Nonnegative Matrix and Tensor Factorizations for Text Mining Applications

IPAM Workshop: Numerical Tools and Fast Algorithms for Massive Data Mining, Search Engines, and Applications

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1 Nonnegative Matrix Factorization (NNMF)

2 Document Parsing and Term Weighting - ASRS

3 NNMF Classification of ASRS Documents

4 NNTF Classification of Enron Email

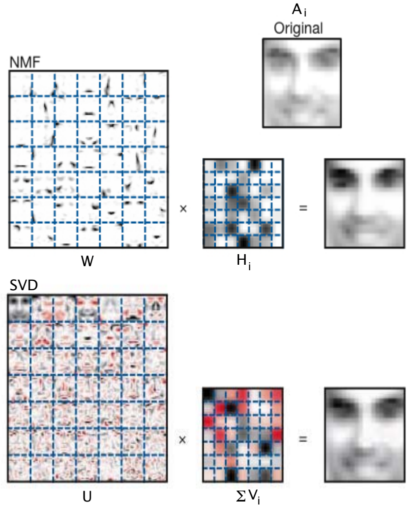
5 Discussion Tracking via PARAFAC/Tensor Factorization

6 Summary and References

NNMF Origins

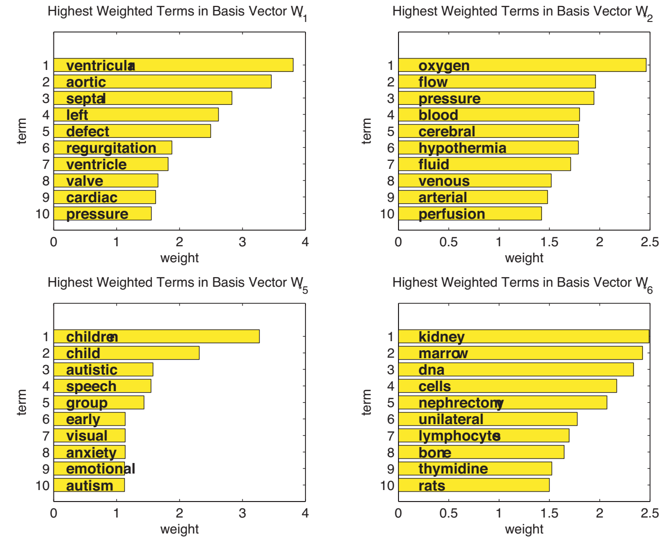
- NNMF (Nonnegative Matrix Factorization) can be used to approximate high-dimensional data having nonnegative components.
- Lee and Seung (1999) demonstrated its use as a *sum-by-parts* representation of image data in order to both identify and classify image *features*.
- Xu et al. (2003) demonstrated how NNMF-based indexing could outperform SVD-based Latent Semantic Indexing (LSI) for some information retrieval tasks.

NNMF for Image Processing



Sparse NNMF verses Dense SVD Bases; Lee and Seung (1999)

NNMF for Text Mining (Medlars)



Interpretable NNMF feature vectors; Langville et al. (2006)

Derivation

- Given an $m \times n$ term-by-document (sparse) matrix X.
- Compute two reduced-dim. matrices W,H so that $X \simeq WH$; W is $m \times r$ and H is $r \times n$, with $r \ll n$.
- **Optimization problem**:

$$\min_{W,H} \|X - WH\|_F^2,$$

subject to $W_{ij} \geq 0$ and $H_{ij} \geq 0$, $\forall i, j$.

- **General approach**: construct initial estimates for W and H and then improve them via alternating iterations.

Minimization Challenges and Formulations [Berry et al., 2007]

- **Local Minima**: Non-convexity of functional $f(W, H) = \frac{1}{2}\|X - WH\|_F^2$ in both W and H.
- **Non-unique Solutions**: $WDD^{-1}H$ is nonnegative for any nonnegative (and invertible) D.
- **NNMF Formulations**:
  - Lee and Seung (2001) – information theoretic formulation based on Kullback-Leibler divergence of X from WH.
  - Guillamet, Bressan, and Vitria (2001) – diagonal weight matrix Q used ($XQ \approx WHQ$) to compensate for feature redundancy (columns of W).
  - Wang, Jiar, Hu, and Turk (2004) – constraint-based formulation using Fisher linear discriminant analysis to improve extraction of spatially localized features.
  - Other Cost Function Formulations – Hamza and Brady (2006), Dhillon and Sra (2005), Cichocki, Zdunek, and Amari (2006)

Multiplicative Method (MM)

- Multiplicative update rules for W and H (Lee and Seung, 1999):
  1. Initialize W and H with nonnegative values, and scale the columns of W to unit norm.
  2. Iterate for each c, j, and i until convergence or after k iterations:
     1. $H_{cj} \leftarrow H_{cj} \frac{(W^T X)_{cj}}{(W^T WH)_{cj} + \epsilon}$
     2. $W_{ic} \leftarrow W_{ic} \frac{(XH^T)_{ic}}{(WHH^T)_{ic} + \epsilon}$
     3. Scale the columns of W to unit norm.
- Setting $\epsilon = 10^{-9}$ will suffice to avoid division by zero [Shahnaz et al., 2006].

Multiplicative Method (MM) contd.

Multiplicative Update MATLAB®Code for NNMF

```
W = rand(m,k);    % W initially random
H = rand(k,n);    % H initially random
for i = 1 : maxiter
      H = H .* (W^T A) ./ (W^T WH + ε);
      W = W .* (AH^T) ./ (WHH^T + ε);
end
```


Lee and Seung MM Convergence

- **Convergence**: when the MM algorithm converges to a limit point in the interior of the feasible region, the point is a *stationary point*. The stationary point **may or may not be a local minimum**. If the limit point lies on the boundary of the feasible region, one cannot determine its stationarity [Berry et al., 2007].
- **Modifications**: Gonzalez and Zhang (2005) accelerated convergence somewhat but stationarity issue remains; Lin (2005) modified the algorithm to guarantee convergence to a stationary point; Dhillon and Sra (2005) derived update rules that incorporate weights for the importance of certain features of the approximation.

Alternating Least Squares Formulation

Basic ALS Approach:
ALS algorithms exploit the convexity of W or H (not both) in the underlying optimization problem. The basic iteration involves

(LS) Solve for H in $W^T WH = W^T X$.
(NN) Set negative elements of H to 0.
(LS) Solve for W in $HH^T W^T = HX^T$.
(NN) Set negative elements of W to 0.

ALS Recovery and Constraints:

- Unlike the MM algorithm, an element of W (or H) that becomes 0 does not have to remain 0; method can escape/recover from a *poor* path.
- Paatero (1999) and Langville et al.(2006) have improved the computational complexity of the ALS approach; sparsity and nonnegativity contraints are enforced.

Alternating Least Squares Algorithms, contd.

ALS Convergence:

- Polak (1971) showed that every limit point of a sequence of alternating variable iterates is a stationary point.
- Lawson and Hanson (1995) produced the Non-Negative Least Squares (NNLS) that was shown to converge to a local minimum.
- The price for convergence of ALS algorithms is the usual high cost per iteration – Bro and de Jong (1997).

