Tales of Rotating Convection

Robert Ecke
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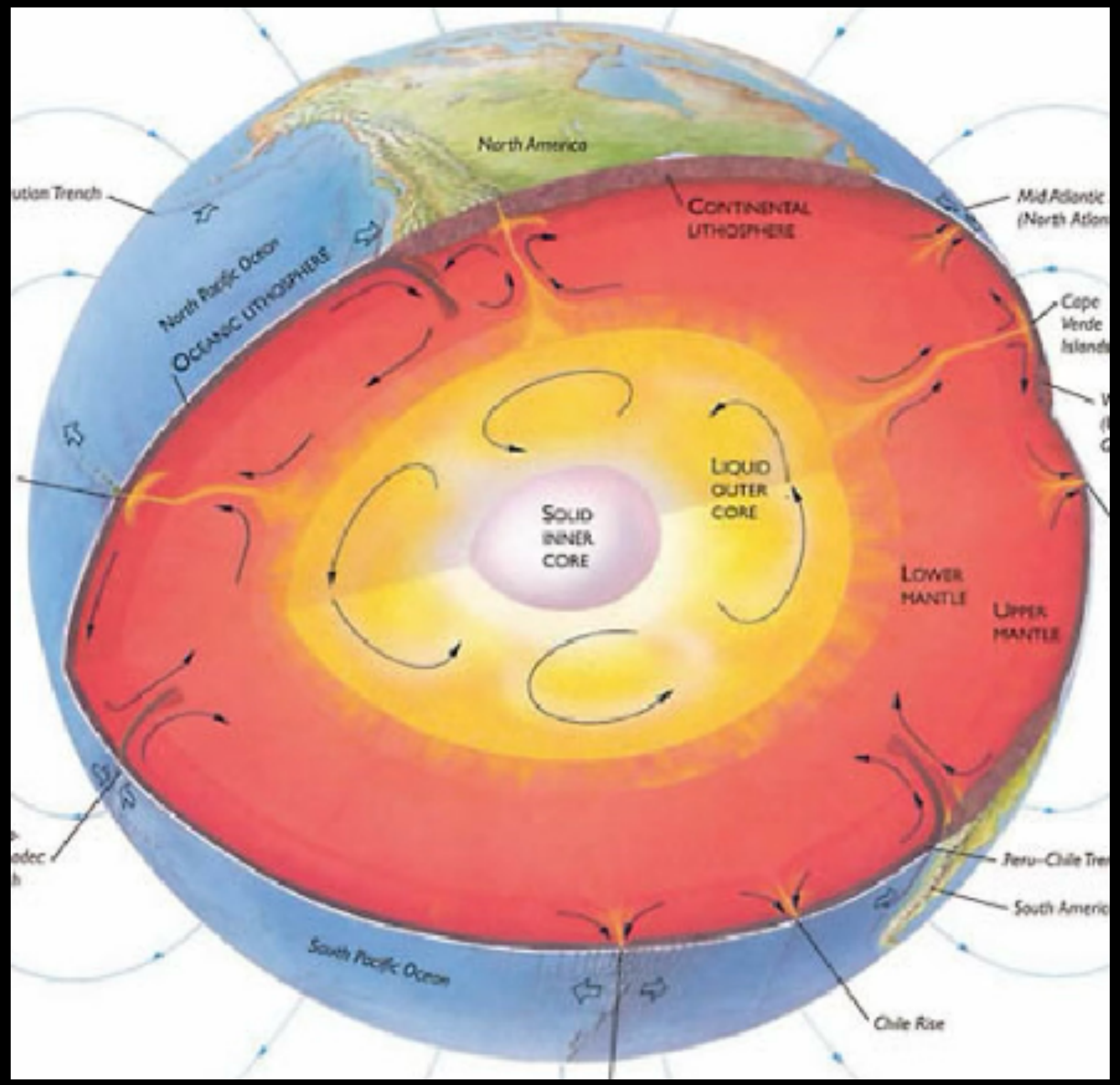
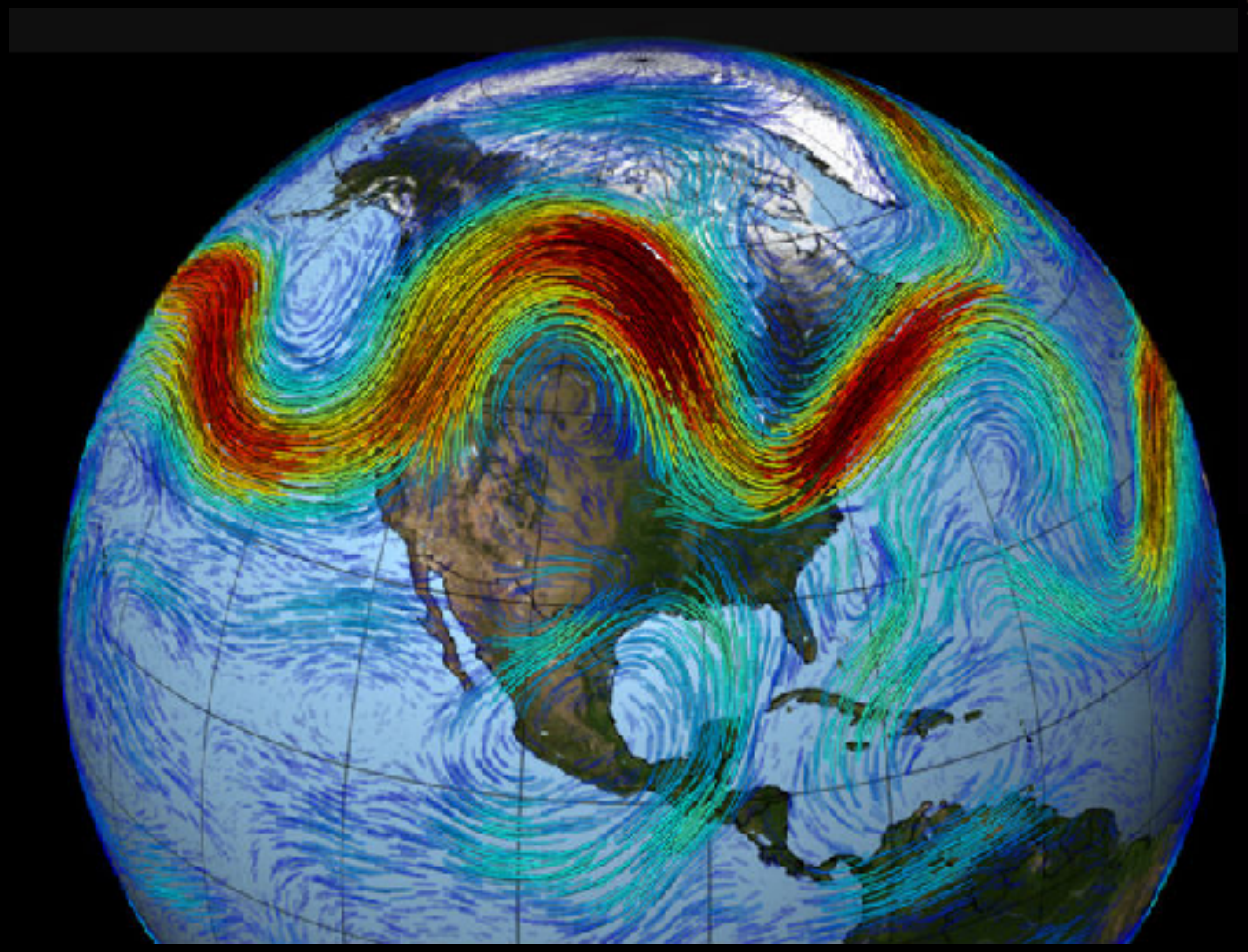
