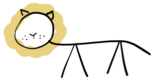
Taming the Dirac fermion

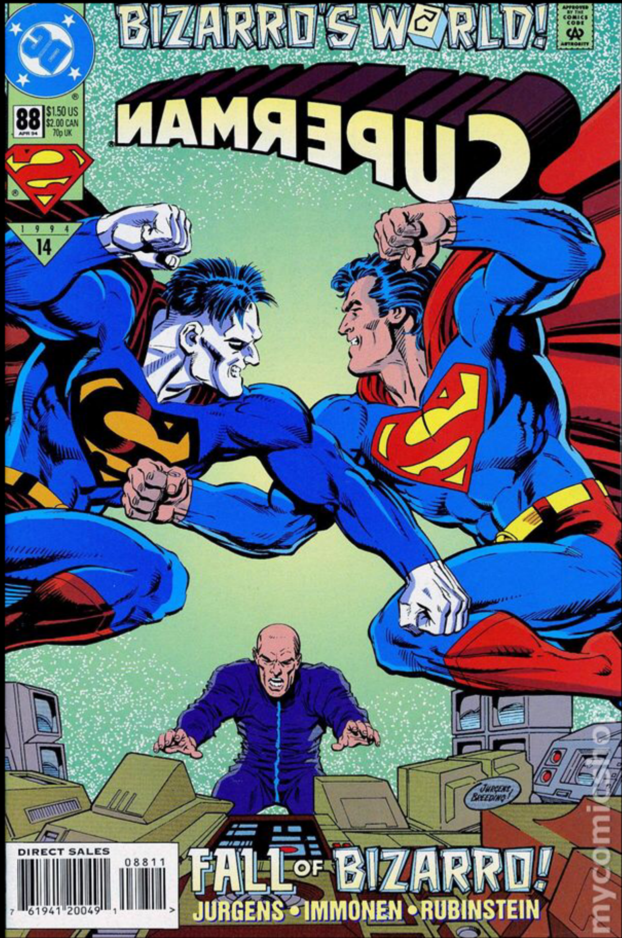


Composite Dirac liquids and surface topological orders

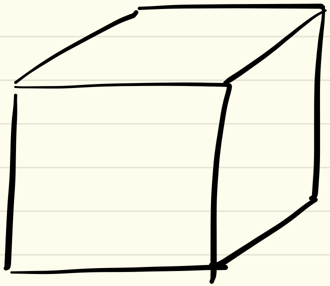
Caltech

Andrew M. Essin

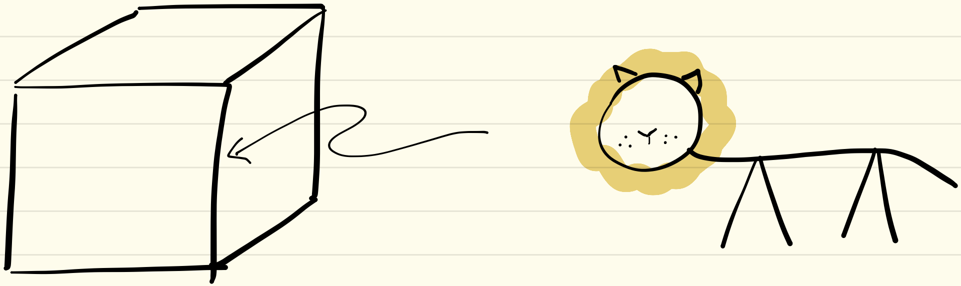
INSTITUTE FOR QUANTUM INFORMATION AND MATTER


BIZARRO'S WORLD!
SUPERMAN
88
APR '94
$1.50 US
$2.00 CAN
70p UK
1994
14
JURGENS BREEDING
DIRECT SALES
FALL OF BIZARRO!
JURGENS • IMMONEN • RUBINSTEIN

TOPOLOGICAL INSULATORS

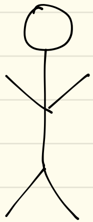


TOPOLOGICAL INSULATORS



WORK WITH

DAVID MROSS



JASON ALICEA

WORK WITH

DAVID MROSS

JASON ALICEA

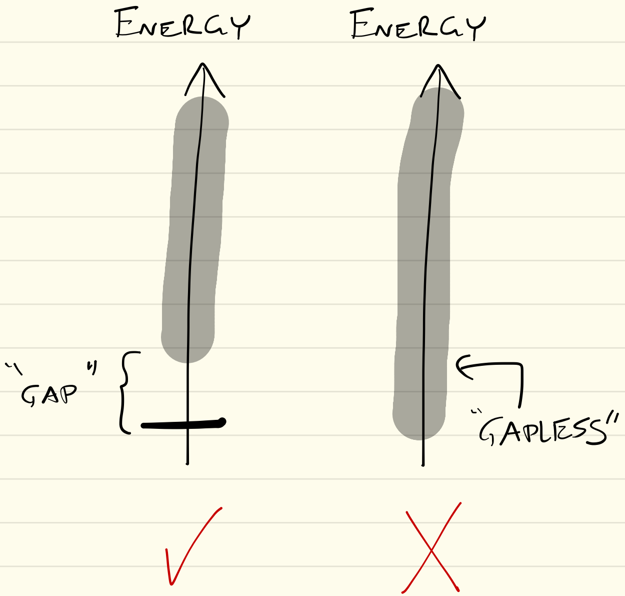
OUTLINE - BACKGROUND

PREHISTORY (~30 YRS.) - FRACTIONAL QUANTUM HALL

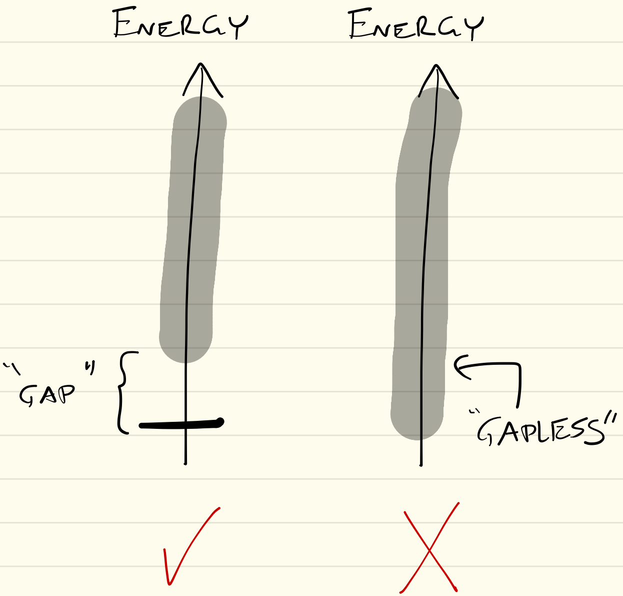
ANCIENT HISTORY (~10 YRS) - TOPOLOGICAL INSULATORS

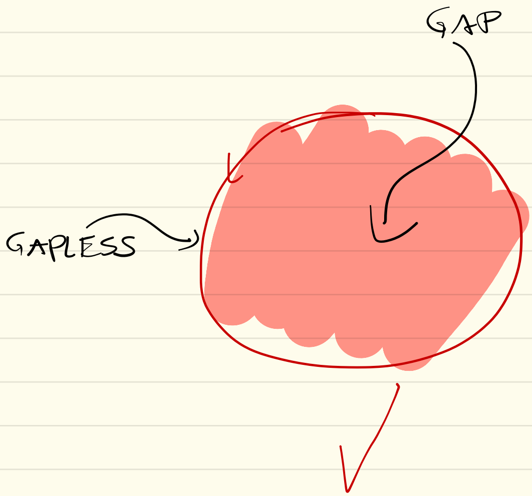
HISTORY (~1 YR) - SYMMETRIC TOPOLOGICAL SURFACES

"Topological" Phases of Matter

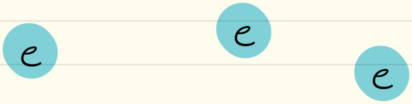


"Topological" Phases of Matter



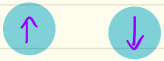


ELECTRONS



PROPERTIES:

- ENERGY
- CHARGE = 1
- SPIN = 1/2



SYMMETRIES

- PHASE ROTATION $\psi \to e^{i\alpha}\psi$
- TIME REVERSAL $\uparrow \longleftrightarrow \downarrow$

- "STATISTICS"

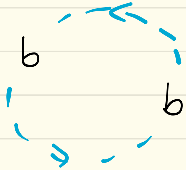
$$\Psi(\text{coords}) \to -\Psi(\text{coords})$$

FERMIONS

MORE GENERALLY,

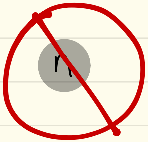
"LOCAL FERMIONS" HAVE CHARGE = ODD INTEGER

LOCAL BOSONS HAVE CHARGE = EVEN



$\Psi \to \Psi$

<u>NOT</u> NEUTRONS, SAY: FERMIONS, CHARGE = 0.



