Blind Calibration of Sensor Networks

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This work was supported by the National Science Foundation

2007 IEEE Statistical Signal Processing Workshop

26-29 August 2007 Madison, Wisconsin

General Chairs: **R. Nowak & H. Krim**
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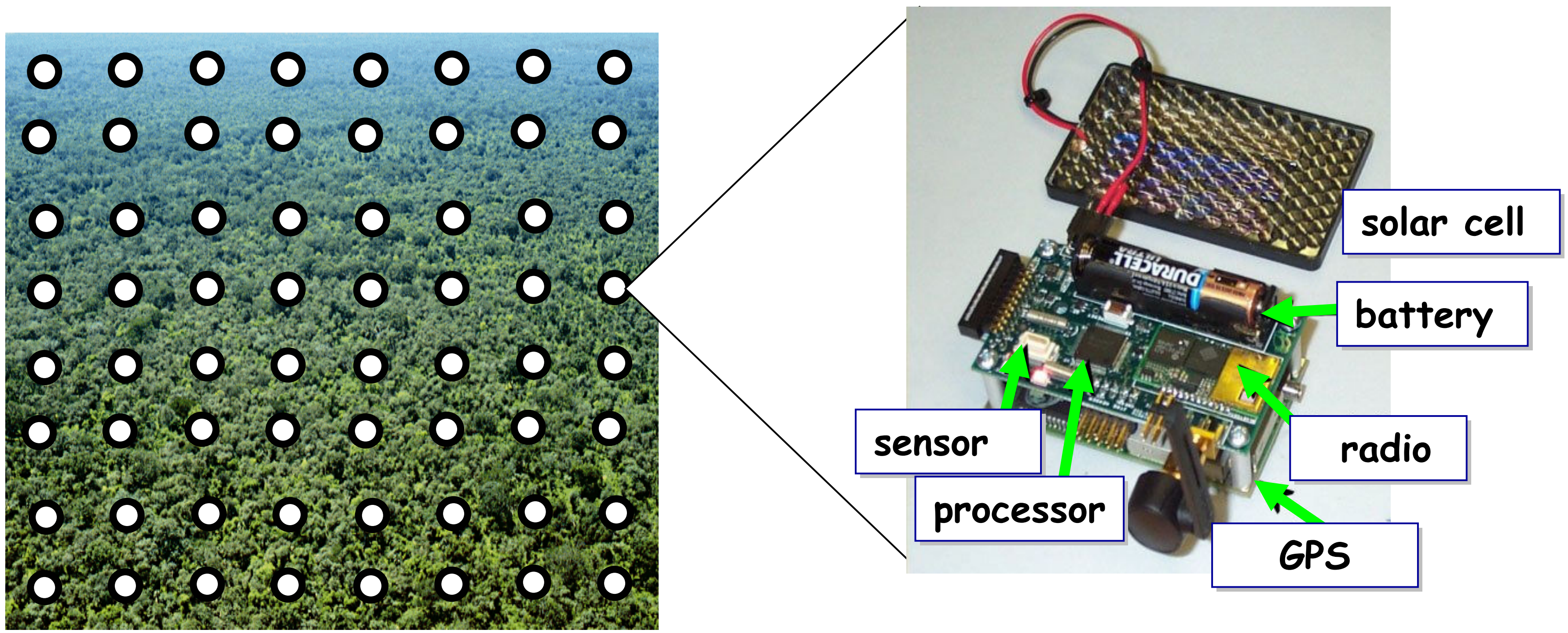
Theoretical topics	**Application areas**
Compressive sampling	Bioinformatics and genomic signal processing
Monte Carlo methods	Automotive and industrial applications
Detection and estimation theory	Array processing, radar and sonar
Distributed signal processing	Communication systems and networks
Learning theory and pattern recognition	**Sensor networks**
Multivariate statistical analysis	Information forensics and security
System identification and calibration	New methods, directions and applications.
Time-frequency and time-scale analysis	Biosignal processing and medical imaging

Wireless Sensing Challenges



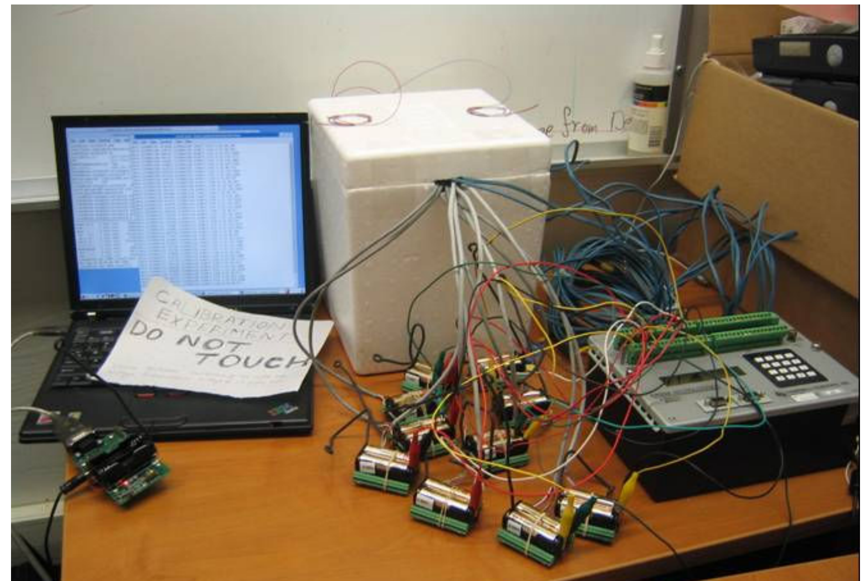
Networking, Communications, Resource Management, Signal Processing

BUT if sensors aren't calibrated,
then all is for naught !

