Simplicity and Swarming in Motion of Pelagic Fish

Discrete and Time-Continuous Models

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Fish Schools

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- The structure can be both simple and exceedingly complex
- Model of a primitive organism?

The Discrete Model

The model was introduced by Vicsek, Czirók, Ben-Jacob, Cohen, and Shochet in 1995 [7, 2, 3, 4, 8]

$$\begin{pmatrix} x_i(t+\Delta t) \\ y_i(t+\Delta t) \end{pmatrix} = \begin{pmatrix} x_i(t) \\ y_i(t) \end{pmatrix} + v_i(t) \begin{pmatrix} \cos(\theta_i(t)) \\ \sin(\theta_i(t)) \end{pmatrix} \Delta t$$

Alignment and Adjustment

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Direction Angle

$$\begin{pmatrix} \cos(\theta_i(t+\Delta t)) \\ \sin(\theta_i(t+\Delta t)) \end{pmatrix} = \frac{1}{n}\sum_{j=1}^{n} \begin{pmatrix} \cos(\theta_j(t)) \\ \sin(\theta_j(t)) \end{pmatrix}$$

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Speed

$$v_i(t+\Delta t) = \frac{1}{n}\sum_{j=1}^{n} v_j(t)$$

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We divide by Δt and send Δt to 0

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Introducing complex coordinates

$$z_k = x_k + i y_k$$

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In polar coordinates

$$z_k = r_k e^{i\theta_k}$$

The Continuous Model

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$$\dot{r}_k = \frac{v}{n} \sum_{j=1}^{n} \cos(\theta_k - \theta_j)$$

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We have made the choice

$$v = \lim_{n \to \infty} \frac{1}{n} \sum_{k=1}^{n} v_k$$

Migratory Solutions

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1-d center manifold. Solutions are *stable* except for 1-d linear motion. Also H-stable

Circling Solutions

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1-d center manifold, 1-d unstable manifold corresponding to circular motion. *Stable*

