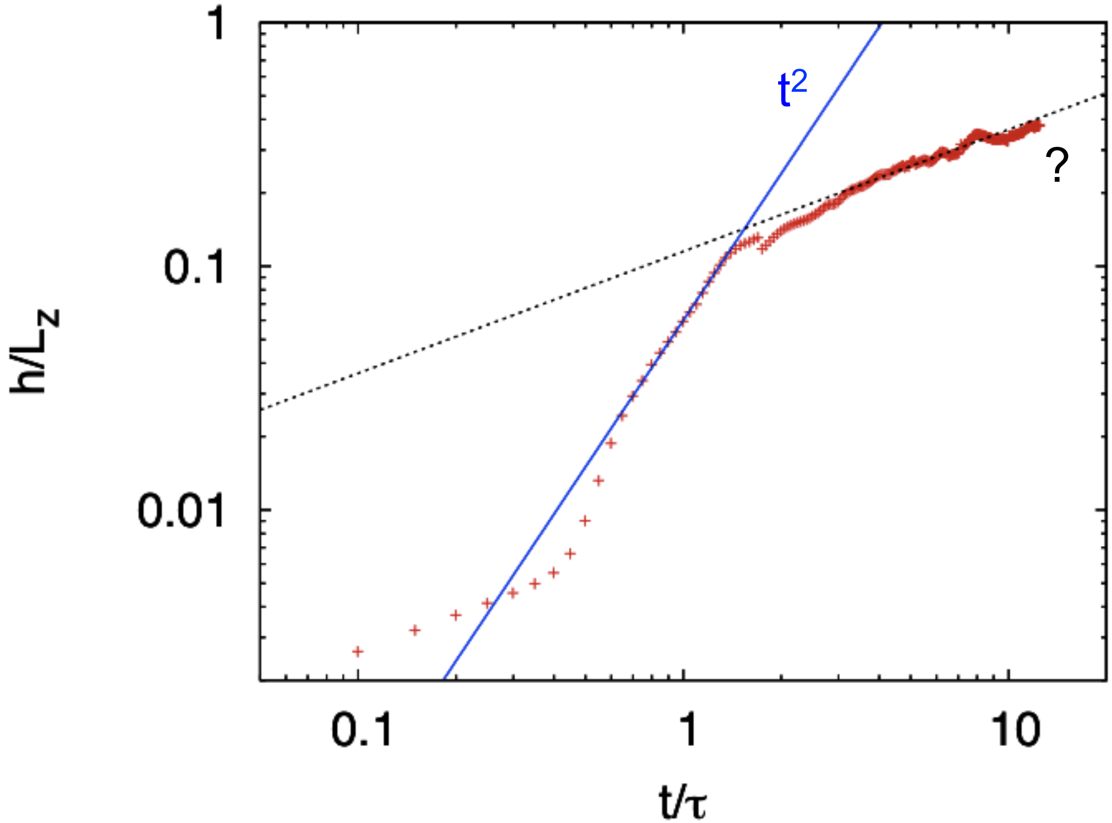
S.B. Dalziel, M.D. Patterson,
C.P. Caulfield, I.A. Coomaraswamy,
POF **20** (2008)

salt water + fresh water
A = 0.01




t
numerical simulations of 1D mixing

Evolution of the mixing layer



* short times ($h << L_x$)

$h(t) \approx t^2$

* long times ($h \approx L_x$)

$h(t) \approx$?

(L_z/L_x = 32)

A model for the growth of the mixing layer

Velocity fluctuations on scales r > L_x are uncorrelated

Evolution of mean temperature profile $\overline{T}(z,t)$

$$\partial_t \overline{T} + \partial_z \overline{wT} = \kappa \partial_z^2 \overline{T}$$

can be modeled in terms of an eddy diffusivity

$$\partial_t \overline{T} = \partial_z \left(K(z,t) \partial_z \overline{T} \right)$$