Srivastava

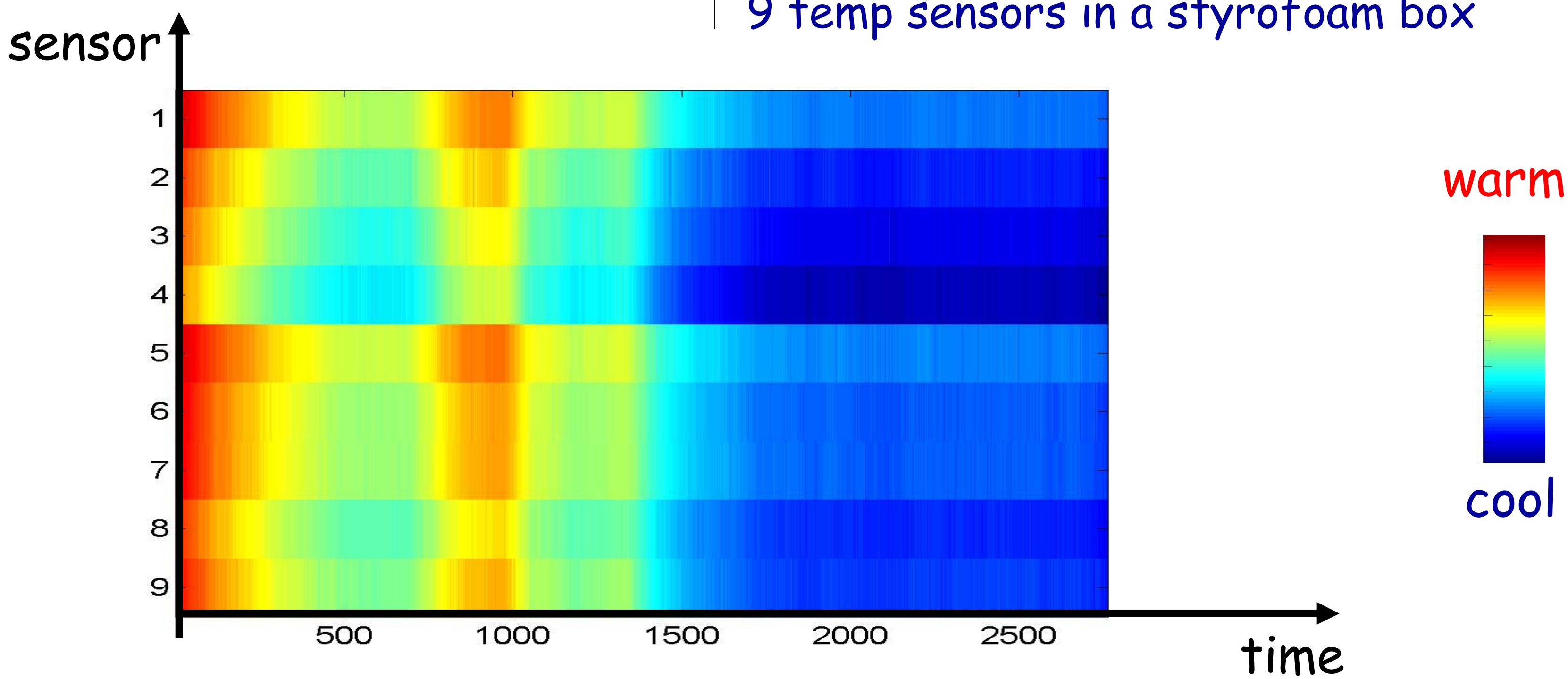
Megerian

Sensing in a Box

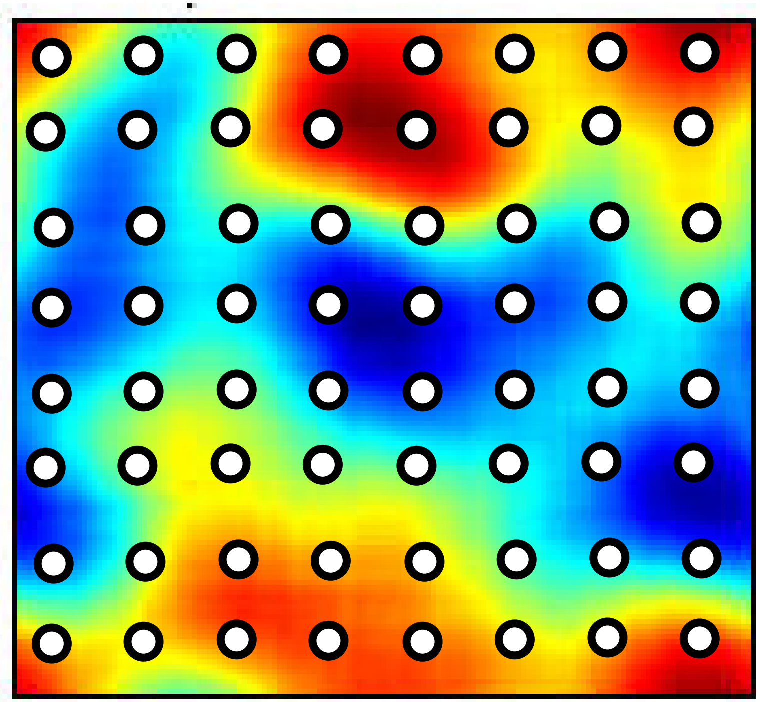


9 temp sensors in a styrofoam box

Actual (uncalibrated) Temperature Readings



Calibration in the Field



Pseudocolor map of
temperature distribution

Neighboring sensors in dense deployment make very similar readings

→ We can automatically calibrate sensor network
by forcing readings to agree locally

V. Bychkovskiy, S. Megerian, D. Estrin, and M. Potkonjak, "A collaborative approach to in-place sensor calibration," Lecture Notes in Computer Science, 2634:301–316, 2003.

Calibration Model

"Uncalibrated" Sensor Measurements:

$$\mathbf{y} = [y(1), \ldots, y(n)]^T$$

Calibrated Measurements:

$$x(j) \;=\; \alpha(j)\, y(j) \;+\; \beta(j)$$

gain correction
for sensor j

offset correction
for sensor j

Vector Notation:

$$\mathbf{x} = \boldsymbol{\alpha} \bullet \mathbf{y} \;+\; \boldsymbol{\beta}$$

Hadamard product

Blind Calibration Problem

Given k uncalibrated "snapshots" (e.g., at different times):

$$\mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_k$$

Estimate Gain and Offset Corrections:

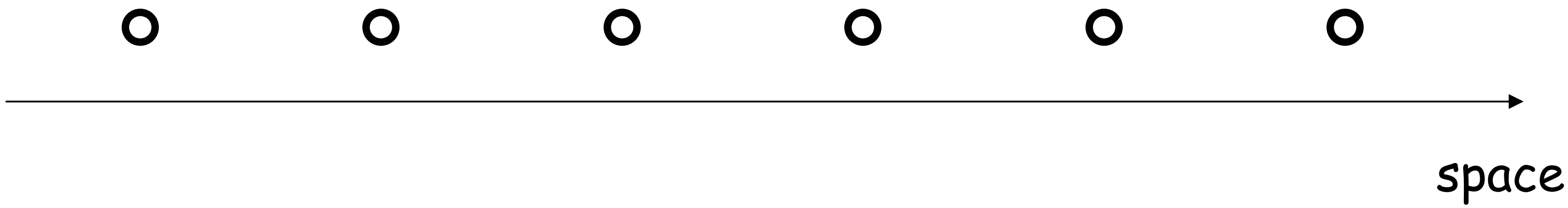
Find α and β such that for i = 1,…,k

