The new Architecture and Solvers in the *Barcelogic* SMT tool

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Overview of the talk

- DPLL and Conflict Analysis
- Satisfiability Modulo Theories (SMT)
- DPLL(T) = DPLL(X) + T-Solver
- Our *Barcelogic* DPLL(T) tool
- What does DPLL(T) need from T-Solver?
- Ongoing work on T-Solvers and Combination
- Some new applications of DPLL(T)
- Other ongoing work

(Abstract) DPLL for propositional SAT

Assmt.:		**Clause set:**		
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Other rules:

- Backjump: (generalizes Backtrack)
- Learn: (learning backjump clauses avoids “similar” conflicts)
- Forget: (removes “inactive” clauses)

Backtrack vs. Backjump

Same example again. Remember: Backtrack gave $1\ 2\ 3\ 4\ \overline{5}$.

But note that decision level $3\ 4$ is unrelated to the conflict $6 \vee \overline{5} \vee \overline{2}$:

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\begin{array}{lcl}
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The Backjump rule is:

$$
M\ l\ N \parallel F, C \ \Rightarrow\ M\ l' \parallel F, C \quad \textbf{IF} \begin{cases} C \text{ is false in } M\ l\ N, \text{ and} \\ \text{there is some clause } C' \vee l' \\ \quad - \text{ entailed by } F, C \\ \quad - \text{ s.t. } C' \text{ is false in } M \end{cases}
$$

$C' \vee l'$ is called the backjump clause. In our example, it is $\overline{2} \vee \overline{5}$.

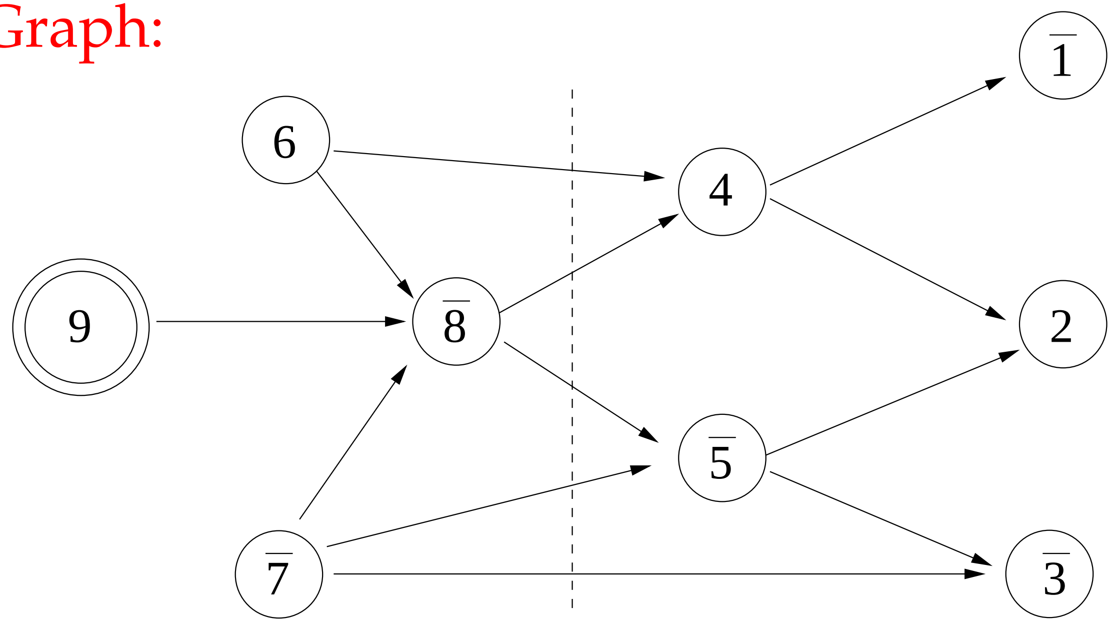
Conflict analysis: find backjump clause

Consider assignment: $\ldots 6 \ldots \overline{7} \ldots 9$ and let F contain:

$\overline{9} \lor \overline{6} \lor 7 \lor \overline{8}$, $8 \lor 7 \lor \overline{5}$, $\overline{6} \lor 8 \lor 4$, $\overline{4} \lor \overline{1}$, $\overline{4} \lor 5 \lor 2$, $5 \lor 7 \lor \overline{3}$, $1 \lor \overline{2} \lor 3$.

