

Towards fast and stable inversion algorithms for the FMM

IPAM Tutorial

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Overview

- ▶ Matrix representation of FMM matrices
- ▶ Recursions of FMM
- ▶ Signal flow-graph representation of FMM
- ▶ Sparse-matrix representation of FMM matrices
- ▶ Fast, stable solvers for FMM matrices
- ▶ Towards super-fast solvers for FMM matrices
 - ▶ Analysis of micro-structure of inverses of FMM matrices
 - ▶ A fast algebra approach: open problems

Greengard & Rokhlin's Fast Multi-pole Method

- ▶ Many matrices that arise in applications can be *covered* by sub-matrices of low numerical rank.
- ▶ This structure can be captured rapidly (without explicitly forming all the matrix entries) in many cases.
- ▶ The structure can be exploited to speed up matrix \times vector multiplication.
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- ▶ (with Starr) they showed that d -dimensional discrete integral operator equations can be solved in $\mathcal{O}\left(n^{\frac{1+d}{2}}\right)$ flops.