$$\mathbf{x}_i = \boldsymbol{\alpha} \bullet \mathbf{y}_i + \boldsymbol{\beta}$$

Without additional assumptions this is an impossible problem

Calibration by Local Agreement

Linear deployment of sensors:



Ideal (calibrated) sensor readings:

$$x_1 \quad x_2 \quad \cdots \quad x_n$$

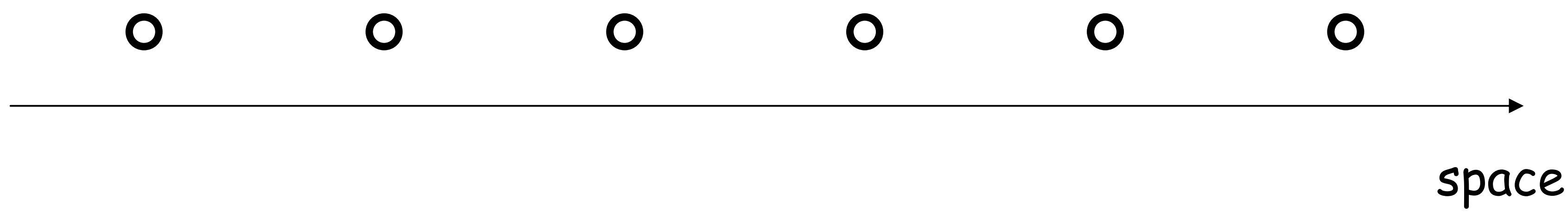
Calibration via local agreement is based on assumption:

$$x_1 - x_2 \approx 0 \quad x_2 - x_3 \approx 0 \quad \cdots \quad x_{n-1} - x_n \approx 0$$

These conditions define a signal subspace (constant functions)

Calibration by Local Agreement

Linear deployment of sensors:



Ideal (calibrated) sensor readings:

$$x_1 \quad x_2 \quad \cdots \quad x_n$$

Calibration assuming second derivatives are approx zero:

$$x_1 - 2x_2 + x_3 \approx 0 \quad \cdots \quad x_{n-2} - 2x_{n-1} + x_n \approx 0$$

These conditions define a signal subspace (linear functions)

Signal Subspaces and Calibration

Ideal (calibrated) sensor readings:

$$\mathbf{x} = [x_1,\ x_2,\ \ldots\ ,\ x_n]^T$$

Calibration via *signal subspace matching* is based on assumption:

$$\mathbf{P}\,\mathbf{x} \approx 0$$

where $\mathbf{P}$ is an orthogonal projection matrix
corresponding to the signal's nullspace

Ex. Bandlimited signals, "smooth" signals

When do solutions exist, how do we find them ?

