Information-driven Inference in Resource-constrained Environments

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Academic Model of a Sensor Network

$p \left( X \mid Z \right)$

- We're not always sure of what "P" is (and it keeps changing)
- We think we know what "Z" is (but it keeps changing)
- We're not always sure of what "X" is (and it keeps changing)
Primary Question

How should we optimize the measurement process in a sensor network for inference problems?

- We can control sensors within resource constraints to obtain different types of information about the underlying phenomenon.
- Control choices impact both quality of inference and resource expenditures.

Applications

- state estimation/tracking
- identification
- random field estimation
Outline

- Distributed Inference in Resource Constrained Environments
  - Theoretical Bounds
    - Optimal schemes are infeasible
    - Can we give performance guarantees of approximation schemes as compared to optimal?
  - In-Network Processing with Dynamic Fusion Centers
    - Explicitly trade-off value of information discounted by resource expenditures
    - Incorporate measurement transmission and selection
    - Incorporate coding/transmission of probabilistic models
In-Network Processing: Dynamic Fusion Centers

Basic Intuition

- Measurements are not equally useful and incur different resource expenditures.
- Moving the fusion center dynamically is a compromise between centralized and decentralized approaches.
- Information regarding many phenomenon is “local”.

Zhao, Shin, Reich (2002)

- Consider a tracking application in which sensors yield noisy range measurements.
- Utilize the single sensor measurement which minimizes the expected uncertainty at the next time step.
- Perform fusion at the sensor with the highest expected uncertainty reduction.
- Implicitly captures the notion that communications and fusion of all measurements is prohibitive relative to the decrease in uncertainty of the kinematic state.
Maximizing Expected Information Gain

- Having incorporated previous measurements (or a subset of those available) to compute a posterior

\[ p(x_k | \{z\}_{0,k-1} = \{Z\}_{0,k-1}) \]

choose the sensor whose measurement yields the highest expected information gain.

- Equivalent information-theoretic criterion:

\[
\begin{align*}
\arg \min_j h(x_k | z^j_k, \{z\}_{0,k-1} = \{Z\}_{0,k-1}) \\
\arg \max_j I(x_k ; z^j_k | \{z\}_{0,k-1} = \{Z\}_{0,k-1}) \\
\arg \max_j D \left( p(x_k | z^j_k, \{z\}_{0,k-1} = \{Z\}_{0,k-1}) \| p(x_k | \{z\}_{0,k-1} = \{Z\}_{0,k-1}) \right) \\
\arg \min_j D \left( p(x_k | z^0_k, \ldots, z^N_k, \{z\}_{0,k-1} = \{Z\}_{0,k-1}) \| p(x_k | \{z\}_{0,k-1} = \{Z\}_{0,k-1}) \right)
\end{align*}
\]

- Such a strategy is myopic (searching over a small number of time steps) and greedy (searching over the single best available measurement at each time step)

Notation
- \( z^i_k \) = measurement of sensor \( i \) at time \( k \)
- \( Z^i_k \) = measurement value of sensor \( i \) at time \( k \)
- \( \{z\}_{i,k} \) = selected measurements from time \( i \) to \( k \)
- \( \{Z\}_{i,k} \) = selected measurement values from time \( i \) to \( k \)
Why Use Information-Theoretic Objective Criteria?

Scheffe's Theorem

\[ \int |p - q| = 2 \sup_{\Lambda} \left| \int_A p - \int_A q \right| \]
\[ = 2 \int_{p>q} (p - q) \]
\[ = 2 \int_{q>p} (q - p) \]
\[ \int_A p - \int_A q \leq \frac{1}{2} \int |p - q| \]

Kullback-Csiszar-Kemperman Inequality

\[ \int |p - q| \leq \min \left\{ \sqrt{2D(p||q)} , \sqrt{2D(q||p)} \right\} \]

Bretagnolle-Huber Inequalities

\[ \int |p - q| \leq 2 \sqrt{1 - e^{-D(p||q)}} \]
\[ \int |p - q| \leq 2 - e^{-D(p||q)} \]
\[ \int \min \{p, q\} \geq \frac{1}{2} e^{-D(p||q)} \]
Value of Long Term Planning?