How to get a gap

HOW TO GET A GAP

- FOR BOSONS, NEED STRONG INTERACTIONS:

ENERGY COST FOR TWO BOSONS IN THE SAME STATE

$$\cancel{bb}$$

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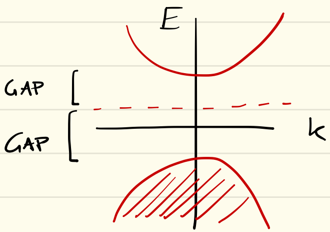
$\cancel{b\,b}$

- FILLED FERMION BAND ("DIRAC SEA")



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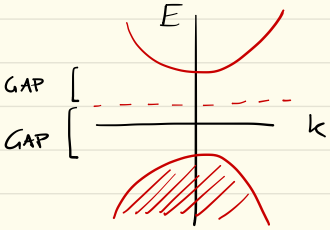
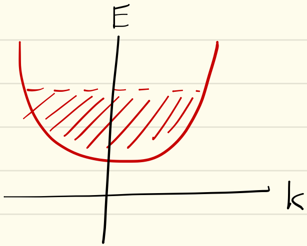
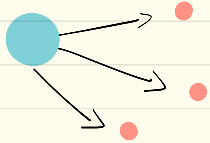


CONTRAST METAL

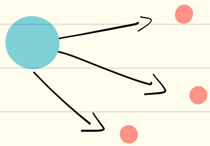


FRACTIONAL EXCITATIONS MAY HAVE
OTHER STATISTICS, CHARGE



CONSIDER FRACTIONAL QUANTUM HALL AT $\nu = 1/2$.

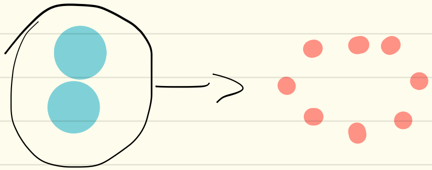
FRACTIONAL EXCITATIONS MAY HAVE OTHER STATISTICS, CHARGE

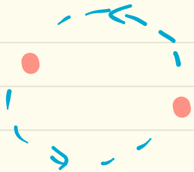


CONSIDER FRACTIONAL QUANTUM HALL AT $\nu = 1/2$.

OLD IDEA OF HALPERIN:

- ELECTRONS PAIR TO FORM CHARGE-2 BOSONS
- THESE PAIRS BREAK INTO 8 PIECES WITH CHARGE $1/4$, STATISTICS $e^{i\pi/8}$





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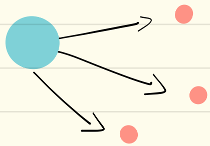

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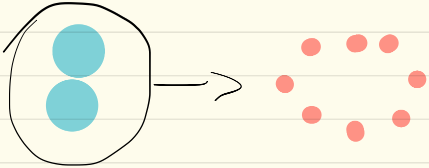
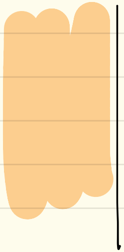


EDGE: CHARGE CONDUCTANCE $\frac{1}{2} = 2^2/8 \longrightarrow \nu = 1/2$



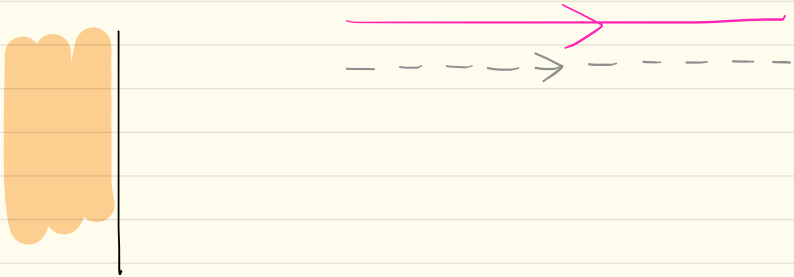
ENERGY CONDUCTANCE 1. $\longrightarrow c = 1$

Later idea (Moore & Read):

Charge $1/4$ anyon carries Majorana bound state,

$\longrightarrow$ Nonabelian fusion, braiding.

Edge



$\nu = \frac{1}{2},\ c = 1$

$\nu = 0,\ c = 1/2$

$\left.\begin{matrix} \nu = \frac{1}{2},\ c = 1 \\ \nu = 0,\ c = 1/2 \end{matrix}\right\}\ \nu = \frac{1}{2},\ c = 3/2.$

EXPERIMENTS SEE A GAPLESS STATE, SORT OF LIKE A METAL

"COMPOSITE FERMI LIQUID"

VERY CRUDELY, ONE CAN IMAGINE FRACTIONAL FERMIONS

EXPERIMENTS SEE A GAPLESS STATE, SORT OF LIKE A METAL

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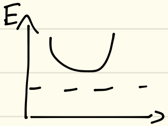
VERY CRUDELY, ONE CAN IMAGINE FRACTIONAL FERMIONS

THE TWO STATES CORRESPOND TO PAIRING OF THESE FERMIONS:

HALPERIN –



MOORE-READ –

⟵ PLAUSIBLE CANDIDATE FOR TOPOLOGICAL STATE AT FILLING $5/2$.

HASTINGS (OSHIKAWA; LIEB, SCHULTZ, MATTIS)

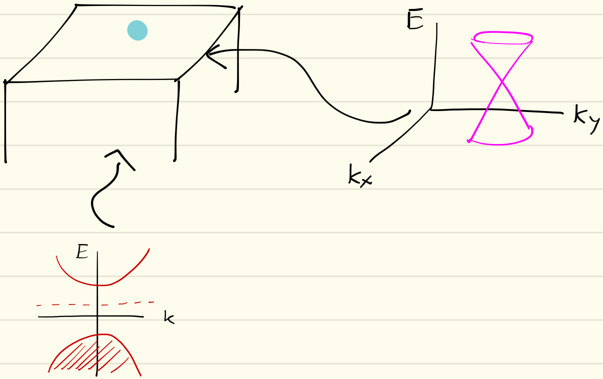
WITH APPROPRIATE SYMMETRIES AND CHARGES:

(0. BREAK SYMMETRY)

1. GAP + FRACTIONAL EXCITATIONS

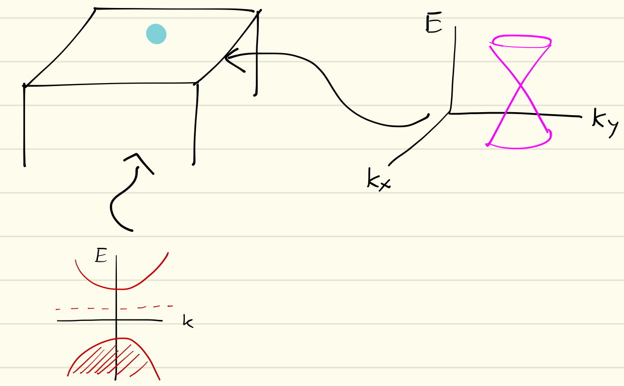
2. GAPLESS

3D Topological Insulator



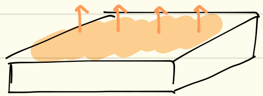
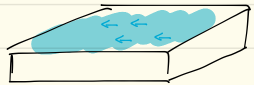
$$H_{2D} = i\,\Psi^{+}\,\vec{\sigma}\cdot\vec{\nabla}\,\Psi \qquad \Psi = \begin{pmatrix}\psi_\uparrow \\ \psi_\downarrow\end{pmatrix}$$

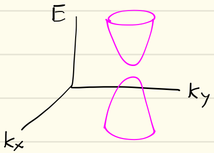
3D Topological Insulator



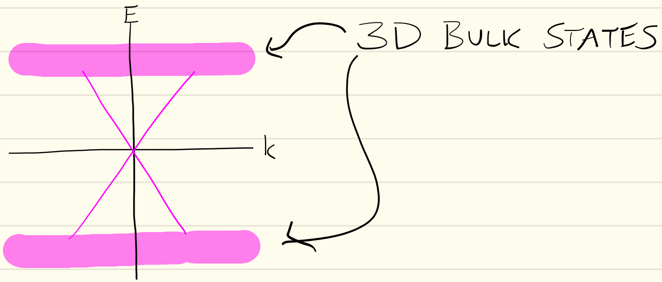
$$H_{2D} = i\,\Psi^{\dagger}\,\vec{\sigma}\cdot\vec{\nabla}\,\Psi \qquad \Psi = \begin{pmatrix}\psi_{\uparrow}\\ \psi_{\downarrow}\end{pmatrix}$$