Hoyer's Method

- From neural network applications, Hoyer (2002) enforced statistical sparsity for the weight matrix H in order to enhance the parts-based data representations in the matrix W.
- Mu et al. (2003) suggested a regularization approach to achieve statistical sparsity in the matrix H: **point count regularization**; penalize the *number* of nonzeros in H rather than $\sum_{ij} H_{ij}$.
- Goal of increased sparsity – better representation of *parts* or *features* spanned by the corpus (X) [Berry and Browne, 2005].

GD-CLS – Hybrid Approach

- First use MM to compute an approximation to W for each iteration – a gradient descent (**GD**) optimization step.
- Then, compute the weight matrix H using a constrained least squares (**CLS**) model to penalize non-smoothness (i.e., non-sparsity) in H – common Tikohonov regularization technique used in image processing (Prasad et al., 2003).
- Convergence to a non-stationary point evidenced (proof still needed).

GD-CLS Algorithm

1. Initialize W and H with nonnegative values, and scale the columns of W to unit norm.
2. Iterate until convergence or after k iterations:
   1. $W_{ic} \leftarrow W_{ic} \frac{(XH^T)_{ic}}{(WHH^T)_{ic} + \epsilon}$, for c and i
   2. Rescale the columns of W to unit norm.
   3. Solve the constrained least squares problem:

$$\min_{H_j} \{ \|X_j - WH_j\|_2^2 + \lambda \|H_j\|_2^2 \},$$

   where the subscript j denotes the j^{th} column, for $j = 1, \ldots, m$.

- Any negative values in H_j are set to zero. The parameter λ is a regularization value that is used to balance the reduction of the metric $\|X_j - WH_j\|_2^2$ with enforcement of smoothness and sparsity in H.

Two Penalty Term Formulation

- Introduce smoothing on W_k (feature vectors) in addition to H^k:
$$\min_{W,H}\{\|X - WH\|_F^2 + \alpha\|W\|_F^2 + \beta\|H\|_F^2\},$$
where $\|\cdot\|_F$ is the Frobenius norm.
- Constrained NNMF (CNMF) iteration:

$$H_{cj} \leftarrow H_{cj}\frac{(W^TX)_{cj} - \beta H_{cj}}{(W^TWH)_{cj} + \epsilon}$$

$$W_{ic} \leftarrow W_{ic}\frac{(XH^T)_{ic} - \alpha W_{ic}}{(WHH^T)_{ic} + \epsilon}$$

Improving Feature Interpretability

Gauging Parameters for Constrained Optimization

How sparse (or smooth) should factors (W, H) be to produce as many interpretable features as possible?

To what extent do different norms (l_1, l_2, l_∞) improve/degradate feature quality or span? At what cost?

Can a nonnegative feature space be built from objects in both images and text? Are there opportunities for multimodal document similarity?

Anomaly Detection (ASRS)

- Classify events described by documents from the Airline Safety Reporting System (ASRS) into 22 anomaly categories; contest from SDM07 Text Mining Workshop.
- General Text Parsing (GTP) Software Environment in C++ [Giles et al., 2003] used to parse both ASRS training set and a combined ASRS training and test set:

Dataset	Terms	ASRS Documents
Training	15,722	21,519
Training+Test	17,994	28,596 (7,077)

- Global and document frequency of required to be at least 2; stoplist of 493 common words used; char length of any term $\in [2, 200]$.
- Download Information:
  GTP: http://www.cs.utk.edu/∼lsi
  ASRS: http://www.cs.utk.edu/tmw07

Term Weighting Schemes

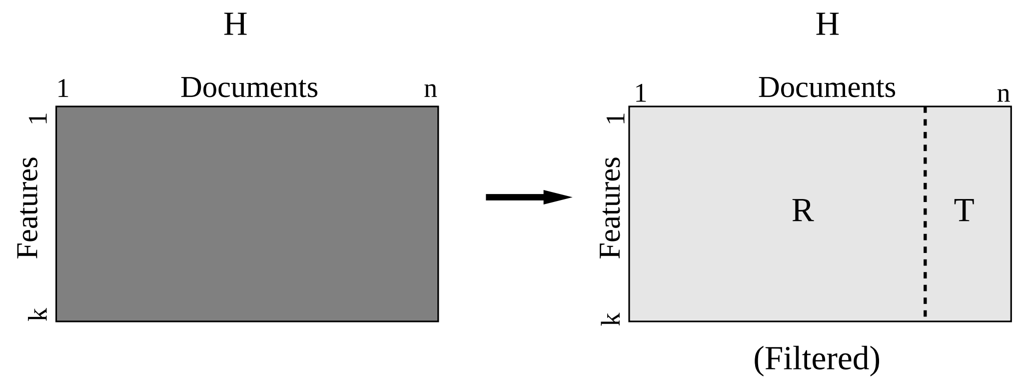
- **Assessment of Term Importance**: for $m \times n$ term-by-message matrix $X = [x_{ij}]$, define
$$x_{ij} = l_{ij} g_i d_j,$$
where l_{ij} is the local weight for term i occurring in message j, g_i is the global weight for term i in the subcollection, and d_j is a document normalization factor (set $d_j = 1$).
- **Common Term Weighting Choices**:

Name	Local	Global
txx	Term Frequency	None
	$l_{ij} = f_{ij}$	$g_i = 1$
lex	Logarithmic	Entropy (Define: $p_{ij} = f_{ij}/\sum_j f_{ij}$)
	$l_{ij} = \log(1 + f_{ij})$	$g_i = 1 + (\sum_j p_{ij} log(p_{ij}))/\log n)$

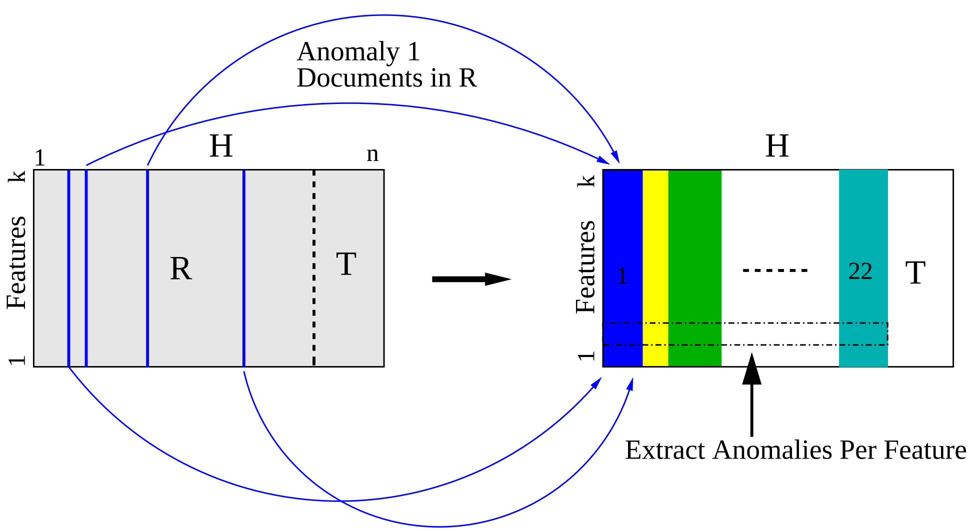
Parameterization

- Important Constants:
  - α, the threshold on the relevance score or (target value) t_{ij} for document i and anomaly/label j; we use **R** submatrix of **H** to cluster documents by the k features — assume documents describing similar anomalies share similar features.
  - δ, the threshold on the column elements of **H**, which will filter out the association of features with both the training (**R**) and test (**T**) documents;
  - σ, the percentage of documents used to define the training set (or number of columns of **R**).

