Orientifolds and Mirror Symmetry

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based on I. Brunner & KH
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4d $\mathcal{N}=1$ Compactification of String theory

* interesting

** relevant for real world physics

Various ways : • Heterotic string on CY^3 with gauge bund

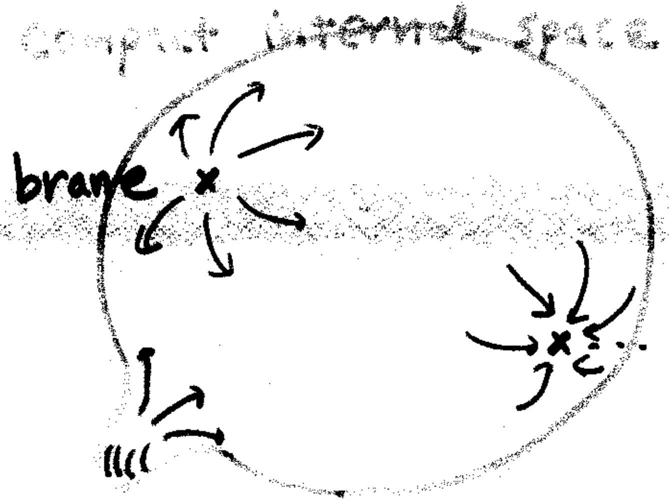
• M theory on G_2 holonomy manifolds

• F theory on CY^4

Many of them are dual to :

Type II string theory with space-filling D-branes

(or fluxes)



flux needs to be absorbe

$\Downarrow$

Orientifold is required

4d $\mathcal{N}=2$ compactifications (Type II on CY^3):

Moduli space looks like

What happens to this picture when Orientifold is imposed & D-branes and fluxes are added ?

We expect a drastic change.

① Orientifold projection

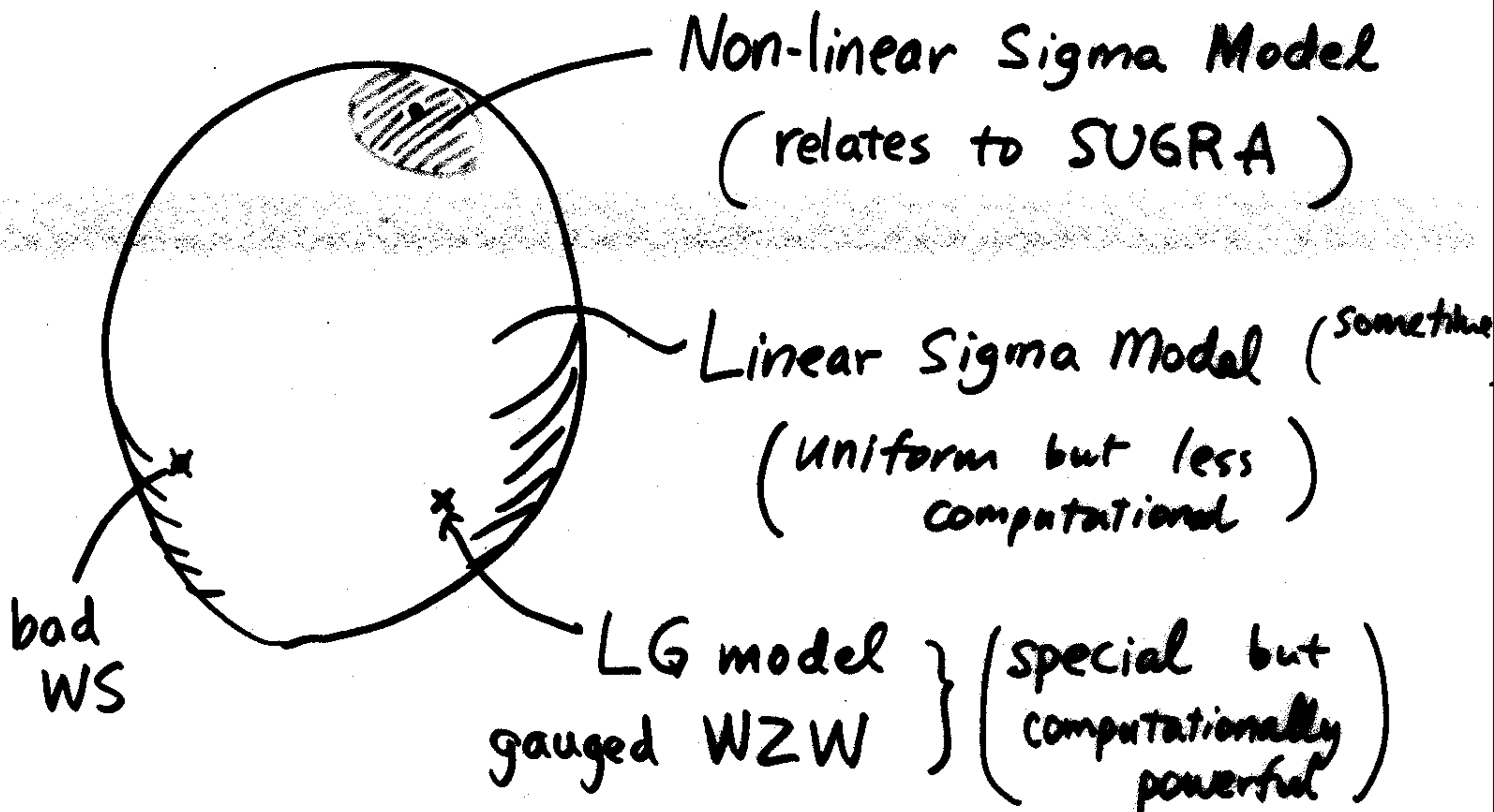
② Potential

We want to understand ①, in particular

> What kinds of orientifolds are possible in which kinds of models ?

We would like to answer to this, first from the <u>worldsheet point</u> of view. (where "orientifold" is originated)

∃ Various descriptions



We want to study orientifolds in each of these descriptions & patch them together to find a global picture

Today's talk is a report of a progress in this direction.

§ Basics ; Non-linear σ-model (NLσM)

§ $\mathcal{N}=2$ minimal model

§ Linear σ-model (LσM)

§ Compact CY

Orientifold is to gauge a Parity symmetry

$$P : X(t,\sigma) \to \tau X(t,-\sigma)$$

of the worldsheet theory.

Relevant for 4d $\mathcal{N}=1$ compactification :

P preserving a half of (2,2) (WS) SUSY ($Q_\pm$,

Two kinds of "a half" (same as for D-branes

A-type $P_A : Q_\pm \to \overline{Q}_\mp$

$$\left.\begin{aligned} Q_A &= \overline{Q}_+ + Q_- \\ Q_A^\dagger &= Q_+ + \overline{Q}_- \end{aligned}\right\} \text{ invariant.}$$

B-type $P_B : Q_\pm \to Q_\mp$

$$\left.\begin{aligned} Q_B &= \overline{Q}_+ + \overline{Q}_- \\ Q_B^\dagger &= Q_+ + Q_- \end{aligned}\right\} \text{ invariant.}$$

superspace description

$$\Omega_A : (t, \sigma, \theta^\pm, \overline{\theta}^\pm) \mapsto (t, -\sigma, -\overline{\theta}^\mp, -\theta^\mp)$$

$$\Omega_B : (t, \sigma, \theta^\pm, \overline{\theta}^\pm) \mapsto (t, -\sigma, \theta^\mp, \overline{\theta}^\mp)$$

$\Phi(x,\theta)$ chiral $(\bar{D}_\pm \Phi = 0)$ $\Rightarrow$ $\Phi(\Omega_A(x,\theta))$ anti-chiral

