Challenges in Detecting & Tracking Moving Objects with Synthetic Aperture Radar (SAR)

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Outline

- Focusing Moving Targets
- Locating Moving Targets
- Separating Moving Targets

Movers In SAR Images

Moving Targets with SAR Challenges

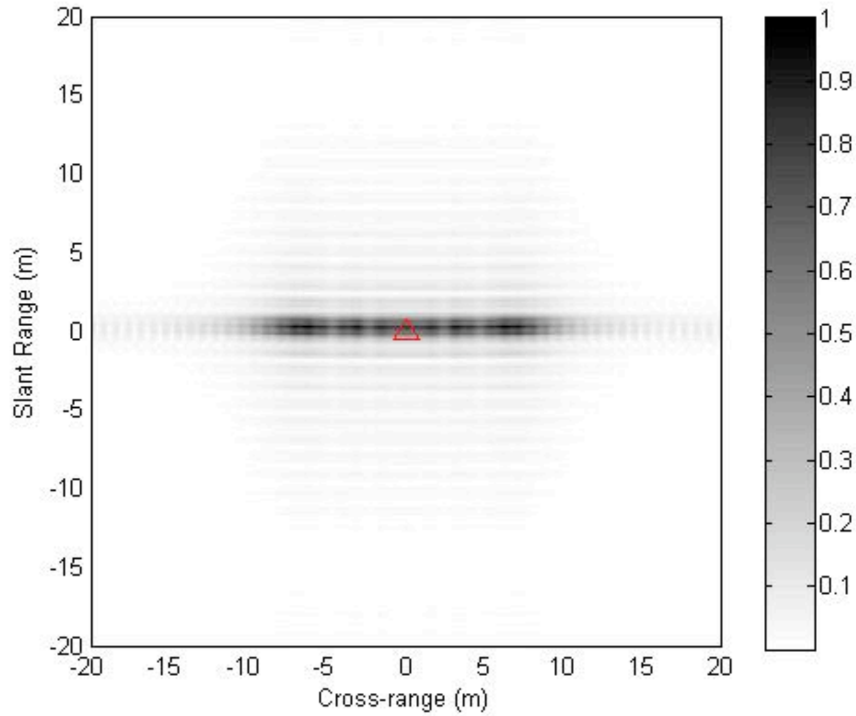
- Three Problems Must be solved to reliably detect and track moving objects with a SAR radar
- The moving objects are defocused into streaks
  - Energy is spread out, dimmer
- The moving objects show up at the wrong place
  - The streaks are rotated through a large angle
  - Angle can change rapidly
  - Range is maintained
- Objects are Mixed in with the bright stationary objects on the earth

Problem 1: Defocusing to Streaks

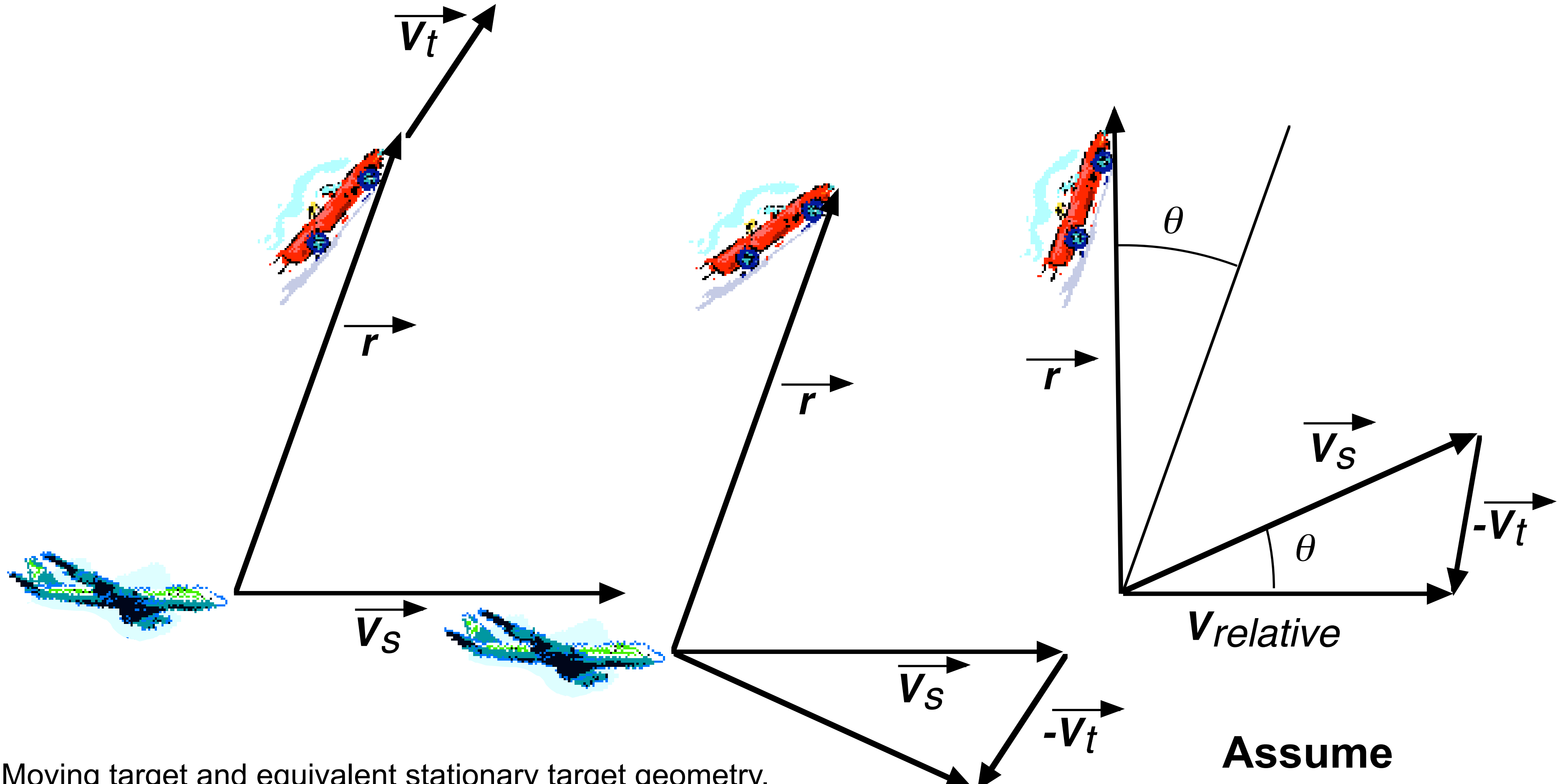
- **Cross range resolution of a SAR image is derived from collecting data from a large synthetic aperture.**
  - **Usually takes several seconds to fly through the aperture**
  - **Every pixel in the SAR image can be thought of as an inner product of the received data and a vector that represents the signal that would be received from an ideal point reflector at that location**
  - **There are many different SAR algorithms and they all do this explicitly or approximate this operation.**
- **Focusing the movers can be achieved by selecting the proper vector**
  - **Must understand the range as a function of time**

Moving Point Target (v_x only)

- Moving Point Target with v_x = 5 m/s
  - Cross-Range Velocity induces Azimuth Smear given by: $$\Delta x_{smear} = 2v_x T_{CPI} = (2)(5\ \text{m/s})(2\ \text{s}) = 20\ \text{m}$$



How we Think of Linear Moving Targets



Moving target and equivalent stationary target geometry.
Both cases produce identical Range as a function of time so both produce identical phase histories. In second case target is stationary, remains at same range but rotates through angle of θ relative to the sensor. Sensor is moving at speed $v_{relative}$ instead of v_s. This holds true for any target position

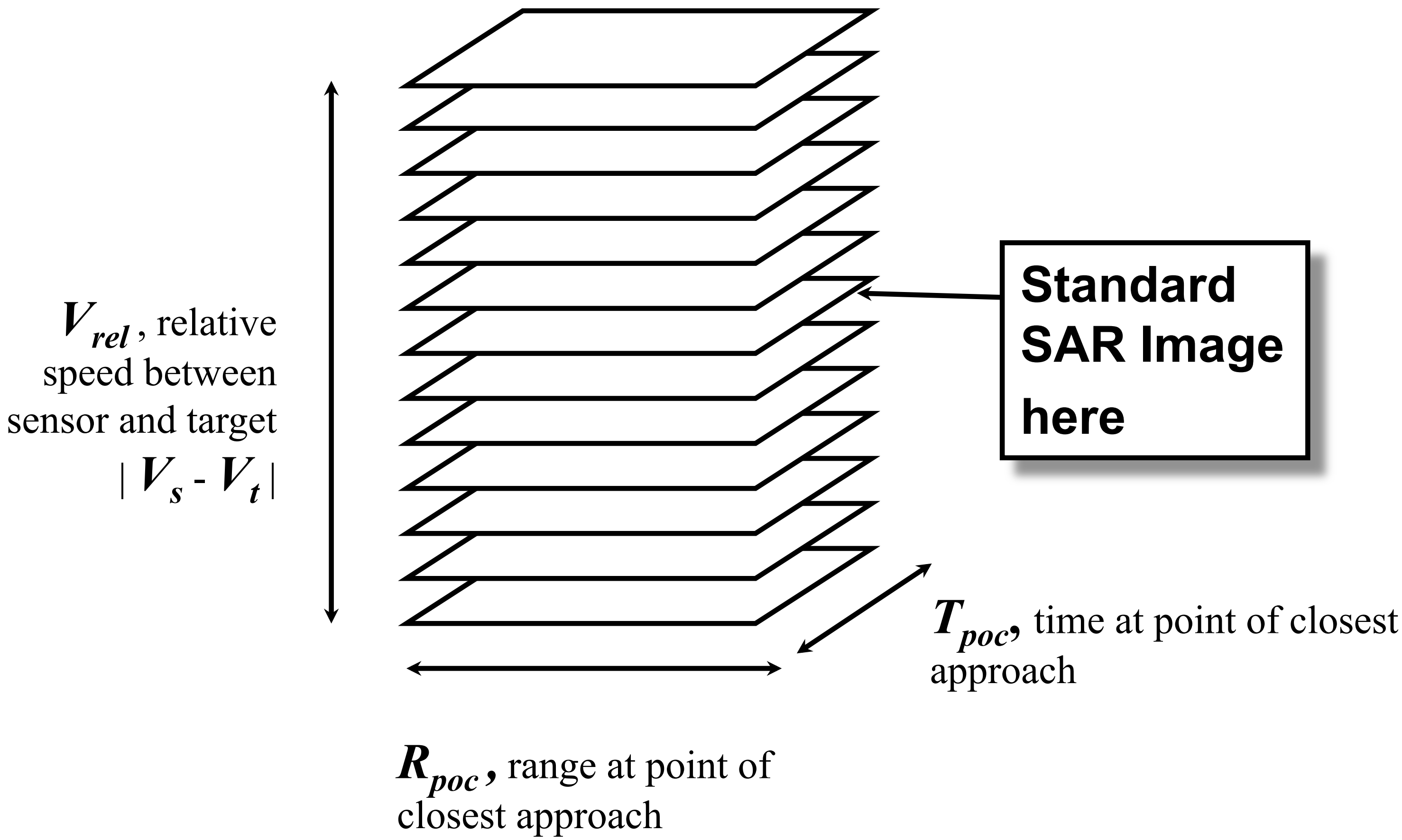
Assume linear motion for targets and radar

