Ground Moving Target Indication (GMTI) with Synthetic Aperture Radar (SAR)
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Signal Processing Contributions

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One of eight image channels
Mover revealed in Clutter Suppressed Imagery

Mover phase as a function of slow time
Ground Moving Targets in SAR Imagery
Outline

• Nominal Moving Target Phase Equations

• Measured Moving Target Phase Examples

• Space Time Adaptive Processing (STAP) Review

• STAP applied to 8 channel SAR image data (element space, phase centers)
  – Single 3 sec coherent integration full azimuth resolution
  – Ten 0.3 second coherent integration times, results in SAR movie of moving vehicle
Airborne X-band Image Example
Azimuth Impulse Response (IPR) and Residual Phase Error Shown

- Image of corner reflector (single look)

- Corner Reflector
  - Azimuth-Doppler IPR
- Corner Reflector
  - Azimuth Phase
    (unwrap phase of FFT of IPR)
- Corner Reflector
  - Azimuth-Amplitude
    (amp of FFT of IPR)

Aircraft data
Ground Moving Target Indication (GMTI)

Moving Point Target Phase Characteristics

Go To Paper on Moving Target Phase
Quadratic Phase & Nonzero Doppler Rate

- Quadratic phase defocuses Doppler/azimuth IPR
- Formula for quadratic expressed for polar format processed data
- Target energy can be focused by phase compensation
- Stationary Targets do not exhibit 1000’s deg of phase

Note: CDP = 2.71 sec.

Quadratic Phase (polar format processing)

\[
Q = 90 \left( \frac{v \cdot u_y T}{\Delta y} + \frac{a \cdot u_x T^2}{\lambda} \right) \text{ (degrees)}
\]

- \(v\) = target velocity vector
- \(a\) = target acceleration vector
- \(u_x\) = slant plane range unit vector
- \(u_y\) = slant plane azimuth unit vector
- \(T\) = coherent dwell period
- \(\Delta y\) = nominal azimuth resolution
- \(\lambda\) = wavelength at mid band

Phase Target 1

Q = -4563 deg  GPS prediction
Q ~ -4000 deg  observed above

Phase Target 5

No GPS data for prediction
Q ~ 8000 deg  observed above

Phase Target 6

Q = 1627 deg  GPS prediction*
Q ~ 1500 deg  observed above

*GPS derived acceleration set = 0

aircraft data
Moving Target Phase: Aircraft Data Observations

Quadratic Phase: target range acceleration & cross range velocity, ...
Cubic Phase: target cross range acceleration & non-constant range acceleration

Stationary targets do not have these phase signatures

aircraft data
GMTI

• Show examples of common processing of multiple channel radar data into image and moving target products
  – Long coherent dwell (e.g., 2 to 3 seconds)
  – Subdivision of dwell into short coherent processing intervals (e.g., 10 CPIs each 0.2 seconds duration)

• Data observations and discussions
  – MTI with and without clutter suppression
  – Target range acceleration and cross range velocity provides phase characteristics not shared stationary targets
    • Energy migration through freq bins, intrinsically more degrees of freedom to match signal, signal processing implications
    • Also hedge against false alarms
Space-Time Adaptive Processing (STAP) Formulated for SAR Imagery

Assume a system with N displaced along track antenna elements or N azimuth beams (N channels)

\( \mathbf{\sigma} = \mathbf{\sigma}(x, y) = (\sigma_1(x, y), \sigma_2(x, y), \ldots, \sigma_N(x, y))^T \) (stack or vector of images, one from each channel)

\( \mathbf{R} = \mathbf{R}_c + \mathbf{R}_n \) (N by N clutter plus receiver noise covariance matrix)

\( \mathbf{w} = [w_1, w_2, \ldots, w_N] \) (channel summation weights)

\( \mathbf{s} = [s_1, s_2, \ldots, s_N] \) (target steering vector, complex exponential of linear phase or azimuth beam weights)

\( s_n = \exp\left( i \frac{2 \pi}{\lambda} (n - 1) \left( \frac{d}{r} \right) (\mathbf{u}_d \cdot \mathbf{x}) \right) \) (nominal antenna element space representation, angle of arrival related)

\( d = \text{dist between antenna elements,} \)

\( r = \text{range,} \)

\( \mathbf{u}_d = \text{unit vector defines direction along which the elements are located in 3D} \)

\[ \text{SINR}(\mathbf{w}) = \frac{\left| \mathbf{w}^H \mathbf{s} \right|^2}{\mathbf{w}^H \mathbf{Rw}} \] (signal - to - interference ratio, include consideration to clutter power)

optimal \( \mathbf{w} = \mathbf{R}^{-1} \mathbf{s} \) (STAP weight vector for maximum SINR, estimate \( \mathbf{R} \) from data, exclude movers)