$\Phi(\Omega_B(x,\theta))$ chiral

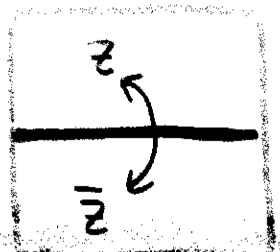
$$P_A : \Phi(x,\theta) \to \overline{\Phi(\Omega_A(x,\theta))}$$

$$P_B : \Phi(x,\theta) \to \Phi(\Omega_B(x,\theta))$$

$\rightsquigarrow$ A-parity : anti-holomorphic

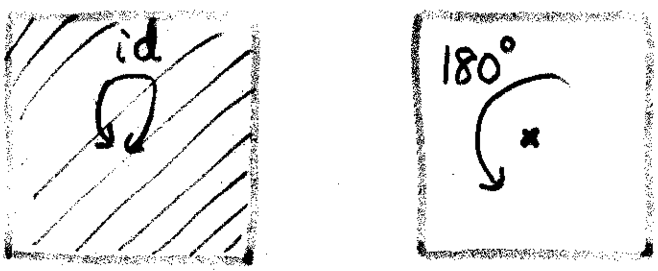
B-parity : holomorphic

e.g. $X = \mathbb{C}$

antiholo: 

$dz\,d\bar{z} \to d\bar{z}\,dz = -dz\,d\bar{z}$

antisymplectic

holo: 

$$dz\,d\bar{z} \to (\pm 1)^2 dz\,d\bar{z} = dz\,d\bar{z}$$

symplectic

SUSY NLσM on a Kähler mfd X (ω : Kähler form)

$$\tau\Omega : \phi^M(t,\sigma) \to \tau^M(\phi(t,-\sigma))$$

is an **A**-parity if $\tau : X \to X$ antiholo. isometry

$$(\Rightarrow \tau^*\omega = -\omega)$$

a **B**-parity if $\tau : X \to X$ holo. isometry

$$(\Rightarrow \tau^*\omega = \omega)$$

Orientifold plane (O-plane):

$$X^\tau = \{ x \in X \mid \tau(x) = x \}$$

τ : antiholo. (A) $\Rightarrow$ X^τ is a Lagrangian submfd.

(c.f Lagrangian submfds are A-branes)

τ : holo. (B) $\Rightarrow$ X^τ is a complex subspace

(c.f cplx subspaces are B-branes)

Parity anomaly

For a map of the worldsheet $\phi : \Sigma \to X$

$$\#\ \psi_- \text{ zero modes} = \#\ \overline{\psi}_+ \text{ zero modes} = \int_\Sigma \phi^* c_1(X) =: k$$

Fermion path-integral measure

$$\mathcal{D}_\phi \Psi = (d\psi_-^{(0)} d\overline{\psi}_+^{(0)})^k \, \mathcal{D}_\phi \Psi^{\text{non-zero}}$$

$$A : \psi_- \leftrightarrow \overline{\psi}_+ \rightsquigarrow \mathcal{D}_\phi \Psi \longrightarrow \underset{\sim\sim}{(-1)^k} \mathcal{D}_\phi \Psi \quad \text{Parity anom}$$

※ absent if $c_1(X)$ even (X : spin)

B : anomaly free

B-field $B \in H^2(X, \mathbb{R})$

$$\tau\Omega : \int_\Sigma \phi^* B \to -\int_\Sigma \phi^* \tau^* B$$

Invariance of $e^{i\int_\Sigma \phi^* B} \Leftrightarrow \tau^*[B] = -[B] \mod H^2(X, 2\pi\mathbb{Z}$

A-parity anomaly can be canceled by $\triangle \int \phi^* B$

if $\tau^*[B] = -[B] + \underset{\sim\sim}{\pi c_1(X)} \mod H^2(X, 2\pi\mathbb{Z}$

Recall A-parity symmetry $\Rightarrow$ $\tau^*\omega = -\omega$

B-parity symmetry $\Rightarrow$ $\tau^*\omega = \omega$

The condition is thus

$$A:\ \tau^*([\omega]-i[B]) = -[\omega]+i[B]+\pi i c_1(X) \mod H^2(X, 2\pi i\mathbb{Z})$$

$$B:\ \tau^*([\omega]-i[B]) = [\omega]+i[B] \mod H^2(X, 2\pi i\mathbb{Z})$$

Introduce the <u>complexified Kähler class</u> moduli t^a

$$[\omega]-i[B] = \sum_a t^a\omega_a \qquad \omega_a : \text{basis of } H^2(X,\mathbb{Z})$$

If $\tau^*\omega_a = \sum_b M_a{}^b\omega_b$,

the symmetry condition is

$$A:\ \underbrace{\sum_b t^b M_b{}^a = -t^a + \pi i c_1(X)^a}_{\text{holomorphic constraint}} \mod 2\pi i\mathbb{Z}$$

$$B:\ \underbrace{\sum_b t^b M_b{}^a = \overline{t^a}}_{\text{antiholomorphic constraint}} \mod 2\pi i\mathbb{Z}$$

t^a : <u>twisted chiral</u> parameter.

$\tilde{\Phi}(x,\theta)$ twisted chiral $\overline{D_+}\tilde{\Phi} = D_-\tilde{\Phi} = 0$

$\Rightarrow$ $\tilde{\Phi}(\Omega_A(x,\theta))$ twisted chiral

$\tilde{\Phi}(\Omega_B(x,\theta))$ twisted <u>anti</u>chiral

$\therefore$ P_A : $\tilde{\Phi}(x,\theta) \to \tilde{\Phi}(\Omega_A(x,\theta))$ **holomorphic**

P_B : $\tilde{\Phi}(x,\theta) \to \overline{\tilde{\Phi}(\Omega_B(x,\theta))}$ **<u>anti</u>holomorphic**

<u>Reduction of moduli</u>

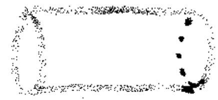
A-parity $\begin{cases} \text{Kähler} : & \tau\text{- dependent (holomorphic)} \\ \text{Complex} : & \text{by } \frac{1}{2} \text{ (real)} \end{cases}$

B-parity $\begin{cases} \text{Kähler} : & \text{by } \frac{1}{2} \text{ (real)} \\ \text{Complex} : & \tau\text{- dependent (holomorphic)} \end{cases}$

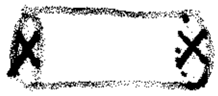
Witten indices

P: parity, a, b: branes (preserving <u>same</u> SUSY)

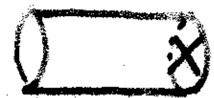
$$I(a,b) = \mathrm{Tr}_{\mathcal{H}_{a,b}} (-1)^F$$



$$I_P = \mathrm{Tr}_{\mathcal{H}_{closed}} P(-1)^F$$



$$I_P(a, P(a)) = \mathrm{Tr}_{\mathcal{H}_{a,P(a)}} P(-1)^F$$



<u>A-type</u> $\tau : X \to X$ antiholo, L_a, L_b : Lagrangian subsp. of X

$$I(L_a, L_b) = \#(L_a \cap L_b)$$

$$I_{\tau\Omega} = \#(X^\tau \cap X^\tau) \qquad \text{intersection } \#\text{'s}$$

$$I_{\tau\Omega}(L, \tau L) = \#(L \cap X^\tau)$$

<u>B-type</u> $\tau : X \to X$ holo, E_a, E_b : holo bundles over X

$$I(E_a, E_b) = \int_X ch(\bar{E}_a \otimes E_b)\, td(X) = \sum_{p=1}^{\hat{n}} (-1)^p \dim H^{0,p}(X, \bar{E}_a \otimes E_b)$$

$$= \chi(E_a, E_b) \qquad \text{Euler characteristic}$$

$$I_{\tau\Omega} = \int_{X^\tau} \frac{L(TX^\tau)}{L(NX^\tau)} e(NX^\tau) = \sum_{p+q=n} tr_{H^{p,q}(X)} (\tau^* \otimes *)$$

$$= \mathrm{Sign}(\tau, X) \qquad \text{Hirzebruch signature}$$

$$I_{\tau\Omega}(E, \tau^* \bar{E}) = \int_{X^\tau} ch(2\bar{E}) \frac{td(X^\tau)}{ch(\Lambda \bar{N}_{X^\tau})} = \sum_{p=1}^{n} (-1)^p \, tr_{H^{0,p}(X, \bar{E} \otimes \tau^* \bar{E})}(\tau)$$

$$= L(\tau, \mathcal{E}^\vee \otimes \tau^* \mathcal{E}^\vee) \qquad \text{holomorphic Lefshetz } \#$$

