Crowd synchronization on the London Millennium bridge

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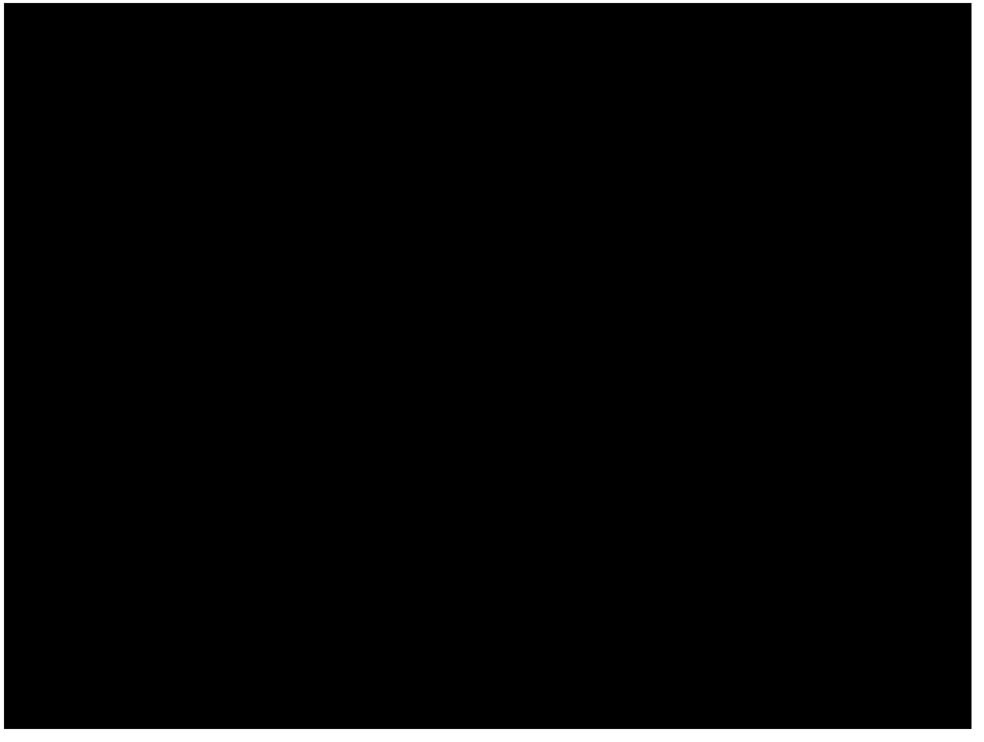
The phenomenon:



London,
Millennium bridge:
Opening day
June 10, 2000

Tacoma narrows bridge

Tacoma,
Pudget Sound
Nov. 7, 1940

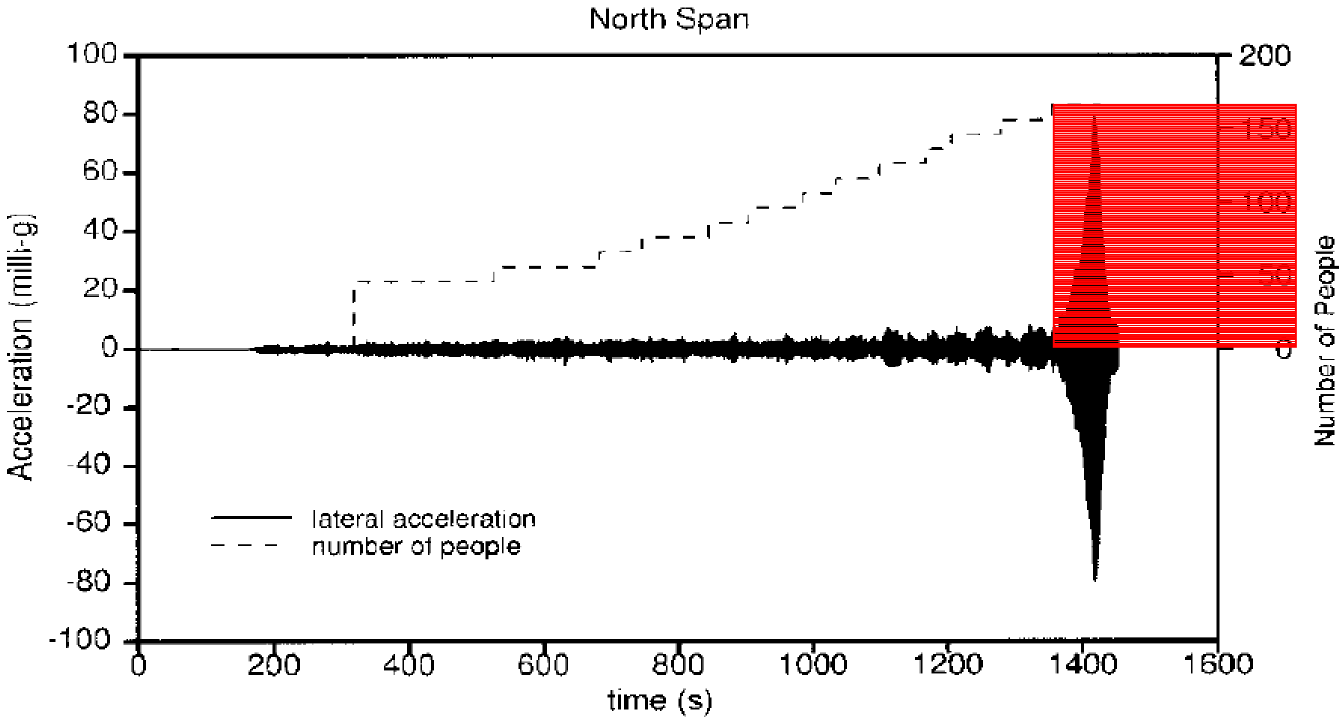
K.Y. Billah and R.H. Scanlan, Am. J. Phys. **29,** 118 (1991)

Differences between MB and TB:

- No resonance near vortex shedding frequency and
- no vibrations of empty bridge

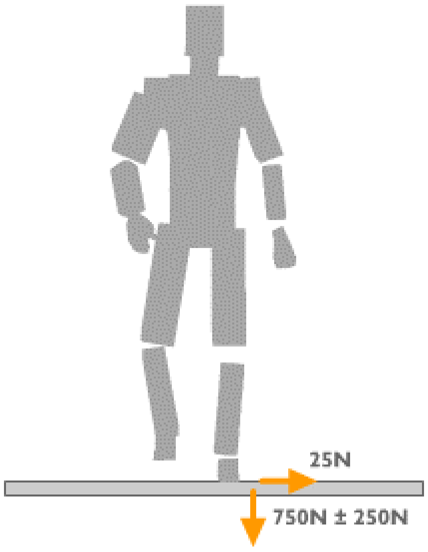
- No swaying with few people
- nor with people standing still
- but onset above a critical number of people in motion

Studies by Arup:

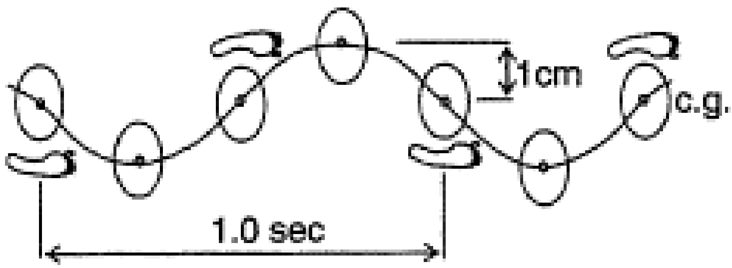


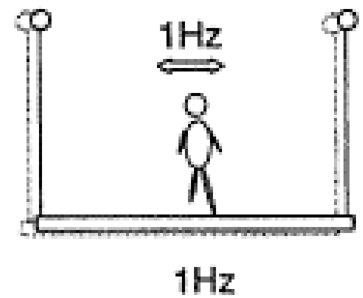
Forces during walking:

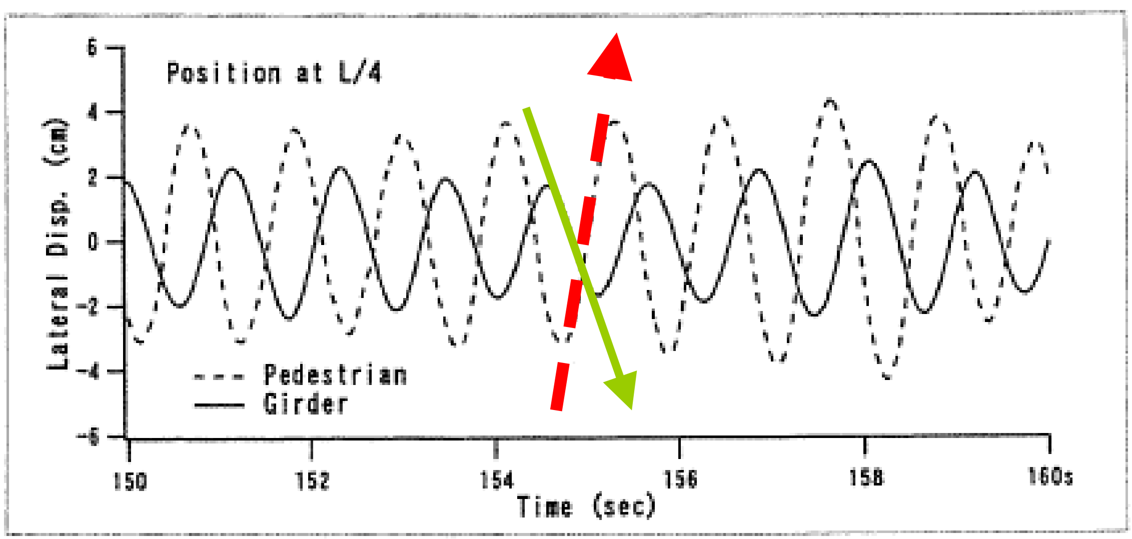
- Downward:
  mg, about 800 N
- forward/backward:
  about mg
- sideways,
  about 25 N



