SPARSE REPRESENTATION AND AUTOFOCUS FOR ISAR AND SPOTLIGHT SAR IMAGING

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FHR: Radar research since 1957

OUTLINE

- Introduction: Challenges for autofocus
  - Introduction to three imaging systems of FHR
  - Examples of compressed sensing applied to radar
- Signal model
  - A review of polar reformatting
- Sparse reconstruction for perfectly known motion / rotation
- Autofocus using sparse reconstruction
- A three step approach to autofocus
  - Rough estimation of the range history
  - Alignment of partial images, improvement of motion estimate
  - Retrieval of phase and motion parameters
- Summary and outlook

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INTRODUCTION: CHALLENGES FOR AUTOFOCUS

Objective:

- ISAR Imaging of moving objects from stationary or moving platforms
- Spotlight SAR with ultra-high resolution

Desired:

- Super resolved images of sparse objects/scenes via compressed sensing
- Here, the emphasis is not on data reduction, but on high quality images

Problem:

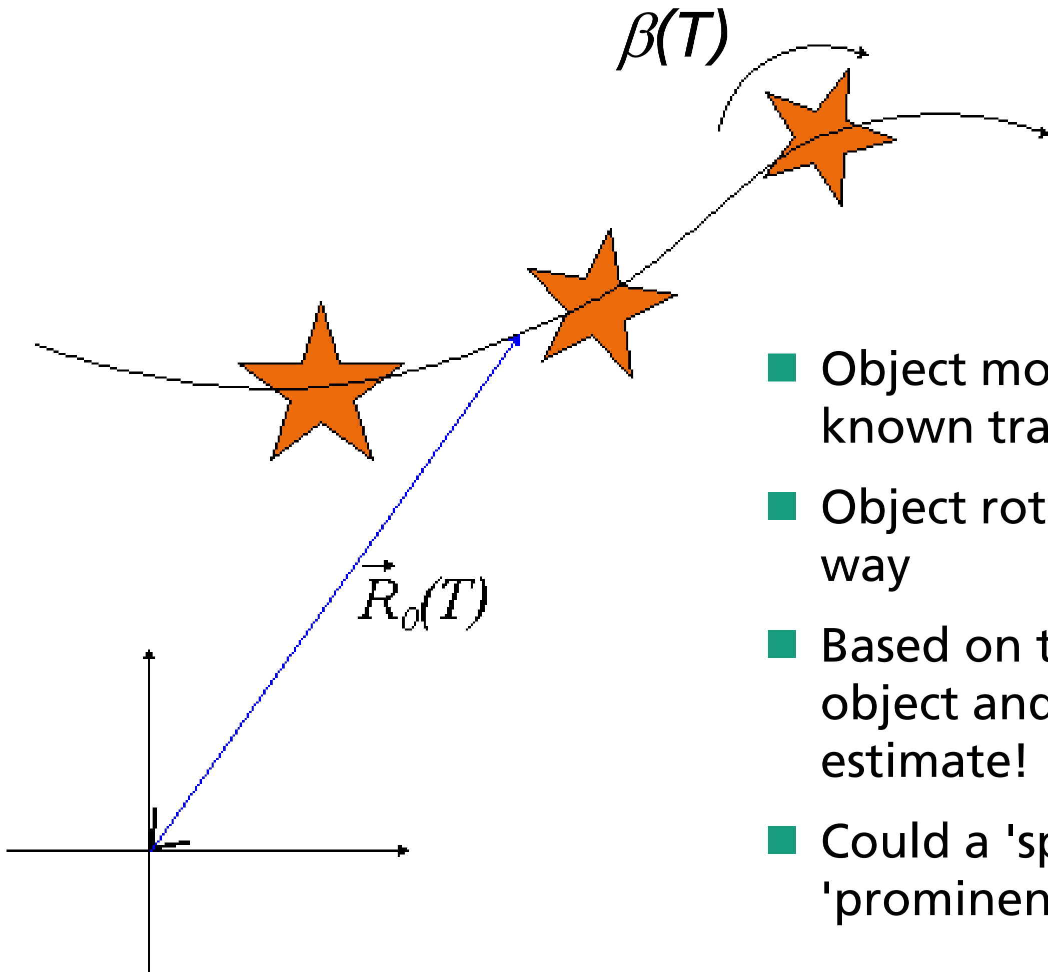
- Translational and rotational motion of object not known
- Trajectory of radar platform not known

Roadmap to solution:

- Use CS and subspace techniques for the retrieval of phase histories and correct motion/rotation estimation

INTRODUCTION: CHALLENGES FOR AUTOFOCUS

Problem statement in a two-dimensional geometry



- Object moves along a not perfectly known trajectory
- Object rotates in a not perfectly known way
- Based on the radar echoes, image this object and improve the motion estimate!
- Could a 'sparse reflectivity' (with some 'prominent' point like scatterers) help?

INTRODUCTION: CHALLENGES FOR AUTOFOCUS
Relations between translational and rotational history

- In real applications, the orientation history may be coupled to the translational history (for instance, the length axis of ground moving vehicles will normally be orientated tangentially to the path).
- For airplanes the situation is a little more complicated since the velocity of the wind has to be taken into account. For this type of target the geometry should be extended to the third dimension, as well as for ships.
- Though the motion of space objects is very smooth and the signals are nearly free from clutter, difficult situations occur e.g. for space objects shortly before immersing into the atmosphere.

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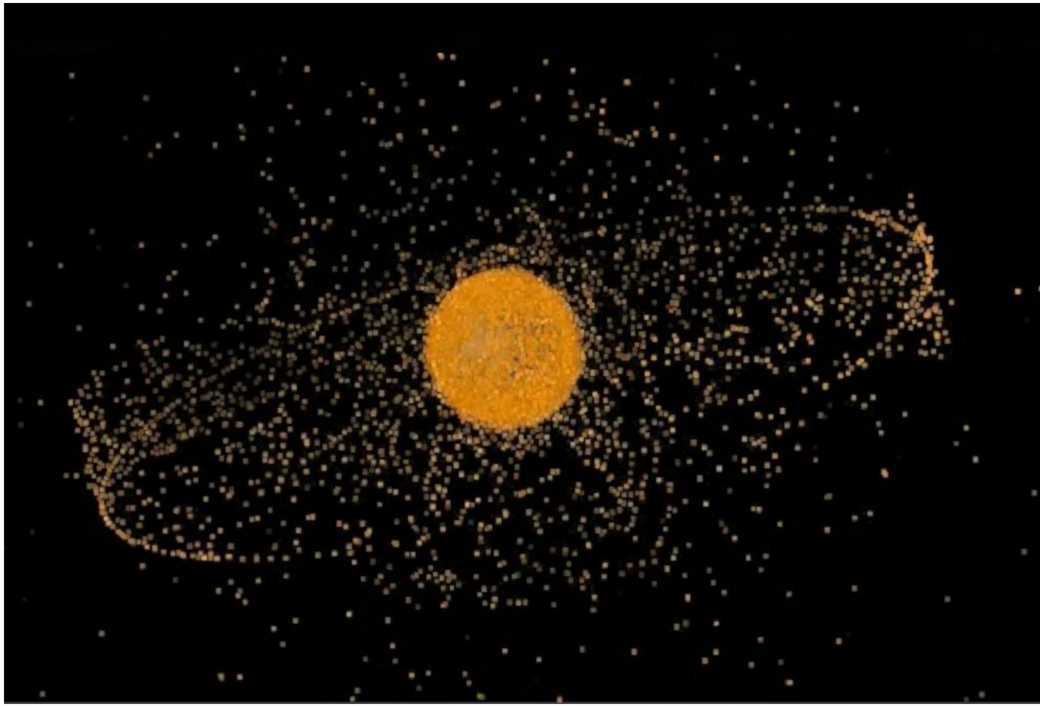
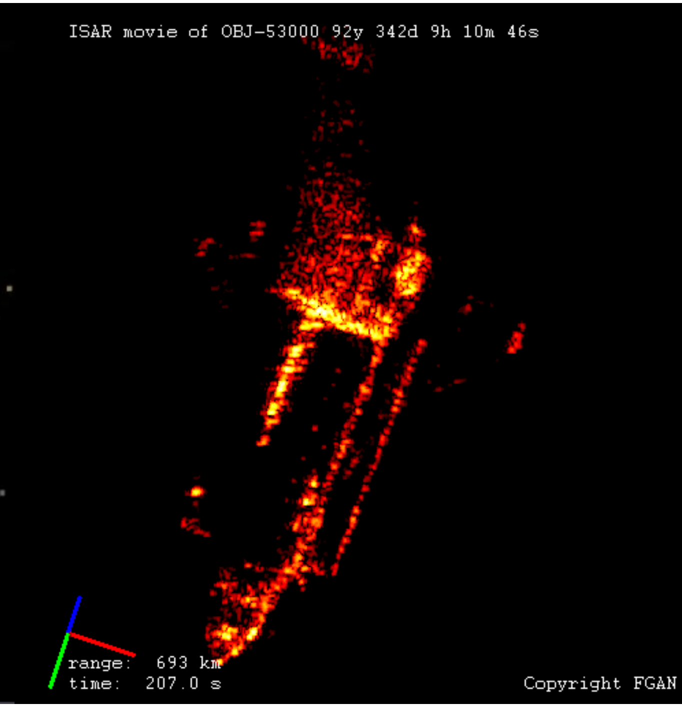
1. FRAUNHOFER RADAR FOR SPACE OBSERVATION

TIRA: Tracking and Imaging Radar





Space Shuttle at a distance > 650 km

TIRA

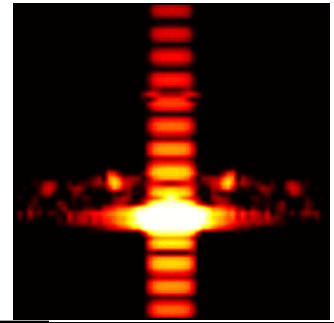
- Analysis of space debris
- Imaging of space objects

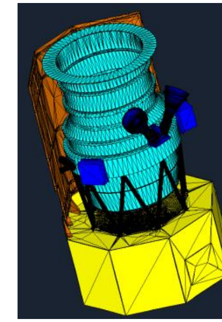
TIRA system

Fraunhofer FHR

1. FRAUNHOFER RADAR FOR SPACE OBSERVATION

TIRA: Tracking and Imaging Radar





L band tracking radar	
Frequency	1.33 GHz
Pulse power	1.5 MW
NERCS range 1 pulse	
2 cm	1,000 km
40 cm	10,000 km

Antenna	
Dish antenna	34 m
Cassegrain system	
Moving mass	240 t

Ku band imaging radar	
Frequency	16.7 GHz
Pulse power	10 kW
Bandwidth	1.6 GHz
Resolution	9.4 cm

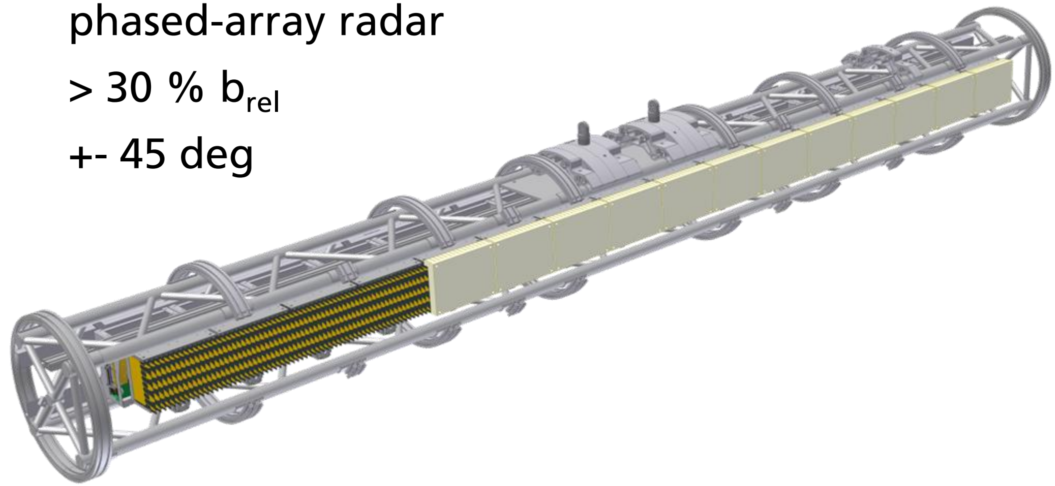
2. FRAUNHOFER MULTIFUNCTION SAR/MTI RADAR

PAMIR (*Phased Array Multifunction Imaging Radar*)

Demonstration of surveillance with multifunction phased-array radar

> 30 % b_{rel}

+- 45 deg

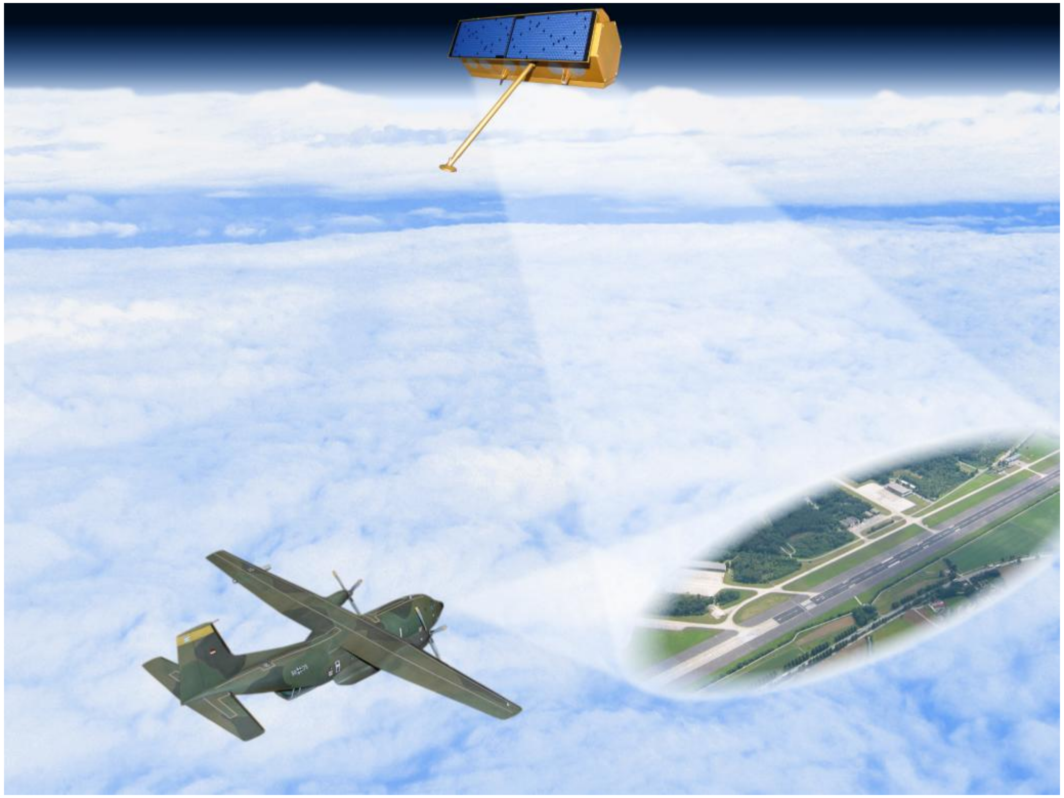


- Stripmap, spotlight and sliding spotlight SAR
- High resolution interferometric SAR
- Ground moving target indication (GMTI)
- GMT imaging (ISAR)
- Bistatic SAR

