# SAR Imaging of Dynamic Scenes
# IPAM SAR Workshop

Brett Borden, Naval Postgraduate School
Margaret Cheney, Rensselaer Polytechnic Institute

7 February, 2012

# Introduction

- Need
  - All weather, day/night, sensor system
  - Situational awareness
  - Extend moving target indicator radar (MTI) to scene reconstruction
- Problem
  - Synthetic aperture radar images assume rigid scenes
  - Moving image elements are mis-located, distorted, or *not imaged at all*

# Introduction

- Need
  - All weather, day/night, sensor system
  - Situational awareness
  - Extend moving target indicator radar (MTI) to scene reconstruction
- Problem
  - Synthetic aperture radar images assume rigid scenes
  - Moving image elements are mis-located, distorted, or *not imaged at all*

# Introduction

- Need
  - All weather, day/night, sensor system
  - Situational awareness
  - Extend moving target indicator radar (MTI) to scene reconstruction
- Problem
  - Synthetic aperture radar images assume rigid scenes
  - Moving image elements are mis-located, distorted, or *not imaged at all*

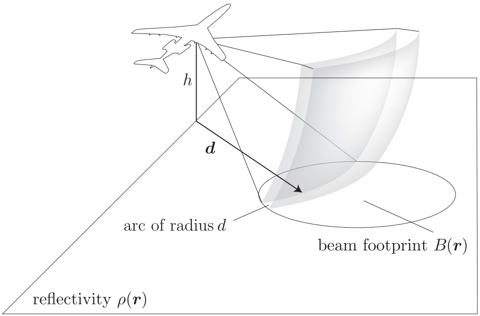
# Introduction

- Need
  - All weather, day/night, sensor system
  - Situational awareness
  - Extend moving target indicator radar (MTI) to scene reconstruction
- Problem
  - Synthetic aperture radar images assume rigid scenes
  - Moving image elements are mis-located, distorted, or *not imaged at all*

# Introduction

- Need
  - All weather, day/night, sensor system
  - Situational awareness
  - Extend moving target indicator radar (MTI) to scene reconstruction
- Problem
  - Synthetic aperture radar images assume rigid scenes
  - Moving image elements are mis-located, distorted, or *not imaged at all*

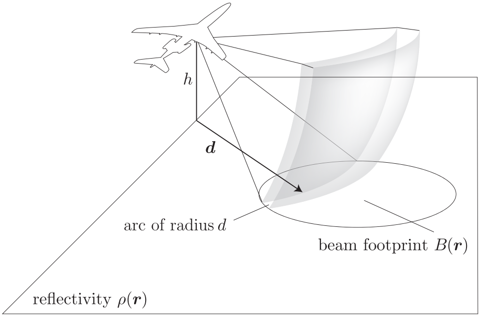
# Radar pulse illumination (high range resolution)




- Spherical pulse radiates at speed $c$
- Echo from all reflectors in arc returns to radar at the same time
- Radar collects energy as function of time:

$$\mathcal{E}(t) \sim \int \rho(\boldsymbol{r}) B(\boldsymbol{r}) \delta\left(t - 2\sqrt{h^2 + d^2}/c\right) \, \mathrm{d}A$$

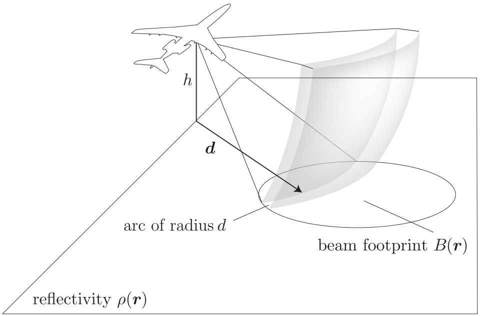
# Radar pulse illumination (high range resolution)




- Spherical pulse radiates at speed $c$
- Echo from all reflectors in arc returns to radar at the same time
- Radar collects energy as function of time:

$$\mathcal{E}(t) \sim \int \rho(\boldsymbol{r}) B(\boldsymbol{r}) \delta\left(t - 2\sqrt{h^2 + d^2}/c\right) \, \mathrm{d}A$$

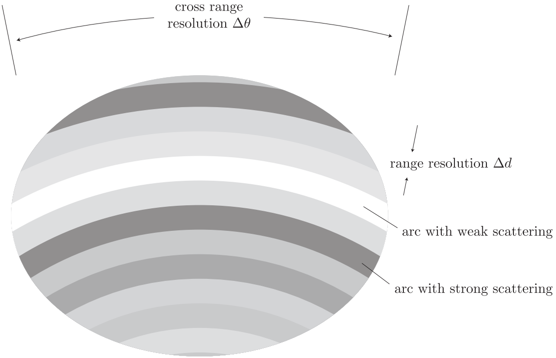
# Radar pulse illumination (high range resolution)




- Spherical pulse radiates at speed $c$
- Echo from all reflectors in arc returns to radar at the same time
- Radar collects energy as function of time:

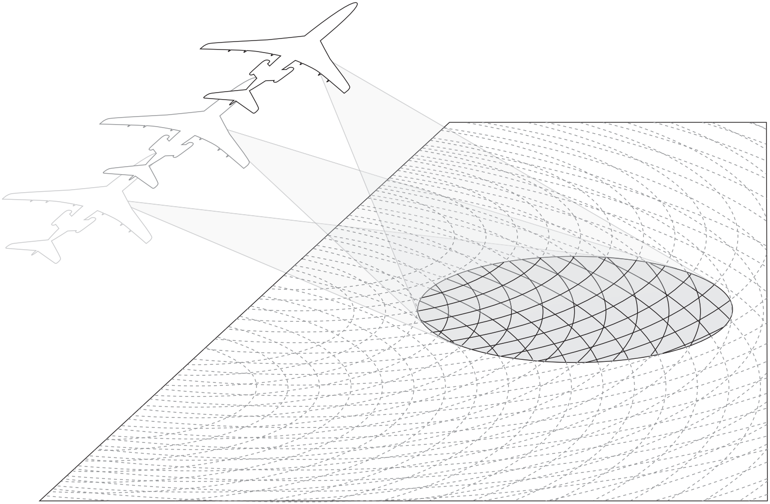
$$\mathcal{E}(t) \sim \int \rho(\boldsymbol{r}) B(\boldsymbol{r}) \delta\left(t - 2\sqrt{h^2 + d^2}/c\right) \mathrm{d}A$$