Assumptions

- The calibrated signals are a linear function of the uncalibrated snapshots:

$$\mathbf{x}_i = \boldsymbol{\alpha} \bullet \mathbf{y}_i \; + \; \boldsymbol{\beta}$$

- The calibrated signals lie in a known r-dimensional subspace of R^n

  Let $\mathbf{P}$ denote the projection on to the orthogonal complement of the signal subspace

Examples: $\mathbf{P}$ could correspond to a projection onto a frequency band, a roughness subspace, or any other subspace where the signal should not be

Blind Calibration

k “snapshots” result in the following system of equations:

$$\mathbf{P}\,\mathbf{x}_i = \mathbf{P}\,(\boldsymbol{\alpha} \bullet \mathbf{y}_i \;+\; \boldsymbol{\beta}) \;=\; \mathbf{0}$$

$$i = 1, \ldots, k$$

Which we can try to solve for α and β

Identifiability

$$\mathbf{P}\mathbf{x}_i = \mathbf{P}\,(\alpha \bullet \mathbf{y}_i \; + \; \beta) \; = \; 0$$

Offsets:

Clearly, we cannot identify the component of β in the signal subspace. This component is indistinguishable from true signal.

Gains:

However, since $\mathbf{y}_i$ "modulates" α, under certain conditions on $\mathbf{P}$ it is possible exactly recover α up to a global gain factor. We cannot distinguish between α and (scalar constant) $\times\, \alpha$.

Matrix-Vector Formulation

Diag Operation:

$$\mathbf{Y} \equiv \mathrm{diag}(\mathbf{y}) = \begin{bmatrix} y(1) & & \\ & \ddots & \\ & & y(n) \end{bmatrix}$$

System of Calibration Equations:

$$\mathbf{P}\,(\mathbf{Y}_i\,\boldsymbol{\alpha} + \boldsymbol{\beta}) = \mathbf{0}, \quad i = 1, \ldots, k$$

Offset Solutions

$$\mathbf{P}\,(\mathbf{Y}_i\,\boldsymbol{\alpha} \;+\; \boldsymbol{\beta}) \;=\; \mathbf{0}, \quad i = 1, \ldots, k$$

$$\boxed{\mathbf{P}\,\boldsymbol{\beta} = -\mathbf{P}\,\bar{\mathbf{Y}}\boldsymbol{\alpha}}$$

where $\bar{\mathbf{Y}} \;=\; \frac{1}{k}\sum_{i=1}^{k} \mathbf{Y}_i$

Note:

- Every offset solution is a simple function of sensor data and gains
- Unique solution only in signal nullspace
- Offset component in signal subspace cannot be blindly recovered (but this may not be too significant if signal subspace is "small")

Gain Solutions

$$\begin{aligned} \mathbf{0} &= \mathrm{P}\left(\mathbf{Y}_i \boldsymbol{\alpha} + \boldsymbol{\beta}\right) \\ &= \mathrm{P}\left(\mathbf{Y}_i - \bar{\mathbf{Y}}\right) \boldsymbol{\alpha}, \quad i = 1, \ldots, k \end{aligned}$$

A unique solution exists iff the matrix

$$\begin{bmatrix} \mathrm{P}\left(\mathbf{Y}_1 - \bar{\mathbf{Y}}\right) \\ \mathrm{P}\left(\mathbf{Y}_2 - \bar{\mathbf{Y}}\right) \\ \vdots \\ \mathrm{P}\left(\mathbf{Y}_k - \bar{\mathbf{Y}}\right) \end{bmatrix}$$

has rank n-1 (i.e., a single singular vector)

Exact Recovery of Calibration Gains

When does $\begin{bmatrix} \mathbf{P}\mathbf{Y}_1 \\ \mathbf{P}\mathbf{Y}_2 \\ \vdots \\ \mathbf{P}\mathbf{Y}_k \end{bmatrix} \boldsymbol{\alpha} = \mathbf{0}$ have a unique solution?

Assumptions:

Oversampling: The ideal sensor network signals lie in a known *r-dimensional* signal subspace

Randomness: The signals are randomly distributed across snapshots according to an unknown density function with support on the signal subspace

Incoherence: The signal subspace is *incoherent* with the canonical spatial basis (i.e., δ basis)

Exact Recovery of Calibration Gains

Theorem 1: Under assumptions *A1*, *A2* and *A3*, the gains can be perfectly recovered from any $k \geq r$ signal measurements by solving the linear system of equations

$$\begin{bmatrix} \mathbf{PY}_1 \\ \mathbf{PY}_2 \\ \vdots \\ \mathbf{PY}_k \end{bmatrix} \boldsymbol{\alpha} = \mathbf{0}$$

Theorem 2: If the signal subspace is defined by a subset of the DFT vectors (i.e., a frequency-domain subspace), then incoherence condition is automatically satisfied, and the gains can be perfectly recovered from any $k \geq r$ signal measurements

Proof Sketch

Oversampling: Signals lie in an r-dimensional subspace of R^n.