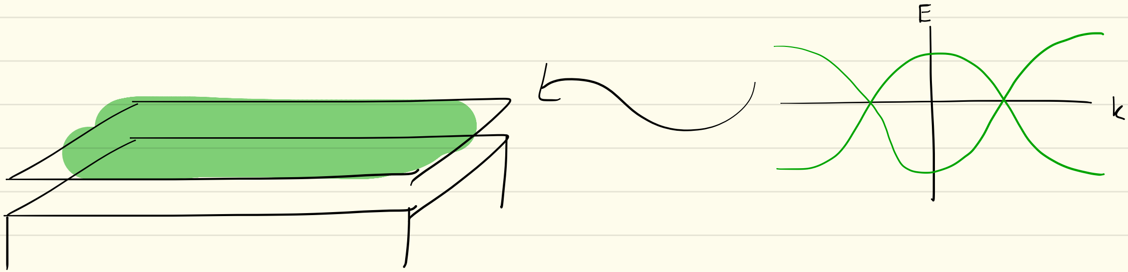
Symmetry

	Time Reversal	Phase Rotation
Symmetry Action	$\uparrow \longleftrightarrow \downarrow$	$\Psi \rightarrow e^{i\alpha}\Psi$
Symmetry-Breaking	$h>0$ 	$\Delta>0$ 
	$h \in \mathbb{R}$	$\Delta \in \mathbb{C}$



2D SURFACE V. 2D BULK



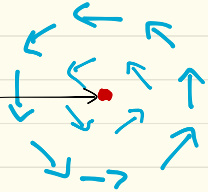


SYMMETRY BREAKING — POINT DEFECTS

SYMMETRY BREAKING – POINT DEFECTS

MAJORANA BOUND STATE

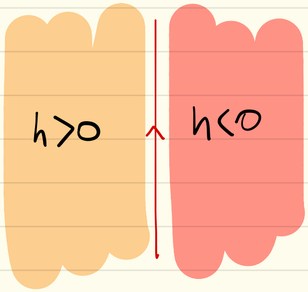
$\gamma = \gamma^{+}$



$\Delta\, e^{i\alpha(\vec{x})}$

Symmetry Breaking - Line defects

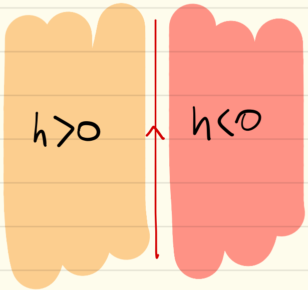
Chiral fermion mode



$\nu = 1, \quad c = 1$

SYMMETRY BREAKING – LINE DEFECTS

CHIRAL FERMION MODE



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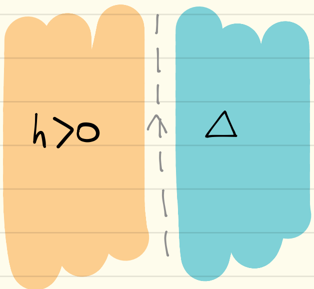
CHIRAL MAJORANA MODE



$c = 1/2$

SYMMETRIC, GAPPED SURFACES

$\left.\begin{array}{l}\text{BONDERSON, NAYAK, QI}\\ \text{METLITSKI, KANE, FISHER}\\ \text{WANG, POTTER, SENTHIL}\end{array}\right\}$ VORTEX CONDENSATION

CHEN, FIDKOWSKI, VISHWANATH — WALKER-WANG MODELS

SYMMETRIC, GAPPED SURFACES

BONDERSON, NAYAK, QI
METLITSKI, KANE, FISHER
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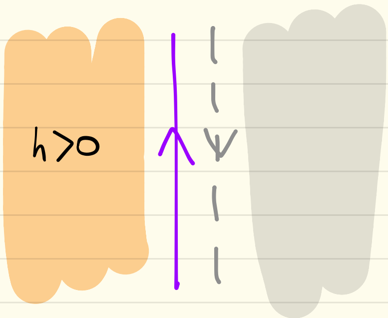
$\Big\}$ VORTEX CONDENSATION

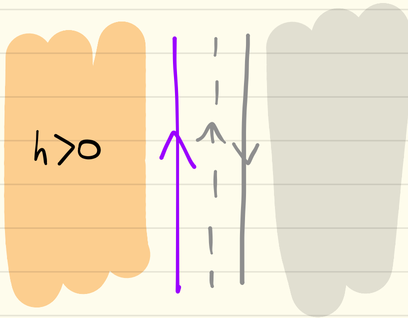
CHEN, FIDKOWSKI, VISHWANATH — WALKER-WANG MODELS

TWO POSSIBILITIES:



$\nu = 1/2, \quad c = 1/2$

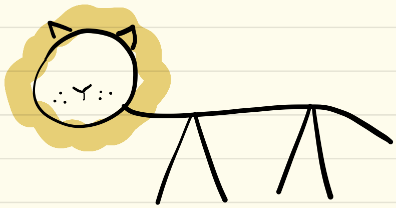
"T-PFAFFIAN"

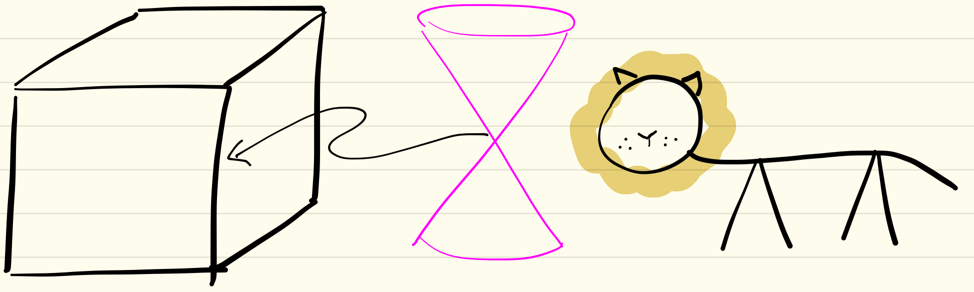


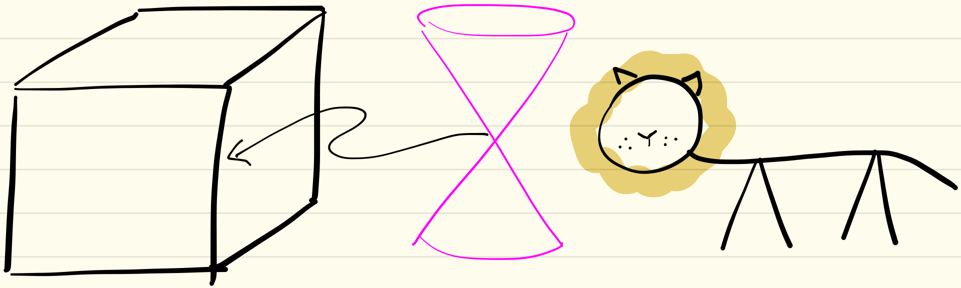
$\nu = 1/2, \quad c = 1/2$

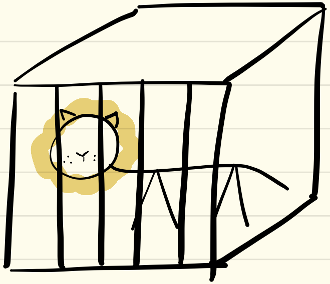
"PFAFFIAN-ANTISEMION"

Questions?

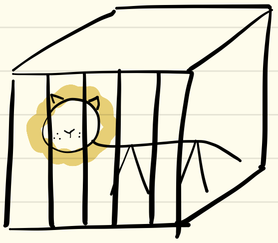








1. CAGE THE DIRAC FERMION



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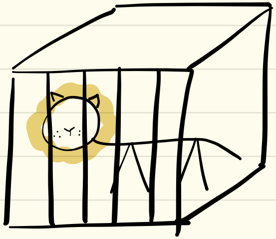
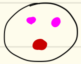


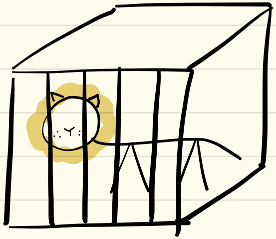
2. ENERGETICALLY BIND CHARGE



→ ALL CHARGES GAPPED.

→ NEUTRAL BOUND STATE → U(1) NOT BROKEN

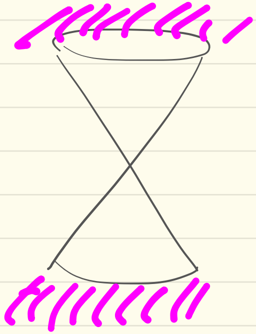
1. CAGE THE DIRAC FERMION



2. ENERGETICALLY BIND CHARGE

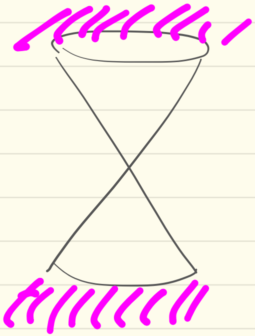
→ ALL CHARGES GAPPED.