Brige-pedestrian phase relation



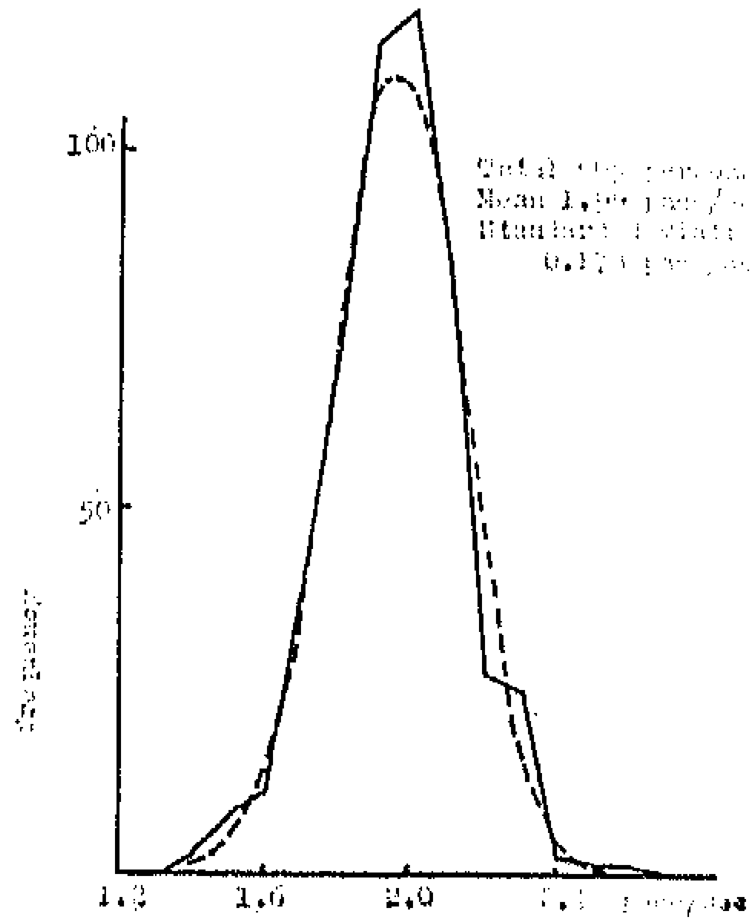




Nakamura, Struct. Eng 2003

The frequency of walking:

People walk at a rate of about 2 steps per second (one step with each foot)



Matsumoto et al, Trans JSCE 5, 50 (1972)

The model

Modal expansion for bridge
plus phase oscillator for pedestrians:

Bridge motion: $$M\ddot{y} + 2M\varsigma\Omega\dot{y} + M\Omega^2 y = \sum_i f_i(t)$$

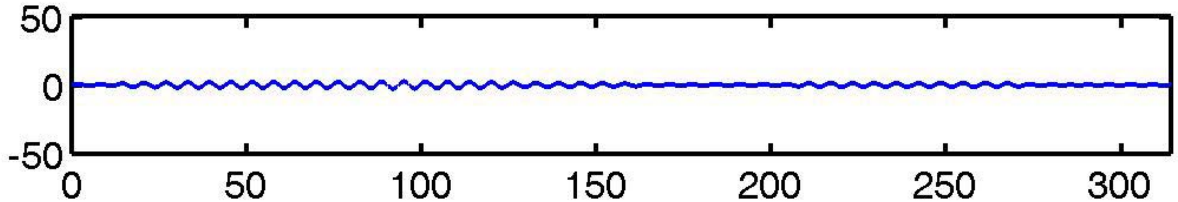
forcing: $$f_i(t) = f_{i0}\cos(\theta_i(t))$$

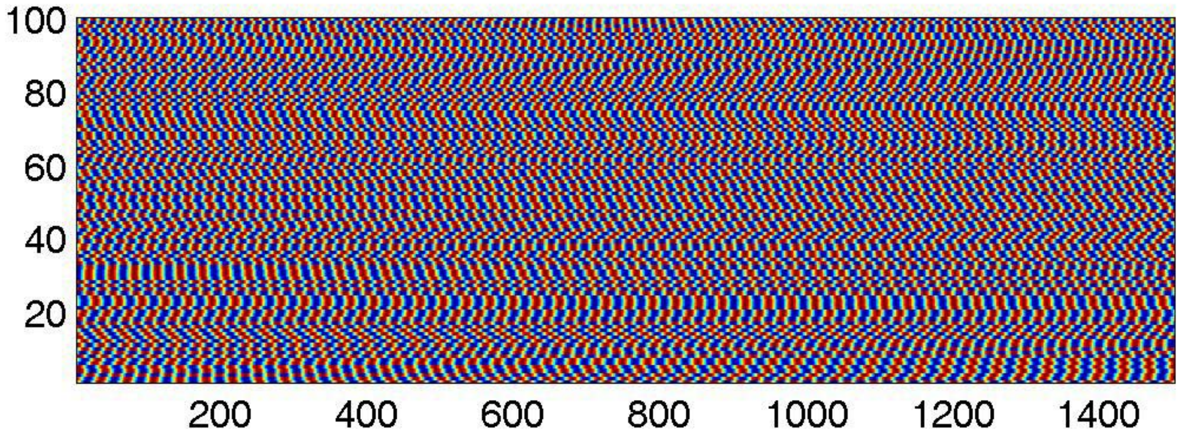
phase oscillator: $$\dot{\theta}_i = \omega_i - b\ddot{y}\cos(\theta_i + \phi)$$