# Image from stationary radar

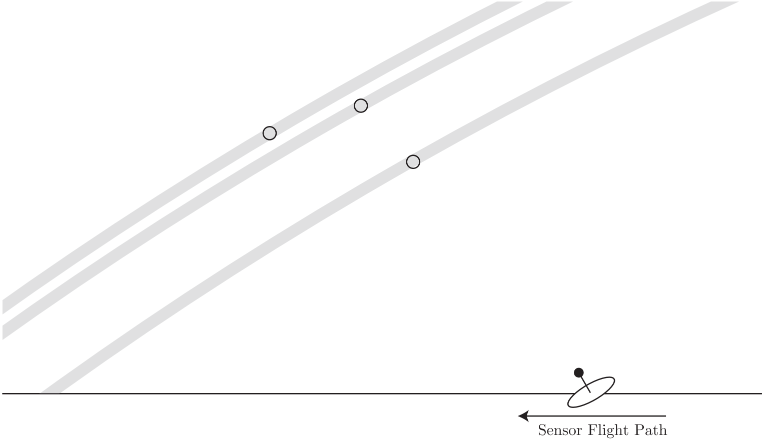



- A (stationary) radar has no knowledge of angular position within the beam
- Best image is arcs "painted" by range intensity values

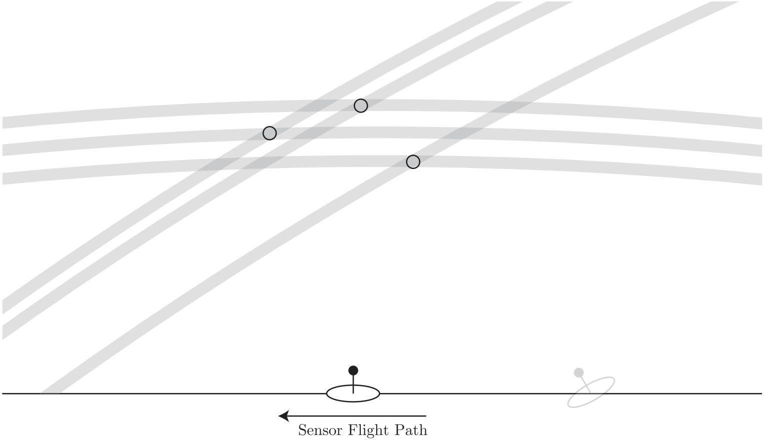
# Spotlight SAR



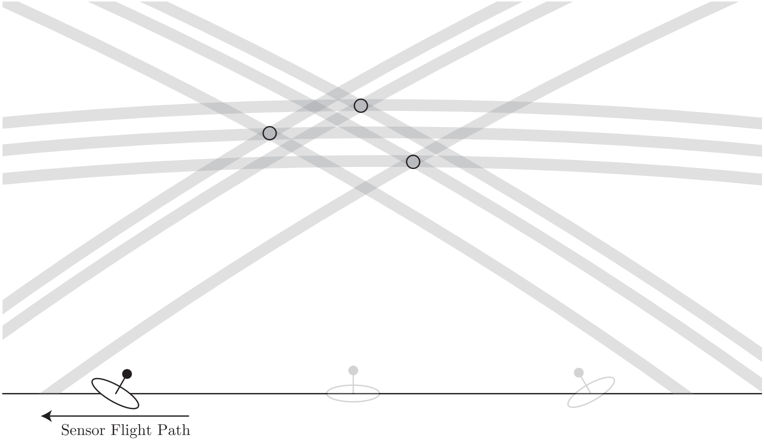
# SAR imaging of stationary scenes



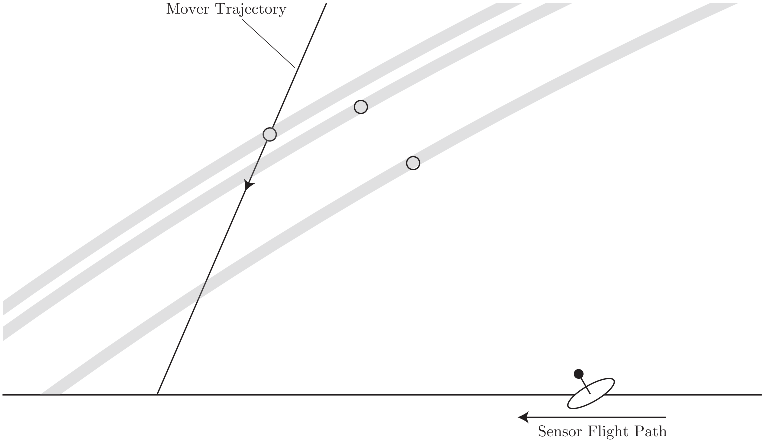
# SAR imaging of stationary scenes



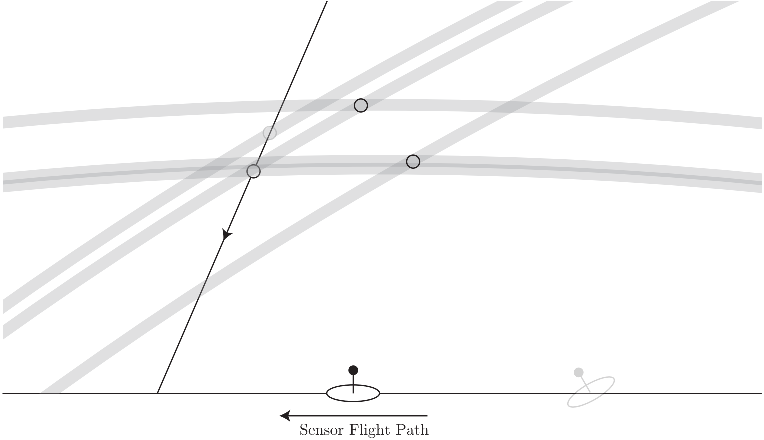
# SAR imaging of stationary scenes



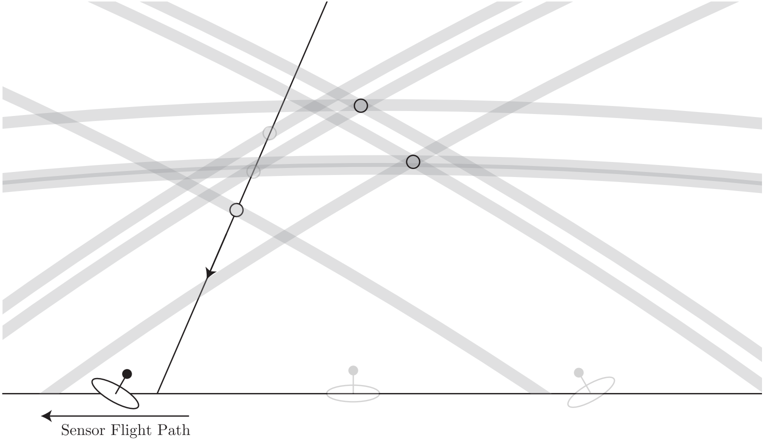
# SAR imaging of dynamic scenes



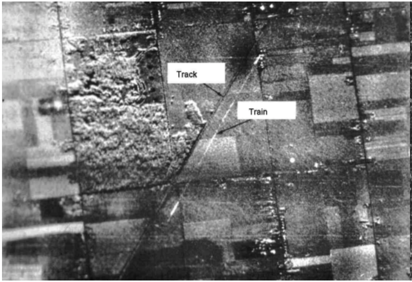
# SAR imaging of dynamic scenes

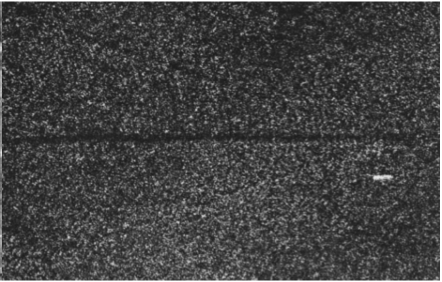


# SAR imaging of dynamic scenes



# Examples



Train off the track

Ship off its wake

# Example

Image from Sandia National Laboratories

## Past approaches

- Don't fix it, *exploit it!*
- Phase history analysis
- Sub-aperture images to isolate motion
- Ground moving target indicator (GMTI), Space-time adaptive processing (STAP), etc...

# Past approaches

- Don't fix it, *exploit it!*
- Phase history analysis
- Sub-aperture images to isolate motion
- Ground moving target indicator (GMTI), Space-time adaptive processing (STAP), etc...

# Past approaches

- Don't fix it, *exploit it!*
- Phase history analysis
- Sub-aperture images to isolate motion
- Ground moving target indicator (GMTI), Space-time adaptive processing (STAP), etc...

# Past approaches

- Don't fix it, *exploit it!*
- Phase history analysis
- Sub-aperture images to isolate motion
- Ground moving target indicator (GMTI), Space-time adaptive processing (STAP), etc...

## The radar problem: What does radar data look like?

- The echo signal at radar receiver

$$s_{\text{rec}}(t) = s_{\text{scatt}}(t) + n(t)$$

- EM radiation decays as $R^{-1}$, energy as $R^{-2}$, and radar echo energy as $R^{-4}$. Signal energy is proportional to field energy.
- Echo signals compete with system noise $n(t)$ (and are often swamped by it).
- "Raw" radar data is usually the result of significant signal processing.

# The radar problem: What does radar data look like?

- The echo signal at radar receiver

$$s_{\text{rec}}(t) = s_{\text{scatt}}(t) + n(t)$$

- EM radiation decays as $R^{-1}$, energy as $R^{-2}$, and radar echo energy as $R^{-4}$. Signal energy is proportional to field energy.
- Echo signals compete with system noise $n(t)$ (and are often swamped by it).