How to focus linear moving targets

- Form a series of images assuming different ground speeds, from v_s-v_{tmax} to v_s+v_{tmax}. Where v_{tmax} is the maximum target velocity of interest
- Each image will capture a different set of target velocities, complete set of images will focus all target velocities less than v_{tmax} regardless of direction and target location
- For the image that focuses the target, the target will remain at same image location as long as velocity is constant

SAR-MTI detection cube

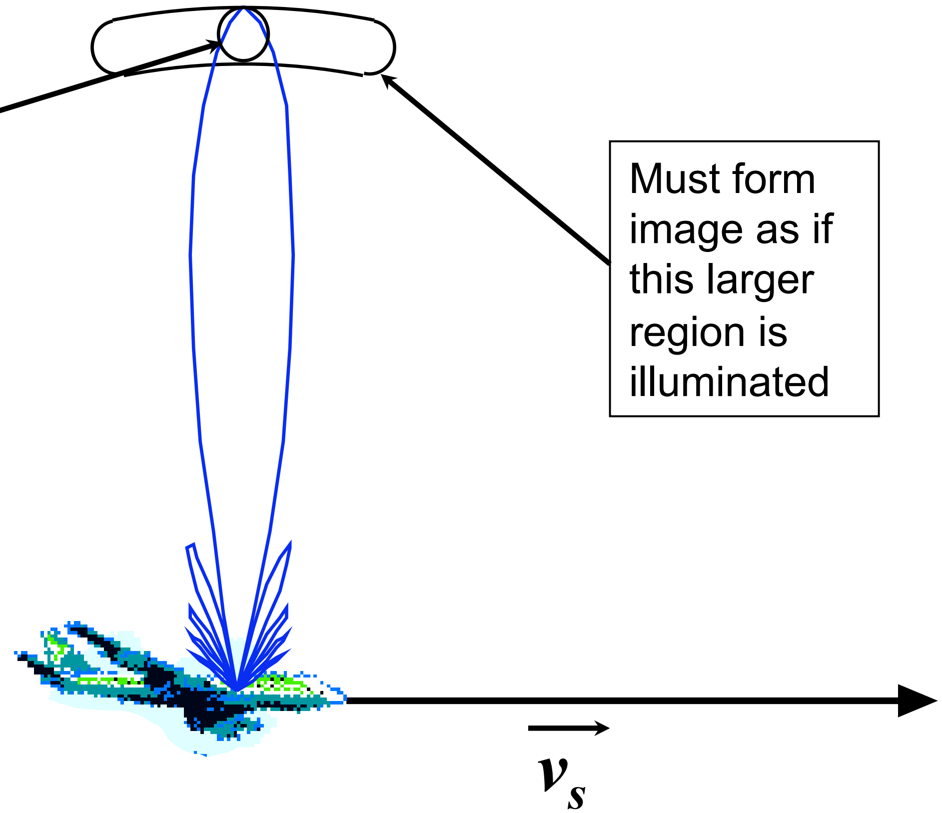
- **Mover space has 3 dimensions, R_{poc}, T_{poc}, $|V_{relative}|$**
- **We may track targets in this space**

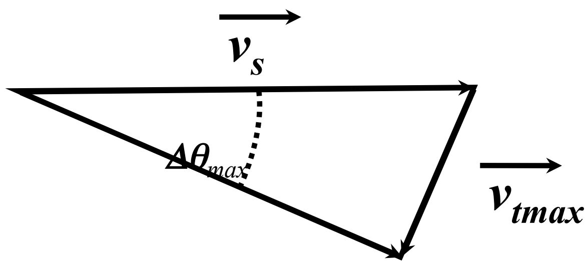


Moving targets appear in larger area due to Azimuthal rotation.

Radar illuminates only this small region of the ground

Must form image as if this larger region is illuminated

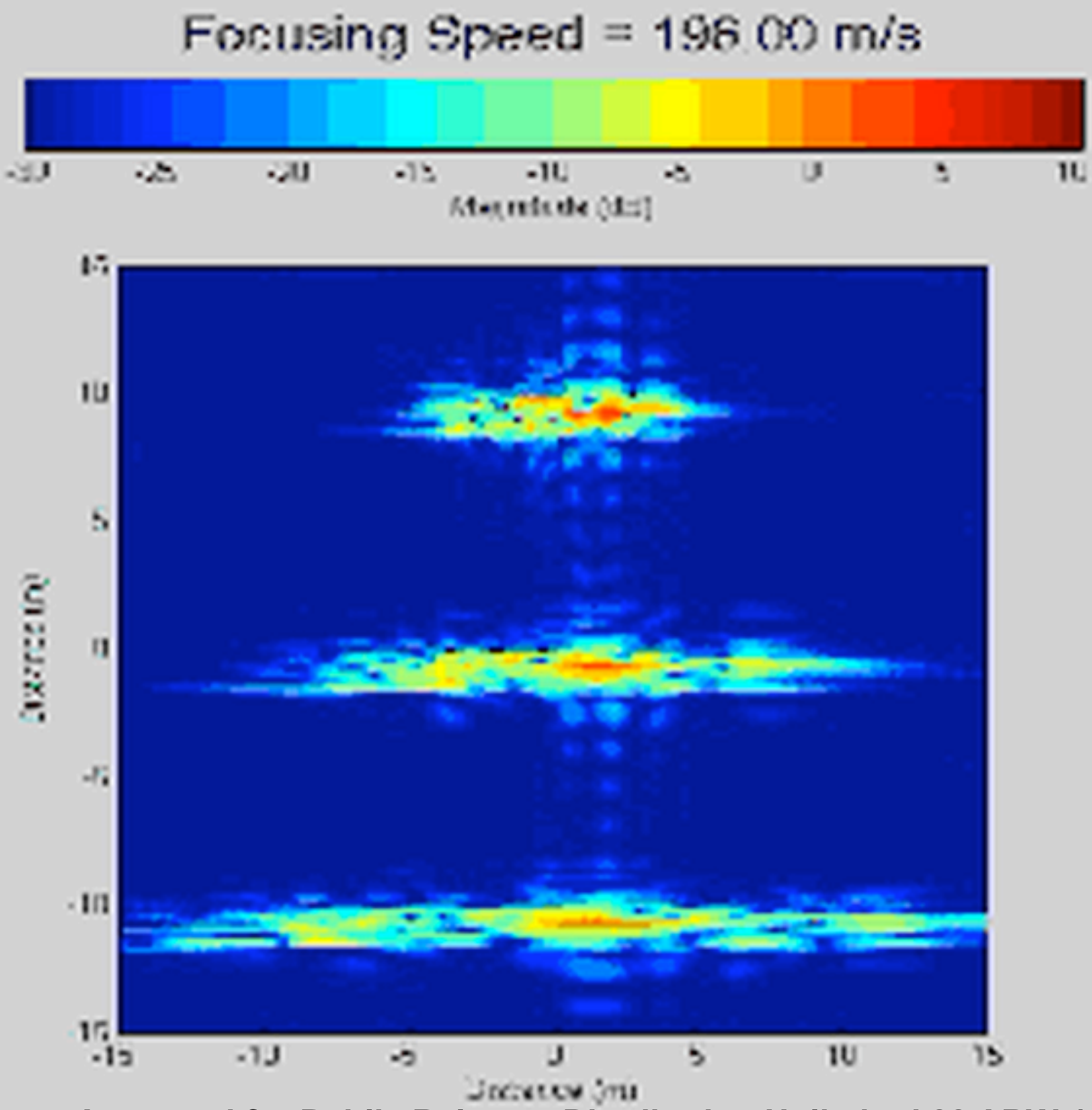




Target velocity which forms the maximum azimuthal rotation

For v_s =150 m/s and v_{tmax} = 30m/s, max spread is +/- 11.3°

Simulated Focusing



Range and Cross-Range Resolution

Cross range resolution can also be thought of as range-rate resolution when considering moving targets. Cross-range resolution is range dependent, range-rate resolution is not

$$\delta_{cr} = \frac{\lambda r}{2 v_{relative} \sin(\theta) T} \qquad \Delta r = \frac{c}{2B}$$

Where T **is the integration time in seconds**

θ **is the squint angle to the target**

r **is the ground range to the target**

δ_{cr} **is cross-range resolution in meters**

Δr **is range resolution in m**

B **is radar Bandwidth in Hz**

c **is the speed of light m/s**

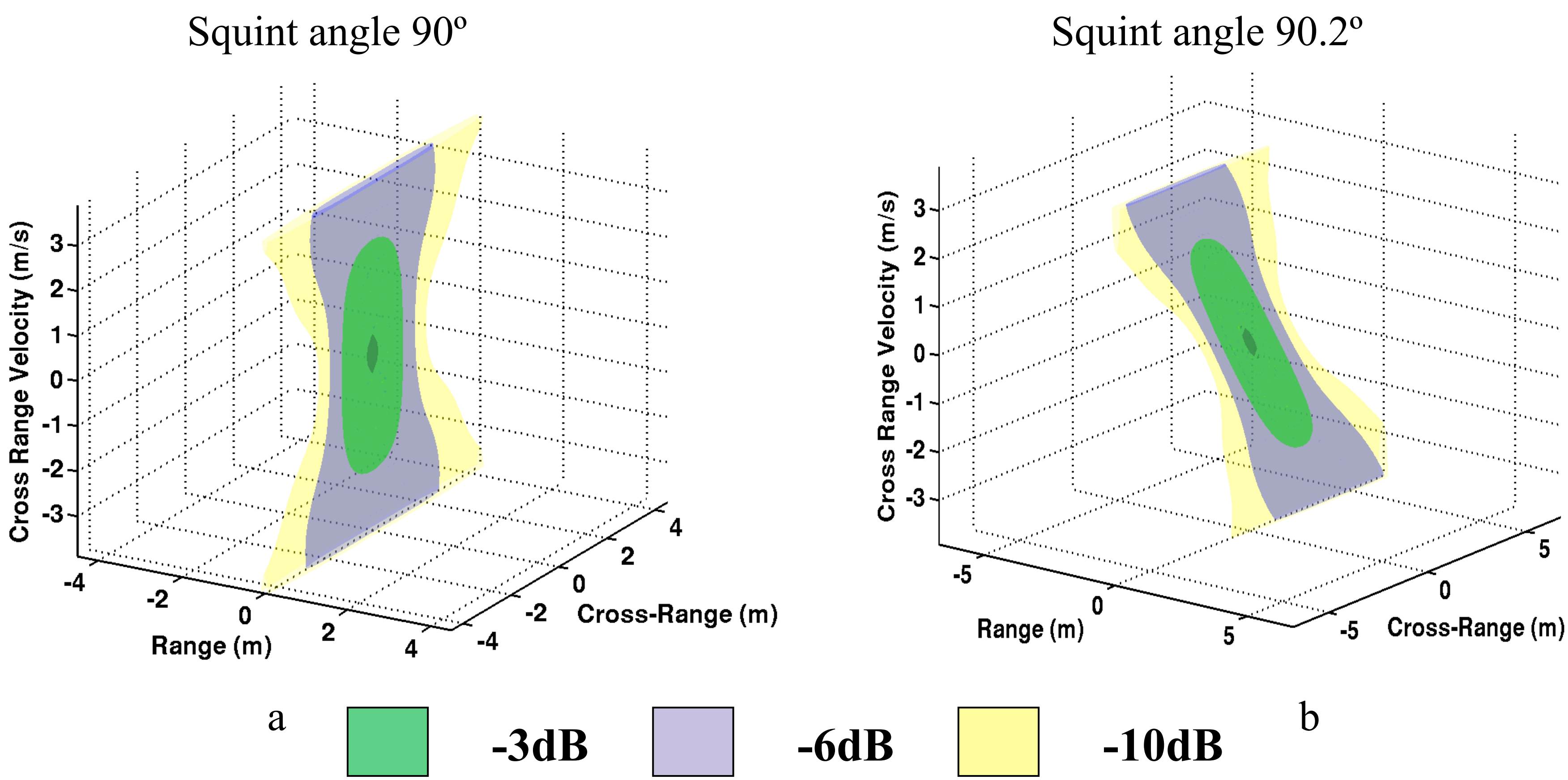
How closely must we sample $V_{relative}$?