Initialization Schematic



Anomaly to Feature Mapping and Scoring Schematic

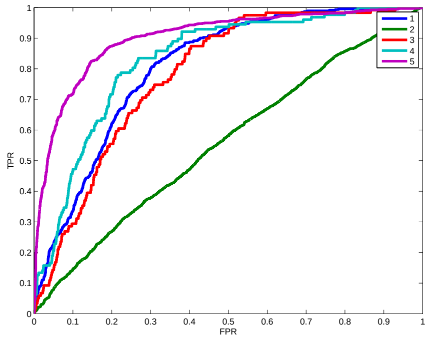


Training/Testing Performance (ROC Curves)

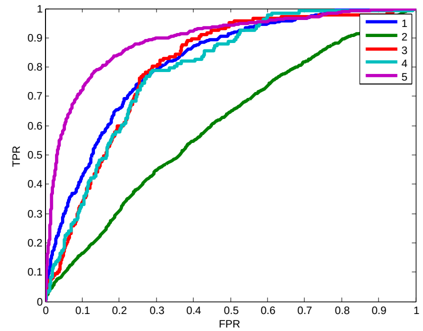
- Best/Worst ROC curves (False Positive Rate versus True Positive Rate)

Anomaly	Type (Description)	ROC Area	
		Training	Contest
22	Security Concern/Threat	.9040	.8925
5	Incursion (collision hazard)	.8977	.8716
4	Excursion (loss of control)	.8296	**.7159**
21	Illness/Injury Event	.8201	.8172
12	Traffic Proximity Event	.7954	.7751
7	Altitude Deviation	.7931	.8085
18	Aircraft Damage/Encounter	.7250	.7261
11	Terrain Proximity Event	.7234	**.7575**
9	Speed Deviation	.7060	.6893
10	Uncommanded (loss of control)	.6784	.6504
13	Weather Issue	.6287	.6018
2	Noncompliance (policy/proc.)	.6009	**.5551**

ROC Curves for Anomalies 1–5 (Test/Training)

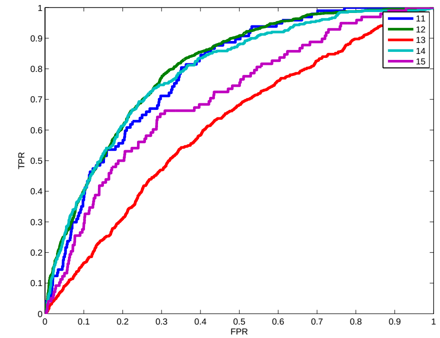


Training

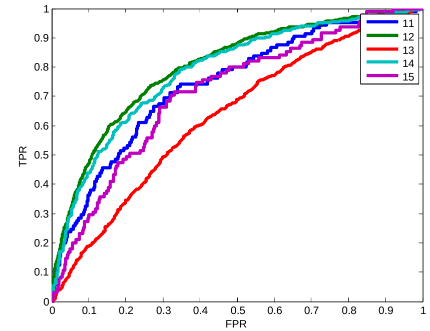


Contest

ROC Curves for Anomalies 11–15 (Test/Training)

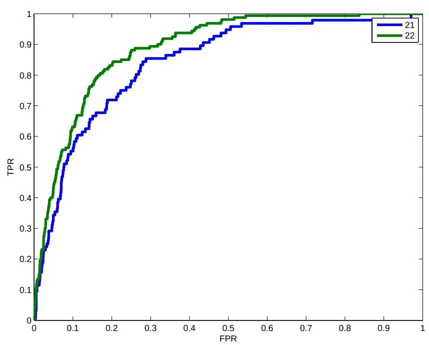


Training

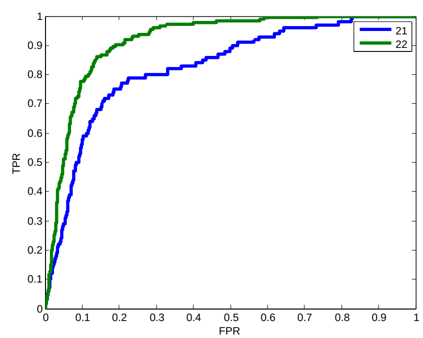


Contest

ROC Curves for Anomalies 21, 22 (Test/Training)