UnitPropagate: $\ldots 6 \ldots \overline{7} \ldots 9\ \overline{8}\ \overline{5}\ 4\ \overline{1}\ 2\ \overline{3}$. Conflict with $1 \lor \overline{2} \lor 3$!

Implication Graph:



Can use $8 \lor 7 \lor \overline{6}$ for Backjump to $\ldots 6 \ldots \overline{7}\ 8$.

Confl. analysis: find backjump clause (2)

Same example: assignment $\ldots 6 \ldots \overline{7} \ldots 9$ and let F contain:

$\overline{9}\vee\overline{6}\vee 7\vee\overline{8}$, $\quad 8\vee 7\vee\overline{5}$, $\quad \overline{6}\vee 8\vee 4$, $\quad \overline{4}\vee\overline{1}$, $\quad \overline{4}\vee 5\vee 2$, $\quad 5\vee 7\vee\overline{3}$, $\quad 1\vee\overline{2}\vee 3$.

UnitPropagate: $\ldots 6 \ldots \overline{7} \ldots 9\ \overline{8}\ \overline{5}\ 4\ \overline{1}\ 2\ \overline{3}$. Conflict with $1\vee\overline{2}\vee 3$!

Do Resolutions in reverse order backwards from conflict:

$$\dfrac{8\vee 7\vee\overline{5} \qquad \dfrac{\overline{6}\vee 8\vee 4 \qquad \dfrac{\overline{4}\vee\overline{1} \qquad \dfrac{\overline{4}\vee 5\vee 2 \qquad \dfrac{5\vee 7\vee\overline{3} \qquad 1\vee\overline{2}\vee 3}{5\vee 7\vee 1\vee\overline{2}}}{\overline{4}\vee 5\vee 7\vee 1}}{5\vee 7\vee\overline{4}}}{\overline{6}\vee 8\vee 7\vee 5}}{8\vee 7\vee\overline{6}}$$

until reaching clause with only 1 lit. of current decision level.

Can use this clause $8\vee 7\vee\overline{6}$ for Backjump to $\ldots 6 \ldots \overline{7}\ 8$.

Abstract DPLL results

A DPLL procedure for F is any derivation: $\emptyset \parallel F \;\Rightarrow\; \ldots \;\Rightarrow\; S$ where S is a final state (no rule applies). It always terminates.

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Abstract DPLL provides formal and uniform proofs of correctness and completeness of many variants, strategies and ... extensions to e.g., SAT Modulo Theories (SMT)...

Overview of the talk

- DPLL and Conflict Analysis
- **Satisfiability Modulo Theories (SMT)** $\Longleftarrow$
- DPLL(T) = DPLL(X) + T-Solver
- Our *Barcelogic* DPLL(T) tool
- What does DPLL(T) need from T-Solver?
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SAT Modulo Theories (SMT)

- Some problems are more naturally expressed in richer logics than just propositional logic, e.g:
  - Software/Hardware verification needs reasoning about equality, arithmetic, data structures, ...
- SMT consists of deciding the satisfiability of a (ground) FO formula with respect to a background theory T
- Example: T is Equality with Uninterpreted Functions (EUF):

$$g(a) = c \quad \land \quad (\ f(g(a)) \neq f(c) \ \lor \ g(a) = d \ ) \quad \land \quad c \neq d$$

- Example: (combined theories)

$$A = write(B, a+1, 4) \ \land \ (\ read(A, b+3) = 2 \lor f(a-1) \neq f(b+1)\ )$$

- Wide range of applications

The Eager approach to SMT

- Methodology: translate problem into equisatisfiable propositional formula and use off-the-shelf SAT solver [Bryant, Velev, Pnueli, Lahiri, Seshia, Strichman, ...]
- Why "eager"?
  Search uses all theory information from the beginning
- Characteristics:
  - \+ Can use best available SAT solver
  - – Sophisticated encodings are needed for each theory
  - – Sometimes translation and/or solving too slow

Main Challenge for alternative approaches is to combine:

– DPLL-based techniques for handling the boolean structure

with

– Efficient theory solvers for conjunctions of T-literals

The Lazy approach to SMT

Same example: consider EUF and

$$\underbrace{g(a)=c}_{1} \quad \wedge \quad (\ \underbrace{f(g(a))\neq f(c)}_{\overline{2}}\ \vee\ \underbrace{g(a)=d}_{3}\ ) \quad \wedge \quad \underbrace{c\neq d}_{\overline{4}}$$

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   SAT solver says UNSAT

Lazy approach (2)

- Why “lazy”?
  Theory information used lazily when checking T-consistency of propositional models
- Characteristics:
  - \+ Modular and flexible
  - – Theory information does not guide the search
- Tools: CVC-Lite, ICS, MathSAT, TSAT+, Verifun, ...