- **Filter spacing required to limit loss due to smearing to -3 dB, I.e. cross-range velocity resolution**

$$\Delta v_{relative} = \frac{6.6^2 \lambda r}{16\pi v_{relative} \sin(\theta) T^2} = \frac{1.72\delta_{cr}}{T}$$

$\boldsymbol{T}$ is the integration time in seconds
$\boldsymbol{\omega}$ is the aspect change in radians/second = $\boldsymbol{v}_{relative}\, sin\theta/\boldsymbol{r}$
$\boldsymbol{\theta}$ is the squint angle to the target
$\boldsymbol{r}$ is the ground range to the target

Examples of point-spread-functions in a SAR-MTI detection cube



Coupling factor: $r \cot\theta / V_{relative}$

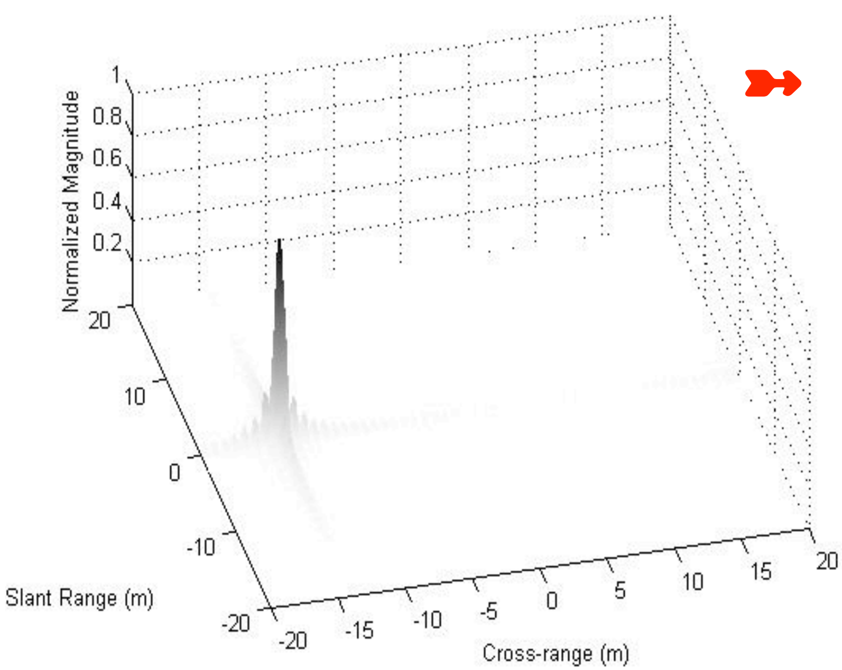
Ambiguities and Geolocation

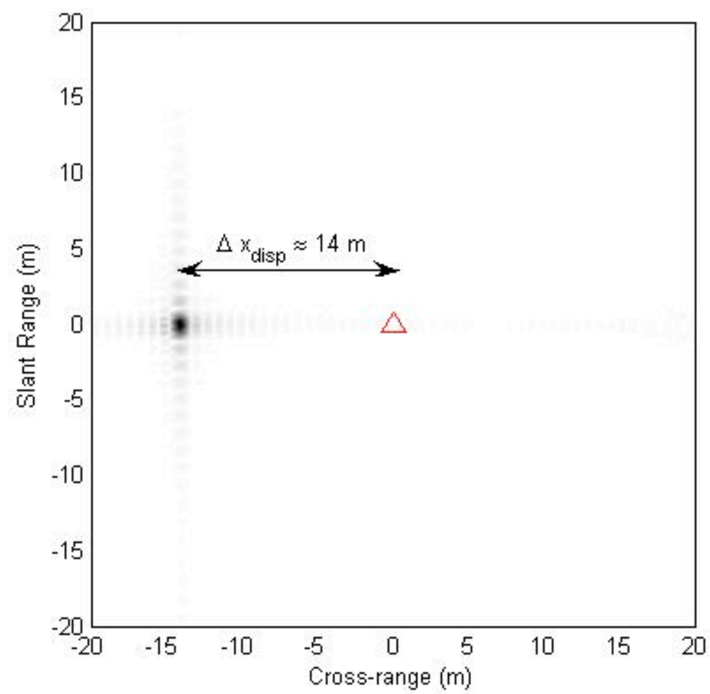
- **Ambiguities are the set of points with their corresponding linear velocities that will focus to the same point in Detection Cube**
- **Four dimension space detected in three dimension space so Ambiguities are unavoidable**
- **Ambiguities occur because different points moving at different velocities can have identical range vs. time functions relative to the sensor.**
- **If a radar has only one phase center (and has an omnidirectional antenna), Range is all that can be measured so:**
  - **Ambiguity cannot be eliminated by changing PRF, Polarization, Waveform coding etc**
  - **Ambiguity will come through any sort of SAR image formation algorithm**
  - **Ambiguity will remain as long as all motion remains linear and constant velocity**
  - **Ambiguity can be unraveled with multiple phase centers**

Moving Point Target (v_y only)

- Moving Point Target with v_y = 1 m/s
  - Range Velocity induces Azimuth Shift given by:

$$\Delta x_{\text{disp}} = \frac{v_y}{v_p} R_{\text{center}} = \frac{1\ (\text{m/s})}{100\ (\text{m/s})}(1414\text{m}) \approx 14\ \text{m}$$





Ambiguity Definition

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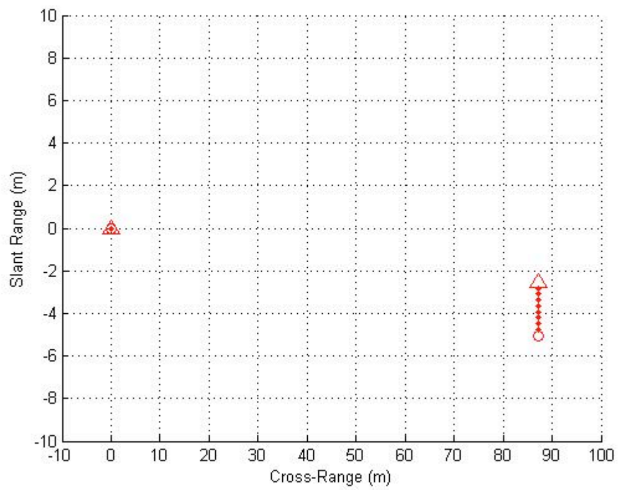
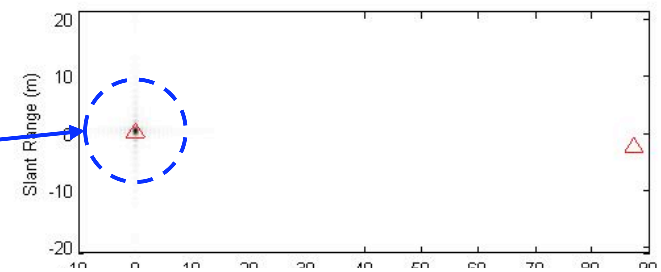
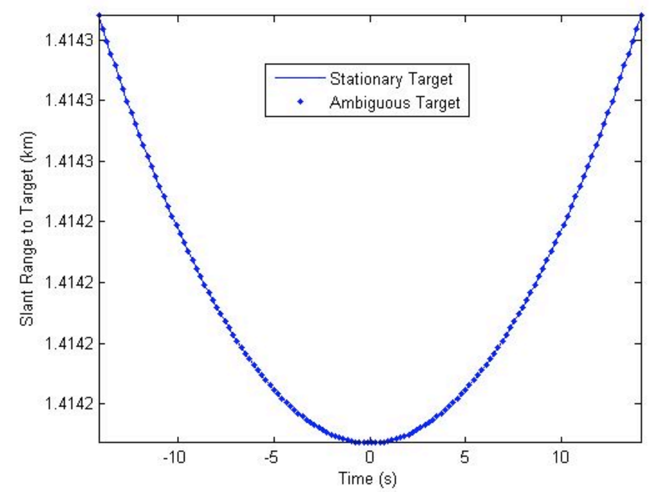
- **Geolocation Ambiguity** … Set of Moving Targets with Corresponding Linear Velocities Focus to Same Point in an Image … ***Identical Range*** **vs.** ***Time Profiles***

SAR Image Focused To Final Position



Both Targets Focused to Here !!



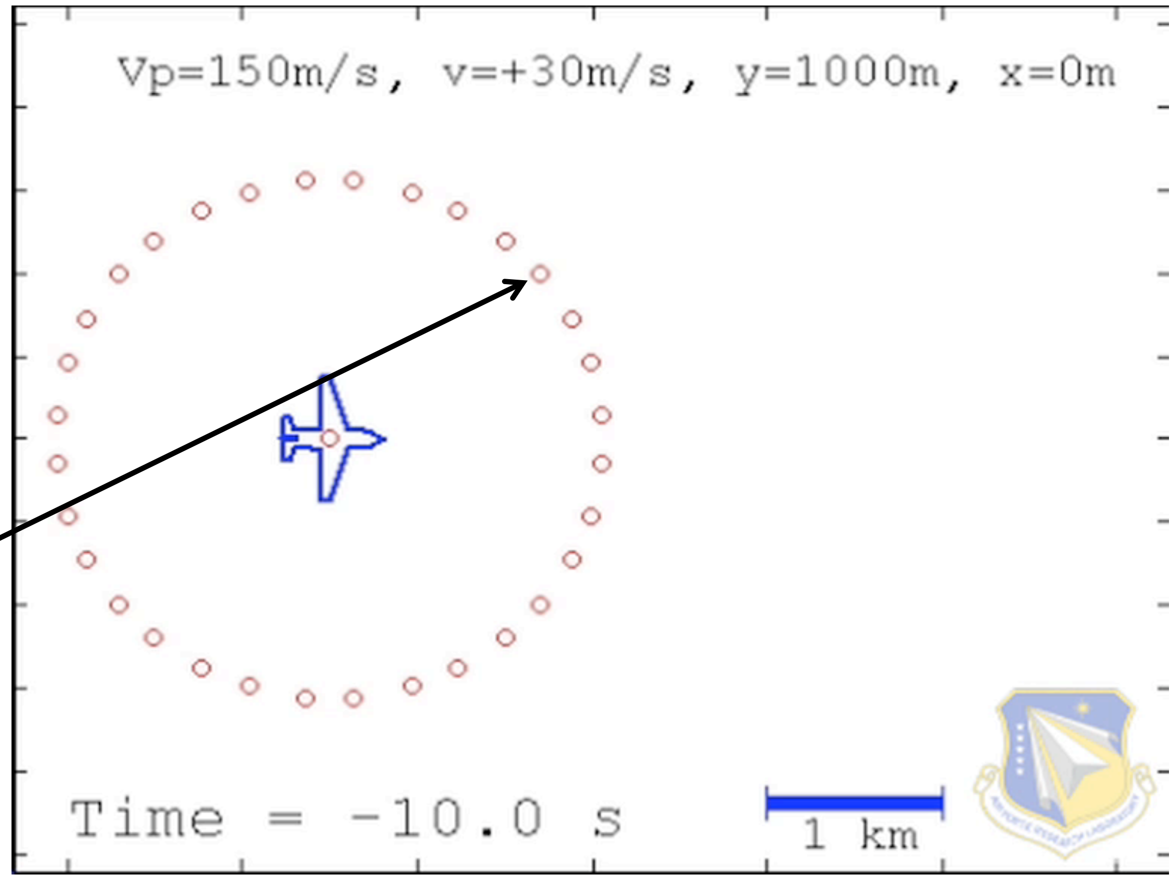


Actual Target Response
… Circle is Starting Position
… Triangle is Final Position

