Sparsity-Driven Radar Imaging

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Sparsity and Compressibility of Signals

Involvement in Sparsity

- Feature-enhanced SAR imaging [1999-2002]
- Evaluation of the impact of sparsity-driven SAR on classification [2000-2003]
- Array signal processing – acoustic source localization [2001-2004]
- HRR profile reconstruction [2002]
- Optimal sparse representations ($l_1$-$l_0$ & $l_p$-$l_0$ equivalence conds.) [2002-2004]
- Homotopy continuation for sparse signal representation [2004-2005]
- Ultrasound imaging for non-destructive evaluation [2003-2006]
- Wide-angle SAR imaging [2003-2005]
- Passive radar imaging [2004-2005]
- Sparse communication channel estimation [2004-2005]
- Anisotropic SAR imaging using structured dictionaries [2004-2008]
- Automatic hyperparameter choice [2007-2009]
- Multiple feature-enhanced imaging [2008-2011]
- Compressed sensing in radar imaging [2008-2009]
- Joint imaging and model error correction [2008-ongoing]
- SAR despeckling & its parallel implementation [2011-ongoing]
- Moving target imaging [2011-ongoing]
Linear Inverse Problems

\[ y = \mathbf{A} \mathbf{\Psi} \mathbf{\alpha} \]

- **\mathbf{A}**: \( M \times N \)
- **\mathbf{\Psi}**: identity matrix
- **\mathbf{\alpha}**: \( L \) non-zero elements

\[ A : M \times N \]
\[ N > M > L \]

- A simple example:
  - \( \mathbf{A} \) : “band-limited” DFT operator
  - \( \mathbf{\Psi} \) : identity matrix

Underdetermined Linear Inverse Problems and Sparsity

- Basic problem: find an estimate of \( f^* \), where
  \[
y = Af^* \quad (A : M \times N, \ N > M)
  \]

- Underdetermined -- non-uniqueness of solutions
- Additional information/constraints needed for a unique solution
- Typical approach: \( \min ||f||_2^2 \) subject to \( y = Af \)
- If we know \( f^* \) is sparse (i.e. has few non-zero elements)?

  \[
  \hat{f}_{\ell_0} = \arg \min ||f||_0^0 \quad \text{subject to} \ y = Af
  \]

  *Number of non-zero elements in \( f^* \)*

- Intractable combinatorial optimization problem
- Recent work on sparse signal representation has produced principled and feasible alternatives
SAR Ground-plane Geometry

- Scalar 2-D complex reflectivity field $f(x, y)$
- Transmitted chirp signal: $s(t) = \Re \left[ e^{j(\omega_0 t + \alpha t^2)} \right]$, $|t| \leq \frac{T_p}{2}$
- Received, demodulated return from circular patch:

$$r_\theta(t) = \left. \int_{|u| \leq L} q_\theta(u) \exp \left\{ -j \frac{2}{c} \left[ \omega_0 + 2\alpha \left( t - \frac{2R}{c} \right) \right] u \right\} du$$

Band-limited Fourier transform of $q_\theta(u)$

Projection of field $f(x, y)$

SAR Observation Model

• Observations are related to projections of the field:

\[
r_\theta(t) = \int_{|u| \leq L} g_\theta(u) \exp \left\{ -j \frac{2}{c} \left[ \omega_0 + 2\alpha \left( t - \frac{2R}{c} \right) \right] u \right\} \, du
\]

- SAR observations are band-limited slices from the 2-D Fourier transform of the reflectivity field:

\[
r_\theta(t) = \int \int_{x^2 + y^2 \leq L^2} f(x, y) \exp \left\{ -j \Omega(t) (x \cos \theta + y \sin \theta) \right\} \, dx \, dy
\]

\[
= F \left[ \Omega(t) \cos \theta, \Omega(t) \sin \theta \right]
\]

• Range profiles: \( y_\theta = \mathcal{F}^{-1} r_\theta \)

• Discrete tomographic SAR observation model:

(combining all measurements)

\[ y = Af + n \]
Conventional Image Formation

- Given SAR returns, create an estimate of the reflectivity field $f$

**Support of observed data in the spatial frequency domain**

**Sample Conventional Image**

Polar format algorithm:
- Each pulse gives slice of 2-D Fourier transform of field
- Polar to rectangular resampling
- 2-D inverse FFT

Outline

1) Sparsity-driven, feature-enhanced radar imaging
   – An analysis-based formulation for point and region-enhanced imaging
   – Making the representation dictionary explicit: multiple feature-enhanced imaging

2) Wide-angle imaging
   – Composite image formation
   – Structured overcomplete dictionaries for anisotropic radar imaging

3) Phase errors
   – Joint imaging and autofocusing
   – Moving target imaging

   • Automatic hyperparameter selection
   • Compressed sensing in radar imaging
Initial motivation for our work

Some challenges for automatic decision-making from SAR images:

- Accurate localization of dominant scatterers
  - Limited resolution
  - Clutter and artifact energy
- Region separability
  - Speckle
  - Object boundaries
- Low SNR, limited apertures
Sparsity-Driven Radar Imaging

\[ J(f) = \| y - Af \|_2^2 + \lambda \| Lf \|_p^p \]

- Complex-valued data and image
- Phase of field spatially uncorrelated
- Magnitude of complex-valued field admits sparse representation
- Typical choices for L:
  - identity (\textit{point-enhanced imaging})
  - gradient (\textit{region-enhanced imaging})
- Optimization problem structure is not identical to common sparse representation problems
- Bayesian interpretation: MAP estimation problem with heavy-tailed priors

[Çetin and Karl, 2001]
Efficient Solution of the Optimization Problem

• Cost functional

\[ J(f) = \|y - Af\|^2_2 + \lambda \|L|f|\|^p_p \]

• Challenging optimization problem
  (non-quadratic, non-convex cost, constraint on \(|f|\), large size)

• Developed a half-quadratic regularization technique for complex-valued, random-phase fields

• Could be viewed as a quasi-Newton algorithm with a special Hessian approximation

• Non-quadratic constraints other than \(l_p\)-norms have also been used.

[Çetin and Karl, 2001, 2002]
Efficient Solution of the Optimization Problem

- Quasi-Newton-based iterative scheme:

\[
H(\hat{f}^{(n)}) \hat{f}^{(n+1)} = (1 - \gamma) H(\hat{f}^{(n)}) \hat{f}^{(n)} + \gamma 2 A^H y
\]

(\(\gamma\): step size)

- Each iteration equivalent to solving a quadratic problem:

\[
\hat{f}^{(n+1)} = \arg \min_{f} \| y - Af \|^2_2 + \lambda f^H W(\hat{f}^{(n)}) f
\]

Feature Preservation \hspace{1cm} Spatially Adaptive Weights

- Can interpret as sequence of non-stationary Gaussian problems

\[
H(f) \triangleq 2A^H A + \lambda \Phi^H(f)L^T V(f) L \Phi(f) \quad V(f) \triangleq \text{diag} \left\{ \frac{p}{((|Lf|) + 1)^{1+p/2}} \right\} \quad \Phi(f) \triangleq \text{diag} \{ \exp(-j \phi((f),l)) \}
\]

\[
W(\hat{f}^{(n)}) \triangleq \frac{1}{2} \Phi^H(\hat{f}^{(n)}) L^T V(\hat{f}^{(n)}) L \Phi(\hat{f}^{(n)})
\]

\(\phi((f),l)\): phase of the complex number \((f)\)
Point-Enhanced Imaging

Synthetic scene

Original

Conventional

Point-Enhanced

Point-Enhanced Imaging

MIT Lincoln Laboratory ADTS data

Conventional

![Conventional Image]

Point-Enhanced

![Point-Enhanced Image]
Region-Enhanced Imaging

MIT Lincoln Laboratory ADTS data

Conventional

Region-Enhanced
Making the representation dictionary explicit: multiple feature-enhanced imaging