Randomness: $k \geq r$ snapshots span signal subspace with probability 1

Incoherence:

First, write system of equations in terms of calibrated signals

$$\mathbf{y} = \mathbf{x} / \boldsymbol{\alpha}^*$$

or equivalently

$$\mathbf{Y} = \mathbf{X}\mathbf{A}$$

$$\mathbf{Y} := \text{diag}(\mathbf{y}), \quad \mathbf{X} := \text{diag}(\mathbf{x}), \quad \mathbf{A} := \text{diag}(1/\boldsymbol{\alpha}^*)$$

Proof Sketch

Calibration equations

$$\begin{aligned} \mathbf{0} &= \mathbf{P}\,\mathbf{Y}\,\boldsymbol{\alpha} \\ &= \mathbf{P}\,\mathbf{X}\,\mathbf{A}\,\boldsymbol{\alpha} \\ &=: \mathbf{P}\,\mathbf{X}\,\mathbf{d} \end{aligned}$$

$\mathbf{d} \propto \mathbf{1}$ is a solution ($\mathbf{d} = \mathbf{1} \Leftrightarrow \boldsymbol{\alpha} = \boldsymbol{\alpha}^*$).
We want to show that it is the only solution.

If $\mathcal{S}$ denotes the signal subspace, then $\mathbf{P}\,\mathbf{X}\,\mathbf{d} = \mathbf{0}$ is equivalent to condition $\mathbf{X}\,\mathbf{d} \in \mathcal{S}$.

Proof Sketch

Oversampling and Randomness imply that the following conditions are equivalent

(i) $\mathbf{X}_i\,\mathbf{d} \;=\; \text{diag}(\mathbf{x}_i)\,\mathbf{d} \;\in\; \mathcal{S}, \quad i = 1, \ldots, k$

(ii) $\text{diag}(\boldsymbol{\phi}_j)\,\mathbf{d} \;\in\; \mathcal{S}, \quad j = 1, \ldots, r$

where $\boldsymbol{\phi}_1, \ldots, \boldsymbol{\phi}_r$ is a basis for $\mathcal{S}$

(iii) $\text{diag}(\phi_j)\,\mathbf{d} \;=\; \boldsymbol{\Phi}\,\boldsymbol{\theta}, \quad j = 1, \ldots, r$

where $\boldsymbol{\Phi} = [\phi_1 \; \phi_2 \; \cdots \; \phi_r]$ and $\boldsymbol{\theta} \in R^r$

Proof Sketch

(iii) $$\text{diag}(\phi_j)\,\mathbf{d} \;=\; \mathbf{\Phi}\,\boldsymbol{\theta}, \quad j = 1, \ldots, r$$

where $\mathbf{\Phi} = [\phi_1 \; \phi_2 \; \cdots \; \phi_r]$ and $\boldsymbol{\theta} \in R^r$

→ Solutions $\mathbf{d}$ must lie in the intersection of sub-spaces spanned by columns of the matrices

$$\mathbf{B}_j \;:=\; [\text{diag}(\phi_j)]^{-1}\mathbf{\Phi}, \quad j = 1, \ldots, r$$

where $[\text{diag}(\phi_j)]^{-1}$ is the pseudoinverse of $\text{diag}(\phi_j)$

Equivalently, solutions $\mathbf{d}$ must lie in the union of the nullspaces of $\mathbf{B}_j$, $j = 1, \ldots, r$

Proof Sketch

Let $\mathbf{V}_j$ be a matrix whose columns span the nullspace of

$$\mathbf{B}_j \;:=\; [\mathrm{diag}(\phi_j)]^{-1}\mathbf{\Phi}, \quad j = 1, \ldots, r$$

"Incoherence" condition:

$$\mathrm{rank}([\mathbf{V}_1 \cdots \mathbf{V}_r]) \;=\; 1$$

(easily checked for a given basis; also holds for almost any frequency subspace)

$$\boxed{\mathbf{d} \;\propto\; \mathbf{1} \quad \Leftrightarrow \quad \boldsymbol{\alpha} \;=\; \boldsymbol{\alpha}^*}$$