Phase dynamics and bridge (I)

below

critical

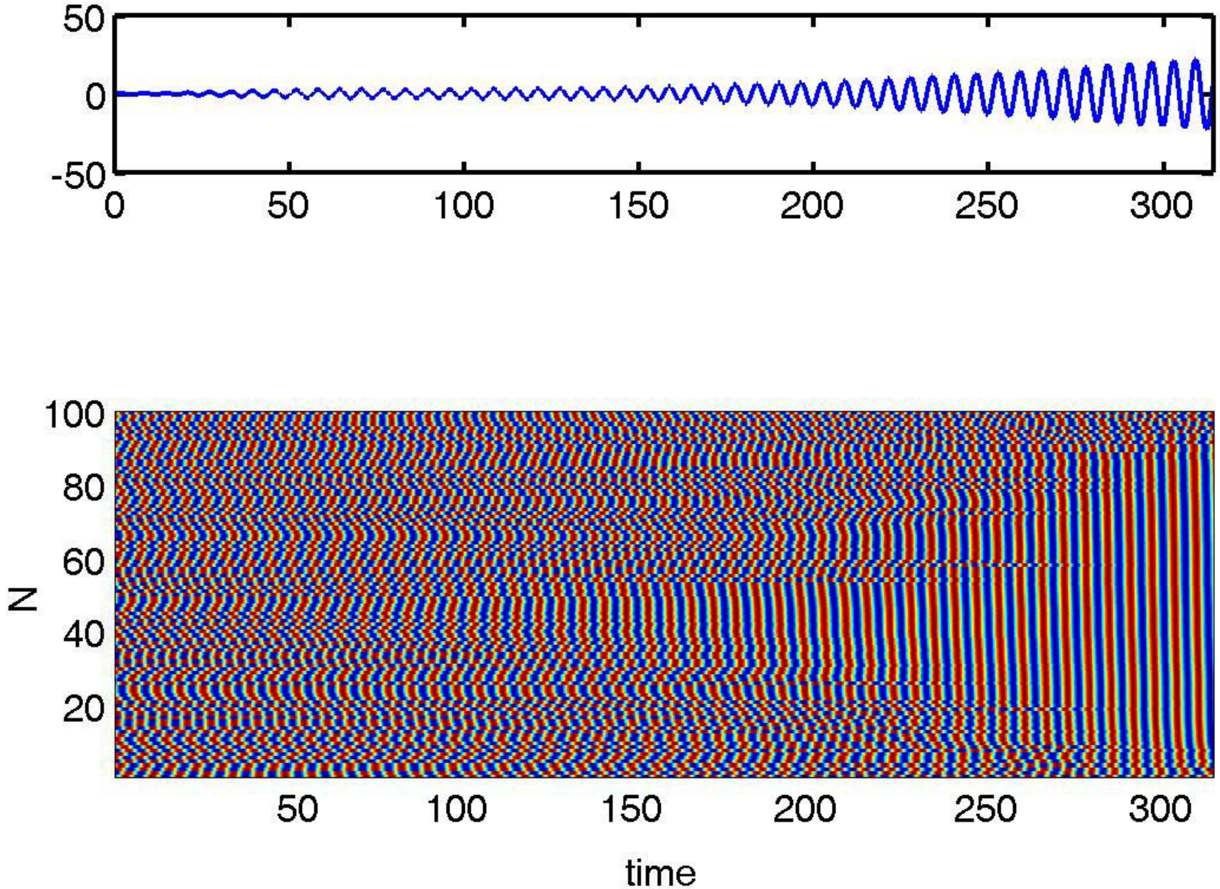




Phase dynamics and bridge (II)