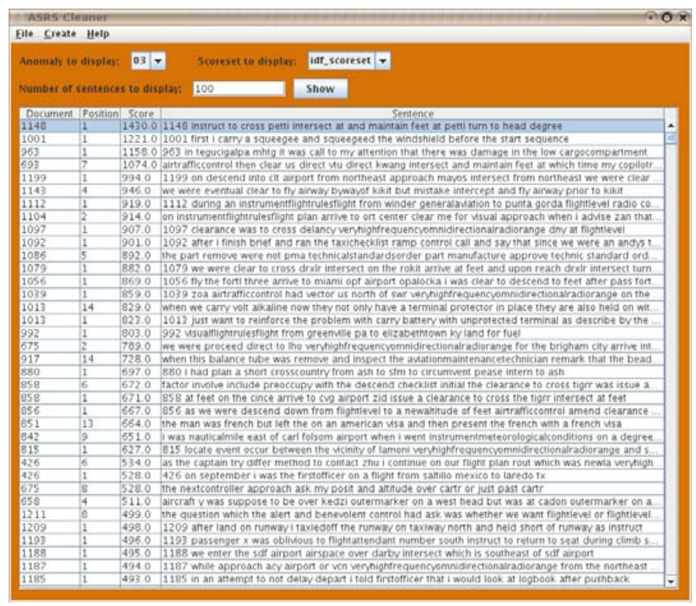


Training



Contest

Anomaly Summarization Prototype - Sentence Ranking



Document	Position	Score	Sentence
1148	1	1430.0	1148 instruct to cross petti intersect at and maintain feet at petti turn to head degree
1001	1	1221.0	1001 first i carry a squeegee and squeegeed the windshield before the start sequence
963	1	1158.0	963 in tegucigalpa mhtg it was call to my attention that there was damage in the low cargocompartment
693	7	1074.0	airtrafficcontrol then clear us direct vtu direct kwang intersect and maintain feet at which time my copilotr...
1199	1	994.0	1199 on descend into clt airport from northeast approach mayos intersect from northeast we were clear ...
1143	4	946.0	we were eventual clear to fly airway bywayof kikit but mistake intercept and fly airway prior to kikit
1112	1	919.0	1112 during an instrumentflightrulesflight from winder generalaviation to punta gorda flightlevel radio co...
1104	2	914.0	on instrumentflightrulesflight plan arrive to ort center clear me for visual approach when i advise zan that...
1097	1	907.0	1097 clearance was to cross delancy veryhighfrequencyomnidirectionalradiorange dny at flightlevel
1092	1	901.0	1092 after i finish brief and ran the taxichecklist ramp control call and say that since we were an andys t...
1086	5	892.0	the part remove were not pma technicalstandardsorder part manufacture approve technic standard ord...
1079	1	882.0	1079 we were clear to cross drxlr intersect on the rokit arrive at feet and upon reach drxlr intersect turn...
1056	1	869.0	1056 fly the forti three arrive to miami opf airport opalocka i was clear to descend to feet after pass fort...
1039	1	859.0	1039 zoa airtrafficcontrol had vector us north of swr veryhighfrequencyomnidirectionalradiorange on the ...
1013	14	829.0	when we carry volt alkaline now they not only have a terminal protector in place they are also held on wit...
1013	1	823.0	1013 just want to reinforce the problem with carry battery with unprotected terminal as describe by the ...
992	1	803.0	992 visualflightrulesflight from greenville pa to elizabethtown ky land for fuel
675	2	789.0	we were proceed direct to lho veryhighfrequencyomnidirectionalradiorange for the brigham city arrive int...
917	14	728.0	when this balance tube was remove and inspect the aviationmaintenancetechnician remark that the bead...
880	1	697.0	880 i had plan a short crosscountry from ash to sfm to circumvent pease intern to ash
858	6	672.0	factor involve include preoccupy with the descend checklist initial the clearance to cross tigrr was issue a...
858	1	671.0	858 at feet on the cince arrive to cvg airport zid issue a clearance to cross the tigrr intersect at feet
856	1	667.0	856 as we were descend down from flightlevel to a newaltitude of feet airtrafficcontrol amend clearance ...
851	13	664.0	the man was french but left the on an american visa and then present the french with a french visa
842	9	651.0	i was nauticalmile east of carl folsom airport when i went instrumentmeteorologicalconditions on a degree...
815	1	627.0	815 locate event occur between the vicinity of lamoni veryhighfrequencyomnidirectionalradiorange and s...
426	6	534.0	as the captain try differ method to contact zhu i continue on our flight plan rout which was newla veryhigh...
426	1	528.0	426 on september i was the firstofficer on a flight from saltillo mexico to laredo tx
675	8	528.0	the nextcontroller approach ask my posit and altitude over cartr or just past cartr
658	4	511.0	aircraft y was suppose to be over kedzi outermarker on a west head but was at cadon outermarker on a...
1211	8	499.0	the question which the alert and benevolent control had ask was whether we want flightlevel or flightlevel...
1209	1	498.0	1209 after land on runway i taxiedoff the runway on taxiway north and held short of runway as instruct
1193	1	496.0	1193 passenger x was oblivious to flightattendant number south instruct to return to seat during climb s...
1188	1	495.0	1188 we enter the sdf airport airspace over darby intersect which is southeast of sdf airport
1187	1	494.0	1187 while approach acy airport or vcn veryhighfrequencyomnidirectionalradiorange from the northeast ...
1185	1	493.0	1185 in an attempt to not delay depart i told firstofficer that i would look at logbook after pushback

Sentence rank = f(global term weights) – B. Lamb

Email Collection

- By-product of the FERC investigation of Enron (originally contained 15 million email messages).
- This study used the improved corpus known as the Enron Email set, which was edited by Dr. William Cohen at CMU.
- This set had over 500,000 email messages. The majority were sent in the 1999 to 2001 timeframe.

Enron Historical 1999-2001

- Ongoing, problematic, development of the Dabhol Power Company (DPC) in the Indian state of Maharashtra.
- Deregulation of the Calif. energy industry, which led to rolling electricity blackouts in the summer of 2000 (and subsequent investigations).
- Revelation of Enron's deceptive business and accounting practices that led to an abrupt collapse of the energy colossus in October, 2001; Enron filed for bankruptcy in December, 2001.

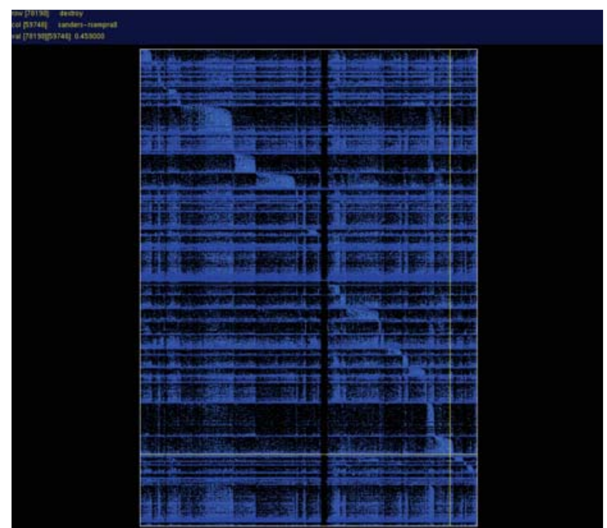
PRIVATE Collection

- Parsed all mail directories (of all 150 accounts) with the exception of `all_documents`, `calendar`, `contacts`, `deleted_items`, `discussion_threads`, `inbox`, `notes_inbox`, `sent`, `sent_items`, and `_sent_mail`; 495-term stoplist used and extracted terms must appear in more than 1 email and more than once globally [Berry and Browne, 2005].
- Distribution of messages sent in the year 2001:

Month	Msgs	Terms	Month	Msgs	Terms
Jan	3,621	17,888	Jul	3,077	17,617
Feb	2,804	16,958	Aug	2,828	16,417
Mar	3,525	20,305	Sep	2,330	15,405
Apr	4,273	24,010	Oct	2,821	20,995
May	4,261	24,335	Nov	2,204	18,693
Jun	4,324	18,599	Dec	1,489	8,097

Visualization of PRIVATE Collection Term-Msg Matrix

- NNMF-generated reordering of $92,133 \times 65,031$ term-by-message matrix (log-entropy weighting) using VISMATRIX [Gleich, 2006]; cluster docs in X according to *arg* $\max_i H_{ij}$, then cluster terms according to *arg* $\max_j W_{ij}$.