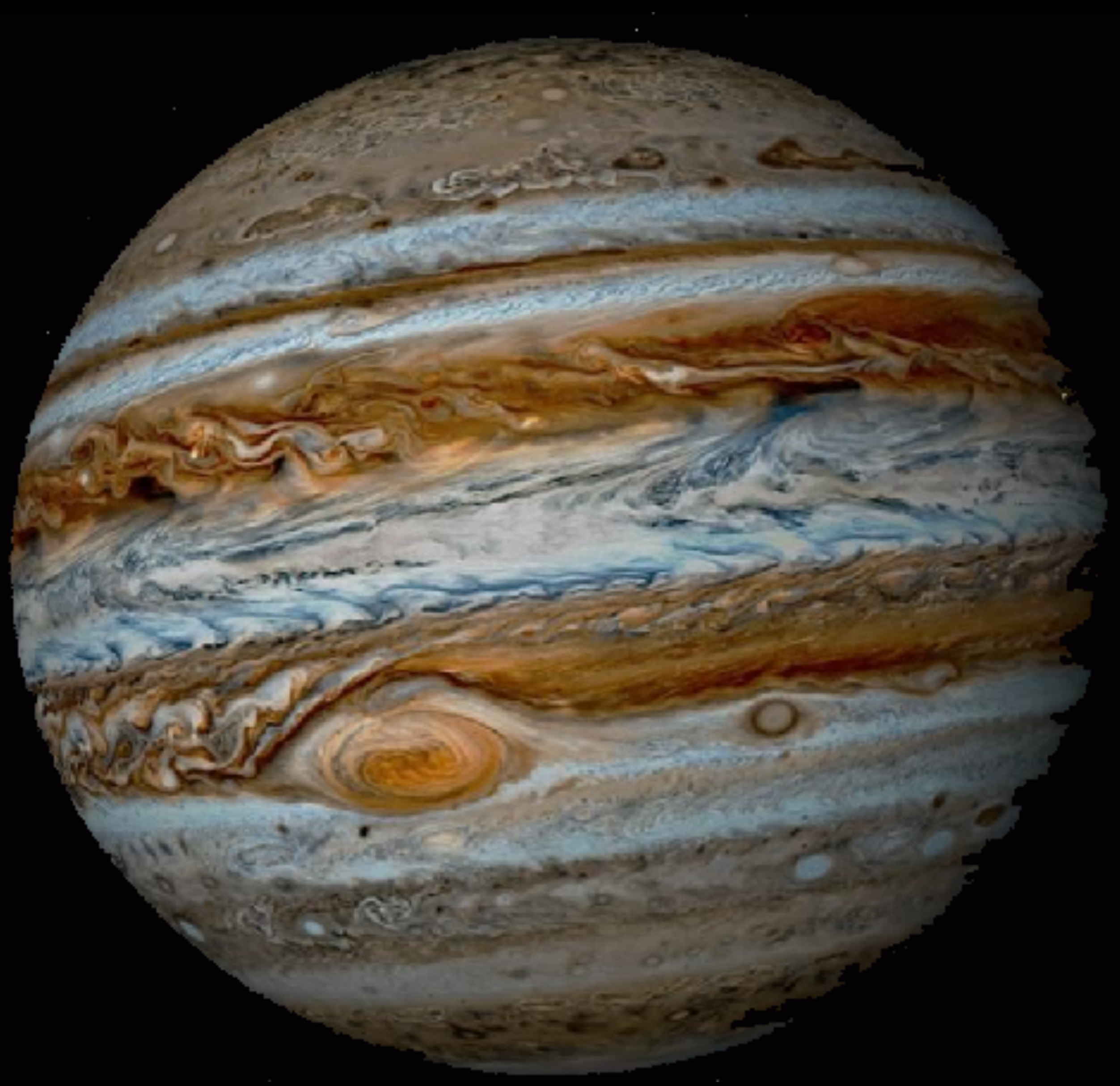
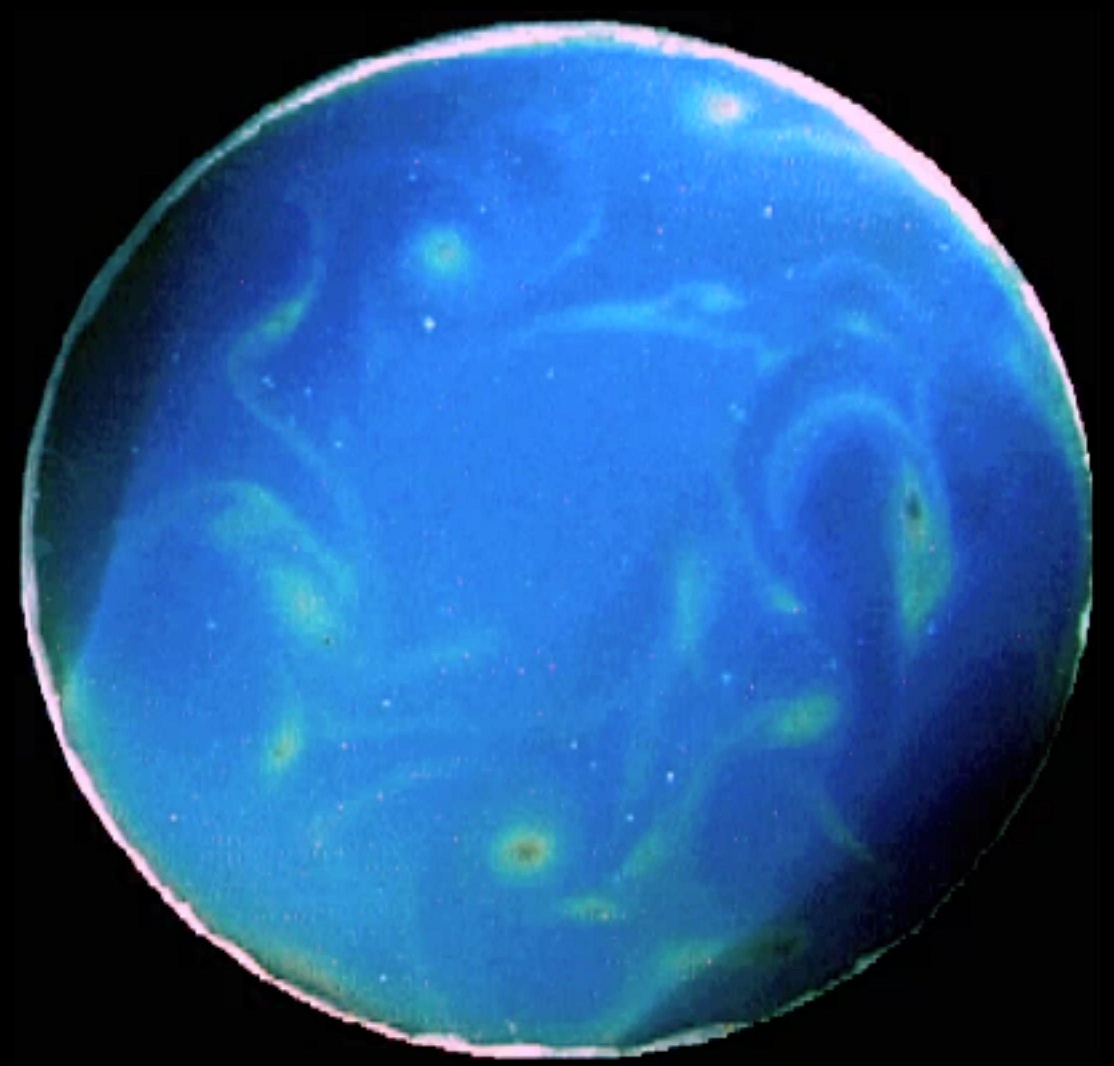
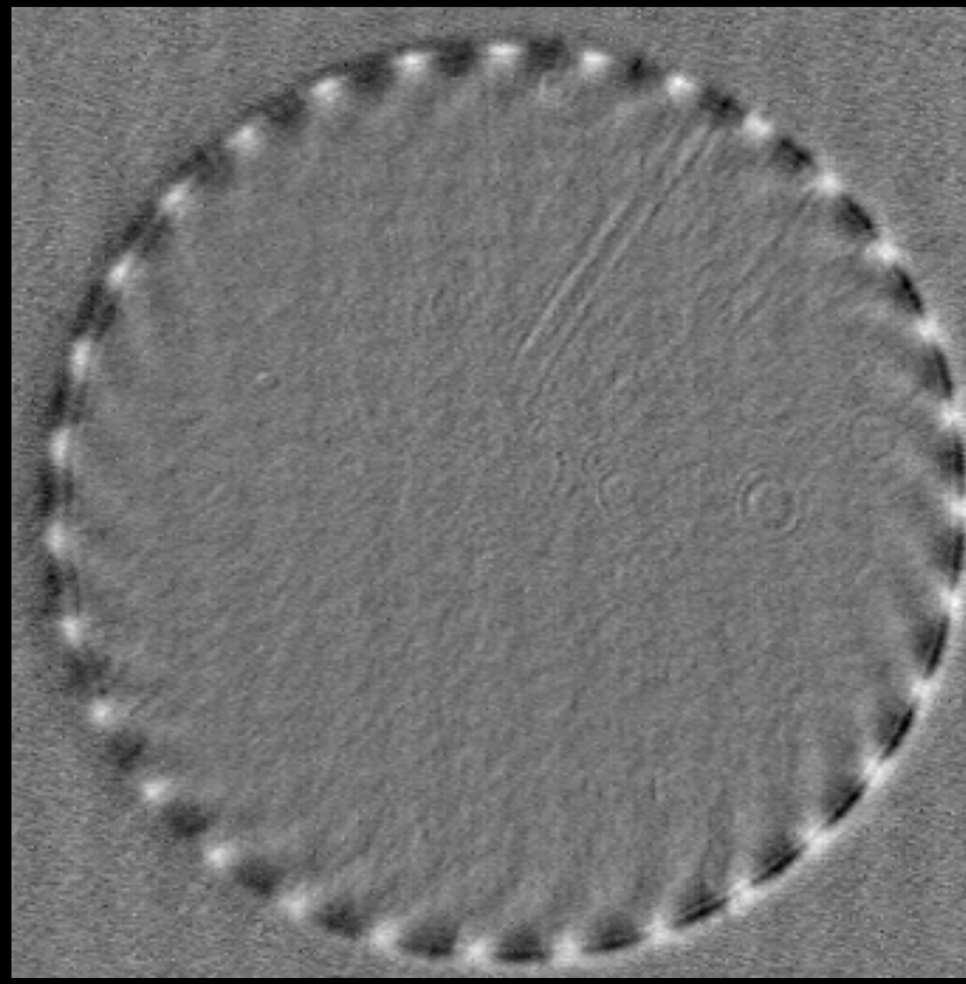
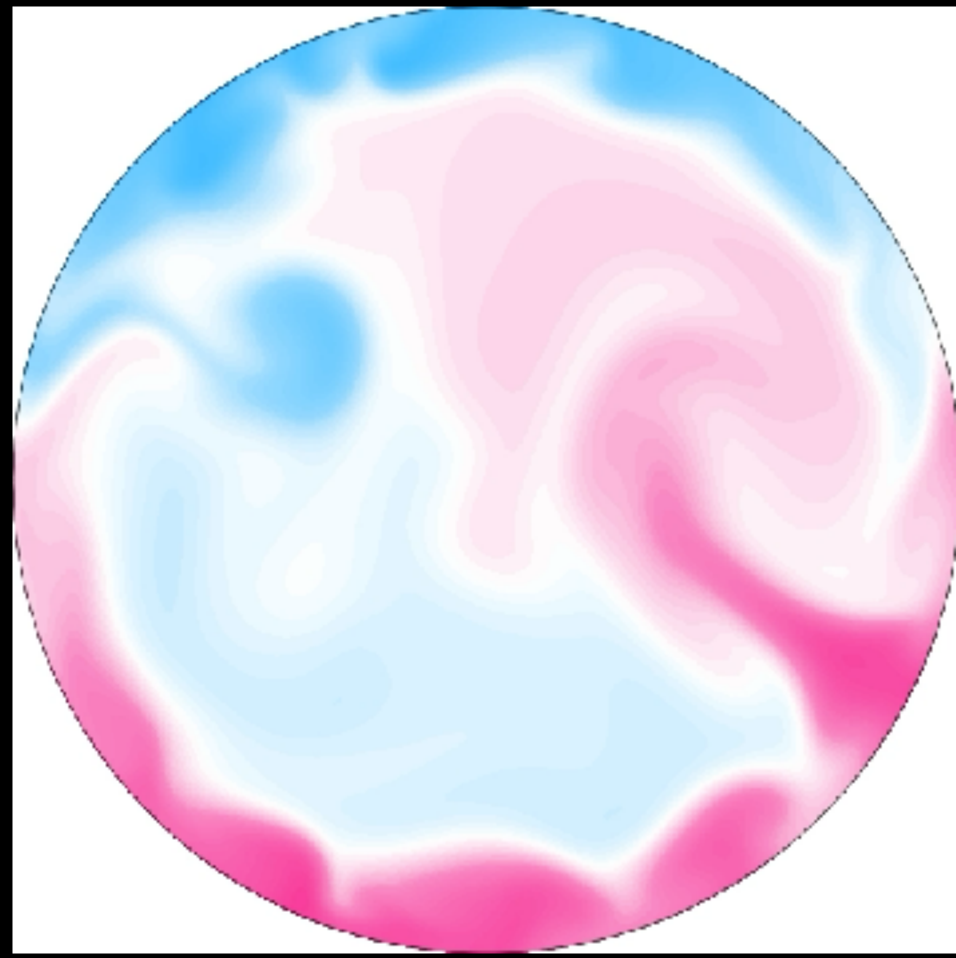