PAMIR: HIGH RESOLUTION SAR INTERFEROMETRY IN URBAN AREAS

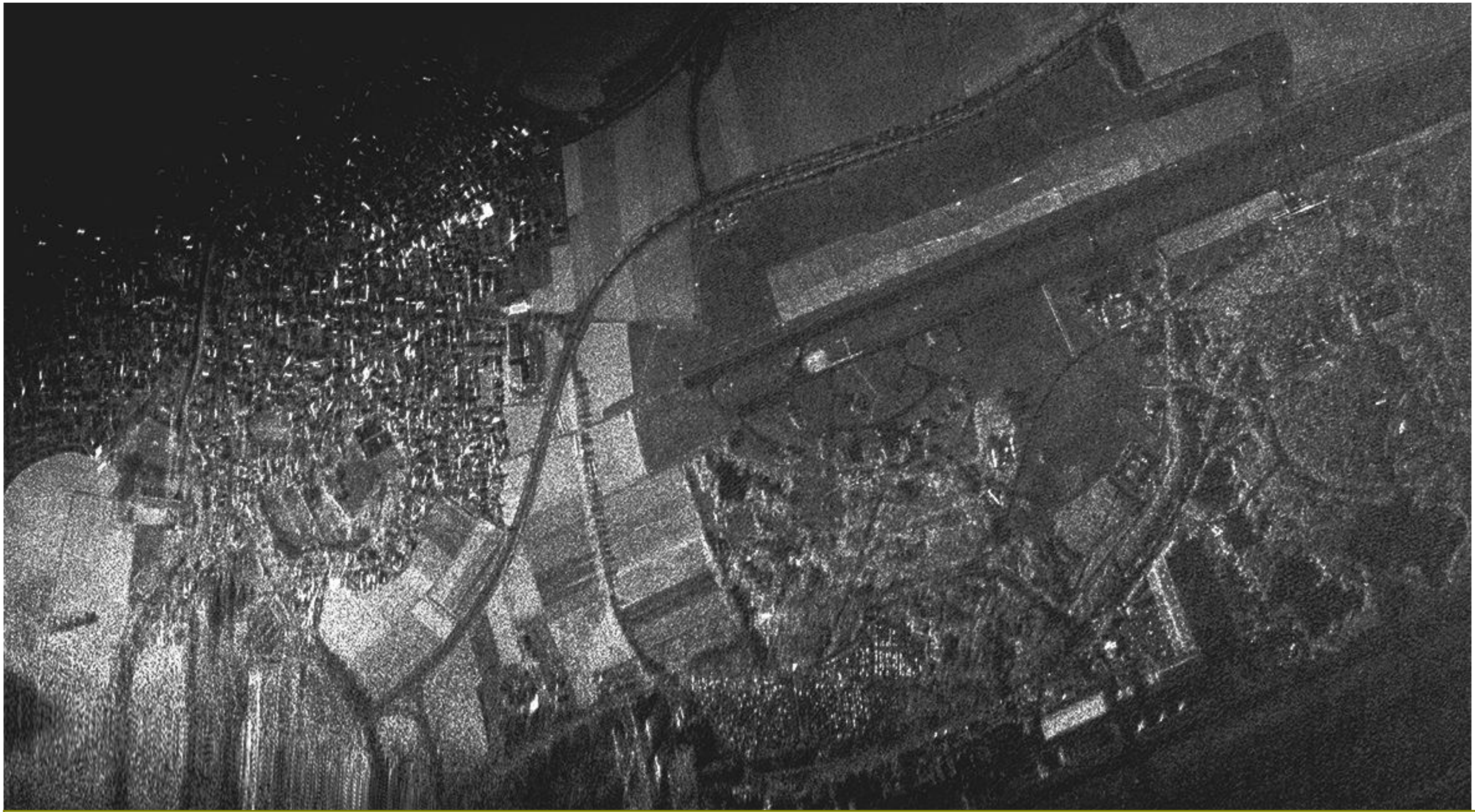
DEM of Karlsruhe

BISTATIC FORWARD LOOKING SAR



- Bistatic SAR opens (in contrary to the monostatic case) the possibility to image in a forward looking geometry
- Interesting application for looking through dense clouds – also at night - and recognize the runway
- Problem: Flight geometry has to be suitable

BISTATIC FORWARD LOOKING SAR
EXPERIMENTAL VERIFICATION (TerraSAR-X / PAMIR)



I. Walterscheid et al: “Bistatic Spaceborne-Airborne Forward-looking SAR“, EUSAR 2010, Aachen

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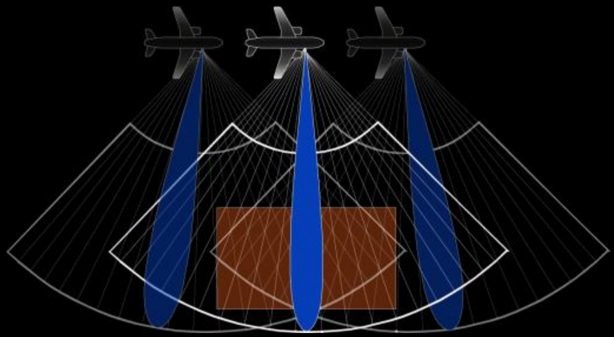
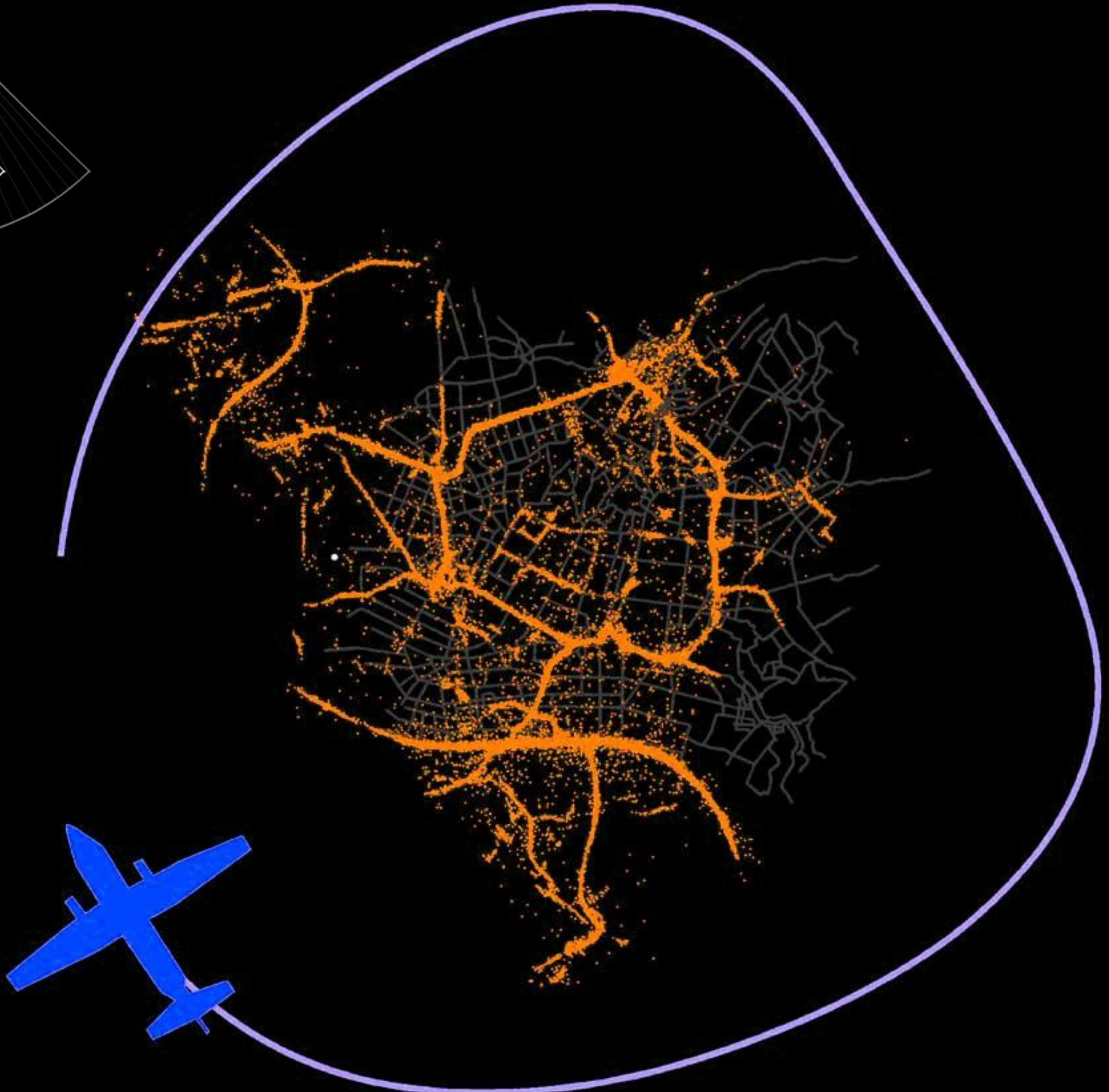
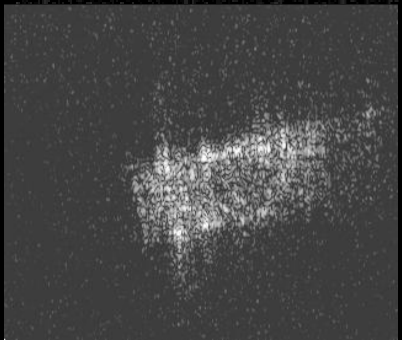




PAMIR: Ground moving target indication & tracking via Scan-MTI and Space-Time Adaptive Processing (STAP)

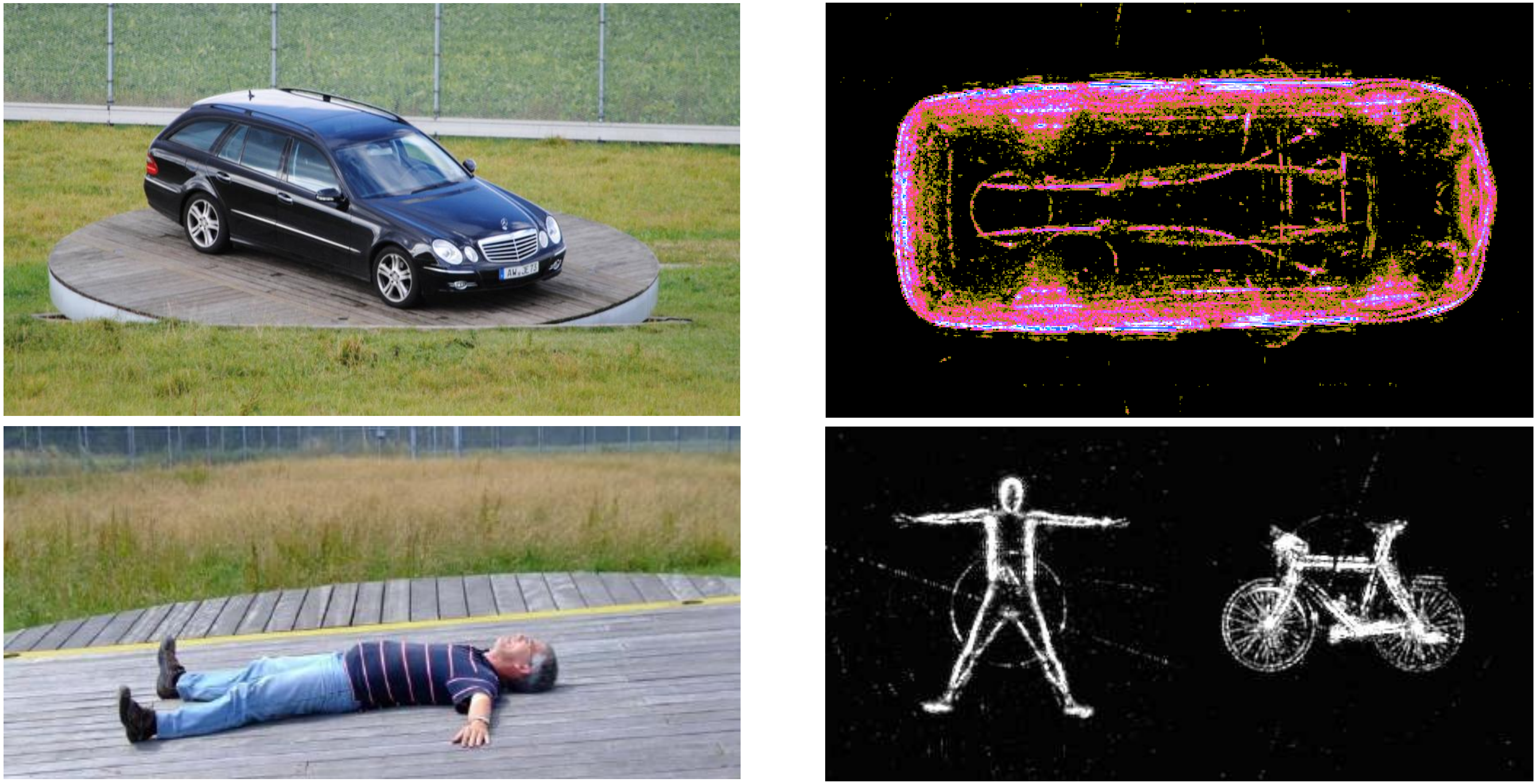
Gallery



ISAR-Image

3. FRAUNHOFER MM-WAVE RADAR COBRA

Ultra high resolution imaging at 220 GHz (Turntable)



ISAR-Imaging at 220 GHz, 8 GHz Bandwidth (1,7 cm resolution), range 170 m

NEAR RANGE MINIATURIZED SAR (1,5 kg) AT 94 GHz

SUMATRA-94 (FHR)

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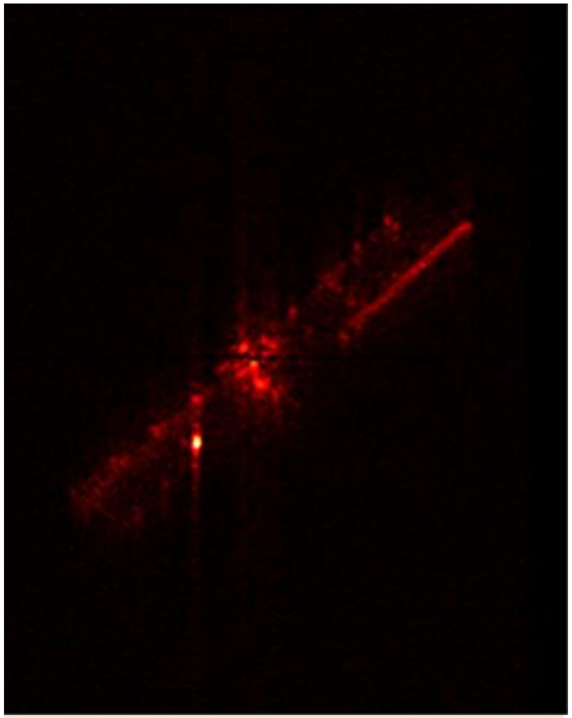
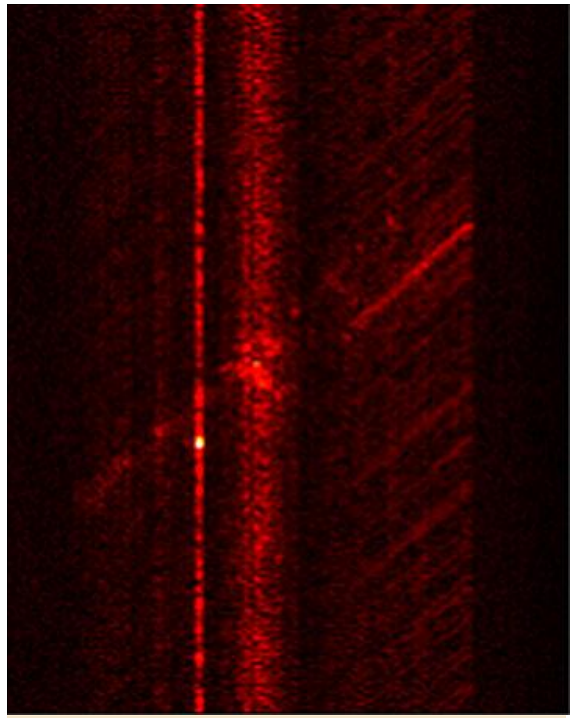
AN EXAMPLE OF CS APPLIED TO RADAR IMAGING

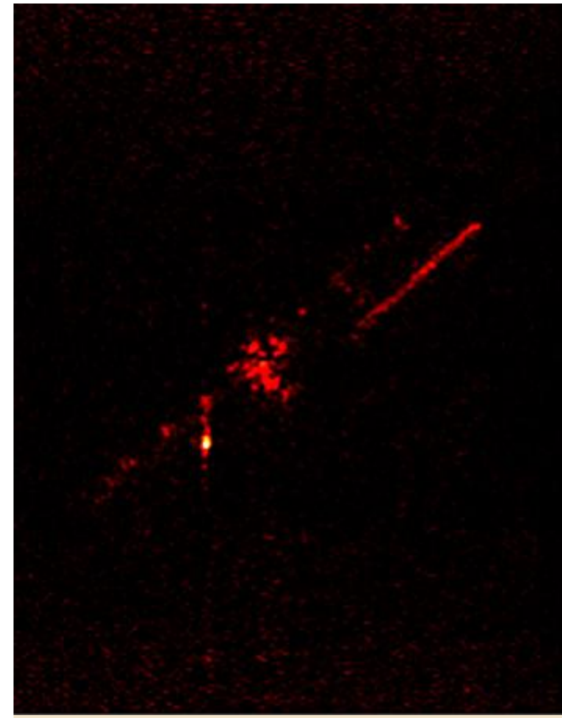
ISAR imaging with CS applied to cross-range (TIRA)



N_range=1024, N_azim=1697
1.737.728 samples!