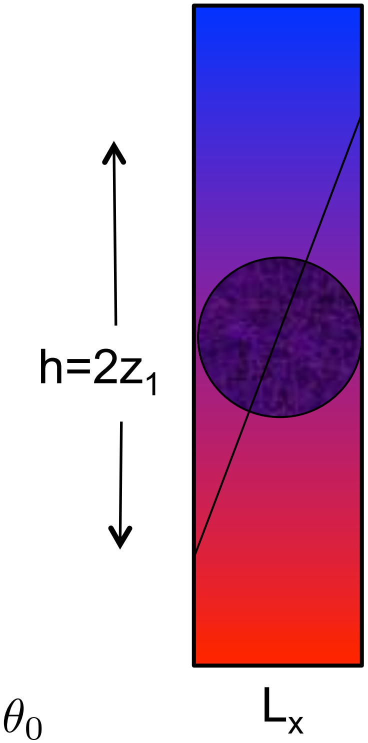
Eddy diffusivity: $K(z,t) \simeq u_{rms} L_u$

where u_{rms} is obtained dimensionally from the balance

$$\frac{u_{rms}^2}{L_u} \simeq \beta g \theta_L$$

and θ_L is the temperature jump at scale L_u

Eddy diffusivity at late times



$$L_u = L_x \qquad \theta_L \simeq \theta_0 \frac{L_x}{h} = L_x \frac{\partial \overline{T}}{\partial z}$$

eddy diffusivity $\quad K = (\beta g)^{1/2} L_x^2 (\partial_z \overline{T})^{1/2}$

$$\partial_t \overline{T} = a(\beta g)^{1/2} L_x^2 \partial_z \left(\partial_z \overline{T}\right)^{3/2}$$

Self-similar solution in the form $\overline{T}(z,t) = f(z/t^{\alpha})$ with $\overline{T}(\pm z_1) = \pm 0.5\theta_0$

$$\overline{T}(z,t) = -\frac{15}{16}\theta_0 \left[\frac{1}{5}\left(\frac{z}{z_1}\right)^5 - \frac{2}{3}\left(\frac{z}{z_1}\right)^3 + \frac{z}{z_1}\right]$$

for $|z| \leq z_1$ $\qquad \alpha = 2/5$

$$h(t) = 2z_1(t) = \gamma L_x^{4/5} (\beta g \theta_0)^{1/5} t^{2/5}$$

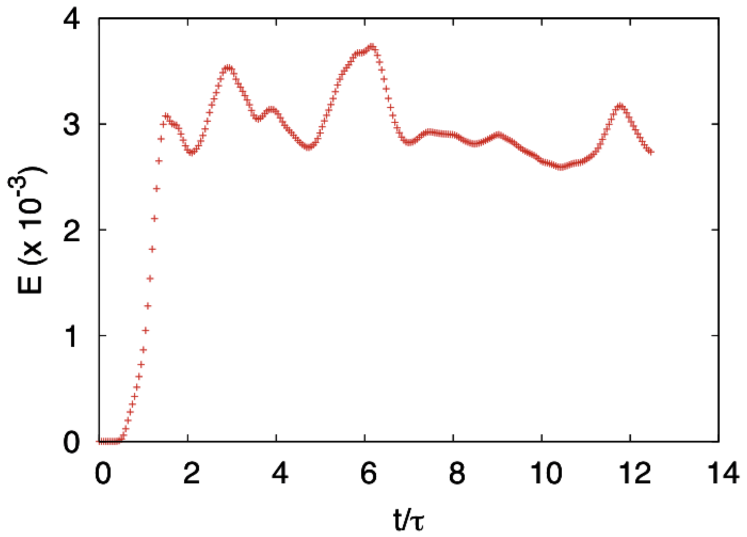
subdiffusive growth of the mixing layer

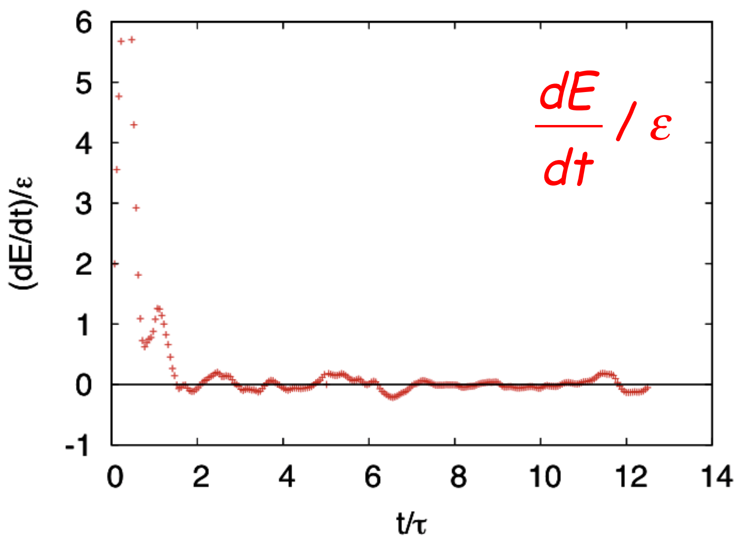
Saturation of kinetic energy

Total kinetic energy $E = \frac{1}{2}\int d^3x|u|^2 \simeq \frac{3}{2}L_x^2 h(t) u_{rms}^2(t)$