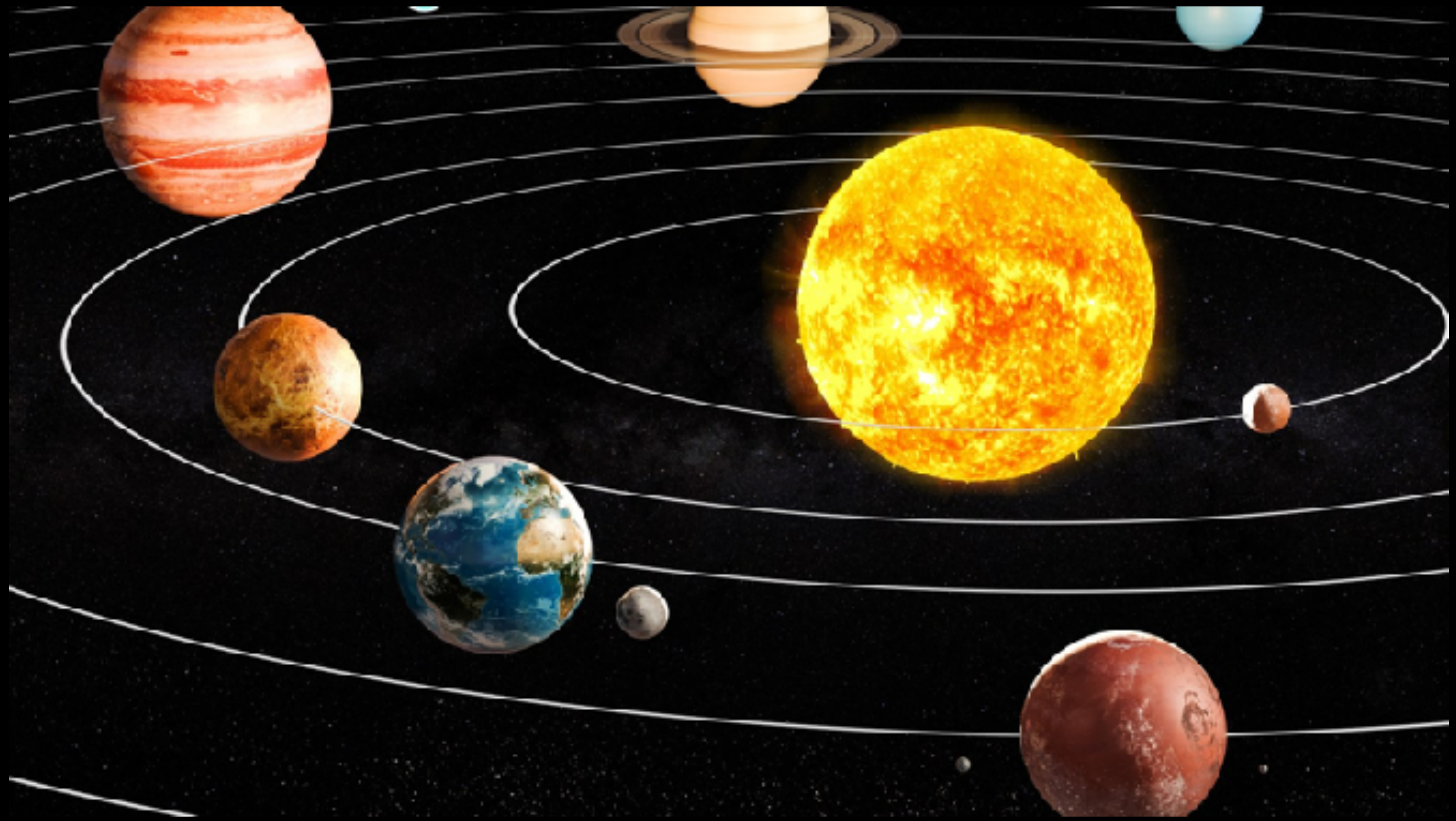


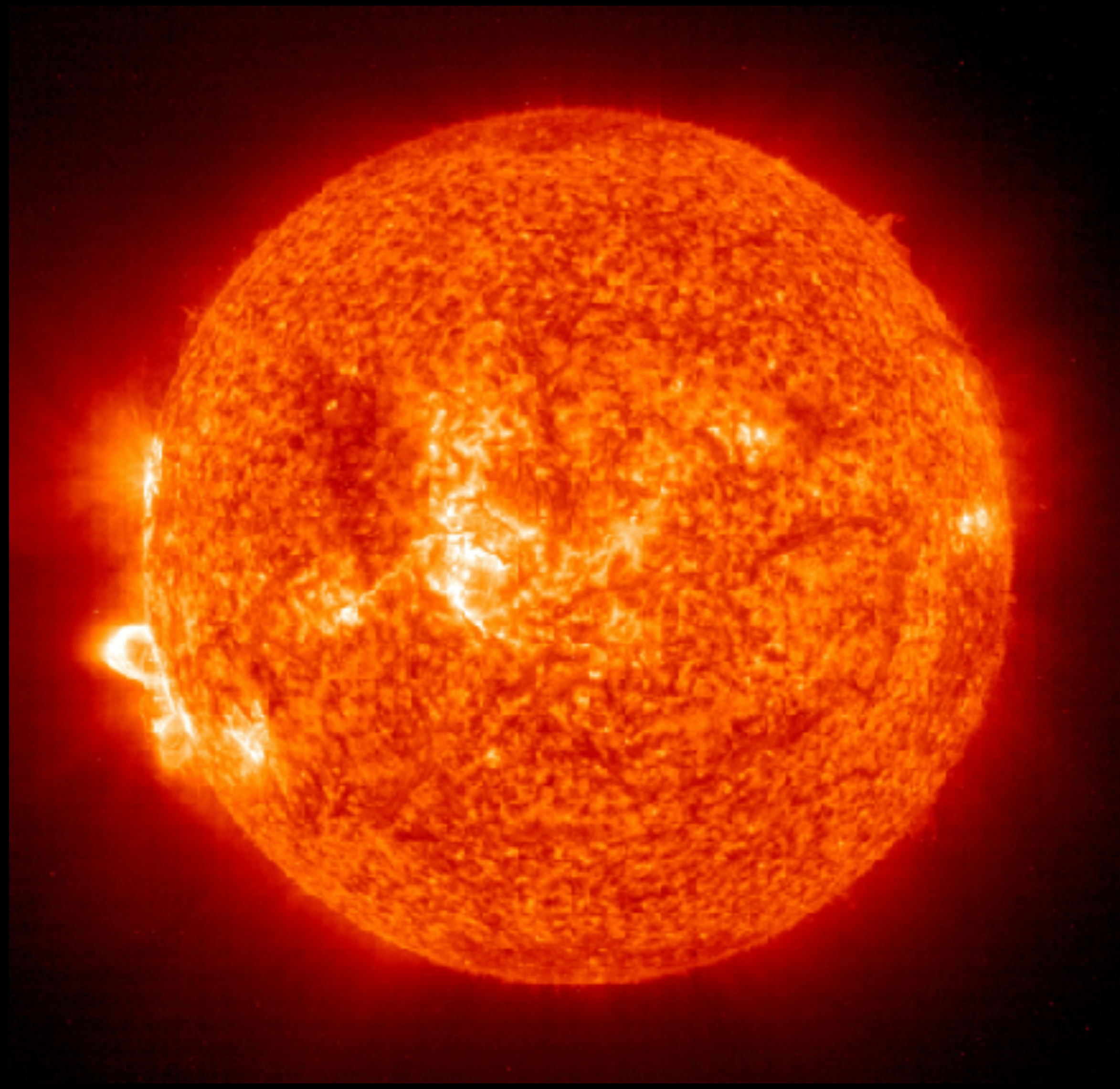


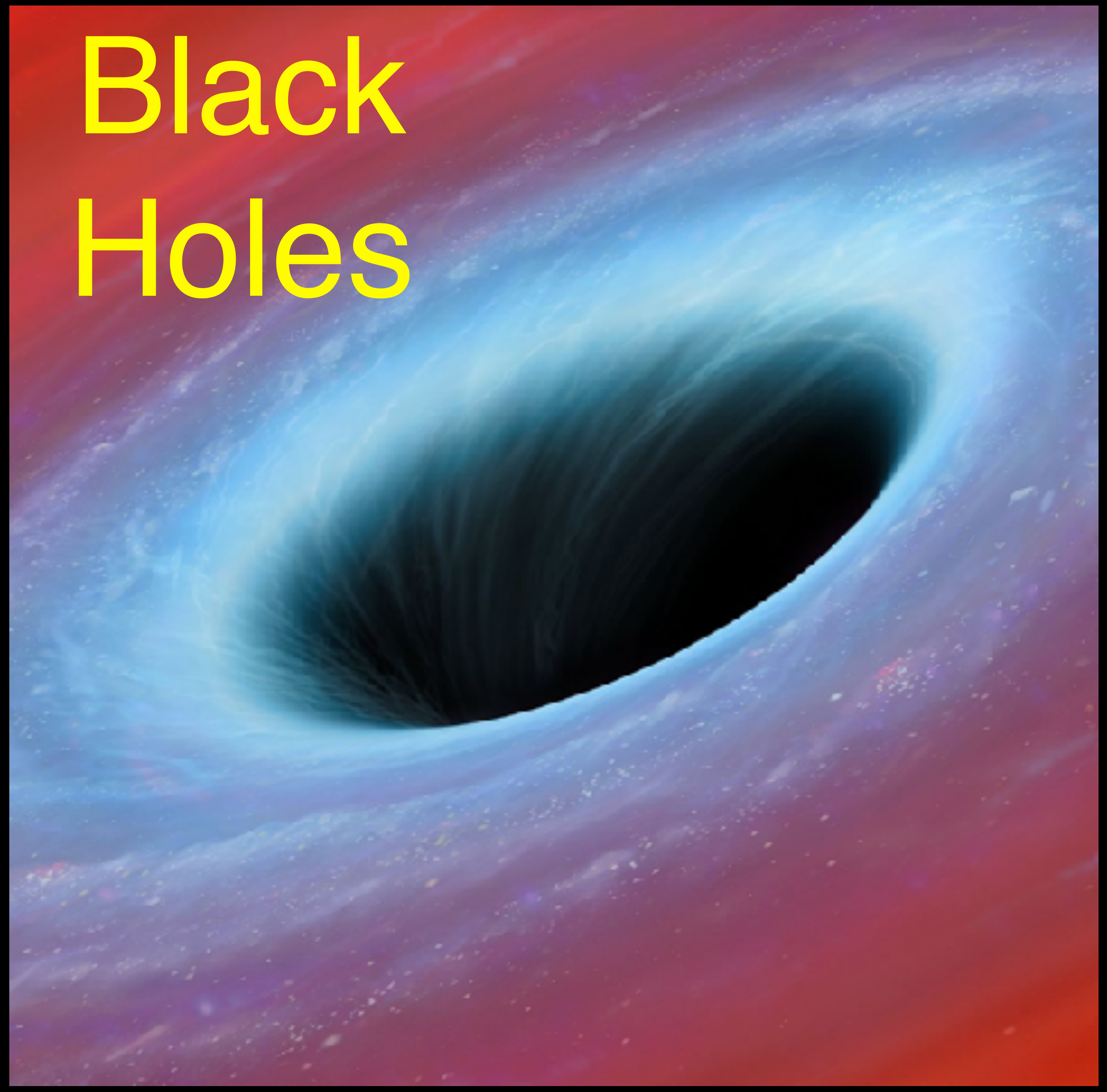




Gravity







Gravity - Stability

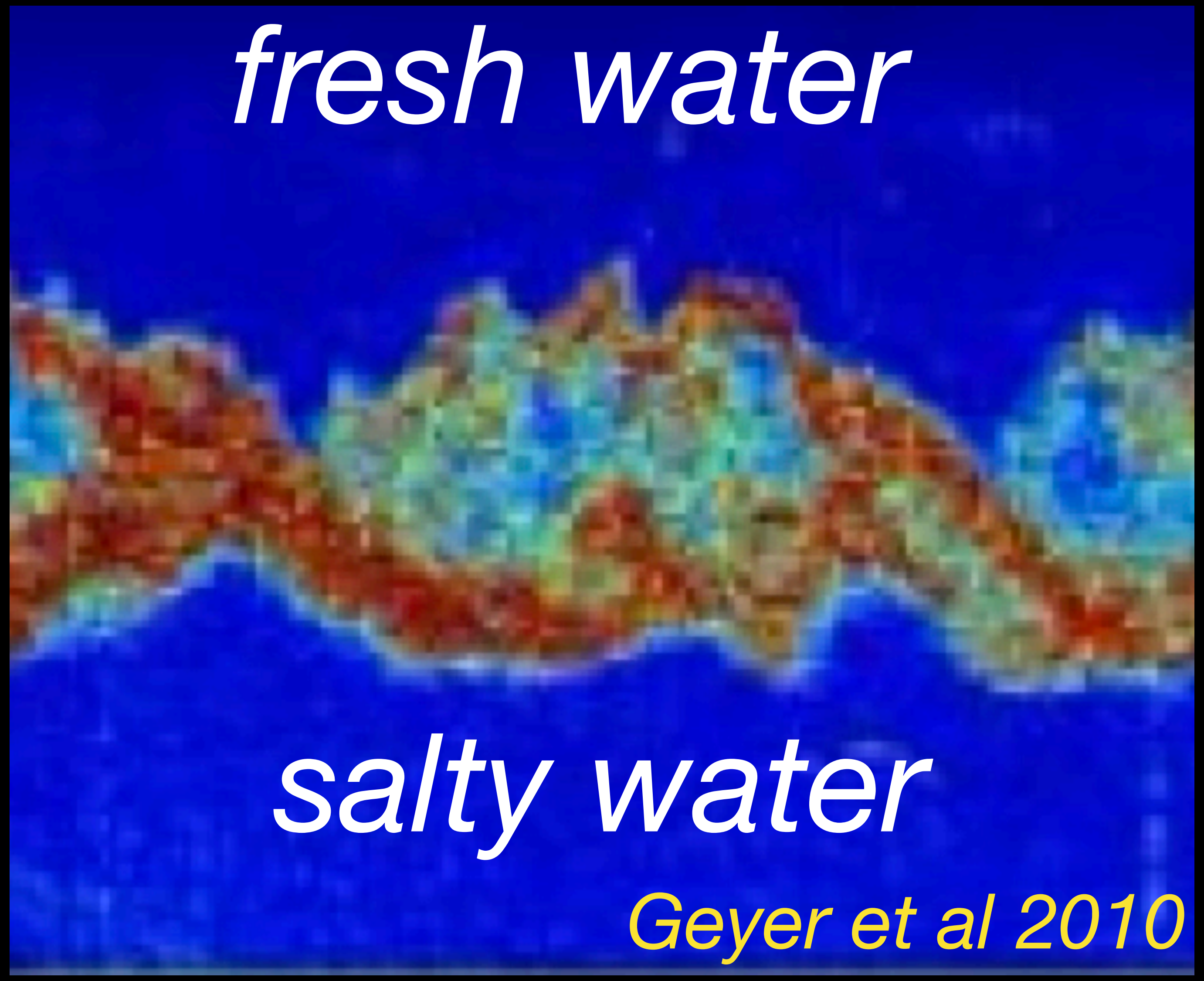
Air *less dense*