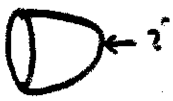
RR charges

$|i\rangle$ RR ground states $\Leftrightarrow$ $\omega_i \in H^{\cdot}(X)$

$\Pi_i^a = \langle B_a | i \rangle$  i disc 1-pt

$\Pi_i^p = \langle C_p | i \rangle$  i RP^2 1-pt

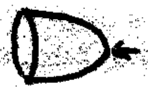
A-type independent of Kähler → exact computation
depends on complex str. $\partial_i \Pi_j \sim C_{ij}^k \Pi_k$

L  i $= \int_L \omega_i$

$\tau\Omega$  i $= \int_{X^\tau} \omega_i$

Period integrals

B-type indep of cplx str
depends on Kähler $\partial_i \Pi_j \sim \tilde{C}_{ij}^k \Pi_k$ corrected

E  i $= \int_X e^{B+i\omega}\, ch(E) \sqrt{\mathrm{td} X}\, \omega_i + \cdots$

$\tau\Omega$  i $= \int_{X^\tau} e^{i\omega} \sqrt{\frac{L(\frac{1}{4} TX^\tau)}{L(\frac{1}{4} NX^\tau)}}\, \omega_i + \cdots$

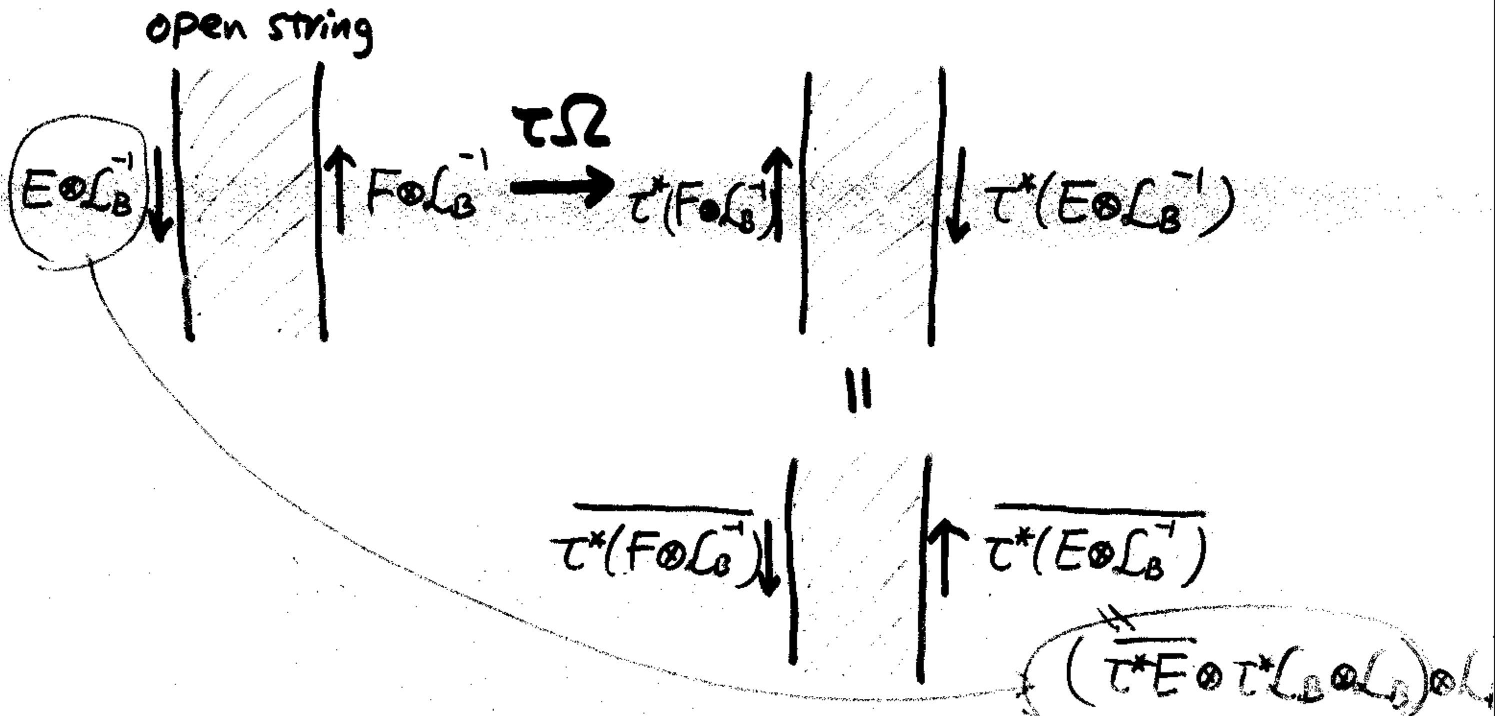
Factorization  $=$  etc

$\Leftrightarrow$ Riemann's bilinear identity

Modification by B-field

A B-field modifies the Chan-Paton factor $E \to E \otimes \mathcal{L}_B^{-1}$

"$c_1(\mathcal{L}_B) = B/2\pi$".



$$\tau\Omega : E \mapsto \tau^*\bar{E} \otimes \mathcal{L}_{\tau^*B+B}$$

one can show $c_1(\mathcal{L}_{\tau^*B+B}) = \frac{\tau^*B+B}{2\pi} \in H^2(X,\mathbb{Z})$

$\therefore$ $\mathcal{L}_{\tau^*B+B}$ is a good line b'dle.

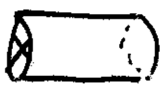
$$\boxed{\text{To specify the parity action of open string, one needs to specify the parity action on } \mathcal{L}_{\tau^*B+B} \text{ over } X^\tau}$$

$$\left.\begin{array}{l}\tau^2 = 1\\ \mathrm{rank}\, \mathcal{L}_{\tau^*B+B} = 1\end{array}\right\} \Rightarrow \tau = \pm 1 \ (=: \varepsilon_B(i))$$

at each component X_i^τ

$$A: \quad I^B_{\tau\Omega}(L, \tau\Omega) = \#\left(L \cap \sum_i \varepsilon_B(i) X_i^\tau\right)$$

$$B: \quad I^B_{\tau\Omega}(E, \tau E) = \int_{X^\tau} \mathrm{ch}(\tau \bar{E})\, \varepsilon_B\, e^{B/\pi} \frac{\mathrm{td}(X^\tau)}{\mathrm{ch}(\Lambda \bar{N}_{X^\tau})}$$

By factorization  =  . we find

$\mathbb{R}P^2$ diagram

 $\leftarrow \phi_i$

$$= \int_{X^\tau} \varepsilon_B\, \omega_i \qquad (A)$$

$$= \int_{X^\tau} \varepsilon_B\, e^{i\omega} \sqrt{\frac{L(\frac{1}{4} TX^\tau)}{L(\frac{1}{4} NX^\tau)}}\, \omega_i + \cdots \qquad (B)$$

The sign function $\varepsilon_B : X^\tau \to \{\pm 1\}$ determines

the type of O-planes (SO or Sp)

§ Orientifold of Minimal Model

$\mathcal{N}=2$ minimal model ···· the building block of Gepner mod

Various descriptions

- SUSY $SU(2)_k$ mod $U(1)$ gauged WZW
- RCFT based on S, T modular matrices
- IR limit of Landau-Ginzburg $W = \Phi^{k+2}$

$SU(2)_k/U(1)$ description

gauge $\quad g \to h g h^{-1} \quad h = \begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{-i\theta} \end{pmatrix}$

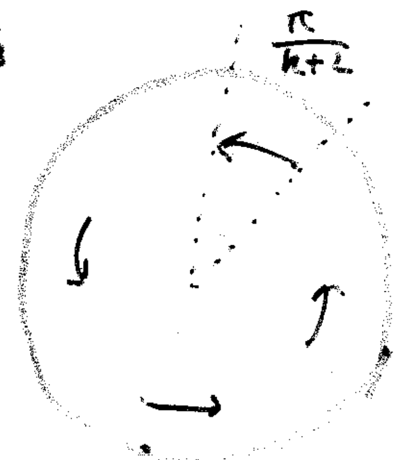
$$g = \begin{pmatrix} z & w \\ -\bar{w} & \bar{z} \end{pmatrix} \quad |z|^2 + |w|^2 = 1 \xrightarrow{/U(1)} z \quad |z|^2 \leq 1$$

disc.

