Motion Coordination for Multi-Agent Networks

Swarming by Nature and by Design
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AFOSR MURI Appers, ARO MURI Swarms, NSF Sensors, ONR YIP

Multi-agent networks

What kind of systems?

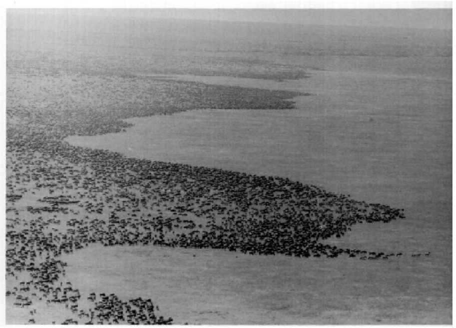
Groups of systems with control, sensing, communication and computing

Individual members in the group can

- sense its immediate environment
- communicate with others
- process the information gathered
- take a local action in response

Example networks from biology and engineering

Biological populations and swarms



Wildebeest herd in the Serengeti

Geese flying in formation

Atlantis aquarium, CDC Conference 2004

Multi-vehicle and sensor networks

embedded systems, distributed robotics

Distributed information systems, large-scale complex systems

intelligent buildings, stock market, self-managed air-traffic systems

Broad challenge

Useful engineering through small, inexpensive, limited-comm vehicles/sensors

Problem lack of understanding of how to assemble and coordinate individual devices into a coherent whole

Distributed feedback rather than "centralized computation for known and static environment"

Approach integration of control, comm, sensing, computing

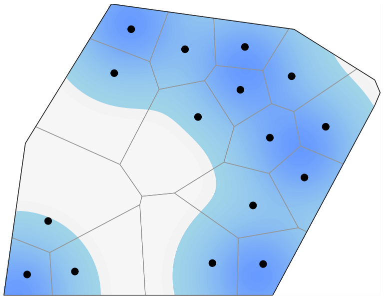
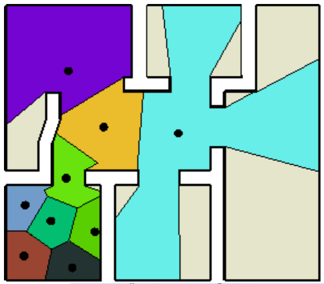
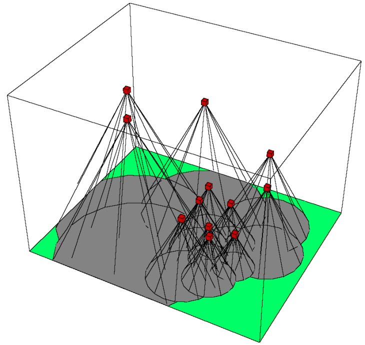
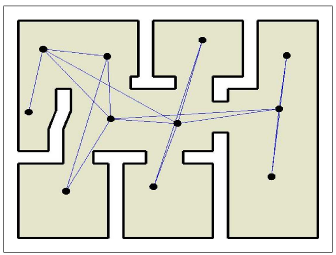
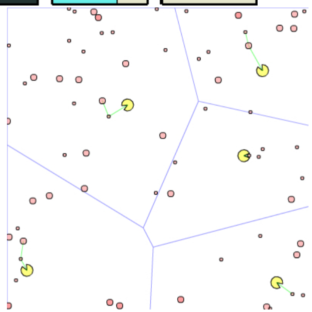
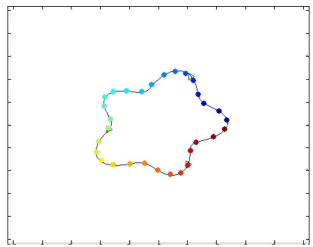
Research in Animation

(i) elementary motion tasks

deployment, rendezvous, flocking, self-assembly

(ii) sensing tasks

detection, localization, visibility, vehicle routing, search, plume tracing

Outline

I: Models for Multi-Agent/Robotic Networks: tools and modeling results

II: Motion Coordination: algorithms for multiple tasks

rendezvous, deployment

III: Sensing Tasks: sensing problems

target servicing, boundary estimation

Part I: Models for Multi-Agent Networks

References

(i) I. Suzuki and M. Yamashita. Distributed anonymous mobile robots: Formation of geometric patterns. *SIAM Journal on Computing*, 28(4):1347–1363, 1999

(ii) N. A. Lynch. *Distributed Algorithms*. Morgan Kaufmann Publishers, San Mateo, CA, 1997. ISBN 1558603484