3. GAPLESS, NEUTRAL DIRAC FERMION REMAINS



<u>COMPOSITE DIRAC LIQUID</u>

Composite Dirac Liquid

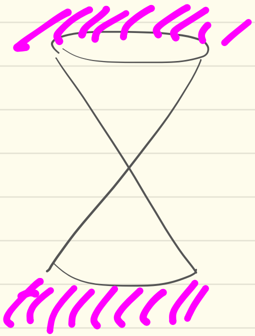


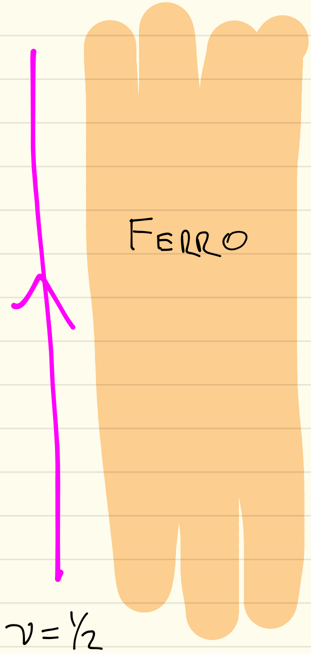
Thermal transport like a Dirac metal

Charge insulator

→ like a gapless spin liquid

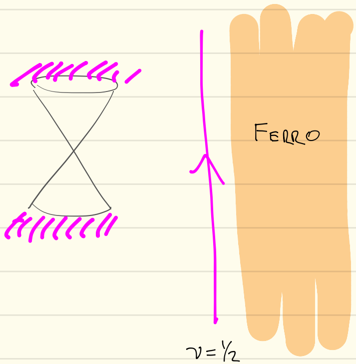
Composite Dirac Liquid





Composite Dirac Liquid

Gapless spectrum enforced by fictitious $\widetilde{U(1)}$ symmetry.



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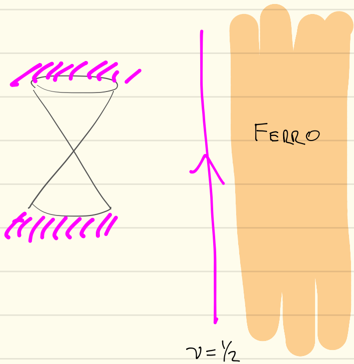
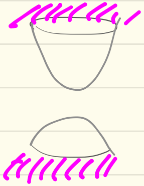


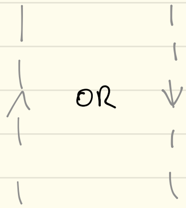
Pair potential $\tilde{\Delta}$ breaks fictitious symmetry

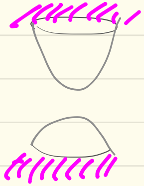
$\rightarrow$ Full gap



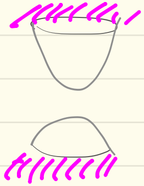
$\rightarrow$ Majorana bound states

$\Rightarrow$ Majorana edge mode





$\sigma_{xy} = 0, \quad \kappa_{xy} = 0$ $\qquad\qquad$ $\sigma_{xy} = 1/2, \quad \kappa_{xy} = 1/2$



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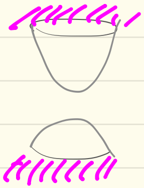
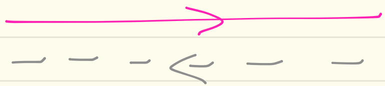


$\sigma_{xy} = 0, \quad \kappa_{xy} = 0$

$\sigma_{xy} = 1/2, \quad \kappa_{xy} = 1/2$

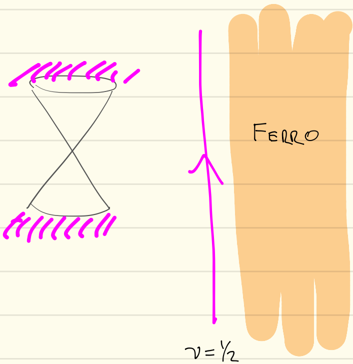
$\nu = 1/2, \quad c = 1/2$



Composite Dirac Liquid

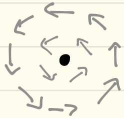
Gapless spectrum enforced by

Fictitious U(1) symmetry.



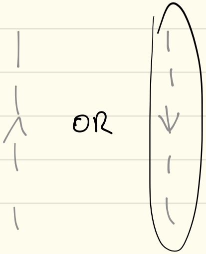
Pair potential $\tilde{\Delta}$ breaks fictitious symmetry