Movie showing ambiguities in action

Circle of points has PCA of 1000 m (t=0) 150 m/s platform velocity (plane not drawn to scale!). All velocities are constant & linear

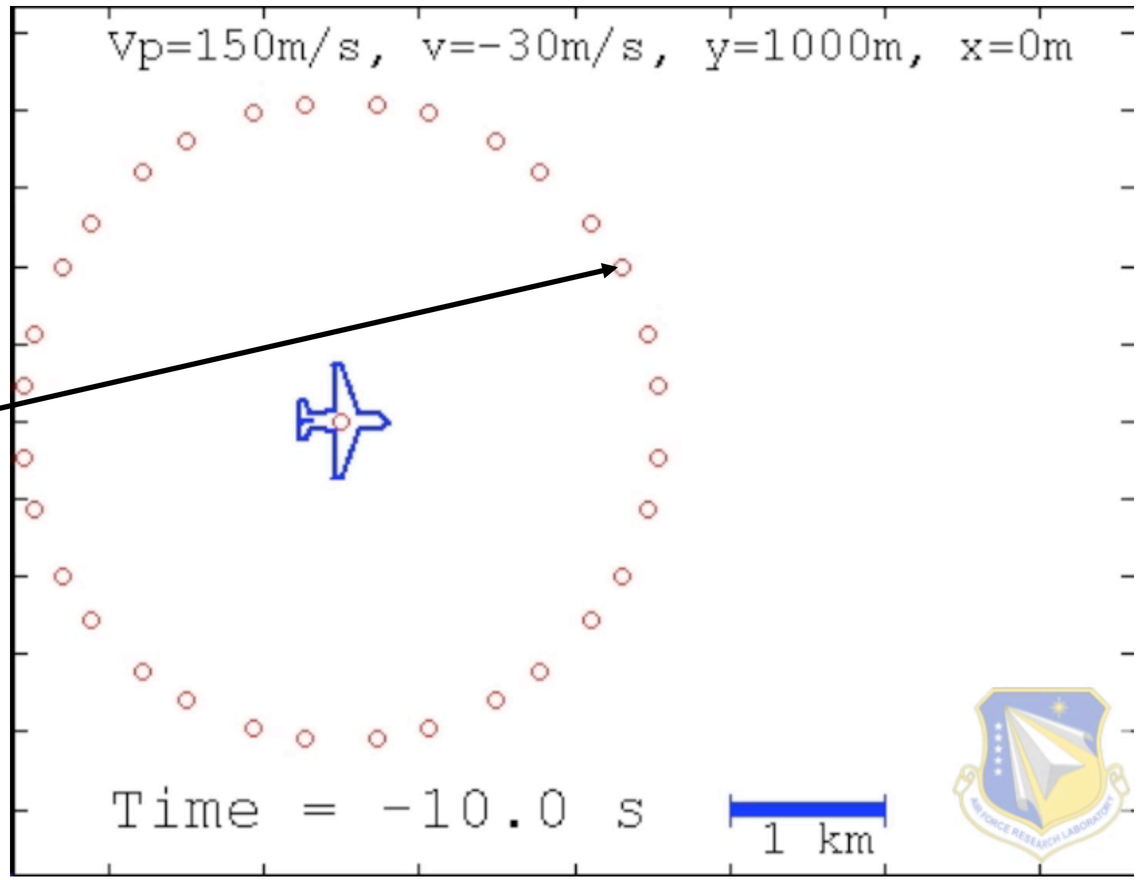
Assuming a virtual ground speed of 120 m/s, standard Image formation will focus all points to this point
which is moving 30 m/s to the right



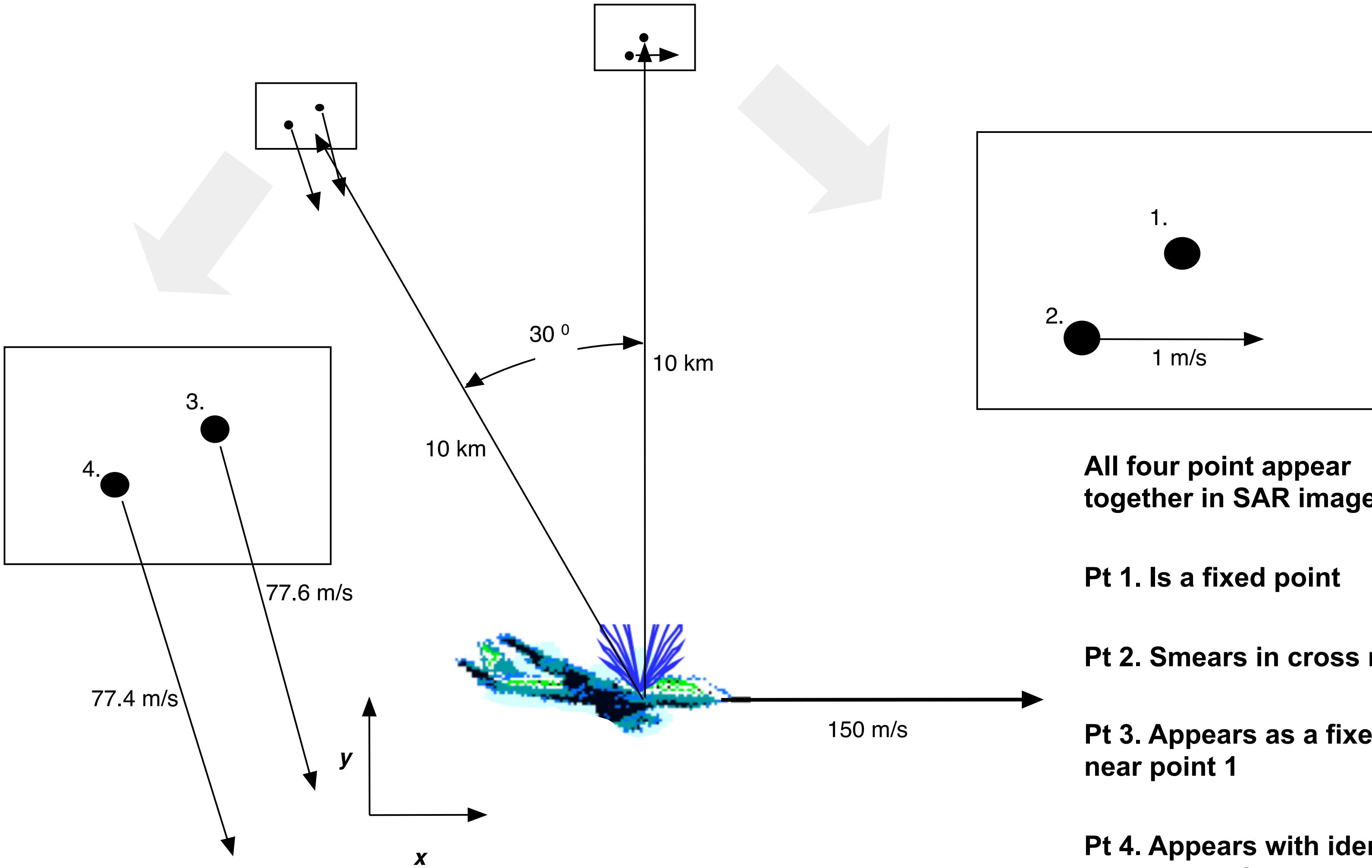
Movie showing ambiguities in action (cont.)

Circle of points has radius of 1000 m at smallest point (t=0)
150 m/s platform velocity (plane not drawn to scale!).
All velocities are constant & linear

Assuming a virtual ground speed of 180 m/s, standard Image formation will focus all points to this point
which is moving 30 m/s to the left



Moving Target Persistence In Detection Cube



All four point appear together in SAR image

Pt 1. Is a fixed point

Pt 2. Smears in cross range

Pt 3. Appears as a fixed pt near point 1

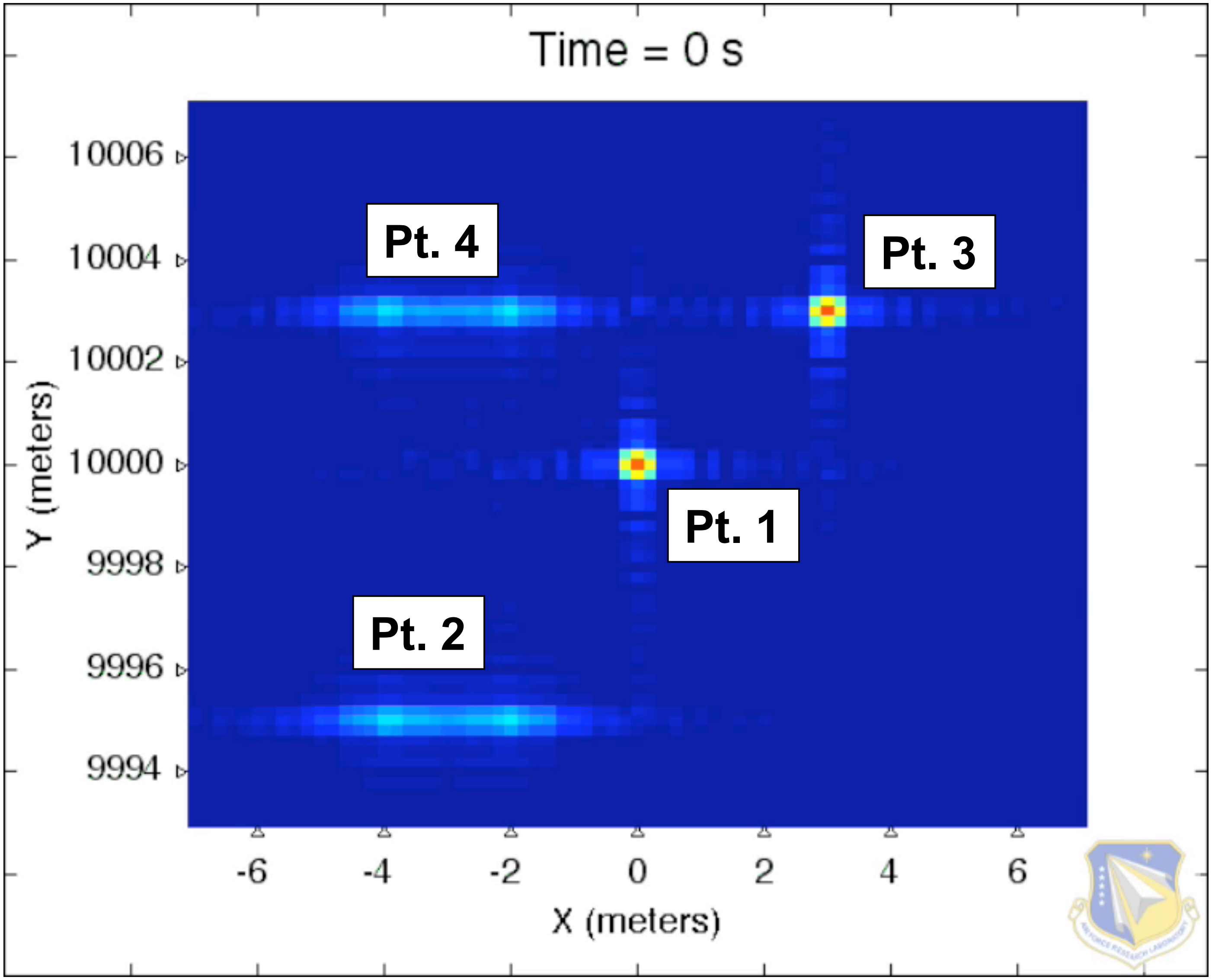
Pt 4. Appears with identical smear to pt 2

Sequence of SAR Images of Scene Shown on Previous slides

Images formed from 2 s apertures

Final frame skips ahead 1 minute. Note that relationship of pt 1 and 3 does not change even though pt 3 has moved almost 5 km!

