Taming the Dirac fermion

Composite Dirac liquids and surface topological orders

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TOPOLOGICAL INSULATORS
Topological Insulators
WORK WITH

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WORK WITH

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**Outline - Background**

Prehistory (-30 yrs.) - Fractional Quantum Hall

Ancient History (-10 yrs) - Topological Insulators

History (-1 yr) - Symmetric Topological Surfaces
"Topological" Phases of Matter

Energy

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"Gap"

 ✓

"Gapless"

 ✗
Topological Phases of Matter

Energy

Gap

Gapless

✓

✗

✓
**Electrons**

Properties:
- Energy
- Charge = 1
- Spin = $\frac{1}{2}$

Symmetries
- Phase rotation
- Time reversal

"Statistics"

$\Psi(\text{coords}) \rightarrow -\Psi(\text{coords})$
More generally,

“Local Fermions” have charge = odd integer

Local Bosons have charge = even

\[ b \xrightarrow{\psi} \bar{\psi} \]

Not neutrons, say: Fermions, charge = 0.
How to Get a Cap
How to get a gap

- For bosons, need strong interactions.

Energy cost for two bosons in the same state
How to get a gap

- For bosons, need strong interactions:

  Energy cost for two bosons in the same state

  \[ \text{\textbf{\textcircled{\textbf{bb}}}} \]

- Filled fermion band ("Dirac sea")

\[ \text{\textbf{\textcircled{\textbf{E}}}} \]

\[ \text{\textbf{\textcircled{\textbf{k}}}} \]
How to get a gap

- For bosons, need strong interactions:

  Energy cost for two bosons in the same state

- Filled fermion band ("Dirac sea")

\[ E \]

\[ \text{GAP} \]

\[ \text{GAP} \]

\[ k \]

Contrast metal

\[ E \]

\[ k \]
Fractional excitations may have other statistics, charge.

Consider fractional quantum Hall at $\nu = 1/2$. 
Fractional excitations may have other statistics, charge

Consider fractional quantum Hall at $v = \frac{1}{2}$.

Old idea of Halperin:
- Electrons pair to form charge-2 bosons
- These pairs break into 8 pieces with charge $\frac{1}{4}$, statistics $e^{\frac{i\pi}{8}}$
Fractional excitations may have other statistics, charge.

Consider fractional quantum Hall at $\nu = \frac{1}{2}$.

Old idea of Halperin:
- Electrons pair to form charge-$\frac{1}{2}$ bosons.
- These pairs break into 8 pieces with charge $\frac{1}{4}$, statistics $e^{-\frac{i\pi}{8}}$.

Edge: charge conductance $\frac{1}{2} = \frac{2^{2/8}}{8} \rightarrow \nu = \frac{1}{2}$

Energy conductance 1. $\rightarrow c = 1$
Later idea (Moore & Read):

Charge $\frac{1}{4}$ anyon carries Majorana bound state,

$\rightarrow$ Non-Abelian fusion, braiding.

\[ \begin{align*}
\left\{ \begin{array}{l}
\nu = \frac{1}{2}, \quad c = 1 \\
\nu = 0, \quad c = \frac{1}{2}
\end{array} \right. \\
\rightarrow \nu = \frac{1}{2}, \quad c = \frac{3}{2}
\end{align*} \]
Experiments see a gapless state, sort of like a metal

"Composite Fermi Liquid"

Very crudely, one can imagine fractional fermions
Experiments see a gapless state, sort of like a metal

"Composite Fermi liquid"

Very crudely, one can imagine fractional fermions

The two states correspond to pairing of these fermions:

Halperin

Moore-Read

Plausible candidate for topological state at filling 5/2.
Hastings (Oshikawa; Lieb, Schultz, Mattis)

With appropriate symmetries and charges:

1. Gap + Fractional excitations

2. Gapless
3D Topological Insulator

\[ H_{2D} = i \Psi^+ \tilde{\sigma} \cdot \nabla \Psi \]

\[ \Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \]
3D Topological Insulator

\[ H_{2D} = i \begin{pmatrix} \sigma^+ & 0 \\ 0 & \sigma^- \end{pmatrix} \Psi \]

\[ \Psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} \]