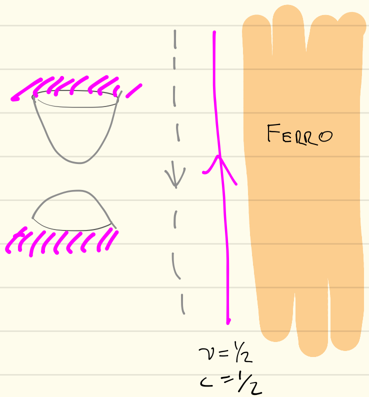
→ Full gap



→ Majorana bound states

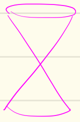
⇒ Majorana edge mode





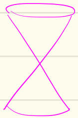
Symmetric, Gapped Surfaces

$U(1)$



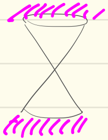
Symmetric, Gapped Surfaces

$U(1)$

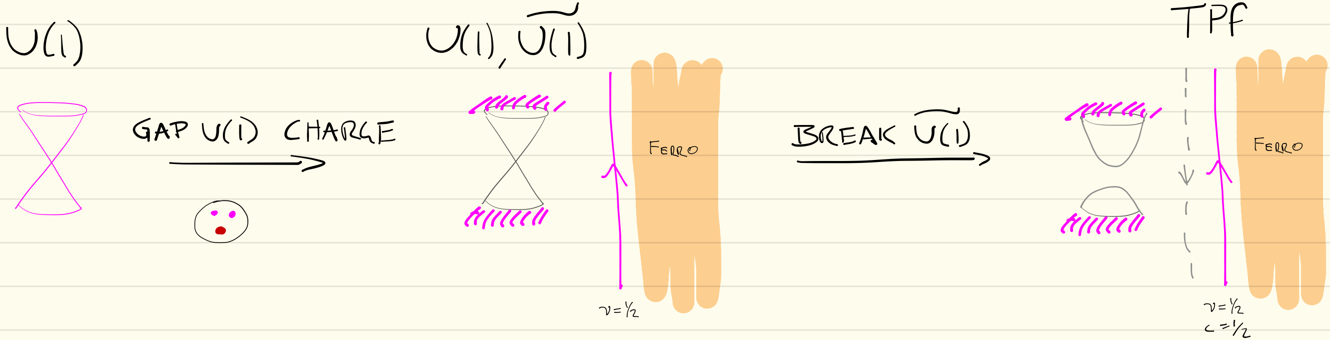


GAP $U(1)$ CHARGE $\Longrightarrow$

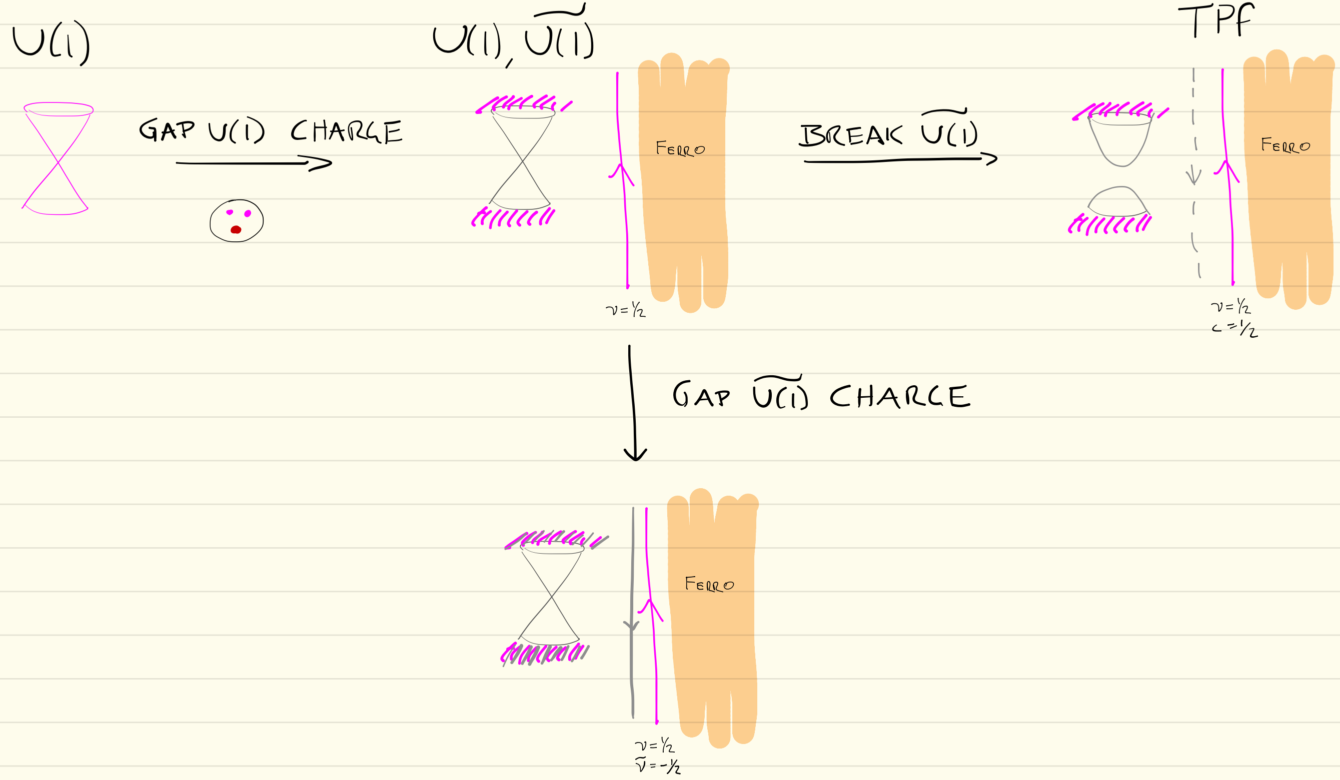
$U(1)$, $\widetilde{U(1)}$



SYMMETRIC, GAPPED SURFACES

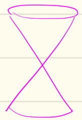


Symmetric, Gapped Surfaces



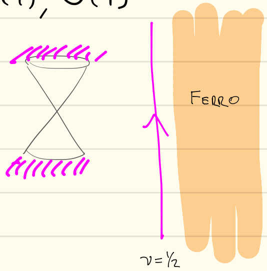
SYMMETRIC, GAPPED SURFACES

U(1)



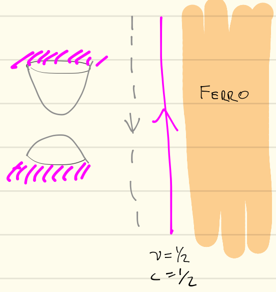
GAP U(1) CHARGE $\longrightarrow$

U(1), $\widetilde{U(1)}$

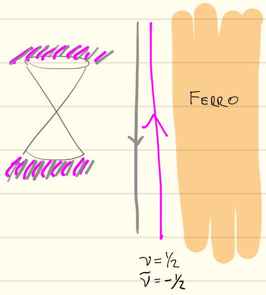


BREAK $\widetilde{U(1)}$ $\longrightarrow$

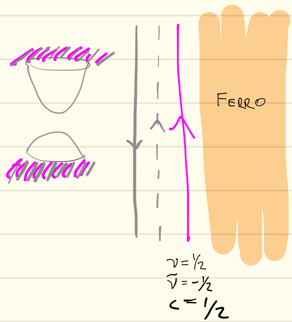
TPF



GAP $\widetilde{U(1)}$ CHARGE $\downarrow$

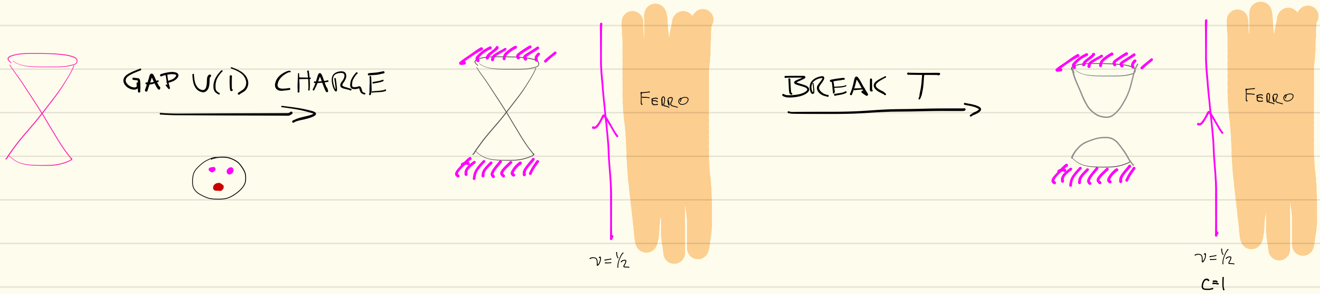


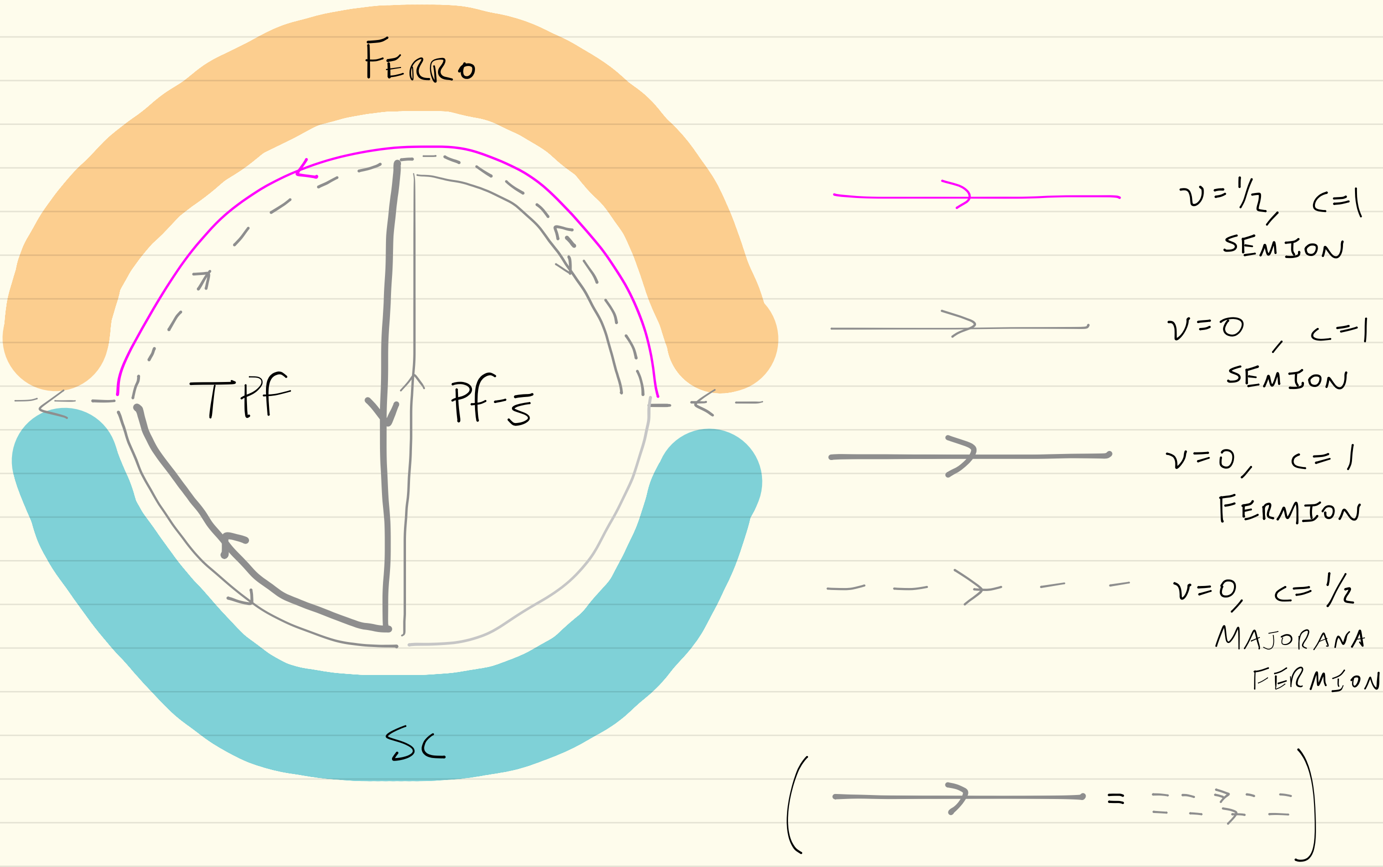
$\longrightarrow$

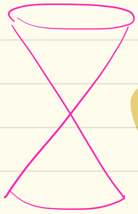
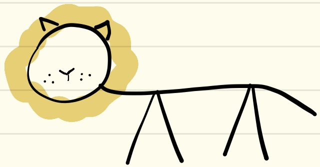
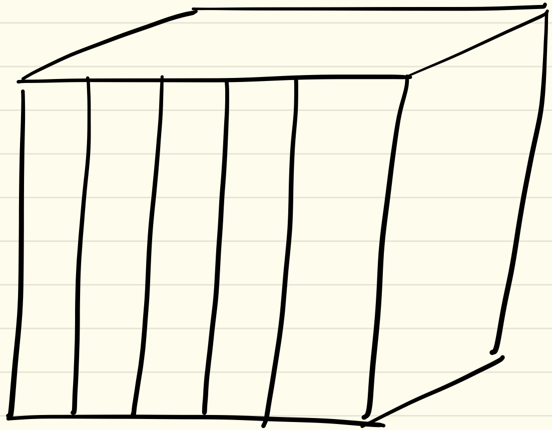


