Learning to Rank for Web Search

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IPAM Workshop on Numerical Tools and Fast Algorithms for Massive Data Mining, Search Engines and Applications: October 22, 2007

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Web Ranking at 100,000 Feet

- Given a user's query, generate a large list of candidate urls
- For each url, compute a feature vector $\in R^n$
- Most features depend on the query and the document
- Feed your feature vectors into your ranker

  $R^n \rightarrow R$
- Sort the urls by the returned scores

Contents

- IR Metrics: Are We Learning the Wrong Thing?
- Brief Overview of Web Search
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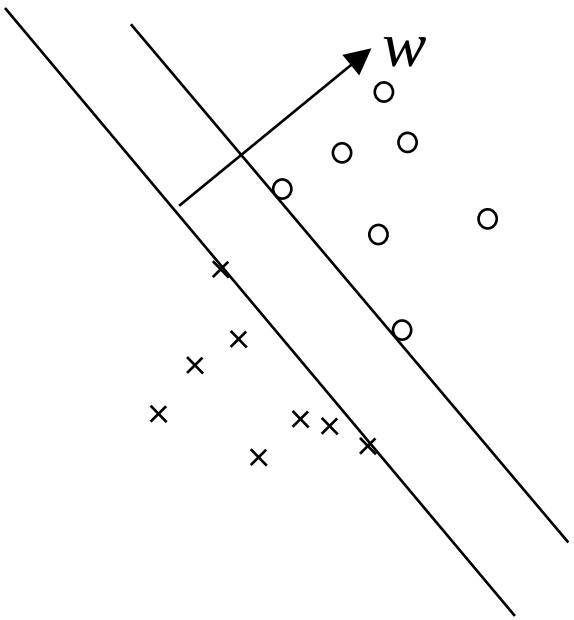
  - Optimal Combiners
  - Simultaneous Perturbation Stochastic Approx.
  - Gradient Boosting, DAGs for Ranking

Are We Learning the Wrong Thing?

An Example: SVMs for Classification



$$Minimize: \frac{1}{2}\|w\|^2 + C\sum_i \xi_i$$

$$subject\ to:\ y_i(x_i \cdot w + b) \geq 1 - \xi_i$$

$$y_i \in \{\pm 1\},\ \ \xi_i \geq 0$$

Information Retrieval Costs / Gains

- Precision: $\frac{number\ correct}{number\ accepted}$ Recall: $\frac{number\ correct}{number\ positives}$
- Average Precision: Compute precision for each positive, average over positions
- Mean Average Precision: Average AP over queries
- Mean Reciprocal Rank (TREC QA) $\frac{1}{N_Q}\sum_{i=1}^{N_Q}\frac{1}{r_i}$
- Winner Takes All: $\frac{1}{N_Q}\sum_{i=1}^{N_Q}\delta(l_{i1},1)$
- Pairwise error and derivatives ('bpref')
- Fraction pairwise correct = area under ROC curve (fraction true positives vs. fraction false positives)

Information Retrieval Gains, cont.

We'd like a measure for several levels of relevance, that emphasizes the top ranked items: *Normalized Discounted Cumulative Gain (Jarvelin & Kekalainen, SIGIR 2000):*

$$DCG(truncation\ level) = \sum_{i=1}^{T} \frac{2^{r(i)} - 1}{\log(1+i)}$$

$$r \in \{0, \ldots, 4\}\ human\ rating$$

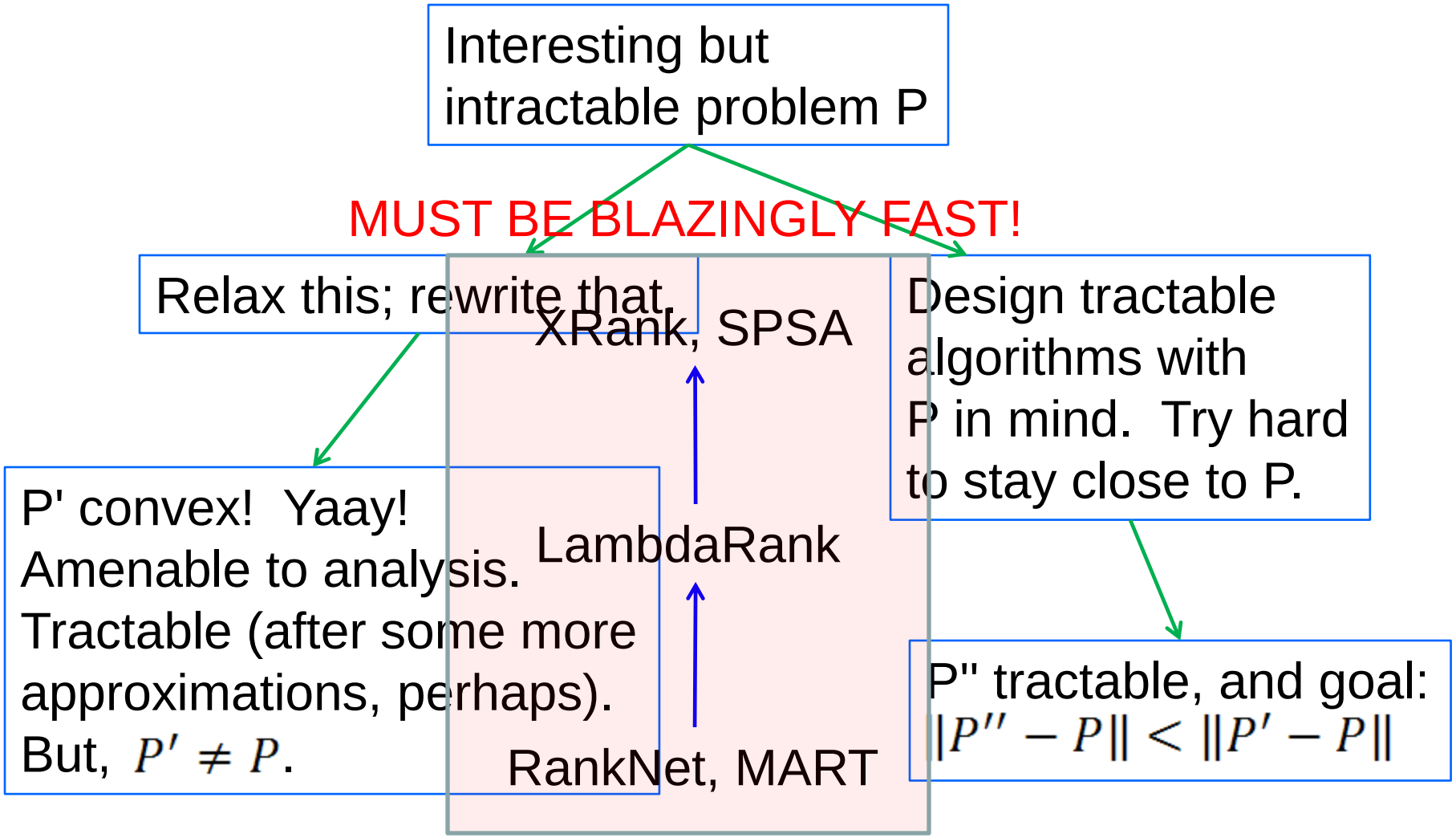
$$NDCG \equiv \frac{DGC(T)}{\max\left(DCG(T)\right)}$$

Information Retrieval Costs, cont.

All of these costs have the following in common:

- They depend only on the labels and the sorted order of the documents
- Viewed as a function of the scores output by some model, the costs are everywhere either:
  - **Flat (zero gradient)**
    or
  - **Discontinuous (gradient not defined)**

One View of the Goal



Web Search: Overview

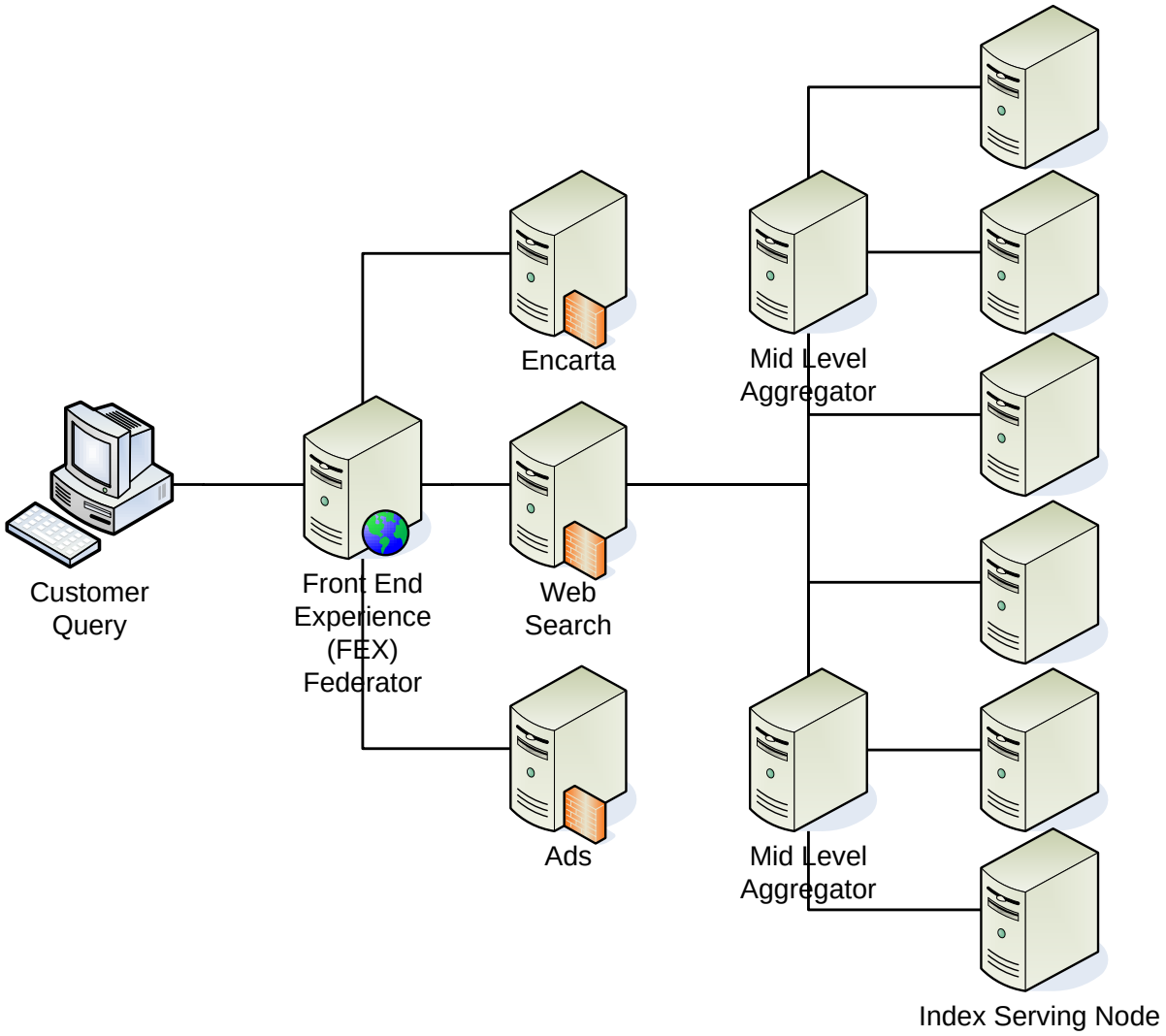
Why is Web Ranking Interesting?

- ***Economic:*** Internet advertising revenues were about $4 billion in 1Q 2006 (IAB, PricewaterhouseCoopers).
- ***Generality:*** Key component of Information Retrieval, Collaborative Filtering, Spam Detection, Question Answering…
- ***Scientific:*** New practical problems are motivating us to consider new machine learning approaches.

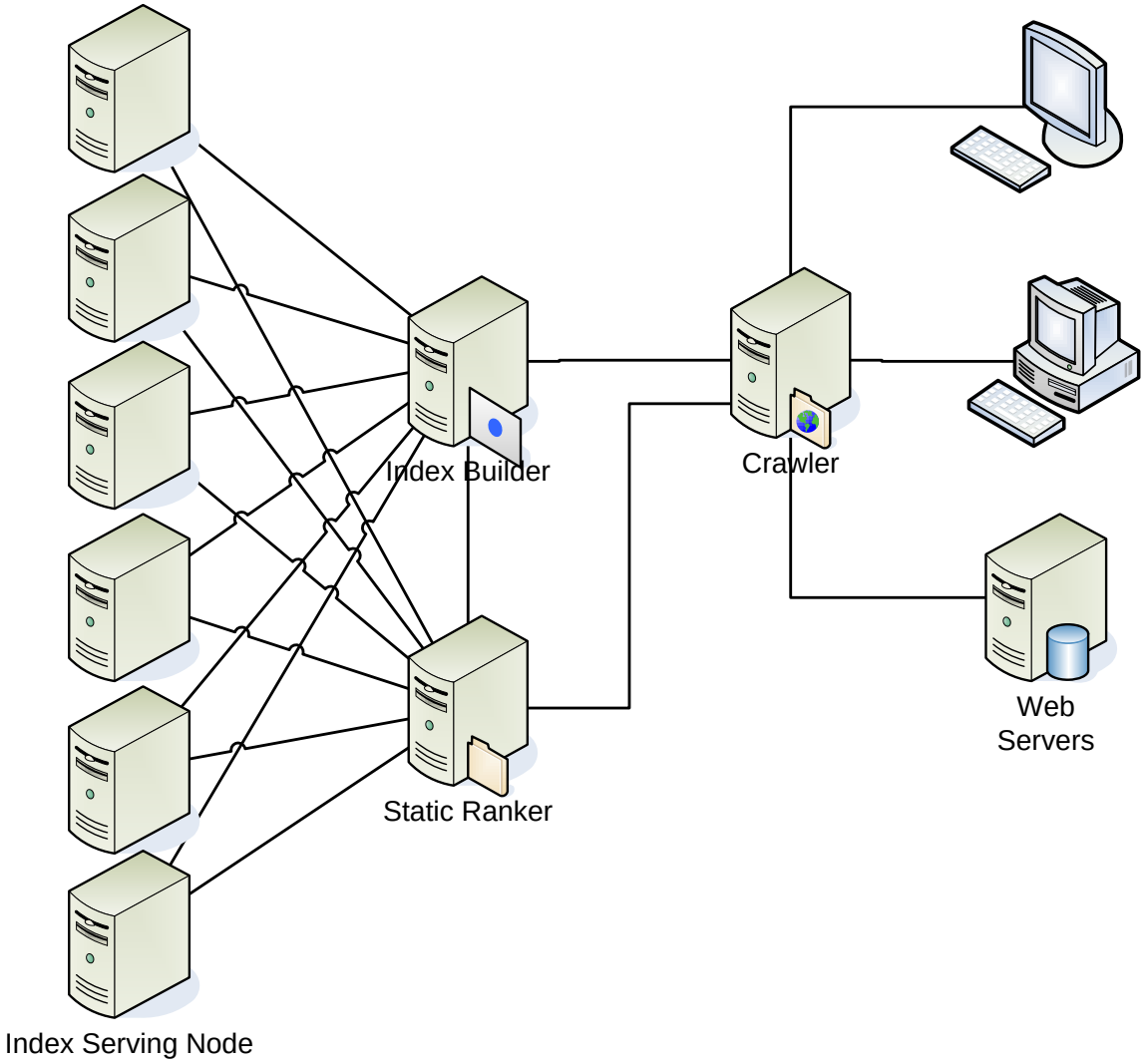
Live Search – Some Statistics

- > 5 billion documents indexed
- 10s of millions of queries handled per day
- Thousands of machines
- Most queries served in less than 100 ms

Live Search Architecture



Live Search Architecture, cont.