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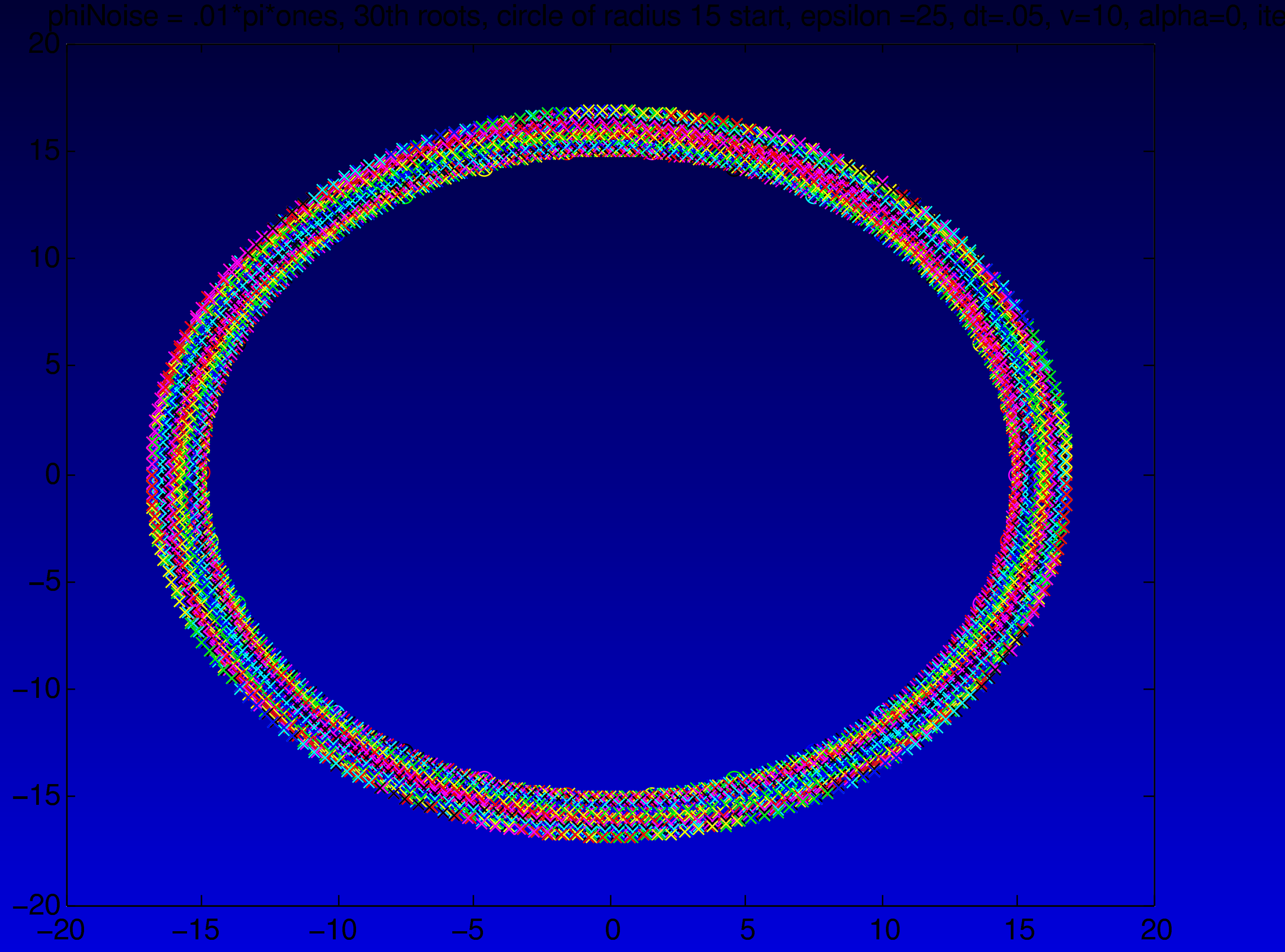
- Other periodic orbits consists of pairs of fish distributed around the circle
- Incoherent oscillators, $\omega_k/\omega_j,\ j \neq k$ irrational, give a complex state as $n \to \infty$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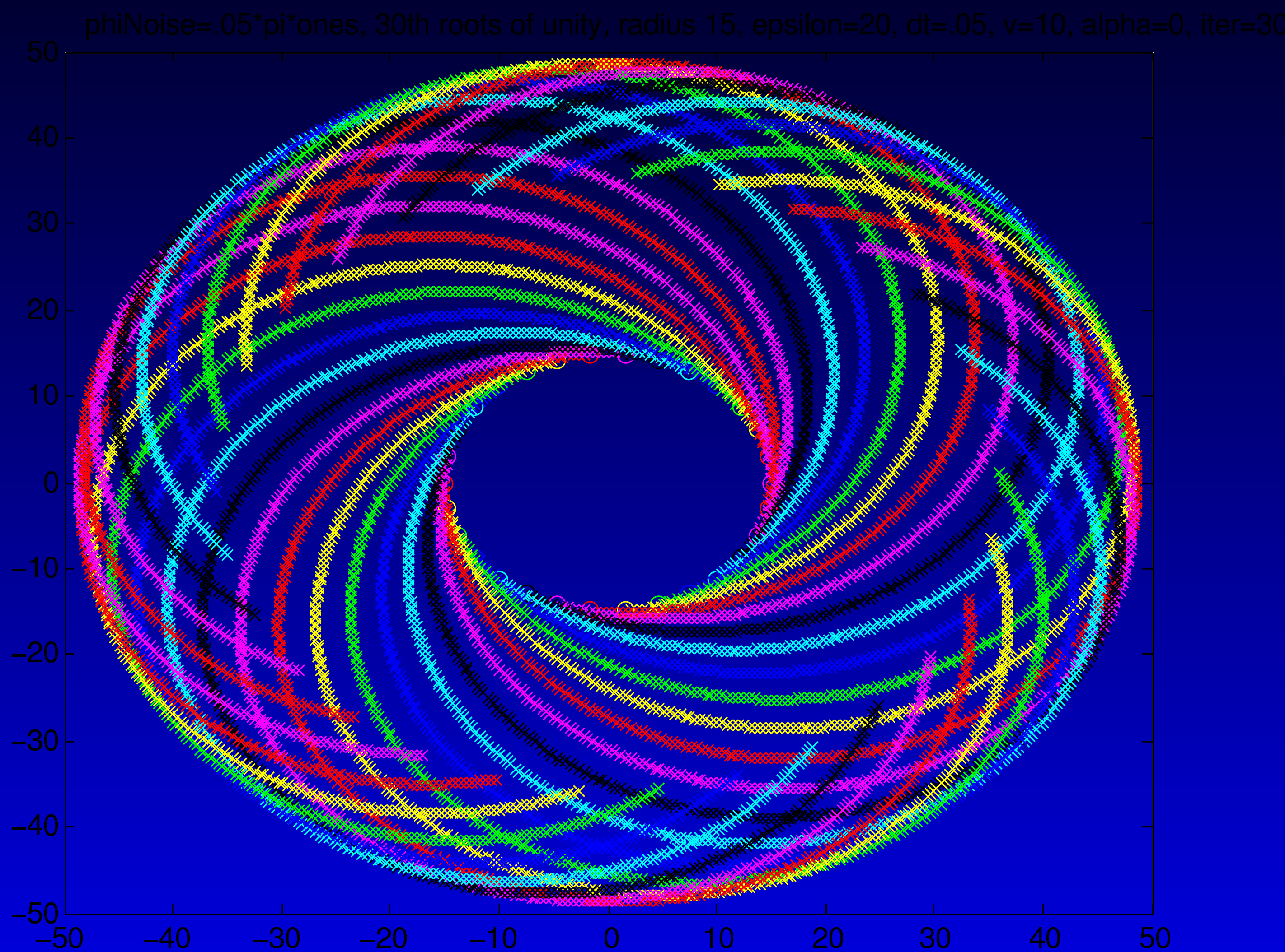
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- They were modeling the migration of the capelin in the North Atlantic
- The capelin is a pelagic species that much of the larger fish, cod etc. feed on