(iii) D. P. Bertsekas and J. N. Tsitsiklis. *Parallel and Distributed Computation: Numerical Methods*. Athena Scientific, Belmont, MA, 1997. ISBN 1886529019

(iv) S. Martínez, F. Bullo, J. Cortés, and E. Frazzoli. On synchronous robotic networks – Part I: Models, tasks and complexity. *IEEE Transactions on Automatic Control*, April 2005. Submitted

Objective

(i) meaningful + tractable model

(ii) feasible operations and their cost

(iii) control/communication tradeoffs

Part I: Synchronous robotic network

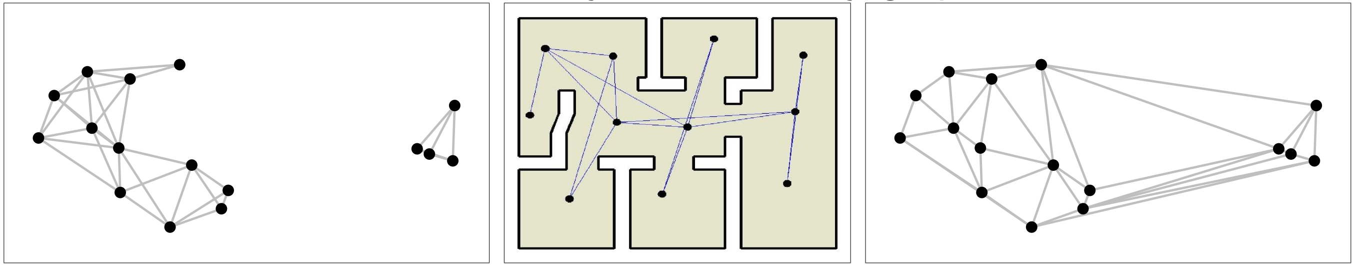
A **uniform/anonymous robotic network** $\mathcal{S}$ is

(i) $I = \{1, \dots, N\}$; **set of unique identifiers (UIDs)**

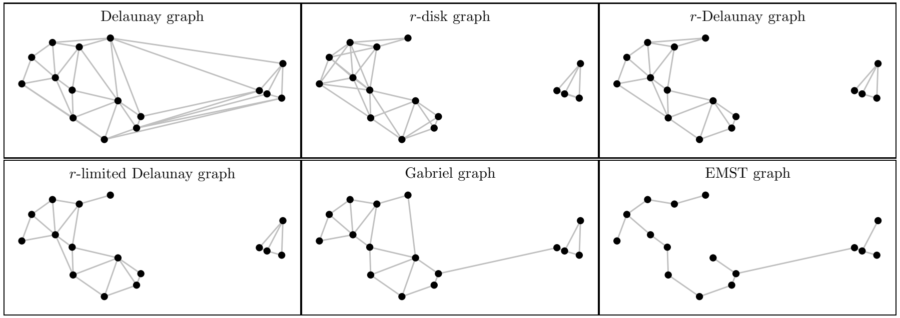
(ii) $\mathcal{A} = \{A_i\}_{i \in I}$, with $A_i = (X, U, X_0, f)$ is a set of identical control systems; **set of physical agents**

(iii) **communication graph**

Disk, visibility and Delauney graphs



Communication models for robotic networks



Relevant graphs

(i) fixed, balanced

(ii) geometric or state-dependent

(iii) switching

(iv) random, random geometric

Message model message, packet, bits; absolute or relative positions

Control and communication law

(i) **communication schedule** — $\mathbb{T} = \{t_\ell\}_{\ell \in \mathbb{N}_0} \subset \overline{\mathbb{R}}_+$

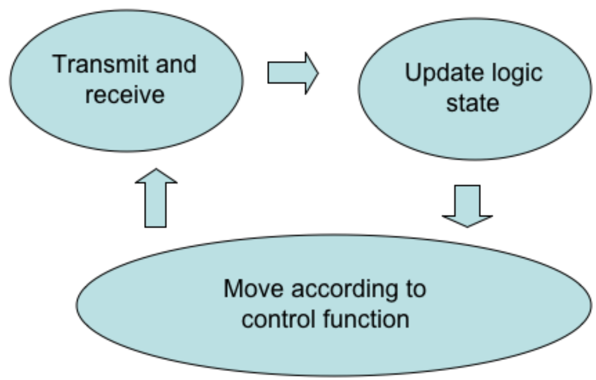
(ii) **communication language** — L including the `null` message