Water *more dense*

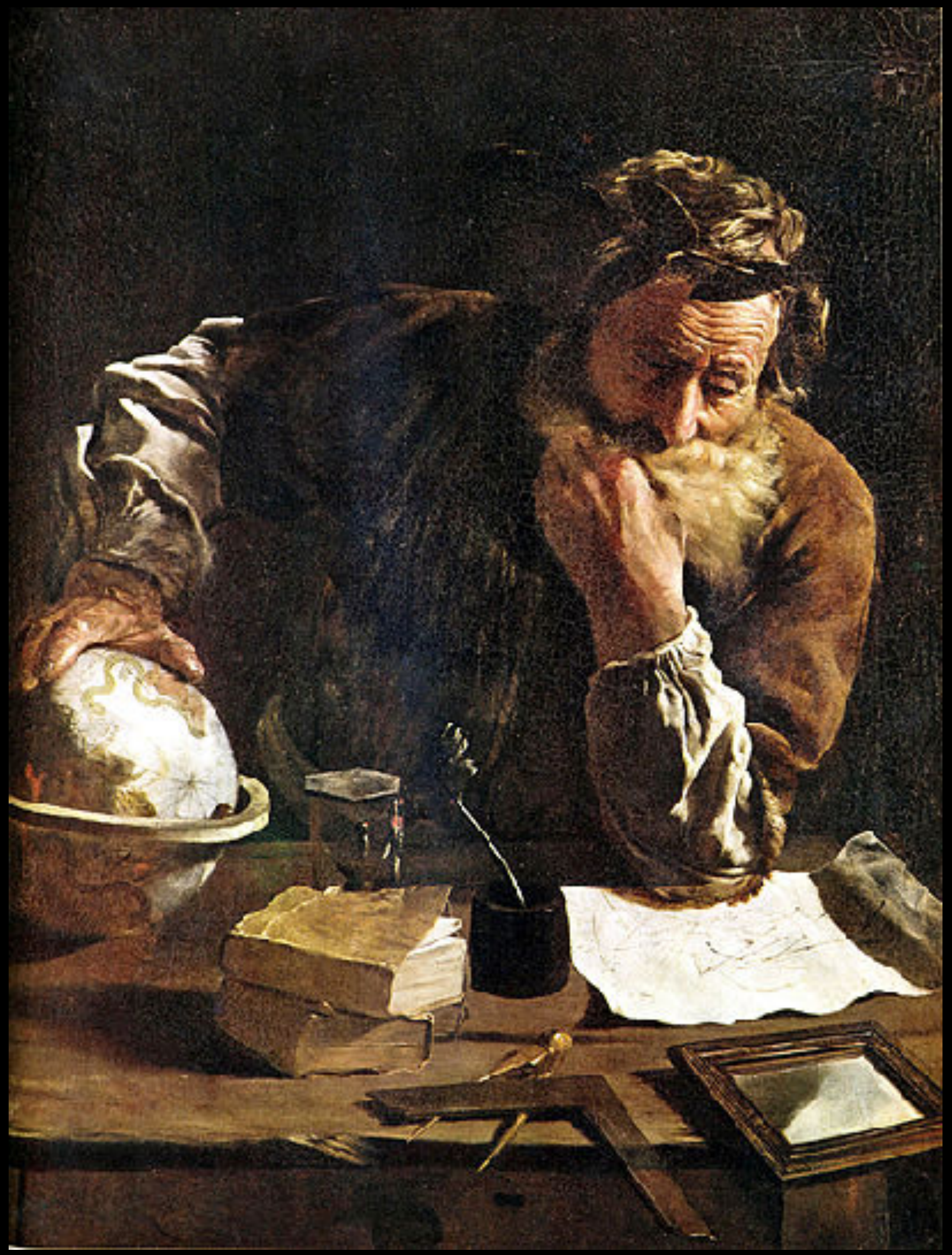
less dense

Atmosphere/Ocean

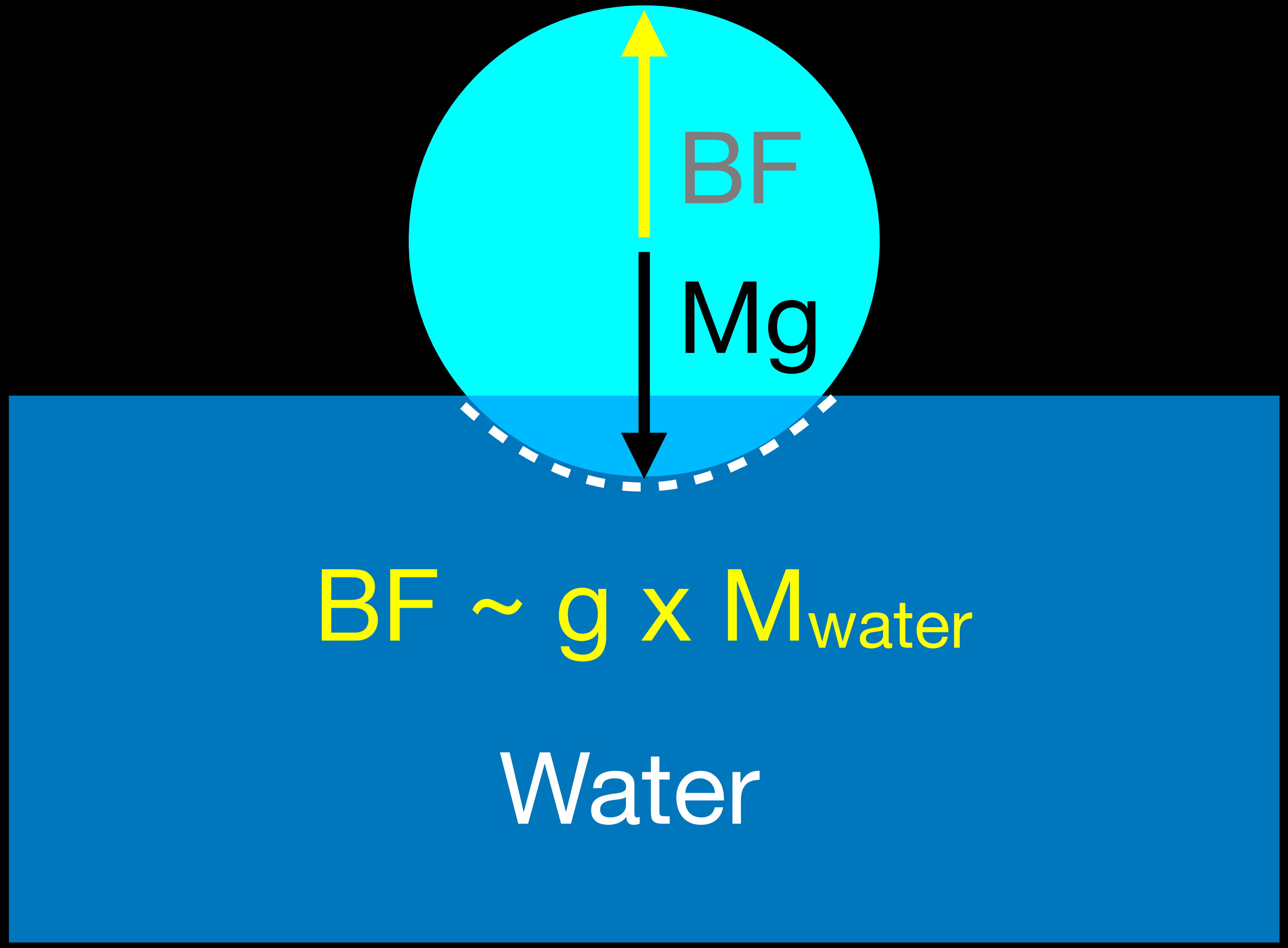
more dense

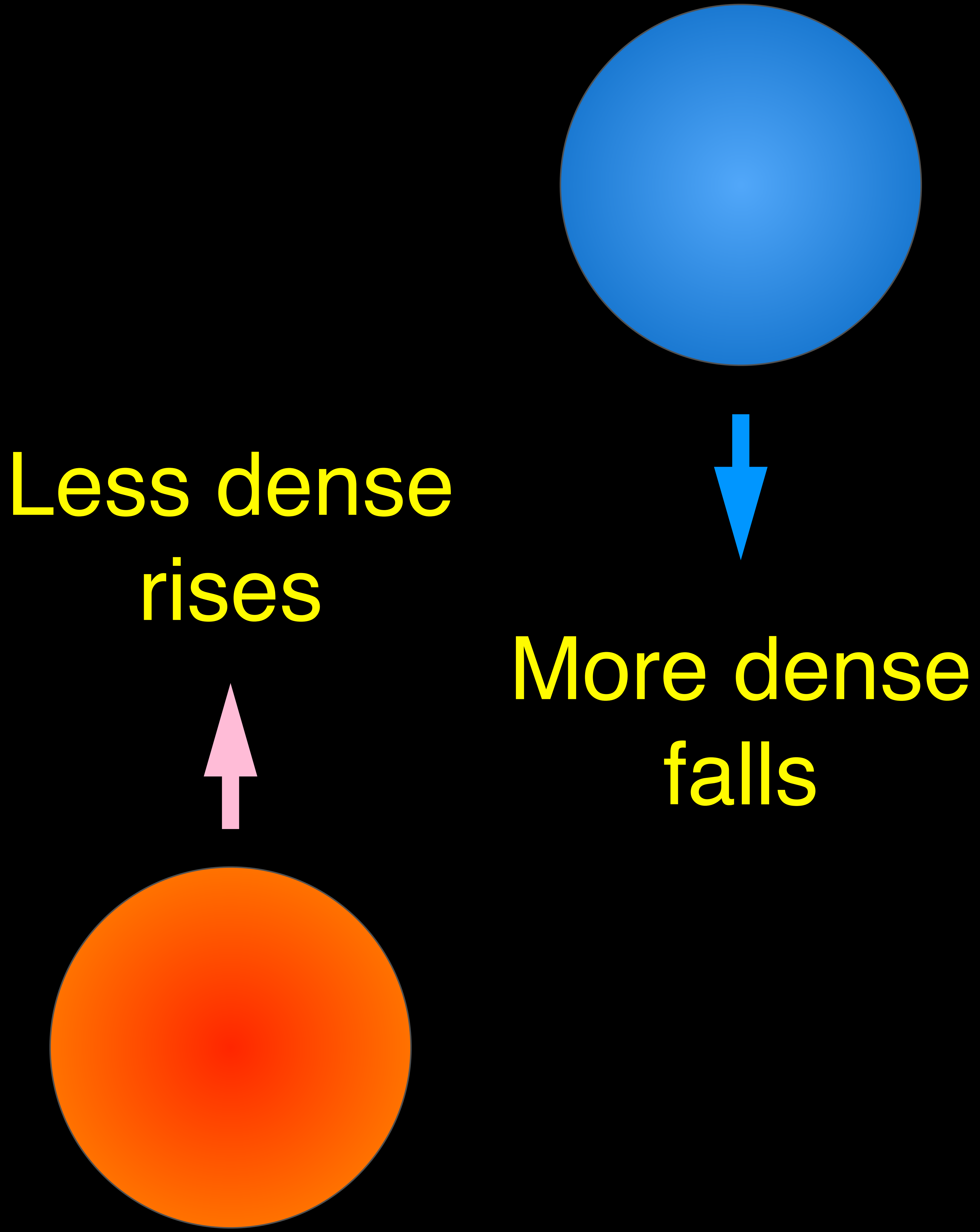


Buoyancy