Some Recent Approaches to Ranking

Recent Work

- "*A Boosting Algorithm for Information Retrieval",* J. Xu and H. Li, *SIGIR '07* ("IR Boost")
- "*Learning to Rank Using Classification and Gradient Boosting",* P. Li, C. Burges and Q. Wu, *MSR-TR-2007-74*
- *"Direct Optimization of Ranking Measures",* Q.V. Le and A.J. Smola, http://arxiv.org/PS_cache/arxiv/pdf/0704/0704.3359v1.pdf, "DORM"
- "*Learning to Rank: From Pairwise to Listwise Approach*", Z. Cao, T. Qin, T-Y Liu, M-F Tsai, H. Li, *ICML 07* ("ListRank")
- "*FRank: A Ranking Method with Fidelity Loss",* M-F. Tsai, T-Y Liu, T. Qin, H-H Chen, W-Y Ma, *SIGIR 07*
- *"A Support Vector Method for Optimizing Average Precision",* Y. Yue, T. Finley, F. Radlinksi and T. Joachims, *SIGIR 07*
- *"Learning to Rank with Nonsmooth Cost Functions",* C.J.C. Burges, R. Ragno and Q.V. Le*, NIPS 06* ("LambdaRank")
- "*Learning to Rank using Gradient Descent*", C.J.C. Burges, T. Shaked, E. Renshaw, A. Lazier, M. Deeds, N. Hamilton, and G. Hullender, *ICML 05* ("RankNet")

Less Recent Work

- "*An Efficient Boosting Algorithm for Combining Preferences*", Y. Freund, R. Iyer, R.E. Schapire and Y. Singer, *JMLR 03* ("RankBoost")
- "*PRanking with Ranking",* K. Crammer and Y. Singer, *KDD 02* ("PRank")
- "*Online Ranking/Collaborative Filtering using the Perceptron Algorithm",* E.F. Harrington, *ICML 03*
- *"Optimizing Search Engines Using Clickthrough Data",* T. Joachims, *KDD 02*
- *"Support Vector Learning for Ordinal Regression",* R. Herbrich, T. Graepel and K. Obermayer, *ICANN 99*
- *"Using the Future to Sort Out the Present: RankProp and Multitask Learning for Medical Risk Evaluation",* R. Caruana, S. Baluja and T. Mitchell, *NIPS 96*

Ranking as Machine Learning

- Given a set of text queries $Q_i, i = 1, \ldots, m$
- Each query has a large set of returned documents $D_{ij}, j = 1, \ldots, n$
- Use query, document, URL, anchor text, and more, to derive set of (several hundred) features
- For each query, rank returned documents in order of relevance
- Most systems map a feature vector to a single score, which is then sorted to obtain the ranking

RankNet: A Starting Point

Joint work with T. Shaked, E. Renshaw, A. Lazier, M. Deeds, N. Hamilton, and G. Hullender, ICML 2005

Ranking with Neural Nets

- Don't need to learn ordinal regression (mapping points to actual rank values); just need to map features to reals.
- Train system on pairs (where first point is to be ranked higher or equal to second).
- However must evaluate on single points.
- Use cross entropy cost → probabilistic model.
- Use gradient descent. Would work for any differentiable function: we chose neural net.

RankNet: Notes

- 5 human judged levels of relevance (“bad” … “perfect”).
- A net with (number of features) inputs and one output
- Sort documents by the score that their feature vectors (which are computed from query + doc + other data) are mapped to
- *Compute NDCG on a set-aside validation set, keep the net that gives the best validation NDCG*

A Probabilistic Ranking Cost Fn.

• Ranking labels tend to be noisier than classification labels

Specify $P(A \rhd B)$ for each train pair $\{A, B\}$

• The pairs of training ranks need not be complete, or consistent, but the test results are

Map to reals: $f(x_1) > f(x_2) \leftrightarrow x_1 \rhd x_2$

Probabilistic Ranking Cost Fn.

Modeled posteriors: $P_{ij} \equiv P(x_i \rhd x_j)$

Target posteriors: $\bar{P}_{ij}$

Define $o_{ij} \equiv f(x_i) - f(x_j)$

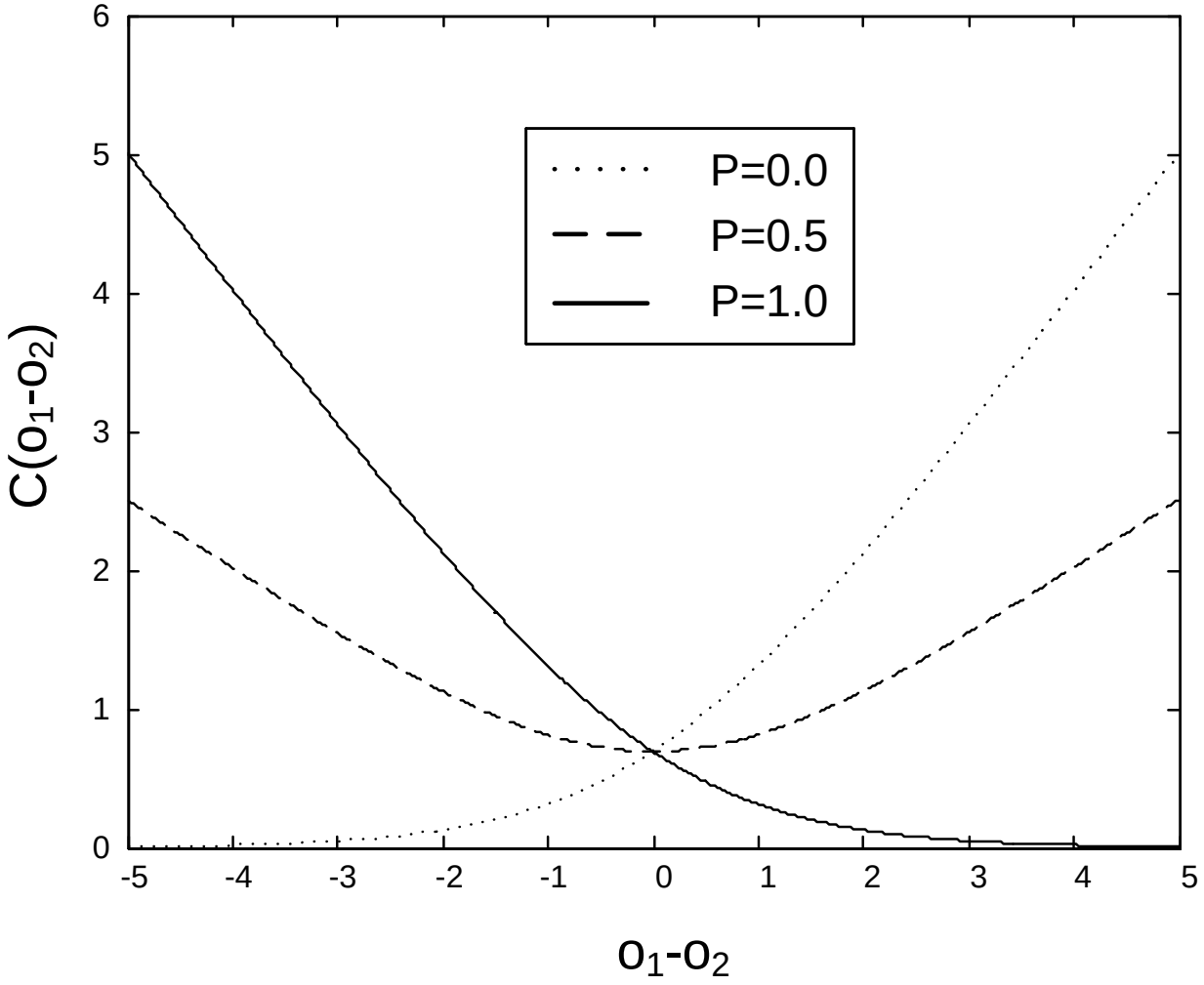
Cross entropy cost:

$$C_{ij} \equiv C(o_{ij}) = -\bar{P}_{ij} \log P_{ij} - (1 - \bar{P}_{ij}) \log(1 - P_{ij})$$

Model output probabilities using logistic:

$$P_{ij} = \frac{\exp(o_{ij})}{1 + \exp(o_{ij})}$$

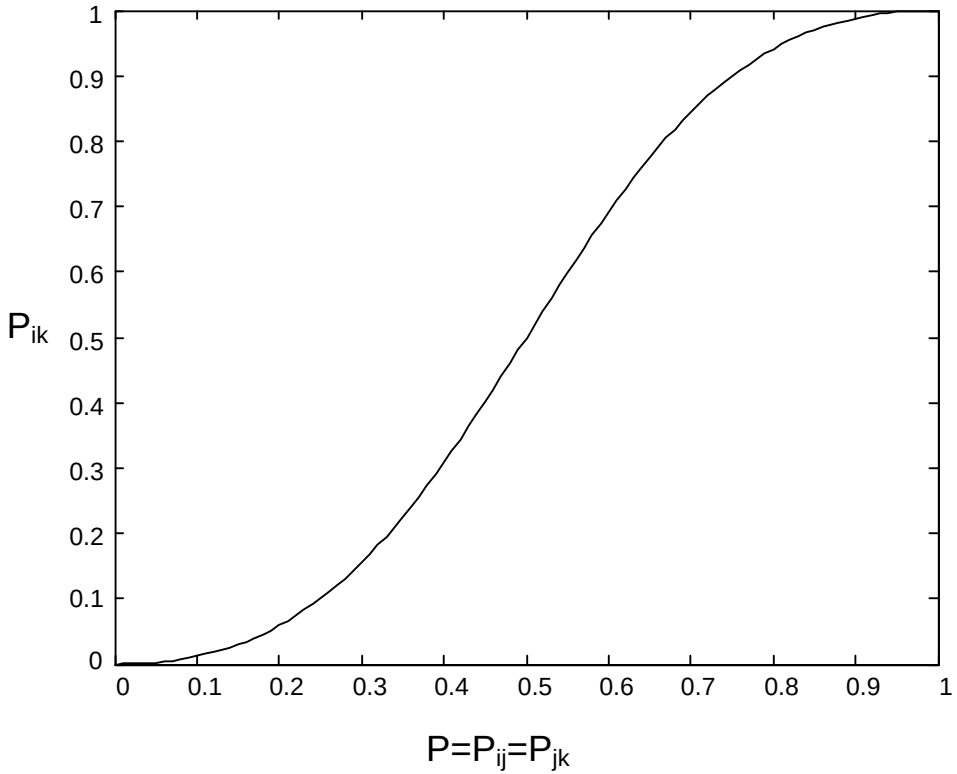
$$\Rightarrow C_{ij} = -\bar{P}_{ij} o_{ij} + \log(1 + \exp o_{ij})$$


P=0.0
P=0.5
P=1.0
C(o1-o2)
o1-o2
6
5
4
3
2
1
0
-5
-4
-3
-2
-1
0
1
2
3
4
5

Consistency Requirements

There must exist $\bar{o}_{ij}$ such that: $\bar{P}_{ij} = \frac{\exp(\bar{o}_{ij})}{1+\exp(\bar{o}_{ij})}$

$$\bar{P}_{ik} = \frac{\bar{P}_{ij}\bar{P}_{jk}}{1+2\bar{P}_{ij}\bar{P}_{jk}-\bar{P}_{ij}-\bar{P}_{jk}}$$



Complete uncertainty propagates, similarly for complete certainty:

$$P(A \triangleright B) = 0.5,\ \ P(B \triangleright C) = 0.5 \Rightarrow P(A \triangleright C) = 0.5$$

Confidences build: for $0 < P < 0.5,\ \ \bar{P}_{ik} < P;$

for $0.5 < P < 1.0,\ \ \bar{P}_{ik} > P$

More Formally:

How free are we to choose the $\bar{P}'s$?

Proposition: Specifying any set of adjacency posteriors is necessary and sufficient to uniquely determine a target posterior for every pair of samples.

Proposition: Let $n > 0$. Then if $\bar{P} > \frac{1}{2}$, then $\bar{P}_{i,i+n} \geq \bar{P}$ with equality when $n = 1$, and $\bar{P}_{i,i+n}$ increases strictly monotonically with n. If $\bar{P} < \frac{1}{2}$, then $\bar{P}_{i,i+n}$ decreases strictly monotonically with n. If $\bar{P} = \frac{1}{2}$, then $\bar{P}_{i,i+n} = \frac{1}{2} \forall\ n$.

The Bradley-Terry Model

Bradley and Terry, Biometrika 1952 consider models:

$P(A_i \mid A_i \vee A_j)$ given, and model $P(A_i \mid A_i \vee A_j) = \dfrac{P_i}{P_i + P_j}$

$(e.g.\ P_i = N \exp(o_i))$.

RankNet: Data

Training on pairs prohibitive? No:

- Docs are only compared to other docs for the same query, and many docs have the same label.

	# Queries	# Documents	# Pairs
Train	11,336	384,314	3,464,289
Valid	2,384	2,726,714	-
Test	2,384	2,715,175	-

- Features from 4 ‘streams’: anchor text, URL, document title, and document body.
- 569 features: most are joint (query/doc dependent).