- Original (analysis-based) imaging formulation:

\[
J(f) = \|y - A f\|_2^2 + \lambda \|L \cdot f\|_p^p
\]

- Let us use the representation \( |f| = \Psi \alpha \)

- Note: \( f = \Phi |f| \), where \( \Phi \) contains reflectivity phases

- Alternate (synthesis-based) formulation:

\[
J(\alpha, \Phi) = \|y - A \Phi \Psi \alpha\|_2^2 + \lambda \|\alpha\|_p^p \quad s.t. \quad \Phi_{ii} = 1 \quad \forall i
\]

- Joint optimization over phase and representation of magnitude

- Opens up the possibility of various dictionaries \( \Psi \)

[Samadi, Çetin, and Shirazi, 2011]
Multiple Feature-Enhanced SAR Imaging: Use of Various Dictionaries

Original Conventional Analysis sparse (point+region)

(a) (b) (c)

Proposed approach with various dictionaries

Spikes + squares Wavelet Spikes + wavelet

(d) (e) (f)

Multiple Feature-Enhanced SAR Imaging: Use of Various Dictionaries

Original

Conventional

Analysis sparse (point+region)

Proposed approach with various dictionaries

Spikes + squares

Wavelet

Spikes + wavelet

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   - Structured overcomplete dictionaries for anisotropic radar imaging

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   - Moving target imaging
     - Automatic hyperparameter selection
     - Compressed sensing in radar imaging
Non-traditional apertures: wide-angle data

- Irregular PSF
- Anisotropic scattering
  - Conventional imaging produces inaccurate reflectivities
  - Conventional imaging does not characterize aspect dependence

Main pieces of proposed approach (first version):
- Sparsity-driven subaperture image reconstruction
- Composite wide-angle image formation

[Moses, Potter, and Çetin 2004]
Composite Image Formation

• Many scatterers won’t persist over wide angles
• Form a composite wide-angle image from narrow-angle subaperture images:

\[ \hat{f}_{ij} = \arg \max_k \hat{f}_{ij}^k \]

- Subaperture index
- Pixel indices
- Composite image
- k-th subaperture image

• Preservation of anisotropic scatterers with short persistence
• Partial characterization of aspect dependence
Wide-angle SAR Imaging

- Angular range = 110°
- Composite images

Reference image

Conventional

Sparsity-driven

[Moses, Potter, and Çetin 2004]
Wide-angle SAR Imaging with Frequency-band Omissions

- Angular range = 110°
- Composite images
- 70% of the band available

Reference image

[Çetin and Moses 2005]
Visualization of aspect dependence in wide-angle imaging

70% band availability

Bandwidth = 4GHz

Bandwidth = 1GHz

Scattering Models

• Model for data collected from $P$ isotropic scatterers:

$$r(\omega, \theta) = \sum_{m=1}^{P} f(x_m, y_m) \exp\left\{ - j \frac{2\omega}{c} (x_m \cos \theta + y_m \sin \theta) \right\}$$

  – Image formation: Recovering $f(x,y)$

• When we consider anisotropy:

$$r(\omega, \theta) = \sum_{m=1}^{P} f(x_m, y_m, \theta) \exp\left\{ - j \frac{2\omega}{c} (x_m \cos \theta + y_m \sin \theta) \right\}$$

  – Joint imaging and anisotropy characterization: Recovering $f(x,y,\theta)$

• The problem becomes much more underdetermined!
Joint Imaging and Sparsity-Driven Anisotropy Characterization

• **Joint Imaging and Anisotropy Characterization**
  – Modeling anisotropy using an overcomplete dictionary
  – Structured overcomplete dictionaries (hierarchy of atoms within molecules)
  – Solution by approximate, graphical algorithm

• **Outcomes:**
  – Accurate reflectivity estimation for scatterers that do not persist over wide angular apertures
  – Characterization of scattering direction and angular width
    • Potentially useful feature for automatic target recognition

