A Formal Practice-Oriented Model for the Analysis of Side-Channel Attacks

F.-X. Standaert, T.G. Malkin, M. Yung

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Dept. of Computer Science, Columbia University.
UCL Crypto Group, Université Catholique de Louvain.
RSA Laboratories.

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Part I:
An Intuitive view of the model

- General context
- Examples of unanswered questions => 1\textsuperscript{st} observation
- Comparison with black box attacks => 2\textsuperscript{nd} observation
- Conclusion
General Context

**Black box cryptanalysis...**
only uses the primitive’s inputs and outputs, *e.g.*, the plaintexts, ciphertexts for block ciphers

**Side-channel attacks...**
additionally takes advantage of physical leakages, *e.g.*, power consumption, timing information, electromagnetic radiation
• Side-channel attacks (the story made short):

– Powerful but specific (usually target a particular implementation rather than an abstract algorithm)
– Still generic (most devices can be targeted)
– Hard to evaluate (many different circuit technologies and statistical distinguishers)
– Hard to prevent (actual countermeasures usually only make the attack more difficult)

⇒ Many open questions
Example of (simple but) unanswered questions

Given one implementation, how to we compare two side-channel adversaries?
Another example of unanswered questions

How to compare two different implementations of the same cryptographic primitive?
First observation

- It is important to distinguish the quality of an implementation from the strength of a side-channel adversary
Comparison with black box attacks, 
*e.g.* linear cryptanalysis
Both attacks work in two steps:
(1) Query the target and get information
(2) Exploit the information with a distinguisher

Both attacks yield two questions:
(1) What is the amount of information provided by the adversarial queries?
(2) How successfully can the adversary turn this information into a successful attack?
Second observation

- Black box attacks mainly care about security. Side-channel attacks don’t, because the same amount of queries to different implementations of the same primitive give rise to different amounts of information

⇒ Information discriminates different implementations
Conclusion

A formal model for the analysis of side-channel attacks should consider:

implementations  \[\uparrow\]  adversaries

information  \[\uparrow\]  security
Intuitive expectations for the model

- **Strong adversary, strong implementation**
  - (little information available, turned into a successful attack)
  - "good leakage prediction / distinguisher and enough queries"

- **Insecure cryptographic implementation**
  - (sufficient information available, turned into a successful attack)

- **Secure cryptographic implementation**
  - (no information available, no successful attack)

- **Weak adversary, weak implementation**
  - (some information available, not exploited/exploitable by the adversary)
  - "bad leakage prediction / distinguisher or not enough queries"
Part II:
Precise description of the model
Motivations and objectives

• A formal practice oriented model:
  – To understand the underlying mechanisms of physically observable cryptography
  – To evaluate (i.e. analyze and compare) actual implementations and adversaries
  – …to construct provably secure designs…
Outline

• Background: Micali & Reyzin model
• Target circuit
• Target block cipher
• Adversarial context & strategy
• Leakage predictions & adversary definition
• Computational restrictions
• Definition of security
• Evaluation criteria
• Evaluation methodology
• Side-channel tradeoffs
• Conclusions & open problems
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Background: Micali & Reyzin model

• TCC 2004:

   Physically Observable Cryptography

• Very general model (but few positive results)
• An open question was to meaningfully restrict the model to realistic adversaries
MR’s Informal axioms

1. Computation and only computation leaks information

2. The same computation leaks differently on different computers

3. The information leakage depend on the chosen measurement

4. The information leakage is local (i.e. the maximum amount of information leaked by a device is the same in any execution of the algorithm with the same inputs)

5. All information leaked through physical observations can be efficiently computed from a computer’s internal configuration
MR’s Definitions

- Abstract computer: $\alpha := \{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ with virtual memory Turing machines $\alpha_i$’s (VTMs)

- Physical computer: $\varphi = (\alpha, \mathcal{L})$, with $\varphi_i = (\mathcal{L}_i, \alpha_i)$

- Leakage functions: $\mathcal{L}(C_\alpha, M, R)$
  - $C_\alpha$: abstract computer internal configuration
  - $M$: specification of the measurement
  - $R$: random string
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Target circuit

\[ \Sigma := \{\sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_s\} \]

\[ \Omega_1 := \{\omega_1^1, \omega_1^2, \omega_1^3, \ldots, \omega_1^{p_1}\} \]
\[ \Omega_2 := \{\omega_2^1, \omega_2^2, \omega_2^3, \ldots, \omega_2^{p_2}\} \]
\[ \Omega_3 := \{\omega_3^1, \omega_3^2, \omega_3^3, \ldots, \omega_3^{p_3}\} \]
\[ \Omega_o := \{\omega_o^1, \omega_o^2, \omega_o^3, \ldots, \omega_o^{p_o}\} \]

Leakage function \( \mathcal{L}(\Sigma, \Omega_f) \)
Fault insertion function \( \mathcal{F}(\Sigma, \Omega_g) \)
Probing functions \( \mathcal{R}(\sigma_i, \sigma_j, \sigma_k) \) \( \mathcal{W}(\sigma_i, \sigma_j, \sigma_k) \)
• With respect to MR:
  – Oracles and operations can be translated into abstract computers and VTM
  – Signals are the VTM’s inputs/outputs
• Abstract computers = cryptographic primitives
• Physical computers = implementations
• Implementations may differ by:
  – Their leakage function
  – Their division into elementary operations
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Target block cipher

\[ \Omega_1 := \{R_1, R_2, R_3, KR_1, KR_2, KR_3\} \]

\[ \Omega_1 := \{\oplus_1, \ldots, \oplus_4, S_A, \ldots, S_F, D_1, \ldots, D_6\} \]
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Adversarial context