Results

Mean NDCG:	Validation	Test (95%)
Quad PRank	0.379	0.327±.011
Linear PRank	0.410	0.412±.010
Large Margin PR	0.455	0.454±.011
1-layer Net	0.479	0.477±.010
2-layer Net	0.489	0.488±.010

Mean NDCG: Training Set	
1-layer Net	0.479±0.005
2-layer Net	0.500±0.005

RankNet Conclusions

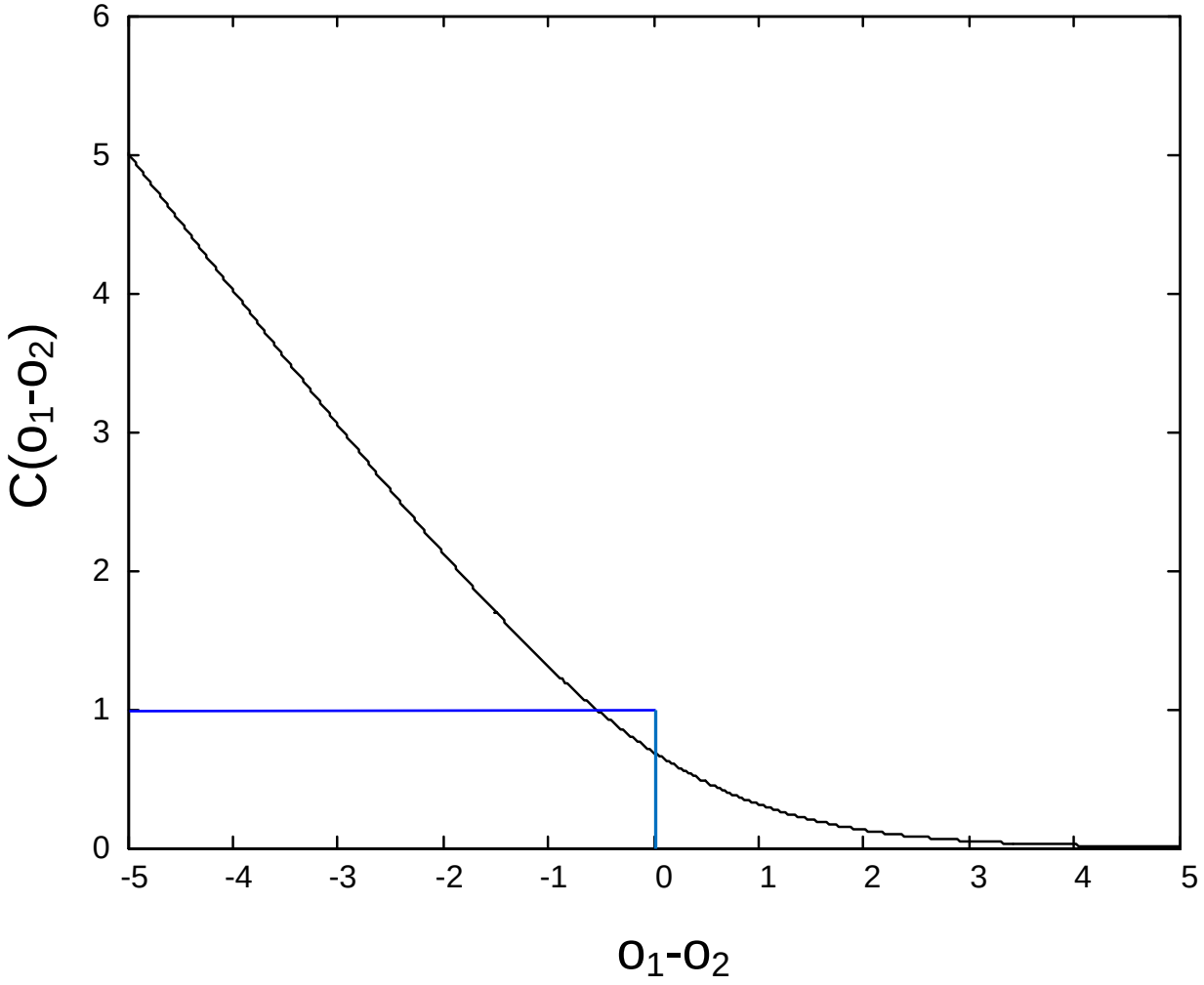
- RankNet is simple to train…
- …fast in test phase…
- and gives good results.
- For pair-based probability costs (e.g. click rates!) it's very well suited to the problem.
- However the cost function used is not NDCG: the latter is optimized only indirectly, using a validation set.

Can we do better?

LambdaRank

Joint work with R. Ragno, Q.V. Le, NIPS 2006

RankNet Cost ~ Pairwise Cost

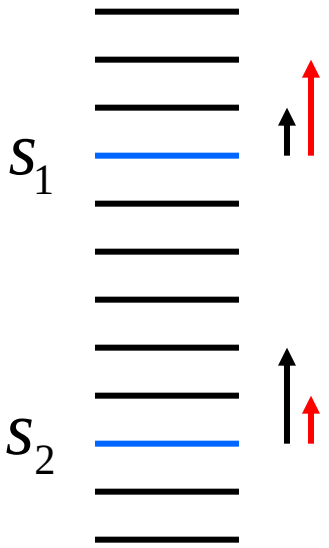


Pairwise Cost Revisited

Pairwise cost fine if no errors, but:

LambdaRank

Instead of using a smooth approximation to the cost, and taking derivatives, write down the derivatives directly.



$$Want: \left|\frac{\partial C}{\partial s_1}\right| \gg \left|\frac{\partial C}{\partial s_2}\right|$$

Then use these derivatives to train a model using gradient descent, as usual.

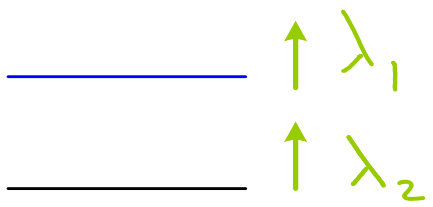
A Simple Example

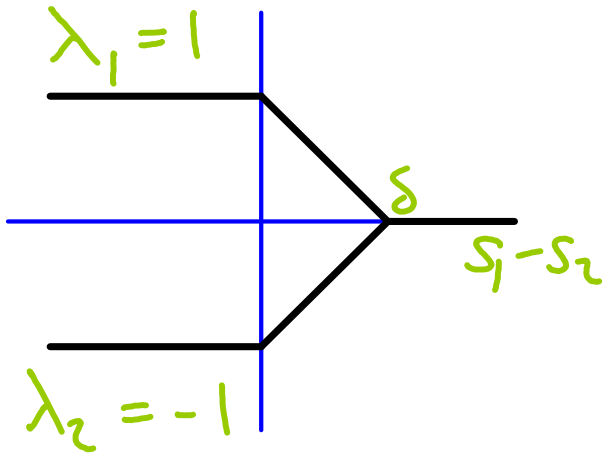
$$D_1, D_2: \; l_1 = 1, l_2 = 0$$

Imagine some cost *C*:

$$\frac{\partial C}{\partial s_1} = -\lambda_1(s_1, l_1, s_2, l_2)$$

$$\frac{\partial C}{\partial s_2} = -\lambda_2(s_1, l_1, s_2, l_2)$$





Letting $x = s_1 - s_2$:

$$x < 0: \quad \lambda_1 = 1 = -\lambda_2$$
$$0 \le x \le \delta: \quad \lambda_1 = \delta - x = -\lambda_2$$
$$x > \delta: \quad \lambda_1 = \lambda_2 = 0$$

Then a cost function C exists:

$$x < 0: \quad C(s_1, l_1, s_2, l_2) = s_2 - s_1 + \tfrac{1}{2}\delta^2$$
$$0 \le x \le \delta: \quad C(s_1, l_1, s_2, l_2) = \tfrac{1}{2}(s_2 - s_1)^2 - \delta(s_1 - s_2) + \tfrac{1}{2}\delta^2$$
$$x > \delta: \quad C(s_1, l_1, s_2, l_2) = 0$$

…furthermore it's convex

LambdaRank

- Choose the $\lambda's$ to model the desired cost. *(Need not use pairs!)*
- Very general. Handles multivariate, non-smooth costs.
- But, how to choose the $\lambda's$?
- When will there exist a cost function *C* for your choice of $\lambda's$?
- When will that *C* be convex?

Some Multilinear Algebra Basics

- An '*n*-form' on a manifold *M* is a totally antisymmetric tensor that lives in the dual of the tangent space of *M*
- You can apply the differential operator *d* to an *n*-form to get an *(n+1)*-form
- A *closed* form *f* is one for which *df=0*
- An *exact* form *g* is one for which *g=dh*, for some form *h*
- *dd=0* (every exact form is closed)

Poincare's Lemma

If $S \subset \mathrm{R}^n$ is an open set that is star-shaped with respect to the origin, then any closed form defined on S is exact.

Hence on such a set, a form is exact iff it is closed.

Define the 1-form $\lambda \equiv \sum_i \lambda_i dx_i$

Then $\lambda = dC$ for some C iff $d\lambda = 0$.

Using classical notation: $\frac{\partial \lambda_i}{\partial x_j} = \frac{\partial \lambda_j}{\partial x_i} \quad \forall i, j$: Jacobian symmetric!

The Jacobian

- Square matrix, of side nDocs
- Family of Jacobians, one for each label set
- Symmetric → cost function exists
- Positive semidefinite → cost function is convex
- (…like a kernel, but more general: depends on all points!)

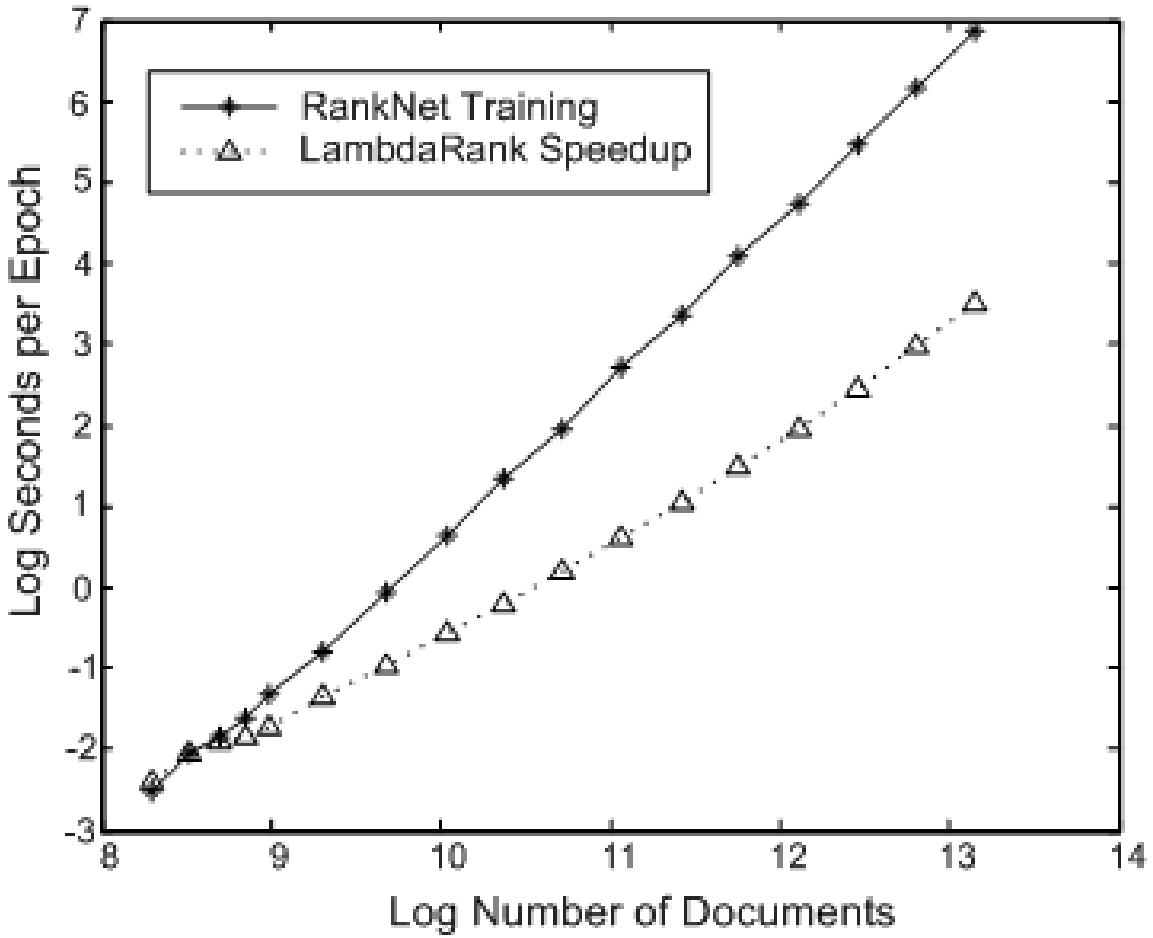
A Physical Analogy

- Think of ranked documents as point masses, $\lambda's$ as forces
- If $\lambda = dC$, the forces are conservative – they derive from a potential
- E.g. choosing the $\lambda's$ to be linear in the scores is equivalent to a spring model

LambdaRank Speedup for RankNet

- Most neural net training is stochastic (update weights after every pattern)
- Here we can compute and increment the gradients for each document (mini batch)
- Batch them, apply fprop and backprop once per doc, per query; *factorize the gradient.*

Speedup Results



The Lambda Function

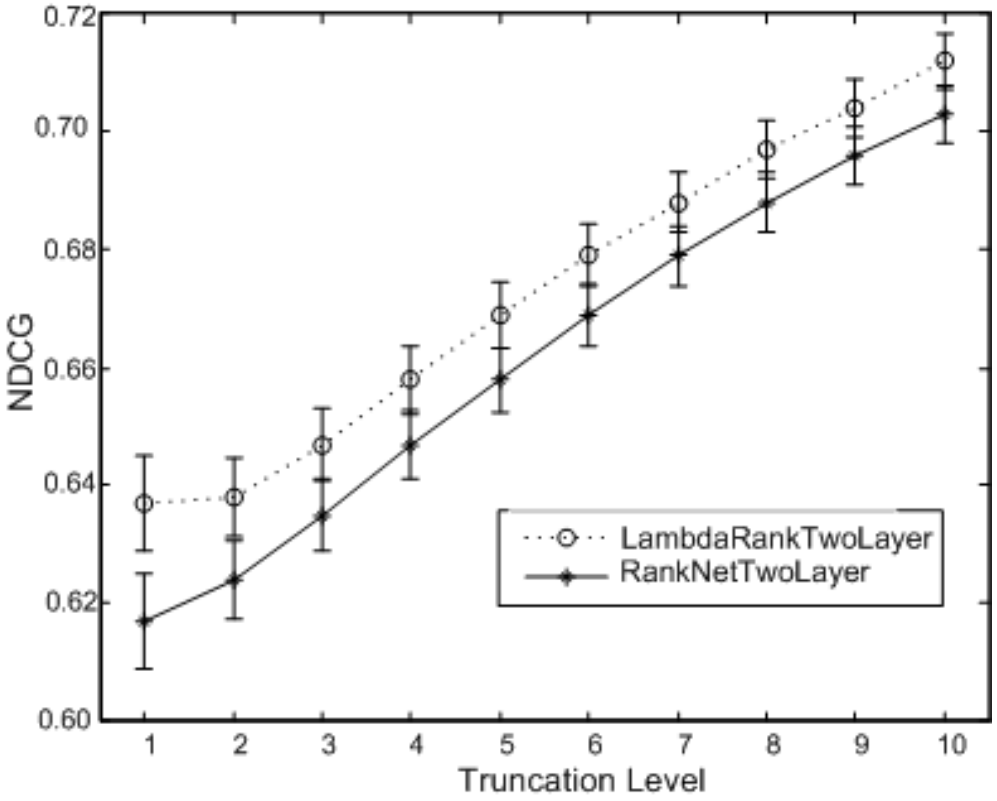
NDCG gain in swapping members of a pair of docs, multiplied by RankNet cost gradient as a smoother:

$$\lambda_{ij} = \frac{N}{1 + e^{s_i - s_j}} \left| \left(2^{l(i)} - 2^{l(j)}\right) \left(\log\left(\frac{1}{1+i}\right) - \log\left(\frac{1}{1+j}\right) \right) \right|$$