PRIVATE with Log-Entropy Weighting

- Identify rows of H from $X \simeq WH$ or H^k with $\lambda = 0.1$; $r = 50$ feature vectors (W_k) generated by GD-CLS:

Feature Index (k)	Cluster Size	Topic Description	Dominant Terms
10	497	California	ca, **cpuc**, **gov**, **socalgas**, sempra, org, sce, gmssr, aelaw, ci
23	43	Louise Kitchen named top woman by Fortune	evp, **fortune**, britain, woman, **ceo**, avon, fiorina, cfo, hewlett, packard
26	231	Fantasy football	game, wr, qb, play, rb, season, injury, updated, fantasy, image

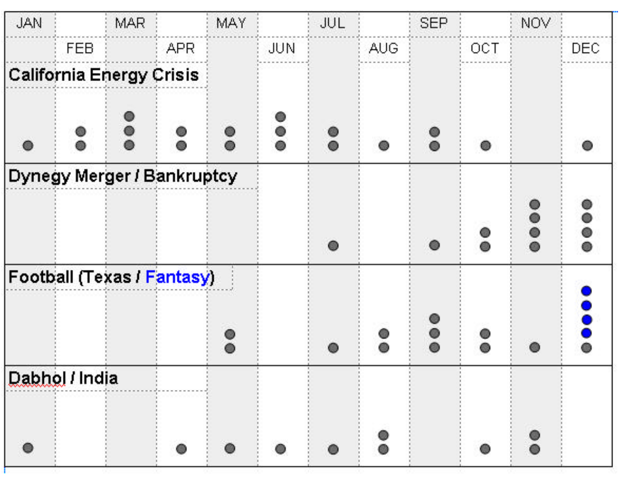
(Cluster size $\equiv$ no. of H^k elements $>$ $row_{max}/10$)

PRIVATE with Log-Entropy Weighting

- Additional topic clusters of significant size:

Feature Index (k)	Cluster Size	Topic Description	Dominant Terms
33	233	Texas longhorn football newsletter	UT, orange, longhorn[s], texas, true, truorange, recruiting, oklahoma, defensive
34	65	Enron collapse	**partnership[s]**, **fastow**, shares, **sec**, stock, shareholder, investors, equity, **lay**
39	235	Emails about India	**dabhol**, **dpc**, **india**, **mseb**, **maharashtra**, indian, lenders, delhi, foreign, minister

2001 Topics Tracked by GD-CLS



$r = 50$ features, **lex** term weighting, $\lambda = 0.1$

(*New York Times*, May 22, 2005)

Term Distribution in Feature Vectors

Terms	Wt	Lambda 0.1	Lambda 0.01	Lambda 0.001	Alpha 0.1	Alpha 0.01	Alpha 0.001	Topics
Blackouts	0.508				4	6	4	Cal
Stocks	0.511				2			Collapse
UT	0.517				2			Texasfoot
Chronicle	0.523				3	2	3	
Indian	0.527				2			India
Fastow	0.531				5	3	4	Collapse
Gas	0.531					2	2	
CFO	0.556				2		2	Kitchen
Californians	0.557					3		Cal
Solar	0.570				2			
Partnerships	0.576				6	2	5	Collapse
Workers	0.577					3	2	
Maharashtra	0.591				2		2	India
Mseb	0.605				2			India
Beach	0.611			2				
Ljm	0.621				3		3	Collapse
Tues	0.626		2	2				
IPPS	0.644			2		2		Cal
Rebates	0.647					2		
Ljm2	0.688				2		2	Collapse

Hoyer Sparsity Constraint

- sparseness($\mathbf{x}$) $= \frac{\sqrt{\mathbf{n}} - \|\mathbf{x}\|_1/\|\mathbf{x}\|_2}{\sqrt{\mathbf{n}}-\mathbf{1}}$, [Hoyer, 2004]
- Imposed as a penalty term of the form

$$J_2(\mathbf{W}) = (\omega\|\text{vec}(\mathbf{W})\|_2 - \|\text{vec}(\mathbf{W})\|_1)^2,$$

  where $\omega = \sqrt{mk} - (\sqrt{mk} - 1)\gamma$ and vec$(\cdot)$ transforms a matrix into a vector by column stacking.
- Desired sparseness in $\mathbf{W}$ is specified by setting $\gamma \in [0, 1]$; *sparseness* is zero iff all vector components are equal (up to signs) and is one iff the vector has a single nonzero.

Sample Benchmarks for Smoothing and Sparsity Constraints

- Elapsed CPU times for CNMF on a 3.2GHz Intel Xeon 3.2GHz (1024KB cache, 4.1GB RAM)
- $k = 50$ feature vectors generated, log-entropy noun-weighting used on $7,424 \times 289,695$ noun-by-message matrix, random $\mathbf{W_0}, \mathbf{H_0}$

W-Constraint	Iterations	Parameters	CPU time
L_2 norm	100	$\alpha = 0.1, \beta = 0$	19.6m
L_2 norm	100	$\alpha = 0.01, \beta = 0$	20.1m
L_2 norm	100	$\alpha = 0.001, \beta = 0$	19.6m
Hoyer	30	$\alpha = 0.01, \beta = 0, \gamma = 0.8$	2.8m
Hoyer	30	$\alpha = 0.001, \beta = 0, \gamma = 0.8$	2.9m

Annotation Project

- Subset of 2001 PRIVATE collection:

Month	Total	Classified	Usable
Jan,Sep	5591	1100	699
Feb	2804	900	460
Mar	3525	1200	533
Apr	4273	1500	705
May	4261	1800	894
June	4324	1025	538
Total	24778	7525	3829

- Approx. 40 topics identified after NNMF initial clustering with $k = 50$ features.

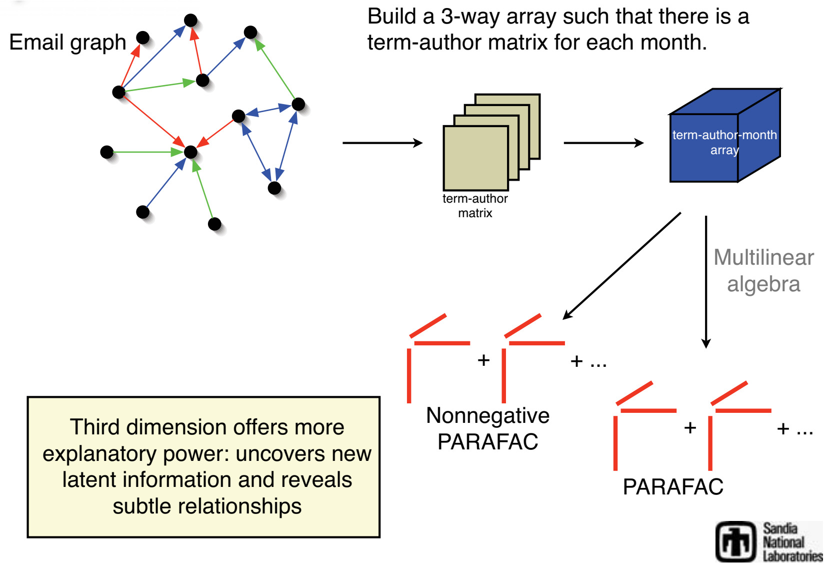
Annotation Project, contd.