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- Upon a T-inconsistency, do conflict analysis of the explanation and Backjump

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Our DPLL(T) approach

DPLL(T) = DPLL(X) engine + T-Solver

- Modular and flexible, as CLP(X) in Constraint Logic Progr.: can plug in any T-Solver into the DPLL(X) engine.
- **Theory Propagation:** more pruning in optimized lazy SMT

  T-Propagate : $\quad M \parallel F \;\Rightarrow\; M\,l \parallel F \quad$ **IF** $\left\{ M \models_T l \right.$

  - T-Solver also guides search, instead of only validating it
  - [Armando et al]: Add $\neg l$. If T-inconsistent then infer l.

    But in DPLL(T):

    – T-Solvers specialized and fast in Theory Propagation

    – Fully exploited in conflict analysis (non-trivial)
    Not any explanation of a theory propagation is ok!

DPLL(T) Example

Notation used: Abstract DPLL Modulo Theories.

Consider again same example with EUF:

$$\underbrace{g(a)=c}_{1} \quad \wedge \quad (\underbrace{f(g(a))\neq f(c)}_{\overline{2}} \vee \underbrace{g(a)=d}_{3}) \quad \wedge \quad \underbrace{c\neq d}_{\overline{4}}$$

$$\emptyset \quad \| \quad 1,\ \overline{2}\vee 3,\ \overline{4} \quad \Rightarrow \quad (\text{UnitPropagate})$$

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$1\ 2$	$\parallel$	$1,\ \overline{2}\vee 3,\ \overline{4}$	$\Rightarrow$	(UnitPropagate)
$1\ 2\ 3$	$\parallel$	$1,\ \overline{2}\vee 3,\ \overline{4}$	$\Rightarrow$	(T-Propagate)
$1\ 2\ 3\ 4$	$\parallel$	$1,\ \overline{2}\vee 3,\ \overline{4}$	$\Rightarrow$	

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1\;\overline{2}\;3\;4 & \| \; 1,\; \overline{2}\vee 3,\; \overline{4} & \Rightarrow & \mathit{fail}
\end{array}
$$

No search in this example

Conflict analysis in DPLL(T)

New kind of arrows (reasons) in implication graph.
Each literal lit is in the partial assignment due to one of:

- Decide (no arrow)
- UnitPropagate with clause C: resolve with C
- T-Propagate: resolve with (small) explanation
  $l_1 \wedge \ldots \wedge l_n \rightarrow lit$ provided by T-Solver
  Too new T-explanations are forbidden!

How should it be implemented?

- UnitPropagate: store a pointer to clause C, as in SAT solvers
- T-Propagate: (pre-)compute explanations at each T-Propagate?
  – If possible, only on demand, during conflict analysis
  – typically only one Explain for every 250 T-Propagate.
  – depends on T

Our *Barcelogic* DPLL(T) tool

- DPLL(X) is a state-of-the-art SAT engine:
  - features à la Chaff: two watched literals, 1UIP learning, VSIDS-like decision heuristics, ...
  - new features: binary clause reasoning, subsumption, lemma simplification, ...
  - see SAT Race 2006
- T−*Solvers* for:
  - Real/Integer Difference Logic (IDL/RDL):
  - Equality with Uninterpreted Functions (EUF)
  - Linear Real Arithmetic (LRA)
  - Linear Integer Arithmetic (LIA) (forthcoming)
  - Arrays

Barcelogic at SMT-COMP'05

Participated in 4 (of 7) divisions:

	top-3 systems	# Pbs solv.	Time (secs.)
EUF (50 pbs.):	Barcelogic	39	8358
	Yices	37	9601
	MathSAT	33	12386
RDL (50):	Barcelogic	41	6341
	Yices	37	9668
	MathSAT	37	10408
IDL (51):	Barcelogic	47	3531
	Yices	47	4283
	MathSAT	46	4295
UFIDL (49):	Barcelogic	45	2705
	Yices	36	9789
	MathSAT	22	17255

Timeout = 600s.