Let D_i^+ (D_i^-) be the set of documents labeled higher (lower) than document i

$$\lambda_i \equiv \sum_{j \in D_i^-} \lambda_{ji} - \sum_{j \in D_i^+} \lambda_{ij}$$

Accuracy Results



10K train, 5K validation, 10K test queries

Robustness to Label Noise

- Query / URL pairs are very hard to label accurately
  - What was the user's intent?
  - If there are several possible intents, how to label? (Ensure diversity of results)
- Can measure label noise by overlapping judgments. Generate confusion matrix: e.g.

$$P(other\ label\ is\ 'bad' | either\ label\ was\ 'excellent')$$

Measure Robustness

- Generate artificial data:
  - 300 features
  - 30 documents per query
  - 40K queries, 10K validation, 10K test
  - Generate clean labels
- Train LambdaRank
- Apply confusion matrix to train+valid data
- Retrain, but test on *clean* data

Robustness Results

	Best NDCG@10 on Validation during training	Test on Clean: NDCG@10/3/1	Test on Confused-Once: NDCG@10/3/1	Test on Confused-Twice NDCG@10/3/1
Train on Clean, Validate on Clean	0.705	0.704/0.634/0.587		
Train on Confused-Once, Validate on Confused-Once	0.529	0.698/0.626/0.578	0.526/0.437/0.389	
Train on Confused-Twice, Validate on Confused-Twice	0.439	0.695/0.624/0.573		0.438/0.347/0.303

- LambdaRank with 2 layer 10 hidden unit nets, is strongly regularized
- More flexible models do better on clean data

LambdaRank Conclusions

- LambdaRank is simple and general (it can handle any cost function) but… how to choose λ ?
- It leverages existing neural net methods (only the training changes)
- It gives a very significant speedup for RankNet
- It gives better accuracy than RankNet
- LambdaRank + 2 layer nets are well suited to the level of noise
- It still does not directly optimize NDCG!

Can we do better?

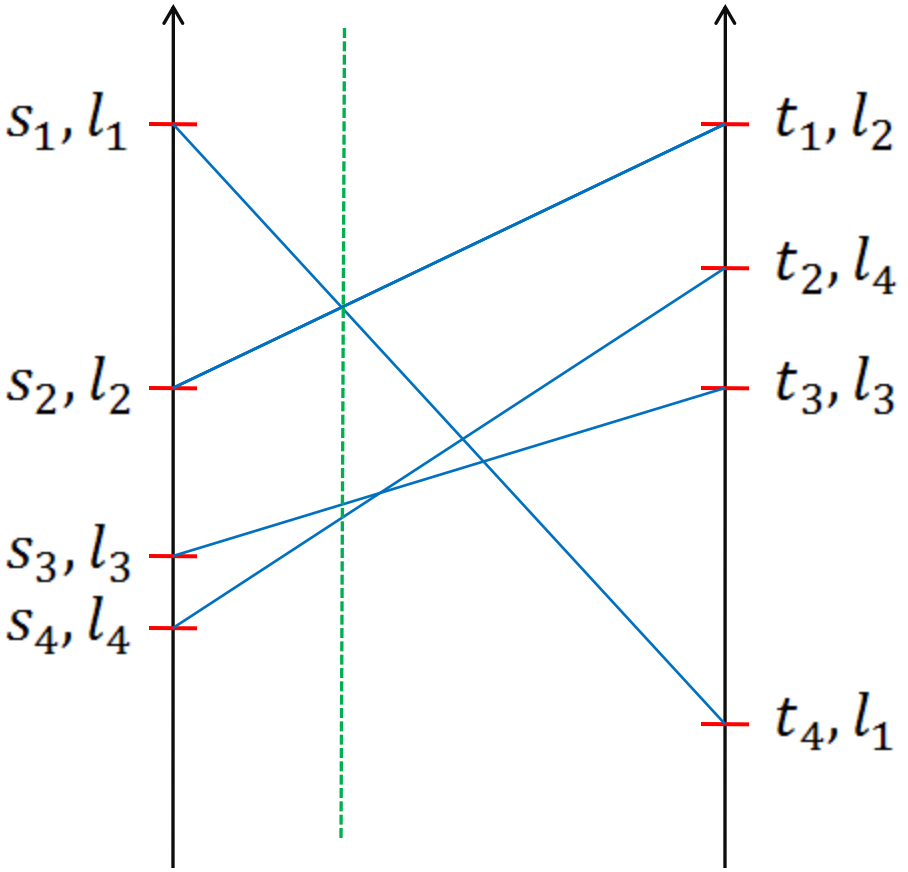
Optimal Combiners: or, Turning the Unpleasantness of IR Metrics to Our Advantage

Key Observations

- The “flat or discontinuous” nature of IR metrics means that when linearly combining two rankers $(s = (1 - \alpha)s_1 + \alpha s_2)$ we only have to examine a finite number of values of α
- This examination is no more expensive than the gradient computation in RankNet $(O(N_Q m^2))$

How does this work?

$$(1-\alpha)s_1 + \alpha t_4 = (1-\alpha)s_2 + \alpha t_1$$



The Algorithm

- Compute α for all pairs for each query.
- Sort values of α.
- Sweep through values of α, computing delta NDCG each time.
- Keep track of ranks as lines cross.
- *Lemma: barring degeneracy (three or more lines meet at a point), ranks will always change by 1.*
- Extend to n rankers by iterating.

How Is This Useful?

- Boosting is an iterative procedure for finding ranking functions of the form

$$F(x) = \sum_{i=1}^{n} w_i f_i(x)$$

- Each f_i may be viewed as a gradient in function space (Friedman, TR, 1999; Mason et al., NIPS 2000)
- Boosting has two steps: find f_i, find w_i
- Can use an optimal combiner to find w_i

Simultaneous Perturbation Stochastic Approximation

Joint work with Yisong Yue

http://research.microsoft.com/~cburges/tech_reports/tr-2007-115-spsa.pdf

J. Spall, *Multivariate Stochastic Approximation using SPSA, IEEE Trans. Autom. Control, 1992*

Back to Basics

So you want to learn NDCG directly. *Why not just use gradient descent?*

The Finite Difference Method

Objective function L, parameters $w \in \mathcal{R}^d$, $c \ll 1$ and $[e_i]_j \equiv \delta_{ij}$

$$\hat{g}(w) = \begin{bmatrix} \frac{L(w+ce_1)-L(w-ce_1)}{2c} \\ \frac{L(w+ce_2)-L(w-ce_2)}{2c} \\ \dots \\ \dots \\ \frac{L(w+ce_{P-1})-L(w-ce_{P-1})}{2c} \\ \frac{L(w+ce_P)-L(w-ce_P)}{2c} \end{bmatrix}$$

… requires *2d* function evaluations.

Simultaneous Perturbation Stochastic Approximation (SPSA)

A general method for performing gradient descent when the gradient is too slow (or is impossible) to compute: $\Delta \in \{\pm 1\}^d, \; P(\Delta_i = 1) = 0.5$

$$\begin{bmatrix} \frac{1}{\Delta_1} \\ \frac{1}{\Delta_2} \\ \vdots \\ \frac{1}{\Delta_{p-1}} \\ \frac{1}{\Delta_p} \end{bmatrix} \left(\frac{L(w + c\Delta) - L(w - c\Delta)}{2c} \right)$$

Why Does This Work?

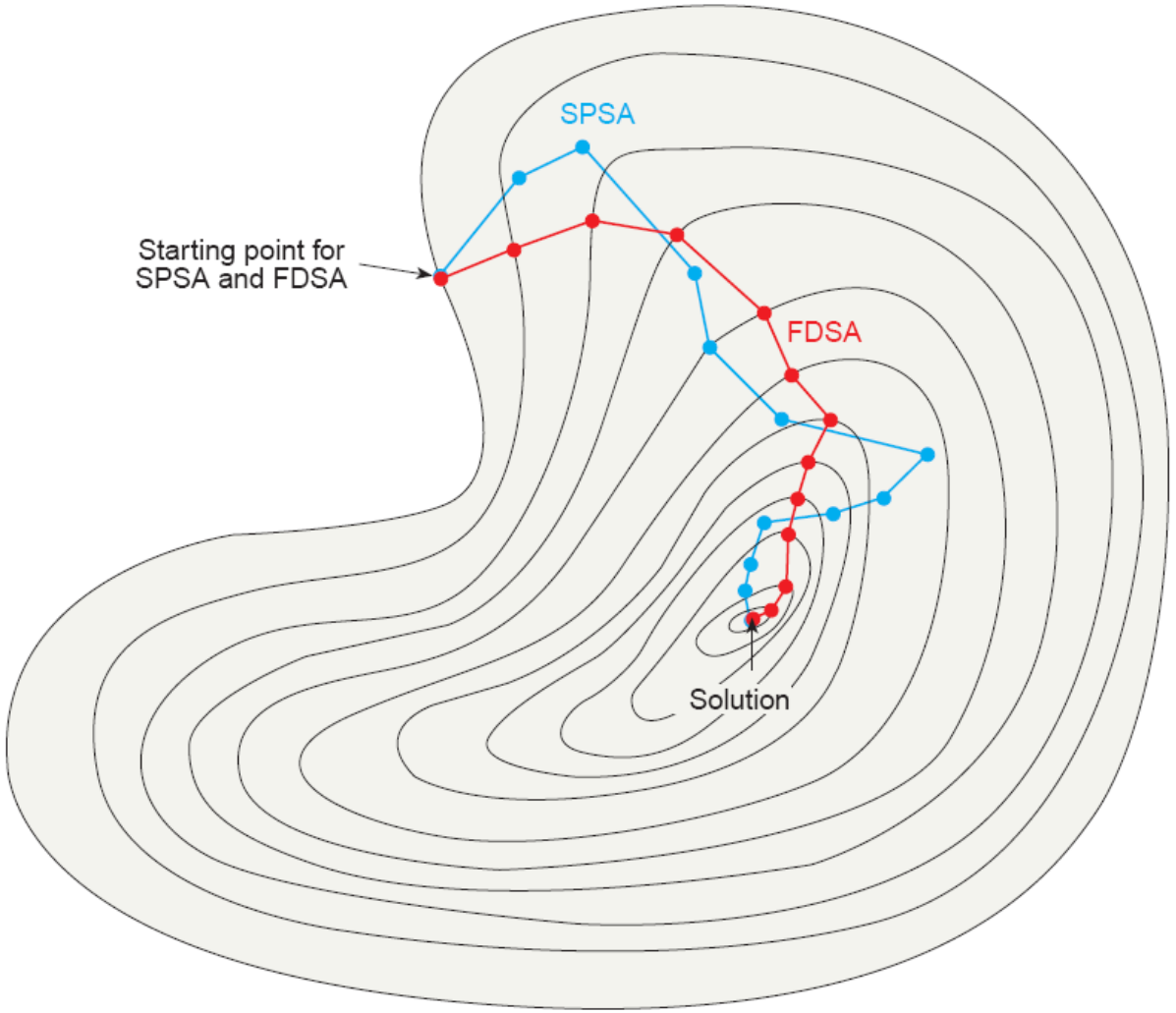
Spall's Lemma 1 (paraphrased): if the cost function's third derivatives are bounded everywhere (independent of iteration number *k*), and if the Δ's are iid component-wise and also satisfy some simple moment conditions (which *are* satisfied by the symmetric Bernoulli distribution), then:

$$E[\hat{g}_k - g_k | w_k] = O(c_k^2)$$

$$|\Delta_{ki}| \leq \alpha_0 \qquad E\left[\left|\Delta_{ki}^{-1}\right|\right] \leq \alpha_1 \qquad \left|\frac{\partial^3 L}{\partial w_i \partial w_j \partial w_k}\right| \leq \alpha_2$$

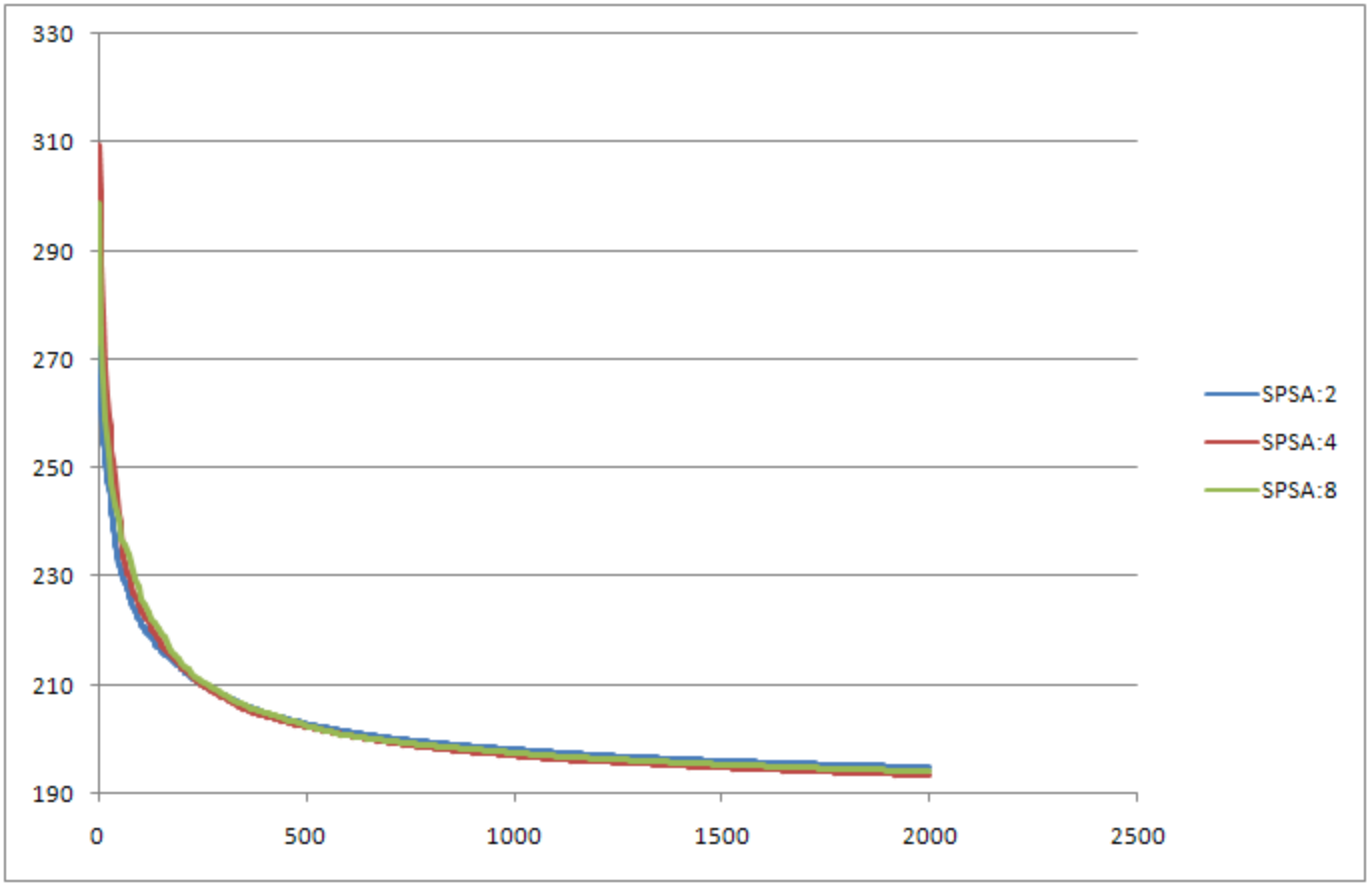
$$E[\hat{g}_k - g_k | w_k] \leq \frac{1}{6}\alpha_2 c_k^2 \{(p^3 - (p-1)^3)\alpha_0^2 + (p-1)^3 \alpha_1 \alpha_0^3\}$$