- Human classfiers: M. Browne (extensive background reading on Enron collapse) and B. Singer (junior Economics major).
- Classify email content versus type (see UC Berkeley Enron Email Analysis Group `http://bailando.sims.berkeley.edu/enron_email.html`
- As of June 2007, distributed by the by U. Penn LDC (Linguistic Data Consortium); see `www.ldc.upenn.edu`

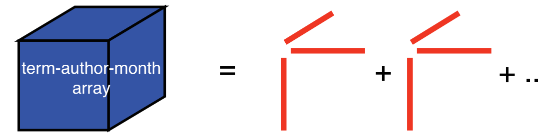
Citation:

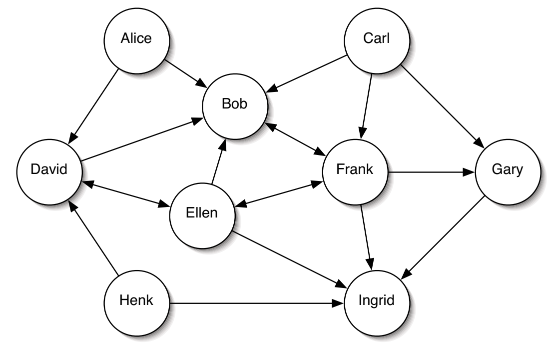
Dr. Michael W. Berry, Murray Browne
and Ben Signer, 2007
2001 Topic Annotated Enron Email Data Set
Linguistic Data Consortium, Philadelphia

Multidimensional Data Analysis via PARAFAC



Temporal Assessment via PARAFAC





Mathematical Notation

- Kronecker product

$$A \otimes B = \begin{pmatrix} A_{11}B & \cdots & A_{1n}B \\ \vdots & \ddots & \vdots \\ A_{m1}B & \cdots & A_{mn}B \end{pmatrix}$$

- Khatri-Rao product (columnwise Kronecker)

$$A \odot B = \begin{pmatrix} A_1 \otimes B_1 & \cdots & A_n \otimes B_n \end{pmatrix}$$

- Outer product

$$A_1 \circ B_1 = \begin{pmatrix} A_{11}B_{11} & \cdots & A_{11}B_{m1} \\ \vdots & \ddots & \vdots \\ A_{m1}B_{11} & \cdots & A_{m1}B_{m1} \end{pmatrix}$$

PARAFAC Representations

- PARAllel FACtors (Harshman, 1970)
- Also known as CANDECOMP (Carroll & Chang, 1970)
- Typically solved by Alternating Least Squares (ALS)

Alternative PARAFAC formulations

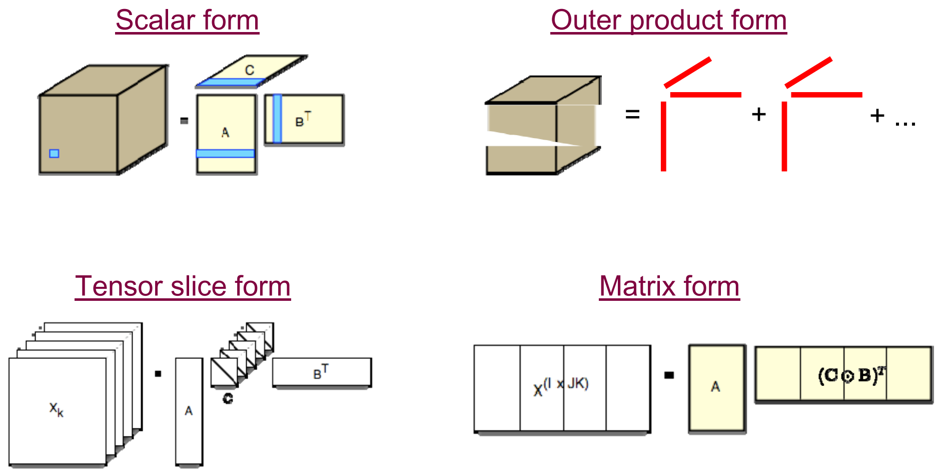
$$X_{ijk} \approx \sum_{i=1}^{r} A_{ir} B_{jr} C_{kr}$$

$$\mathcal{X} \approx \sum_{i=1}^{r} A_i \circ B_i \circ C_i, \text{ where } \mathcal{X} \text{ is a 3-way array (tensor).}$$

$$X_k \approx A \operatorname{diag}(C_{k:})\, B^T, \text{ where } X_k \text{ is a tensor slice.}$$

$$X^{I \times JK} \approx A (C \odot B)^T, \text{ where } X \text{ is matricized.}$$

PARAFAC (Visual) Representations



Nonnegative PARAFAC Algorithm

- Adapted from (Mørup, 2005) and based on NNMF by (Lee and Seung, 2001)

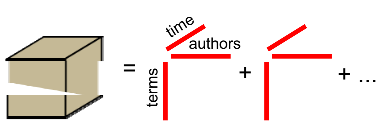
$$
\begin{aligned}
\|X^{I\times JK} - A(C\odot B)^T\|_F &= \|X^{J\times IK} - B(C\odot A)^T\|_F \\
&= \|X^{K\times IJ} - C(B\odot A)^T\|_F
\end{aligned}
$$

- Minimize over A, B, C using multiplicative update rule:

$$
\begin{aligned}
A_{i\rho} &\leftarrow A_{i\rho}\frac{(X^{I\times JK}Z)_{i\rho}}{(AZ^TZ)_{i\rho}+\epsilon}, \quad Z=(C\odot B)\\
B_{j\rho} &\leftarrow B_{j\rho}\frac{(X^{J\times IK}Z)_{j\rho}}{(BZ^TZ)_{j\rho}+\epsilon}, \quad Z=(C\odot A)\\
C_{k\rho} &\leftarrow C_{k\rho}\frac{(X^{K\times IJ}Z)_{k\rho}}{(CZ^TZ)_{k\rho}+\epsilon}, \quad Z=(B\odot A)
\end{aligned}
$$

Discussion Tracking Using Year 2001 Subset

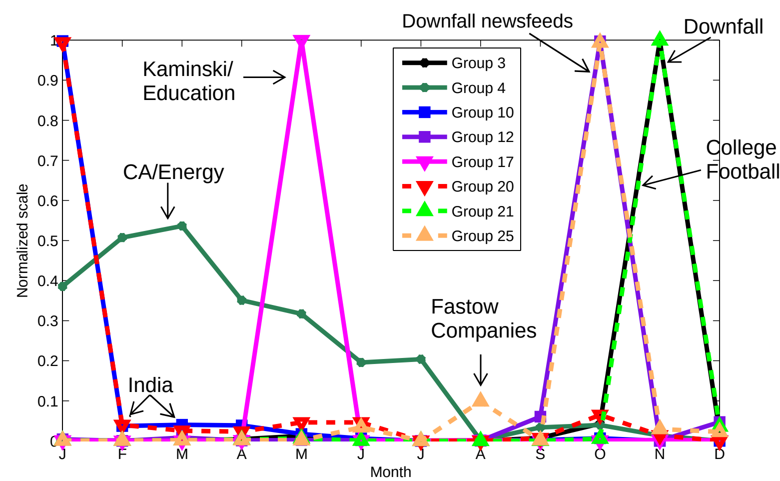
- 197 authors (`From:user_id@enron.com`) monitored over 12 months;
- Parsing 34,427 email subset with a base dictionary of 121,393 terms (derived from 517,431 emails) produced 69,157 unique terms; (term-author-month) array X has ~ 1 million nonzeros.
- Term frequency weighting with constraints (global frequency ≥ 10 and email frequency ≥ 2); expert-generated stoplist of 47,154 words (M. Browne)
- Rank-25 tensor: A $(69,157 \times 25)$, B (197×25), C (12×25)