Robust Recovery of Calibration Gains

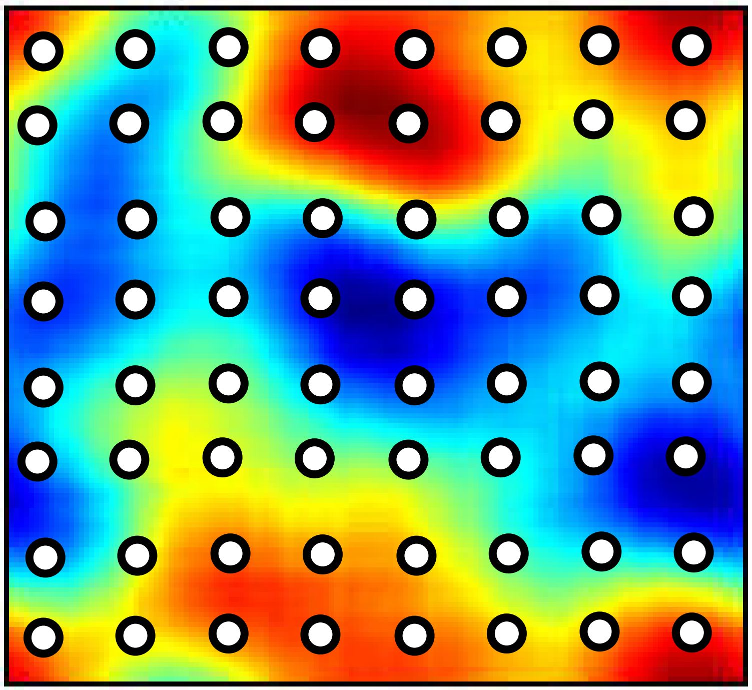
Gain equations may hold only approximately due to noise/errors:

$$\underbrace{\begin{bmatrix} \mathbf{P}\left(\mathbf{Y}_1 - \bar{\mathbf{Y}}\right) \\ \mathbf{P}\left(\mathbf{Y}_2 - \bar{\mathbf{Y}}\right) \\ \vdots \\ \mathbf{P}\left(\mathbf{Y}_k - \bar{\mathbf{Y}}\right) \end{bmatrix}}_{\mathbf{C}} \boldsymbol{\alpha} \approx 0$$

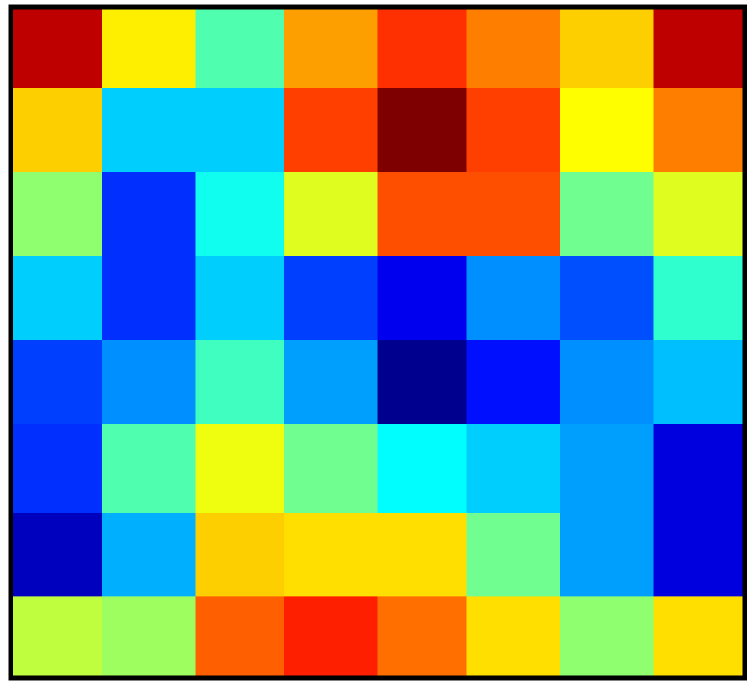
Robust solutions:

$$\hat{\boldsymbol{\alpha}} = \arg\min_{\boldsymbol{\alpha}} \|\mathrm{C}\,\boldsymbol{\alpha}\|_2^2$$