- "Raw" radar data is usually the result of significant signal processing.

# The radar problem: What does radar data look like?

- The echo signal at radar receiver

$$s_{\text{rec}}(t) = s_{\text{scatt}}(t) + n(t)$$

- EM radiation decays as $R^{-1}$, energy as $R^{-2}$, and radar echo energy as $R^{-4}$. Signal energy is proportional to field energy.
- Echo signals compete with system noise $n(t)$ (and are often swamped by it).
- "Raw" radar data is usually the result of significant signal processing.

# The radar problem: What does radar data look like?

- The echo signal at radar receiver

$$s_{\mathrm{rec}}(t) = s_{\mathrm{scatt}}(t) + n(t)$$

- EM radiation decays as $R^{-1}$, energy as $R^{-2}$, and radar echo energy as $R^{-4}$. Signal energy is proportional to field energy.
- Echo signals compete with system noise $n(t)$ (and are often swamped by it).
- "Raw" radar data is usually the result of significant signal processing.

# Correlation reception

- Usually, the received signal is compared to the ideal echo signal model

$$s_{\mathrm{scatt}}(t) = \iint \rho(\tau,\nu) s_{\mathrm{inc}}(t-\tau) \mathrm{e}^{\mathrm{i}\nu(t-\tau)} \,\mathrm{d}\tau\,\mathrm{d}\nu$$

  where $\rho(\tau,\nu)$ is the reflectivity of a point isotropic scatterer at distance $R = c\tau/2$ and radial velocity $\dot{R} = \nu\lambda/2$.
- Output of correlation receiver is

$$\eta(\tau,\nu) = \int s_{\mathrm{rec}}(t) s^*_{\mathrm{inc}}(t-\tau) \mathrm{e}^{-\mathrm{i}\nu(t-\tau)} \,\mathrm{d}t$$

- Local maxima of $|\eta(\tau,\nu)|^2$ define target's range and radial velocity.
- The parameters $\tau$ and $\nu$ define a two-dimensional search space *appropriate to single pulses.*

# Correlation reception

- Usually, the received signal is compared to the ideal echo signal model

$$s_{\mathrm{scatt}}(t) = \iint \rho(\tau,\nu) s_{\mathrm{inc}}(t-\tau) \mathrm{e}^{\mathrm{i}\nu(t-\tau)} \, \mathrm{d}\tau \, \mathrm{d}\nu$$

  where $\rho(\tau,\nu)$ is the reflectivity of a point isotropic scatterer at distance $R = c\tau/2$ and radial velocity $\dot{R} = \nu\lambda/2$.
- Output of correlation receiver is

$$\eta(\tau,\nu) = \int s_{\mathrm{rec}}(t) s^{*}_{\mathrm{inc}}(t-\tau) \mathrm{e}^{-\mathrm{i}\nu(t-\tau)} \, \mathrm{d}t$$

- Local maxima of $|\eta(\tau,\nu)|^2$ define target's range and radial velocity.
- The parameters $\tau$ and $\nu$ define a two-dimensional search space *appropriate to single pulses.*