Greenland
Jan Mayen
Iceland
Faroe Islands
Norway

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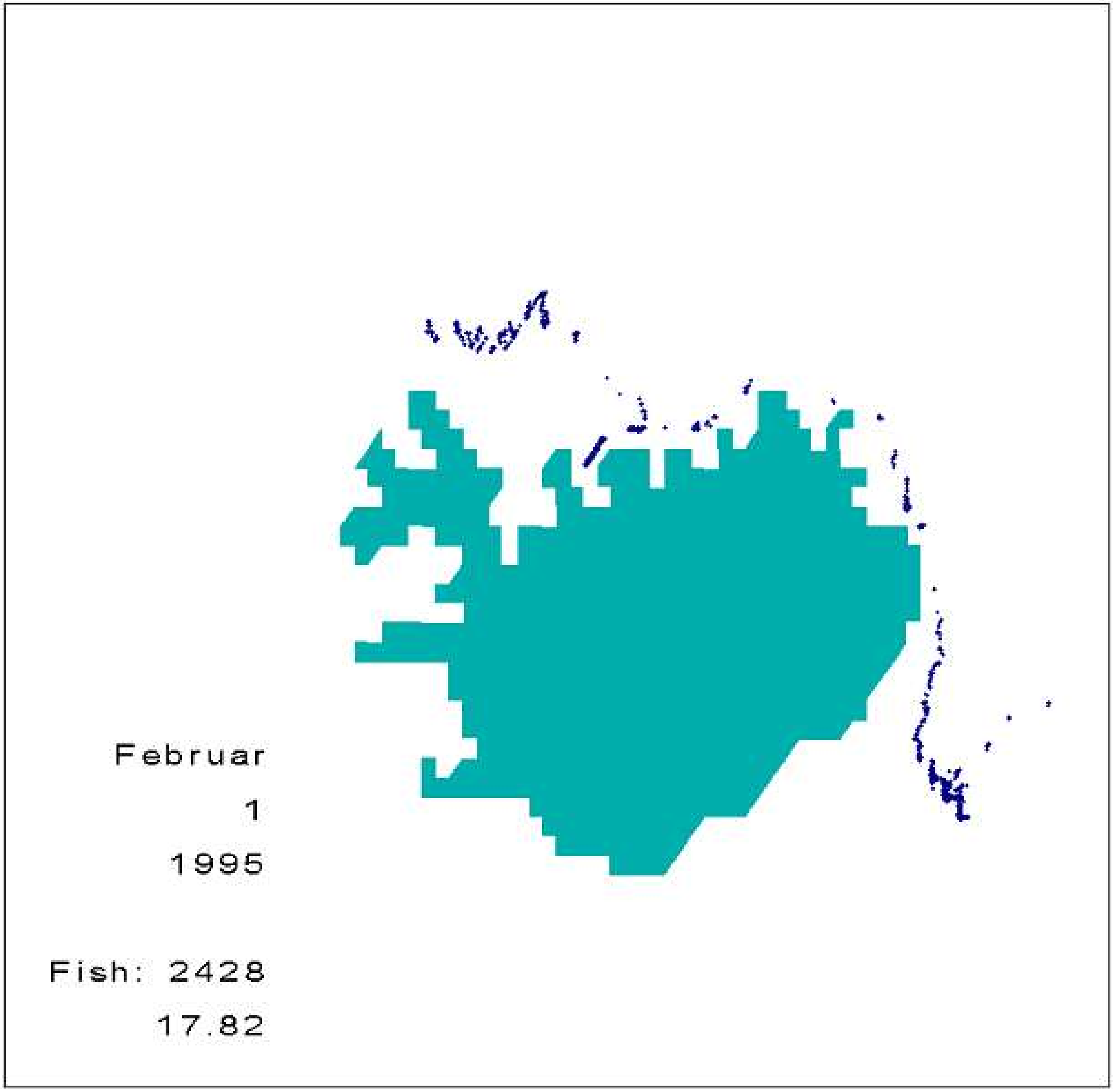
- It migrates along the *west coast* of Iceland in the spring and returns along the *east coast* in the winter, after spawning it dies
- Is this migration entirely controlled by the environment, currents, food availability, temperature gradients and landmasses or is it genetic?