Simulation Experiment



simulated temperature field

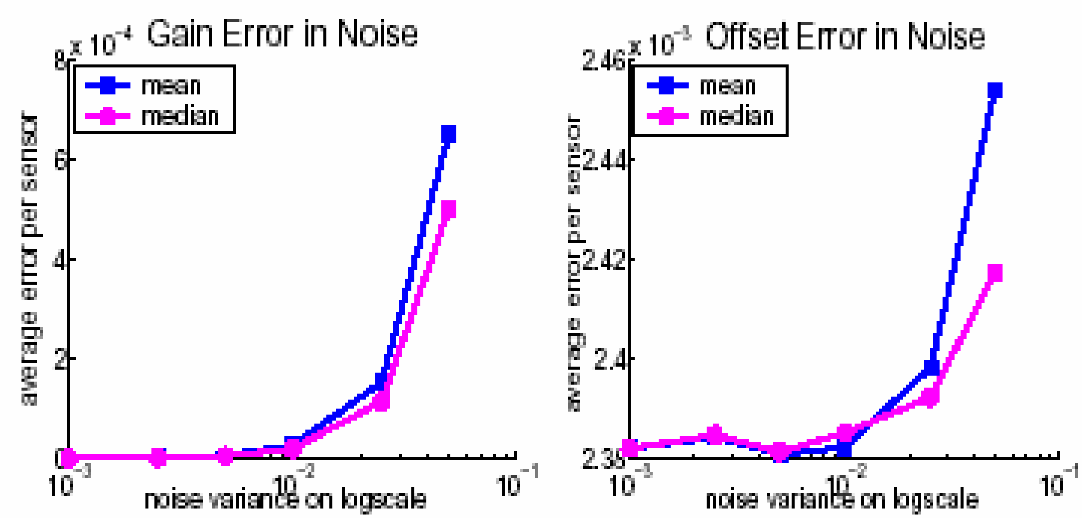


8x8 sensor readings

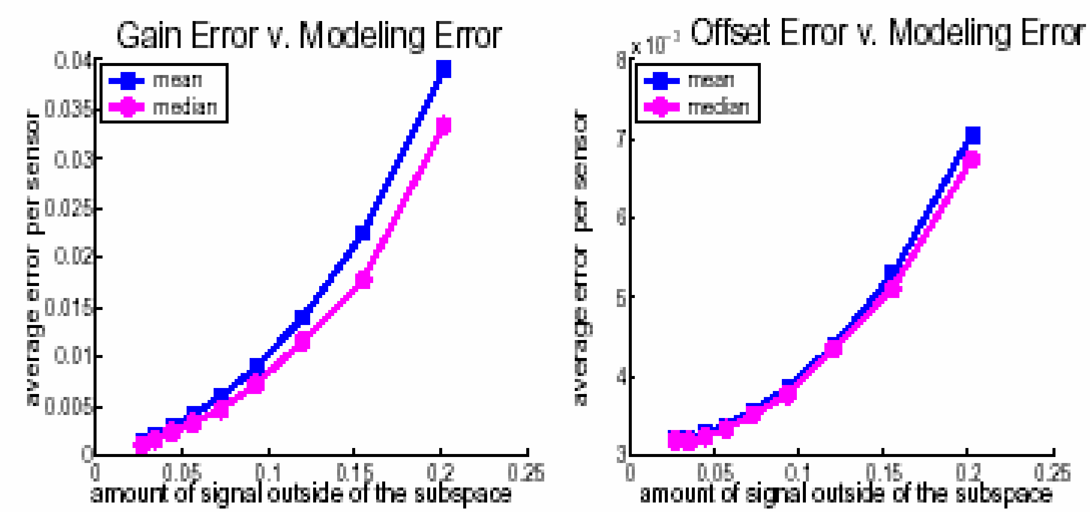
- field is smoothed GWN process
- approximate signal subspace = span of lowpass DFT vectors