Other tools:

- CVC-Lite (Barrett)
- Ario (Sakallah)
- Sateen (Somenzi)
- ...

Overview of the talk

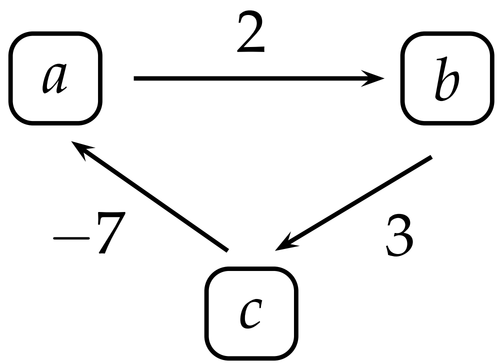
- DPLL and Conflict Analysis
- Satisfiability Modulo Theories (SMT)
- DPLL(T) = DPLL(X) + T-Solver
- Our *Barcelogic* DPLL(T) tool
- **What does DPLL(T) need from T-Solver?** $\Longleftarrow$
- **Ongoing work on T-Solvers and Combination** $\Longleftarrow$
- Some new applications of DPLL(T)
- Other ongoing work

What does DPLL(T) need from T-Solver?

- T-consistency check of a set of literals M, with:
  - Explain of T-inconsistency: find (small) T-inconsistent subset of M [minimal wrt. size?, wrt. $\subseteq$?]
  - Incrementality: if l is added to M, check for $M\,l$ faster than reprocessing $M\,l$ from scratch.
- Theory propagation: find input T-consequences of M, with:
  - Explain T-Propagate of l: find (small) subset of M that T-entails l (needed in conflict analysis).
- Backtrack n: undo last n literals added

A standard Difference Logic solver

- Given $M = \{a - b \leq 2,\ b - c \leq 3,\ c - a \leq -7\}$, construct weighted graph $\mathcal{G}(M)$



- M is T-inconsistent iff $\mathcal{G}(M)$ has a negative cycle
- Bellmann-Ford-like algorithms to find such cycles in $O(nm)$
- Irredundant inconsistent subsets are negative cycles
- What about theory propagation?

Our CAV'05 DL Solver

- Key idea: exhaustive theory propagation avoids consistency checks: $M\,l$ is T-inconsistent iff $M \models_T \neg l$. Hence we would have added $\neg l$ right after M.
- For detecting all consequences of a new literal $a - b \leq k$:

  $c - d \leq k_1$ is T-entailed iff there is a path form c to d with length at most k_1. Hence *T-Solver* checks all shortest paths

  $$c \xrightarrow{k'} * \quad a \xrightarrow{k} b \xrightarrow{k''} * \quad d$$

  and finds all input literals entailed by $c - d \leq k' + k + k''$
- Complexity: $O(nm + N)$, being N the number of input literals
- Irredundant explanations for $c - d \leq k$ given by the shortest path from c to d

Analyzing our CAV'05 solver

CHARACTERISTICS:

- TheoryProp is invoked even if UnitProp still applicable
- Cannot get rid of the exhaustiveness requirement if TheoryProp is too expensive

IDEAL SITUATION:

- Cheaper reasoning should be done first:
  1. Apply UnitProp exhaustively
  2. If no conflict, then check T-consistency of model
  3. If model T-consistent apply TheoryProp (if wanted)
- Some of the computations of the consistency check should be reused in TheoryProp

Our new solver: [Cotton&Maler,SAT06]

CHECK CONSISTENCY:

- Check T-consistency of model using Bellmand-Ford-like algorithm (each newly added literal in $O(m + n \log n)$)
- Gives potential function π s.t. for each each edge $a \xrightarrow{k} b$ we have $\underbrace{\pi(a) + k - \pi(b)}_{reduced\ cost} \geq 0$

THEORY PROPAGATION:

- Addition of $a \xrightarrow{k} b$ entails $c - d \leq k'$ only if

$$\underbrace{c \longrightarrow * \;\; a \xrightarrow{k} b}_{shortest} \longrightarrow * \;\; d$$