# Correlation reception

- Usually, the received signal is compared to the ideal echo signal model
$$s_{\text{scatt}}(t) = \iint \rho(\tau,\nu) s_{\text{inc}}(t-\tau) \mathrm{e}^{\mathrm{i}\nu(t-\tau)}\,\mathrm{d}\tau\,\mathrm{d}\nu$$
where $\rho(\tau,\nu)$ is the reflectivity of a point isotropic scatterer at distance $R = c\tau/2$ and radial velocity $\dot{R} = \nu\lambda/2$.
- Output of correlation receiver is
$$\eta(\tau,\nu) = \int s_{\text{rec}}(t) s^*_{\text{inc}}(t-\tau) \mathrm{e}^{-\mathrm{i}\nu(t-\tau)}\,\mathrm{d}t$$
- Local maxima of $|\eta(\tau,\nu)|^2$ define target's range and radial velocity.
- The parameters $\tau$ and $\nu$ define a two-dimensional search space *appropriate to single pulses.*

# Correlation reception

- Usually, the received signal is compared to the ideal echo signal model
$$s_{\mathrm{scatt}}(t) = \iint \rho(\tau,\nu) s_{\mathrm{inc}}(t-\tau) \mathrm{e}^{\mathrm{i}\nu(t-\tau)}\, \mathrm{d}\tau\, \mathrm{d}\nu$$
where $\rho(\tau,\nu)$ is the reflectivity of a point isotropic scatterer at distance $R = c\tau/2$ and radial velocity $\dot{R} = \nu\lambda/2$.
- Output of correlation receiver is
$$\eta(\tau,\nu) = \int s_{\mathrm{rec}}(t) s^{*}_{\mathrm{inc}}(t-\tau) \mathrm{e}^{-\mathrm{i}\nu(t-\tau)}\, \mathrm{d}t$$
- Local maxima of $|\eta(\tau,\nu)|^2$ define target's range and radial velocity.
- The parameters $\tau$ and $\nu$ define a two-dimensional search space *appropriate to single pulses.*

# Ambiguity function

- If the ideal scattering model is the actual model, the substitution for $s_{\text{rec}}(t) = s_{\text{scatt}}(t) + n(t)$ yields

$$\eta(\tau, \nu) = \iint \rho(\tau', \nu')\chi(\tau - \tau'; \nu - \nu')\, \mathrm{d}\tau'\, \mathrm{d}\nu' + \underbrace{\text{correlation noise}}_{\text{small}}$$

(up to a phase factor in the integrand)

- The kernel here is the *radar ambiguity function:*

$$\chi(\tau; \nu) = \int s_{\text{inc}}(t - \tfrac{1}{2}\tau)s^*_{\text{inc}}(t + \tfrac{1}{2}\tau)\mathrm{e}^{\mathrm{i}\nu t}\, \mathrm{d}t$$

- $\chi(\tau; \nu)$ characterizes the radar's ability to estimate $R$ and $\dot{R}$
- "Radar data" usually means

$$\eta(\tau, \nu) = \iint \rho(\tau', \nu')\chi(\tau - \tau'; \nu - \nu')\, \mathrm{d}\tau'\, \mathrm{d}\nu'$$

# Ambiguity function

- If the ideal scattering model is the actual model, the substitution for $s_{\text{rec}}(t) = s_{\text{scatt}}(t) + n(t)$ yields

$$\eta(\tau, \nu) = \iint \rho(\tau', \nu')\chi(\tau - \tau'; \nu - \nu')\,\mathrm{d}\tau'\,\mathrm{d}\nu' + \underbrace{\text{correlation noise}}_{\text{small}}$$

  (up to a phase factor in the integrand)
- The kernel here is the *radar ambiguity function:*

$$\chi(\tau; \nu) = \int s_{\text{inc}}(t - \tfrac{1}{2}\tau)s^*_{\text{inc}}(t + \tfrac{1}{2}\tau)\mathrm{e}^{\mathrm{i}\nu t}\,\mathrm{d}t$$

- $\chi(\tau; \nu)$ characterizes the radar's ability to estimate $R$ and $\dot{R}$
- “Radar data" usually means

$$\eta(\tau, \nu) = \iint \rho(\tau', \nu')\chi(\tau - \tau'; \nu - \nu')\,\mathrm{d}\tau'\,\mathrm{d}\nu'$$

# Ambiguity function

- If the ideal scattering model is the actual model, the substitution for $s_{\mathrm{rec}}(t) = s_{\mathrm{scatt}}(t) + n(t)$ yields

$$\eta(\tau,\nu) = \iint \rho(\tau',\nu')\chi(\tau-\tau';\nu-\nu')\,\mathrm{d}\tau'\,\mathrm{d}\nu' + \underbrace{\text{correlation noise}}_{\text{small}}$$

(up to a phase factor in the integrand)

- The kernel here is the *radar ambiguity function:*

$$\chi(\tau;\nu) = \int s_{\mathrm{inc}}(t-\tfrac{1}{2}\tau)s^*_{\mathrm{inc}}(t+\tfrac{1}{2}\tau)\mathrm{e}^{\mathrm{i}\nu t}\,\mathrm{d}t$$

- $\chi(\tau;\nu)$ characterizes the radar's ability to estimate $R$ and $\dot{R}$
- "Radar data" usually means

$$\eta(\tau,\nu) = \iint \rho(\tau',\nu')\chi(\tau-\tau';\nu-\nu')\,\mathrm{d}\tau'\,\mathrm{d}\nu'$$

# Ambiguity function

- If the ideal scattering model is the actual model, the substitution for $s_{\mathrm{rec}}(t) = s_{\mathrm{scatt}}(t) + n(t)$ yields

$$\eta(\tau,\nu) = \iint \rho(\tau',\nu')\chi(\tau-\tau';\nu-\nu')\,\mathrm{d}\tau'\,\mathrm{d}\nu' + \underbrace{\text{correlation noise}}_{\text{small}}$$

(up to a phase factor in the integrand)

- The kernel here is the *radar ambiguity function:*

$$\chi(\tau;\nu) = \int s_{\mathrm{inc}}(t-\tfrac{1}{2}\tau)s^*_{\mathrm{inc}}(t+\tfrac{1}{2}\tau)\mathrm{e}^{\mathrm{i}\nu t}\,\mathrm{d}t$$

- $\chi(\tau;\nu)$ characterizes the radar's ability to estimate $R$ and $\dot{R}$
- "Radar data" usually means

$$\eta(\tau,\nu) = \iint \rho(\tau',\nu')\chi(\tau-\tau';\nu-\nu')\,\mathrm{d}\tau'\,\mathrm{d}\nu'$$