Simulation Experiment

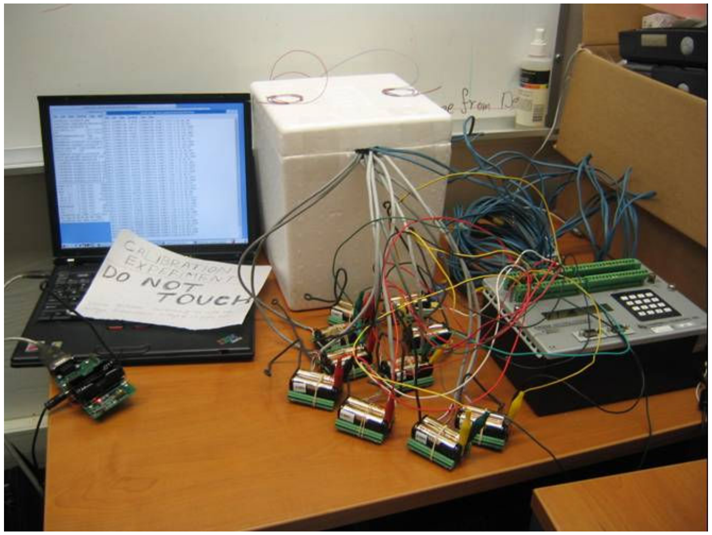
robust to noise



robust to mismodeling

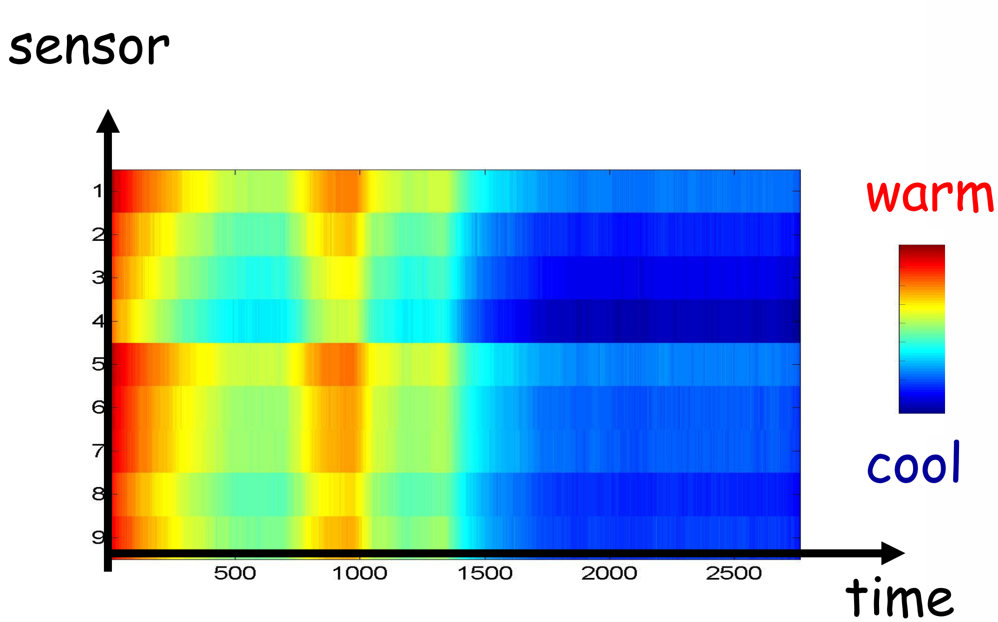


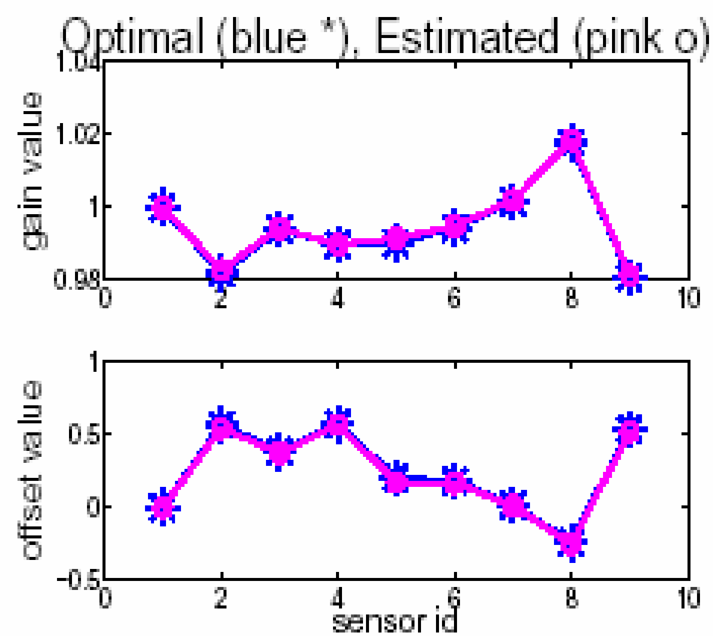
Sensing in the Box



Ideally all sensors should read the same temperature

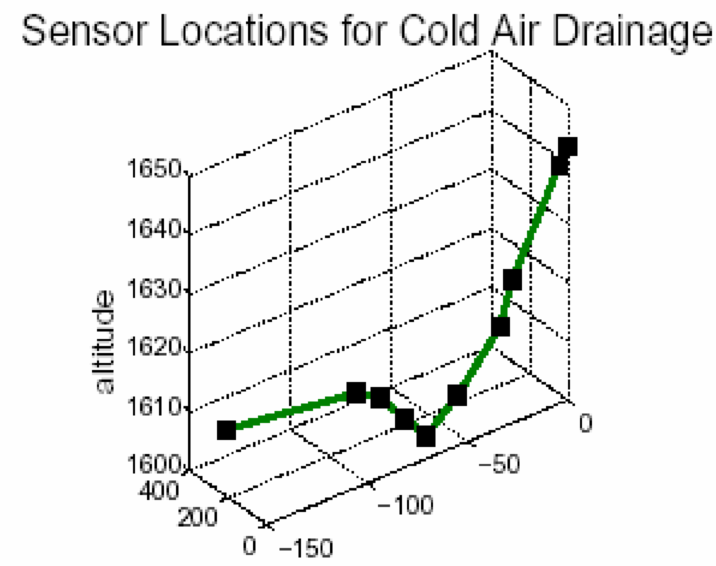
→ 1-d signal subspace of constant functions



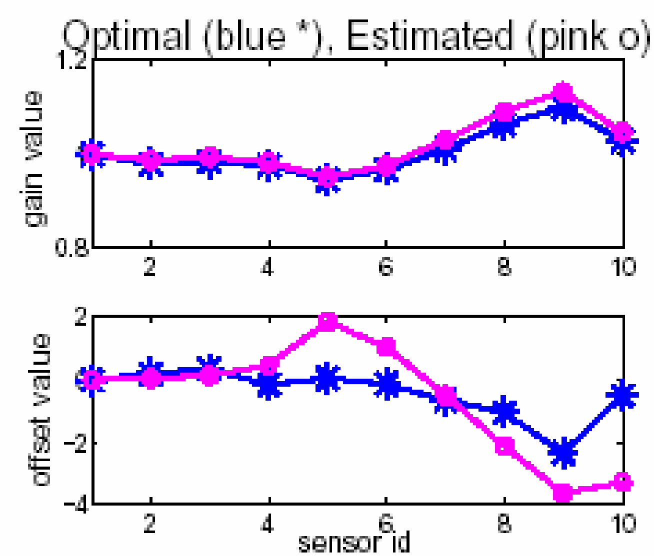


Sensing in the Wild

temperature sensing
at James Reserve



4-dimensional signal
subspace determined from
calibrated sensor data



Conclusions

- Sensor calibration gains and (partially) offsets can be determined from routine sensor measurements with some knowledge of calibrated signal characteristics (e.g., frequency band, smoothness)
- key necessary condition is "incoherence" between signal subspace and canonical (spatial) basis; the condition is a nonlinear requirement on the signal basis and is easy to check in general
- our experience is that solutions are robust to noise and mismodeling in some cases, and sensitive in others; we do not have a good understanding of the robustness of the methodology at this time
- extensions are possible to handle more exotic calibration functions (e.g., nonlinear)

More info: Balzano and Nowak, "Blind Calibration of Sensor Networks," submitted to IPSN 2007