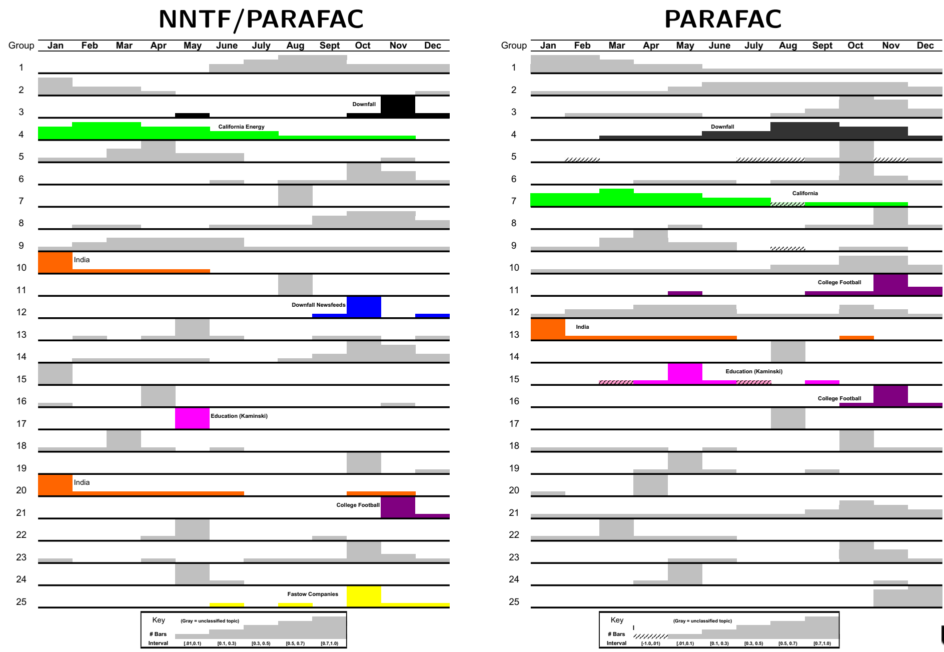
Month	Emails	Month	Emails
Jan	7,050	Jul	2,166
Feb	6,387	Aug	2,074
Mar	6,871	Sep	2,192
Apr	7,382	Oct	5,719
May	5,989	Nov	4,011
Jun	2,510	Dec	1,382

Tensor-Generated Group Discussions

- NNTF Group Discussions in 2001
- 197 authors; 8 distinguishable discussions
- "Kaminski/Education" topic previously unseen

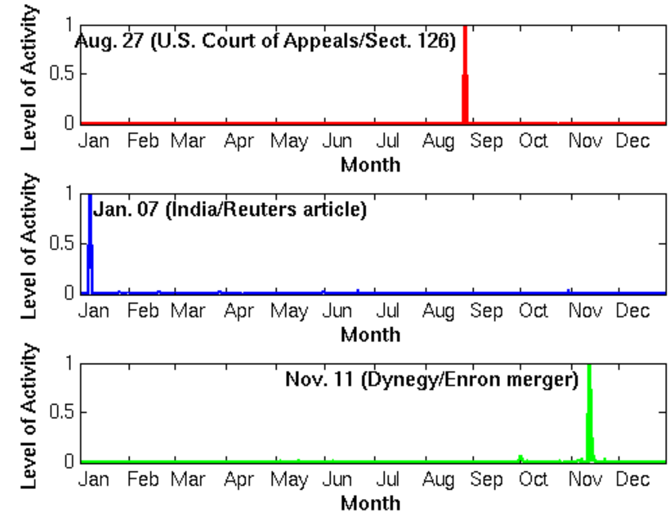


Gantt Charts from PARAFAC Models



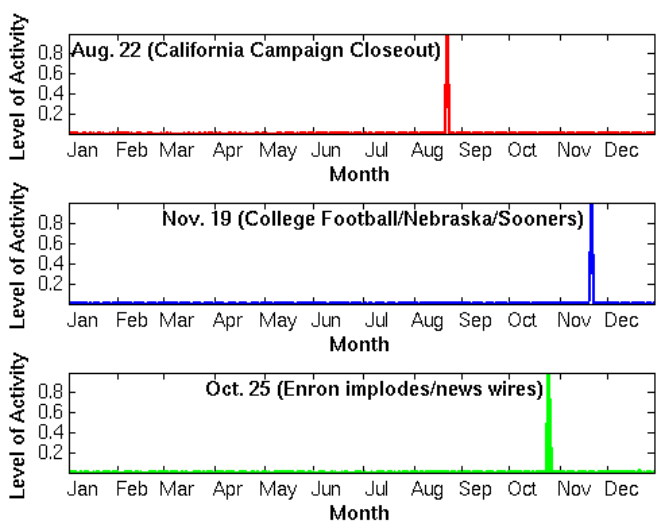
Day-level Analysis for PARAFAC (Three Groups)

- Rank-25 tensor for 357 out of 365 days of 2001: A $(69,157 \times 25)$, B (197×25), C (357×25)
- Groups 3,4,5:



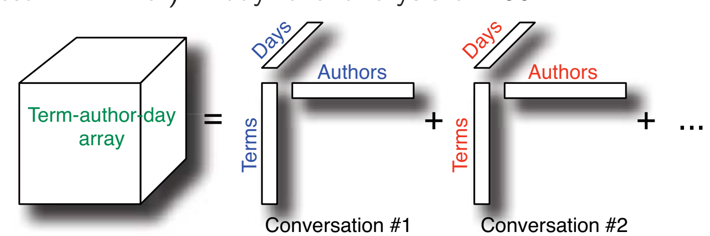
Day-level Analysis for NN-PARAFAC (Three Groups)

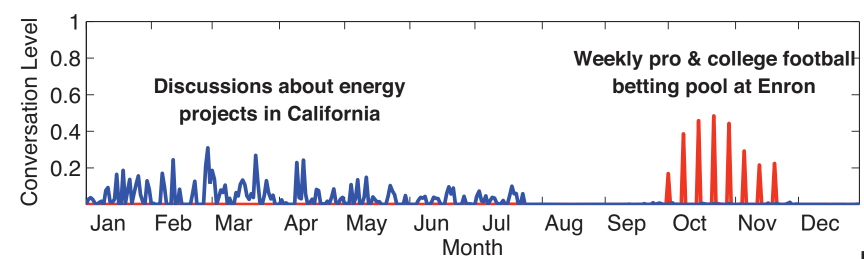
- Rank-25 tensor (best minimizer) for 357 out of 365 days of 2001: A $(69,157 \times 25)$, B (197×25), C (357×25)
- Groups 1,7,8:



Day-level Analysis for NN-PARAFAC (Two Groups)