(iii) set of values for **logic variables** — W

(iv) **message-generation function** — $\mathsf{msg}: \mathbb{T} \times X \times W \times I \to L$

(v) **state-transition functions** — $\mathsf{stf}: \mathbb{T} \times W \times L^N \to W$

(vi) **control function** — $\mathsf{ctrl}: \overline{\mathbb{R}}_+ \times X \times W \times L^N \to U$



Task and complexity

- **Coordination task** is $(\mathcal{W}, \mathcal{T})$ where $\mathcal{T}$: $X^N \times \mathcal{W}^N \to \{\texttt{true}, \texttt{false}\}$
- For $\{\mathcal{S}, \mathcal{T}, \mathcal{CC}\}$, define costs/complexity:
  control effort, communication packets, computational cost
- **time complexity to achieve $\mathcal{T}$ with $\mathcal{CC}$**

$$\mathrm{TC}(\mathcal{T}, \mathcal{CC}, x_0, w_0) = \inf \{\ell \mid \mathcal{T}(x(t_k), w(t_k)) = \texttt{true}\,,\ \text{for all } k \geq \ell\}$$

$$\mathrm{TC}(\mathcal{T}, \mathcal{CC}) = \sup \left\{ \mathrm{TC}(\mathcal{T}, \mathcal{CC}, x_0, w_0) \mid (x_0, w_0) \in X^N \times \mathcal{W}^N \right\}$$

$$\mathrm{TC}(\mathcal{T}) = \inf \{\mathrm{TC}(\mathcal{T}, \mathcal{CC}) \mid \mathcal{CC} \text{ achieves } \mathcal{T}\}$$

Example tasks / control objective

Motion: deploy, gather, flock, reach pattern

Logic-based: achieve consensus, synchronize, form a team

Sensor-based: search, estimate, identify, track, map

Open problems in Part I

(i) complexity analysis (time/energy)
(ii) models/algorithms for asynchronous networks with agent arrival/departures
(iii) parallel, sequential, hierarchical composition of behaviors

Part II: Motion Coordination

Scenarios examples of networks, tasks, ctrl+comm laws

(i) rendezvous
(ii) deployment

Rendezvous

(i) H. Ando, Y. Oasa, I. Suzuki, and M. Yamashita. Distributed memoryless point convergence algorithm for mobile robots with limited visibility. *IEEE Transactions on Robotics and Automation*, 15(5):818–828, 1999

(ii) J. Lin, A. S. Morse, and B. D. O. Anderson. The multi-agent rendezvous problem. In *IEEE Conf. on Decision and Control*, pages 1508–1513, Maui, HI, December 2003

(iii) J. Cortés, S. Martínez, and F. Bullo. Robust rendezvous for mobile autonomous agents via proximity graphs in arbitrary dimensions. *IEEE Transactions on Automatic Control*, 51(6), 2006. To appear

Deployment

(i) J. Cortés, S. Martínez, T. Karatas, and F. Bullo. Coverage control for mobile sensing networks. *IEEE Transactions on Robotics and Automation*, 20(2):243–255, 2004

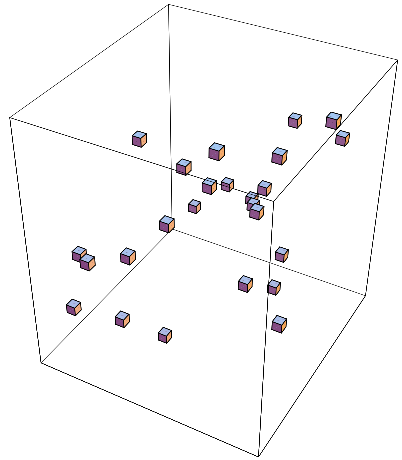
(ii) J. Cortés, S. Martínez, and F. Bullo. Spatially-distributed coverage optimization and control with limited-range interactions. *ESAIM. Control, Optimisation & Calculus of Variations*, 11:691–719, 2005

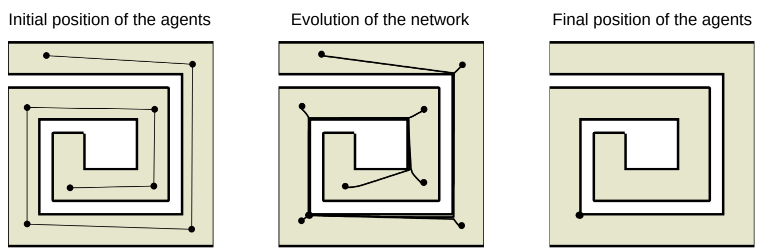
Scenario 1: aggregation laws for rendezvous