# SAR imaging

- Coordinate change from radar-centric to geo-centric coordinates $\boldsymbol{r}$. If $\gamma(\theta)$ denotes radar flight path, then

$$\boldsymbol{d} \to \gamma(\theta) - \boldsymbol{r}$$
$$\chi(\tau;\nu) \to \chi_\theta(\tau(\boldsymbol{r});\nu(\boldsymbol{r}))$$

$$\Rightarrow \;\; \eta(\tau,\nu) \to \eta_\theta(\tau(\boldsymbol{r}),\nu(\boldsymbol{r})) = \iint \rho(\mathbf{r}')\chi_\theta(2(r - \mathrm{r}')/c;0)\,\mathrm{d}x'\,\mathrm{d}y'$$

- The image is formed as

$$\begin{aligned} I(\boldsymbol{r}) &= \int \eta_\theta(\tau(\boldsymbol{r}),\nu(\boldsymbol{r}))\,\mathrm{d}\theta \\ &= \iiint \rho(\mathbf{r}')\chi_\theta(2\hat{\boldsymbol{d}}\cdot(\boldsymbol{r}-\mathbf{r}')/c;0)\,\mathrm{d}x'\,\mathrm{d}y'\,\mathrm{d}\theta \end{aligned}$$

# SAR imaging

- Coordinate change from radar-centric to geo-centric coordinates $\boldsymbol{r}$. If $\gamma(\theta)$ denotes radar flight path, then

$$\boldsymbol{d} \to \gamma(\theta) - \boldsymbol{r}$$
$$\chi(\tau;\nu) \to \chi_\theta(\tau(\boldsymbol{r});\nu(\boldsymbol{r}))$$
$$\Rightarrow \quad \eta(\tau,\nu) \to \eta_\theta(\tau(\boldsymbol{r}),\nu(\boldsymbol{r})) = \iint \rho(\mathbf{r}')\chi_\theta(2(r - r')/c;0)\,\mathrm{d}x'\,\mathrm{d}y'$$

- The image is formed as

$$\begin{aligned} I(\boldsymbol{r}) &= \int \eta_\theta(\tau(\boldsymbol{r}),\nu(\boldsymbol{r}))\,\mathrm{d}\theta \\ &= \iiint \rho(\mathbf{r}')\chi_\theta(2\hat{\mathbf{d}}\cdot(\mathbf{r}-\mathbf{r}')/c;0)\,\mathrm{d}x'\,\mathrm{d}y'\,\mathrm{d}\theta \end{aligned}$$

# Imaging via filtered adjoint

- SAR correlates echo field with model at each step of the data collection process. What if we wait until the end?
- Point scattering model:

$$s_{\mathrm{scatt}}^{(\theta)}(t) = s_{\mathrm{inc}}^{(\theta)}(t - \tau_\theta) e^{\mathrm{i}\nu(t-\tau_\theta)}$$

where $\tau_{\theta,\boldsymbol{r}} = 2|\gamma(\theta) - \boldsymbol{r}|/c$

- If $s_{\mathrm{rec}}^{(\theta)}(t)$ denotes scattered field measurements along $\gamma(\theta)$ then

$$\begin{aligned} I(\boldsymbol{r}) &= \iint s_{\mathrm{scatt}}^{(\theta)}(t') s_{\mathrm{rec}}^{(\theta)*}(t') \,\mathrm{d}t' \,\mathrm{d}\theta \\ &= \cdots \\ &= \iiint \rho(\boldsymbol{r}') \chi_\theta(2\hat{\boldsymbol{d}} \cdot (\boldsymbol{r} - \boldsymbol{r}')/c, 0) \,\mathrm{d}x' \,\mathrm{d}y' \,\mathrm{d}\theta \end{aligned}$$

(same result)

# Imaging via filtered adjoint

- SAR correlates echo field with model at each step of the data collection process. What if we wait until the end?
- Point scattering model:

$$s^{(\theta)}_{\text{scatt}}(t) = s^{(\theta)}_{\text{inc}}(t - \tau_\theta) e^{i\nu(t-\tau_\theta)}$$

where $\tau_{\theta,\boldsymbol{r}} = 2|\gamma(\theta) - \boldsymbol{r}|/c$

- If $s^{(\theta)}_{\text{rec}}(t)$ denotes scattered field measurements along $\gamma(\theta)$ then

$$\begin{aligned} I(\boldsymbol{r}) &= \iint s^{(\theta)}_{\text{scatt}}(t') s^{(\theta)*}_{\text{rec}}(t') \, dt' \, d\theta \\ &= \cdots \\ &= \iiint \rho(\boldsymbol{r}') \chi_\theta(2\hat{\boldsymbol{d}} \cdot (\boldsymbol{r} - \boldsymbol{r}')/c, 0) \, dx' \, dy' \, d\theta \end{aligned}$$

(same result)

# Imaging via filtered adjoint

- SAR correlates echo field with model at each step of the data collection process. What if we wait until the end?
- Point scattering model:

$$s^{(\theta)}_{\mathrm{scatt}}(t) = s^{(\theta)}_{\mathrm{inc}}(t - \tau_\theta)\mathrm{e}^{\mathrm{i}\nu(t-\tau_\theta)}$$

  where $\tau_{\theta,\boldsymbol{r}} = 2|\boldsymbol{\gamma}(\theta) - \boldsymbol{r}|/c$
- If $s^{(\theta)}_{\mathrm{rec}}(t)$ denotes scattered field measurements along $\boldsymbol{\gamma}(\theta)$ then

$$\begin{aligned} I(\boldsymbol{r}) &= \iint s^{(\theta)}_{\mathrm{scatt}}(t') s^{(\theta)*}_{\mathrm{rec}}(t')\,\mathrm{d}t'\,\mathrm{d}\theta \\ &= \cdots \\ &= \iiint \rho(\boldsymbol{r}')\chi_\theta(2\hat{\boldsymbol{d}}\cdot(\boldsymbol{r}-\boldsymbol{r}')/c, 0)\,\mathrm{d}x'\,\mathrm{d}y'\,\mathrm{d}\theta \end{aligned}$$

  (same result)

# Revised scattering model

- The two parameter model can be modified to include any *image-dependent* parameters.
- Target-specific velocities:

$$\tau_\theta \to \tau_{\theta,\boldsymbol{r},\boldsymbol{v}} = 2|\gamma(\theta) - \boldsymbol{r} - \boldsymbol{v}t|/c$$

$$\rho(\boldsymbol{r}) \to \rho_{\boldsymbol{v}}(\boldsymbol{r})$$

$$\chi_\theta(\tau(\boldsymbol{r}), \nu(\boldsymbol{r})) \to \chi_\theta(\tau(\boldsymbol{r}, \boldsymbol{v}); \nu(\boldsymbol{r}, \boldsymbol{v}))$$