The perceived relationship between scatterers remain fixed as long as all motion is linear and constant velocity



Geolocation of movers

- Range to target is known.
- Azimuth is Problematic, there is an azimuth ambiguity that cannot be resolved with a single phase center Change in Velocity may admit solution based on most likely path between Ambiguities
- Multiple phase centers would allow monopulse or STAP type processing
- Heuristic reasoning may help. E.g. assume target is traveling on a road

Integration Time Effects

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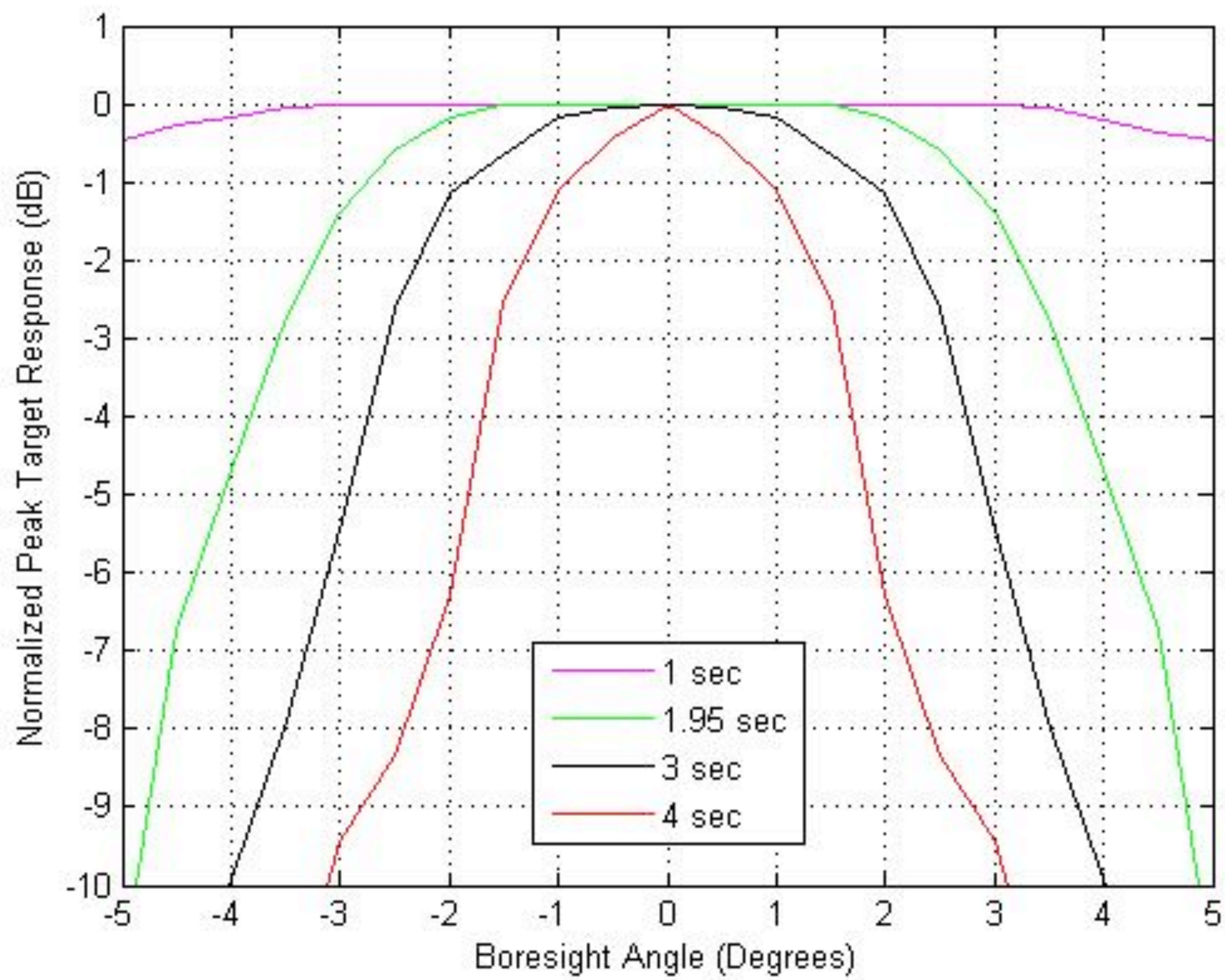