N_range=1024, N_azim=70
matched filter



N_range=1024, N_azim=70
compressed sensing

Reduction factor 25!

Joachim H.G. Ender, On compressive sensing applied to radar, Signal Processing, Volume 90, Issue 5, May 2010, Pages 1402-1414

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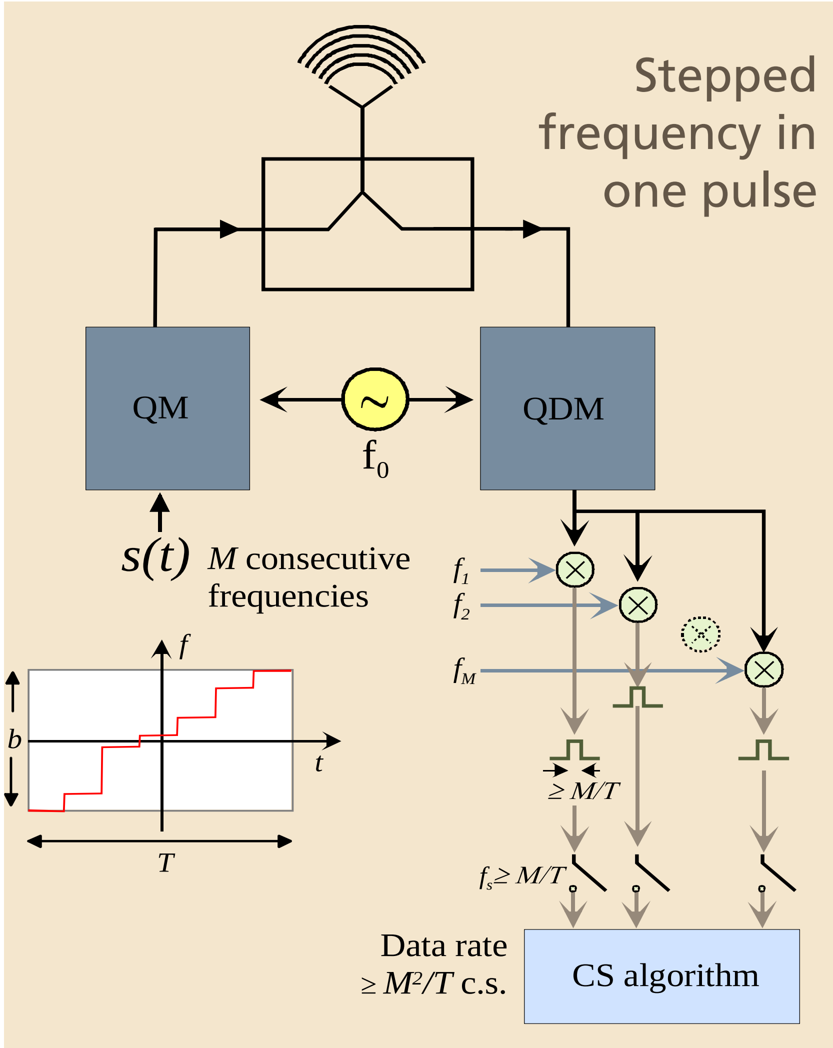
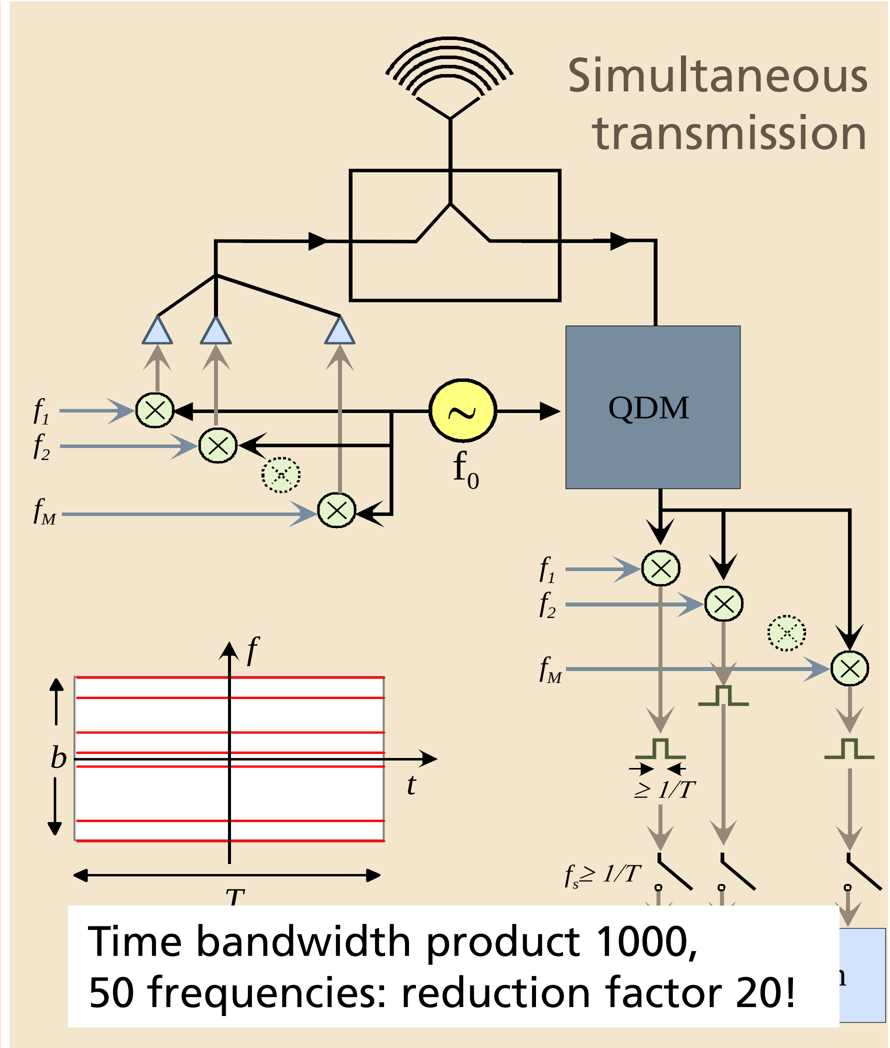
COMPRESSIVE RADAR

Hypothetic radar systems with CS for range compression





Time bandwidth product 1000,
50 frequencies: reduction factor 20!

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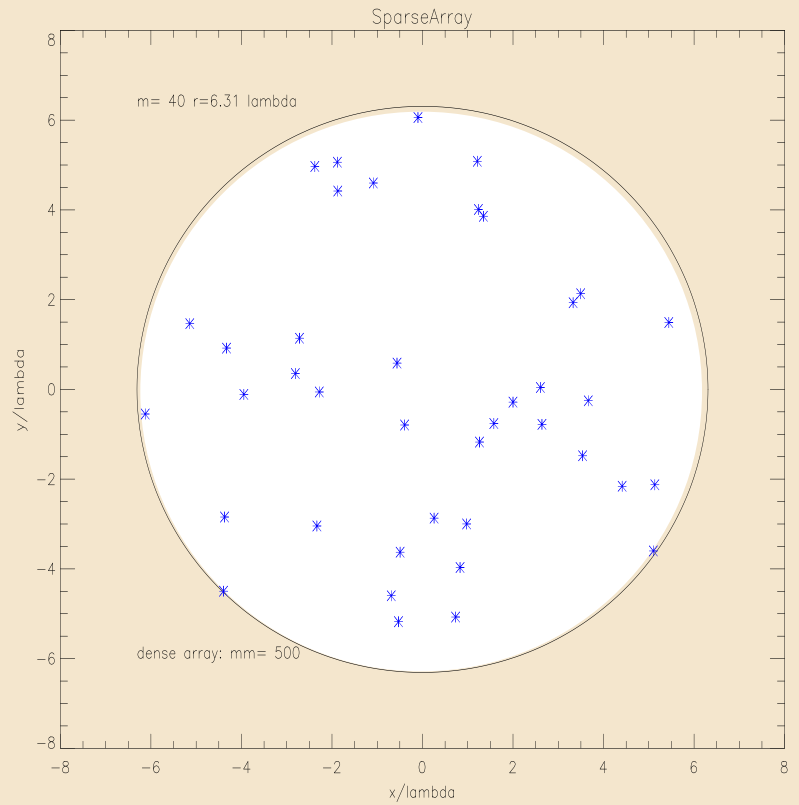
COMPRESSIVE RADAR

Example: DOA estimation using a thinned array - simulation

Circular aperture, randomly chosen element positions with uniform distribution

Radius	6.3 λ
Number elements (M)	40
Number of elements for Nyquist array	500
Number of targets (S)	5
Surveillance area	|u| <= 0.5
Raster spacing in directional plane	0.25 beamwidths
Number of grid points	2000
SNR	30 dB

Element distribution of CS array



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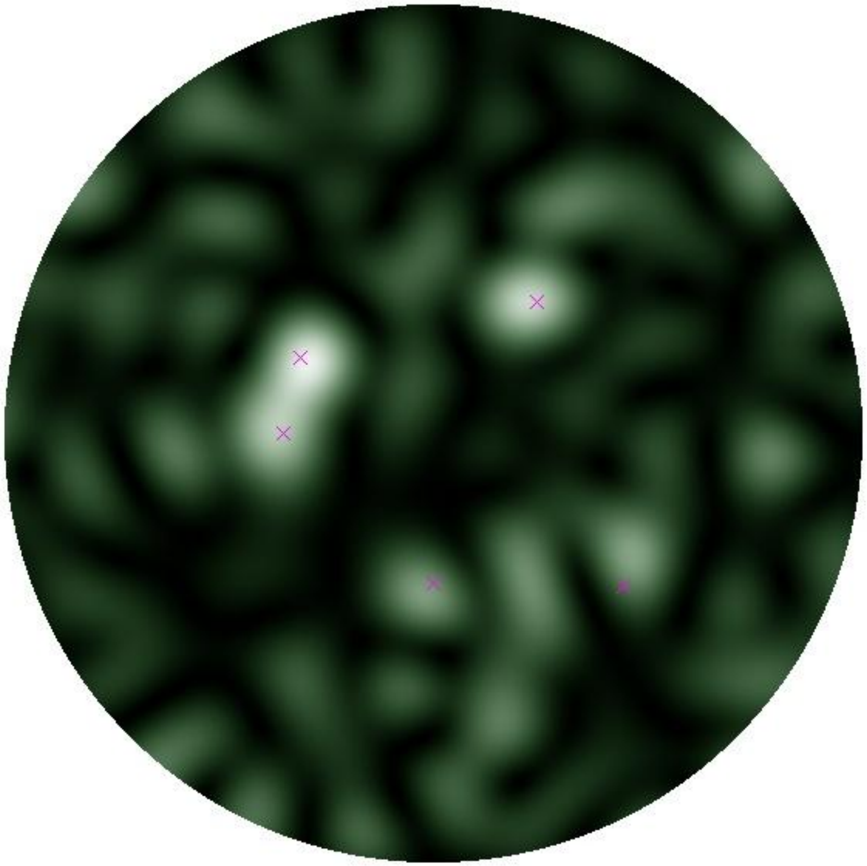
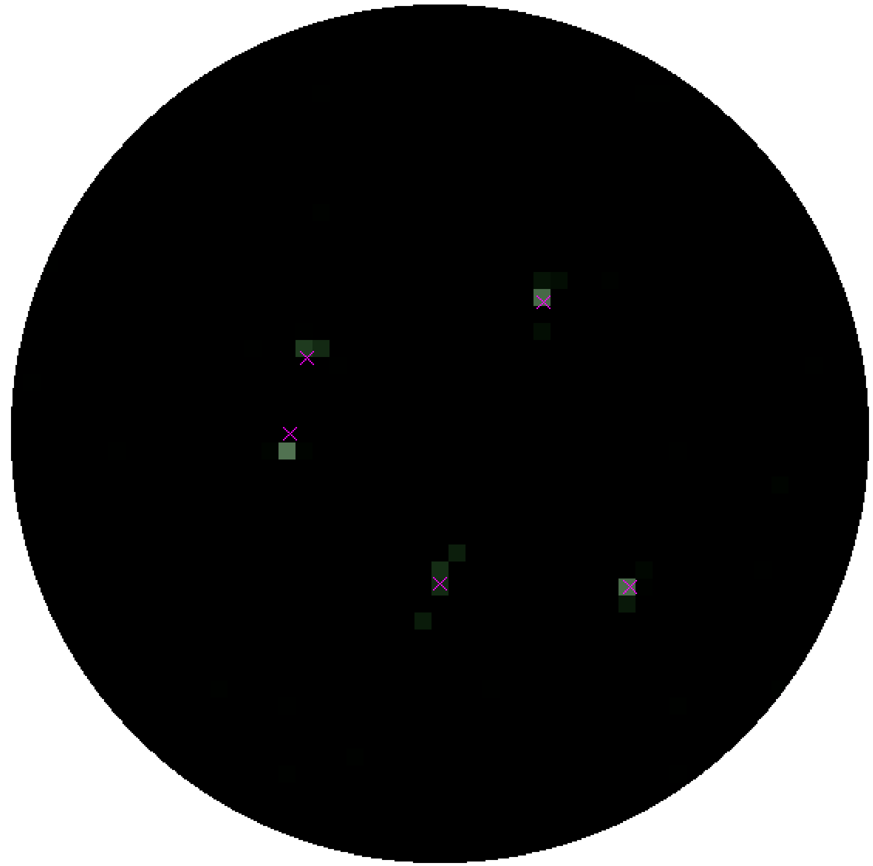
COMPRESSIVE RADAR

Example: DOA estimation of five flying airplanes (simulation)



Conventional beamformer



Compressive Sensing

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COMPRESSIVE RADAR

Sparsity of radar tasks, applicability of CS

Radar task	Scene sparsity	Acquisition sparsity	Remarks	Applic ability
Airspace surveillance	Only a few targets	Thinned array	Wide transmit beam	+++
MTI, also airborne	Only a few targets	Thinned sampling in slow time	Clutter suppression before	++
Range profiles of isolated vehicles	Only a few dominant scatterers	RF frequency band thinned to a few distinct frequencies	If necessary, preceding clutter suppression	+
ISAR for isolated vehicles	Only a few dominant scatterers	Thinning of RF frequencies and/or pulses	If necessary, preceding clutter suppression	++
SAR, basic mode	Generally not sparse	-	Image compr.	-
SAR tomography	Only a few elevation angles with reflections	Across track array, number of tracks	Calibration?	+++

COMPRESSIVE RADAR
Sparsity of radar tasks, applicability of CS

Radar task	Scene sparsity	Acquisition sparsity	Remarks	Applicability
MIMO SAR/ISAR	Single scatterers	Limited number of Tx/Rx units	Putting together MN images	++
ATR	A few features			+