Since $u_{rms}^2 \propto \theta_L \propto h^{-1}$
E becomes constant for h(t)>L_x:

$$E \simeq \frac{3}{2}\beta g \theta_0 L_x^4$$

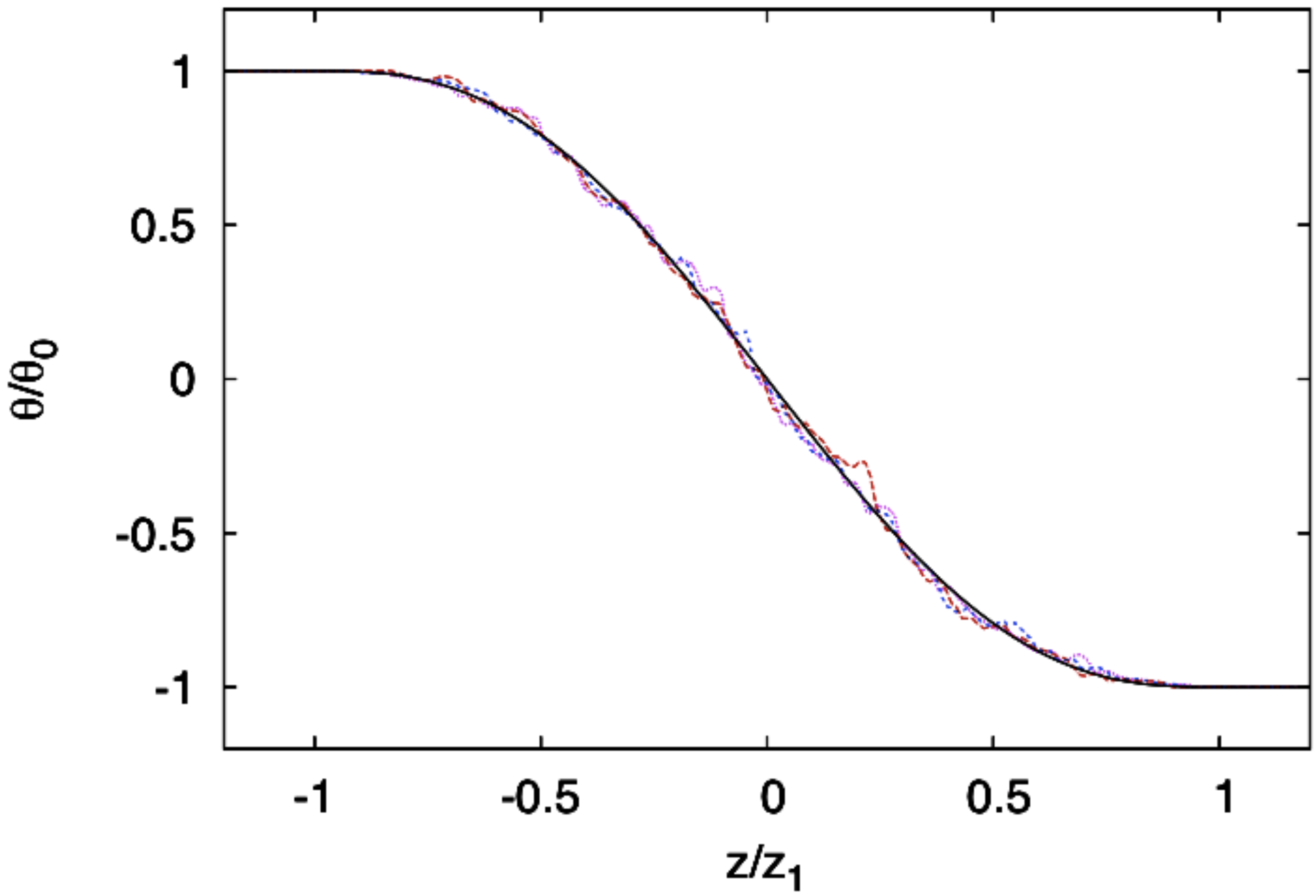




Energy balance:
all potential energy is dissipated by viscosity

$$\frac{dE}{dt} = -\frac{dP}{dt} - \varepsilon = 0$$

Self-similar evolution of the mixing layer



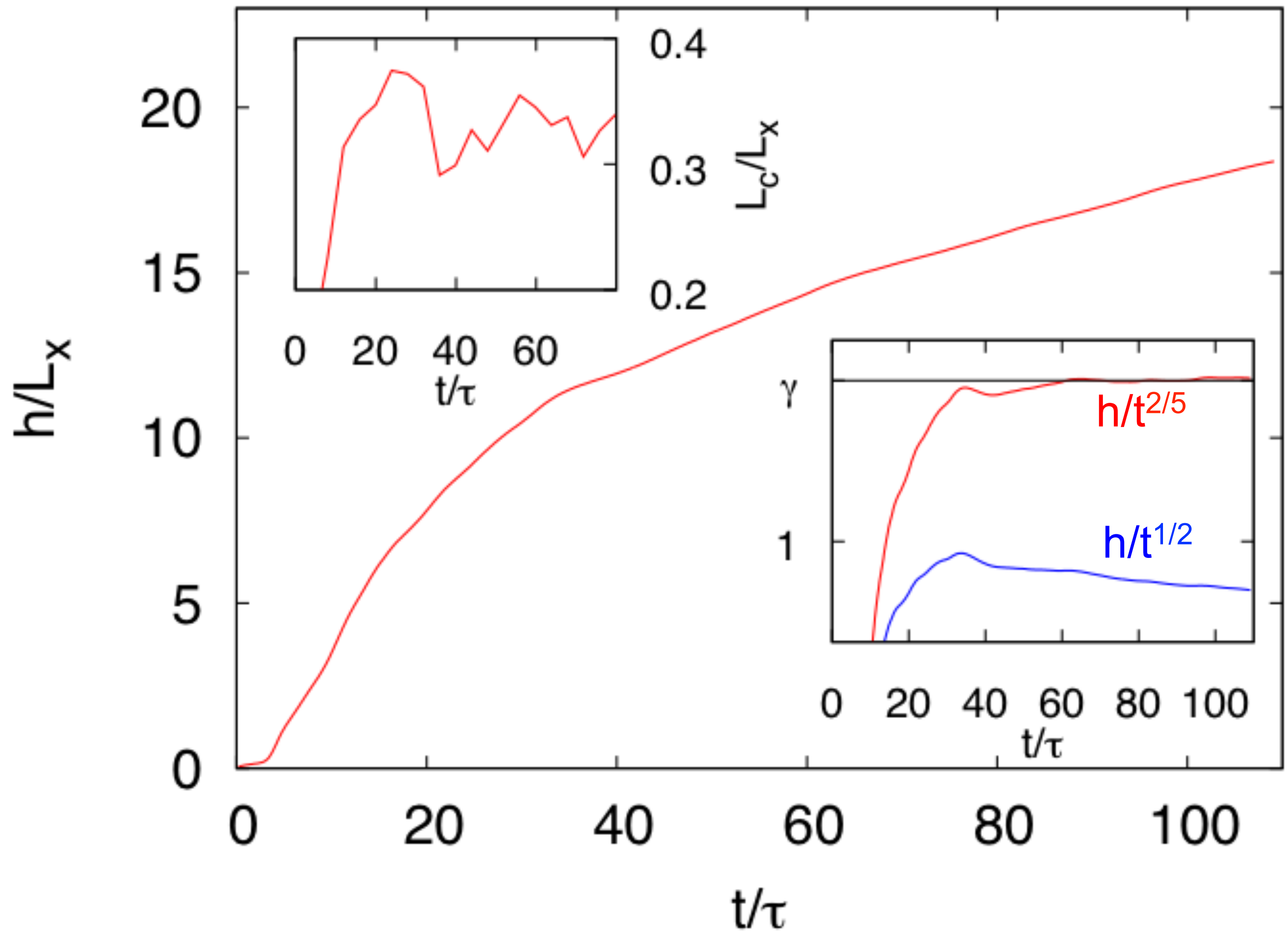
$\overline{T}(z/z_1, t)$

for different t

Fitting with $$\overline{T}(z,t) = -\frac{15}{16}\theta_0 \left[\frac{1}{5}\left(\frac{z}{z_1}\right)^5 - \frac{2}{3}\left(\frac{z}{z_1}\right)^3 + \frac{z}{z_1} \right]$$ gives $z_1(t)$

Simulations at 256x256x8192

Time evolution of h(t)



the nonlinear model allows for a precise determination of the temporal scaling exponent $h(t) = \gamma L_x^{4/5}(\beta g \theta_0)^{1/5} t^{2/5}$

Experiments: Lawrie & Dalziel (2011)

Thank you

G. Boffetta and A. Mazzino, *Incompressible Rayleigh-Taylor Turbulence*, Annual Review of Fluid Mechanics **49**, 119 (2017).