\( \max \text{SINR}(\mathbf{w}) = \text{SINR}(\mathbf{R}^{-1} \mathbf{s}) = \left| (\mathbf{R}^{-1} \mathbf{s})^H \mathbf{s} \right| \) (max SINR, application of Cauchy - Schwarz inequality)

STAP filter output = \( \mathbf{w}^H \mathbf{\sigma} = (\mathbf{R}^{-1} \mathbf{s})^H \mathbf{\sigma} = \mathbf{s}^H \mathbf{R}^{-1} \mathbf{\sigma} \) (for a given target steering vector hypothesis)

application of inverse covariance matrix to image stack apparently suppresses clutter

clutter suppressed image stack = \( (\mathbf{R}^{-1} \mathbf{\sigma})(x, y) \), (for analysis, now apply hypothesis space of steering vectors)

Note: Missing wavelength factor in steering vector definition corrected above on February 10, 2012.
Element Space: 1 and 2 of 8 Clutter Images

1 and 2 of 8 channels/images shown

single look complex

image 1

image 2

aircraft data
R\(^{-1}\) Applied - Clutter Suppression Apparent

1 and 2 of 8 channels/images shown
steering vector not applied

image 1

image 2

aircraft data
R^{-1} Applied - Clutter Suppression Apparent
1 and 2 of 8 channels/images shown
steering vector not (yet) applied

image 1

image 2

moving target
energy observed

aircraft data
Digital Beam Steer to Upper Part of Image

Application of beam steering vector shown

Complex-valued beam steering vector components are complex sinusoids a phase which is linear with antenna element index

Analogous to discrete Fourier transform

\[ s = [s_1, s_2, ..., s_N] \]
\[ s_n = \exp\left(i \left(\frac{2\pi}{\lambda}\right)(n - 1)(d/r)(u_d \cdot x)\right) \]

Note: Missing wavelength factor in steering vector definition corrected above on February 10, 2012.
Digital Beam Steer to Center Part of Image

Application of beam steering vector shown

Complex-valued beam steering vector components are complex sinusoids a phase which is linear with antenna element index

Analogous to discrete Fourier transform

\[ s = [s_1, s_2, \ldots, s_N] \]
\[ s_n = \exp\left( i \left( \frac{2 \pi}{\lambda} (n-1) \left( \frac{d}{r} \right) \cdot (u_d \cdot x) \right) \right) \]

Note: Missing wavelength factor in steering vector definition corrected above on February 10, 2012.

aircraft data
Application of beam steering vector shown

Complex-valued beam steering vector components are complex sinusoids a phase which is linear with antenna element index

Analogous to discrete Fourier transform

\[ s = [s_1, s_2, \ldots, s_N] \]
\[ s_n = \exp\left( i \left( \frac{2\pi}{\lambda} \right) (n - 1) \left( \frac{d}{r} \right) (\mathbf{u}_d \cdot \mathbf{x}) \right) \]

Note: Missing wavelength factor in steering vector definition corrected above on February 10, 2012.
Left image shows area of interest outlined in green.

Right image is a zoom into region of interest.

Image formed with 2.7 seconds of radar dwell time.
Left image shows area of interest outlined in green.

Right image results from clutter suppression processing (STAP) reveals two movers. Note energy is spread over many Doppler cells.

Multiple channels necessary for clutter suppression.
Left image shows area of interest outlined in green.

Yellow arrows point to Doppler shifted location of targets.

Red arrows point to actual target location as determined by radar processing algorithm (STAP).
Phase Characteristics

Clutter Suppressed Output
Nominal Element Space
Adaptive Processing (STAP)
- 8 channels
  - No quadratic phase compensation
Quadratic Phase Frequently Observed in Aircraft Data

clutter suppressed - channels combined
aircraft data
Space-Time Adaptive Processing (STAP)
Target Signal Loss

\[ \text{SINR}(w) = \frac{|w^H s|^2}{w^H Rw} \] (signal - to - interference ratio, include consideration to clutter power)

optimal \( w = R^{-1} s \) (STAP weight vector for maximum SINR, estimate \( R \) from data, exclude movers)

\[ \max \text{SINR}(w) = \text{SINR}(R^{-1} s) = \left| (R^{-1} s)^H s \right| \] (max SINR, application of Cauchy - Schwarz inequality)

STAP filter output = \( w^H \sigma = (R^{-1} s)^H \sigma = s^H R^{-1} \sigma \) (for a given target steering vector hypothesis)

\[ \max \text{SNR} = \left| (R_n^{-1} s)^H s \right| \] (gives no consideration to clutter power)

\[ \text{SINR}(R^{-1} s) = L_{\text{sinr}} \text{ SNR} \] (SINR loss defined relative to SNR)