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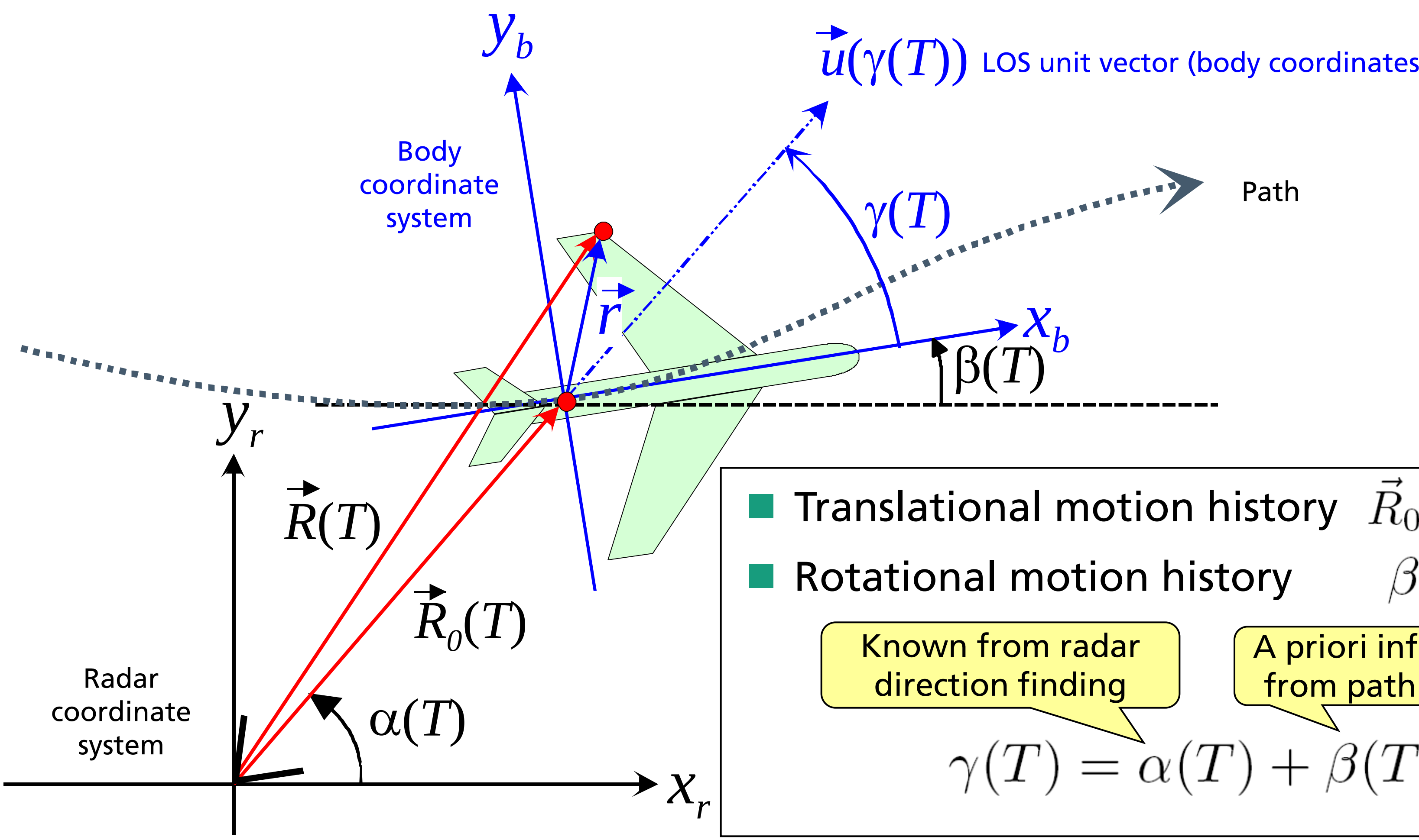
SIGNAL MODEL FOR ISAR

Geometric relations in two dimensions



- Translational motion history $\vec{R}_0(T)$
- Rotational motion history $\beta(T)$

Known from radar direction finding

A priori inf. from path

$$\gamma(T) = \alpha(T) + \beta(T)$$

SIGNAL MODEL
Geometric relations in two dimensions

Notation		
$\vec{u}(\theta) = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$	$\vec{u}_\perp(\theta) = \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}$	$\mathbf{\Phi}(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} = (\vec{u}(\theta), \vec{u}_\perp(\theta))$

- Range history for a specific scatterer

$$R(T;\vec{r}) = \left| \vec{R}_0(T) + \mathbf{\Phi}^t(\gamma(T))\vec{r} \right|$$

$$\approx R_0(T) + \langle \vec{u}(\gamma(T)), \vec{r} \rangle$$

Slow time

if the far field condition is fulfilled

Range history of origin

LOS vector in body coordinates

SIGNAL MODEL
Equivalent transformations

$$R(T;\vec{r}) = R_0(T) + \langle \vec{u}(\gamma(T)), \vec{r} \rangle$$

- If the body coordinate system is shifted by a vector $\Delta\vec{r}$ and then rotated by an angle $\Delta\gamma$ the new coordinates are

$$\vec{r'} = \mathbf{\Phi}(\Delta\gamma)\,(\vec{r} - \Delta\vec{r})$$

- The new range histories are

$$\begin{aligned} R'(T;\vec{r}) &= R_0(T) + \left\langle \vec{u}(\gamma(T)), \mathbf{\Phi}(-\Delta\gamma)\vec{r'} + \Delta\vec{r} \right\rangle \\ &= R_0(T) + \langle \vec{u}(\gamma(T)), \Delta\vec{r} \rangle + \left\langle \vec{u}(\gamma(T) + \Delta\gamma), \vec{r'} \right\rangle \end{aligned}$$

- So the transformations

$$R_0'(T) = R_0(T) + \langle \vec{u}(\gamma(T)), \Delta\vec{r} \rangle$$

Due to rotation of the new origin

$$\gamma'(T) = \gamma(T) + \Delta\gamma$$

Constant rotation angle offset

describe the same body in shifted and rotated coordinates. The new range histories are equivalent to the old.

SIGNAL MODEL
Continuous formulation

$$k = 2\pi/\lambda$$

Wave number

- Elementary signal $$s(T, k; \vec{r}) = \exp\{-j2kR(T; \vec{r})\}$$
- Superposition $$y(T, k; a) = \int a(\vec{r}) s(T, k; \vec{r}) d\vec{r}.$$

Reflectivity

Noise

- Noisy measurement $$z(T, k; a) = y(T, k; a) + N(T, k)$$

Estimate of range history

- Motion compen-sation: $$\begin{aligned} z_{mc}(T, k; a) &:= \exp\left\{j2k\hat{R}_0(T)\right\} z(T, k; a) \\ &= y_{mc}(T, k; a) + N_{mc}(T, k) \\ &= \int a(\vec{r}) s_{mc}(T, k; \vec{r}) d\vec{r} + N_{mc}(T, k) \end{aligned}$$

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A CLASSICAL IMAGING METHOD: POLAR REFORMATTING
Variable substitution to spatial frequency vectors

- For perfect motion compensation the signal is written as

$$s_{mc}(T,k;\vec{r}) \quad = \quad \exp\left\{-j2k\left\langle \vec{u}(\gamma(T)),\vec{r}\right\rangle\right\}$$

- Variable substitution $(T,k) \rightarrow \vec{K} = 2k\vec{u}(\gamma(T))$

$$s_{pr}(\vec{K};\vec{r}) \quad = \quad \exp\left\{-j\left\langle \vec{K},\vec{r}\right\rangle\right\}$$

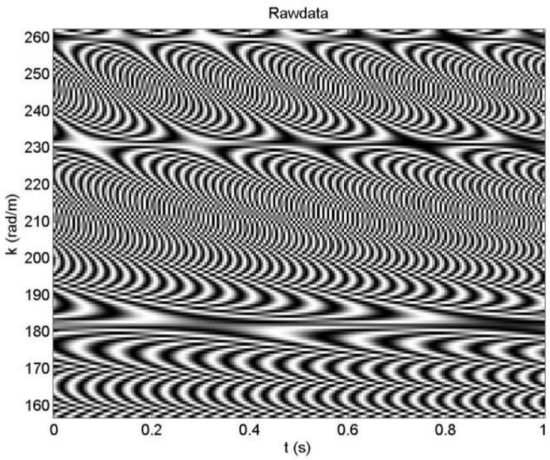
Fourier term

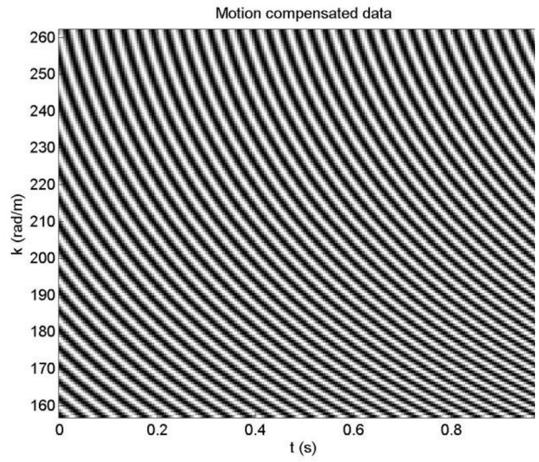
$$\boxed{z_{pr}(\vec{K};a) \quad = \quad \int a(\vec{r})\exp\left\{-j\left\langle \vec{K},\vec{r}\right\rangle\right\}d\vec{r} + N_{pr}(\vec{K})}$$

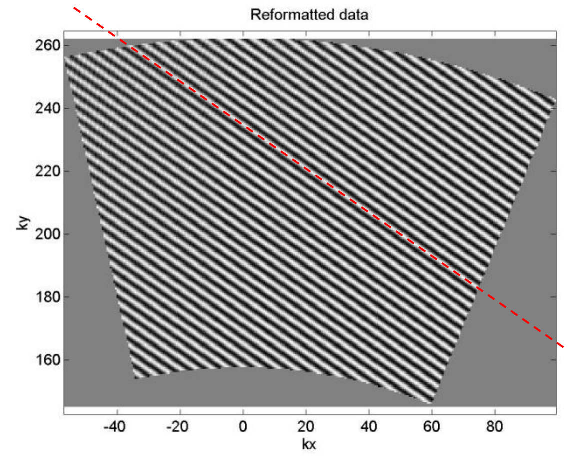
- Reconstruction of $a(\vec{r})$ by a simple inverse Fourier-transform

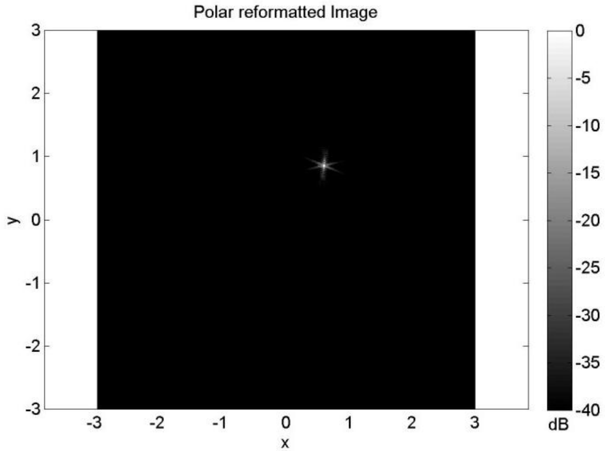
A CLASSICAL IMAGING METHOD: POLAR REFORMATTING

Illustration for only one scatterer





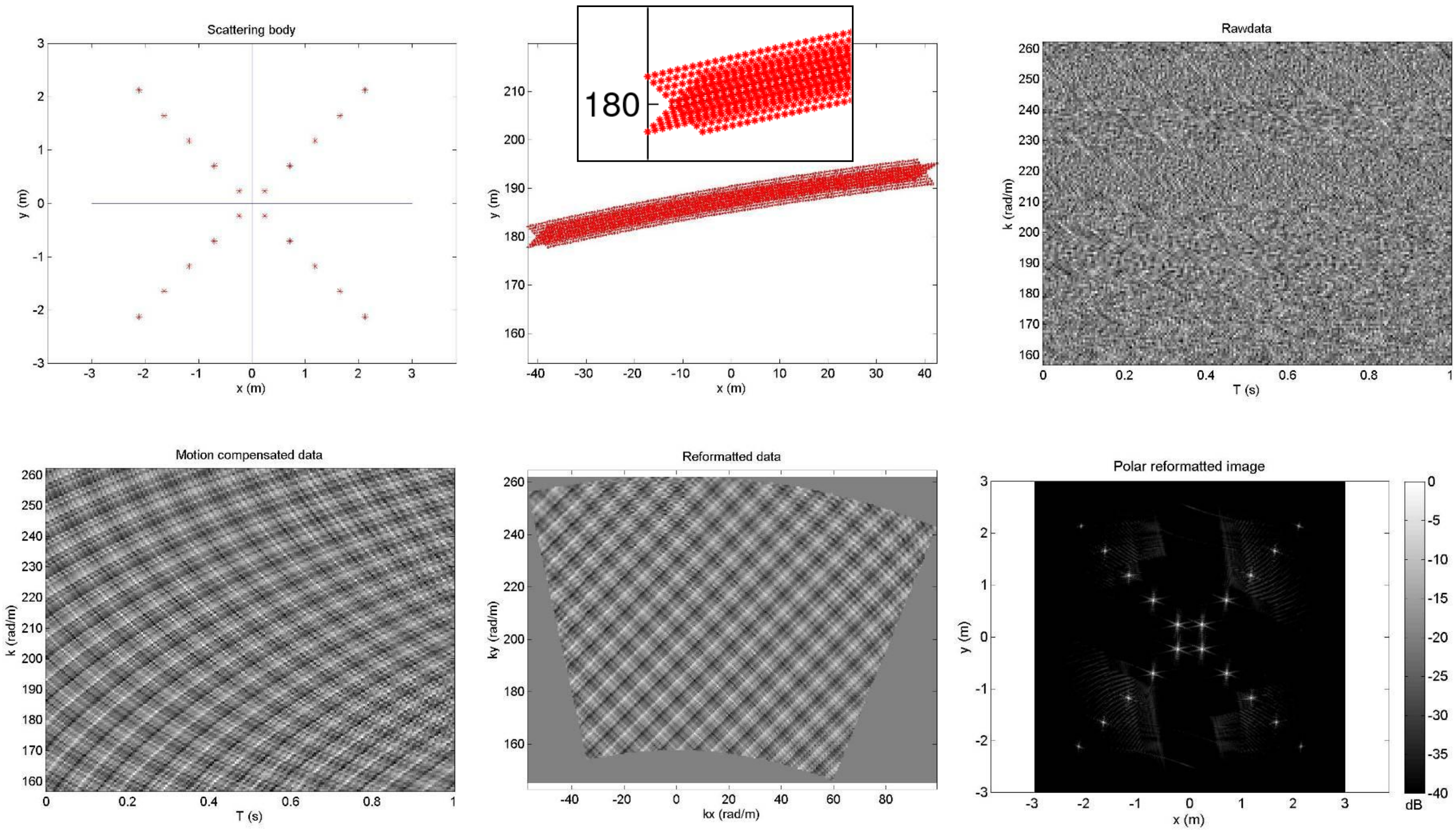




- Point spread function = Fourier transform of the indicator function of the k-set