- Moving point scattering model:

$$s_{\text{scatt}}^{(\theta,\boldsymbol{r},\boldsymbol{v})}(t) = s_{\text{inc}}^{(\theta)}(t - \tau_{\theta,\boldsymbol{r},\boldsymbol{v}})\mathrm{e}^{\mathrm{i}\nu(t-\tau_{\theta,\boldsymbol{r},\boldsymbol{v}})}$$

# Revised scattering model

- The two parameter model can be modified to include any *image-dependent* parameters.
- Target-specific velocities:

$$\begin{aligned}\tau_\theta &\to \tau_{\theta,\boldsymbol{r},\boldsymbol{v}} = 2|\gamma(\theta) - \boldsymbol{r} - \boldsymbol{v}t|/c \\ \rho(\boldsymbol{r}) &\to \rho_{\boldsymbol{v}}(\boldsymbol{r}) \\ \chi_\theta(\tau(\boldsymbol{r}), \nu(\boldsymbol{r})) &\to \chi_\theta(\tau(\boldsymbol{r}, \boldsymbol{v}); \nu(\boldsymbol{r}, \boldsymbol{v}))\end{aligned}$$

- Moving point scattering model:

$$s_{\text{scatt}}^{(\theta,\boldsymbol{r},\boldsymbol{v})}(t) = s_{\text{inc}}^{(\theta)}(t - \tau_{\theta,\boldsymbol{r},\boldsymbol{v}})\mathrm{e}^{\mathrm{i}\nu(t-\tau_{\theta,\boldsymbol{r},\boldsymbol{v}})}$$

# Revised scattering model

- The two parameter model can be modified to include any *image-dependent* parameters.
- Target-specific velocities:

$$\begin{aligned} \tau_\theta &\to \tau_{\theta,\boldsymbol{r},\boldsymbol{v}} = 2|\gamma(\theta) - \boldsymbol{r} - \boldsymbol{v}t|/c \\ \rho(\boldsymbol{r}) &\to \rho_{\boldsymbol{v}}(\boldsymbol{r}) \\ \chi_\theta(\tau(\boldsymbol{r}), \nu(\boldsymbol{r})) &\to \chi_\theta(\tau(\boldsymbol{r}, \boldsymbol{v}); \nu(\boldsymbol{r}, \boldsymbol{v})) \end{aligned}$$

- Moving point scattering model:

$$s_{\text{scatt}}^{(\theta,\boldsymbol{r},\boldsymbol{v})}(t) = s_{\text{inc}}^{(\theta)}(t - \tau_{\theta,\boldsymbol{r},\boldsymbol{v}})\mathrm{e}^{\mathrm{i}\nu(t-\tau_{\theta,\boldsymbol{r},\boldsymbol{v}})}$$

# Extended image generation

- (Hyper) image as:

$$I(\boldsymbol{r};\boldsymbol{v}) = \iint s_{\mathrm{scatt}}^{(\theta,\boldsymbol{r},\boldsymbol{v})}(t')\, s_{\mathrm{rec}}^{(\theta)*}(t')\,\mathrm{d}t'\,\mathrm{d}\theta$$

- Define the imaging kernel

$$K(\boldsymbol{r},\boldsymbol{r}',\boldsymbol{v},\boldsymbol{v}') \equiv \int s_{\mathrm{inc}}^{(\theta)}(t-\tau_{\theta,\boldsymbol{r},\boldsymbol{v}})\mathrm{e}^{\mathrm{i}\nu(t-\tau_{\theta,\boldsymbol{r},\boldsymbol{v}})} s_{\mathrm{inc}}^{(\theta)*}(t'-\tau_{\theta,\boldsymbol{r}',\boldsymbol{v}'})\mathrm{e}^{-\mathrm{i}\nu(t'-\tau_{\theta,\boldsymbol{r}',\boldsymbol{v}'})}\,\mathrm{d}\theta$$

- Can show (slow mover approximation)

$$K(\boldsymbol{r},\boldsymbol{r}',\boldsymbol{v},\boldsymbol{v}') = \cdots = \int \chi_\theta(2\hat{\boldsymbol{d}}\cdot(\boldsymbol{r}-\boldsymbol{r}')/c,\, 2\hat{\boldsymbol{d}}\cdot(\boldsymbol{v}-\boldsymbol{v}')/\lambda)\,\mathrm{d}\theta$$

$$I(\boldsymbol{r};\boldsymbol{v}) = \iint \rho_{\boldsymbol{v}}(\boldsymbol{r}')\tilde{K}(\boldsymbol{r}-\boldsymbol{r}',\boldsymbol{v}-\boldsymbol{v}')\,\mathrm{d}x'\,\mathrm{d}y'\,\mathrm{d}v_x'\,\mathrm{d}v_y'$$

# Extended image generation

- (Hyper) image as:

$$I(\boldsymbol{r};\boldsymbol{v}) = \iint s_{\text{scatt}}^{(\theta,\boldsymbol{r},\boldsymbol{v})}(t')\, s_{\text{rec}}^{(\theta)*}(t')\,\mathrm{d}t'\,\mathrm{d}\theta$$

- Define the imaging kernel

$$K(\boldsymbol{r},\boldsymbol{r}',\boldsymbol{v},\boldsymbol{v}') \equiv \int s_{\text{inc}}^{(\theta)}(t-\tau_{\theta,\boldsymbol{r},\boldsymbol{v}})\mathrm{e}^{\mathrm{i}\nu(t-\tau_{\theta,\boldsymbol{r},\boldsymbol{v}})}\, s_{\text{inc}}^{(\theta)*}(t'-\tau_{\theta,\boldsymbol{r}',\boldsymbol{v}'})\mathrm{e}^{-\mathrm{i}\nu(t'-\tau_{\theta,\boldsymbol{r}',\boldsymbol{v}'})}\,\mathrm{d}\theta$$

- Can show (slow mover approximation)

$$K(\boldsymbol{r},\boldsymbol{r}',\boldsymbol{v},\boldsymbol{v}') = \cdots = \int \chi_\theta(2\hat{\boldsymbol{d}}\cdot(\boldsymbol{r}-\boldsymbol{r}')/c, 2\hat{\boldsymbol{d}}\cdot(\boldsymbol{v}-\boldsymbol{v}')/\lambda)\,\mathrm{d}\theta$$

$$I(\boldsymbol{r};\boldsymbol{v}) = \iint \rho_{\boldsymbol{v}}(\boldsymbol{r}')\tilde{K}(\boldsymbol{r}-\boldsymbol{r}',\boldsymbol{v}-\boldsymbol{v}')\,\mathrm{d}x'\,\mathrm{d}y'\,\mathrm{d}v_x'\,\mathrm{d}v_y'$$