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**STAP SINR Loss**
8 antenna element aircraft data
CNR per channel approx 18 dB

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![Graph](image-url)
Space-Time Adaptive Processing (STAP)  
Clutter Loss (Suppression)

\[
\text{SINR}(w) = \frac{|w^H s|^2}{w^H R w}
\]

(signal - to - interference ratio, include consideration to clutter power)

optimal \( w = R^{-1} s \)

(STAP weight vector for maximum SINR, estimate \( R \) from data, exclude movers)

\[
\text{SINR}(R^{-1} s) = L_{\text{sir}} \text{ SNR}
\]

(SINR loss defined relative to SNR)

\[
\text{CNR}(w) = \frac{w^H R_c w}{w^H R_n w} = \frac{w^H (R - R_n) w}{w^H R_n w} = \frac{w^H R w}{w^H R_n w} - 1
\]

\( R e_k = \lambda_k e_k \quad k = 1, 2, ..., N \)

(eigenvectors and eigenvalues)

\[
\text{CNR}(w) \leq \text{CNR}(e_1)
\]

(max CNR using eigenvector with max eigenvalue as channel summation weights)

\[
\text{CNR}(R^{-1} s) = L_e \text{ CNR}(e_1)
\]

(defines the clutter suppression, post channel summation, relative to max CNR)

STAP Clutter Suppression

8 element aircraft data  
CNR per channel approx 18 dB
Analysis: Relation Between Space Time Adaptive Processing (STAP) and Eigen Image Decomposition

\[ \mathbf{R} = \text{N by N covariance matrix, } \text{N = number of channels} \]

\[ \mathbf{R} \mathbf{e}_k = \lambda_k \mathbf{e}_k \quad k = 1, 2, \ldots, \text{N} \]

\[ \mathbf{R}^{-1} \mathbf{e}_k = \lambda_k^{-1} \mathbf{e}_k \]

\[ \mathbf{\sigma} = \mathbf{\sigma}(x, y) = [\sigma_1(x, y), \sigma_2(x, y), \ldots, \sigma_N(x, y)]^T \quad \text{(data/image vector)} \]

\[ \mathbf{\sigma}(x, y) = \sum_k \left[ \mathbf{e}_k^H \mathbf{\sigma}(x, y) \right] \mathbf{e}_k \quad \text{(eigenvector expansion, orthonormal basis)} \]

\[ \mathbf{e}_k^H \mathbf{\sigma}(x, y) = \text{eigen - image computed as kth eigenvector projected onto image vector, } k = 1, 2, \ldots, \text{N.} \]

\[ \mathbf{w}^H \mathbf{\sigma} = (\mathbf{R}^{-1} \mathbf{s})^H \mathbf{\sigma} = \mathbf{s}^H \mathbf{R}^{-1} \mathbf{\sigma} = \sum_k \left( \frac{\mathbf{e}_k^H \mathbf{\sigma}}{\lambda_k} \right) (\mathbf{s}^H \mathbf{e}_k) \quad \text{(adaptive processor & eigenvector expansion)} \]

Inverse covariance applied to image stack results in eigen image vector sum as above

Moving target energy in eigen images, helps explain why targets are observed in suppressed imagery, following application of inverse covariance matrix to image stack

Optimal adaptive processor (STAP) is expressed as weighted sum of eigen images, as given above (applies to element & beam space)
Relation Between Space Time Adaptive Processing (STAP) and Eigen Image Decomposition

Analysis: Optimal Adaptive Processor is Weighted Sum of Eigen-Images
Observation: Variability of Eigenvalues with CNR

Beam Edge

Bright Uniform Clutter

Eigenvalues of R

5.9598164
0.46193165
0.31020248
0.28341122
0.26182939
0.25569636
0.24116470
0.22594751

Eigenvalues of R

7.6169138
0.12200656
0.081824081
0.051896852
0.042732919
0.030645195
0.027344463
0.026636630

ratio
largest/smallest
eigenvalues = 26

ratio
largest/smallest
eigenvalues = 286

Greater range of variability of eigenvalues observed for bright clutter aircraft data
Summary Space-Time Adaptive Processing (STAP) Formulated for SAR Imagery

Assume a system with $N$ displaced along track antenna elements or $N$ azimuth beams (N channels)

Signal-to-Interference Ratio (SINR)

$$\text{SINR}(w) = \frac{|w^H s|^2}{w^H R_w w}$$

(signal-to-interference ratio, include consideration to clutter power)

Optimal weight vector for maximum SINR

$$\text{optimal } w = R^{-1} s$$

(STAP weight vector for maximum SINR, estimate $R$ from data, exclude movers)

Maximum SINR

$$\max \text{SINR}(w) = \text{SINR}(R^{-1} s) = \frac{|(R^{-1} s)^H s|}{|s^H R^{-1} s|}$$