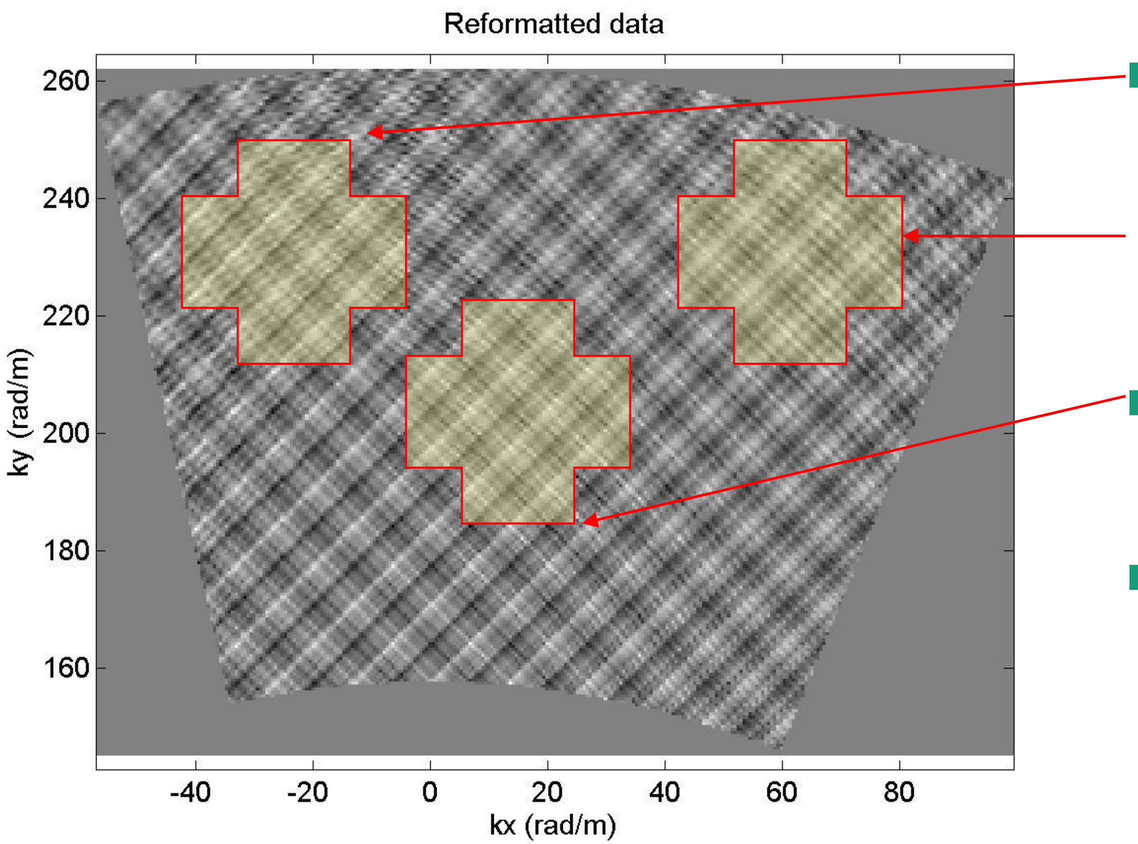
A CLASSICAL IMAGING METHOD: POLAR REFORMATTING

No noise, perfect motion compensation



A CLASSICAL IMAGING METHOD: POLAR REFORMATTING

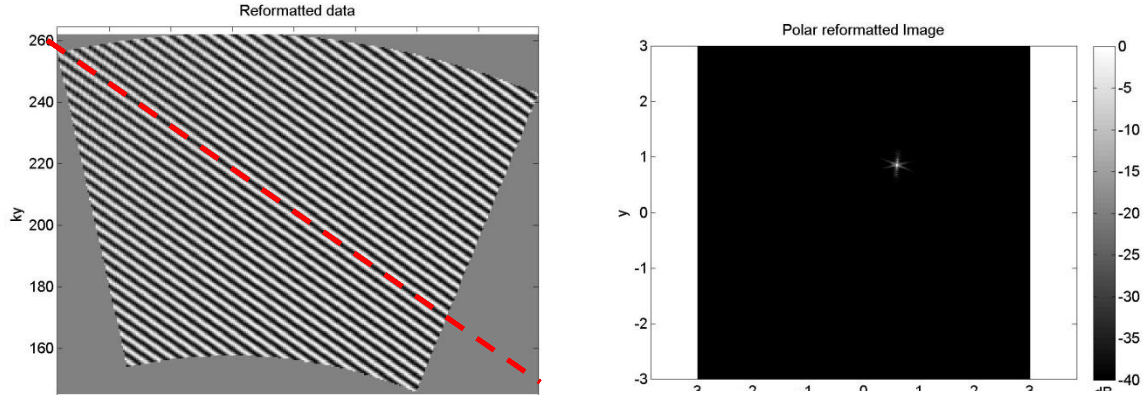
A remark on the reformatted data



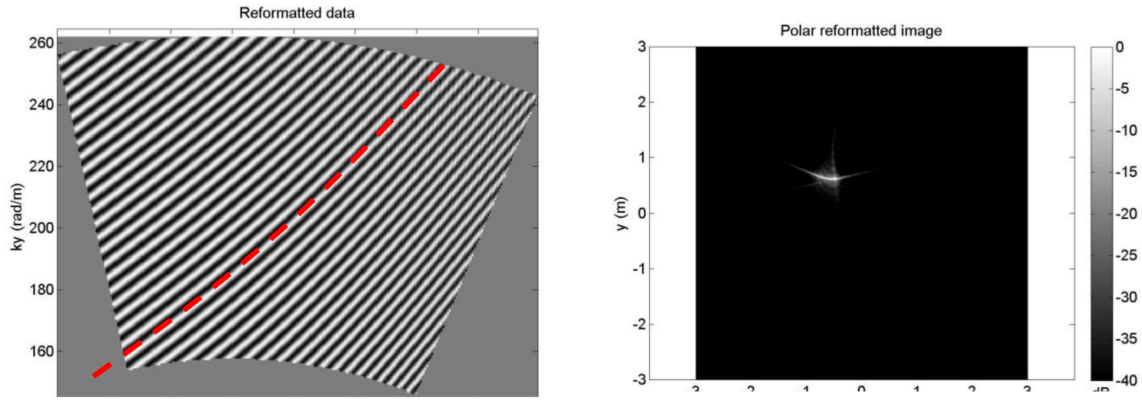
- If the motion compensation is perfect, each of the vectors of samples in shifted subsets are elements of the same subspace
- This is not true if the motion compensation is imperfect!
- Deformation of the k-space

EFFECT OF MOTION ERRORS
Distortion of the k-domain data

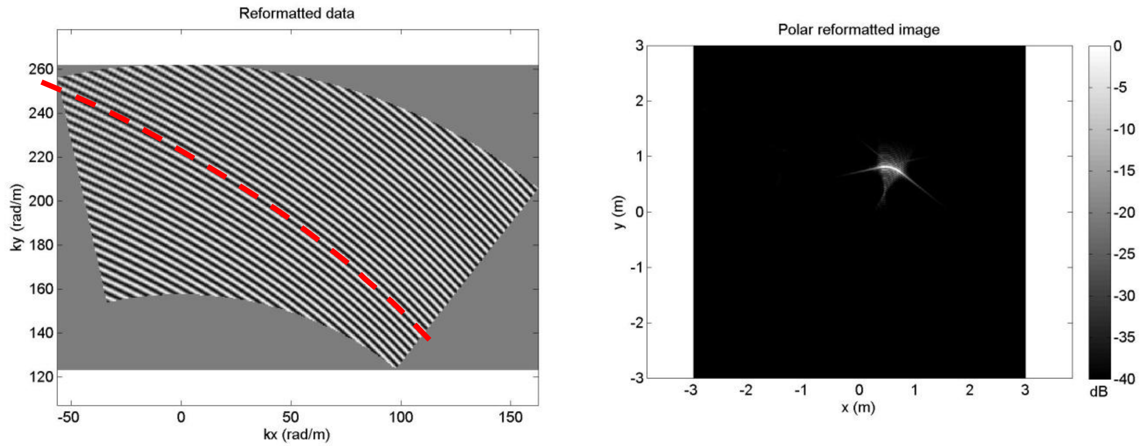
No errors



Range error
(linear)



Rotation error
(linear)



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FHR

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SIGNAL MODEL FOR SPARSE RECONSTRUCTION
Discretization in time and wavenumber

- Wavenumbers used for measurement $k_1, \ldots, k_Q$
- Pulse transmit times $T_1, \ldots, T_P$
- Resulting k-set $\mathcal{K} = \{2k_q \vec{u}(\gamma(T_p)) : p = 1, \ldots, P, q = 1, \ldots, Q\}$

- Signal matrix (P x Q) at measurement points

$$\Sigma(\vec{r}) = \begin{pmatrix} s(T_1, k_1, \vec{r}) & \ldots & s(T_1, k_Q, \vec{r}) \\ \ldots & \ldots & \ldots \\ s(T_P, k_1, \vec{r}) & \ldots & s(T_P, k_Q, \vec{r}) \end{pmatrix}$$

- Written as large M=P Q - dimensional vector

$$\mathrm{s}(\vec{r}) \quad = \quad \mathrm{vec}\left(\Sigma(\vec{r})\right) = \begin{pmatrix} s(T_1, k_1, \vec{r}) \\ \ldots \\ s(T_P, k_1, \vec{r}) \\ \ldots \\ s(T_1, k_Q, \vec{r}) \\ \ldots \\ s(T_P, k_Q, \vec{r}) \end{pmatrix}$$

SIGNAL MODEL FOR SPARSE RECONSTRUCTION
Discretization in time and wavenumber

- Analogue for noiseless superposition, noisy measurements, and noise:

$$
\begin{aligned}
y(T,k;a) &\rightarrow \Upsilon(a) \rightarrow \mathbf{y}(a) \\
z(T,k;a) &\rightarrow \Psi(a) \rightarrow \mathbf{z}(a) \\
N(T,k) &\rightarrow \Lambda(a) \rightarrow \mathbf{N}
\end{aligned}
$$

- The subscript 'mc' will always indicate motion compensated data, 'pr' stands for polar re-formatted

- Re-written signal model

$$
\begin{aligned}
\mathbf{y}(a) &= \int a(\vec{r})\mathbf{s}(\vec{r})d\vec{r} \qquad & \mathbf{z}(a) = \mathbf{y}(a) + \mathbf{N} \\
\mathbf{y}_{mc}(a) &= \int a(\vec{r})\mathbf{s}_{mc}(\vec{r})d\vec{r} \qquad & \mathbf{z}_{mc}(a) = \mathbf{y}_{mc}(a) + \mathbf{N}_{mc} \\
\mathbf{y}_{pr}(a) &= \int a(\vec{r})\mathbf{s}_{pr}(\vec{r})d\vec{r} \qquad & \mathbf{z}_{pr}(a) = \mathbf{y}_{pr}(a) + \mathbf{N}_{pr}
\end{aligned}
$$

SIGNAL MODEL FOR SPARSE RECONSTRUCTION
Discretization in the position - image grid

- N representative points $\vec{r}_1, \ldots, \vec{r}_N$ in the scene are chosen, where scatterers may be present with $N > M$.
- The signals now are written as

$$\begin{aligned} \mathbf{y}(\mathbf{a}) &= \sum_{n=1}^{N} \mathrm{a}_n \mathrm{s}(\vec{r}_n) = \mathbf{S}\mathbf{a} \\ \mathbf{z}(\mathbf{a}) &= \mathbf{y}(\mathbf{a}) + \mathbf{N} \end{aligned}$$

- The reflectivity is called L-sparse', if maximum L of the coefficients are unequal to zero.

$$\mathbf{a} = \begin{pmatrix} \mathrm{a}_1 \\ \ldots \\ \mathrm{a}_N \end{pmatrix}$$

- Sensing matrix

$$\mathbf{S} = (\mathrm{s}(\vec{r}_1), \ldots, \mathrm{s}(\vec{r}_N))$$

SIGNAL MODEL FOR SPARSE RECONSTRUCTION
Properties of the sensing matrix

- In the context of imaging a moving, rotating object, the sensing matrix is strongly dependent on the motion parameters.
- Remember: By using the data in the motion compensated form, the signal in the continuous description is

$$s_{mc}(T, k; \vec{r}) \quad = \quad \exp\left\{-j2k \left\langle \vec{u}(\gamma(T)), \vec{r} \right\rangle\right\}$$

- Restricted isometry or nullspace properties will be difficult to investigate.
- In the T-domain, the signal will have a certain Doppler-bandwidth, determined by the angular velocity and the extension of the scene. The image pixel spacing in cross-range should be related to this.
- In the polar reformatted domain the signal is given by the Fourier terms

$$s_{pr}(\vec{K}; \vec{r}) \quad = \quad \exp\left\{-j \left\langle \vec{K}, \vec{r} \right\rangle\right\}$$

and the image grid has to be related to the 2dim Nyquist rate, maybe supersampled by a certain degree.

SPARSE RECONSTRUCTION
ℓ1-minimization

- Method A: Minimization of the cost function

$$\begin{aligned} J(\mathbf{a}) &= \|\mathbf{z} - \mathbf{S}\mathbf{a}\|_2^2 + \lambda \|\mathbf{a}\|_1 \\ \hat{\mathbf{a}} &= \operatorname{argmin} \left\{ J(\mathbf{a}) : \mathbf{a} \in \mathbb{C}^N \right\} \end{aligned}$$

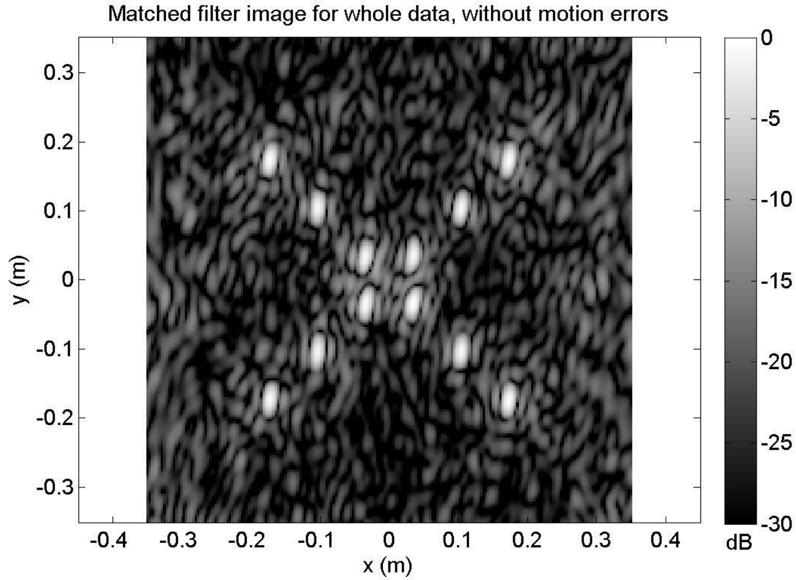
- Method B: Minimization of the cost function under constraints

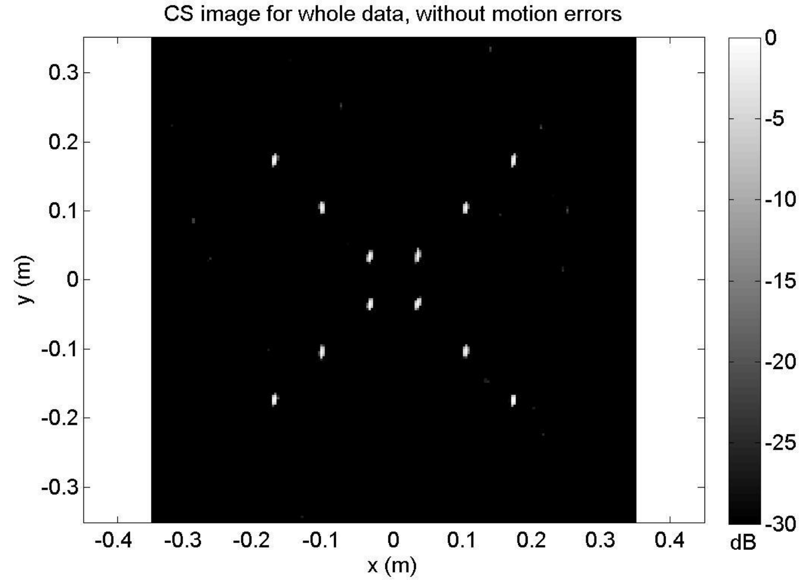
$$\begin{aligned} J(\mathbf{a}) &= \|\mathbf{a}\|_1 \\ \widehat{\mathbf{a}} &= \operatorname{argmin} \left\{ J(\mathbf{a}) : \mathbf{a} \in \mathbb{C}^N, \|\mathbf{z} - \mathbf{S}\mathbf{a}\|_2^2 \leq \sigma^2 \right\} \end{aligned}$$

- Both solutions can be obtained by the SPGL1 algorithm, see

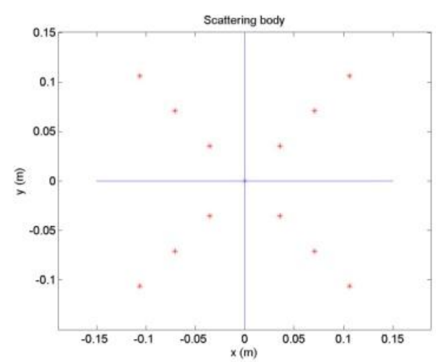
Ewout van den Berg and Michael P. Friedlander. SPGL1: A solver for large-scale sparse reconstruction. program code, Jun 2007.