Choosing the optimal set of measurements (sensors) is exponential in the planning horizon.

- Can we bound the difference in performance of approximate (tractable) algorithms as compared to optimal?
- Is there a performance gain if we incorporating planning over a longer time-horizon (non-myopic)
Capturing the Notion of Diminishing Returns

- Assuming that observations are independent conditioned on the state, mutual information is submodular

\[
\text{If } \mathcal{A} \subseteq \mathcal{B} \text{ then: } I(X; z^C | z^B) = H(z^C | z^B) - H(z^C | z^B, X) = H(z^C | z^B) - H(z^C | X) \leq H(z^C | z^A) - H(z^C | X) = I(X; z^C | z^A)
\]

\[
\mathcal{B} = \left\{ x_1, x_2, x_3, x_4, x_5, x_6, \ldots, x_N \right\} \quad \mathcal{C} = \left\{ x_{N+1}, \ldots \right\}
\]
A simple bound...

• Suppose we have $M$ stages
  ▪ Each stage involves selection of an observation for a different sensor or a different time step

• Suppose we use the greedy heuristic to select each observation:

$$g_j = \arg \max_{g \in \{1, \ldots, n_j\}} I(X; z_j^g | z_1^{g_1}, \ldots, z_{j-1}^{g_{j-1}})$$

Then...

$$I(X; z_1^{o_1}, \ldots, z_M^{o_M}) \leq 2I(X; z_1^{g_1}, \ldots, z_M^{g_M})$$
A simple bound...

\[ I(X; z_1^{o1}, \ldots, z_M^{oM}) \]
\[ \leq I(X; z_1^{g1}, \ldots, z_M^{gM}, z_1^{o1}, \ldots, z_M^{oM}) \]

\[ = \sum_{j=1}^{M} \left\{ I(X; z_j^{gj} | z_1^{g1}, \ldots, z_{j-1}^{gj-1}) + I(X; z_j^{oj} | z_1^{g1}, \ldots, z_M^{gM}, z_1^{o1}, \ldots, z_{j-1}^{oj-1}) \right\} \]
\[ \leq \sum_{j=1}^{M} \left\{ I(X; z_j^{gj} | z_1^{g1}, \ldots, z_{j-1}^{gj-1}) + I(X; z_j^{oj} | z_1^{g1}, \ldots, z_j^{gj-1}) \right\} \]
\[ \leq 2 \sum_{j=1}^{M} I(X; z_j^{gj} | z_1^{g1}, \ldots, z_{j-1}^{gj-1}) \]
\[ = 2I(X; z_1^{g1}, \ldots, z_M^{gM}) \]
Online computable bound

• While the bound is tight, the proof gives rise to an online computable version which may be stronger in particular circumstances

• Let \( \bar{g}_i = \arg \max_{\bar{g} \in \{1, \ldots, n_i\}} I(X; z_{i1}^{\bar{g}}, \ldots, z_{Mi}^{\bar{g}}) \)

• Then:

\[
I(X; z_{11}^{O1}, \ldots, z_{M1}^{OM}) \leq I(X; z_{11}^{g1}, \ldots, z_{M1}^{gM}) + \sum_{i=1}^{M} I(X; z_{i1}^{\bar{g}_i} | z_{11}^{g1}, \ldots, z_{M1}^{gM})
\]

• This bound is tight in situations where the greedy selection leaves little information behind
Online computable bound – example

• Suppose we model the depth of the ocean as a Gauss-Markov random field thin membrane model (500 × 100 cells)

• We seek to measure ocean depth using a surface vehicle traveling along a fixed path
  ▪ Available observations depend on current vehicle position

• We need to choose observations at each time
Online computable bound – example

![Graphs showing achieved performance and bound, and factor of optimality.](image)

- Achieved performance and bound
  - Total accrued MI vs Time step
  - Realized performance
  - Upper bound on optimal

- Factor of optimality
  - Ratio of greedy to opt. bound vs Time step
  - Values range from 0.7 to 1.0
Feasible Information Driven Sensor Planning