Aggregation laws

At each comm round:

1: acquire neighbors' positions
2: compute connectivity constraint set
3: move towards circumcenter of neighbors (while remaining connected)

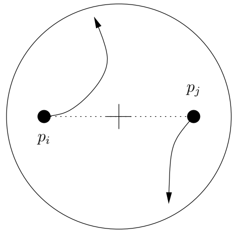


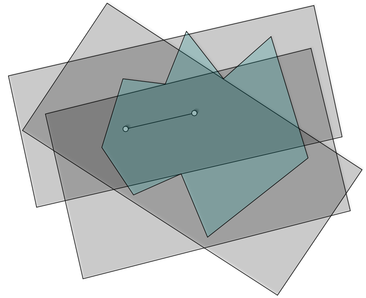


Task: rendezvous with connectivity constraint

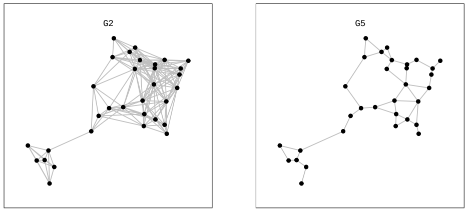
Scenario 1: aggregation laws for rendezous, cont'd

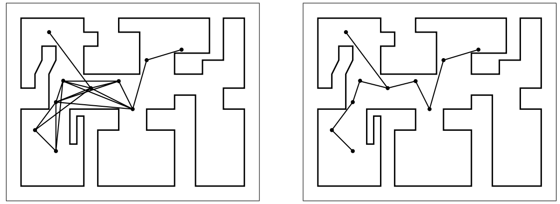
Pair-wise motion constraint set for connectivity maintenance



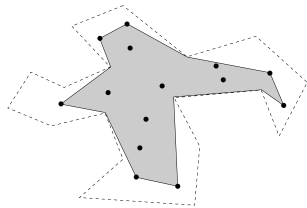


Reducing number of constraints





Lyapunov function



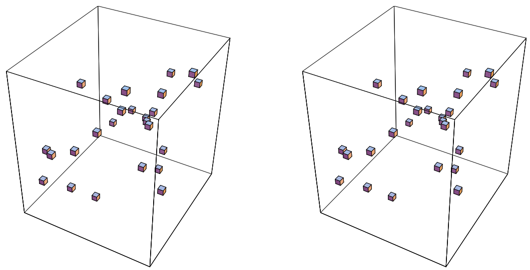
Scenario 1: Example complexity analysis

(i) first-order agents with disk graph, for $d = 1$,

$$\mathrm{TC}(\mathcal{T}_{\mathsf{rendezvous}}, \mathcal{CC}_{\mathsf{circumcenter}}) \in \Theta(N)$$

(ii) first-order agents with limited Delaunay, for $d = 1$,

$$\mathrm{TC}(\mathcal{T}_{(r\epsilon)\text{-}\mathsf{rendezvous}}, \mathcal{CC}_{\mathsf{circumcenter}}) \in \Theta(N^2 \log(N\epsilon^{-1}))$$



Tridiagonal Toeplitz and circulant systems

Let $N \geq 2$, $\epsilon \in]0,1[$, and $a, b, c \in \mathbb{R}$. Let $x, y \colon \mathbb{N}_0 \to \mathbb{R}^N$ solve:

$$x(\ell+1) = \mathrm{Trid}_N(a,b,c)\, x(\ell), \qquad x(0) = x_0,$$
$$y(\ell+1) = \mathrm{Circ}_N(a,b,c)\, y(\ell), \qquad y(0) = y_0.$$

(i) if $a = c \neq 0$ and $|b| + 2|a| = 1$, then $\lim_{\ell \to +\infty} x(\ell) = \mathbf{0}$, and the maximum time required for $\|x(\ell)\|_2 \leq \epsilon \|x_0\|_2$ is $\Theta\big(N^2 \log \epsilon^{-1}\big)$;

(ii) if $a \neq 0$, $c = 0$ and $0 < |b| < 1$, then $\lim_{\ell \to +\infty} x(\ell) = \mathbf{0}$, and the maximum time required for $\|x(\ell)\|_2 \leq \epsilon \|x_0\|_2$ is $O\big(N \log N + \log \epsilon^{-1}\big)$;

(iii) if $a \geq 0$, $c \geq 0$, $b > 0$, and $a + b + c = 1$, then $\lim_{\ell \to +\infty} y(\ell) = y_{\text{ave}} \mathbf{1}$, where $y_{\text{ave}} = \frac{1}{N} \mathbf{1}^T y_0$, and the maximum time required for $\|y(\ell) - y_{\text{ave}} \mathbf{1}\|_2 \leq \epsilon \|y_0 - y_{\text{ave}} \mathbf{1}\|_2$ is $\Theta\big(N^2 \log \epsilon^{-1}\big)$.