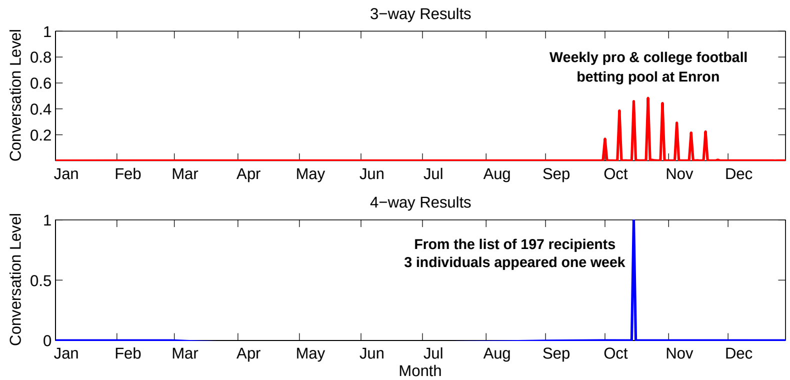
- Groups 20 (California Energy) and 9 (Football) (from C factor of best minimizer) in day-level analysis of 2001:





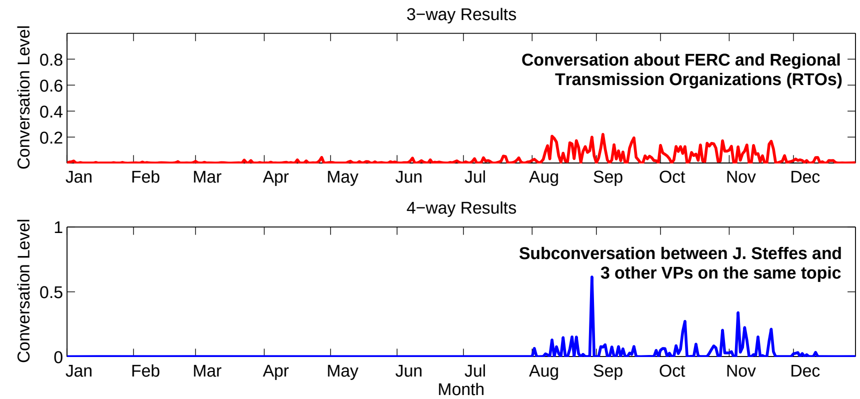
Four-way Tensor Results (Sept. 2007)

- Apply NN-PARAFAC to term-author-recipient-day array ($39,573 \times 197 \times 197 \times 357$); construct a rank-25 tensor (best minimizer among 10 runs).
- Goal: track more focused discussions between individuals/ small groups; for example, betting pool (football).



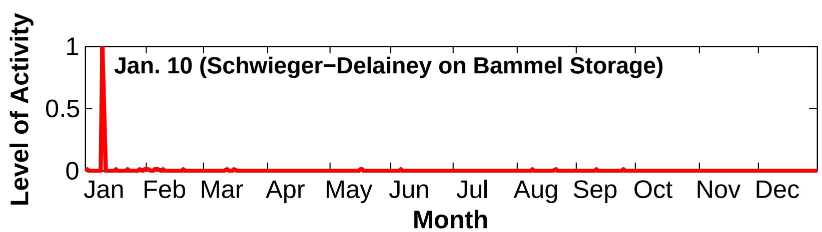
Four-way Tensor Results (Sept. 2007)

- Four-way tensor may track subconversation already found by three-way tensor; for example, RTO (Regional Transmission Organization) discussions.



Four-way Tensor Results (October 2007)

- Four-way tensor exposed conversation confirming bank fraud related to the natural gas reserves in the Bammel Storage field (Texas)—"The Enron whistle-blower who wasn't" by G. Farrell, **USA Today**, Oct. 11, 2007



NNTF Optimal Rank?

- No known algorithm for computing the rank of a k-way array for $k \geq 3$ [Kruskal, 1989].
- The maximum rank is **not a closed set** for a given random tensor.
- The maximum rank of a $m \times n \times k$ tensor is unknown; one weak inequality is given by

$$\max\{m, n, k\} \leq \text{rank} \leq \min\{m \times n, m \times k, n \times k\}$$

- For our rank-25 NNTF, the size of the relative residual norm suggests we are still far from the maximum rank of the 3-way and 4-way arrays.

Conclusions (NNMF for ASRS)

- Training phase was a good predictor of performance (for most anomalies).
- Obvious room for improvement in matching certain anomalies (e.g., 2. Noncompliance).
- Summarization of anomalies using NNMF features needs further work.
- Effects of sparsity contraints on NNMF versus element-wise filtering of **H** should be studied.

Conclusions (NNMF/NNTF for Enron)

- GD-CLS/NNMF Algorithm can effectively produce a *parts-based* approximation $X \simeq WH$ of a sparse term-by-message matrix X.
- Smoothing on the features matrix (W) as opposed to the weight matrix H forces more reuse of higher weighted (log-entropy) terms; yields potential **control vocabulary** for topic tracking.
- Surveillance systems based on NNMF/NNTF algorithms show promise for monitoring discussions without the need to isolate or perhaps incriminate individuals.
- Potential applications include the monitoring/tracking of company morale, employee feedback to policy decisions, extracurricular activities, and blog discussions.

Future Work

- Further work needed in determining effects of alternative term weighting schemes (for X) and choices of control parameters (e.g., α, β) for CNMF and NNTF/PARAFAC.
- How does document (or message) clustering change with different ranks (r) in GD-CLS and NNTF/PARAFAC?
- How many dimensions (factors) for NNTF/PARAFAC are really needed for mining electronic mail and similar corpora? And, at what **scale** should each dimension be measured (e.g., **time**)?

Improving Summarization and Steering

What versus why:

Extraction of textual concepts still requires human interpretation (in the absence of ontologies or domain-specific classifications).

How can previous knowledge or experience be captured for feature matching (or pruning)?

To what extent can feature vectors be annotated for future use or as the text collection is updated? What is the cost for updating NNMF/NNTF models?

For Further Reading

- M. Berry, M. Browne, A. Langville, V. Pauca, and R. Plemmons.
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  Comput. Stat. & Data Anal. 52(1):155-173, 2007.
- F. Shahnaz, M.W. Berry, V.P. Pauca, and R.J. Plemmons.
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  Info. Proc. & Management 42(2):373-386, 2006.
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  Comp. & Math. Org. Theory 11:249-264, 2005.
- J.T. Giles and L. Wo and M.W. Berry.
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  Software for Text Mining, in Statistical Data Mining and Knowledge Discovery. CRC Press, Boca Raton, FL, 2003, pp. 455-471.

For Further Reading (contd.)

- P. Hoyer.
  Nonnegative Matrix Factorization with Sparseness Constraints.
  J. Machine Learning Research 5:1457-1469, 2004.
- W. Xu, X. Liu, and Y. Gong.
  Document-Clustering based on Nonneg. Matrix Factorization.
  Proceedings of SIGIR'03, Toronto, CA, 2003, pp. 267-273.
- J.B. Kruskal.
  Rank, Decomposition, and Uniqueness for 3-way and n-way Arrays.
  In *Multiway Data Analysis*, Eds. R. Coppi and S. Bolaso, Elsevier 1989, pp. 7-18.