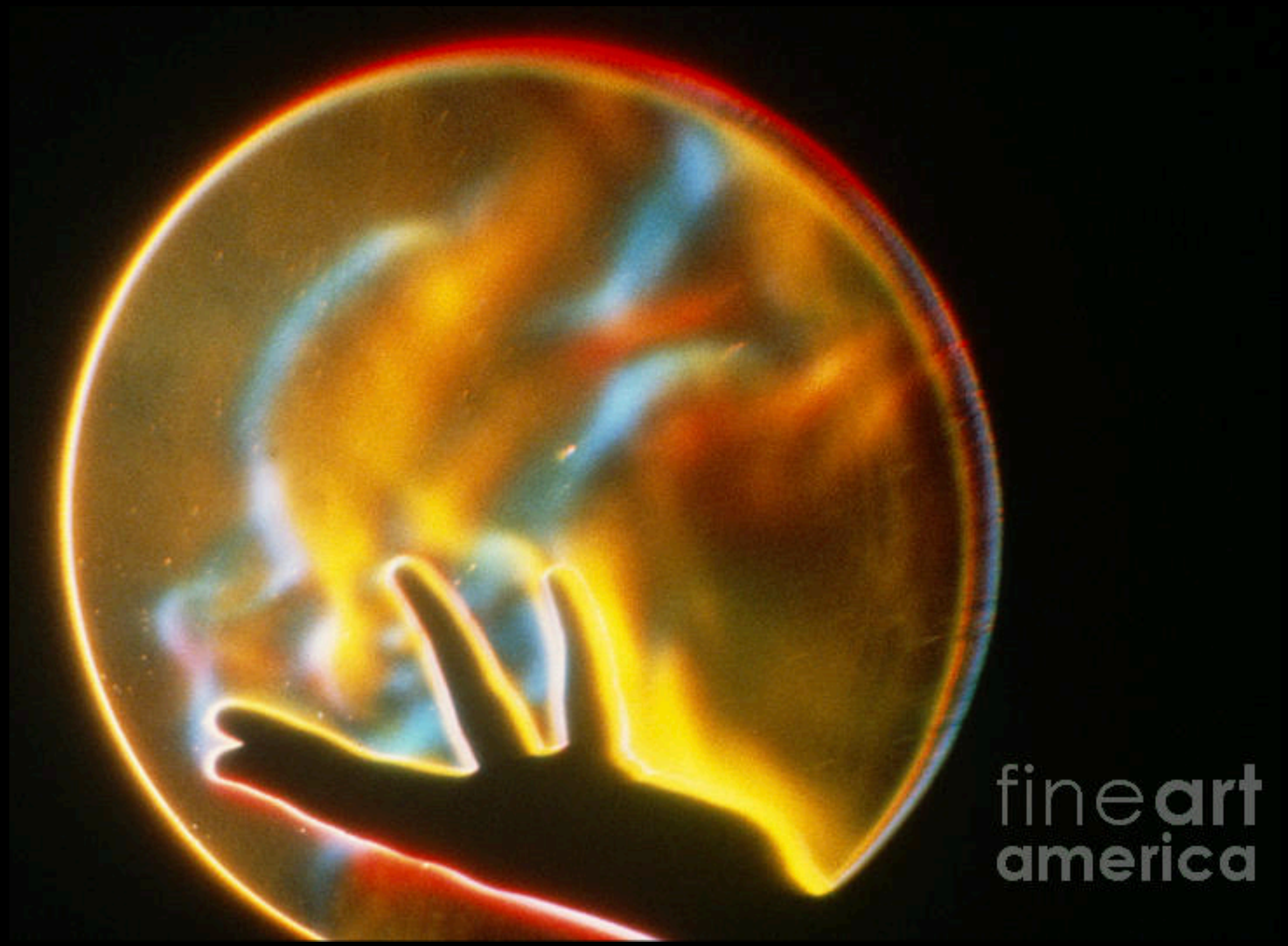
Archimedes
250 BC



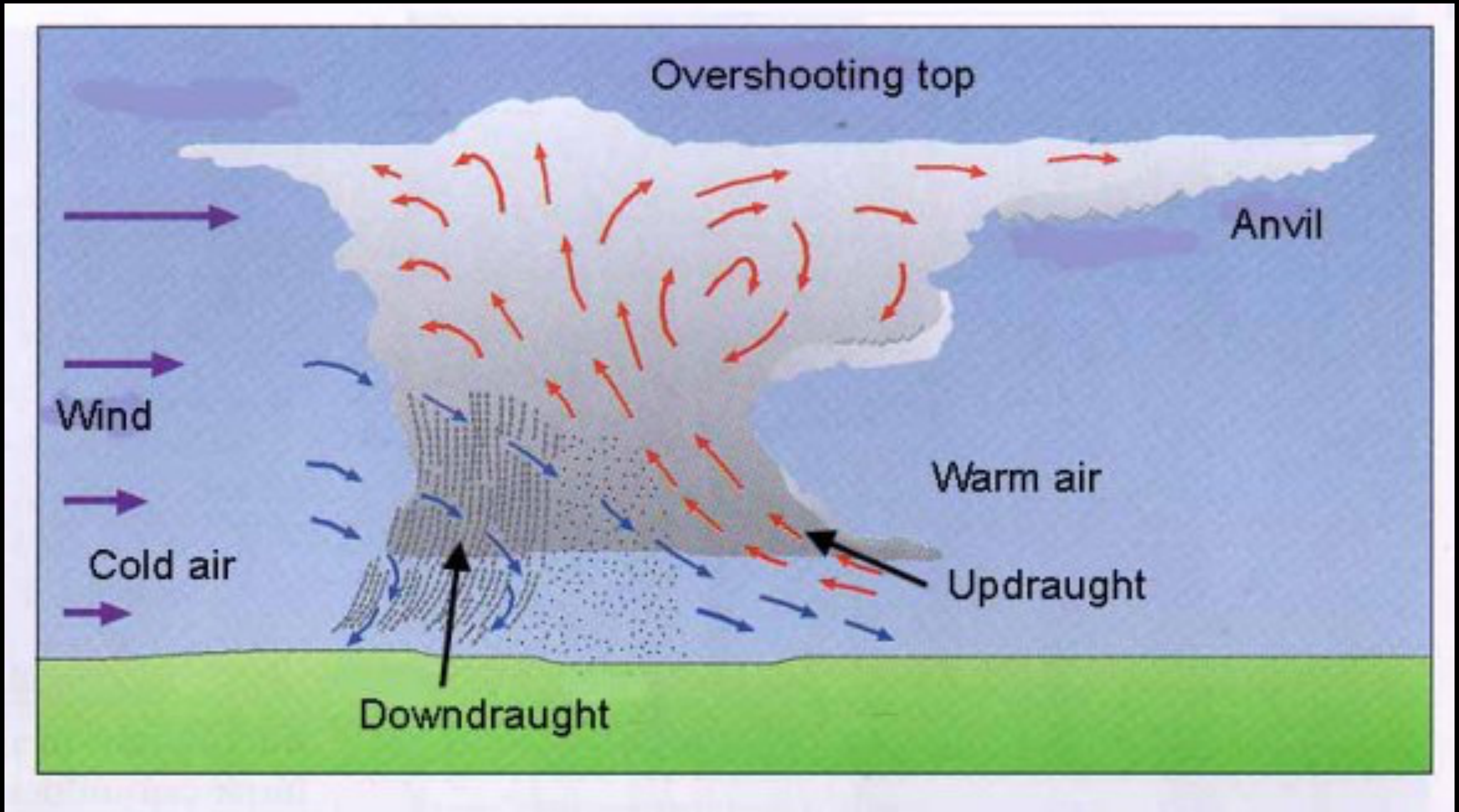


Gravity - Instability

Hot air rises

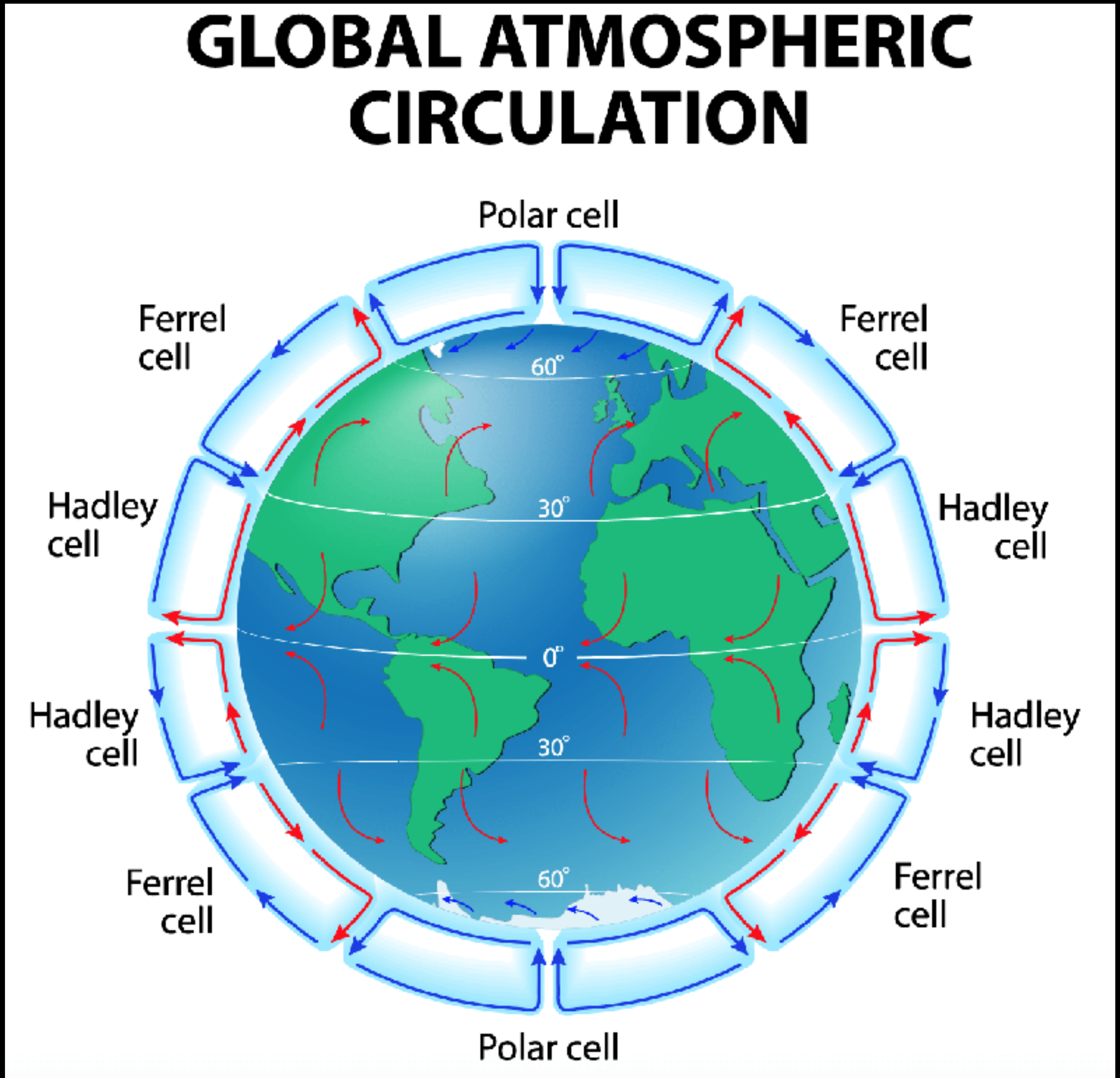


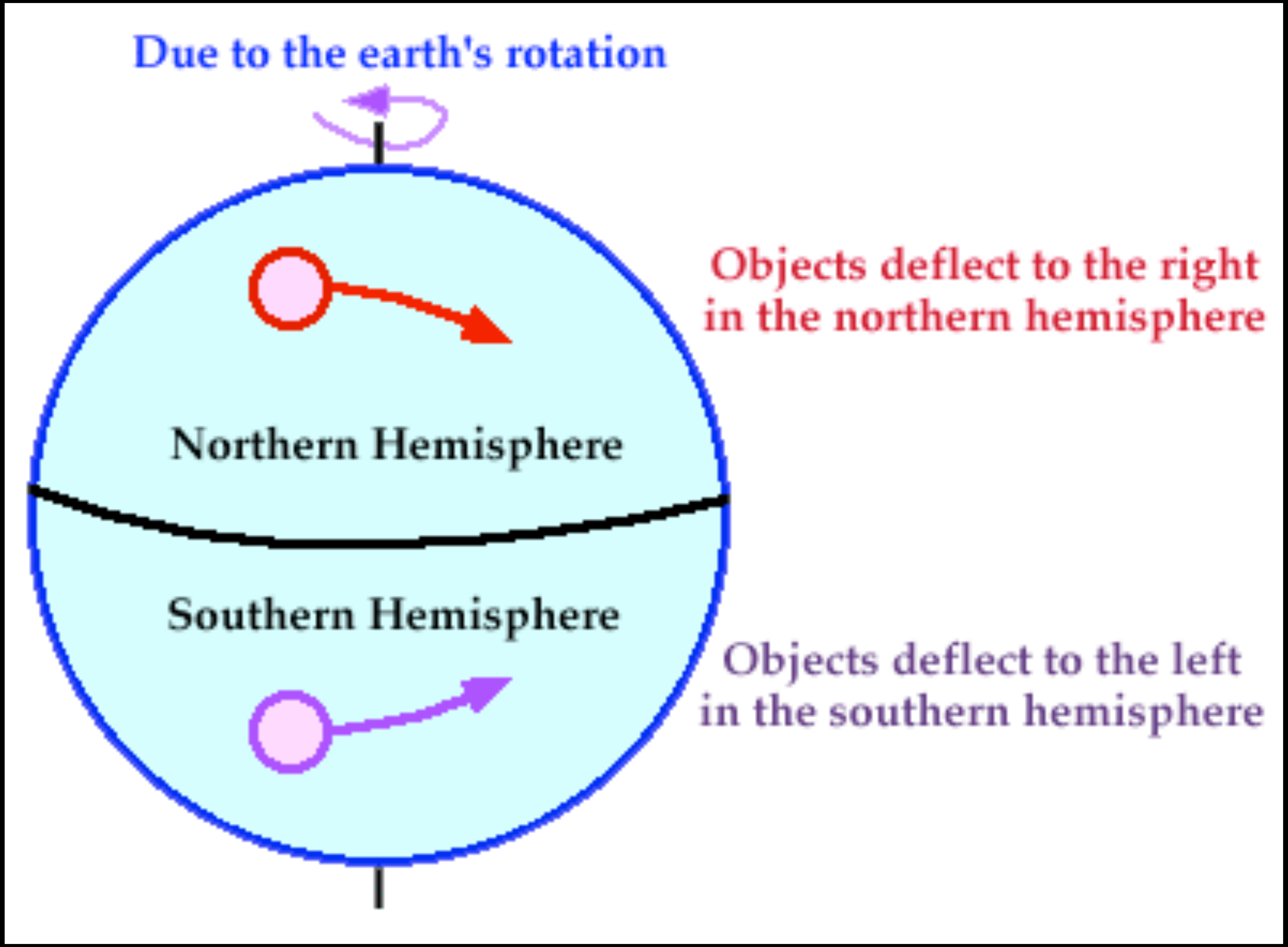