[Varshney, Çetin, Fisher, and Willsky, 2008]
Modeling angular anisotropy

- Model \( f(x, y, \theta) \) by the dictionary \( \{b_1(\theta), b_2(\theta), \ldots, b_M(\theta)\} \)

\[
r(\omega, \theta) = \sum_{m=1}^{P} f(x_m, y_m, \theta) \exp \left\{ -j \frac{2\omega}{c} (x_m \cos \theta + y_m \sin \theta) \right\}
\]

- Problem: determine unknown coefficients \( a_{m,i} \)
- How do we choose the \( b_i(\theta) \)?
Dictionary selection

- Want to choose dictionary to allow parsimonious representation
- Observation: Non-zero scattering usually in one contiguous interval of $\theta$
- Dictionary elements: contiguous intervals of anisotropy – all widths and all shifts
Dictionary selection

- Incorporates some prior information about scattering structure
- Consider rectangular pulse shape (Hamming, triangular, sinc, etc. are other possibilities)

$$b_1(\theta) \quad b_2(\theta) \quad \ldots \quad b_M(\theta)$$

$\begin{bmatrix}
\theta_1 \\
\theta_2 \\
\theta_3 \\
\vdots \\
\theta_N \\
\end{bmatrix}$

$\text{NxM matrix } M = \frac{N(N+1)}{2}$

- Can solve the resulting sparse representation problem in principle
- Exact solution is memory intensive
- Use structure to develop less expensive approximate algorithm
Illustration of graph-structured algorithm
Quick Example on Anisotropy Characterization

<table>
<thead>
<tr>
<th>generalized inversion</th>
<th>sparse solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(\theta) )</td>
<td></td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\| a_1^{\text{g}} \|_{0.8} & = |a_1^{\text{s}}|_{0.8} \\
\| a_2^{\text{g}} \|_{0.8} & = |a_2^{\text{s}}|_{0.8}
\end{align*}
\]
Anisotropic Imaging Results on the Backhoe Data

Sample result of analysis-based formulation for anisotropy characterization

Aspect dependent scattering behavior in indicated sub-area

[Stojanovic, Çetin, and Karl, 2008]
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Phase Errors and Defocusing

• Phase errors in the SAR data → Defocusing of the reconstructed image

  ▶ Phase errors
  ▶ Inexact knowledge of the round-trip propagation time of the transmitted signal
  ▶ Uncertainties on the SAR platform position
  ▶ Delays in the transmitted signal due to atmospheric effects
  ▶ Moving targets in the scene

  space-invariant defocusing
  the amount of defocusing is same for all points in the scene

  space-variant defocusing
  defocusing in the corresponding spatial region in the reconstructed image

1D phase errors
due to demodulation time errors

- Measurement errors on the round-trip propagation time of the transmitted signal
- After certain approximations, the relationship between the erroneous and error-free discrete data can be expressed as
  \[ \bar{r}_{\varepsilon_m} = e^{j\varepsilon_m \omega_0} \bar{r}_m = e^{j\phi_D(m)} \bar{r}_m \]
- 1D phase errors cause defocusing in the reconstructed image in the cross-range direction

\[ \phi_{1D} = [\phi_{1D}(1), \phi_{1D}(2), \ldots, \phi_{1D}(M)]^T \]

\[ A_m(\phi_{1D}) f \rightarrow e^{j\phi_D(m)} A_m f \quad \text{for} \quad m = 1, 2, \ldots, M \]

The observation model matrix which takes phase errors into account

Joint imaging and model error correction: Sparsity-Driven Autofocus (SDA)

• SAR observation model may not be known perfectly, due to, e.g., uncertainties in platform location
• This leads to phase errors in observed data
• Proposed approach: joint optimization over the reflectivities and model parameters:

\[
J(f, \phi) = \|y - A(\phi)f\|^2_2 + \lambda \|f\|_1
\]

• Solution through coordinate descent
  – Closed-form solution for the phase error estimation step