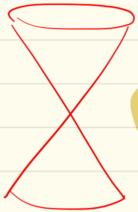
Pf-$\bar{5}$

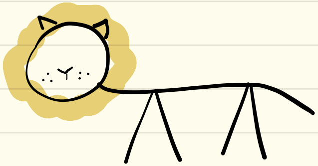
OTHER GAPPED SURFACES ~ 2D STATES

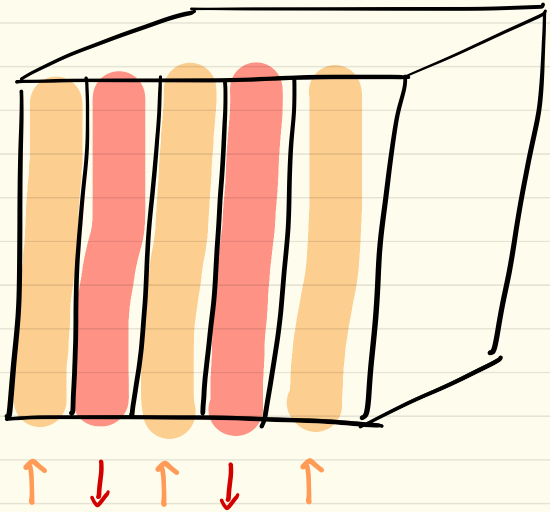


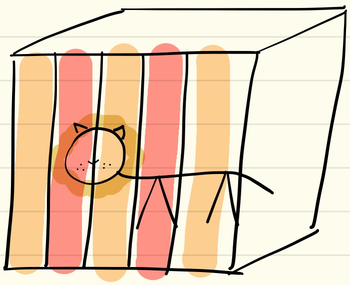

FERRO
TPf
Pf-$\bar{5}$
SC
$\nu = 1/2$, $c = 1$
SEMION
$\nu = 0$, $c = 1$
SEMION
$\nu = 0$, $c = 1$
FERMION
$\nu = 0$, $c = 1/2$
MAJORANA
FERMION
=

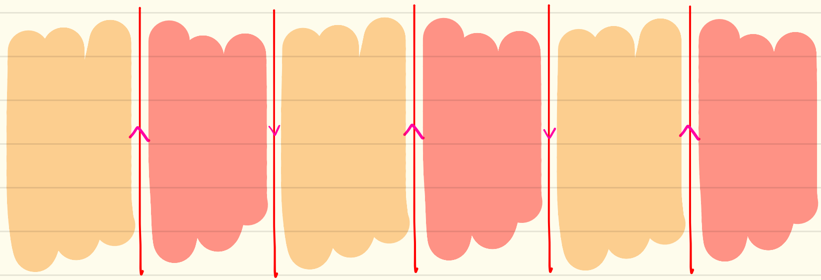












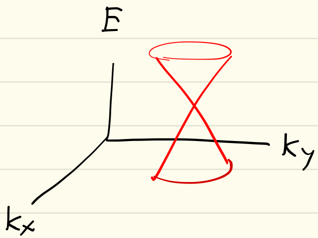


MODIFY TIME-REVERSAL SYMMETRY

$\uparrow \quad \downarrow \quad \uparrow \quad \downarrow \quad \uparrow \quad \downarrow \quad \uparrow \quad \downarrow$

$\psi_\uparrow \to \psi_\downarrow, \ \psi_\downarrow \to -\psi_\uparrow$

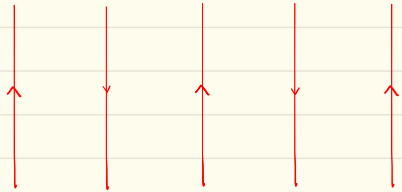
$\psi_\uparrow(\vec{r}) \to \psi_\downarrow(\vec{r}+\hat{x}), \ \psi_\downarrow(\vec{r}) \to \psi_\uparrow(\vec{r}+\hat{x})$