Symmetry

Symmetry Action

Time Reversal

\[ \uparrow \leftrightarrow \downarrow \]

Phase Rotation

\[ \Psi \rightarrow e^{i\phi} \Psi \]

Symmetry-Breaking

\[ h > 0 \]

\[ h \in \mathbb{R} \]

\[ \Delta > 0 \]

\[ \Delta \in \mathbb{C} \]
2D SURFACE v. 2D BULK

3D BULK STATES

E k

E k
Symmetry Breaking: Point Defects
Symmetry Breaking - Point Defects

Majorana Bound State

\[ \gamma = \gamma^+ \]

[Diagram with arrows and \( \Delta e^{i\alpha(x)} \)]
Symmetry Breaking - Line Defects

Chiral Fermion Mode

$\nu = 1$, $c = 1$
Symmetry Breaking - Line Defects

Chiral Fermion Mode

\[ h > 0 \quad h < 0 \]

\[ V = 1, \quad c = 1 \]

Chiral Majorana Mode

\[ h > 0 \quad \Delta \]

\[ c = \frac{1}{2} \]
Symmetric, Gapped Surfaces

Bonderson, Nayak, Qi
Metlitski, Kane, Fisher
Wang, Potter, Senthil

Chen, Fidkowski, Vishwanath — Walker-Wang Models
Symmetric, Gapped Surfaces

Bonderson, Nayak, Qi
Metlitski, Kane, Fisher
Wang, Potter, Senthil

CHEN, FIDKOWSKI, VISHWANATH – WALKER-WANG MODELS

Two possibilities:

\[ h > 0 \]

\[ \nu = \frac{1}{2}, \ c = \frac{1}{2} \]

“T-Pfaffian”

\[ h > 0 \]

\[ \nu = \frac{1}{2}, \ c = \frac{1}{2} \]

“Pfaffian - Antisemion”
Questions?
1. CAGE THE DIRAC FERMION
1. Cage the Dirac fermion

2. Energetically bind charge

→ all charges capped.

→ Neutral bound state → U(1) not broken
1. CAGE THE DIRAC FERMION

2. ENERGETICALLY BIND CHARGE

   -> ALL CHARGES GAPPED.

3. GAPLESS, NEUTRAL DIRAC FERMION REMAINS

   **COMPOSITE DIRAC LIQUID**
Composite Dirac Liquid

Thermal transport like a Dirac metal

Charge Insulator

$\Rightarrow$ Like a gapless spin liquid
COMPOSITE DIRAC LIQUID

\[ \nu = \frac{1}{2} \]

Ferro
COMPOSITE DIRAC LIQUID

Gapless spectrum enforced by fictitious $U(1)$ symmetry.
Composite Dirac Liquid

Gapless spectrum enforced by fictitious $U(1)$ symmetry.

Pair potential $\Delta$ breaks fictitious symmetry

→ Full gap

→ Majorana bound states

→ Majorana edge mode

$\nu = \frac{1}{2}$
\[ \sigma_{xy} = 0, \quad K_{xy} = 0 \]

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\[ \nu = \frac{1}{2}, \quad \xi = \frac{1}{2} \]
\[ \sigma_{xy} = 0, \quad K_{xy} = 0 \]

\[ \sigma_{xy} = \frac{1}{2}, \quad K_{xy} = \frac{1}{2} \]

\[ \nu = \frac{1}{2}, \quad c = \frac{1}{2} \]
Composite Dirac Liquid

Gapless spectrum enforced by fictitious $U(1)$ symmetry.