$\mathbb{Z}_{2(k+2)}$ symmetry

$$a : g \to e^{-\frac{\pi i}{2(k+2)}\sigma_3} \, g \, e^{-\frac{\pi i}{2(k+2)}\sigma_3}$$

$$z \to e^{\frac{\pi i}{k+2}} z$$



$$a Q_\pm a^{-1} = \pm Q_\pm$$

$$[a^2, Q_\pm] = 0$$

Parity $\quad I\Omega$

$I_A: \; g \to g^{-1} \qquad z \to \bar{z}$

$I_B: \; g \to \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix} g^{-1} \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \qquad z \to z$

- $\underline{P_A = I_A \Omega}$ is an exact symmetry
- $I_B \Omega$ has an anomaly (as in NLσM)

  ... cancelled by $(-1)^{F_R}$

  $\underline{P_B = (-1)^{F_R} I_B \Omega}$ is a symmetry

One may consider dressing by discrete symmetry:

$\underline{\text{A-parities are } \; a^{2m} P_A} \qquad m = 1, 2, \cdots, k+2$

$\underline{\text{B-parities are } \; a^{2m+1} P_B} \qquad m = 1, 2, \cdots, k+2$

All A-parities are <u>involutive</u> $\left(a^{2m} P_A\right)^2 = 1$

Not so for B: $\left(a^{2m+1} P_B\right)^2 = a^{2(2m+1)}$

involutive only if $\underline{2m+1 = k+2}$ (possible only if k odd)

LG description

$$\int d^2\theta\, W(\Phi) + \int \overline{d^2\theta}\; \overline{W(\Phi)}$$

A-parity: $d^2\theta = d\theta^- d\theta^+ \longrightarrow d\overline{\theta}^+ d\overline{\theta}^- = \overline{d^2\theta}$

$\Phi(x,\theta) \longrightarrow f\, \overline{\Phi(\Omega_A(x,\theta))}$ is an A-parity iff

$$\boxed{W(f\overline{\Phi}) = \overline{W(\Phi)}}$$

B-parity: $d^2\theta = d\theta^- d\theta^+ \longrightarrow d\theta^+ d\theta^- = \underset{\sim}{-}\, d^2\theta$

$\Phi(x,\theta) \rightarrow f\,\Phi(\Omega_B(x,\theta))$ is a B-parity iff

$$\boxed{W(f\Phi) = -W(\Phi)}$$

The case $W = \Phi^{k+2}$:

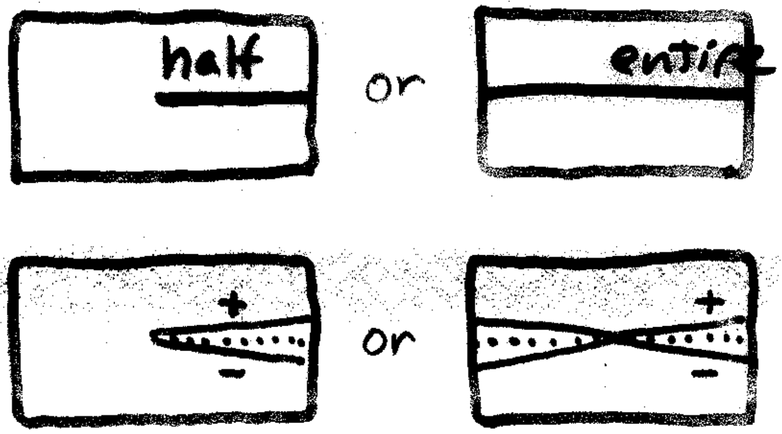
A-parity $\quad \Phi(x,\theta) \rightarrow e^{\frac{2\pi i}{k+2} m}\, \overline{\Phi(\Omega_A(x,\theta))} \quad \Leftrightarrow \quad a^{2m} P_A$

B-parity $\quad \Phi(x,\theta) \rightarrow e^{\frac{\pi i (2m+1)}{k+2}}\, \Phi(\Omega_B(x,\theta)) \quad \Leftrightarrow \quad a^{2m+1} P_B$

c.f. $\Phi(x,\theta) \rightarrow \Phi(\Omega_B(x,\theta))$ is <u>NOT a symmetry</u> $\Leftrightarrow$ $I_B \Omega$ <u>anoma</u>

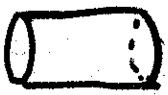
Witten indices & RR-charges (A-type)

D-brane, O-plane $\Big\}\ \gamma \xrightarrow{W} W(\gamma)$ horizontal line



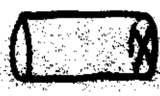
small deformation $\big\downarrow$

$\gamma^{\pm} \xrightarrow{W}$

L_a  $L_b \;=\; \#(L_a^- \cap L_b^+)$

$O_\tau = X^\tau$ O-plane

$\tau\Omega$  $\tau\Omega \;=\; \#(O_\tau^- \cap O_\tau^+)$

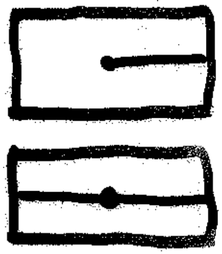
L  $\tau\Omega \;=\; \#(L^- \cap O_\tau^+)$

L  $\leftarrow i \;=\; \int_{L^-} e^{-iW} \phi_i \Omega$

$\tau\Omega$  $\leftarrow i \;=\; \int_{O_\tau^-} e^{-iW} \phi_i \Omega$

$W = \Phi^{k+2}$. $P_A : \Phi(x,\theta) \to \overline{\Phi(\Omega_A(x,\theta))}$

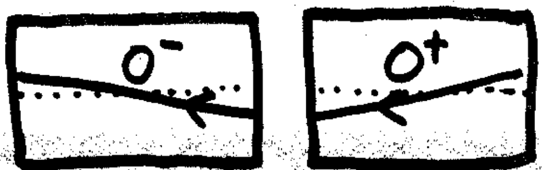
O-plane $\mathcal{O} = \{\phi = \overline{\phi}\} \xrightarrow{W} W(\mathcal{O}) =$



k even

k odd

k : even

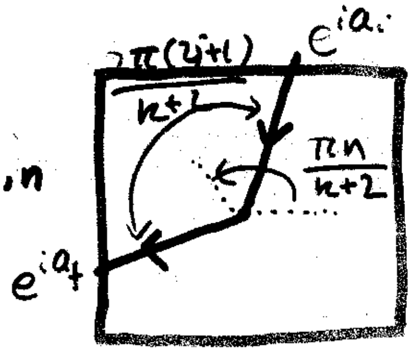


$I_{P_A} = \#(O^- \cap O^+) = 1$

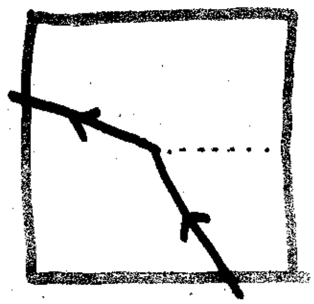
k : odd

$I_{P_A} = \#(O^- \cap O^+) = 0$

brane $L_{j,n}$



$\xrightarrow{P_A}$



$$I_{P_A}(L_{j,n}, P_A L_{j,n}) = \begin{cases} 1 & 0 < a_f \le \pi < a_i \le 2\pi \\ -1 & 0 < a_i \le \pi < a_f \le 2\pi \\ 0 & \text{others} \end{cases}$$

$$= \tilde{N}_{jj}^{\,n-1}$$

SU(2) fusion coefficient
with suitable extension

P_A  j

$$= c_j \int_{O^-} e^{i\phi^{k+2}} \phi^{2j} d\phi = \begin{cases} i e^{-\frac{\pi i(2j+1)}{k+2}} \dfrac{1 + (-1)^{2j}}{\sqrt{2(k+2)}\sin\left[\frac{\pi(2j+1)}{k+2}\right]} & k \text{ even} \\[2ex] i e^{-\frac{\pi i(2j+1)}{k+2}} \dfrac{e^{-\frac{\pi i(2j+1)}{2(k+2)}} + (-1)^{2j} e^{\frac{\pi i(2j+1)}{2(k+2)}}}{\sqrt{2(k+2)}\sin\left[\frac{\pi(2j+1)}{k+2}\right]} & k \text{ odd} \end{cases}$$