Scenario 2: dispersion laws for deployment

Dispersion laws

```
At each comm round:
 1: acquire neighbors' positions
 2: compute own dominance region
 3: move towards incenter /
    circumcenter / centroid of own
    dominance region
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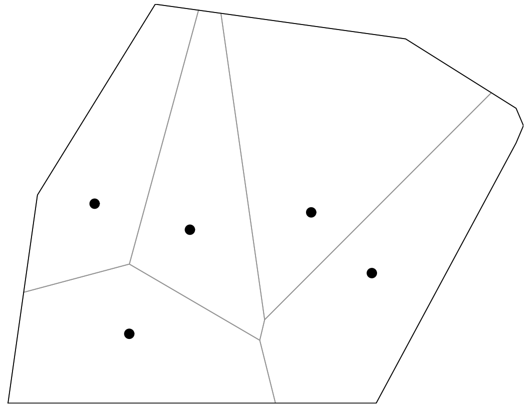
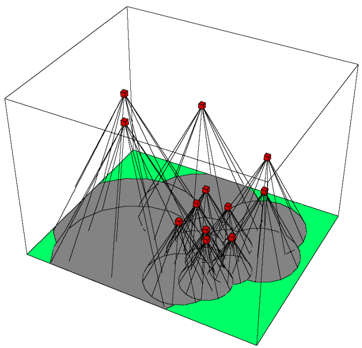
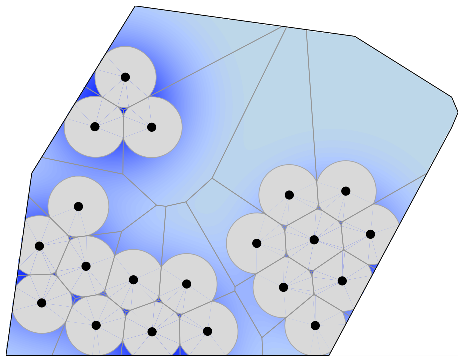
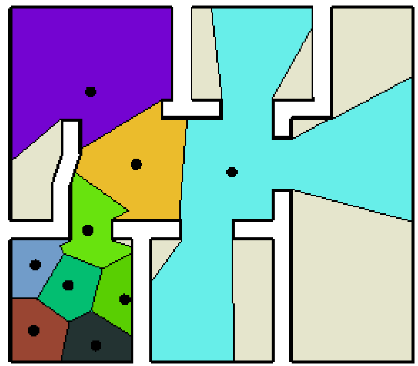




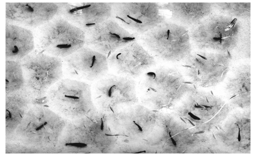




Scenarios: optimal deployment

ANALYSIS of cooperative distributed behaviors

(i) how do animals share territory?
what if every fish in a swarm goes
toward center of own dominance region?



Barlow, Hexagonal territories. Anim. Behav. '74

(ii) what if each vehicle moves toward center of mass of own Voronoi cell?

(iii) what if each vehicle moves away from closest vehicle?

DESIGN of performance metric

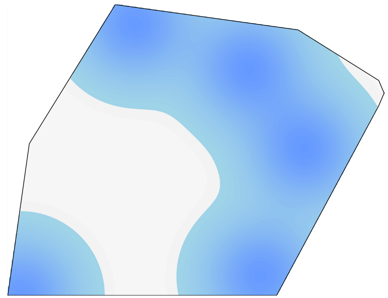
(iv) how to cover a region with n minimum radius overlapping disks?

(v) how to design a minimum-distorsion (fixed-rate) vector quantizer? (Lloyd '57)

(vi) where to place mailboxes in a city / cache servers on the internet?

Scenario 2: general multi-center function

Objective: Given agents $(p_1, \dots, p_n)$ in convex environment Q unspecified comm graph, achieve optimal coverage



Expected environment coverage

- let ϕ be distribution density function
- let f be a **performance/penalty function**

$$f(\|q - p_i|) \text{ is price for } p_i \text{ to service } q$$

- define **multi-center function**

$$\mathcal{H}_{\mathsf{C}}(p_1, \dots, p_n) = E_\phi \left[\min_i f(\|q - p_i\|) \right]$$

Scenario 2: distributed gradient result

For a general non-decreasing $f \colon \overline{\mathbb{R}}_+ \to \mathbb{R}$
piecewise differentiable with finite-jump discontinuities at $R_1 < \cdots < R_m$

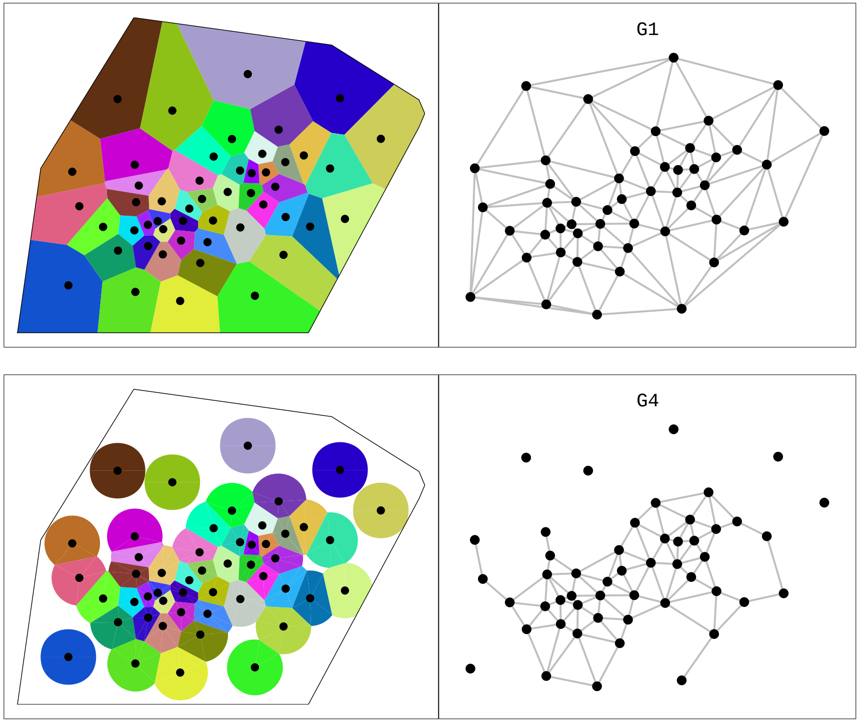
Thm:

$$\frac{\partial \mathcal{H}_{\mathsf{C}}}{\partial p_i}(p_1, \dots, p_n) = \int_{V_i} \frac{\partial}{\partial p_i} f(\|q - p_i\|)\phi(q)dq + \sum_{\alpha=1}^{m} \Delta f_\alpha(R_\alpha) \Big(\sum_{k=1}^{M_i(2R_\alpha)} \int_{\mathrm{arc}_{i,k}(2R_\alpha)} n_{B_{R_\alpha}(p_i)} d\phi \Big)$$

$=$ **integral over** V_i $+$ **integral along arcs inside** V_i

Gradient depends on information contained in V_i

On Voronoi and limited-Voronoi partitions



$\frac{\partial \mathcal{H}_C}{\partial p_i}$ is distributed over Delaunay graph, but not disk graph

Scenario 2: truncation

problem $\partial\mathcal{H}_{\mathsf{C}}$ distributed over Delaunay graph, but comm. is disk graph

approach truncate $f_{\frac{r}{2}}(x) = f(x)\ 1_{[0,\frac{r}{2})}(x) + (\sup_Q f) \cdot 1_{[\frac{r}{2},+\infty)}(x)$,

$$\mathcal{H}_{\frac{r}{2}}(p_1,\dots,p_n) = E_\phi\left[\min_i f_{\frac{r}{2}}(\|q-p_i\|)\right]$$

Result 1: $\mathcal{H}_{\mathsf{C}}$ **constant-factor approximation**

$$\beta\,\mathcal{H}_{\frac{r}{2}}(P) \leq \mathcal{H}_{\mathsf{C}}(P) \leq \mathcal{H}_{\frac{r}{2}}(P)\,, \quad \beta = \left(\frac{r}{2\,\mathrm{diam}(Q)}\right)^2$$

Result 2 Gradient of $\mathcal{H}_{\frac{r}{2}}$ is **distributed over limited-range Delaunay**

$$\frac{\partial\mathcal{H}_{\frac{r}{2}}}{\partial p_i} = 2M_{V_i(P)\cap B_{\frac{r}{2}}(p_i)}(C_{V_i(P)\cap B_{\frac{r}{2}}(p_i)} - p_i) - \left(\left(\frac{r}{2}\right)^2 - \mathrm{diam}(Q)^2\right)\sum_{k=1}^{\mathcal{M}_i(r)}\int_{\mathrm{arc}_{i,k}(r)} n_{B_{\frac{r}{2}}(p_i)}\,\phi$$

Aggregate objective functions

design of aggregate network-wide cost/objective/utility functions

- objective functions to encode motion coordination objective
- objective functions as Lyapunov functions
- objective functions for gradient flows

	$\mathcal{H}_{\mathrm{C}}$	$\mathcal{H}_{\mathrm{area}}$	$\mathcal{H}_{\mathrm{diam}}$
DEFINITION	$E\left[\min d(q, p_i)\right]$	$\mathrm{area}_\phi(\cup_i B_{r/2}(p_i))$	$\max_{i,j} \lVert p_i - p_j \rVert$
SMOOTHNESS	C^1	globally Lipschitz	continuous, locally Lipschitz