A Schematic View

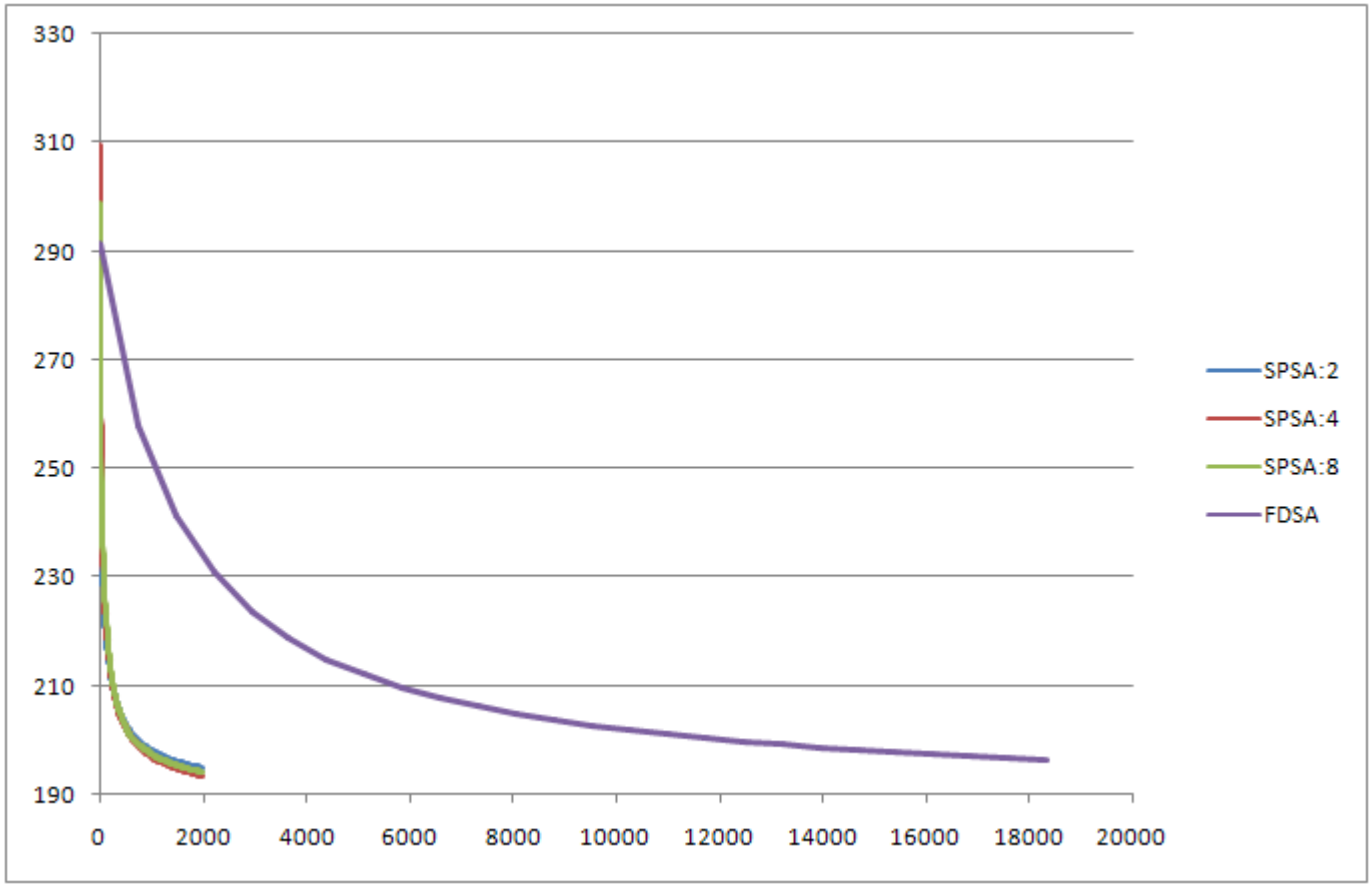


From Spall 1998, with permission

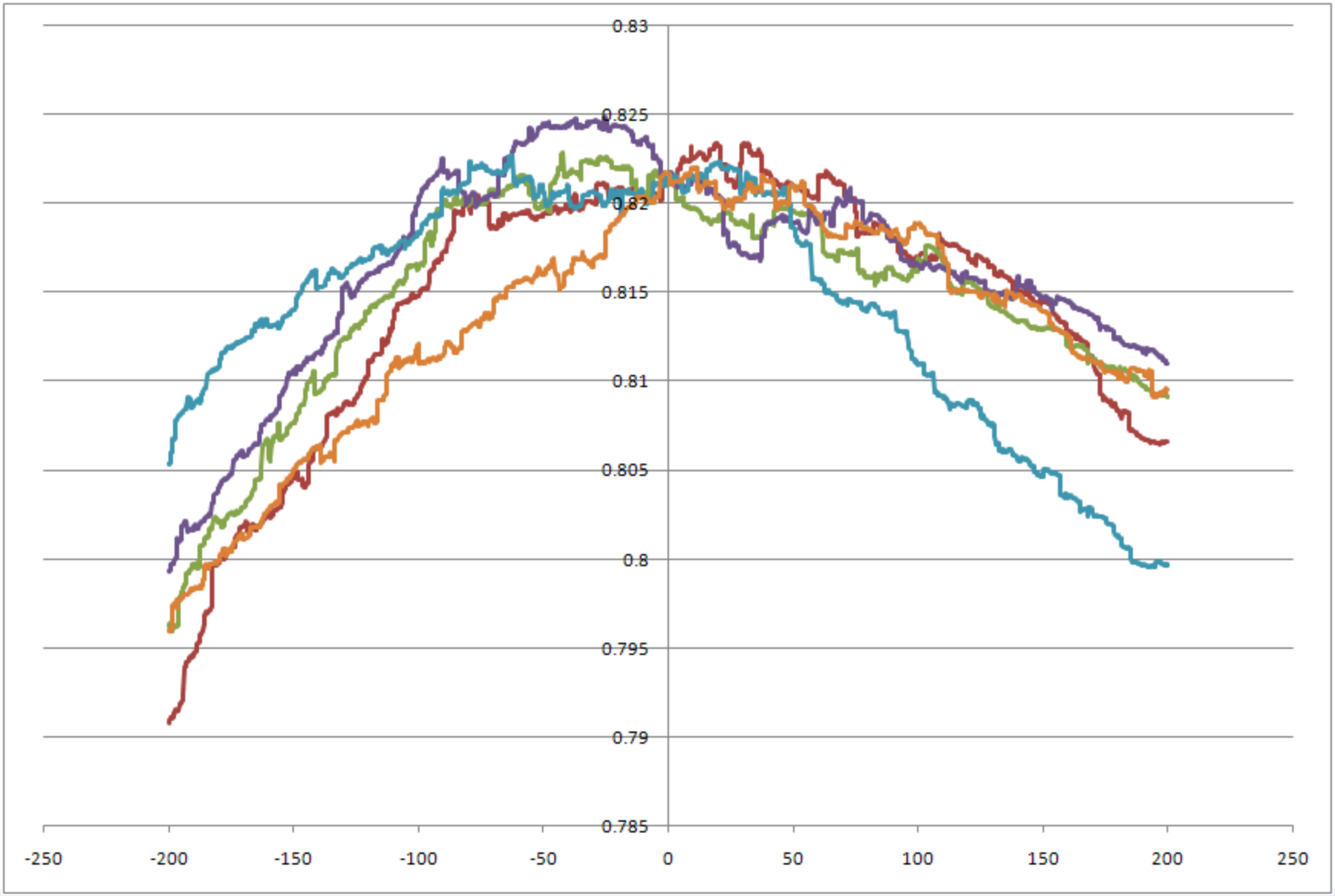
SPSA Results: Cross Entropy, Web



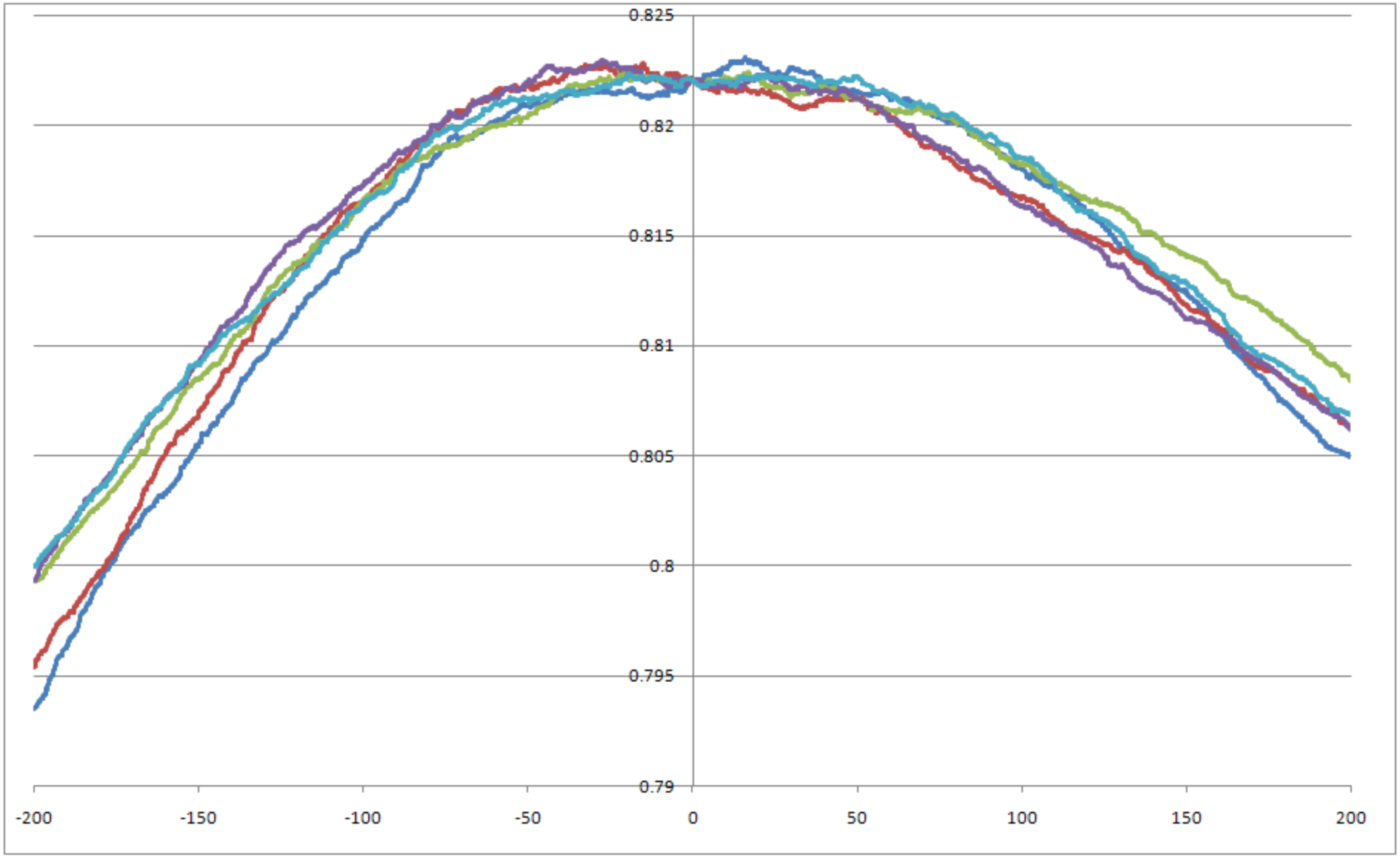
SPSA versus FDSA



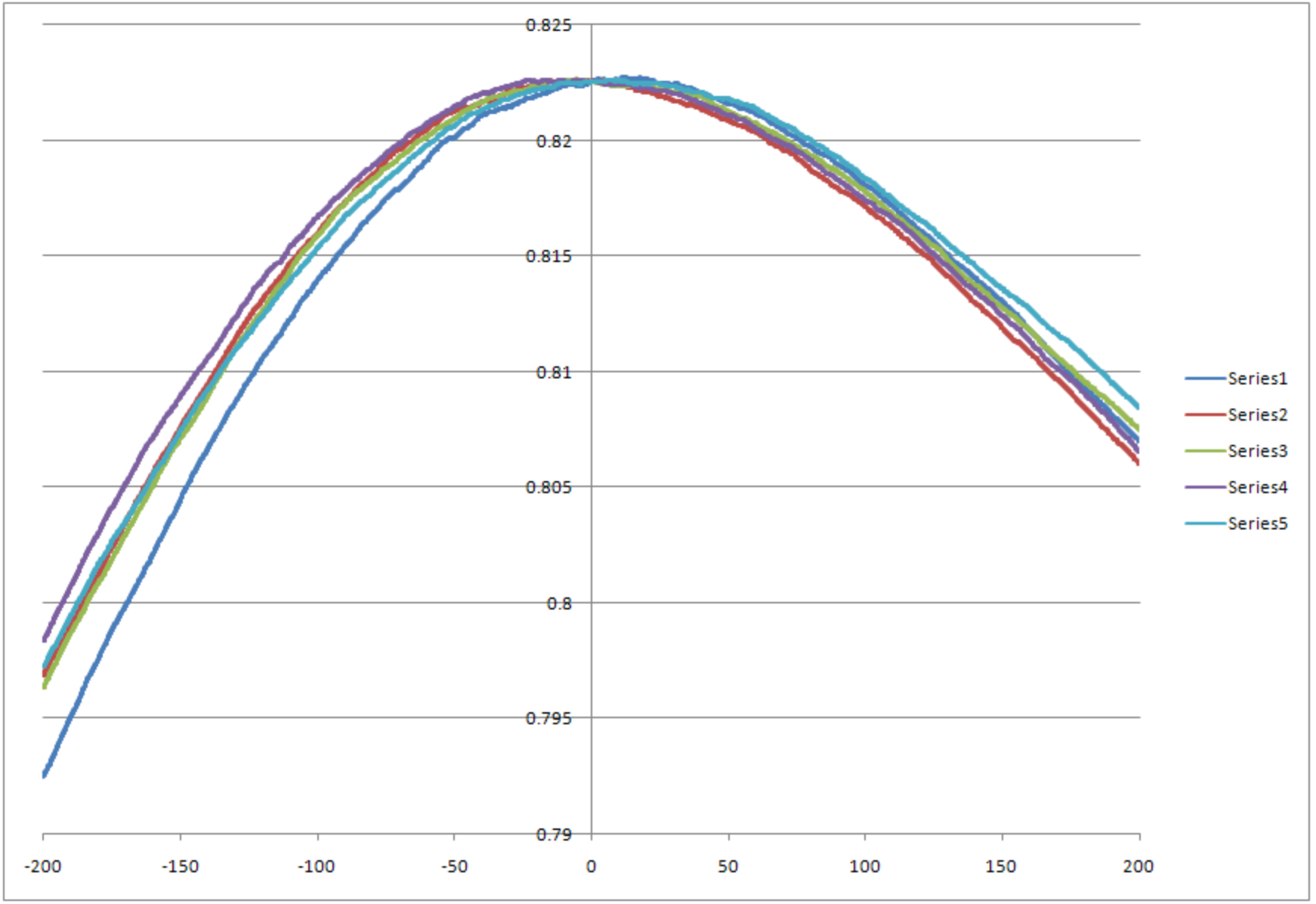
Smoothness Tests: 100 Queries, Toy



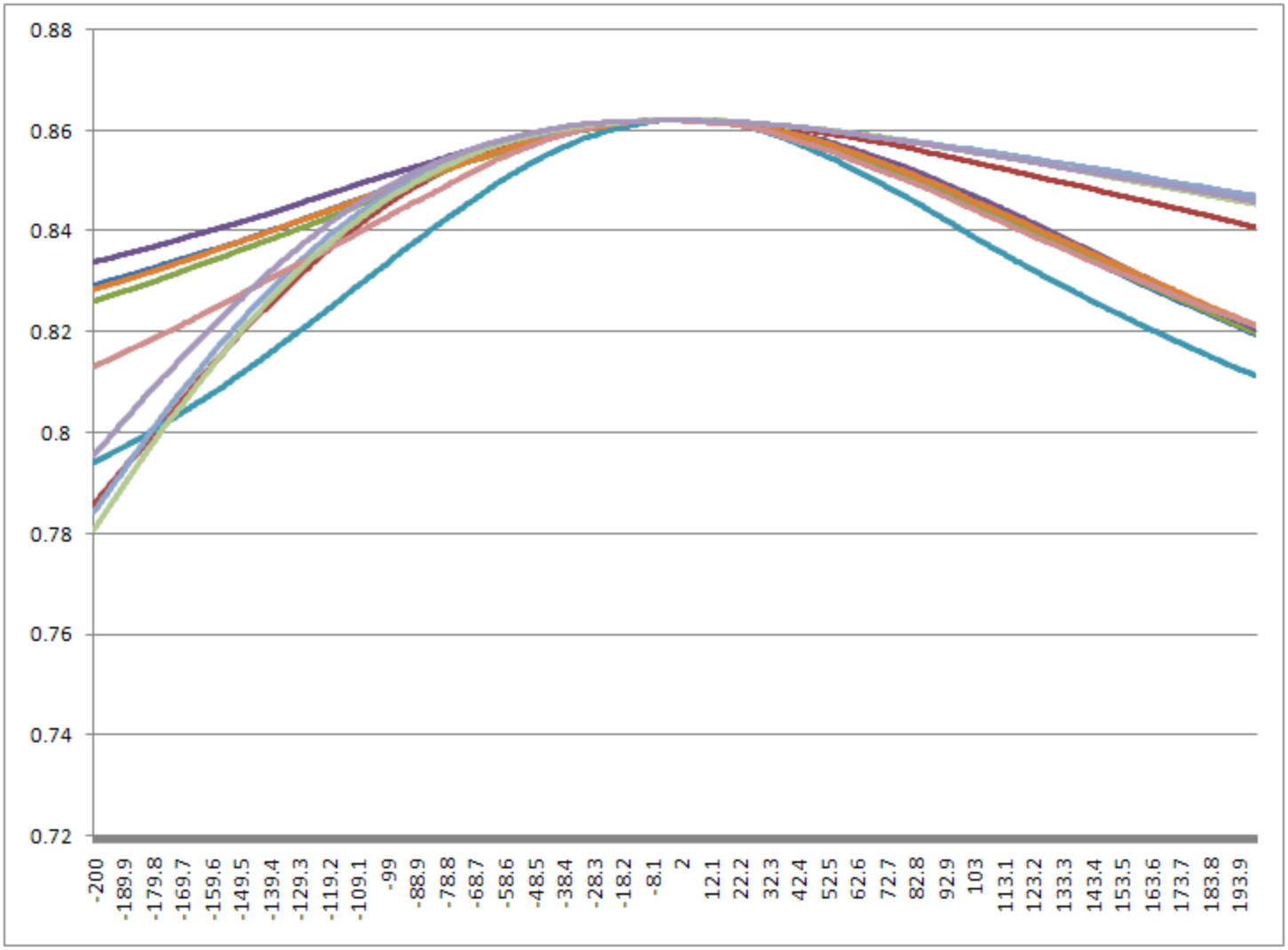
Smoothness: 1000 Queries



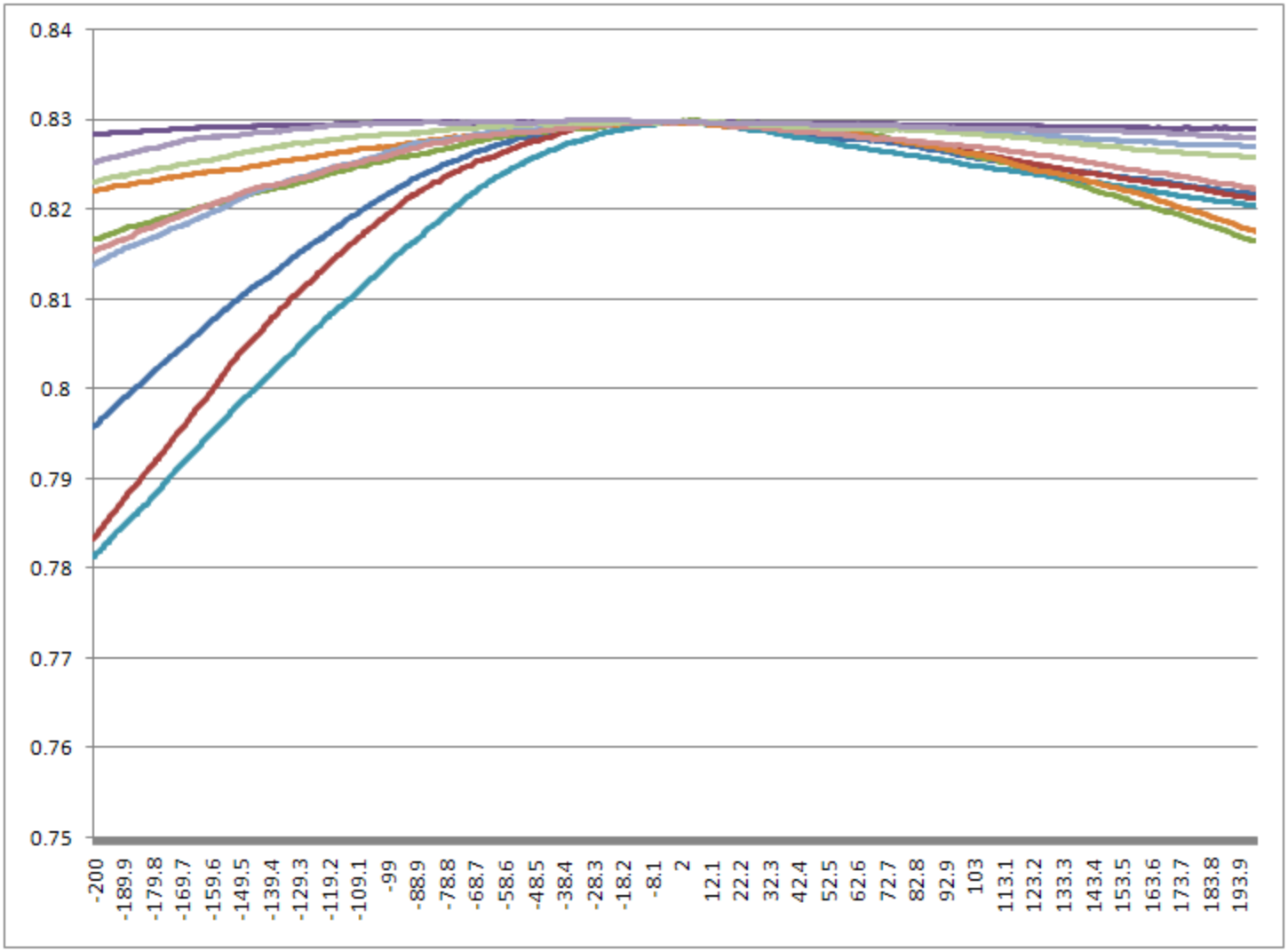
Smoothness: 10,000 Queries



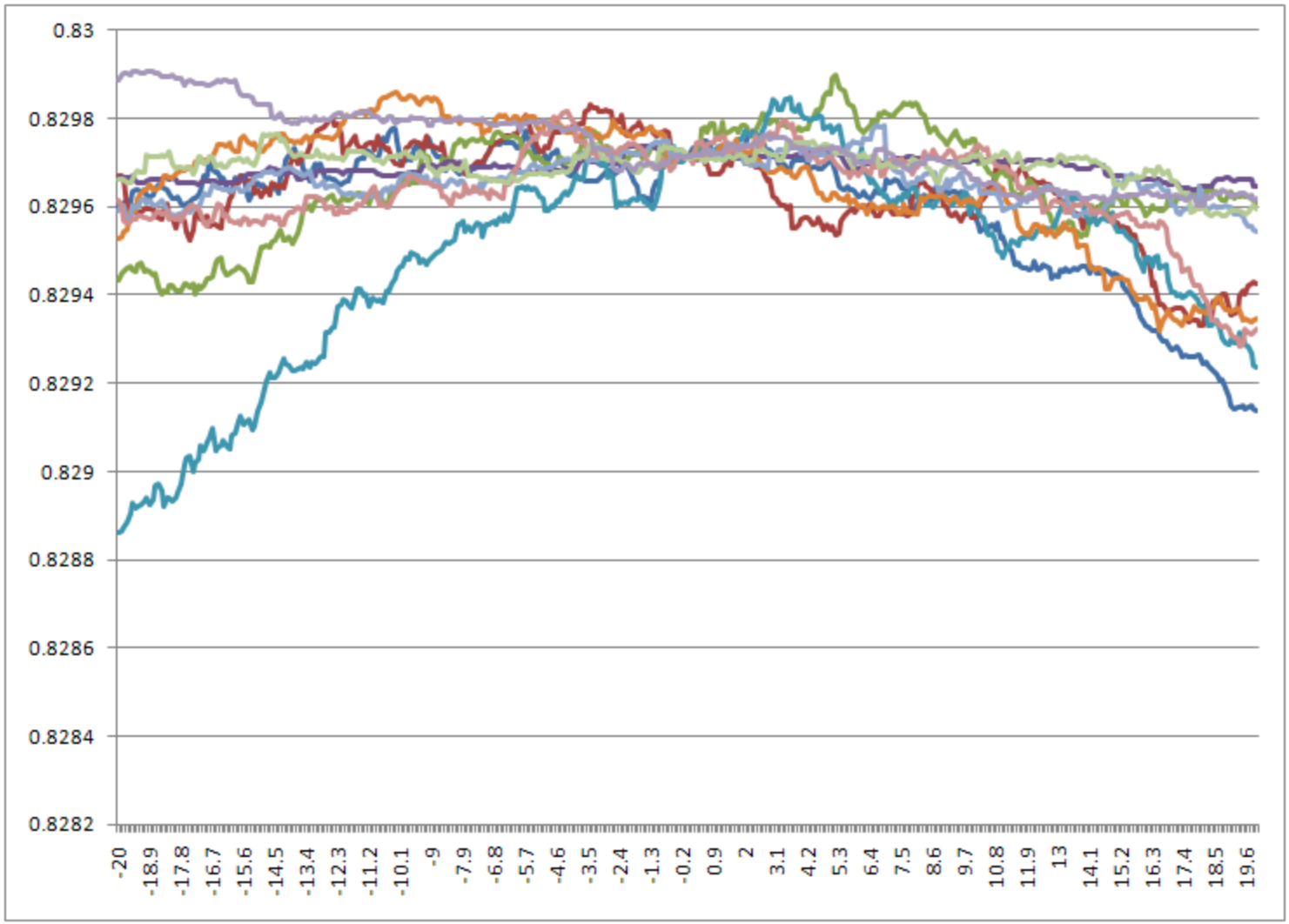
Two Layers, Artificial



Two Layers, Web Data



Zoom In


0.83
0.8298
0.8296
0.8294
0.8292
0.829
0.8288
0.8286
0.8284
0.8282
-20
-18.9
-17.8
-16.7
-15.6
-14.5
-13.4
-12.3
-11.2
-10.1
-9
-7.9
-6.8
-5.7
-4.6
-3.5
-2.4
-1.3
-0.2
0.9
2
3.1
4.2
5.3
6.4
7.5
8.6
9.7
10.8
11.9
13
14.1
15.2
16.3
17.4
18.5
19.6

MART

Joint work with Ping Li and Qiang Wu, "Learning to Rank Using Classification and Gradient Boosting"
In NIPS 2007 at
http://research.microsoft.com/~cburges/pubs.htm

Classification / Regression?

- Challenge our assumptions! Powerful, standard methods are available for classification and regression (in particular, boosted trees).
- So: let's treat this as a classification, ordinal classification or regression problem.
- Why classification? Perfect (and some imperfect) classifications imply max DCG.