These results agrees (of course) with the one from RCFT description (<u>after resolving GSO</u>)

a  $b = \sum_c N_{\bar{a}b}^{\bar{c}} \chi_c$

$g\Omega$  $h\Omega = \sum_c Y_{ch}^{g} \chi_c$

a  $g\Omega = \sum_c Y_{\bar{a}g}^{\bar{c}} \hat{\chi}_c$

$$N_{ab}^{c} = \sum_d \frac{S_{ad} S_{bd} S_{cd}^*}{S_{0d}}$$

$$Y_{ab}^{c} = \sum_d \frac{S_{ad} P_{bd} P_{cd}^*}{S_{0d}} \qquad P = \sqrt{T} S T^2 S \sqrt{T}$$

a  $i = \frac{S_{ai}}{\sqrt{S_{0i}}}$

$g\Omega$  $i = \frac{P_{gi}}{\sqrt{S_{0i}}}$

§ Orientifolds of Linear σ-Model

LσM — Cover the whole moduli space (except conifolds) including NLσM & L.G. orbifold points

— used in the derivation of Mirror Symmetry

$U(1)^k$ gauge theory, gauge field $V_{a=1,\cdots,k}$

$\Phi_1,\ldots,\Phi_N$ chiral matter, charge $Q_1^a,\ldots,Q_N^a$

$$L = \int d^4\theta \left[\sum_{i=1}^{N} \overline{\Phi}_i e^{Q_i V} \Phi_i - \sum_{a=1}^{k} \frac{1}{e_a^2} |\Sigma_a|^2 \right]$$

$$+ \mathrm{Re} \int d^2\tilde{\theta} \sum_{a=1}^{k} (-t^a \Sigma_a) \qquad \Sigma_a = \overline{D}_+ D_- V_a \quad \text{field strength}$$

$$+ \mathrm{Re} \int d^2\theta \, W(\Phi_1,\ldots,\Phi_N) \qquad t^a = \underset{\substack{\uparrow \\ \text{FI}}}{r^a} - i \underset{\substack{\uparrow \\ \text{Theta}}}{\theta^a}$$

At $E \ll e_a$: NLσM on

$$X = \left\{ (\phi_1,\cdots,\phi_N) \,\middle|\, \begin{array}{c} \sum_{i=1}^{N} Q_i^a |\phi_i|^2 = r^a \\ \partial_i W = 0 \end{array} \right\} \Big/ U(1)^k$$

$$\text{B-field} = \sum_a \theta^a \omega_a \qquad \{\omega_a\} \text{ basis of } H^2(X:\mathbb{Z})$$

A-parity

$$\begin{cases} \Phi_i \to \overline{\Phi_i \circ \Omega_A} \\ V_a \to V_a \circ \Omega_A \end{cases}$$

Permutation $i \mapsto \sigma(i)$: compatible with gauge sym. iff $\underline{Q^a_{\sigma(i)} = \sum_b Q_i^b \sigma_b^a}$

$$\left.\begin{array}{l} \Phi_i \to \overline{\Phi_{\sigma(i)} \circ \Omega_A} \\ V_a \to \sigma_a^{\ b} V_b \circ \Omega_A \end{array}\right\}$$ This is a symmetry under

the condition • $W(\overline{\Phi_{\sigma(i)}}) = \overline{W(\Phi_i)}$

• $t^b \sigma_b^{\ a} = \overline{t^a} + \pi i b_1^a \mod 2\pi i \mathbb{Z}$

$\quad \sum_{i=1}^N Q_i^a \leftrightarrow c_1(X)$

B-parity

$$\left.\begin{array}{l} \Phi_i \to e^{i\theta_i} \Phi_{\sigma(i)} \circ \Omega_B \\ V_a \to \sigma_a^{\ b} V_b \circ \Omega_B \end{array}\right\}$$

symmetry iff • $W(e^{i\theta_i} \Phi_{\sigma(i)}) = -W(\Phi_i)$

• $t^b \sigma_b^{\ a} = \overline{t^a} \mod 2\pi i \mathbb{Z}$

Mirror Description (for $W(\Phi_1, \cdots, \Phi_N) = 0$)

$$\arg(\Phi_i) \xleftrightarrow{T} Y_i \equiv Y_i + 2\pi i \quad \text{(twisted chiral)}$$

$$\widetilde{W} = \sum_{a=1}^{k} \left(\sum_{i=1}^{N} Q_i^a Y_i - t^a \right) \Sigma_a + \sum_{i=1}^{N} e^{-Y_i}$$

Integrat - Out $\Sigma_a \Rightarrow$

constraint $\displaystyle\sum_{i=1}^{N} Q_i^a Y_i = t^a \mod 2\pi i \mathbb{Z}$ $\quad (\to (\mathbb{C}^\times)^{N-k})$

superpotential $\widetilde{W} = e^{-Y_1} + \cdots + e^{-Y_N}$

A-parity $\left.\begin{array}{l} \Phi_i \to \overline{\Phi_{\sigma(i)} \circ \Omega_A} \\ V_a \to \sigma_a{}^b V_b \circ \Omega_A \end{array}\right\} \longleftrightarrow Y_i \to Y_{\sigma(i)} \circ \Omega_A + \pi i$

$t^b \sigma_b{}^a \equiv t^a + \pi i b_i^a \iff$ compatibility with the constraint on Y_i

(parity anomaly) $\longleftrightarrow$ (classical non-invariance)

B-parity $\left.\begin{array}{l} \Phi_i \to \Phi_{\sigma(i)} \circ \Omega_B \\ V_a \to \sigma_a{}^b V_b \circ \Omega_B \end{array}\right\} \longleftrightarrow Y_i \to \overline{Y_{\sigma(i)} \circ \Omega_B}$

$$\tau_{r_1\cdots r_N} : \Phi_i \to \underbrace{(-1)^{r_i}}\Phi_i$$

$$\begin{cases} \text{action on } X \xrightarrow{\tau_r} X \\ \text{action on } \mathcal{L}_{\tau_r^* B + B} \end{cases}$$

mirror

$$\tilde{\tau}_{r_1\cdots r_N} : Y_i \to \overline{Y_i}$$

fixed point locus $Y_i = \overline{Y_i}$ mod $2\pi i$

$$L_{p_1\cdots p_N} = \{ Y_i \in \mathbb{R} + \pi\sqrt{-1}\,p_i \} \quad \left(\sum_{i=1}^{N} Q_i^a p_i = -\frac{\theta^a}{\pi} \right) \quad p_i \in \mathbb{Z}$$

orientation related by translation

$$Y^{\tilde{\tau}_r} = \sum_p \underbrace{(-1)^{p_1 r_1 + \cdots + p_N r_N}}_{\text{type of O-plane}} L_{p_1\cdots p_N}$$

eg. $X = \mathbb{CP}^1$

$$\Phi_1, \Phi_2 \longleftrightarrow Y_1, Y_2 \ ; \ Y_1 + Y_2 = t, \ W = e^{-Y_1} + e^{-Y_2}$$