Thunderstorms



Thunderstorms: Lightning & Tornadoes

Atmospheric Circulation



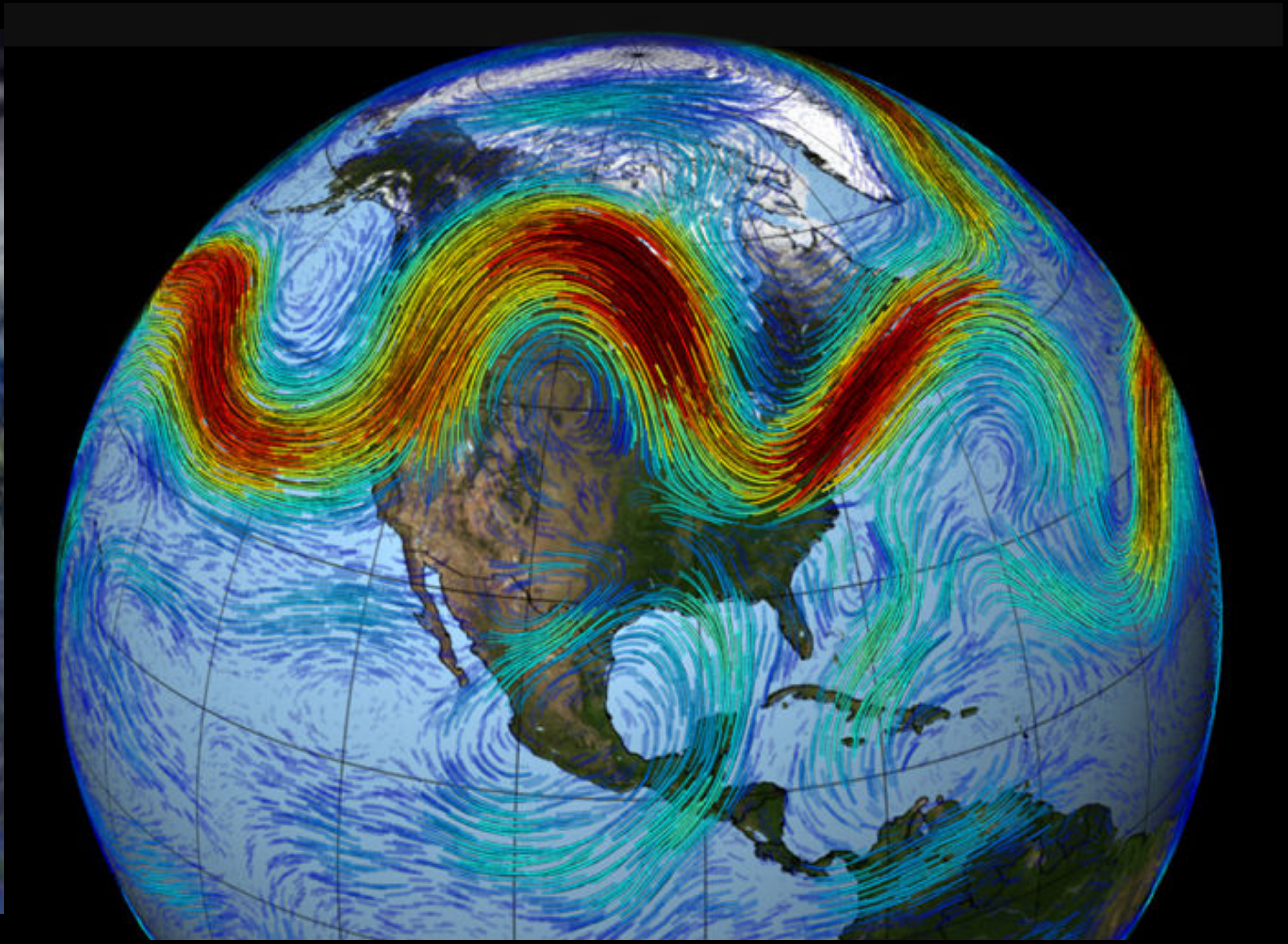


Coriolis Force: Effect of Rotation

Image/Text/Data from the University of Illinois WW2010 Project."