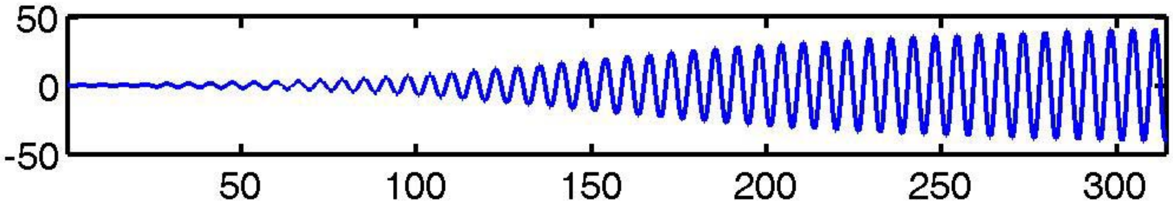
near

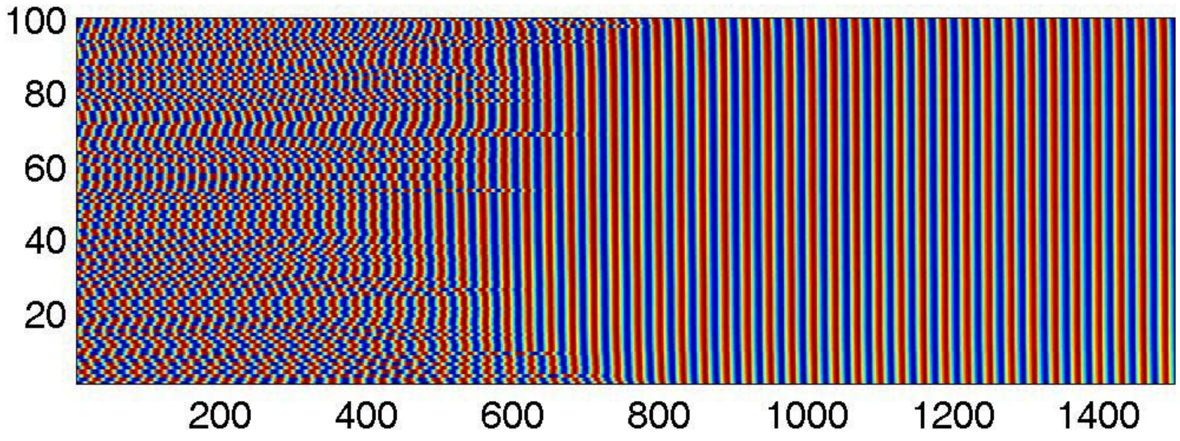