Fixed

R_{center}

Look Angle

f_o

Trend

$\uparrow$ T_{CPI}

$\downarrow$ θ_{amb}

Finding Direction with Multiple Antennas

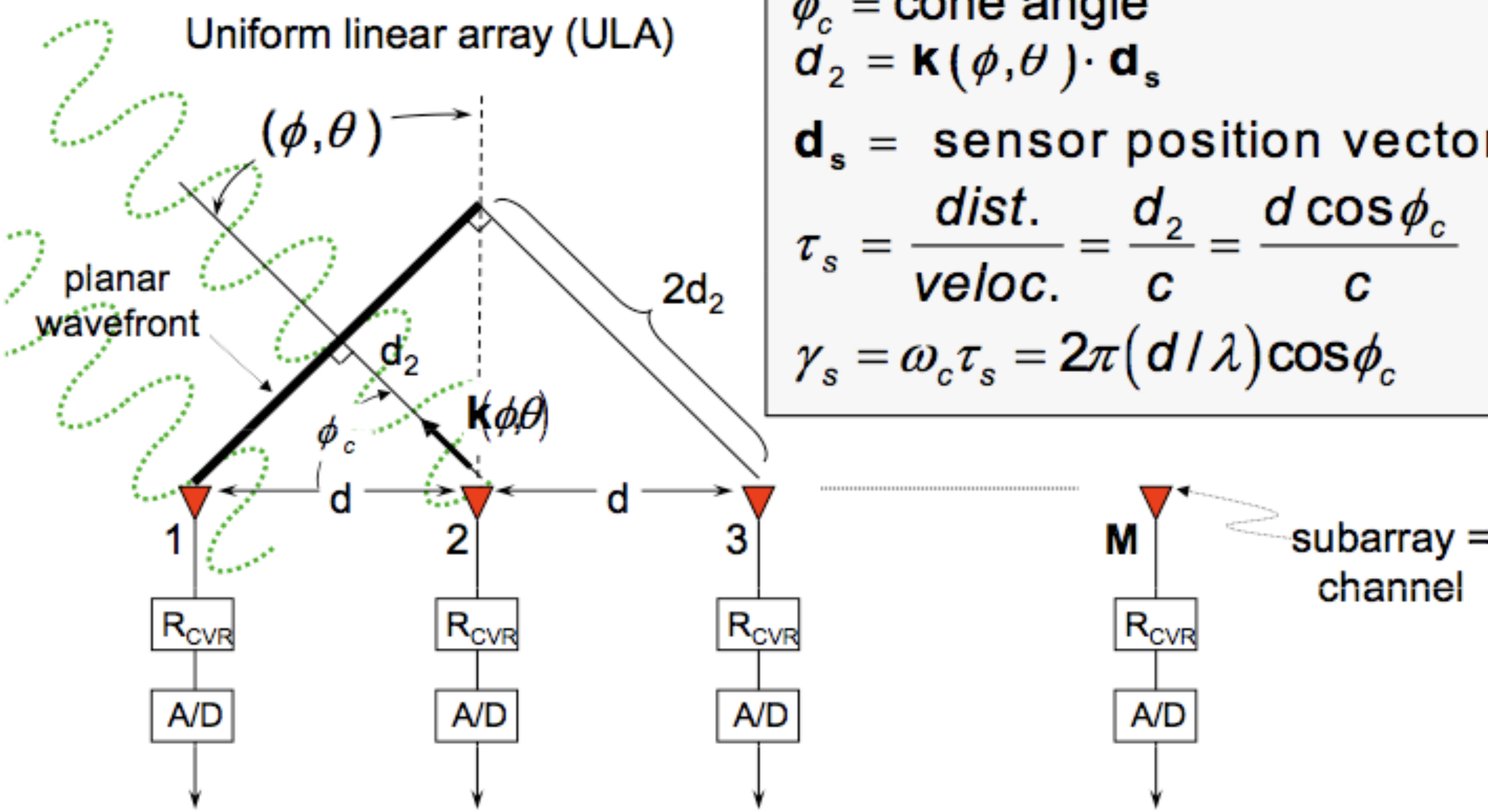
$$\phi_c = \text{cone angle}$$

$$d_2 = \mathbf{k}(\phi,\theta)\cdot\mathbf{d}_s$$

$$\mathbf{d}_s = \text{sensor position vector}$$

$$\tau_s = \frac{dist.}{veloc.} = \frac{d_2}{c} = \frac{d\cos\phi_c}{c}$$

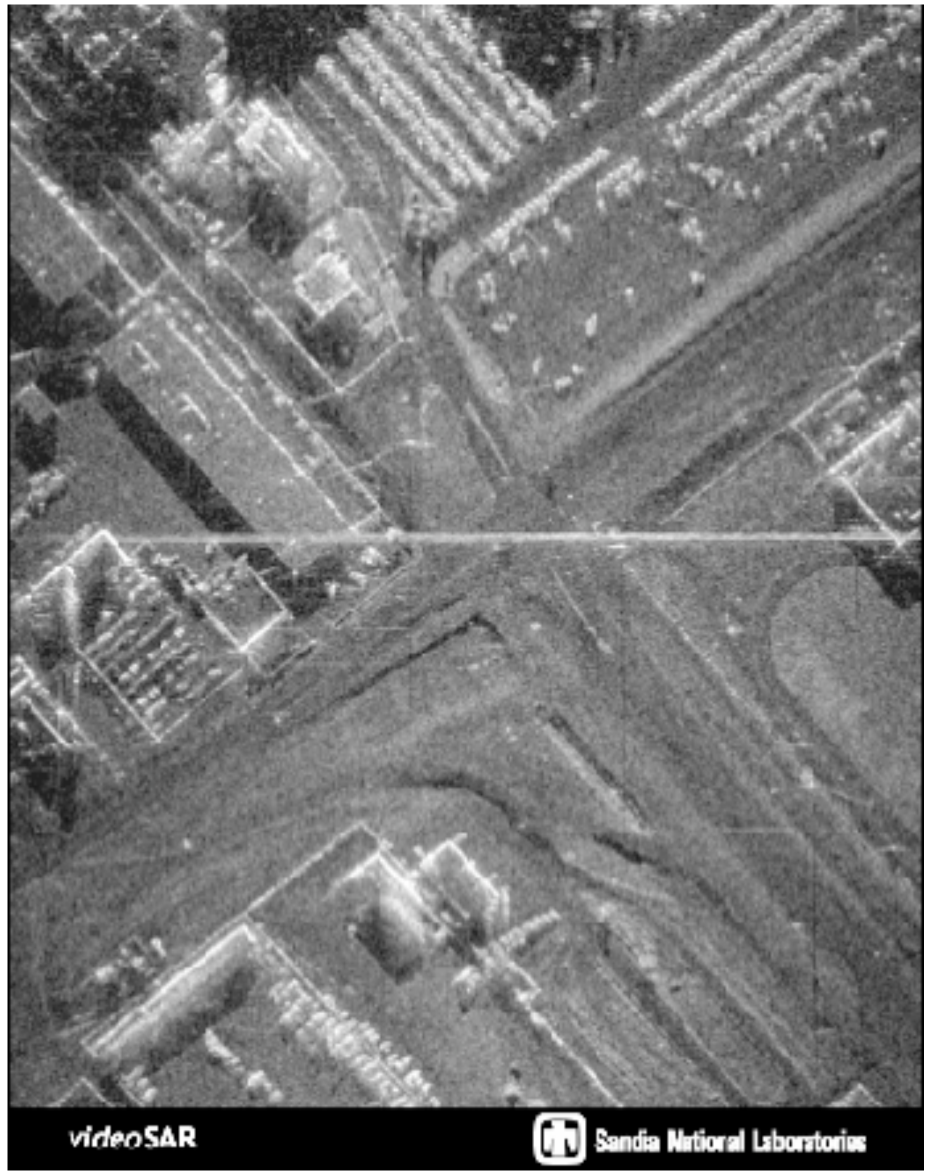
$$\gamma_s = \omega_c\tau_s = 2\pi(d/\lambda)\cos\phi_c$$



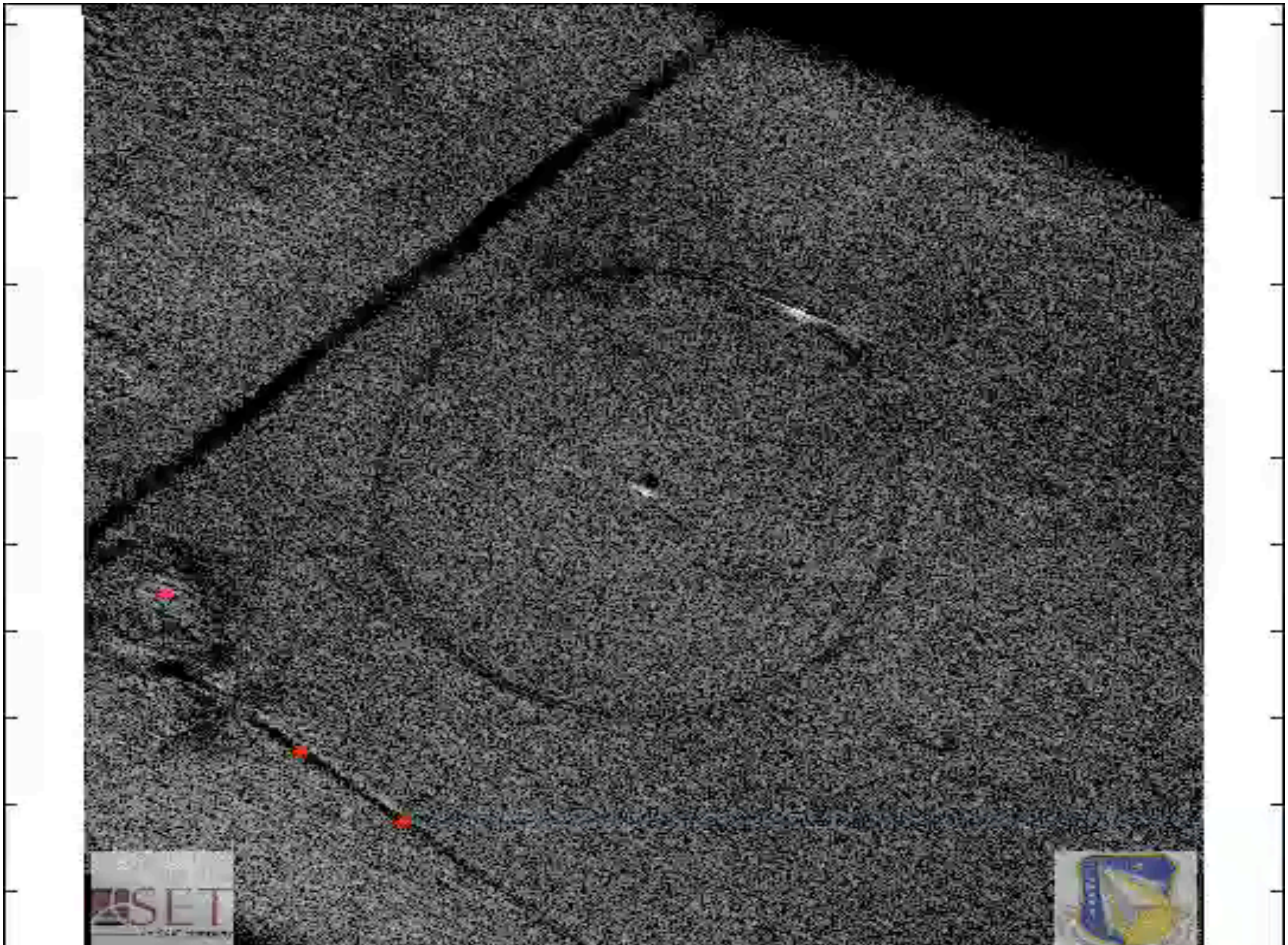
Courtesy Dr. Murali Rangaswamy AFRL/RYAP

Heuristic Geolocation Approaches

- **Find Shadows**
- **Target Orientation**
- **Assume Road travel**
- **Combination of above approaches**



Shadow Tracker

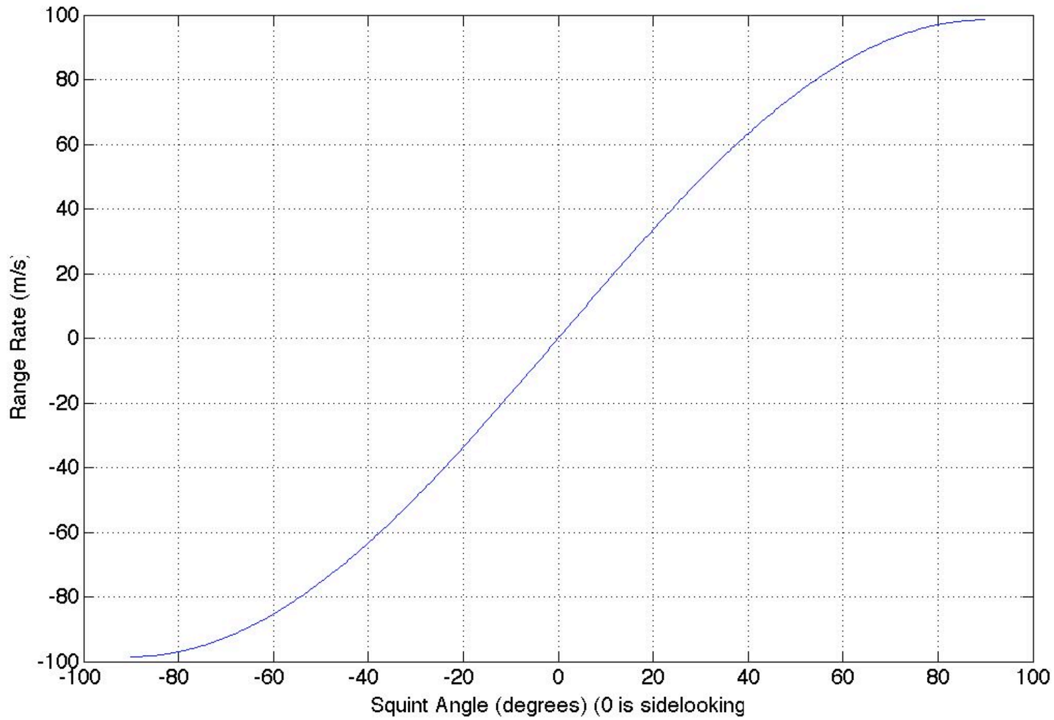


Movers Mixed in with the Bright Earth

Restrict search to movers with range rates faster than any stationary object in the beam

Simple solution, works better with narrow beams and beams squinted fore or aft.

SAR works better with a side looking antenna with a wide beam



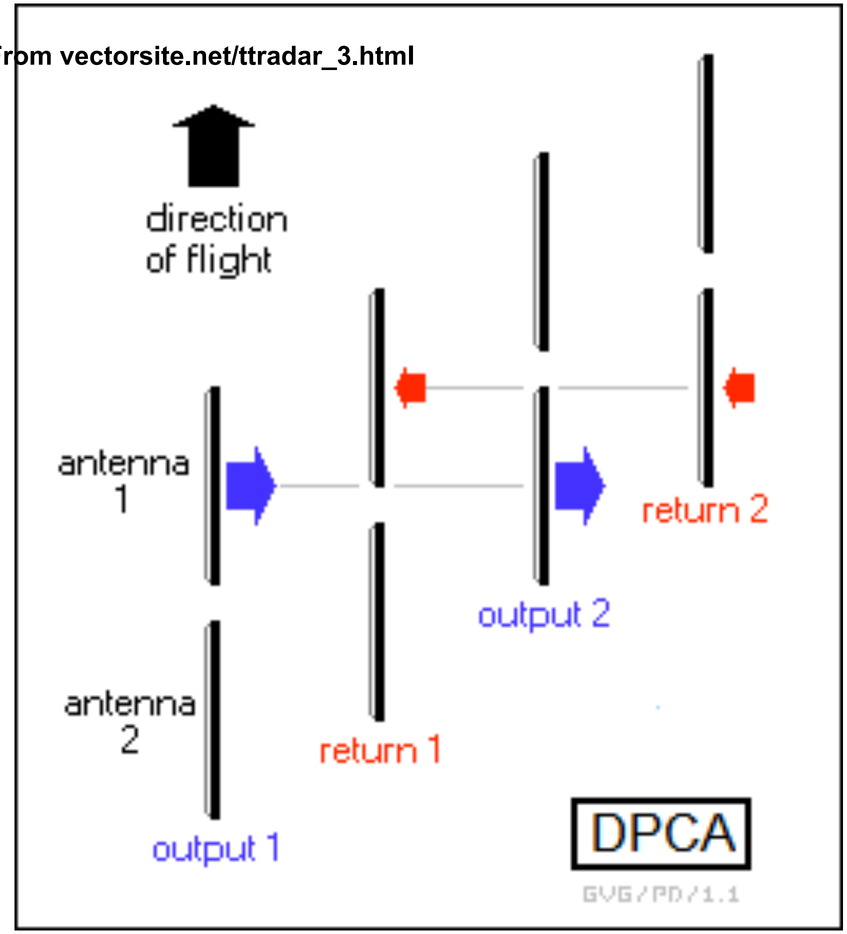
Displaced Phase Center Antenna

Multiple Antennas can be used to Cancel Stationary objects

Subtract return 2 from return 1

Very sensitive to aircraft velocity And receiver characteristics

From vectorsite.net/ttradar_3.html



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Steering Vectors

- **Originally used to describe the spatial response of the array to signal with a specific direction of arrival (DOA)**
  - **The conjugate "steers" the beam**
  - **Terminology also used to describe temporal response**
  - **Matched filter**