# Extended image generation

- (Hyper) image as:

$$I(\boldsymbol{r};\boldsymbol{v}) = \iint s_{\mathrm{scatt}}^{(\theta,\boldsymbol{r},\boldsymbol{v})}(t') s_{\mathrm{rec}}^{(\theta)*}(t')\,\mathrm{d}t'\,\mathrm{d}\theta$$

- Define the imaging kernel

$$K(\boldsymbol{r},\boldsymbol{r}',\boldsymbol{v},\boldsymbol{v}') \equiv \int s_{\mathrm{inc}}^{(\theta)}(t-\tau_{\theta,\boldsymbol{r},\boldsymbol{v}})\mathrm{e}^{\mathrm{i}\nu(t-\tau_{\theta,\boldsymbol{r},\boldsymbol{v}})}\, s_{\mathrm{inc}}^{(\theta)*}(t'-\tau_{\theta,\boldsymbol{r}',\boldsymbol{v}'})\mathrm{e}^{-\mathrm{i}\nu(t'-\tau_{\theta,\boldsymbol{r}',\boldsymbol{v}'})}\,\mathrm{d}\theta$$

- Can show (slow mover approximation)

$$K(\boldsymbol{r},\boldsymbol{r}',\boldsymbol{v},\boldsymbol{v}') = \cdots = \int \chi_\theta(2\hat{\boldsymbol{d}}\cdot(\boldsymbol{r}-\boldsymbol{r}')/c, 2\hat{\boldsymbol{d}}\cdot(\boldsymbol{v}-\boldsymbol{v}')/\lambda)\,\mathrm{d}\theta$$

$$I(\boldsymbol{r};\boldsymbol{v}) = \iint \rho_{\boldsymbol{v}}(\boldsymbol{r}')\tilde{K}(\boldsymbol{r}-\boldsymbol{r}',\boldsymbol{v}-\boldsymbol{v}')\,\mathrm{d}x'\,\mathrm{d}y'\,\mathrm{d}v_x'\,\mathrm{d}v_y'$$

# Implementation: Choice of $s_{\text{inc}}(t)$?

- "Impulse radar" uses

$$s_{\text{inc}}(t) \sim \delta(t) \quad \text{and so} \quad \chi(\tau,\nu) \sim \delta(\tau)$$

$\Rightarrow$ May not be the best choice (especially for slow movers).

- Same appears to be true for high Doppler resolution waveforms (for which $\chi(\tau,\nu) \sim \delta(\nu)$)
- Ideally, we want

$$K(\boldsymbol{r},\boldsymbol{r}',\boldsymbol{v},\boldsymbol{v}') = \delta(\boldsymbol{r}-\boldsymbol{r}',\boldsymbol{v}-\boldsymbol{v}')$$

- But new (extended) imaging kernel is no longer even guaranteed to be shift-invariant.
- Design depends on $s_{\text{inc}}(t)$ *as well as on sensor geometry* $\gamma(\theta)$

# Implementation: Choice of $s_{\text{inc}}(t)$?

- "Impulse radar" uses

$$s_{\text{inc}}(t) \sim \delta(t) \quad \text{and so} \quad \chi(\tau, \nu) \sim \delta(\tau)$$

  $\Rightarrow$ May not be the best choice (especially for slow movers).
- Same appears to be true for high Doppler resolution waveforms (for which $\chi(\tau, \nu) \sim \delta(\nu)$)
- Ideally, we want

$$K(\boldsymbol{r}, \boldsymbol{r}', \boldsymbol{v}, \boldsymbol{v}') = \delta(\boldsymbol{r} - \boldsymbol{r}', \boldsymbol{v} - \boldsymbol{v}')$$

- But new (extended) imaging kernel is no longer even guaranteed to be shift-invariant.
- Design depends on $s_{\text{inc}}(t)$ *as well as on sensor geometry* $\gamma(\theta)$

# Implementation: Choice of $s_{\text{inc}}(t)$?

- "Impulse radar" uses

$$s_{\text{inc}}(t) \sim \delta(t) \quad \text{and so} \quad \chi(\tau, \nu) \sim \delta(\tau)$$

  $\Rightarrow$ May not be the best choice (especially for slow movers).

- Same appears to be true for high Doppler resolution waveforms (for which $\chi(\tau, \nu) \sim \delta(\nu)$)
- Ideally, we want

$$K(\boldsymbol{r}, \boldsymbol{r}', \boldsymbol{v}, \boldsymbol{v}') = \delta(\boldsymbol{r} - \boldsymbol{r}', \boldsymbol{v} - \boldsymbol{v}')$$

- But new (extended) imaging kernel is no longer even guaranteed to be shift-invariant.
- Design depends on $s_{\text{inc}}(t)$ *as well as on sensor geometry* $\gamma(\theta)$

## Implementation: Choice of $s_{\rm inc}(t)$?

- "Impulse radar" uses

$$s_{\rm inc}(t) \sim \delta(t) \quad \text{and so} \quad \chi(\tau, \nu) \sim \delta(\tau)$$

  $\Rightarrow$ May not be the best choice (especially for slow movers).

- Same appears to be true for high Doppler resolution waveforms (for which $\chi(\tau, \nu) \sim \delta(\nu)$)
- Ideally, we want

$$K(\boldsymbol{r}, \boldsymbol{r}', \boldsymbol{v}, \boldsymbol{v}') = \delta(\boldsymbol{r} - \boldsymbol{r}', \boldsymbol{v} - \boldsymbol{v}')$$

- But new (extended) imaging kernel is no longer even guaranteed to be shift-invariant.
- Design depends on $s_{\rm inc}(t)$ *as well as on sensor geometry* $\gamma(\theta)$

# Implementation: Choice of $s_{\text{inc}}(t)$?

- "Impulse radar" uses

$$s_{\text{inc}}(t) \sim \delta(t) \quad \text{and so} \quad \chi(\tau,\nu) \sim \delta(\tau)$$

  $\Rightarrow$ May not be the best choice (especially for slow movers).