critical



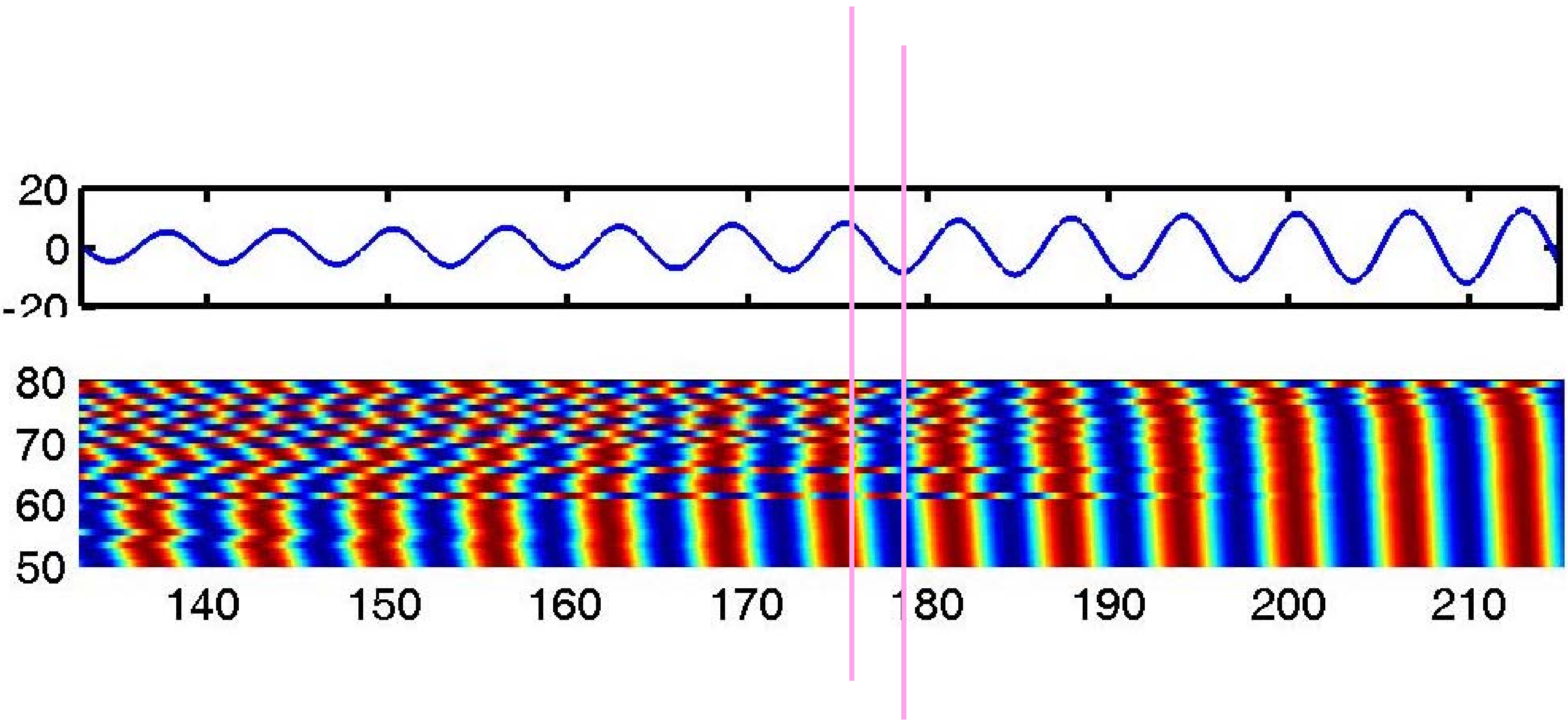
Phase dynamics and bridge (III)