CRITICAL POINTS MINIMA	Centroidal Voronoi configurations	r-limited Voronoi configurations*	common location for p_i
HEURISTIC DESCRIPTION	expected distortion	area covered	diameter connected component

Open problems in Part II

(i) general pattern formation problem
(ii) static and dynamic motion patterns
(iii) algorithms for line-of-sight 3D networks
(iv) connectivity and collision avoidance algorithms

Part III on Sensing Tasks

Problems of interest

- optimal sensor placement
- localization, estimation
- distributed sensing tasks:
  search, exploration, map building, target identification

References

Target Servicing

(i) R. W. Beard, T. W. McLain, M. A. Goodrich, and E. P. Anderson. Coordinated target assignment and intercept for unmanned air vehicles. *IEEE Transactions on Robotics and Automation*, 18(6):911–922, 2002

(ii) A. E. Gil, K. M. Passino, and A. Sparks. Cooperative scheduling of tasks for networked uninhabted autonomous vehicles. In *IEEE Conf. on Decision and Control*, pages 522–527, Maui, Hawaii, December 2003

(iii) W. Li and C. G. Cassandras. Stability properties of a cooperative receding horizon controller. In *IEEE Conf. on Decision and Control*, pages 492–497, Maui, HI, December 2003

(iv) E. Frazzoli and F. Bullo. Decentralized algorithms for vehicle routing in a stochastic time-varying environment. In *IEEE Conf. on Decision and Control*, pages 3357–3363, Paradise Island, Bahamas, December 2004

Boundary Estimation

(i) D. Marthaler and A. L. Bertozzi. Tracking environmental level sets with autonomous vehicles. In *Proc. of the Conference on Cooperative Control and Optimization*, Gainesville, FL, December 2002

(ii) J. Clark and R. Fierro. Cooperative hybrid control of robotic sensors for perimeter detection and tracking. In *American Control Conference*, pages 3500–3505, Portland, OR, June 2005

(iii) D. W. Casbeer, S.-M. Li, R. W. Beard, R. K. Mehra, and T. W. McLain. Forest fire monitoring with multiple small UAVs. In *American Control Conference*, pages 3530–3535, Portland, OR, June 2005