[Önhon and Çetin, 2009]
SDA on a synthetic scene

Without phase errors
- Original scene
- Conventional
- Sparsity-driven imaging

With phase errors
- Phase error
- Conventional
- Proposed SDA

SDA on slicy

Without phase errors

Picture of slicy

Conventional

Sparsity-driven imaging

Quadratic phase error

Conventional

Sparsity-driven imaging

Proposed SDA

Random, uniform phase error

Conventional

Sparsity-driven imaging

Proposed SDA

SDA on backhoe

Without phase errors

Conventional imaging

Sparsity-driven imaging

Random, uniform phase error

Conventional imaging

Sparsity-driven imaging

Proposed SDA

1D Phase error uniform. dist. in $\left[-\frac{\pi}{2}, +\frac{\pi}{2}\right]$
Existing Autofocus Techniques

- **Phase Gradient Autofocus (PGA)**
  PGA estimates phase errors using the data obtained by isolating many single defocused targets via center-shifting and windowing operations.

- **Techniques based on the Optimization of the Sharpness Metrics of the Image Intensity**
  These techniques optimize different sharpness metrics of the conventionally reconstructed defocused image intensity.

- **Multi-Channel Autofocus (MCA)**
  MCA is based on a non-iterative algorithm which finds the focused image in terms of a basis formed from the defocused image, relying on a condition on the image support to obtain a unique solution.
Comparison of autofocus methods on a synthetic scene

Phase error uniform. dist. in $[-\pi, +\pi]$
Quantitative analysis

\[ \text{MSE} = \frac{1}{I} \left\| f - \hat{f} \right\|^2 \]

\[ \text{TBR} = 20 \log_{10} \left( \frac{\max_{i \in T} |\hat{f}_i|}{\frac{1}{I_B} \sum_{j \in B} |\hat{f}_j|} \right) \]

\[ \phi_e = \phi - \hat{\phi} \]

\[ \text{MSE}_{PE} = \frac{1}{M-1} \left\| \nabla \phi_e \right\|^2 \]
Comparison of autofocus methods on backhoe

1D Phase error uniform. dist. in $[-\pi, +\pi]$
Moving-target imaging

- SAR platform position uncertainties cause space-invariant defocusing of the reconstructed image, i.e., the amount of defocusing is the same for all points in the scene.
- Motion of a target in the scene can also be modeled as a phase error over the phase history data corresponding to a stationary scene.
- Moving targets in the scene cause artifacts including defocusing around the spatial neighborhood of the target in the scene.
  - Space-variant defocusing
  - Need to keep an account of the contributions from each spatial location to the phase error at each aperture position
Sparsity-Driven Moving-Target Imaging

\[
\arg \min_{f, \beta} J(f, \beta) = \arg \min_{f, \beta} \left\| y - A(\phi) f \right\|_2^2 + \lambda_1 \left\| f \right\|_1 + \lambda_2 \left\| \beta - 1 \right\|_1 \\
\text{s.t.}\quad |\beta(i)| = 1 \quad \forall i
\]

\[
\beta_m = \left[ e^{j\phi_1(m)}, e^{j\phi_2(m)}, \ldots, e^{j\phi_l(m)} \right]^T
\]

- Involves sparsity constraints both on the reflectivity field and on the motion field.
- Have also developed a more efficient and potentially robust version based on constructing ROIs and performing space-invariant focusing within them.
Moving-target imaging results

Original scene

\[ v_{1_{cr}} = 8 \text{ m/s} \]
\[ v_{2_{cr}} = 5 \text{ m/s} \]

Conventional imaging

Sparsity-driven imaging

Proposed method

Moving-target imaging results

$v_{cr} = 8 m / s$

Original scene

Conventional imaging

Sparsity-driven imaging

Proposed method

Summary

• Radar imaging presents interesting sparse representation problems involving some variations of basic sparse representation formulations

• Significant amount of work exists on sparsity-driven algorithms as well as associated results with potential impact on applications

• We have examined and contributed to
  – various sparse representation formulations and algorithms
  – stationary and moving target imaging
  – wide-angle and passive radar imaging
  – handling model errors
  – hyperparameter choice
  – impact on target recognition
  – compressed sensing for monostatic and multistatic SAR

• Empirical successes in regime where sufficient sparsity/CS conditions not satisfied