SPARSE RECONSTRUCTION WITH PERFECT MOTION COMPENSATION





Matched filter



Sparse reconstruction

Sim07
N = 2401
M = 150
L = 12
sigma_noise = -25 dB

OUTLINE

- Introduction: Challenges for autofocus
  - Introduction to three imaging systems of FHR
  - Examples of compressed sensing applied to radar
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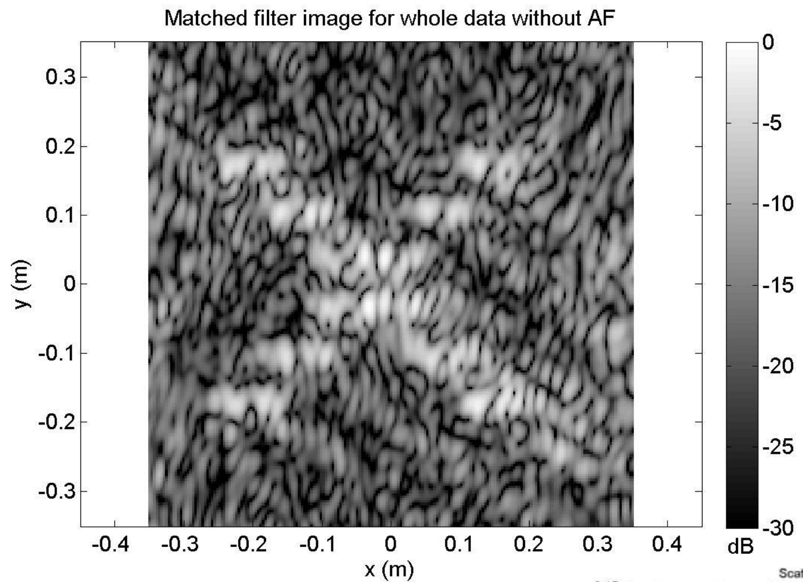
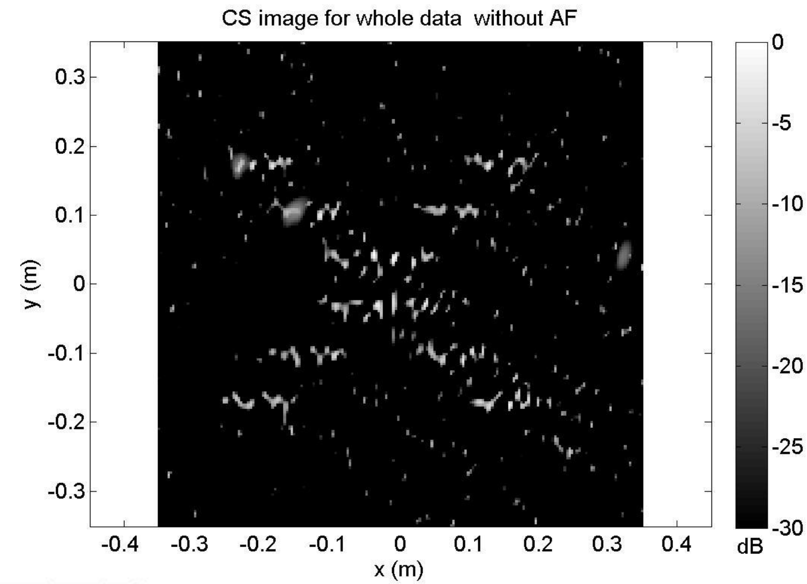
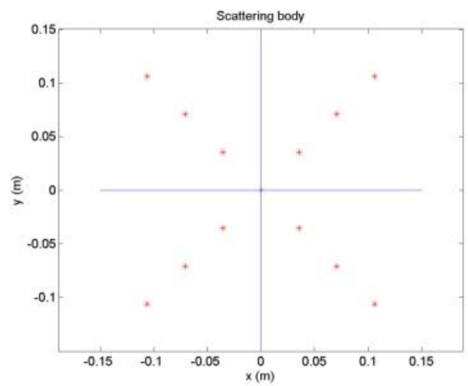
SPARSE RECONSTRUCTION WITH IMPERFECT MOTION COMPENSATION

Example for matched filter and CS reconstruction





Matched filter



Sparse reconstruction

Sim07

N = 2401

M = 150

L = 12

sigma_noise = -25 dB

Fraunhofer FHR

AUTOFOCUS WITH SPARSE RECONSTRUCTION

Problem formulation and approaches

Problem:

- Translational and rotational motion, described by a parameter vector $\boldsymbol{\vartheta}$ are not perfectly known. An a prior estimation is available.
- *Based on the radar data, find an improved estimate $\widehat{\boldsymbol{\vartheta}}$ which makes possible a high quality imaging!*

Remark

- The sensing matrix will be dependent on $\boldsymbol{\vartheta}$.
- For motion compensation, also the data depend on $\boldsymbol{\vartheta}$.
- For matched filtering the result is independent on the transformation. For CS: not!

$$\mathbf{S}_{..}(\boldsymbol{\vartheta})$$

'..' stands for subscripts!

$$\begin{aligned}\mathbf{z}_{mc}(\boldsymbol{\vartheta}) &= \mathbf{B}_{mc}(\boldsymbol{\vartheta})\mathbf{z}\\ \mathbf{B}_{mc}(\boldsymbol{\vartheta}) &= \mathrm{diag}\left(e^{j\Delta\varphi_1},\ldots,e^{j\Delta\varphi_M}\right)\end{aligned}$$

AUTOFOCUS WITH SPARSE RECONSTRUCTION
Approaches

- Method A: Minimization of the cost function

$$
\begin{aligned}
J(\boldsymbol{\vartheta}, \mathbf{a}) &= \|\mathbf{z}_{..}(\boldsymbol{\vartheta}) - \mathbf{S}_{..}(\boldsymbol{\vartheta})\mathbf{a}\|_2^2 + \lambda\|\mathbf{a}\|_1 \\
(\widehat{\boldsymbol{\vartheta}}, \widehat{\mathbf{a}}) &= \operatorname{argmin}\{J(\boldsymbol{\vartheta}, \mathbf{a}) : \boldsymbol{\vartheta} \in \Theta, \mathbf{a} \in \mathbb{C}^N\}
\end{aligned}
$$

Onhon, N. O., Cetin, M.: "A Sparsity-driven Approach for Joint SAR Imaging and Phase Error Correction", Image Processing, IEEE Transactions on, December 2011

- Method B: Minimization of the cost function under constraints

$$
\begin{aligned}
J(\boldsymbol{\vartheta}, \mathbf{a}) &= \lambda\|\mathbf{a}\|_1 \\
(\widehat{\boldsymbol{\vartheta}}, \widehat{\mathbf{a}}) &= \operatorname{argmin}\{J(\boldsymbol{\vartheta}, \mathbf{a}) : \boldsymbol{\vartheta} \in \Theta, \mathbf{a} \in \mathbb{C}^N, \|\mathbf{z}_{..}(\boldsymbol{\vartheta}) - \mathbf{S}_{..}(\boldsymbol{\vartheta})\mathbf{a}\|_2^2 \leq \sigma^2\}
\end{aligned}
$$

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AUTOFOCUS WITH SPARSE RECONSTRUCTION
Problem formulation and approaches

- Both approaches look mathematically elegant!
- It has been proven in the cited article that the reconstruction of the correct phases is possible by an iterative method.
- But:
  - for larger motion errors and large aspect angle change it may fail.
  - only the phases are corrected, a recovery of range and aspect history is not performed -> the images become sharp, but distorted.

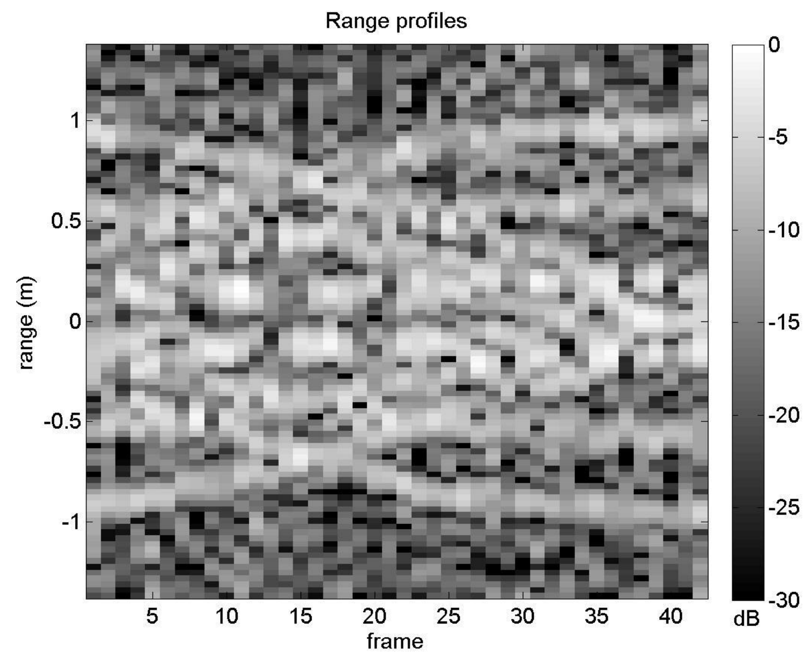
We propose a third method:

- Method C: The "three step procedure"
  1. Range history estimation
  2. Partial image alignment
  3. Phase retrieval

AUTOFOCUS WITH SPARSE RECONSTRUCTION

1. Average range history estimation

- The range center of gravity is tracked over the time.
- A smoothed version is used as first range history estimation.
- If an angular track is available, a priori information on the rotation can be implemented into the first rotation estimate.
- Experimental praxis shows that this is by far not sufficient to achieve a sharp image!



Range profiles after first motion compensation

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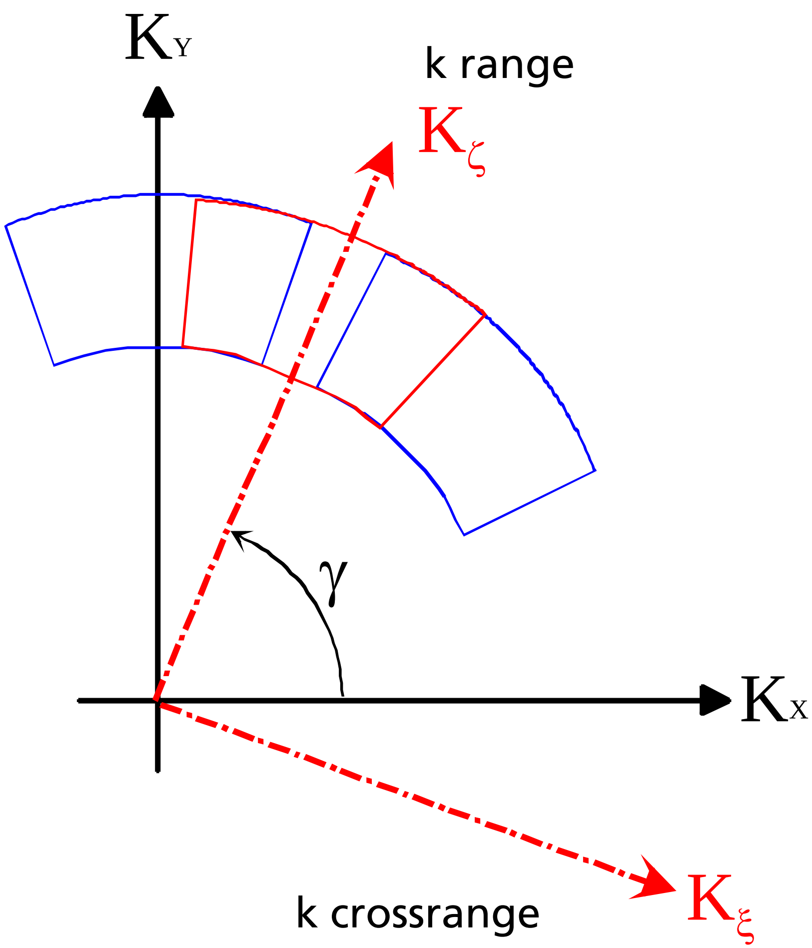
AUTOFOCUS WITH SPARSE RECONSTRUCTION

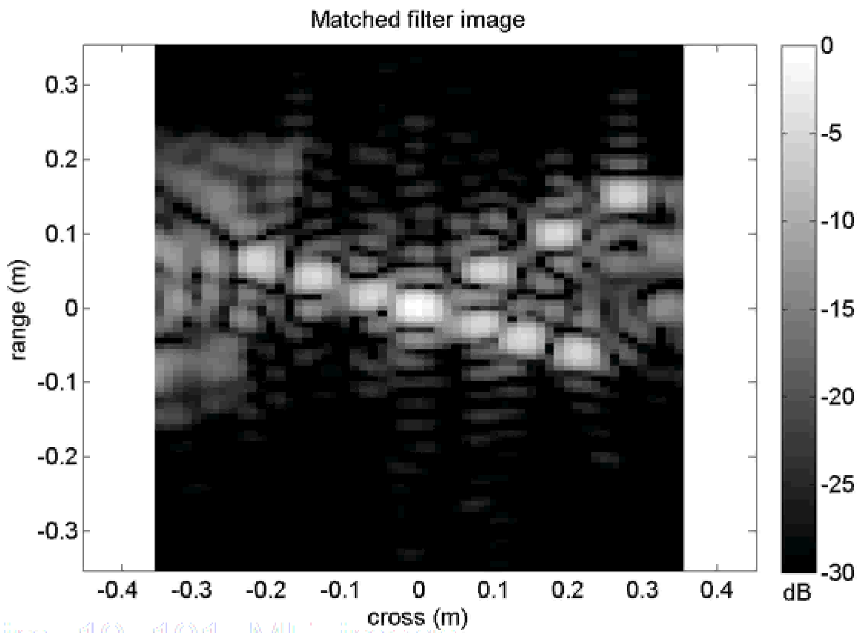
2. Partial image alignment



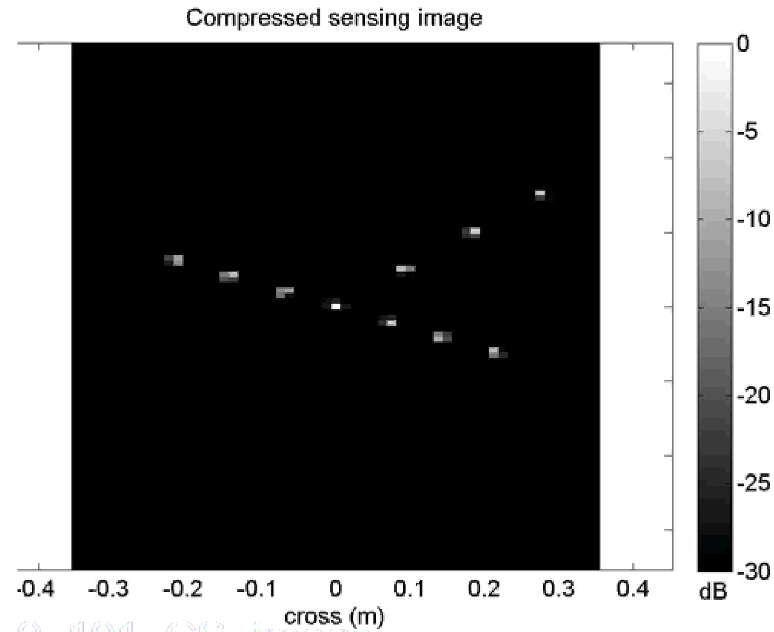
- The first stage motion compensated data are partitioned into segments, where the temporal / angular intervals are small enough to avoid range / crossrange migration larger than a classical resolution cell
- The segments may overlap
- For each segment a sparse representation is found by $\ell 1$ minimization
- To take into account the different resolutions in range and cross-range, the k-set has to be rotated to its main axes, as well as the image grid.