Pair potential $\Delta$ breaks fictitious symmetry

$\rightarrow$ Full gap

$\rightarrow$ Majorana bound states

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$\Rightarrow \ \nu = \frac{1}{2}$

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Symmetric, Gapped Surfaces

U(1)
Symmetric, Gapped Surfaces

$U(1)$ \quad $U(1), \tilde{U}(1)$

GAP $U(1)$ CHARGE

$\nu = \frac{1}{2}$
Symmetric, Gapped Surfaces

\[ U(1) \quad U(1), \tilde{U}(1) \]
Symmetric, Gapped Surfaces

\( U(1) \)

\( U(1), \tilde{U}(1) \)

GAP \( U(1) \) CHARGE

\( \nu = \frac{1}{2} \)

BREAK \( \tilde{U}(1) \)

\( \nu = \frac{1}{2} \)

\( \nu < \frac{1}{2} \)

TPF

GAP \( \tilde{U}(1) \) CHARGE

\( \nu = \frac{1}{2} \)

\( \nu = -\frac{1}{2} \)

Ferro
Symmetric, Gapped Surfaces

\( U(1) \quad U(1), \bar{U}(1) \quad TPF \)

\[ \begin{align*}
\text{GAP } U(1) & \quad \text{CHARGE} \\
\nu = \frac{1}{2} & \quad \nu = \frac{1}{2} \\
\phi = 0 & \quad \phi = 0 \\
\end{align*} \]
Other Gapped Surfaces ~ 2D States

\[ \text{Gap } \mathbb{U}(1) \text{ Charge } \xrightarrow{\mathcal{M}} \text{ Ferro} \quad \xrightarrow{T \text{ broken}} \text{ Ferro} \]

\( \nu = \frac{1}{2} \)
\[ \begin{array}{ccc}
\text{Ferro} & \rightarrow & v = \frac{1}{2}, \ c = 1 \\
\text{SEMION} & \rightarrow & v = 0, \ c = 1 \\
\text{FERMION} & \rightarrow & v = 0, \ c = \frac{1}{2} \\
\text{MAJORANA FERMION} & \rightarrow & \text{...}
\end{array} \]
Modify time-reversal symmetry

\[ \psi_n \rightarrow \psi_n, \quad \psi_{\bar{n}} \rightarrow -\psi_{\bar{n}} \]

\[ \psi_n(\vec{r}) \rightarrow \psi_n(\vec{r} + \vec{x}), \quad \psi_{\bar{n}}(\vec{r}) \rightarrow \psi_{\bar{n}}(\vec{r} + \vec{x}) \]
Now: Condense Bosons to Gap Charge

- Nonchiral, Neutral

Don't Break Antiferro Symmetry

Not Strong Enough
Now: Condense bosons to gap charge

- Nonchiral, neutral

Don't break AF symmetry

Not strong enough
Demand \( \nu_{\text{TOTAL}} = 0 \)  \( \Rightarrow \)  \( \nu = \frac{1}{2} \)
\( v = \frac{1}{2} \) - Any Abelian \( v = \frac{1}{2} \) state.

- Moore - read

- Not TPf, Pf-5

- Spinless electron wire

Quasi-1D approach pioneered by

Kane, Mukhopadhyay, Lubensky
Teo & Kane
$\sum_i v_i = 0 \quad - \text{NEUTRAL}$

$\sum_i (-1)^i v_i^2 = 0 \quad - \text{NONCHIRAL}$

$\sum_i (-1)^i v_i v_{i+3} = 0 \quad - \text{MUTUALLY LOCAL AND SYMMETRIC}$

$\vec{v} = (\cdots \ 001 -3 4 -3 100 \cdots)$
BIND

1 Fermion, charge = 1
2 Semions, charge = $-\frac{1}{2}$
BIND

1. Fermion, charge = 1
2. Semions, charge = $-\frac{1}{2}$

NB: Physical process must take 4 Semions from

So really condense
Neutral Fermion

(Note: Chirality Reversed)
Neutral Fermion

(Note: Chirality Reversed)
COMPOSITE DIRAC LIQUID
How ABOUT \( TPF \)?

\[ c = 1 \quad \rightarrow \quad c = \frac{1}{2} \]

\[ f = \chi + i\eta \]
\( T - \text{Pfaffian} \)
AND Pf - 3?
MAJORANA BOUND STATES?
Open Questions

Prove deconfined nonabelian anyons

Continuum Description

$T^2 = ? \quad (\bar{T}^2 = \text{translation})$

Other topological bulks

- Weak topological insulator
- Topological superconductor?
THANK YOU