- Non adaptive known plaintext
- Non adaptive known ciphertext
- Non adaptive chosen plaintext
- Non adaptive chosen ciphertext
- Adaptive chosen plaintext
- Adaptive chosen ciphertext
& Strategy

• Hard strategy:

Given some physical observations and a resulting classification of the key candidates, select the best classified key only

• Soft strategy

Given some physical observations and a resulting classification of the key candidates, select the $h$ best classified keys
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Generic side-channel attack

- According to some approximation of the leakage function, an adversary *predicts* (a part/function of) the key dependent leakages emanated from a device.
- It then *measures* the actual leakages from the target physical implementation.
- It finally *compares* the actual leakages with the key dependent predictions. If the attack is successful, it is expected that the correct key candidate gives rise to the best leakage prediction which can be detected with a side-channel distinguisher.
Leakage predictions

Univaritate

Multivaritate
Classification

- Non profiled
- Device profiled
- Key profiled
Adversary definition

\[ A_{f_K,L}^C(\tau, q) \]

\[ \mathcal{P} \quad \mathcal{D} \]

prediction function  distinguisher
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• Memory complexity:
  – Size of the key guess: $G$

• Time and data complexity: $\tau$, $q$
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Definition of security: Side-channel key recovery

Experiment $\text{Exp}_{f_K,\mathcal{L}}^{\text{sc-kr}}$

$K \xleftarrow{\text{R}} \{0, 1, 2, \ldots, G - 1\}$;
$K^* \leftarrow A_C^{f_K,\mathcal{L}}(\tau, q)$;
\textbf{if} $K = K^*$ \textbf{then} return 1;
\textbf{else} return 0;

- Key recovery advantage of the adversary

$$\text{Adv}_{A}^{\text{sc-kr}}(\tau, q) = P\left[\text{Exp}_{f_K,\mathcal{L}}^{\text{sc-kr}} = 1\right]$$

- Key recovery advantage of the implementation

$$\text{Adv}_{f_K,\mathcal{L}}^{\text{sc-kr}}(\tau, q) = \max_A\{\text{Adv}_{A}^{\text{sc-kr}}(\tau, q)\}$$
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Evaluation of a side-channel attack
• Two questions to face:
  – What is the amount of information provided by a given leakage function?
  – How successfully can an adversary turn this information into a successful attack?
Definitions

• $L^q_{S_g} = \mathcal{L}(S_g)^q$: an observation generated by a secret $S_g$ and $q$ queries to the target device

• $P^q_S = \mathcal{P}(S)^q$: the adversary’s predictions

• $\mathcal{D}(L^q_{S_g}, P^q_S)$: the distinguisher used by the adversary to compare an actual observation of a leaking device with its key dependent predictions
Security metric: average success rate

• Keys selected by the adversary (hard strategy):

\[ M^q_{S_g} = \{ \hat{s} \mid \hat{s} = \arg \max_S \mathcal{D}(L^q_{S_g}, P^q_S) \}, \]

• Index matrix:

\[ I^q_{S_g, S} = \frac{1}{|M^q_{S_g}|} \text{ if } S \in M^q_{S_g}, \text{ else } 0 \]

• Success rate:

\[ S_R(S_g, q) = \mathbb{E}_{L^q_{S_g}} I^q_{S_g, S_g}, \quad \overline{S_R}(q) = \mathbb{E}_{S_g} \mathbb{E}_{L^q_{S_g}} I^q_{S_g, S_g} \]
Example: Bayesian classifier

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<th>S_g=2</th>
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\[ S_R(S_g = 2, q) \approx 58\% \]
Information theoretic metric: mutual information

- Entropy matrix:
  \[ H^q_{S_g,S} = \mathbb{E} - \log_2 P[S|L^q_{S_g}] \]

- Conditional entropy:
  \[ H[S_g|L^q_{S_g}] = \mathbb{E}_{S_g} H^q_{S_g,S_g} \]

- Leakage matrix:
  \[ \Lambda^q_{S_g,S} = H[S_g] - H^q_{S_g,S} \]

- Mutual information:
  \[ I(S_g; L^q_{S_g}) = H[S_g] - H[S_g|L^q_{S_g}] = \mathbb{E}_{S_g} \Lambda^q_{S_g,S_g} \]
Example

\[ \Lambda_{S_g,S}^q = 2 - H_{S_g,S}^q \]

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-0.43, -0.43, 0.77, -0.76
• Definition: a leakage function is sound

\[ \iff \max_{S} \wedge_{S_g,S}^{q} = \wedge_{S_g,S_g}^{q}, \forall S_g, q. \]

• If provided with a sound leakage function, a Bayesian adversary with unlimited queries to the target device will eventually be successful
  – Intuitive meaning: there is enough information in the side-channel observations
Combining security and IT metrics

- **Strong adversary, strong implementation**
  - (little information available, turned into a successful attack)
  - « good leakage prediction / distinguisher and enough queries »

- **Insecure cryptographic implementation**
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- **Weak adversary, weak implementation**
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- **Information theoretic metric**

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Exemplary applications

- The security metric can be used to compare different adversaries
- The information theoretic metric can be used to compare different implementations
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Side-channel tradeoffs

• Flexibility vs. Efficiency

  DPA … correlation … templates

• Information vs. Computation

  hard strategy … soft strategy
Conclusions and open problems

• Introduction of a formal model for the analysis of cryptographic primitives against side-channel attacks as a specialization of Micali and Reyzin
• Both theoretical and practical contribution
• Open problems:
  – More analyzes of practical contexts
  – Design of provably secure implementations
  – Optimal leakage prediction functions / distinguishers
-THANKS-

Send comments to:

fstandae@uclouvain.be

More information on:

http://www.dice.ucl.ac.be/~fstandae/tsca/