AUTOFOCUS WITH SPARSE RECONSTRUCTION

2. Partial image alignment -> movie of patial images



Matched filter



Sparse reconstruction

- Sparse reconstruction is working very well also for short time segments.

Fraunhofer FHR

AUTOFOCUS WITH SPARSE RECONSTRUCTION

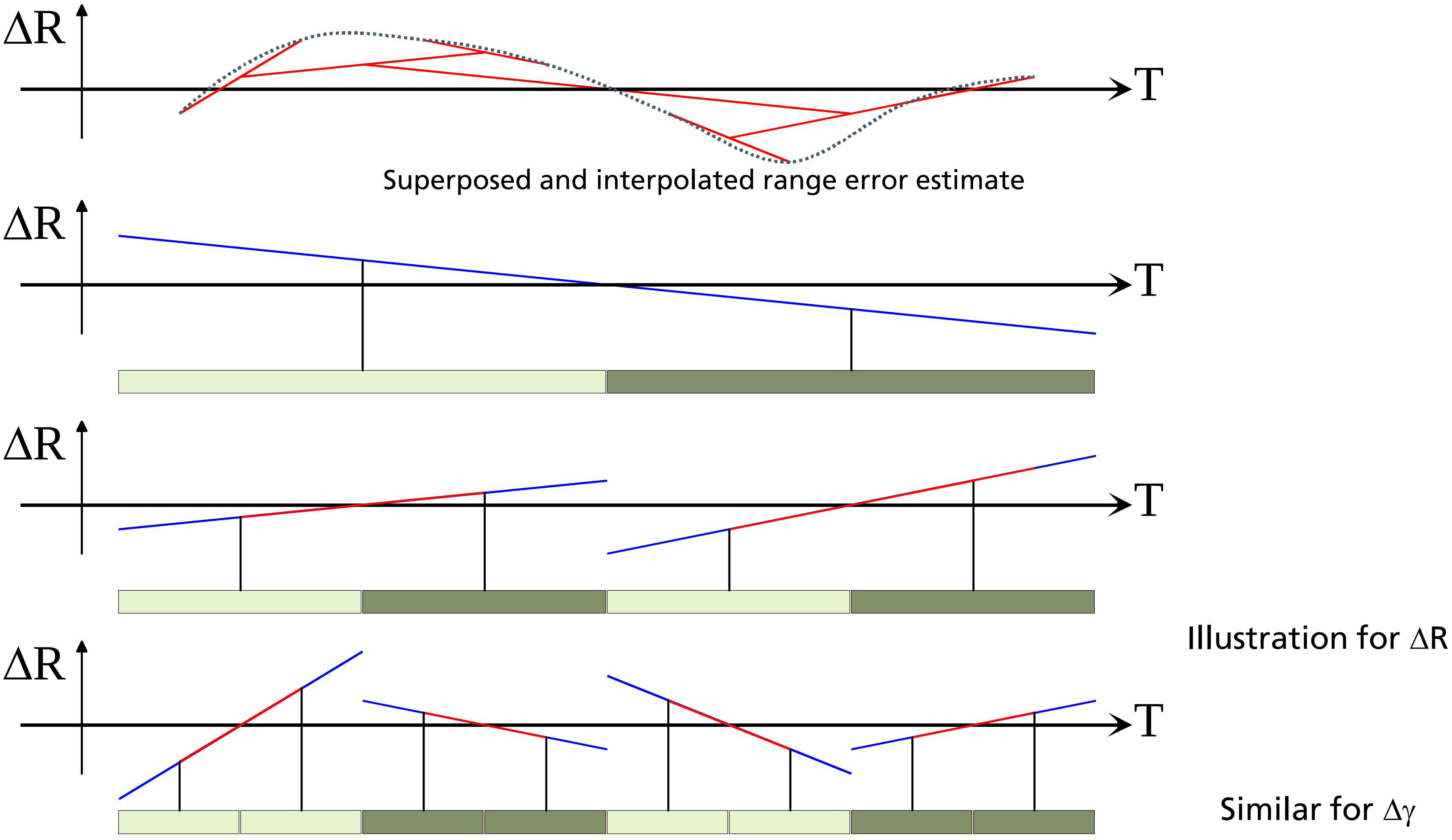
2. Partial image alignment -> Images processed for two center times

$$\underbrace{\Phi(\Delta\gamma/2)}_{\text{Turning}}\left[\underbrace{\begin{pmatrix} \frac{\hat{\omega}_-}{\omega_-} & 0 \\ 0 & 1 \end{pmatrix}}_{\text{Scaling cross range}}\begin{pmatrix} \hat{\xi}_- \\ \hat{\zeta}_- \end{pmatrix} + \underbrace{\begin{pmatrix} \frac{\Delta v_-}{\omega_-} \\ \Delta R_- \end{pmatrix}}_{\text{Shifting}}\right] = \Phi(-\Delta\gamma/2)\left[\begin{pmatrix} \frac{\hat{\omega}_+}{\omega_+} & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \hat{\xi}_+ \\ \hat{\zeta}_+ \end{pmatrix} + \begin{pmatrix} \frac{\Delta v_+}{\omega_+} \\ \Delta R_+ \end{pmatrix}\right]$$

- This equation holds for all related prominent points in the images. Rotation, shift and cross range scaling can be estimated by MMS fit.
- So, information about the error of translational and rotational motion between the two points of time can be gained by aligning the two images, if a constant velocity error history and a constant angular velocity error is assumed.
- Putting together these information for adjacent pairs, an estimate of these histories can be obtained.

AUTOFOCUS WITH SPARSE RECONSTRUCTION

2. Partial image alignment -> Putting together the estimated values



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AUTOFOCUS WITH SPARSE RECONSTRUCTION

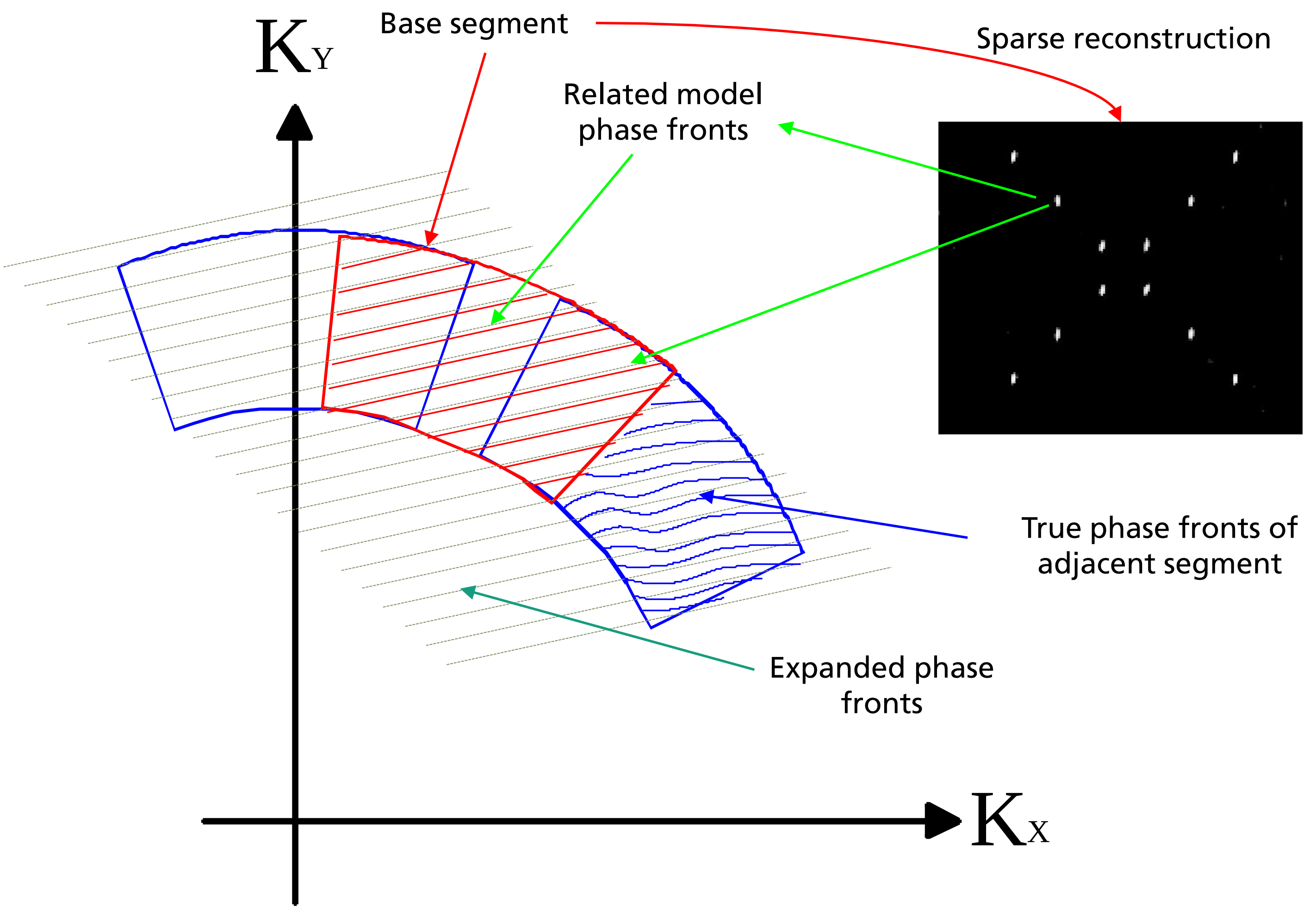
3. Phase retrieval -> Principle

- The approach with sparse reconstruction via $\ell 1$ minimization is superior to classical methods, if the scene itself is sparse:
  - Sparse reconstruction yields a much better basis to analyze the measured raw data in comparison to the reconstructed data based on the estimated sparse scene.
  - This comparison serves as a bases for the elimination of phase errors.
    - Here, we are not only interested in a phase correction (like for the phase gradient algorithm) but also in the improved reconstruction of range and rotation estimation errors
  - The prediction of data for a subsequent data segment based on the estimated sparse scene is superior to classical methods.

▶ see the next but one viewgraph

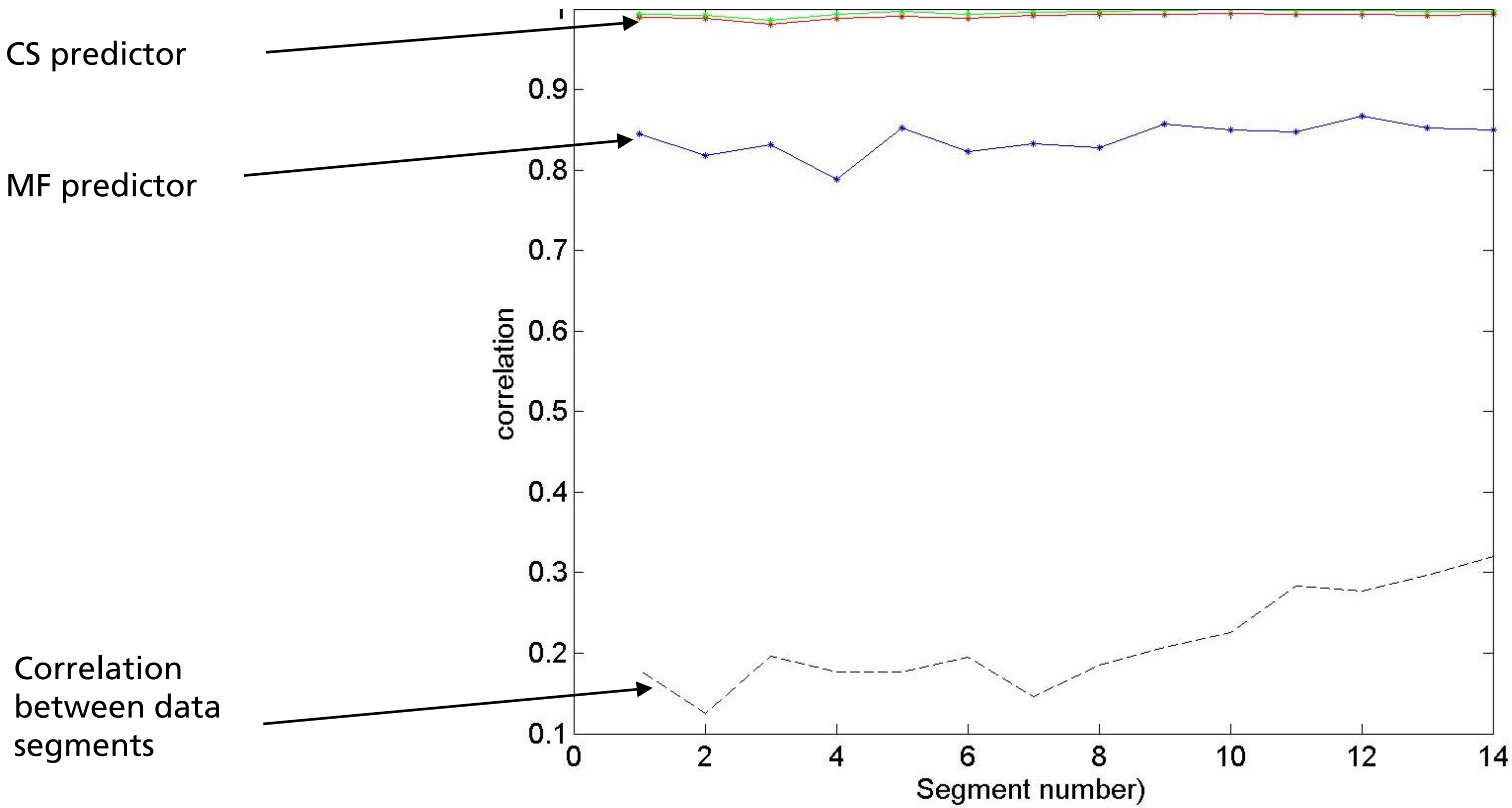
AUTOFOCUS WITH SPARSE RECONSTRUCTION

3. Phase retrieval -> Principle



AUTOFOCUS WITH SPARSE RECONSTRUCTION

3. Phase retrieval -> Correlation between predicted and measured data



- The reconstructed image from data segment *p* is used to predict the data for segment *p*+1

AUTOFOCUS WITH SPARSE RECONSTRUCTION

3. Phase retrieval -> Phase correction after alignment – the idea

- The model $$\mathbf{y} = \mathbf{S}\mathbf{a}$$
- The measurement $$\tilde{\mathbf{y}} = \tilde{\mathbf{S}}\mathbf{a}$$
- So both, model and measurement are elements of an L-dimensional subspace, the coefficients are equal, only the vectors spanning the subspace are different.
- Since the deformed signals are not known, the reconstruction is performed with the model for the sensing matrix:

$$\begin{aligned}\hat{\mathbf{a}} &= \operatorname{argmin}\{\|\mathbf{a}'\|_1 : \|\tilde{\mathbf{y}} - \mathbf{S}\mathbf{a}'\|_2 \leq \sigma\} \\ &= \operatorname{argmin}\left\{\|\mathbf{a}'\|_1 : \|\tilde{\mathbf{S}}\mathbf{a} - \mathbf{S}\mathbf{a}'\|_2 \leq \sigma\right\}\end{aligned}$$

- We can express the measurements also via the model sensing matrix:

$$\tilde{\mathbf{y}} = \mathbf{S}\mathbf{a} + \left(\tilde{\mathbf{S}} - \mathbf{S}\right)\mathbf{a} = \mathbf{S}\mathbf{a} + \mathbf{e}$$

AUTOFOCUS WITH SPARSE RECONSTRUCTION

3. Phase retrieval -> Phase correction after alignment – the idea

- Though the true coefficient vector is L-sparse, the estimated vector will normally be not L-sparse, since the model does not fit the reality perfectly.
- Nevertheless, we can expect that **â** is not too far away from the true **a** if the error is small enough (Theorems for this robustness exist).
- Search the P indices for the coefficients in **â** with the largest magnitudes.
- To each of these (hopefully) there corresponds a scatterer with non-zero component in **a**.
- For each of these indices, calculate $$\mathbf{s}_p^* \odot \widetilde{\mathbf{y}} = \sum_{l \in \mathcal{L}} a_l \mathbf{s}_p^* \odot \widetilde{\mathbf{s}}_l$$
- The signal $\widetilde{\mathbf{s}}_l$ which is 'most similar' to $\mathbf{s}_p$ will have the slowest variation in the differential phase.
- A low pass filter can separate this pair and increases the SNR.