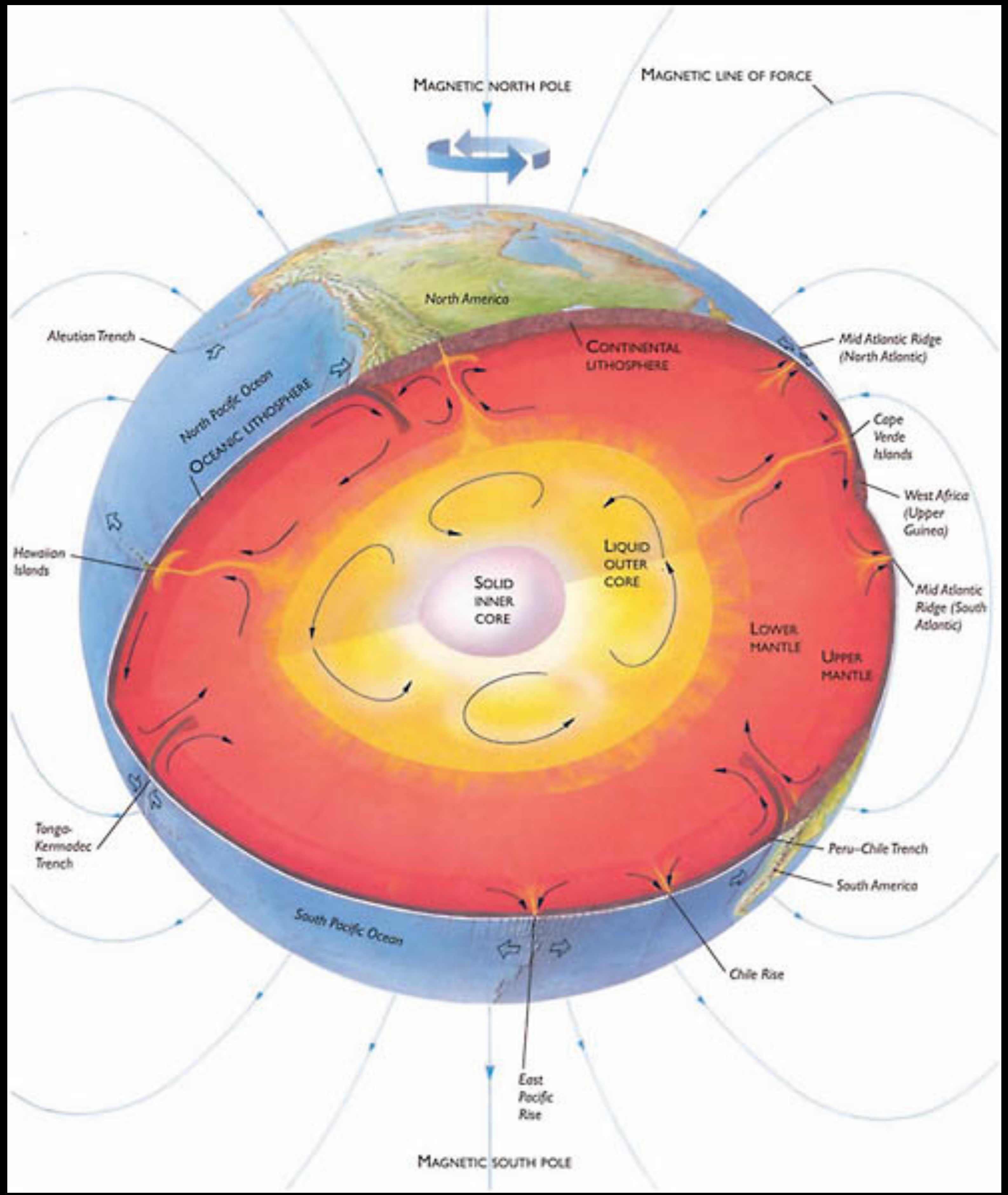
Gravity + Instability + Rotation

Weather & Climate



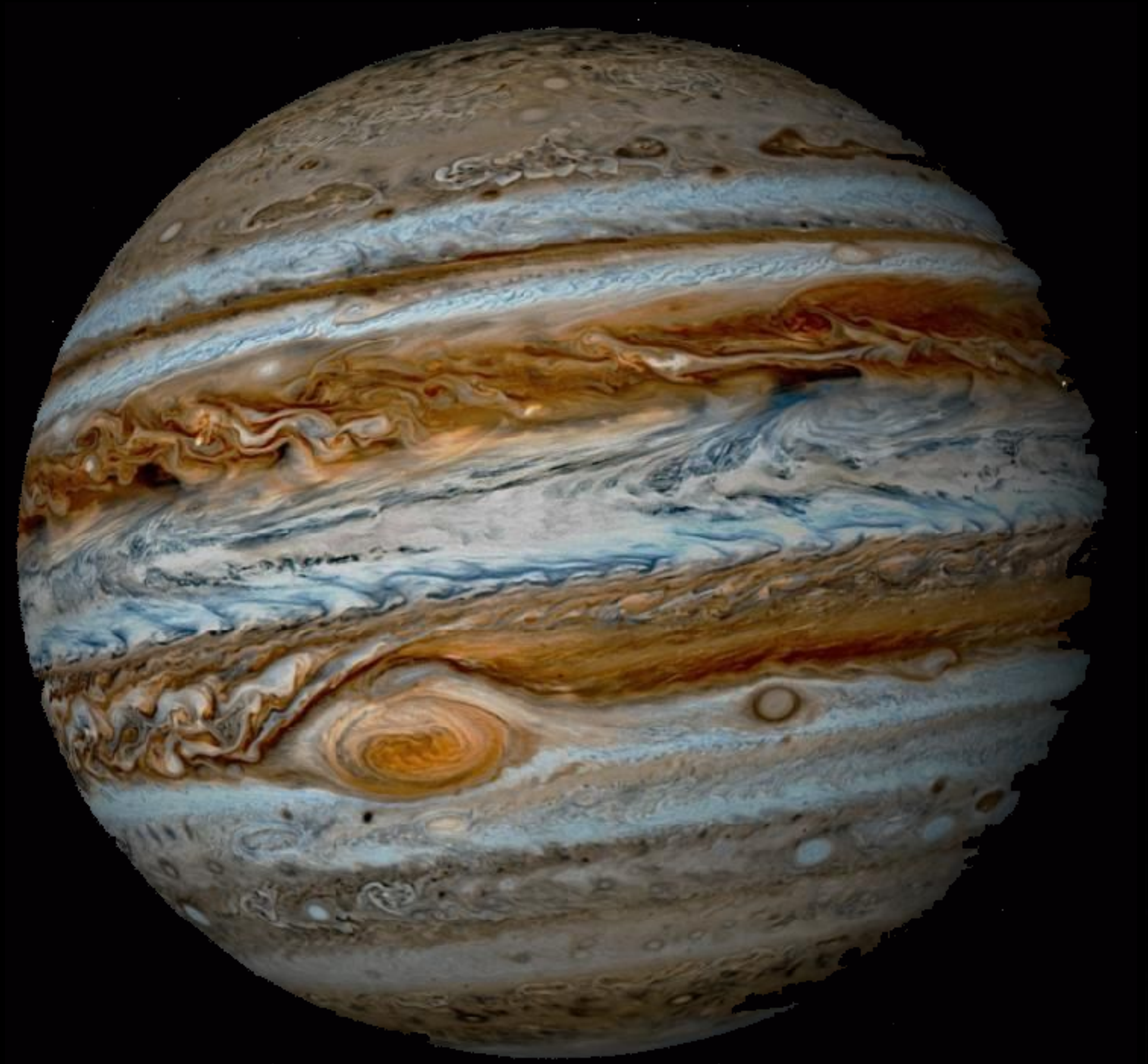
Other

Earth Interior



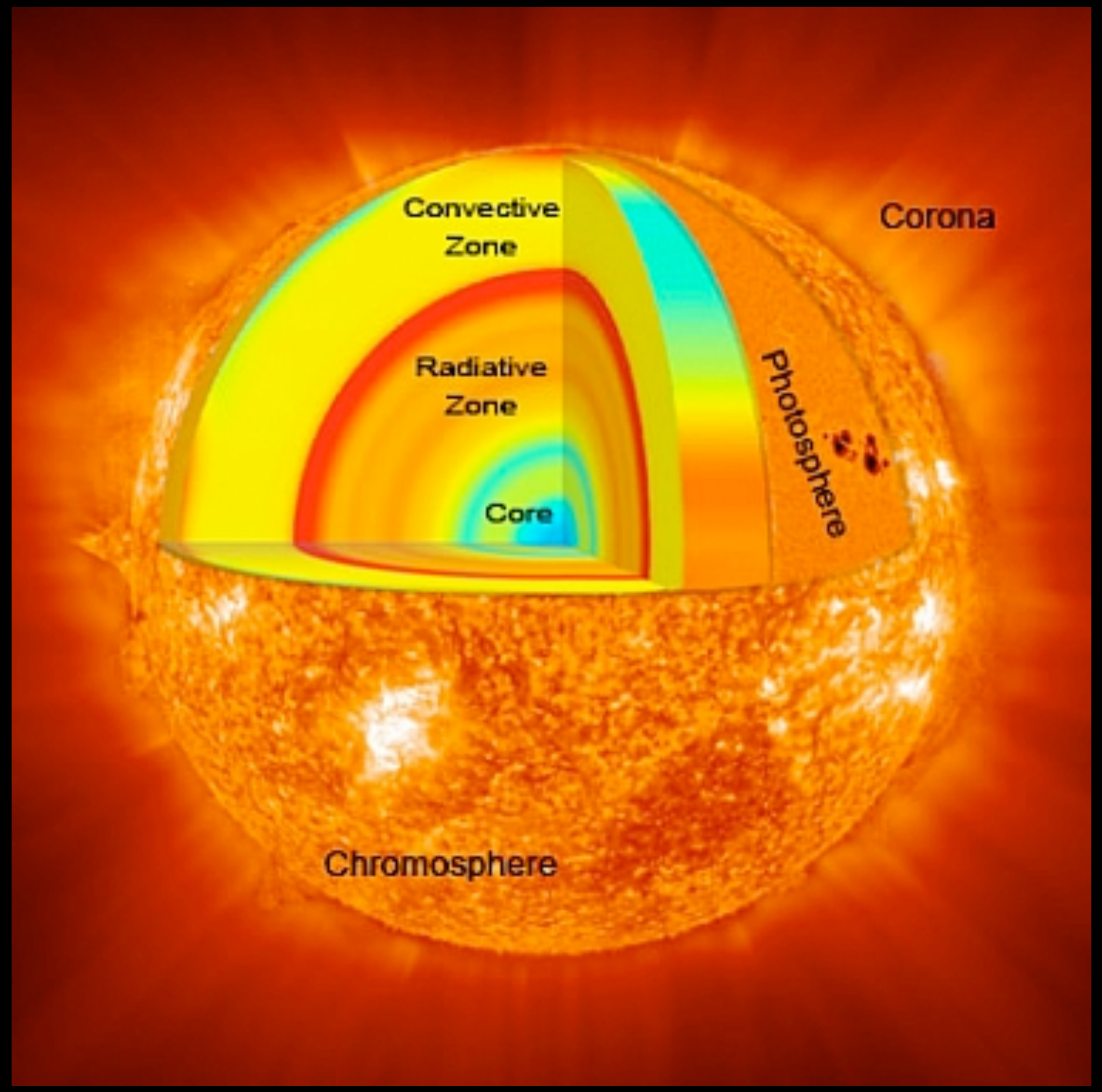
Magnetic field
Earthquakes
Volcanos

Planetary Atmospheres



Jupiter

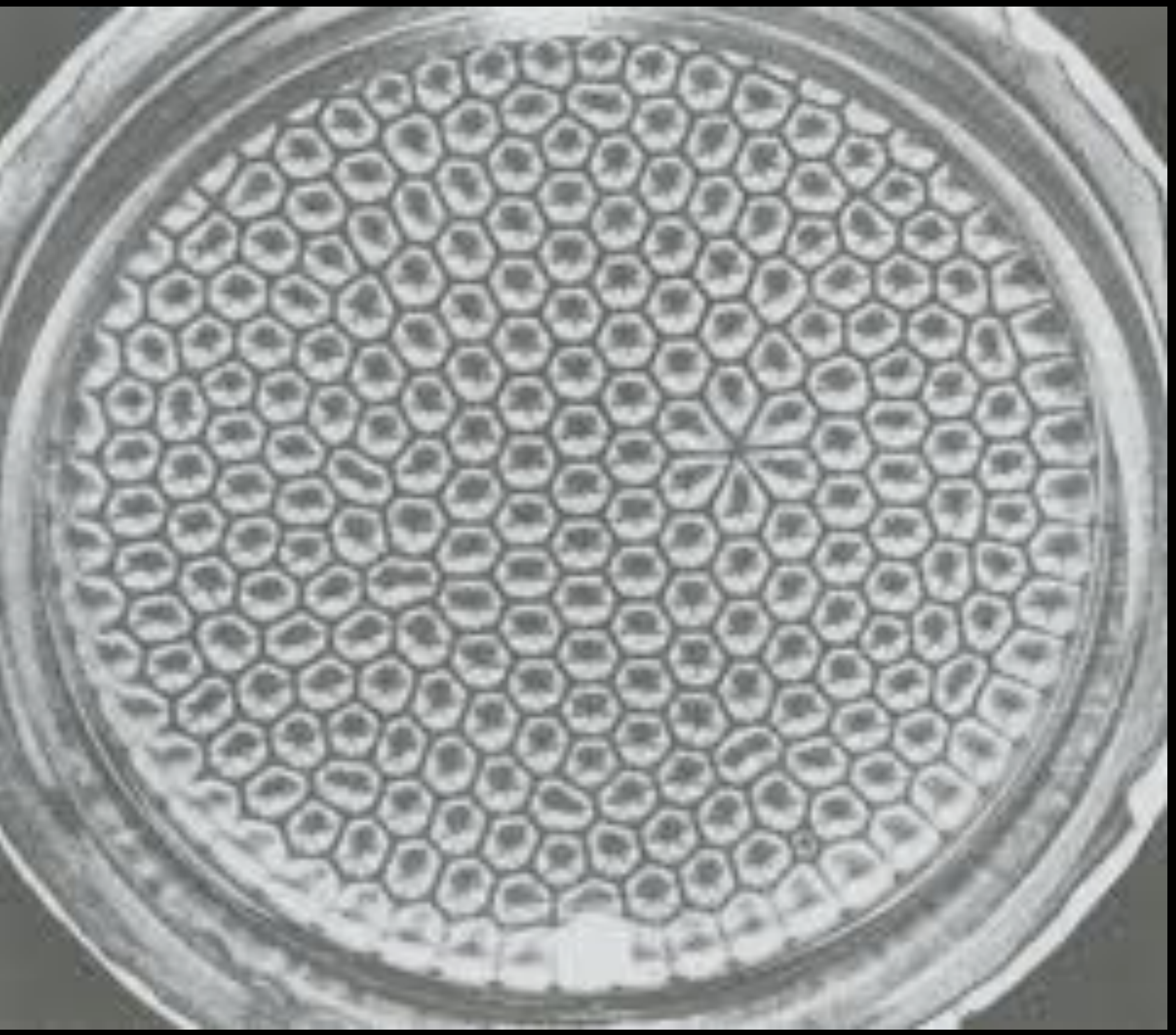
Sun

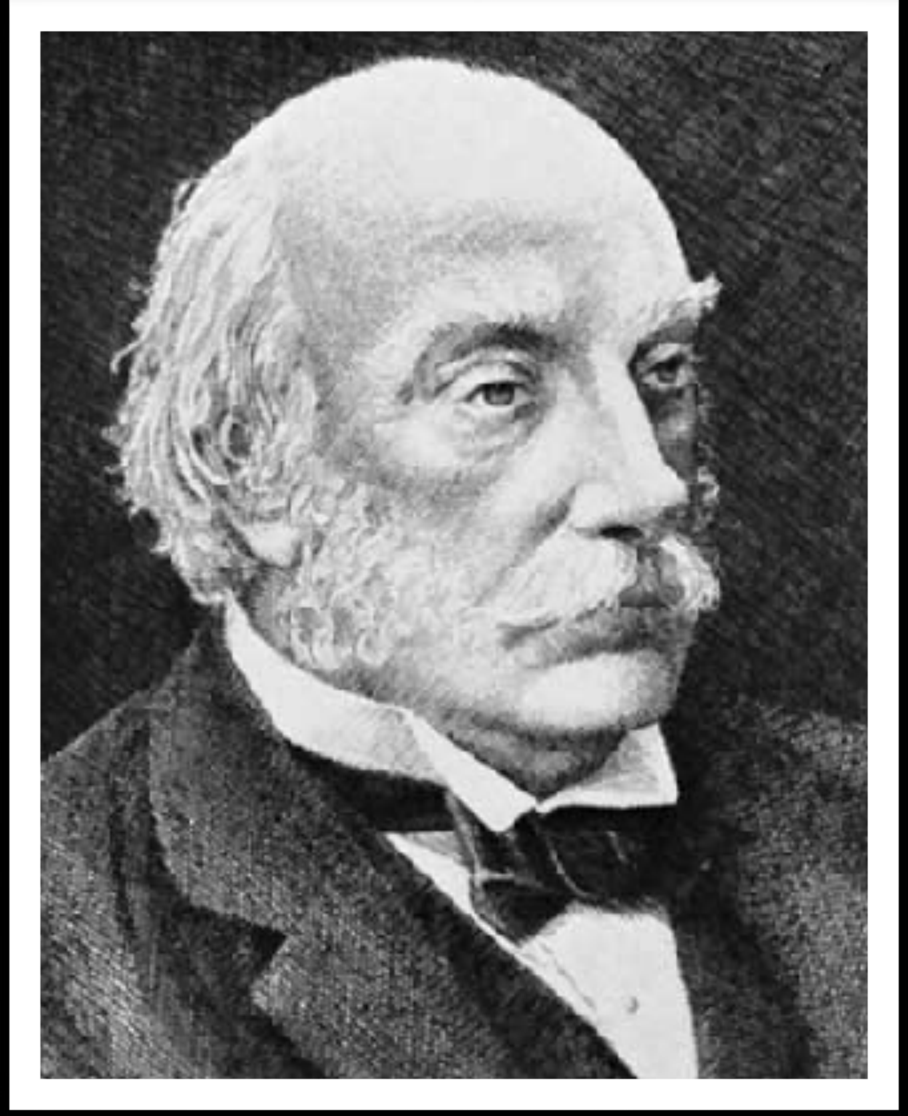


Supernova

Model Experiment - Beginnings

Henri Bénard
1900





Lord Rayleigh
1916

THE

LONDON, EDINBURGH, AND DUBLIN

PHILOSOPHICAL MAGAZINE

AND

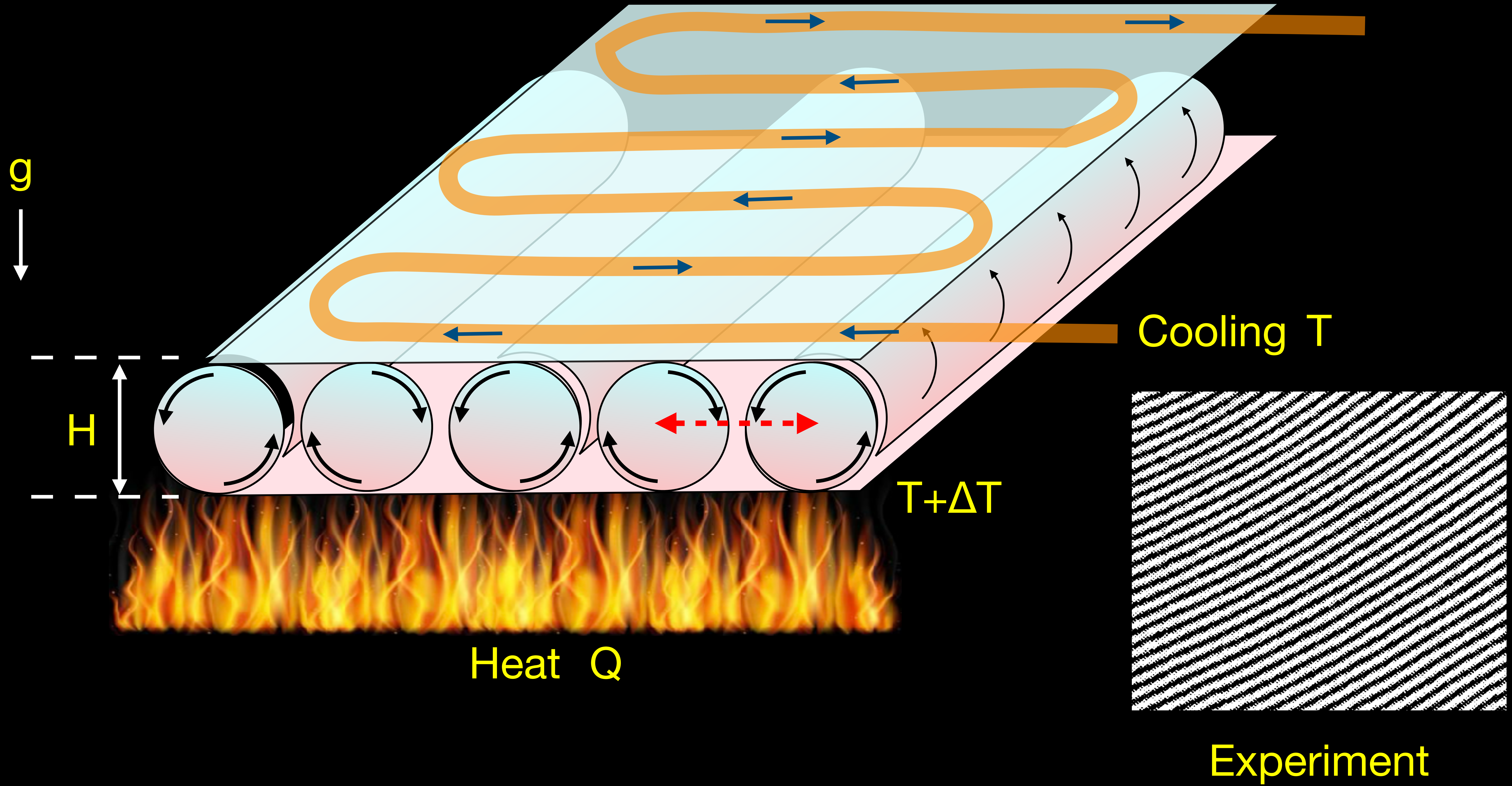
JOURNAL OF SCIENCE.

[SIXTH SERIES.]

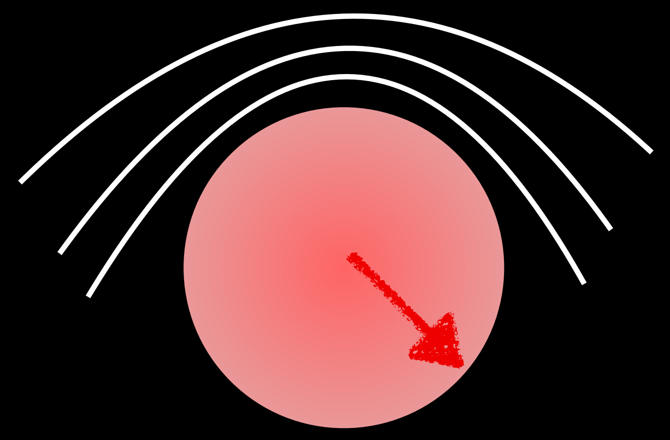
DECEMBER 1916.