Februar
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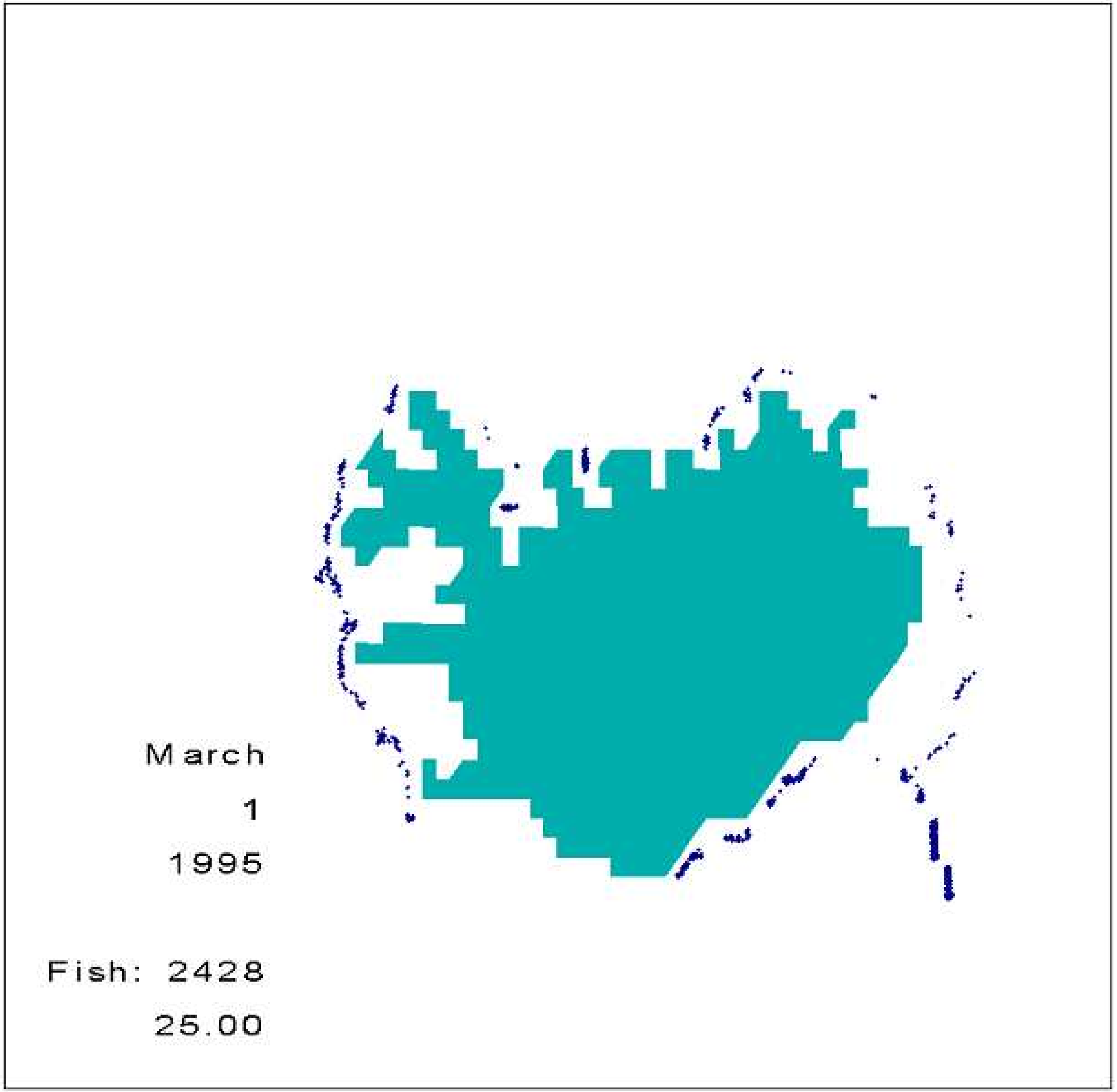
1995
Fish: 2428
17.82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- Migratory and circling solutions are easily found
- The migratory solutions consist of stable linear motion with constant speed
- The circling solutions consist of stable or unstable periodic orbits that exist for any number of fish
- Complex, incoherent, noise-driven, circling states exist as the number of fish in the school becomes large

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