- Motivation & assumptions
- Constrained Markov Decision Process formulation
  - Communication constraint
  - Entropy constraint
- Approximations
  - Linearized Gaussian
  - Greedy sensor subset selection
  - $n$-Scan pruning
- Simulation results
Incorporating Resource Constraints

- **Object tracking using a network of sensors**
  - Sensors provide localization in close vicinity
  - Sensors can communicate locally at cost $\propto f(\text{range})$

- **Energy is a limited resource**
  - Consumed by communication, sensing and computation
  - Communication is orders of magnitude more expensive than other tasks

- **Sensor management algorithm must determine**
  - Which sensors to activate at each time
  - Where the probabilistic model should be stored at each time
  - While trading off the competing objectives of estimation performance and communication cost
Greedy Myopic Planning

• Model transmission occurs with little benefit to the inference problem.
• Single measurement used at each time.
• Resources are only implicitly incorporated into the inference problem.

Can we benefit by considering expected information gain discounted by resource expenditures over long horizons?
**Notation**

Time index: $k$

Position of sensor $s$: $l^s$

Leader node: $l_k$

Activated sensors: $S_k \subseteq S$

Object state / location: $x_k / \mathbf{L}x_k$

Probabilistic model: $X_k = p(x_k | z_{0:k-1})$

Sensor $s$ measurement: $z_k^s$

History of incorporated measurements: $z_{0:k-1}$
Example – observation/communications

Observation model:

\[ z_k^s = \frac{a}{\| L x_k - l^s \|^2 + b} + v_k^s \]

Communications cost:

\[ \tilde{C}_{ij} = \| l^i - l^j \|^2 \]

\[ C_{ij} = \sum_{k=1}^{n} \tilde{C}_{i_{k-1} i_k} \]
Formulation

- We formulate as a constrained Markov Decision Process
  - Minimize uncertainty subject to a constraint on communication cost
  - Minimize communication cost subject to a constraint on estimation performance
- State is PDF \( \mathbb{X}_k = p(x_k|z_{0:k-1}) \) and previous leader node \( l_{k-1} \)
  - Dynamic programming equation for an \( N \)-step rolling horizon:
    \[
    J_i(\mathbb{X}_i, l_{i-1}) = \min_{u_i} \left\{ g(\mathbb{X}_i, l_{i-1}, u_i) + \mathbb{E}_{\mathbb{X}_{i+1} | \mathbb{X}_i, u_i} J_{i+1}(\mathbb{X}_{i+1}, l_i) \right\}, \quad i \in \{k : k + N - 1\}
    \]
    \[
    J_{k+N}(\mathbb{X}_{k+N}, l_{k+N-1}) = 0
    \]
  - such that \( \mathbb{E}\{G(\mathbb{X}_k, l_{k-1}, u_{k:k+N-1}) | \mathbb{X}_k, l_{k-1}\} \leq 0 \)
**Communication-constrained formulation**

- Cost-per-stage is such that the system minimizes joint expected conditional entropy of object state over planning horizon:

\[
g(X_k, l_{k-1}, u_k) = - \sum_{j=1}^{|S_k|} I(x_k; z_k^{S_k^j} | z_{0:k-1}, z_k^{S_k^{1:j-1}})
\]

- Constraint applies to expected communication cost over planning horizon:

\[
G(X_k, l_{k-1}, u_{k:k+N-1}) = \sum_{i=k}^{k+N-1} \left[ C_{l_{i-1}l_i} + \sum_{j \in S_i} rC_{l_ij} \right] - C_{\text{max}}
\]
Communication-constrained formulation

• Integrating the communication costs into the per-stage cost:

\[
\bar{g}(x_k, l_{k-1}, u_k, \lambda) = -\sum_{j=1}^{\lvert S_k \rvert} I(x_k; z_{k}^{S_k^j} | z_{0:k-1}, z_{k}^{S_k^{1:j-1}}) + \lambda \left[ C_{l_{k-1}l_k} + \sum_{j \in S_k} r C_{l_kj} \right]
\]