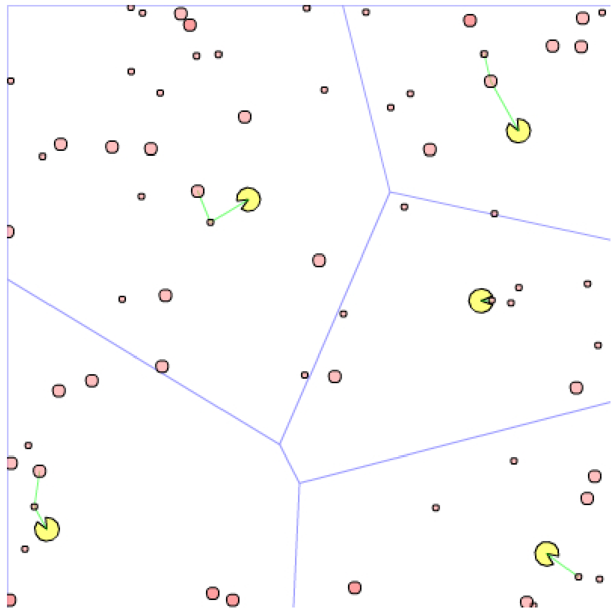
(iv) F. Zhang and N. E. Leonard. Generating contour plots using multiple sensor platforms. In *IEEE Swarm Intelligence Symposium*, pages 309–316, Pasadena, CA, June 2005

Scenario 3: Vehicle Routing

Objective: Given agents $(p_1, \ldots, p_n)$ moving in environment Q service targets in environment

Model:

- targets arise randomly in space/time
- vehicle know of targets arrivals



Scenario 3 —min expected waiting time

Scenario 3: receding-horizon TSP algorithm, I

Name: (Single Vehicle) Receding-horizon TSP

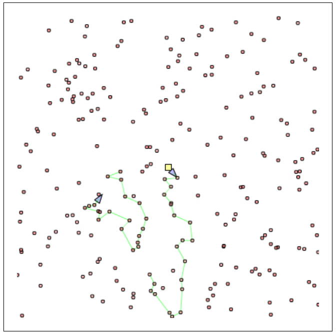
For $\eta \in (0, 1]$, single agent performs:

1: while no targets, dispersion/coverage algorithm

2: while targets waiting,

(i) compute optimal TSP tour through all targets

(ii) service the η-fraction of tour with maximal number of targets



Asymptotically optimal in light and high traffic

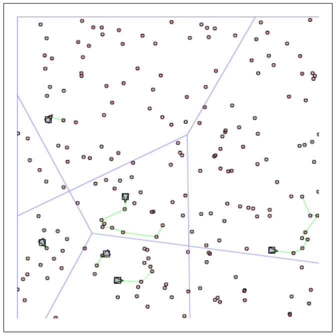
Scenario 3: receding-horizon TSP algorithm, II

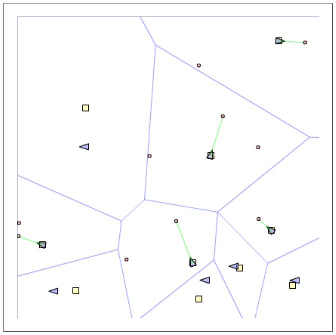
Name: Receding-horizon TSP

For $\eta \in (0, 1]$, agent i performs:

1: compute own Voronoi cell V_i
2: apply Single-Vehicle RH-TSP policy on V_i

Asymptotically optimal in light and high traffic (simulations only)

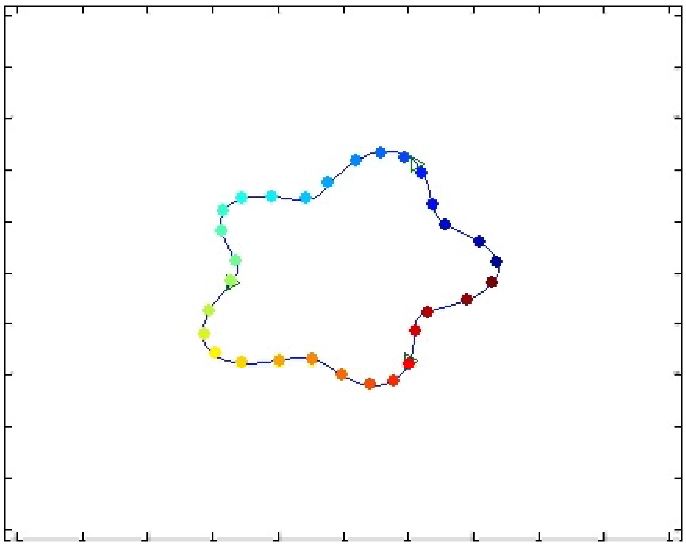




Scenario 4: Boundary Estimation

Objective: estimate/interpolate moving boundary
Model:

- UAVs can locally sense and track
- comm. graph is ring
- interpolation via waypoints



Emerging Motion Coordination Discipline

(i) **network modeling**

network, ctrl+comm algorithm, task, complexity

coordination algorithm

optimal deployment, rendezvous, vehicle routing
scalable, adaptive, asynchronous, agent arrival/departure

(ii) **Systematic algorithm design**

- meaningful aggregate cost functions
- class of (gradient) algorithms local, distributed
- geometric graphs
- stability theory for networked hybrid systems