- Same appears to be true for high Doppler resolution waveforms (for which $\chi(\tau,\nu) \sim \delta(\nu)$)
- Ideally, we want

$$K(\boldsymbol{r},\boldsymbol{r}',\boldsymbol{v},\boldsymbol{v}') = \delta(\boldsymbol{r}-\boldsymbol{r}',\boldsymbol{v}-\boldsymbol{v}')$$

- But new (extended) imaging kernel is no longer even guaranteed to be shift-invariant.
- Design depends on $s_{\text{inc}}(t)$ *as well as on sensor geometry* $\gamma(\theta)$

# Other applications: “Start-Stop’ error

- Scheme allows for straightforward introduction of a variety of complex imaging configurations
- Start-stop accuracy?

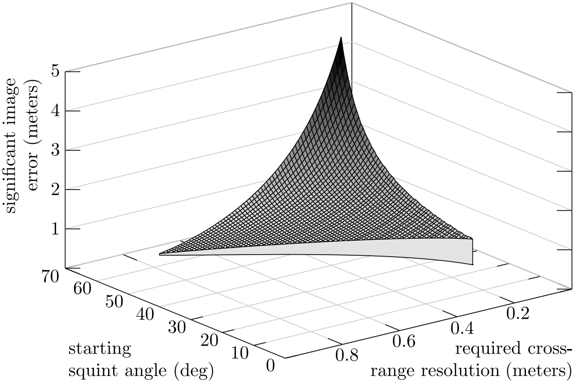



Figure: Error as a function of squint and resolution for a target traveling 100km/hr in the direction opposite to the platform velocity (8 km/sec). (X-band illumination.)

# Other applications: Multistatic imaging

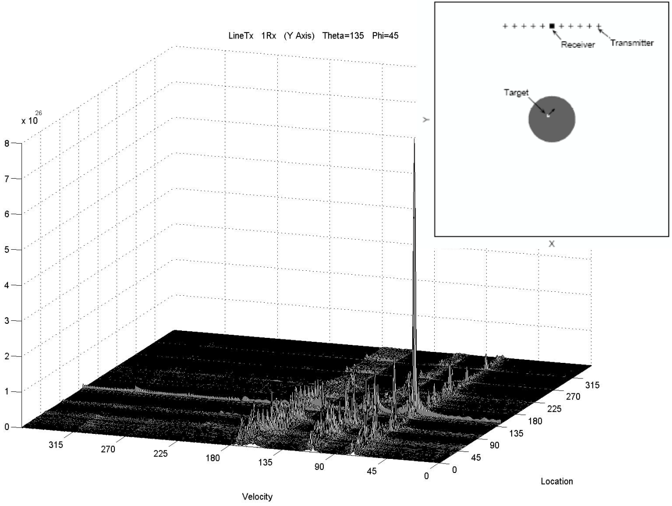



Figure: The geometry (not to scale) for a linear array 11 transmitters and a single receiver and the corresponding combined point-spread function.

# Future work

- Understand $K(\boldsymbol{r}, \boldsymbol{r}', \boldsymbol{v}, \boldsymbol{v}')$
  - Choice of $s_{\text{inc}}(t)$ vs. $\gamma(\theta)$
  - Nature of its shift-variance.
  - Expect that $K$ will also characterize effects of non-uniqueness of solutions.
- Extend to accelerating image elements
- Can other target-specific parameters be introduced?

$$\rho_{\boldsymbol{v}}(\boldsymbol{r}) \to \rho_{\boldsymbol{v}}(\boldsymbol{r}, \theta, \ldots)$$

- What about multiple scattering?

# Future work

- Understand $K(\boldsymbol{r}, \boldsymbol{r}', \boldsymbol{v}, \boldsymbol{v}')$
  - Choice of $s_{\text{inc}}(t)$ vs. $\gamma(\theta)$
  - Nature of its shift-variance.
  - Expect that $K$ will also characterize effects of non-uniqueness of solutions.
- Extend to accelerating image elements
- Can other target-specific parameters be introduced?

$$\rho_{\boldsymbol{v}}(\boldsymbol{r}) \to \rho_{\boldsymbol{v}}(\boldsymbol{r}, \theta, \dots)$$

- What about multiple scattering?

# Future work

- Understand $K(\boldsymbol{r}, \boldsymbol{r}', \boldsymbol{v}, \boldsymbol{v}')$
  - Choice of $s_{\text{inc}}(t)$ vs. $\gamma(\theta)$
  - Nature of its shift-variance.
  - Expect that $K$ will also characterize effects of non-uniqueness of solutions.
- Extend to accelerating image elements
- Can other target-specific parameters be introduced?

$$\rho_{\boldsymbol{v}}(\boldsymbol{r}) \to \rho_{\boldsymbol{v}}(\boldsymbol{r}, \theta, \ldots)$$

- What about multiple scattering?

# Future work

- Understand $K(\boldsymbol{r}, \boldsymbol{r}', \boldsymbol{v}, \boldsymbol{v}')$
  - Choice of $s_{\text{inc}}(t)$ vs. $\gamma(\theta)$
  - Nature of its shift-variance.
  - Expect that $K$ will also characterize effects of non-uniqueness of solutions.
- Extend to accelerating image elements
- Can other target-specific parameters be introduced?

$$\rho_{\boldsymbol{v}}(\boldsymbol{r}) \to \rho_{\boldsymbol{v}}(\boldsymbol{r}, \theta, \ldots)$$

- What about multiple scattering?

## Future work

- Understand $K(\boldsymbol{r}, \boldsymbol{r}', \boldsymbol{v}, \boldsymbol{v}')$
  - Choice of $s_{\text{inc}}(t)$ vs. $\gamma(\theta)$
  - Nature of its shift-variance.
  - Expect that $K$ will also characterize effects of non-uniqueness of solutions.
- Extend to accelerating image elements
- Can other target-specific parameters be introduced?

$$\rho_{\boldsymbol{v}}(\boldsymbol{r}) \to \rho_{\boldsymbol{v}}(\boldsymbol{r}, \theta, \ldots)$$

- What about multiple scattering?

# Future work

- Understand $K(\boldsymbol{r}, \boldsymbol{r}', \boldsymbol{v}, \boldsymbol{v}')$
  - Choice of $s_{\mathrm{inc}}(t)$ vs. $\gamma(\theta)$
  - Nature of its shift-variance.
  - Expect that $K$ will also characterize effects of non-uniqueness of solutions.
- Extend to accelerating image elements
- Can other target-specific parameters be introduced?

$$\rho_{\boldsymbol{v}}(\boldsymbol{r}) \to \rho_{\boldsymbol{v}}(\boldsymbol{r}, \theta, \ldots)$$

- What about multiple scattering?