Three Basic Models

- Multiclass Classification: $P(y_i = k|x_i)$

- Ordinal Classification: $P(y_i \leq k|x_i)$

$$P(y_i = k|x_i) = P(y_i \leq k|x_i) - P(y_i \leq k-1|x_i)$$

- Regression: model targets $2^{y_i} - 1$ using least squares *(cf. Cossock and Zhang, Colt '06)*

Classification and the DCG

Lemma: Given *n* urls, originally ordered as $\{1,2,\ldots,n\}$. Suppose a classifier assigns a relevance level $\hat{y}_i \in \{1,\ldots,k\}$ to the i^{th} url, for all *n* urls. Let a permutation mapping π rank the urls according to $\hat{y}_i$. The corresponding DCG error is bounded by the square root of the classification error:

$$DCG_m - DCG_\pi \leq (2^{k_m} - 2^1)\sqrt{2}\left(\sum_{i=1}^{n} c_i^2\right)^{\frac{1}{2}}\left(\sum_{i=1}^{n} 1_{y_i \neq \hat{y}_i}\right)^{\frac{1}{2}}$$

From Classification to Ranking

We need a ranking score. Use the expected relevance:

$$score = \sum_{k=1}^{K} kP(y_i = k|x_i)$$

Could use any monotonic function of *k*: simplest (*k*) gave best results.

Use cross entropy loss:

$$\Psi \equiv loss = \sum_{i=1}^{N}\sum_{k=1}^{K} -\log P(y_i = k|x_i)1_{k=k_i}$$

Gradient Boosting: MART

$$(\beta_m, a_m) = \underset{(\beta, a)}{argmin} \sum_{i=1}^{N} \Psi(y_i, F_{m-1}(x_i) + \beta h(x_i, a))$$

Estimate gradient: $$a_m = \underset{(\beta, a_m)}{argmin} \sum_{i=1}^{N} \left(-g_m(x_i) + \beta h(x_i, a_m)\right)^2$$

Perform line search:

$$\rho_m = \underset{(\rho, a)}{argmin} \sum_{i=1}^{N} \Psi(y_i, F_{m-1}(x_i) + \rho h(x_i, a))$$

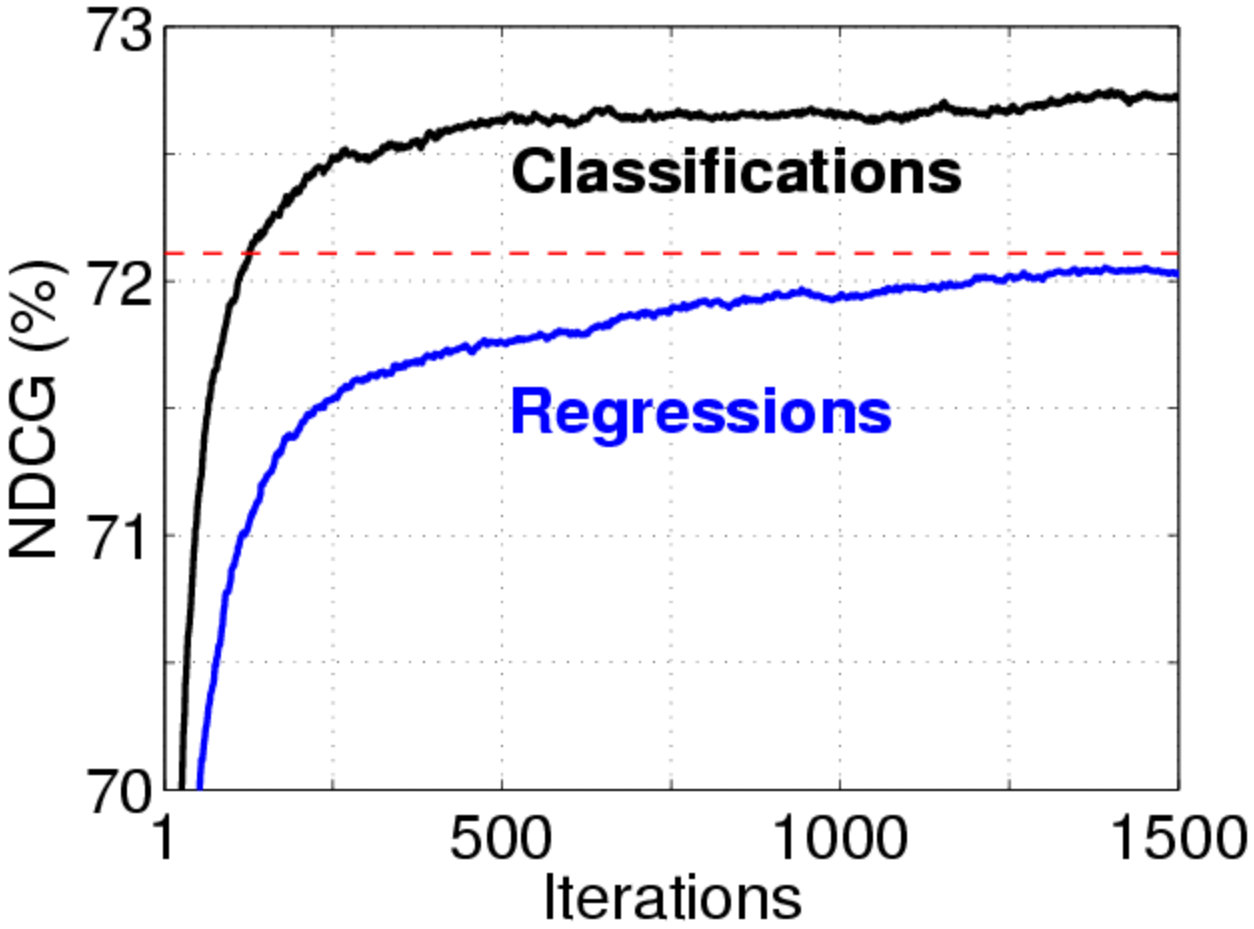
Update: $$F_m(x) = F_{m-1}(x) + \rho_m h(x, a_m)$$

J. Friedman, *Greedy Function Approximation: A Gradient Boosting Machine,* Inst. Math. Statistics, 2001

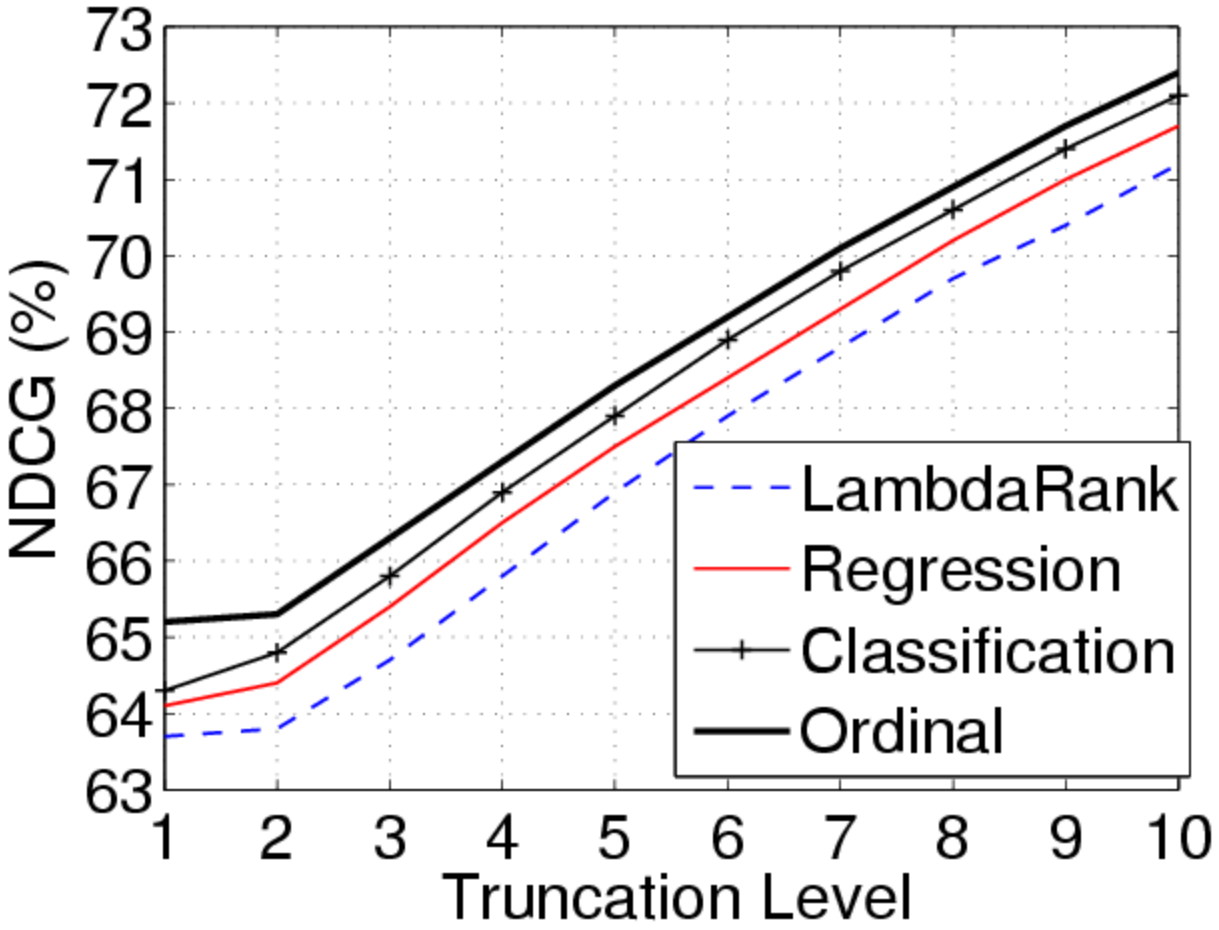
MART for Ranking: Notes

- *K* trees per boosting iteration
- Each tree fits gradient estimate using least squares
- Line search is performed for *each leaf*, using a Newton-Raphson step
- Tree outputs converted to probs using logistic function
- MART builds a multiclass classifier from regression trees (that fit residuals)

2 layer results: 16K train, 10K test



Results, cont.



MART: Conclusions

- MART gives great results, but it's not optimizing the cost directly (and it's a little slow).

Building on boosting sounds like a good direction: can we build weak learners that more directly solve the problem at hand?

XRank

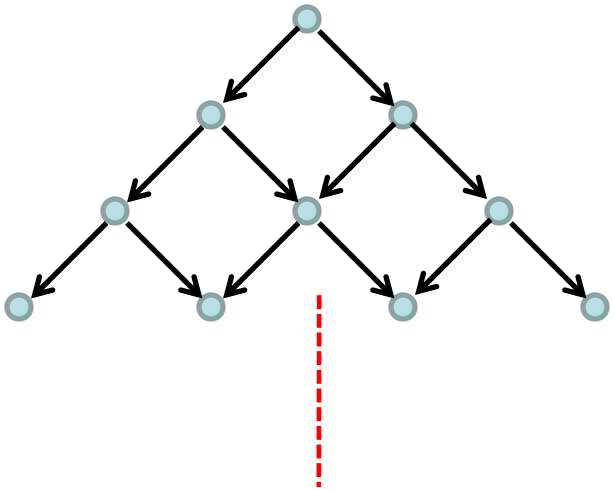
Joint work with Robert Rounthwaite and Qiang Wu

XRank

- An attempt to directly optimize the (non-differentiable) cost function we care about
- Build a planar, directed acyclic graph, with a single root node.
- Like a decision tree, but it's a DAG, and has a different interpretation.

Martingale Boosting for Classification

P. Long and R. Servedio, COLT 2005



XRank

- Instead of classifying by position, encode the rank of the sample by the *position* of the leaf node it winds up at
- Long and Servedio give exponential bound on the learning error rate for classification. We can extend this to a bound for the training error for pairwise ranking, for an arbitrary number of levels of relevance.

Martingale Bound for Ranking

Training samples $x_i \in \mathbb{R}^n$, $x_i \sim D$; set of training pairs $\{x_i, x_j\}$ such that $x_i \rhd x_j$

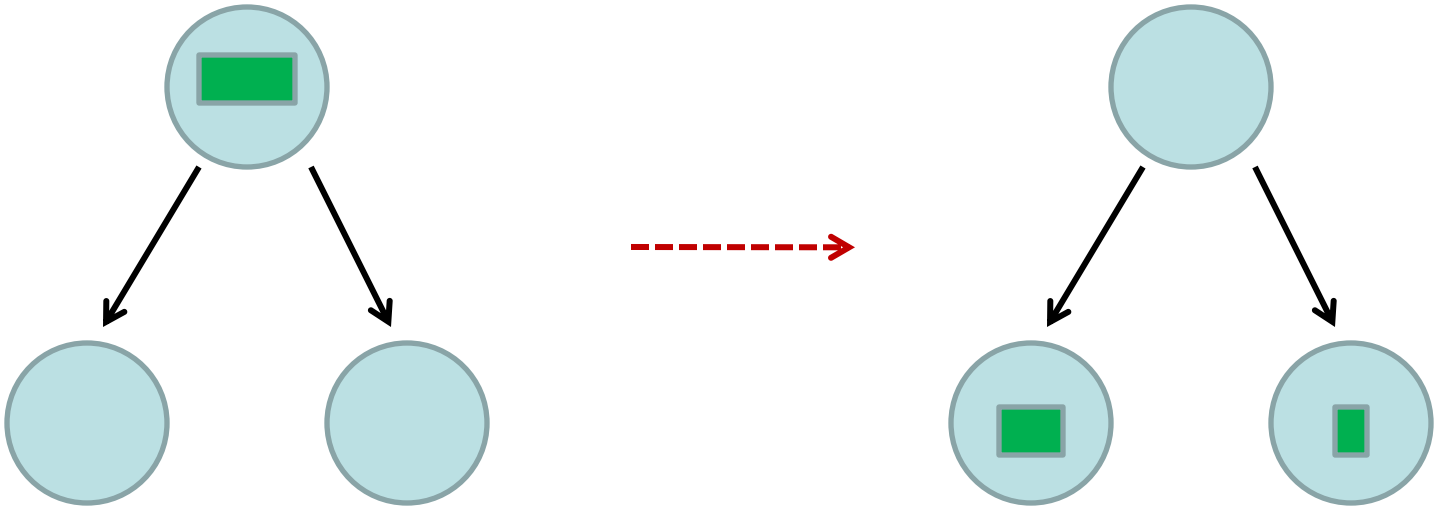
Assume:given D, hypotheses $h_t: \mathbb{R}^n \to \{0,1\}$ such that $\forall\, t, k, l$

$$P(h_{tk}(x_i) = 1) - P(h_{tl}(x_j) = 1) \geq \gamma_t$$

Theorem: then, for T levels, the final output hypotheses satisfy:

$$P\left(rank(x_i) < rank(x_j)\right) \leq \exp\left(-\frac{(\sum_{t=1}^{T} \gamma_t)^2}{4T}\right)$$

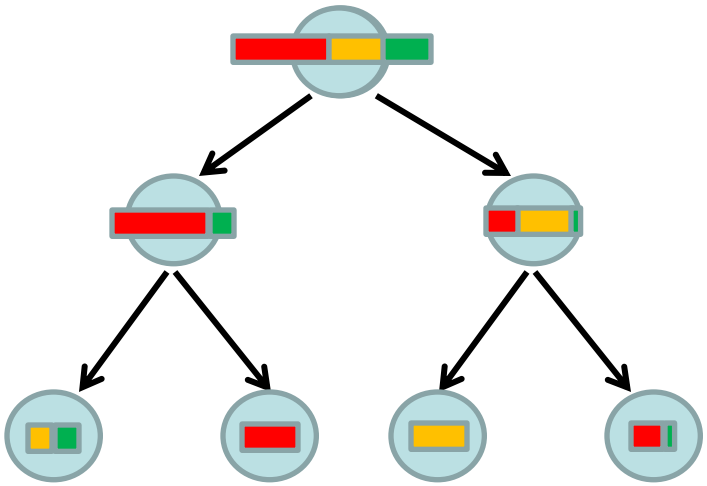
A Model: (Mini)Max NDCG



Start with some ordering in parent node.