1-D Spatial Steering Vector (uniformly-spaced linear array, or ULA)

$$\mathbf{s}_{\mathbf{s}}(\gamma_s) = [1 \quad e^{j\gamma_s} \quad e^{j2\gamma_s} \quad \ldots \quad e^{j(M-1)\gamma_s}]^T \quad \left(= \mathbf{v}_{\mathbf{s}}(\gamma_s)\right)$$

Temporal Steering Vector

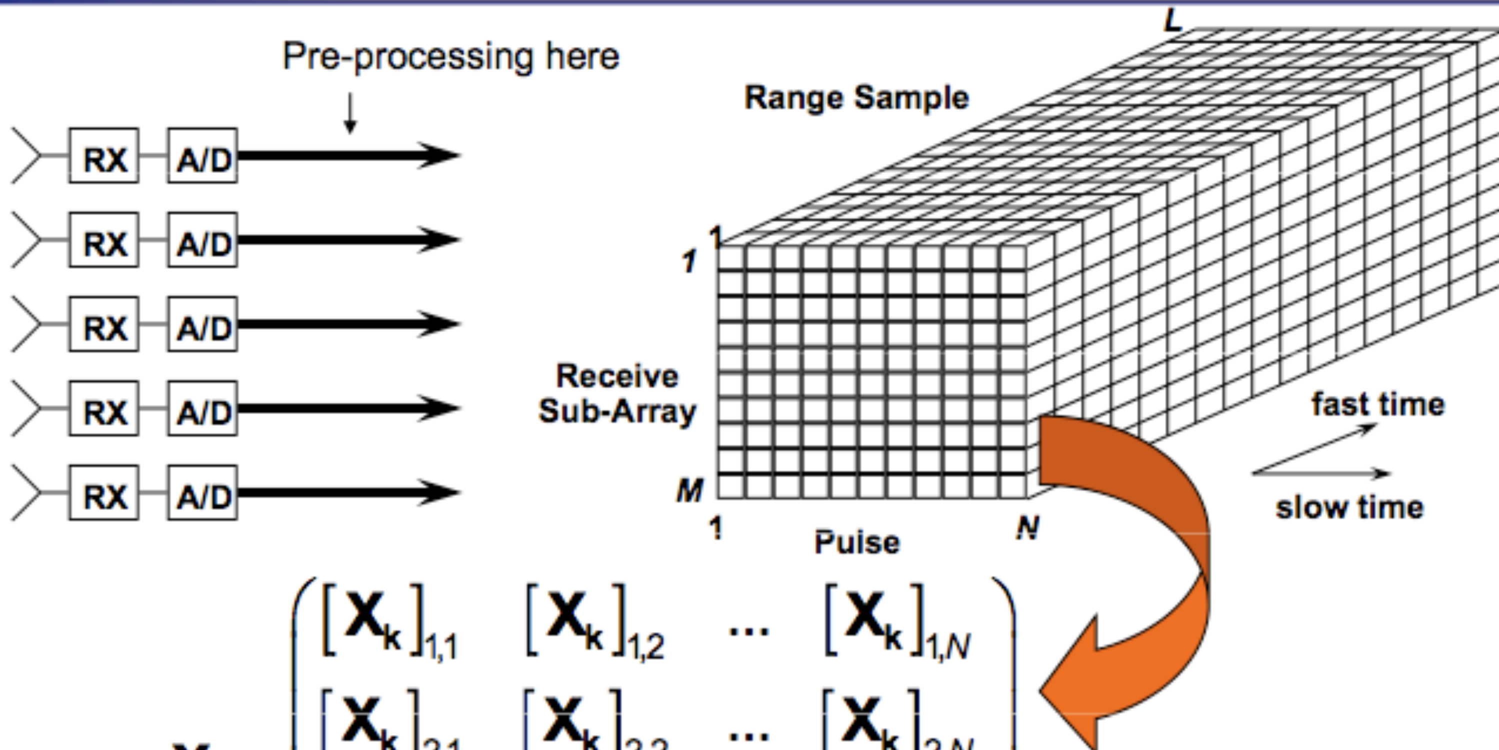
$$\mathbf{s}_{\mathbf{t}}(\tilde{f}_d) = [1 \quad e^{j\tilde{f}_d} \quad e^{j2\tilde{f}_d} \quad \ldots \quad e^{j(N-1)\tilde{f}_d}]^T \quad \left(= \mathbf{v}_{\mathbf{t}}(\tilde{f}_d)\right)$$

(Normalized Doppler)

$$\tilde{f}_d = 2\pi f_d T = 2\pi \left(\frac{2v}{\lambda}\right) T = 2\pi \left(\frac{2\Delta R}{\lambda}\right)$$

Courtesy Dr. Murali Rangaswamy AFRL/RYAP

The Radar Data Cube



$$\mathbf{X}_k = \begin{pmatrix} [\mathbf{X}_k]_{1,1} & [\mathbf{X}_k]_{1,2} & \cdots & [\mathbf{X}_k]_{1,N} \\ [\mathbf{X}_k]_{2,1} & [\mathbf{X}_k]_{2,2} & \cdots & [\mathbf{X}_k]_{2,N} \\ | & | & \cdots & | \\ [\mathbf{X}_k]_{M,1} & [\mathbf{X}_k]_{M,2} & \cdots & [\mathbf{X}_k]_{M,N} \end{pmatrix}$$

“Space-time snapshot”

Courtesy Dr. Murali Rangaswamy AFRL/RYAP

Maximum SINR Filter

Signal and I+N snapshots: $\mathbf{s}, \mathbf{x}_\mathbf{n} \in C^{NMx1}$

Target signal: $\mathbf{s} = \alpha \mathbf{s}_{\text{s-t}}(\gamma_s, \tilde{f}_d);\ \sigma_s^2 = E\left[\left|\alpha / \sqrt{2}\right|^2\right]$

Interference-plus-noise signal: $\mathbf{x}_\mathbf{n} \sim CN(\mathbf{0}, \mathbf{R}_\mathbf{n})$

$$SINR = \frac{P_s}{P_n} = \frac{E\left[y_s y_s^*\right]}{E\left[y_n y_n^*\right]} = \frac{E\left[\mathbf{w}^H \mathbf{s}\mathbf{s}^H \mathbf{w}\right]}{E\left[\mathbf{w}^H \mathbf{x}_\mathbf{n} \mathbf{x}_\mathbf{n}^H \mathbf{w}\right]} = \frac{\mathbf{w}^H \mathbf{R}_\mathbf{s} \mathbf{w}}{\mathbf{w}^H \mathbf{R}_\mathbf{n} \mathbf{w}}$$

$$= \sigma_s^2 \frac{\left|\tilde{\mathbf{w}}^H \tilde{\mathbf{s}}\right|^2}{\tilde{\mathbf{w}}^H \tilde{\mathbf{w}}} \leq \sigma_s^2 \frac{\left(\tilde{\mathbf{w}}^H \tilde{\mathbf{w}}\right)\left(\tilde{\mathbf{s}}^H \tilde{\mathbf{s}}\right)}{\tilde{\mathbf{w}}^H \tilde{\mathbf{w}}}$$

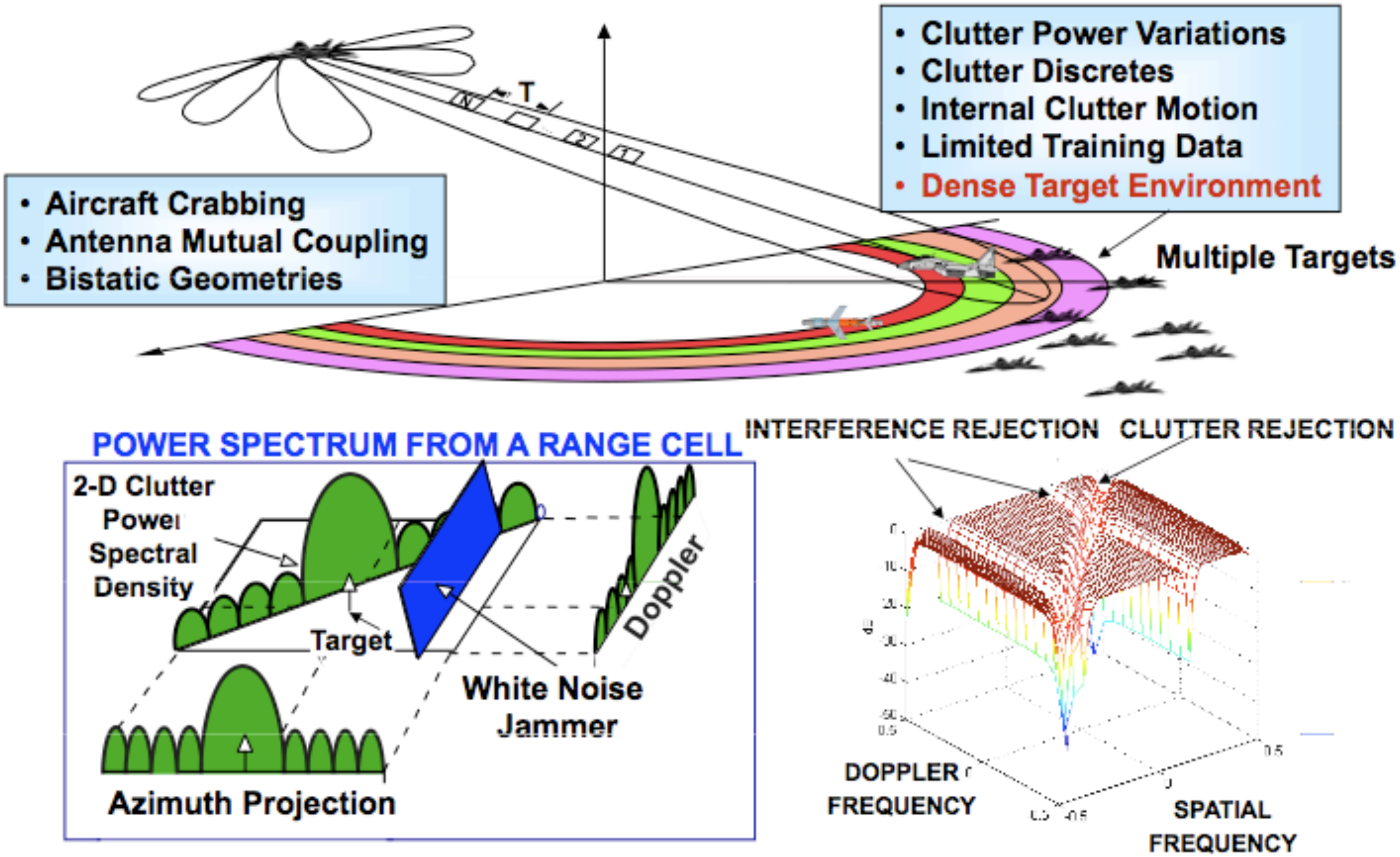
$$\tilde{\mathbf{w}} = \mathbf{R}_\mathbf{n}^{1/2} \mathbf{w};\quad \tilde{\mathbf{s}} = \mathbf{R}_\mathbf{n}^{-1/2} \mathbf{s}_{\text{s-t}}(\gamma_s, \tilde{f}_d)$$