• Per-stage cost now contains both information gain and communication cost
Information-constrained formulation

- Cost-per-stage is such that the system minimizes the energy consumed over the planning horizon:

\[ g(\mathbf{X}_k, l_{k-1}, u_k) = C_{l_{k-1}} l_k + \sum_{j \in S_k} rC_{l_{k,j}} \]

- Constraint ensures that the joint entropy over the planning horizon is less than given threshold:

\[ G(\mathbf{X}_k, l_{k-1}, u_{k:k+N-1}) = - \left\{ \sum_{i=k}^{k+N-1} \sum_{j=1}^{\left| S_i \right|} I(\mathbf{x}_i; z_{S_i}^{S_i^{j}} | z_{0:i-1}, z_{S_i}^{S_i^{1:j-1}}) \right\} - I_{\min} \]
Evaluating the DP

- DP has infinite state space, hence it cannot be evaluated exactly
- Conceptually, it could be evaluated through simulation

\[ \text{Complexity is } O\left([N_s 2^{N_s}]^N N_p^N\right) \]
Branching due to measurement values

• The first source of branching is that due to different values of measurement

• If we approximate the measurement model as linear Gaussian locally around a nominal trajectory, then the future costs are dependent only on the control choices, not on the measurement values

• Hence this source of branching can be eliminated entirely
**Greedy sensor subset selection**

- For large sensor networks complexity is high even for a single look-ahead step due to consideration of sensor subsets.
- We decompose each decision stage into a generalized stopping problem, where at each substage we can:
  - Add unselected sensor to the current selection
  - Terminate with the current selection
- The per-stage cost can be conveniently decomposed into a per-substage cost which directly trades off the cost of obtaining each measurement against the information it returns:

\[
\bar{g}(X_i, l_i, S_{i,i'}, s_{i,i'}, \lambda) = \lambda r_{C_{l_i S_{i,i'}}} - I(x_i; z_i^{s_{i,i'}} | z_{0:i-1}, z_i^{S_{i,i'}})
\]
Greedy sensor subset selection

- **Outer DP recursion**

\[
\tilde{J}_i(X_i, l_{i-1}, \lambda) = \min_{l_i} \left\{ \lambda C_{l_{i-1}l_i} + \tilde{J}_{i,0}(X_i, l_i, \emptyset, \lambda) \right\}
\]

- **Inner DP sub-problem**

\[
\tilde{J}_{i,i'}(X_i, l_i, S_{i,i'}, \lambda) = \min \left\{ \mathbb{E}_{X_{i+1}|X_{i}, S_{i,i'}} \tilde{J}_{i+1}(X_{i+1}, l_i, \lambda), \right. \\
\left. \min_{s_{i,i'} \in S \setminus S_{i,i'}} \left\{ \bar{g}(X_i, l_i, S_{i,i'}, s_{i,i'}, \lambda) + \tilde{J}_{i,i'+1}(X_i, l_i, S_{i,i'} \cup s_{i,i'}, \lambda) \right\} \right\}
\]
$n$-Scan approximation

- The greedy subset selection is embedded within a $n$-scan pruning method (similar to the MHT) which addresses the growth due to different choices of leader node sequence
Simulation results

Position entropy vs communication cost

Average position entropy

Accrued communication cost

- Triangle: DP CC N=5
- Square: DP CC N=10
- Circle: DP CC N=25
- Triangle: DP IC N=5
- Square: DP IC N=10
- Circle: DP IC N=25
- Pink: Greedy MI
- Red: Min Expect Dist

x 10^4

0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2

0.5 1 1.5 2 2.5 3

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**Significant Cost Reduction**

- Decomposed cost structure into a form in which greedy approximations are able to capture the trade-off
  - Complexity $O([N_s 2^{N_s}]^{N N_p} N)$ reduced to $O(N N_s^3)$
  - $N_s = 20 / N_p = 50 / N = 10$: $1.6 \times 10^{90} \rightarrow 8 \times 10^5$
  - Strong non-myopic planning for horizon lengths > 20 steps possible