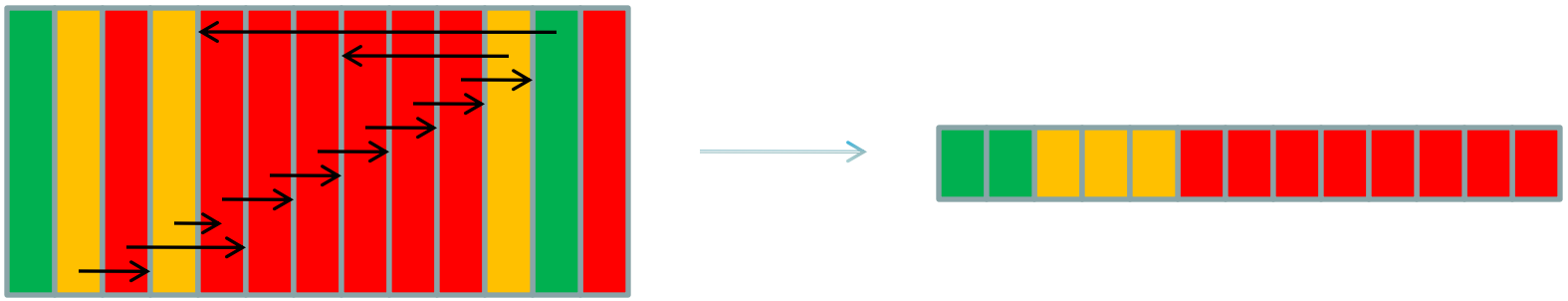
Choose split to maximize the gain in NDCG, given that the ordering within each child node is unchanged.

Directly Optimizing NDCG



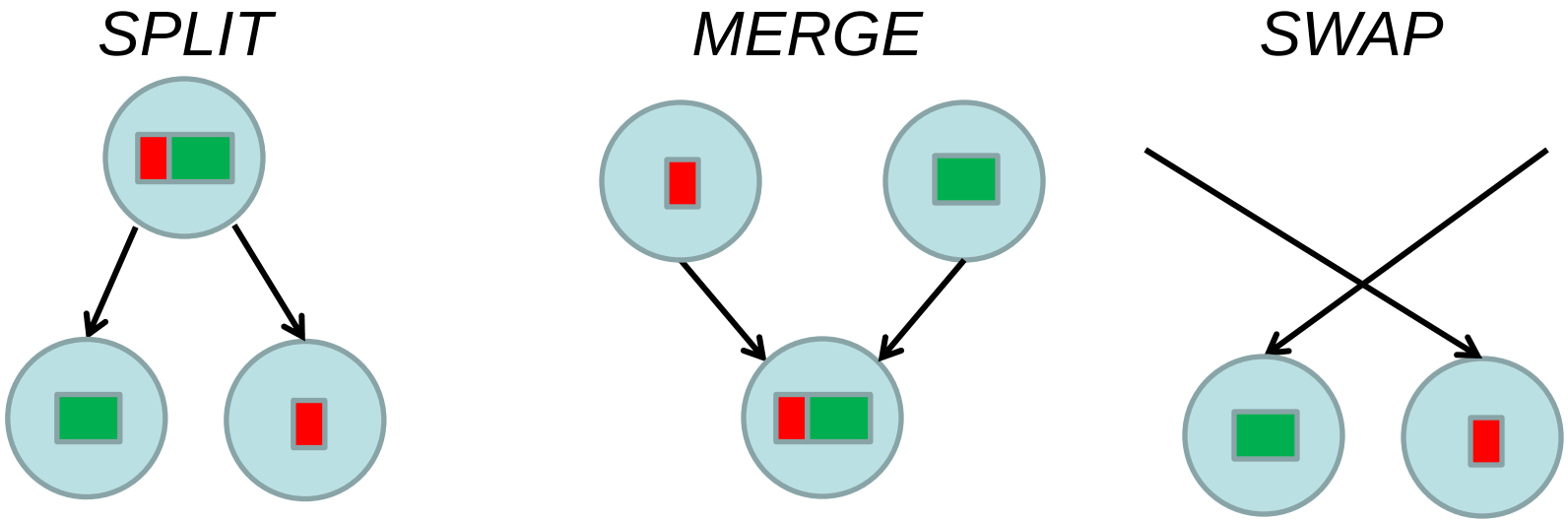
- Loop through thresholds; track which queries affected; compute their NDCG. (Relevance: Green/Orange/Red)
- *Monotonically increases NDCG!*
- … but, does not learn to completion: the Martingale bound fails: *Query Fragmentation*
- *Too local!*

An Energy-Based Model



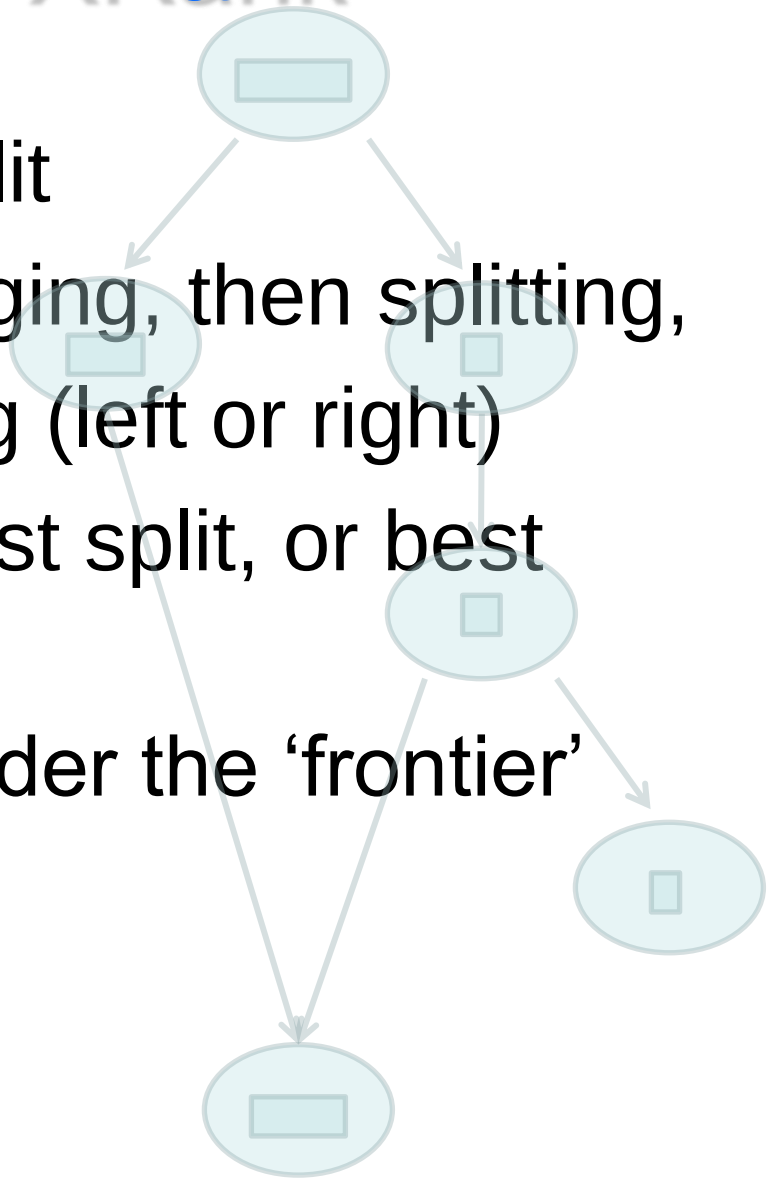
- All forces equal and opposite (they sum to zero)
- Force between two samples is proportional to NDCG gain for swapping those two samples
- The sum of absolute values of the forces is a useful objective function

XRank: Three Basic Operations



Learning with XRank

- Compute gain from a split
- Compare gain from merging, then splitting, to gain from just splitting (left or right)
- At each step, choose best split, or best merge
- After every change, reorder the ‘frontier’

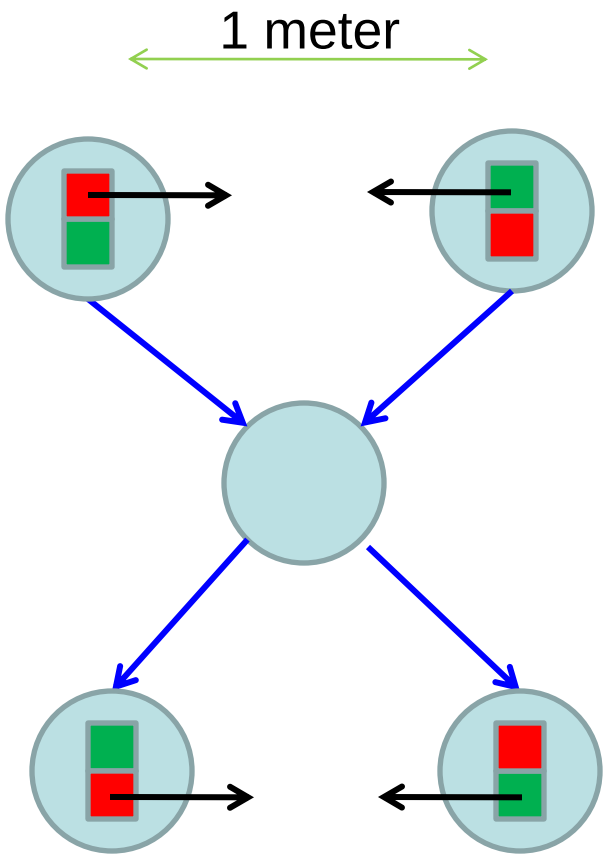


Why Use a Directed Acyclic Graph?

- Can grow linearly with depth instead of exponentially (less overfitting, less query fragmentation)
- Allows samples to migrate back to where they should be, if an error is made
- Can rebalance, and rearrange nodes after learning a level, to further reduce cost
- DAG advantages: they boost well, no learning rate, easy to interpret (e.g. to find most important features), fast in test phase

Physical Models Can be Tricky

Pitfall I: Oscillations



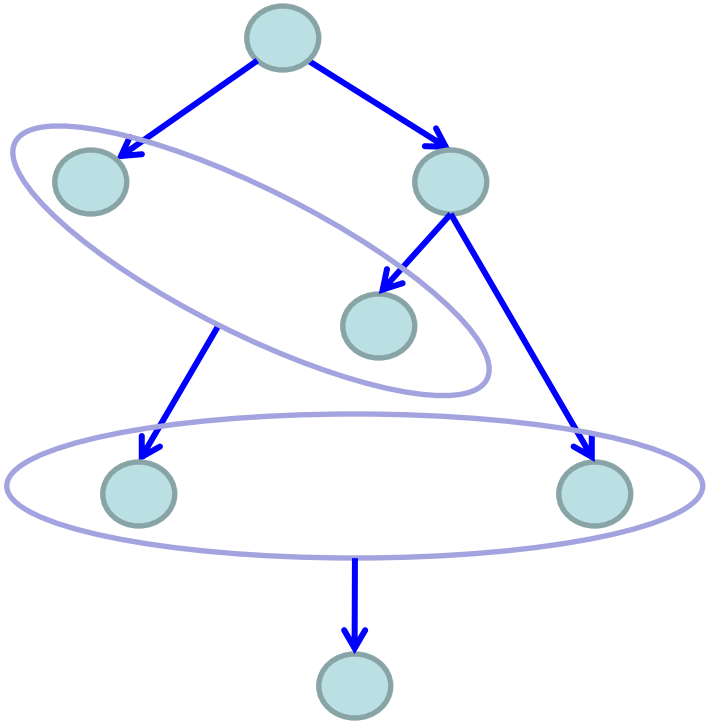
Physical Models Can be Tricky II

Pitfall 2: Consistency

Three samples $\{s_1, s_2, s_3\}$ with forces $\{f_1, f_2, f_3\}$ with $\sum_{i=1}^{3} f_i = 0$

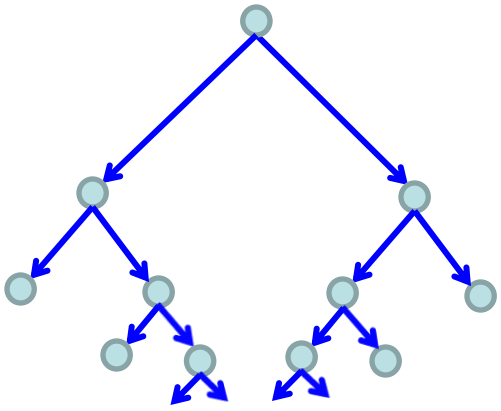
Can choose splits so that net gain does not vanish (e.g. $G = f_1 + f_3$)

Going to higher dimensions (e.g. n-dimensional simplex) does not help: we'd like 1-d forces!



Physical Models Can Be Tricky III

Pitfall 3: Bunching



Some Simple Theorems

Simplified XRank: only splits and swaps; also negative gain splits are allowed (i.e. if no positive gain splits exist, take the best non-positive split for the heaviest node): then we have:

Theorem: The training procedure cannot result in oscillations.

Theorem: If every pair of samples differ in at least one (binned) feature, then given sufficient iterations, XRank will learn the training data perfectly (despite not necessarily being monotonic in NDCG).

Theorem: The computational complexity of computing the best split for a node is O(KFTN) (why not $O(KFTN^2)$?)

Parting Notes

- Learning to Rank, with cost measures typically used in information retrieval, presents many opportunities for developing useful new machine learning solutions.
- For given features, eventually methods will likely converge to having similar performance.
- The ‘speed in test phase’ constraint is not typically the main focus of current research, but it also motivates interesting new research directions.

Thank You.