Achieves the upper bound when $\tilde{\mathbf{w}} = \tilde{\mathbf{s}}$, or $\mathbf{w} = \mathbf{R}_\mathbf{n}^{-1} \mathbf{s}_{\text{s-t}}(\gamma_s, \tilde{f}_d)$...

$$SINR_{\max} = \sigma_s^2 \mathbf{s}_{\text{s-t}}^H \mathbf{R}_\mathbf{n}^{-1} \mathbf{s}_{\text{s-t}}$$

Courtesy Dr. Murali Rangaswamy AFRL/RYAP

Airborne Radar Scenario



J.H. Michels, STAP Seminar, Malvern, U.K., 1999

Courtesy Dr. Murali Rangaswamy AFRL/RYAP

Move-Stop-Move Problem

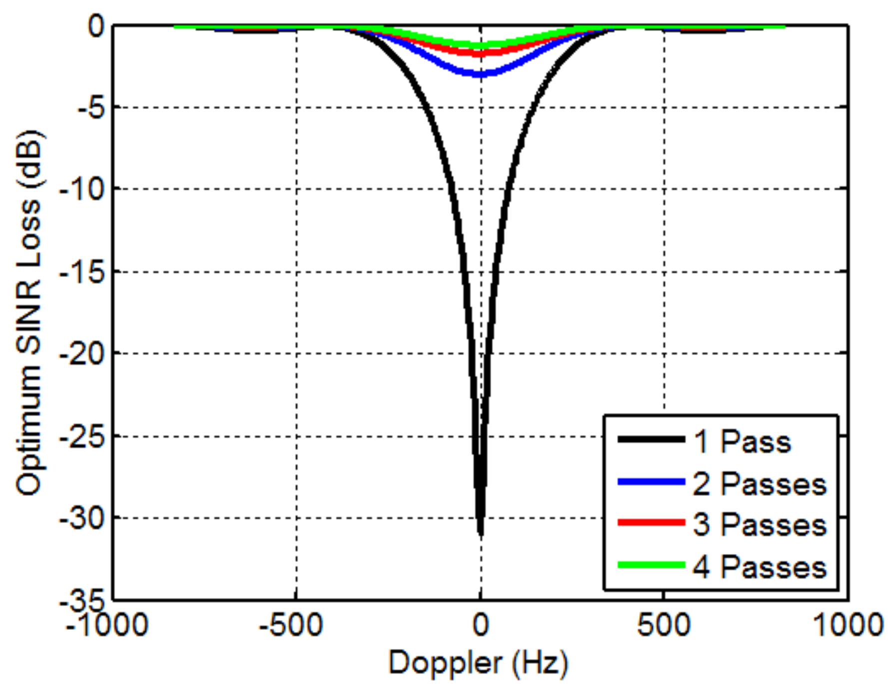
- STAP and DPCA suffer from Move-Stop-move problem
- If targets stop they disappear,
- If no range velocity (velocity orthogonal to the line of sight) – Target disappears

- Solution – Use previous passes

SINR Loss with Multiple Passes

- Gotcha 1 system
- 64 Pulses
- No ICM
- Platform flies the same path in each pass
- Losses from additional noise at clutter center:

$$L_{s,ccd} = \frac{N_{pass} - 1}{N_{pass}} \quad N > 1$$



- **Weights at clutter center**

$$\mathbf{w}_{pass} \approx \left[\frac{N_{pass} - 1}{N_{pass}} \quad \frac{-1}{N_{pass}} \quad \frac{-1}{N_{pass}} \quad \cdots \quad \frac{-1}{N_{pass}} \right]^T$$

- **Weights away from clutter**

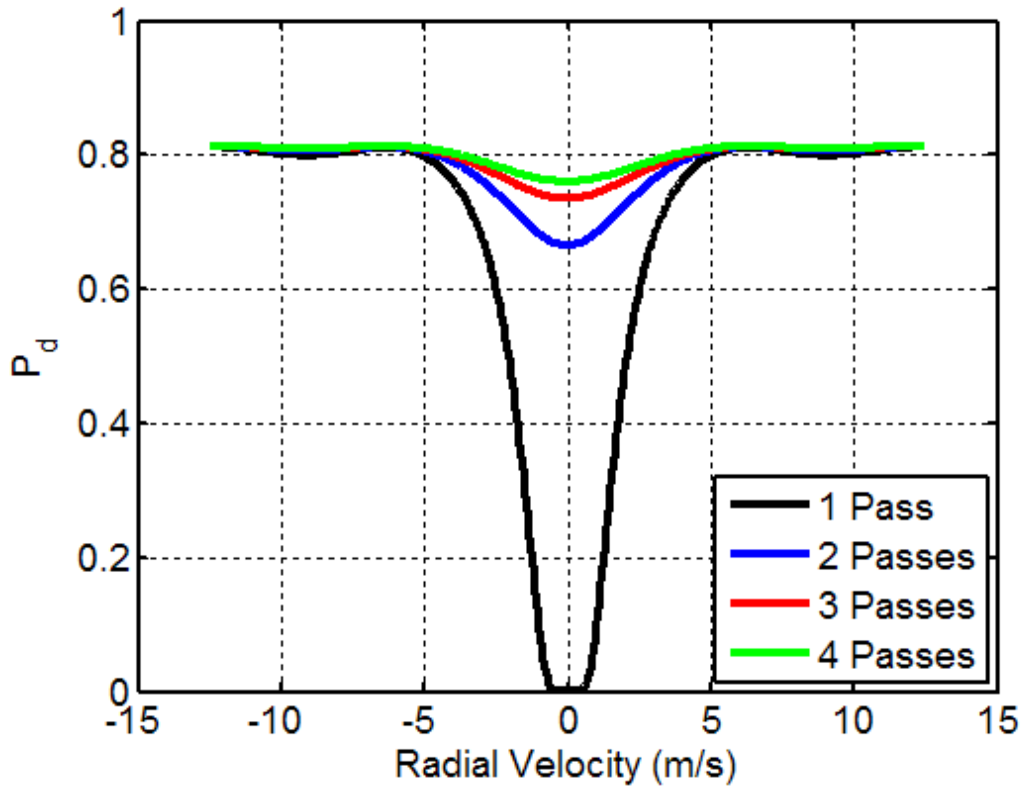
$$\mathbf{w}_{pass} \approx \begin{bmatrix} 1 & 0 & 0 & \ldots & 0 \end{bmatrix}^T$$

Courtesy Dr. William Melvin GTRI

Probability of Detection With Multiple Passes

- **0 dBsm target**
- **Target present on only one pass**

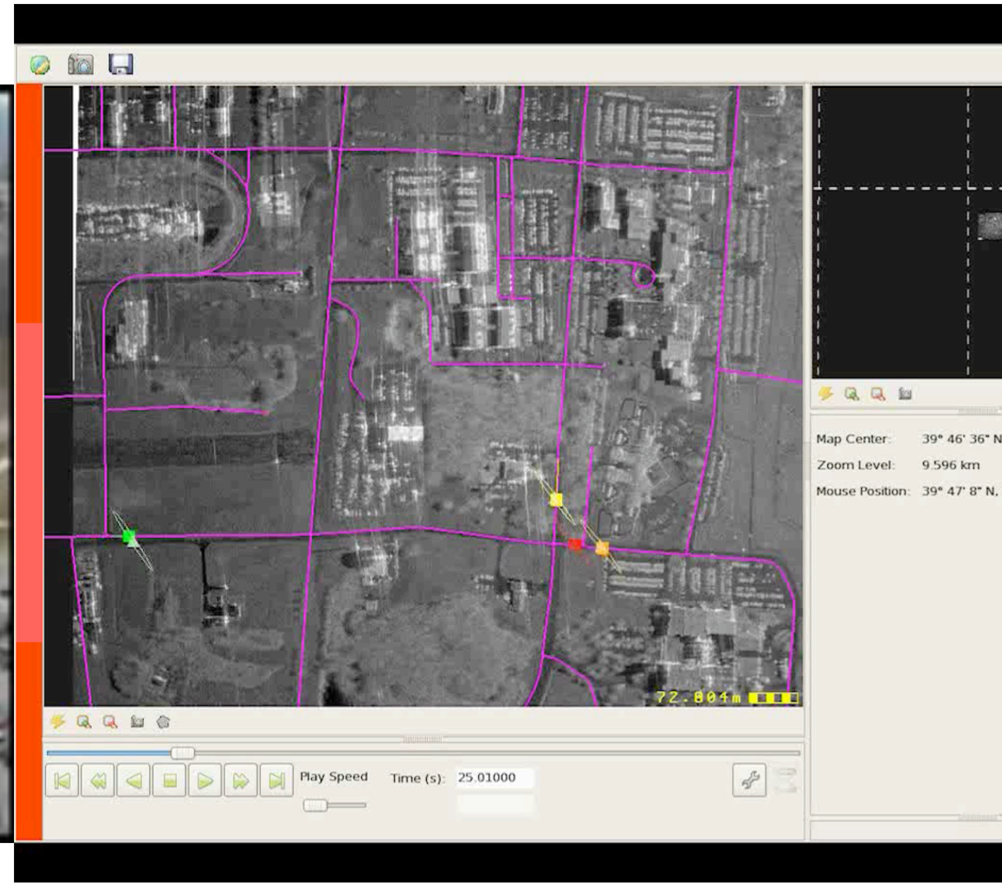
- **Single-pass MDV = 3 m/s**
  - **PD ≥ 75%**
- **CCD PD ≥ 65%**
  - **Over all radial velocities for two or more passes**



Courtesy Dr. William Melvin GTRI

Tracking using a single antenna and 2 passes

Tracking in a Dense Urban Environment



Future Problems

- ***…….Pedestrians?***

Previous Work

1) Mehrdad Soumekh, “Moving target detection and imaging using an X band along-track monopulse SAR”, IEEE Transactions on Aerospace and Electronic Systems, Volume **38**, Issue 1, pp. 315-333, January 2002.

2) Jen King Jao, “Theory of Synthetic Aperture Radar Imaging of a Moving Target”, IEEE Transactions on Geoscience and Remote Sensing, Volume **39**, Number 9, pp. 1984-1992, September 2001.

3) Mats I. Pettersson, “Detection of moving targets in wide band SAR”, IEEE Transactions on Aerospace and Electronic Systems, Volume **40**, Issue 3, pp. 780-796, July 2004.

4) J.R. Fienup, “Detecting moving targets in SAR imagery by focusing”, IEEE Transactions on Aerospace and Electronic Systems, Volume **37**, Issue 3, pp. 794-809, July 2001.

5) Charles V. Jakowatz, Jr., Daniel E. Wahl, and Paul H. Eichel, “Refocus of constant velocity moving targets in synthetic aperture radar imagery”, SPIE Conference on Algorithms for Synthetic Aperture Radar Imagery V, SPIE Volume 3370, pp. 85-95, April 1996.