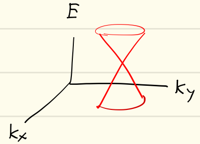


NOW: CONDENSE BOSONS TO GAP CHARGE

— NONCHIRAL, NEUTRAL

DON'T BREAK ANTIFERRO SYMMETRY



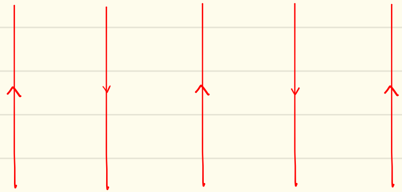


NOT STRONG ENOUGH

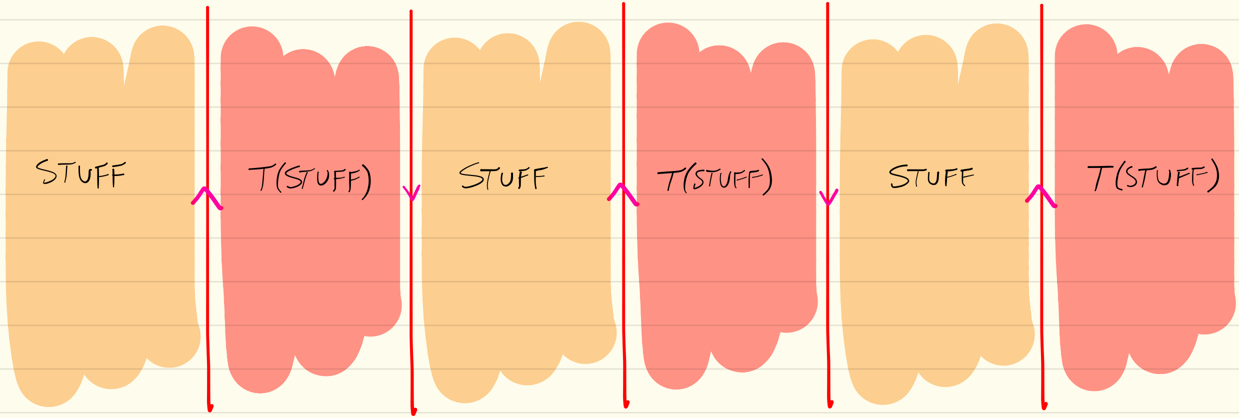
NOW: CONDENSE BOSONS TO GAP CHARGE

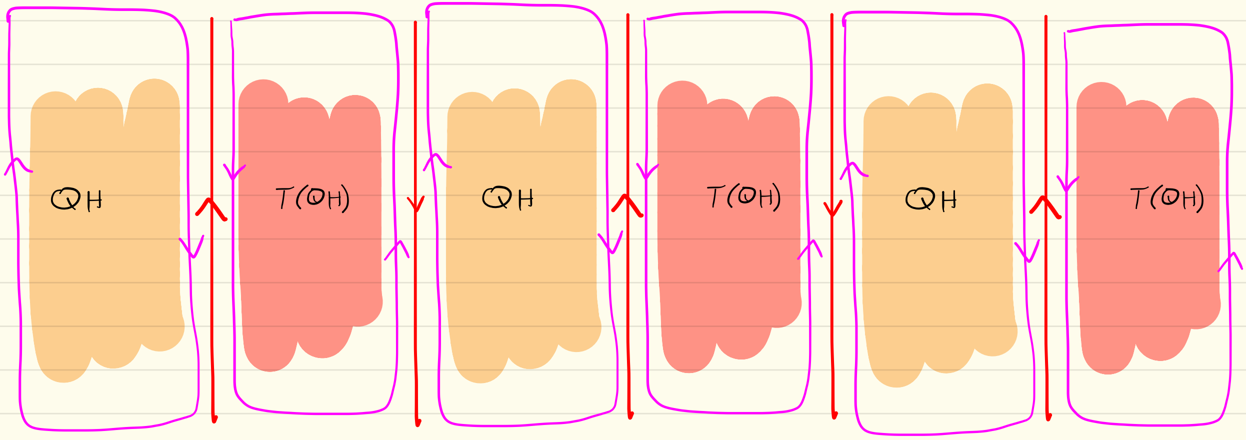
– NONCHIRAL, NEUTRAL

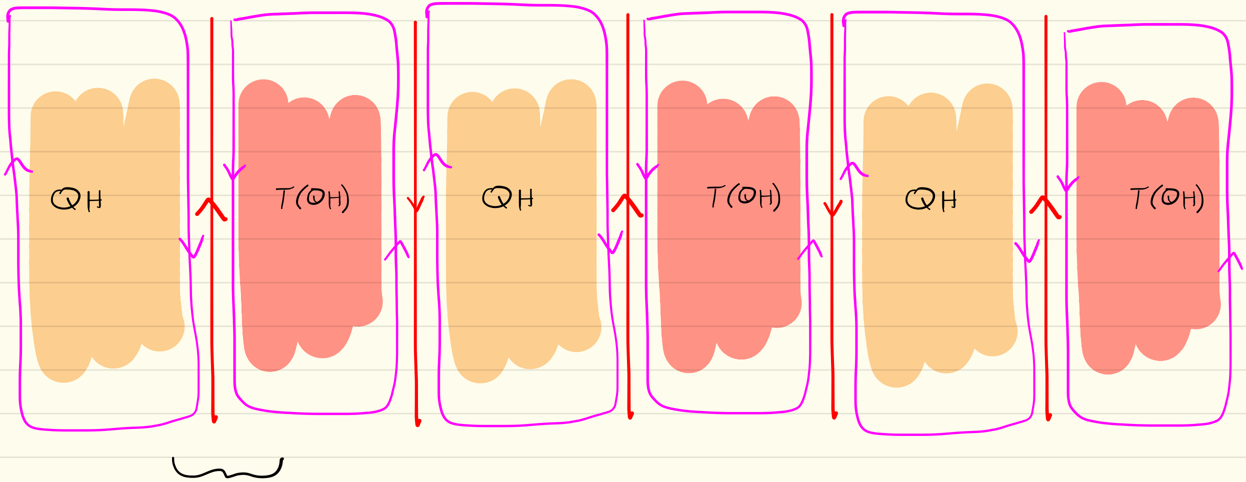
DON'T BREAK AF SYMMETRY



NOT STRONG ENOUGH



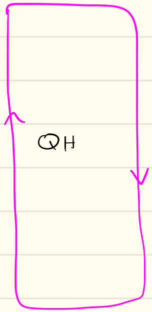

QH
T(QH)
QH
T(QH)
QH
T(QH)



DEMAND $\nu_{TOTAL} = 0$

$\longrightarrow \quad \nu = 1$

$\Rightarrow \quad \longrightarrow \quad \nu = 1/2$



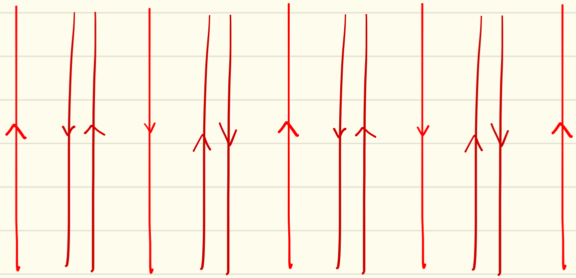
$\nu = 1/2$

- Any Abelian $\nu = 1/2$ state.
- Moore-Read
- <u>Not</u> TPf, Pf-$\bar{5}$

- Spinless electron wire

Quasi-1D approach pioneered by

Kane, Mukhopadhyay, Lubensky
Teo & Kane



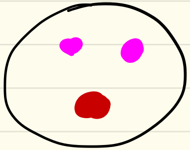
$\sum_i v_i = 0$ — NEUTRAL

$\sum_i (-1)^i v_i^2 = 0$ — NONCHIRAL

$\sum_i (-1)^i v_i v_{i+3} = 0$ — MUTUALLY LOCAL } SYMMETRIC

$$\vec{v} = (\cdots \; 0 \; 0 \; 1 \; -3 \; 4 \; -3 \; 1 \; 0 \; 0 \; \cdots)$$

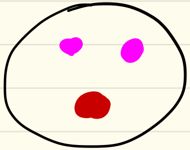
BIND



1 FERMION, CHARGE = 1

2 SEMIONS, CHARGE = $-1/2$

BIND

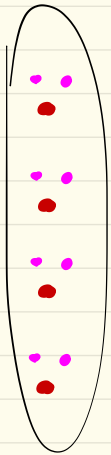


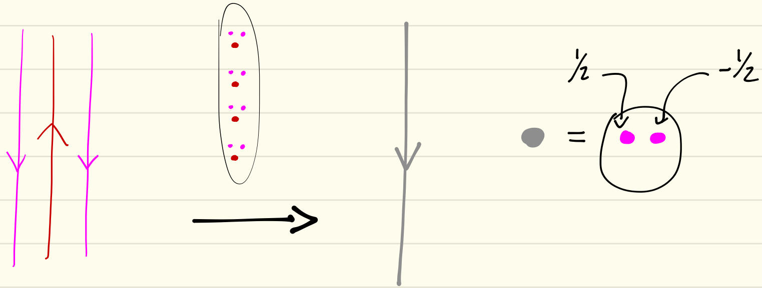
1 FERMION, CHARGE = 1

2 SEMIONS, CHARGE = $-\frac{1}{2}$

N.B.: PHYSICAL PROCESS MUST TAKE 4 SEMIONS FROM

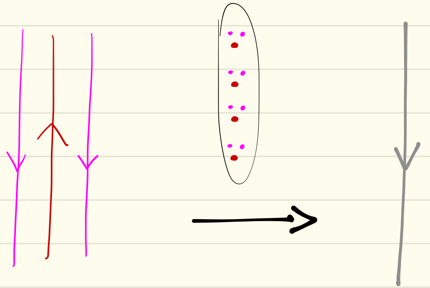
SO REALLY CONDENSE

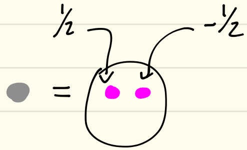




NEUTRAL FERMION

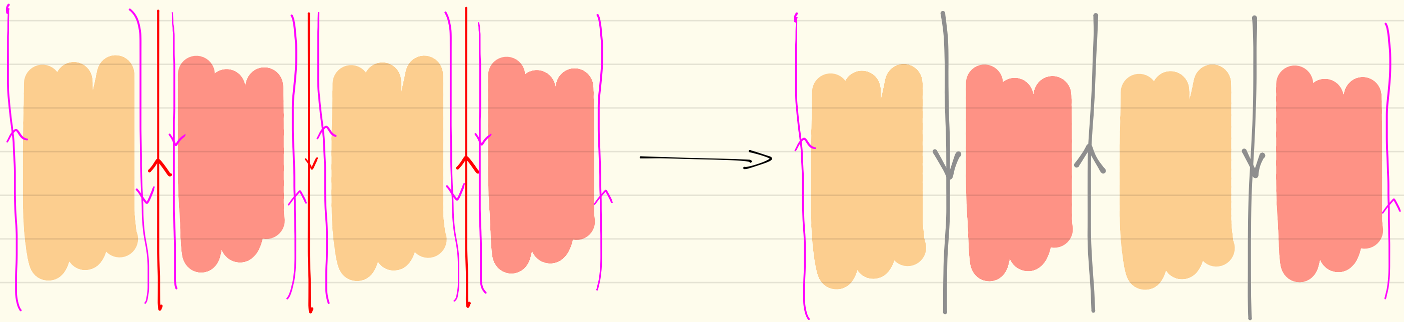
(NOTE: CHIRALITY
REVERSED)

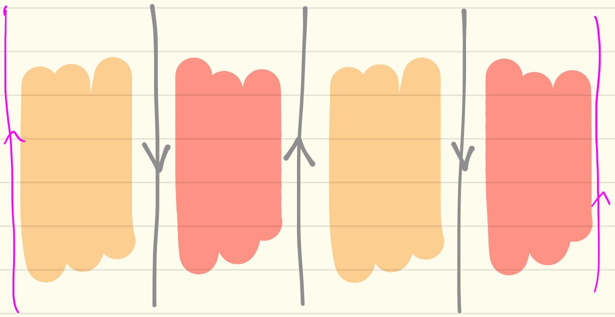


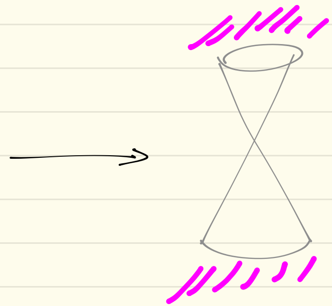


NEUTRAL FERMION

(NOTE: CHIRALITY REVERSED)



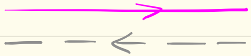




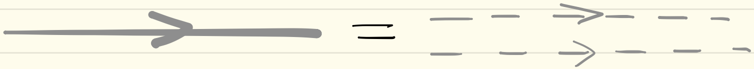
COMPOSITE
DIRAC
LIQUID

COMPOSITE DIRAC LIQUID

HOW ABOUT TPf ?