- Shortest path computation more efficient using reduced costs, since they are non-negative

Linear Arithmetic Solver: Ongoing work

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  - It is efficient in practice
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- Most solvers implement the tableau simplex method:
  - Pivoting is expensive as it requires to update all coefficients of the linear program
- Our alternative: revised simplex method
  - Pivoting is cheap as it just needs to incrementally update the inverse matrix corresponding to dependent variables.
  - Method of choice for LP community (if sparse)

Benchmark-goaled Linear Arithmetic

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  1. Propagate bounds of the model

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  2. Propagate difference logic fragment of the model

$$x - y \leq 1 \;\wedge\; y - z \leq 2 \;\wedge\; y = x + 2z \implies x - z \leq 3$$

New ideas to be added soon

[Dutertre&DeMoura,CAV'06]:

- Very nice simple ideas, extremely good results
- Initial translation into equalities + bounds. E.g., replace $2x - 3y + 5z \leq 12$ by $2x - 3y + 5z = s$ and $s \leq 12$
- The equalities never change, atoms sent to (and retracted from) T-Solver are bounds.
- Allows for initial simplifications
- Little work on backtracking
- Can identify cheap T-Propagate cases

Expensive Theories, Combination

Splitting on demand [Barrett N O Tinelli, LPAR'06]:

- Some T-Solvers need internal case splits (non-convex T)
- Idea: T-Solver must request DPLL(X) engine to do them.
  Advantages:
  - DPLL(X) is much better in doing case splits
  - Centralized decision heuristic not disturbed by other ones
  - T-Solver simpler: no splitting infrastructure needed
  - Weaker requirements for T-Solver:
    complete "if all demanded splits have been done"
- Resulting architecture naturally includes an efficient DPLL($T_1 \ldots T_n$) Nelson-Oppen-based combination

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Application to Predicate Abstraction

Predicate Abstraction:

- gives finite-state abstractions from infinite-state systems
- abstraction efficiently analyzed using Boolean techniques
- many applications to verification

Key operation:

- INPUT: a formula φ and set of predicates P
- OUTPUT: the most precise approximation of φ using P, either
  $\mathcal{F}_P(\varphi)$: weakest formula over P T-entailing φ or
  $\mathcal{G}_P(\varphi)$: strongest formula over P T-entailed by φ.

Use of *B*arcelogic: See CAV'06 for details!

- Use All-SAT SMT + BDD to get all models over P of φ
- Extract (compact) approximation from BDD

Experimental results for P.Abstraction

Microsoft SLAM (device drivers verification):

- Initially, ZAP [Ball et al, CAV'04] was used for p. abstraction
- Specialized Symbolic Decision Procedures (SDPs) [Lahiri et al, CAV'05] obtained 100x speedup factor over ZAP
- *B*arcelogic gives another 100x speedup over SDPs

Hardware and protocol verification problems
(70pbs, over $\approx$ 25 preds) [Lahiri and Bryant, CAV'04]:

- *B*arcelogic gives 25x – 100x speedup over UCLID

Benchmarks from the verification of programs with linked lists
(30pbs, $\approx$ 20 preds) [Qaader and Lahiri, POPL'06]:

- *B*arcelogic gives 30x – 40x speedup over UCLID

Application to optimization problems

- Aim: find SAT/SMT models M with minimal $cost(M)$.
- Branch and bound in *B*arcelogic: See SAT'06 for details!
  - Theory T = function $cost \ \wedge$ best M so far.
    After each new solution, T is strengthened
- (Weighted) Max-SAT:
  - $cost(M)$ = sum of weights of clauses that are false in M
  - Specialized rules, e.g: if units l and $\neg l$ detected, add smallest of their weights to cost
  - *B*arcelogic improves best Weighted CSP/PB solvers on most larger problems
- Max-SMT: Modeled and solved well-known hard Radio Freq. Assignment Problems with distance constraints: Diff. Logic.
  - *B*arcelogic with no specialized heuristics beats best Weighted CSP solver (with its best heuristic)

Other ongoing/future work

- Bit vector arithmetic
- Adding support for quantifiers
- Efficient interpolation modulo T
- Other less-standard applications of SMT: e.g., CSP's, FO finite model finding, ...

Thank you!