$$\text{or } W = e^{-Y} + e^{-t+Y} \qquad Y \equiv Y + 2\pi i$$

$$(\Phi_1, \Phi_2) \xrightarrow{\tau_r} ((-1)^{r_1}\Phi_1, (-1)^{r_2}\Phi_2) \xleftrightarrow{\text{mirror}} Y \to \overline{Y}$$

fixed pt locus

$$L_0 = \{ Y \in \mathbb{R} \}$$

$$L_1 = \{ Y \in \mathbb{R} + \pi i \}$$

$\theta = 0$

$\tau = \tau_{00} = id$: $X^{\tau} = \mathbb{C}P^1$ itself

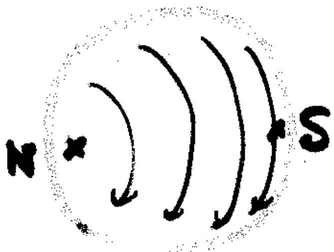
$$\Omega : \mathcal{O}(n) \to \overline{\mathcal{O}(n)} = \mathcal{O}(-n)$$

$$I_{\Omega} = \int_{\mathbb{C}P^1} L(\mathbb{C}P^1) = 0$$

$$I_{\Omega}(\mathcal{O}(n), \mathcal{O}(-n)) = \int_{\mathbb{C}P^1} ch(2\overline{\mathcal{O}(n)}) \frac{td(\mathbb{C}P^1)}{ch(\Lambda \overline{N}_{\mathbb{C}P^1})}$$

$$= \int_{\mathbb{C}P^1} e^{-2nH}(1+H) = 1 - 2n$$

$\tau = \tau_{10}$:



$X^{\tau_{10}} = \{N, S\}$

$$\tau_{10}\Omega : \mathcal{O}(n) \to \mathcal{O}(-n)$$

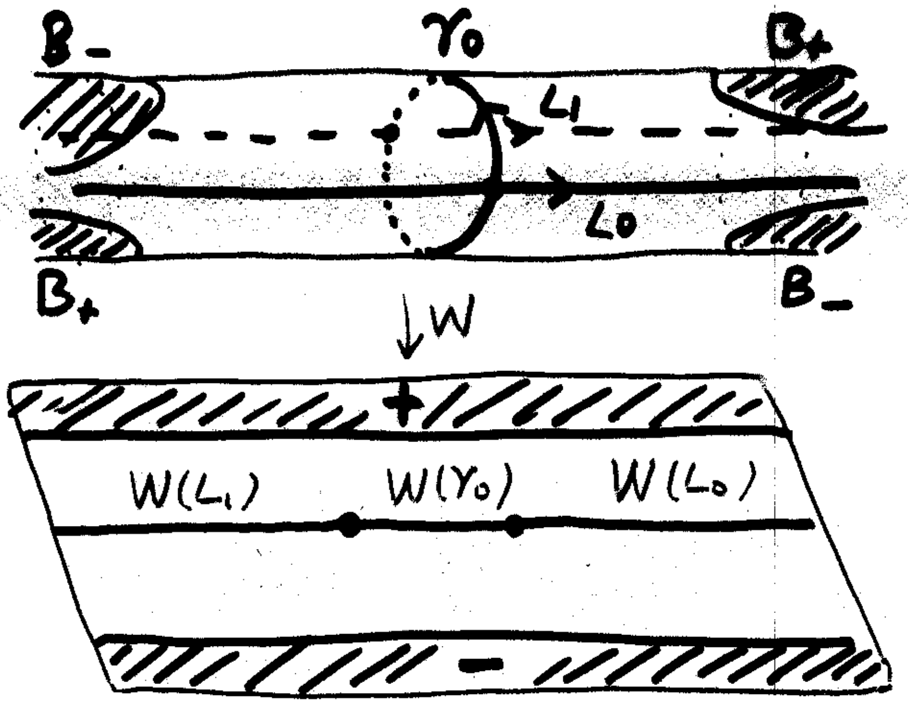
$$I_{\tau_{10}\Omega} = 0$$

$$I_{\tau_{10}\Omega}(\mathcal{O}(n), \mathcal{O}(-n)) = \sum_{p=N,S} \int_p 1 \cdot \frac{1}{2} = \frac{1}{2} + \frac{1}{2} = 1$$

mirror side crit pt : $Y = \frac{r}{2}, \frac{r}{2}+\pi i = \frac{r}{2}, \frac{r}{2}+\pi i$

$$Y^{\widetilde{id}} = L_0 + L_1 \qquad \gamma_{\mathcal{O}(n)} = L_0 + n\gamma_0$$

$$Y^{\widetilde{\tau_{10}}} = L_0 - L_1$$



$\#(L_0^- \cap L_0^+) = 1$

$\#(L_1^- \cap L_1^+) = -1$

$\#(L_i^- \cap L_j^+) = 0 \quad i \neq$

$\#(\gamma_0^- \cap L_j^+) = -1, \ j = 0.$

$$I_{\tilde{\Omega}} = \#((Y^{\widetilde{id}})^- \cap (Y^{\widetilde{id}})^+) = 1 - 1 = 0$$

$$I_{\tilde{\Omega}}(\gamma_{\mathcal{O}(n)}, \gamma_{\mathcal{O}(-n)}) = \#(\gamma_{\mathcal{O}(n)}^- \cap (Y^{\widetilde{id}})^+) = 1 + n(-1) + n(-1) = 1 -$$

$$I_{\widetilde{\tau_{10}}\Omega} = \#((Y^{\widetilde{\tau_{10}}})^- \cap (Y^{\widetilde{\tau_{10}}})^+) = 1 - 1 = 0$$

$$I_{\widetilde{\tau_{10}}\Omega}(\gamma_{\mathcal{O}(n)}, \gamma_{\mathcal{O}(-n)}) = \#(\gamma_{\mathcal{O}(n)}^- \cap (Y^{\widetilde{\tau_{10}}})^+) = 1 + n(-1) - n(-1) = 1.$$

$\theta = \pi$

$$\mathcal{L}_{\tau^* B + B} = \mathcal{L}_{2B} = \mathcal{O}(1)$$

$$\tau\Omega : \mathcal{O}(n) \to \overline{\mathcal{O}(n)} \otimes \mathcal{O}(1) = \mathcal{O}(1-n)$$

$\tau = \tau_{00}$: $X^{id} = \mathbb{C}P^1$ itself $\quad \varepsilon_B^{id} = 1$

$$I_\Omega = 0$$

$$I_\Omega(\mathcal{O}(n), \mathcal{O}(-n)) = \int_{\mathbb{C}P^1} ch(2\overline{\mathcal{O}(n)})\, \varepsilon_B^{id}\, e^{B/\pi} \frac{td(\mathbb{C}P^1)}{ch(\Lambda \overline{N}_{\mathbb{C}P^1})}$$

$$= \int_{\mathbb{C}P^1} e^{-2nH} \cdot 1 \cdot e^{H} \cdot (1+H) = 2 - 2n$$

$\tau = \tau_{10}$: $X^{\tau_{10}} = \{N, S\}$

$(\Phi_1, \Phi_2) \to (-\Phi_1, \Phi_2)$

$$\varepsilon_B^{\tau_{10}} = \begin{cases} 1 & \text{at } N \\ -1 & \text{at } S \end{cases}$$