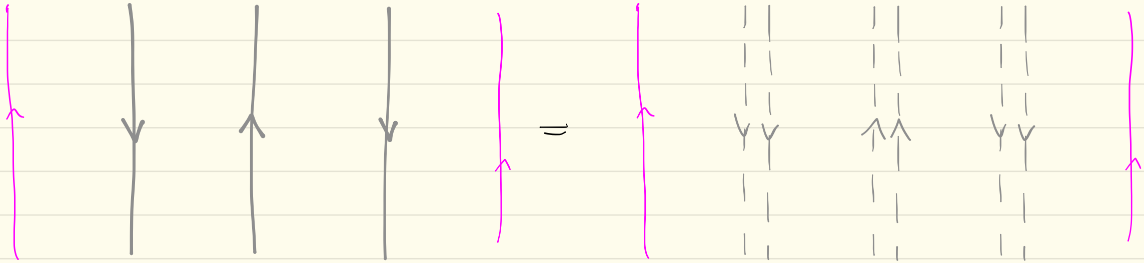
$c = 1$



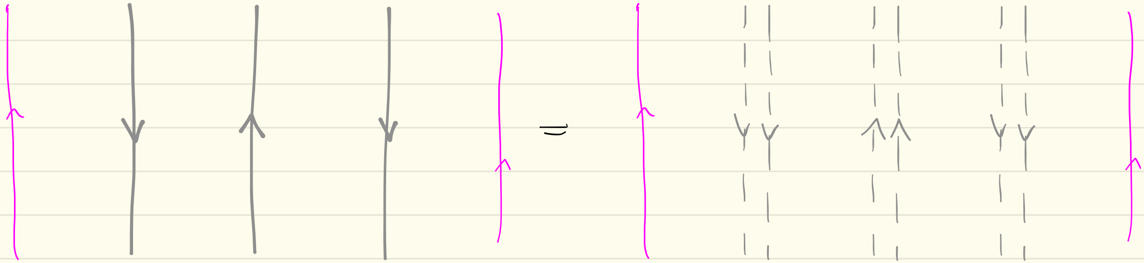
$c = 1/2$

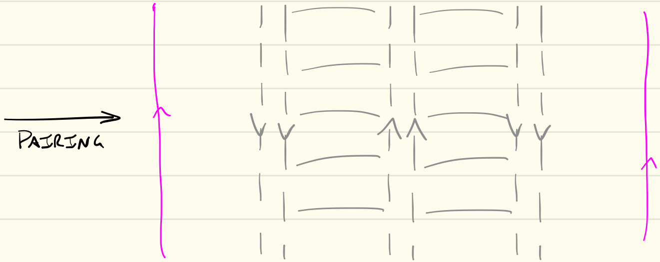
$c = 1/2$

$$f = \chi + i\eta$$

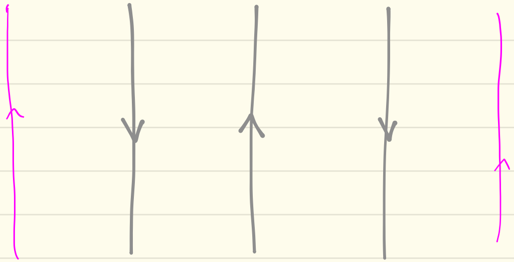


T-PFAFFIAN





T-PFAFFIAN



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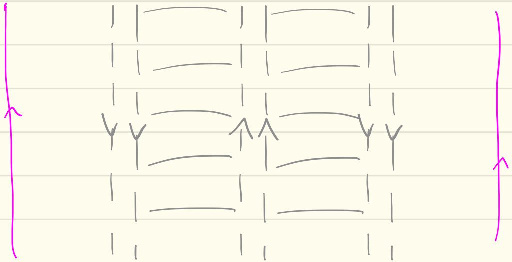
PAIRING



T-PFAFFIAN

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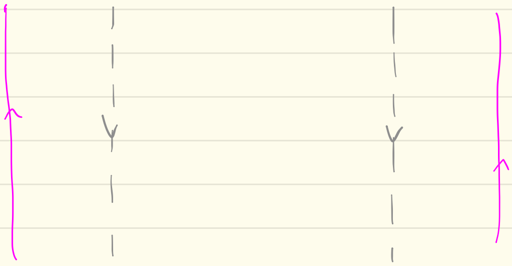
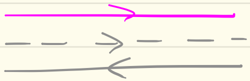
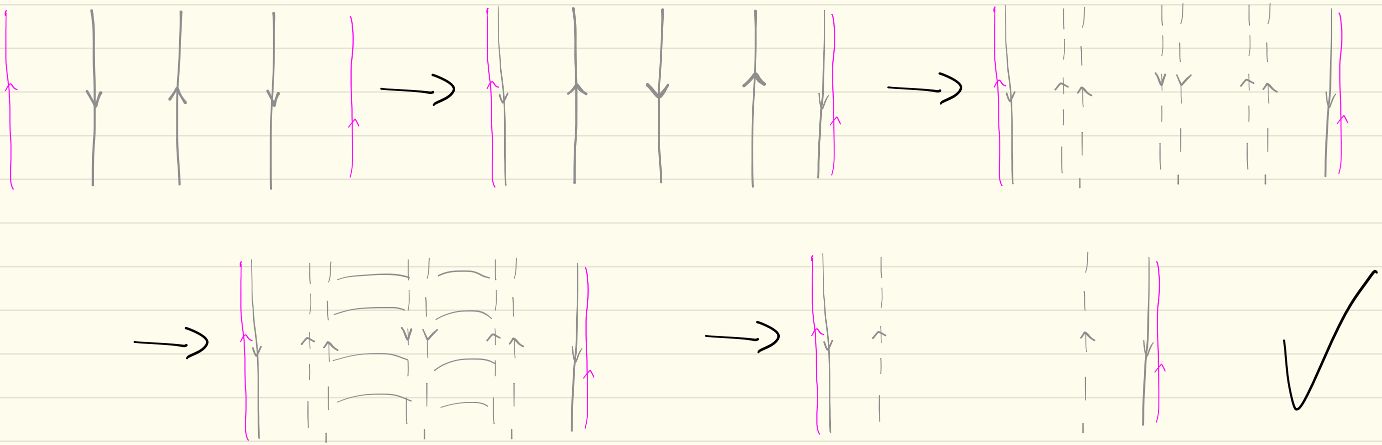
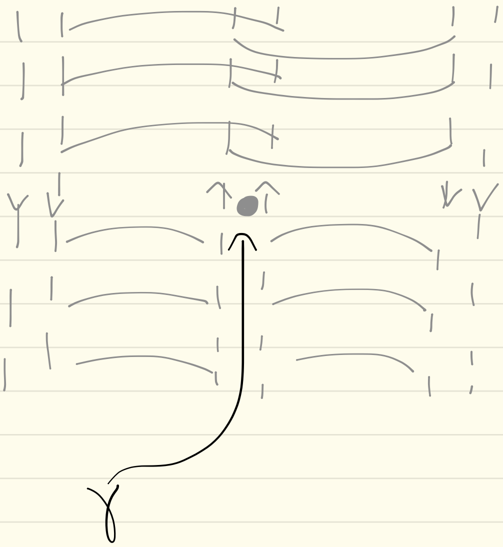


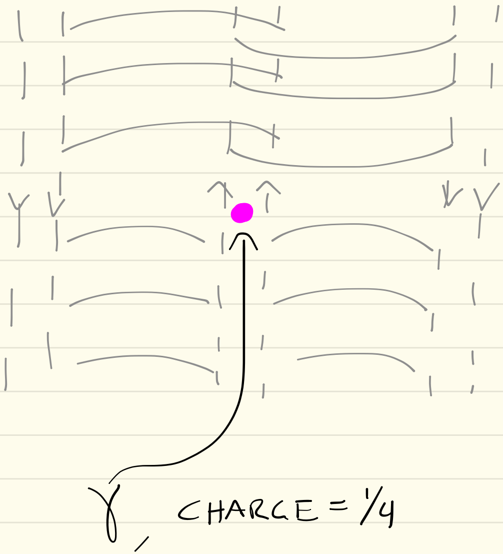
AND Pf-5?





MAJORANA BOUND STATES?





CHARGE = 1/4

Open Questions

Prove deconfined nonabelian anyons

Continuum description

$T^2 = ?$ $(\tilde{T}^2 = \text{translation})$

Other topological bulks

- Weak topological insulator
- Topological superconductor ?

THANK YOU