AUTOFOCUS WITH SPARSE RECONSTRUCTION

3. Phase retrieval -> Phase correction after alignment – the idea

- The difference phase can be evaluated to

independent on scatterer

$$\delta\varphi_p(T,k) \quad = \quad \varphi_{p0} - 2k\left[g(T) + \left\langle \vec{v}(T), \widehat{\vec{r}_p} \right\rangle\right]$$

with

remaining range error

equivalent transformation

$$g(T) \quad := \quad -\Delta R(T) + \vec{u}^t(\gamma(T))\mathbf{Q}^{-1}\vec{q}$$

$$\vec{v}(T) \quad := \quad \mathbf{Q}^{-1}\vec{u}(\gamma(T)) - \vec{u}(\widehat{\gamma}(T))$$

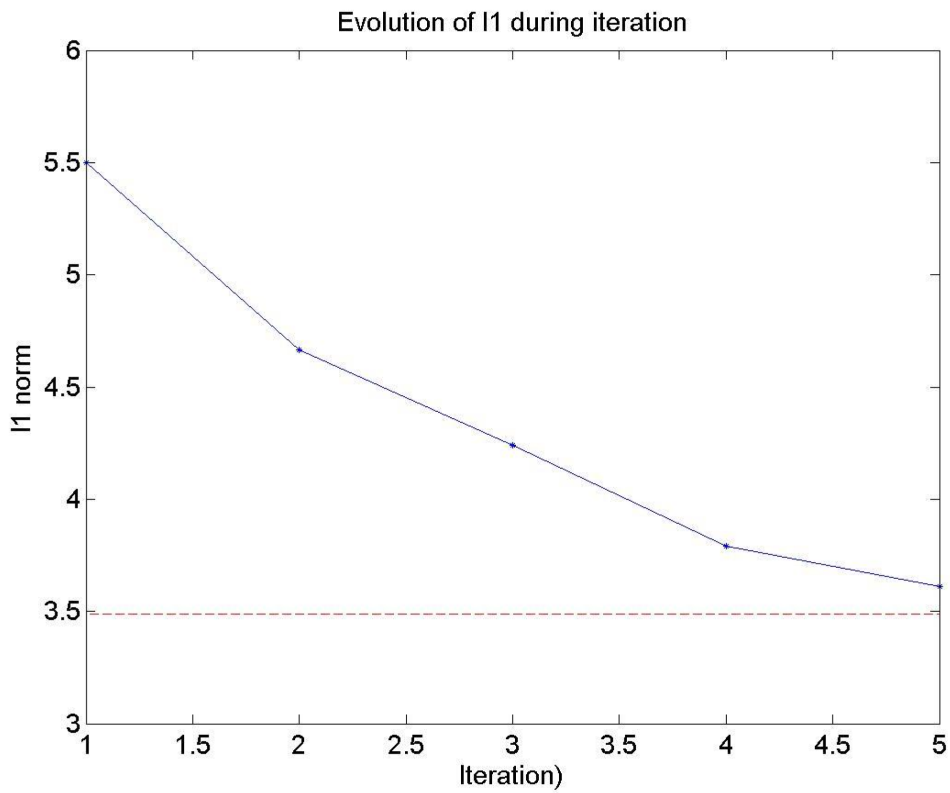
geometry matrix close to $\mathbf{I}$

- If these two functions are known, range and orientation error can be computed and compensated (up to an equivalent transformation)
- Extraction similar to the phase gradient method, but 3D.

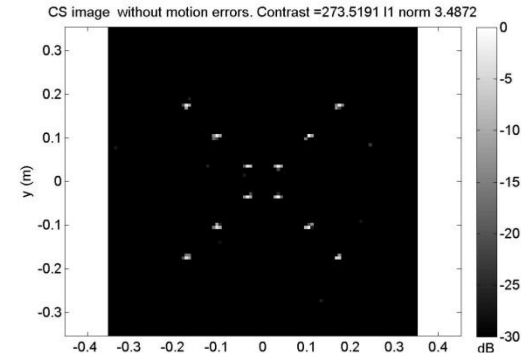
AUTOFOCUS WITH SPARSE RECONSTRUCTION

3. Phase retrieval -> simulated example

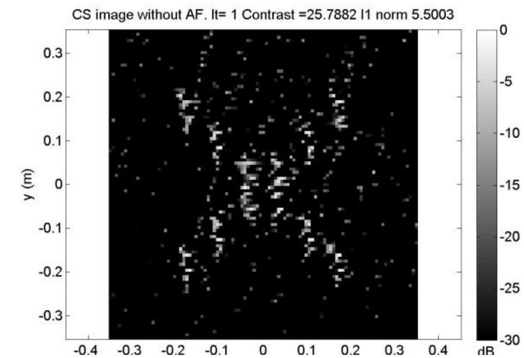
- The procedure of phase retrieval has to be iterated



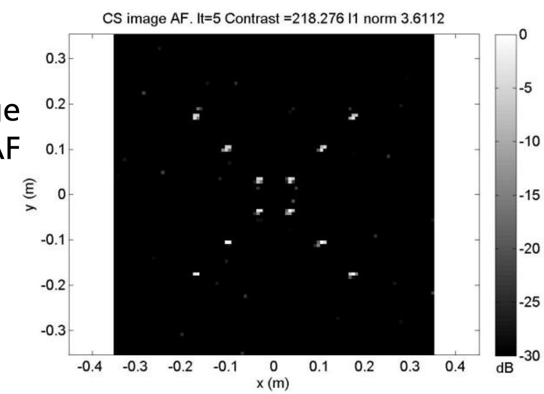
CS image without motion errors



CS image with motion errors



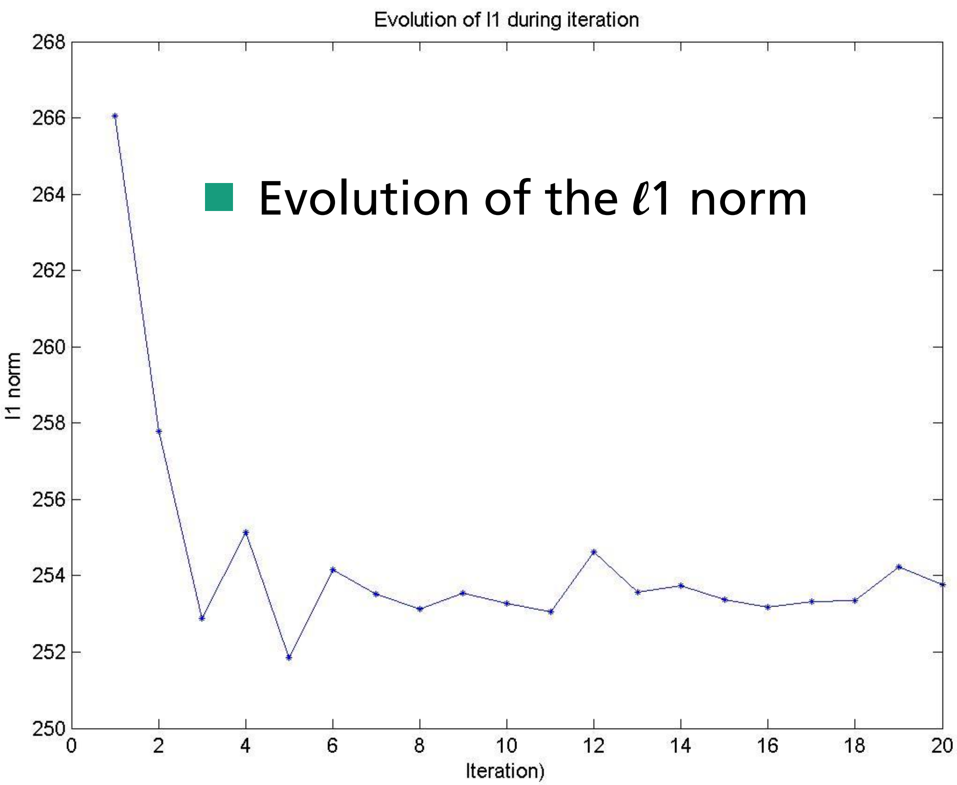
CS image after AF



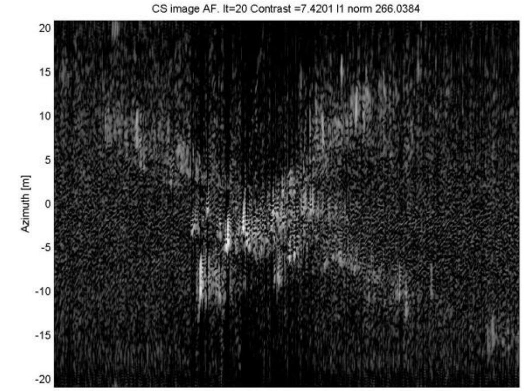
Fraunhofer FHR

AUTOFOCUS WITH SPARSE RECONSTRUCTION
3. Phase retrieval -> Example for spotlight SAR

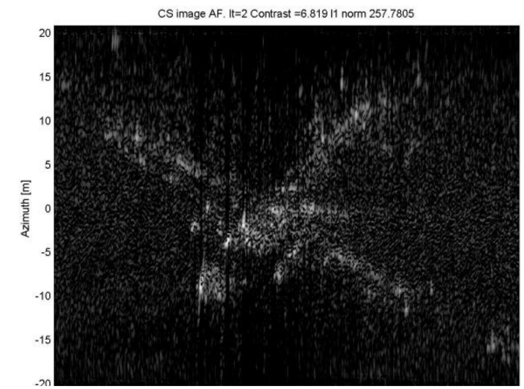
- Evolution of the ℓ1 norm



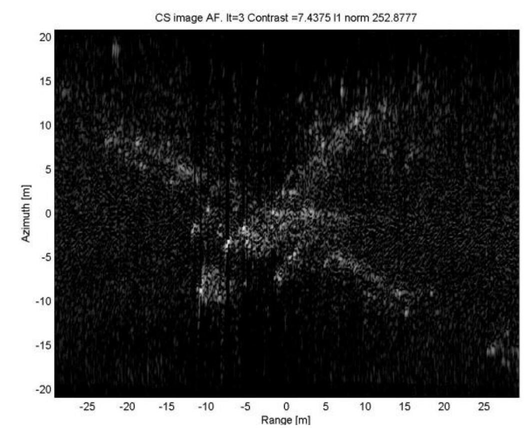
CS image without motion errors



CS image with motion errors



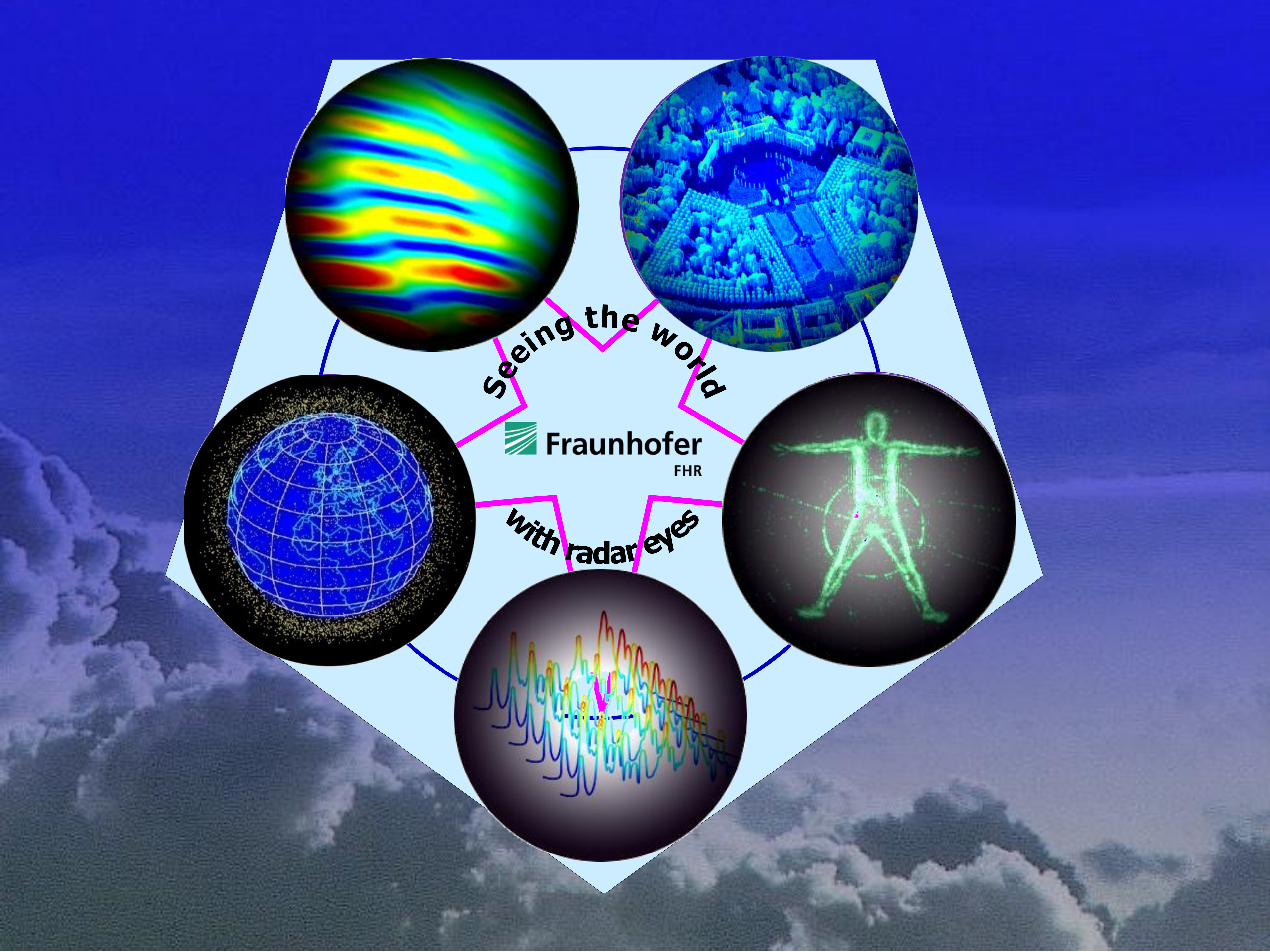
CS image after AF



Fraunhofer FHR

SUMMARY AND OUTLOOK

- We presented an investigation on autofocus using CS methods
- Sparse representation for a sparse reflecting body yields
    - an improved prediction of the data without distortion
    - a basis for autofocus by comparing the prediction with the measurements
- To cover a large angular sector, a three-step method is preferable:
    - Coarse estimation of the range history
    - Alignment of partial images
    - Phase retrieval with improved estimation of range and rotation motion
- What has to be done ...
    - Analysis of the performance over a large simulation basis
    - Comparison to other methods
    - Derivation of theorems (?)


Seeing the world
Fraunhofer
FHR
with radar eyes