$$I_{\tau_{10}\Omega} = 0$$

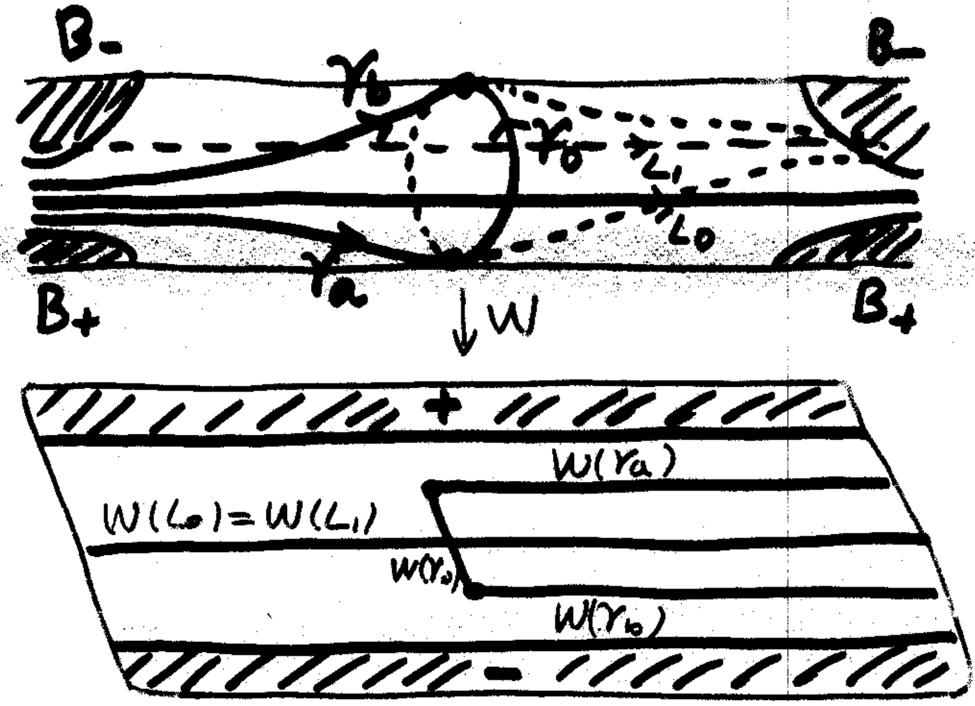
$$I_{\tau_{10}\Omega}(\mathcal{O}(n), \mathcal{O}(1-n)) = \sum_{p=N,S} \int_p 1 \cdot \varepsilon_B^{\tau_{10}} \cdot 1 \cdot \frac{1}{2} = \frac{1}{2} - \frac{1}{2} = 0$$

Mirror Side: Crit pt : $Y = \frac{t}{2}, \frac{t}{2} + \pi i = \frac{r}{2} \pm \frac{\pi i}{2}$

$$Y^{\widetilde{id}} = L_0 + L_1$$

$$Y^{\widetilde{\tau_{10}}} = L_0 - L_1$$

$$\gamma_{\mathcal{O}(n)} = \gamma_a + n\gamma_0$$



$\#(L_i^- \cap L_j^+) = 0 \quad \forall i, \forall j$

$\#(\gamma_a^- \cap L_i^+) = 1 \quad i = 0, 1$

$\#(\gamma_0^- \cap L_i^+) = -1 \quad i = 0, 1$

$$\gamma_{\mathcal{O}(n)} = \gamma_a + n\gamma_0 \to \gamma_b - n\gamma_0 \underset{\uparrow}{=} \gamma_a + (1-n)\gamma_0 = \gamma_{\mathcal{O}(1-n)}$$

$$\gamma_b = \gamma_a + \gamma_0$$

$$I_{\tilde{\Omega}} = \#((Y^{\widetilde{id}})^- \cap (Y^{\widetilde{id}})^+) = 0$$

$$I_{\tilde{\Omega}}(\gamma_{\mathcal{O}(n)}, \gamma_{\mathcal{O}(1-n)}) = \#(\gamma_{\mathcal{O}(n)}^- \cap (Y^{\widetilde{id}})^+) = 1 + 1 + n(-1) + n(-1) = 2 - 2n$$

$$I_{\widetilde{\tau_{10}}\Omega} = \#((Y^{\widetilde{\tau_{10}}})^- \cap (Y^{\widetilde{\tau_{10}}})^+) = 0$$

$$I_{\widetilde{\tau_{10}}\Omega}(\gamma_{\mathcal{O}(n)}, \gamma_{\mathcal{O}(1-n)}) = \#(\gamma_{\mathcal{O}(n)}^- \cap (Y^{\widetilde{\tau_{10}}})^+) = 1 - 1 + n(-1) - n(-1) = 0.$$

Compact Calabi-Yau

$X =$ Quintic $\quad z_1^5 + \cdots + z_5^5 = 0$

Kähler moduli $\quad 1_{\mathbb{C}}$ $\qquad$ # pdy $\quad GL(5,\mathbb{C})$

mirror $\quad$ Complex str. moduli $\quad 101_{\mathbb{C}}$ $\qquad (126-25)$

$\tilde{X} =$ Resolution of the orbifold of

$$\tilde{z}_1^5 + \cdots + \tilde{z}_5^5 + e^{t/5}\,\tilde{z}_1 \cdots \tilde{z}_5 = 0$$

by $\mathbb{Z}_5^3 : \tilde{z}_i \to \omega_i \tilde{z}_i \quad \omega_i^5 = \omega_1 \cdots \omega_5 = 1$

Kähler moduli $\quad 101_{\mathbb{C}}$ $\qquad \left(1 + \binom{5}{3}\times 6 + \binom{5}{2}\times 4\right)$

Cplx str. moduli $\quad 1_{\mathbb{C}}$ $\qquad \mathbb{C}^3/\mathbb{Z}_5\times\mathbb{Z}_5$ blow ups $\qquad \mathbb{C}^2/\mathbb{Z}_5$ blow ups

Greene-Plesser

HV : $\arg z_i \leftrightarrow Y_i$, $\quad \tilde{z}_i^5 \propto e^{-Y_i}$

$$z_i \to \overline{z_{\sigma(i)}} \quad \overset{\text{mirror}}{\longleftrightarrow} \quad \tilde{z}_i \to \tilde{z}_{\sigma(i)}$$

$$z_i \to z_{\sigma(i)} \quad \longleftrightarrow \quad \tilde{z}_i \to \overline{\tilde{z}_{\sigma(i)}}$$

Another derivation : at Gepner point

$$W = X^N \xleftrightarrow{\text{mirror}} \tilde{W} = \tilde{X}^N / \mathbb{Z}_N$$

$\tilde{X} \to \omega \tilde{X}$, ω^N

$$X \to \overline{X \circ \Omega_A} \qquad \tilde{X} \to e^{\frac{\pi i}{N}} \tilde{X} \circ \Omega_B$$

involutiv

Gepner Model

$X_i \to \omega X_i$, $\omega^N = 1$

$$\| $$

$$(X_1^N + \cdots + X_N^N)/\mathbb{Z}_N \xleftrightarrow{\text{mirror}} (\tilde{X}_1^N/\mathbb{Z}_N \times \cdots \times \tilde{X}_N^N/\mathbb{Z}_N)/\mathbb{Z}_N$$

$$X_i \to \overline{X_i \circ \Omega_A} \qquad \tilde{X}_i \to e^{\frac{\pi i}{N}} \tilde{X}_i \circ \Omega_B$$

$$\|$$

$\tilde{X}_i \to \omega_i \tilde{X}_i$

$\omega_i^N = \omega_1 \cdots \omega_N =$

$$(\tilde{X}_1^N + \cdots + \tilde{X}_N^N)/\mathbb{Z}_N^{N-1} \cong (\tilde{X}_1^N + \cdots + \tilde{X}_N^N)/\mathbb{Z}_N^{N-1}/\mathbb{Z}_N/\mathbb{Z}$$

quantum symm

$$\tilde{X}_i \to e^{\frac{\pi i}{N}} \tilde{X}_i \circ \Omega_B \qquad \tilde{X}_i \to e^{\frac{\pi i}{N}} \tilde{X}_i \circ \Omega_B$$

A-parity

① $\tau_A : z_i \to \bar{z}_i$

$$K = 1_{\mathbb{C}}$$
$$C = 101_{\mathbb{R}}$$

$$X^{\tau_A} = \{ (x_1, \cdots, x_5) \} \cong \mathbb{RP}^3 \quad (\text{O6-plane})$$