(max SINR, application of Cauchy - Schwarz inequality)

STAP filter output

$$w^H \sigma = (R^{-1} s)^H \sigma = s^H R^{-1} \sigma$$

(for a given target steering vector hypothesis)

Application of inverse covariance matrix to image stack apparently suppresses clutter

Clutter suppressed image stack

$$\text{max SNR} = \frac{|(R^{-1}_n s)^H s|}{|s^H R^{-1} s|}$$

(gives no consideration to clutter power)

Signal-to-Noise Ratio (SNR)

$$\text{SNR} = \frac{|s^H R^{-1} s|}{|s^H R^{-1} s|}$$

(SINR loss defined relative to SNR)

CNR

$$\text{CNR}(w) = \frac{w^H R_c w}{w^H R_w w} = \frac{w^H (R - R_n) w}{w^H R_n w} = \frac{w^H R w}{w^H R_n w} - 1$$

(eigenvectors and eigenvalues)

$$R e_k = \lambda_k e_k \quad k = 1, 2, ..., N$$

(max CNR using eigenvector with max eigenvalue as channel summation weights)

CNR for eigenvector

$$\text{CNR}(R^{-1} s) = L_{c} \text{CNR}(e_1)$$

(defines the clutter suppression, post channel summation, relative to max CNR)

$$R^{-1} e_k = \lambda_k^{-1} e_k$$

(eigenvector expansion, orthonormal basis)

$$\sigma(x, y) = \sum_k [e_k^H \sigma(x, y)] e_k$$

(eigen - image computed as kth eigenvector projected onto image vector, $k = 1, 2, ..., N$)

$$w^H \sigma = (R^{-1} s)^H \sigma = s^H R^{-1} \sigma = \sum_k \frac{e_k^H \sigma}{\lambda_k} (s^H e_k)$$

(adaptive processor & eigenvector expansion)
Last Example: Show Moving Target Quadratic Phase Causes Target Energy to Migrate Through Doppler/Azimuth Cells

Single Coherent Data Collection Period (2.7 sec.) Divided into 13 Time Intervals

For each time interval 8 channels are processed via STAP and Displayed as a Time Sequenced Movie of 13 Images (azimuth resolution more coarse by factor of 13)

Note that SAR image formation applied to the full data collection period, then subdividing the data, mitigates moving target migration through range bins
Moving Target Observed in Clutter Suppressed Imagery

Stationary Target

Moving Target

aircraft data
Moving Target Observed in Clutter Suppressed Imagery

Stationary Target

Moving Target

Doppler

range

aircraft data
Moving Target Observed in Clutter Suppressed Imagery

Stationary Target

Moving Target

aircraft data
Moving Target Observed in Clutter Suppressed Imagery

Stationary Target

Moving Target

aircraft data
Moving Target Observed in Clutter Suppressed Imagery

Stationary Target

Moving Target

Doppler

range

aircraft data
Moving Target Observed in Clutter Suppressed Imagery

Doppler

Stationary Target

Moving Target

range

aircraft data
Moving Target Observed in Clutter Suppressed Imagery
Moving Target Observed in Clutter Suppressed Imagery

Stationary Target

Moving Target

Doppler

range
Moving Target Observed in Clutter Suppressed Imagery

Stationary Target

Moving Target

Doppler

range
Moving Target Observed in Clutter Suppressed Imagery
Moving Target Observed in Clutter Suppressed Imagery
Moving Target Observed in Clutter Suppressed Imagery

Stationary Target

Moving Target

aircraft data
Moving Target Observed in Clutter Suppressed Imagery
Ground Moving Targets in SAR Imagery

Summary

• Nominal Moving Target Phase Equations
  – Uncompensated motion leads to quadratic, cubic, .. phase

• Measured Moving Target Phase Examples
  – Cross range motion causes phase characteristics not displayed by stationary targets

• Space Time Adaptive Processing (STAP) Review
  – SINR, SINR loss, clutter suppression

• STAP applied to 8 channel SAR image data (element space, phase centers)
  – Single 2.7 sec coherent processing interval (CPI)
  – Thirteen, 2.7/13 = 0.2077 sec. CPIs, results in SAR movie of moving vehicle, target migration through Doppler cells observed
Appreciation

• UCLA’s Institute for Pure and Applied Mathematics
• Margaret Cheney
• The Aerospace Corporation
References


References


Backup Charts
Azimuth Beam pattern estimated using

\[ b(y) = \sum x |\sigma(x, y)|^2, \quad x = \text{range}, \ y = \text{azimuth} \]

\[ B(y) = 10 \log_{10} \left( b(y)/b_{\text{max}} \right) \]

\[ b_{\text{max}} = \max(b), \quad \text{max computed excluding large discretes} \]