above
critical

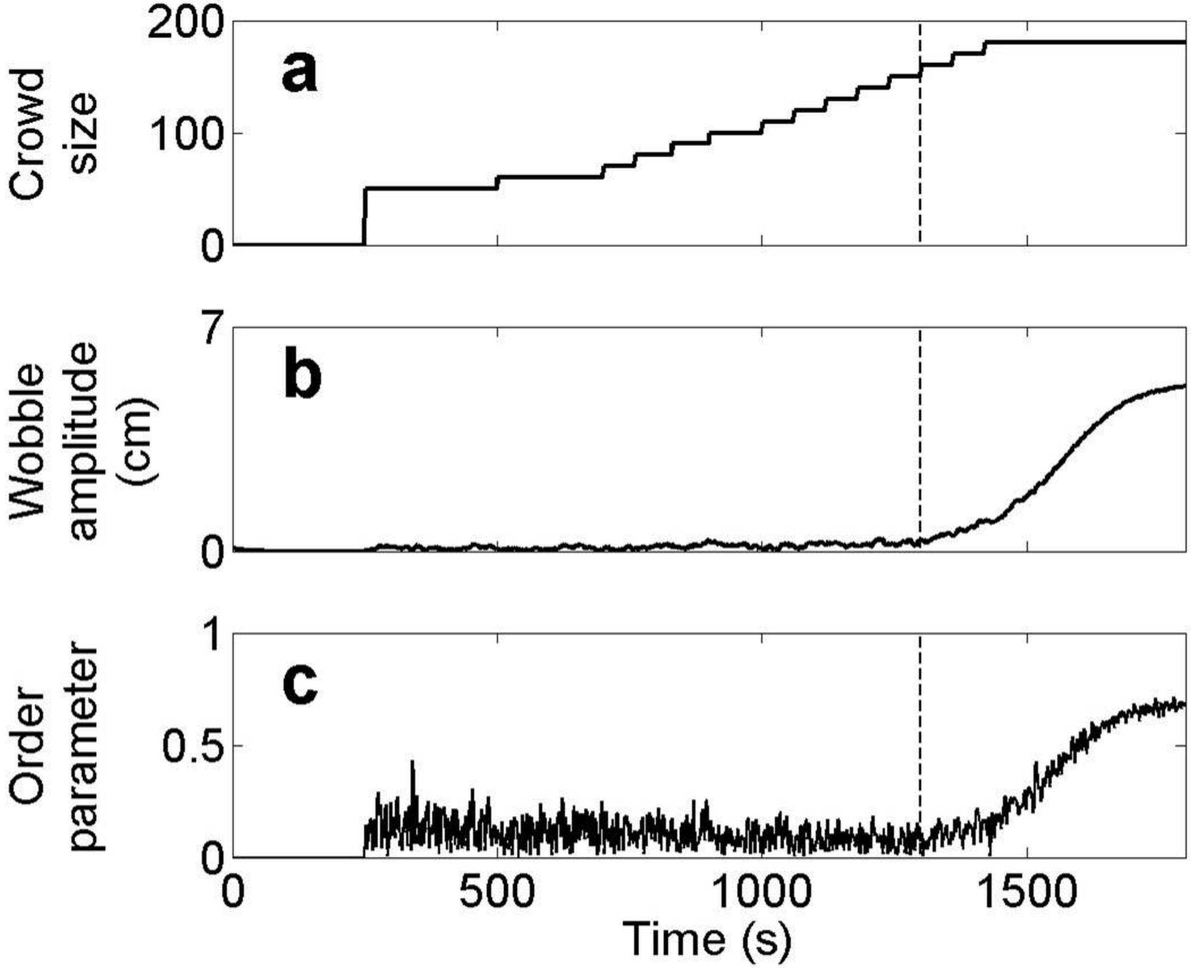




Phase relation



Dynamical simulation



The model

Dimensionless model:

Bridge motion:

$$\ddot{y} + 2\varsigma\dot{y} + y = \langle \cos(\theta_i) \rangle$$

phase oscillator:

$$\dot{\theta}_i = \omega_i - B\ddot{y}\cos(\theta_i + \phi)$$

coupling parameter:

$$B = \frac{Nf}{M\Omega(g_0\tau)}$$

Amplitude vs. Damping

Force vs. velocity:

In equilibrium:

$$\cancel{\ddot{y}} + 2\varsigma\dot{y} + \cancel{y} = \langle \cos(\theta_i) \rangle$$

acceleration and
restoring force vanish,
velocity and forcing remain !

force

velocity

Comparing parameters:

Experimental finding (Dallard et al):

$$N_c = 4\varsigma M\Omega / k$$

$$k = 300\,\mathrm{Ns/m}$$

Theoretical result:

$$k = \frac{\pi P(1) f}{2 g_0 \tau}$$

$$f = 25\,\mathrm{N}$$

$$P(1) = 1/(\sqrt{2\pi}\sigma)$$

$$\sigma = 0.09\,\Omega$$

$$g_0 = 0.3\,\mathrm{m^2/s}$$

$$\tau = 1.25\,\mathrm{s}$$

People and Publications:

Steven H Strogatz, Cornell
Daniel M Abrams, Cornell
Alan McRobie, Cambridge
Ed Ott, Maryland
Bruno Eckhardt, Maryland and Marburg

Nature, **438**, 43 (2005)
and in preparation