$\overset{\text{mirror}}{\longleftrightarrow}$ $\tilde{\tau}_B : \tilde{z}_i \to \tilde{z}_i$

$$\tilde{X}^{\tilde{\tau}_B} = \tilde{X} \text{ itself} \quad (\text{O9})$$

② $\tau_A^{1,2} : z_1, z_2, z_3, z_4, z_5 \to \bar{z}_2, \bar{z}_1, \bar{z}_3, \bar{z}_4, \bar{z}_5$

$$K = 1_{\mathbb{C}}$$
$$C = 101_{\mathbb{R}}$$

$$X^{\tau_A^{12}} = \{ (z, \bar{z}, x_3, x_4, x_5) \} \cong \mathbb{RP}^3 \quad (\text{O6})$$

$\leftrightarrow$ $\tilde{\tau}_B^{12} : \tilde{z}_1 \tilde{z}_2 \tilde{z}_3 \tilde{z}_4 \tilde{z}_5 \to \tilde{z}_2 \tilde{z}_1 \tilde{z}_3 \tilde{z}_4 \tilde{z}_5$

$$\tilde{X}^{\tilde{\tau}_B} = \underset{(\text{O3})}{\{ (1, -1, 0, 0, 0) \}} \cup \underset{(\text{O7})}{\{ (\tilde{z}, \tilde{z}, \tilde{w}, \tilde{w}, \tilde{u}) \}}$$

③ $\tau_A^{12,34} : z_1, z_2, z_3, z_4, z_5 \to \bar{z}_2, \bar{z}_1, \bar{z}_4, \bar{z}_3, \bar{z}_5$

$$K = 1_{\mathbb{C}}$$
$$C = 101_{\mathbb{R}}$$

$$X^{\tau_A^{12,34}} = \{(z, \bar{z}, w, \bar{w}, x)\} \cong \mathbb{R}P^3 \quad (06)$$

$\leftrightarrow \tilde{\tau}_B^{12,34} : \tilde{z}_1, \tilde{z}_2, \tilde{z}_3, \tilde{z}_4, \tilde{z}_5 \to \tilde{z}_2, \tilde{z}_1, \tilde{z}_4, \tilde{z}_3, \tilde{z}_5$

$$\tilde{X}^{\tilde{\tau}_B^{12,34}} = \underbrace{\{(\tilde{z}, -\tilde{z}, \tilde{w}, -\tilde{w}, 0)\}}_{(105)} \cup \underbrace{\{(\tilde{z}, \tilde{z}, \tilde{w}, \tilde{w}, \tilde{u})\}}_{(05)}$$

line　　　　genus 6

B-parity

④ $\tau_B = id : z_i \to z_i$

$$K = 1_{\mathbb{R}}$$
$$C = 101_{\mathbb{C}}$$

$$X^{\tau_B} = X \text{ itself} \quad (09)$$

$\leftrightarrow \tilde{\tau}_A : \tilde{z}_i \to \overline{\tilde{z}_i}$

⑤ $\tau_B^{12} : z_1, z_2, z_3, z_4, z_5 \to \bar{z}_2, \bar{z}_1, \bar{z}_3, \bar{z}_4, \bar{z}_5$

$$K = 1_{\mathbb{R}}$$

$$C = 63_{\mathbb{C}} \qquad \Big(\overset{(z_1 z_2)^i z_3^j z_4^k z_5^l}{34} + \frac{126-34}{2} \Big) - 17$$

$$X^{\tau_B^{12}} = \{(1,-1,0,0,0)\} \cup \{(z, \bar{z}, z_3, z_4, z_5)\}$$

$$(O3) \qquad\qquad (O7)$$

$\leftrightarrow \tilde{\tau}_A^{12} : \tilde{z}_1 \tilde{z}_2 \tilde{z}_3 \tilde{z}_4 \tilde{z}_5 \to \overline{\tilde{z}}_2 \overline{\tilde{z}}_1 \overline{\tilde{z}}_3 \overline{\tilde{z}}_4 \overline{\tilde{z}}_5$

$\tilde{K} = 63_{\mathbb{C}} = 1 + \#$ anti-inv. blow up modes

⑥ $\tau_B^{12,34} : z_1, z_2, z_3, z_4, z_5 \to \bar{z}_2, \bar{z}_1, \bar{z}_4, \bar{z}_3, \bar{z}_5$

$$K = 1_{\mathbb{R}}$$

$$C = 53_{\mathbb{C}} \qquad \Big(\overset{(z_1 z_2)^i (z_3 z_4)^j z_5^k}{6} + \frac{126-6}{2} \Big) - 13$$

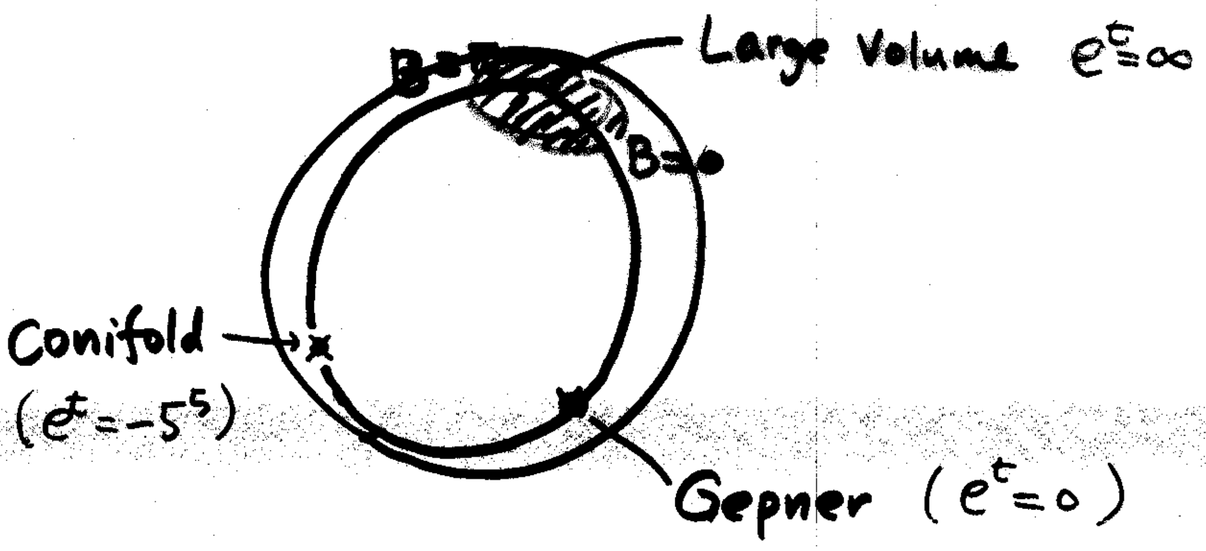
$$X^{\tau_B^{12,34}} = \{(z, -\bar{z}, w, -\bar{w}, 0)\} \cup \{(z, \bar{z}, w, \bar{w}, u)\}$$

(O5 on a line) (O5 on genus 6 curve)

$\leftrightarrow \tilde{\tau}_A^{12,34} : \tilde{z}_1, \tilde{z}_2, \tilde{z}_3, \tilde{z}_4, \tilde{z}_5 \to \overline{\tilde{z}}_2, \overline{\tilde{z}}_1, \overline{\tilde{z}}_4, \overline{\tilde{z}}_3, \overline{\tilde{z}}_5$

$\tilde{K} = 53_{\mathbb{C}} = 1 + \#$ anti-inv. blow up modes

Kähler moduli space of IIB Orientifolds ④,⑤,⑥



★ Combined with RR holonomy $\begin{cases} A_2 & O9 \ldots O5 \\ A_4^+ & O7/O3 \end{cases}$

it forms a <u>complex moduli</u>

★ superpotential may or may not be generated.

★ It passes through the conifold point
(where worldsheet description breaks down)

In the mirror, O-plane passes the conifold singularity $(\tilde{z}_i = 1)$