LIX. *On Convection Currents in a Horizontal Layer of Fluid, when the Higher Temperature is on the Under Side.* By Lord RAYLEIGH, *O.M., F.R.S.**

Rayleigh-Bénard Convection

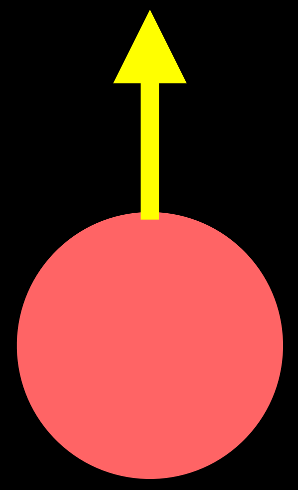


Non-Dimensional Numbers



Buoyancy resisted by drag and thermal diffusion

F ~ Δρ/ρ ~ α ΔT



$$v = \sqrt{gH\alpha\Delta T}$$

Buoyancy

Time Scales

$$t_{drag} = H^2/\nu$$

$$t_{diffusion} = H^2/\kappa$$

$$t_{buoyancy} = H/v = \sqrt{H/(g\alpha\Delta T)}$$

$$Ra = (t_{drag}\ t_{diffusion})/t^2_{buoyancy} = \frac{g\alpha\Delta T H^3}{\nu\kappa}$$

$Ra < Ra_c \approx 1700$ No motion

$Ra > Ra_c$ motion

Heat Transport - Nusselt Number

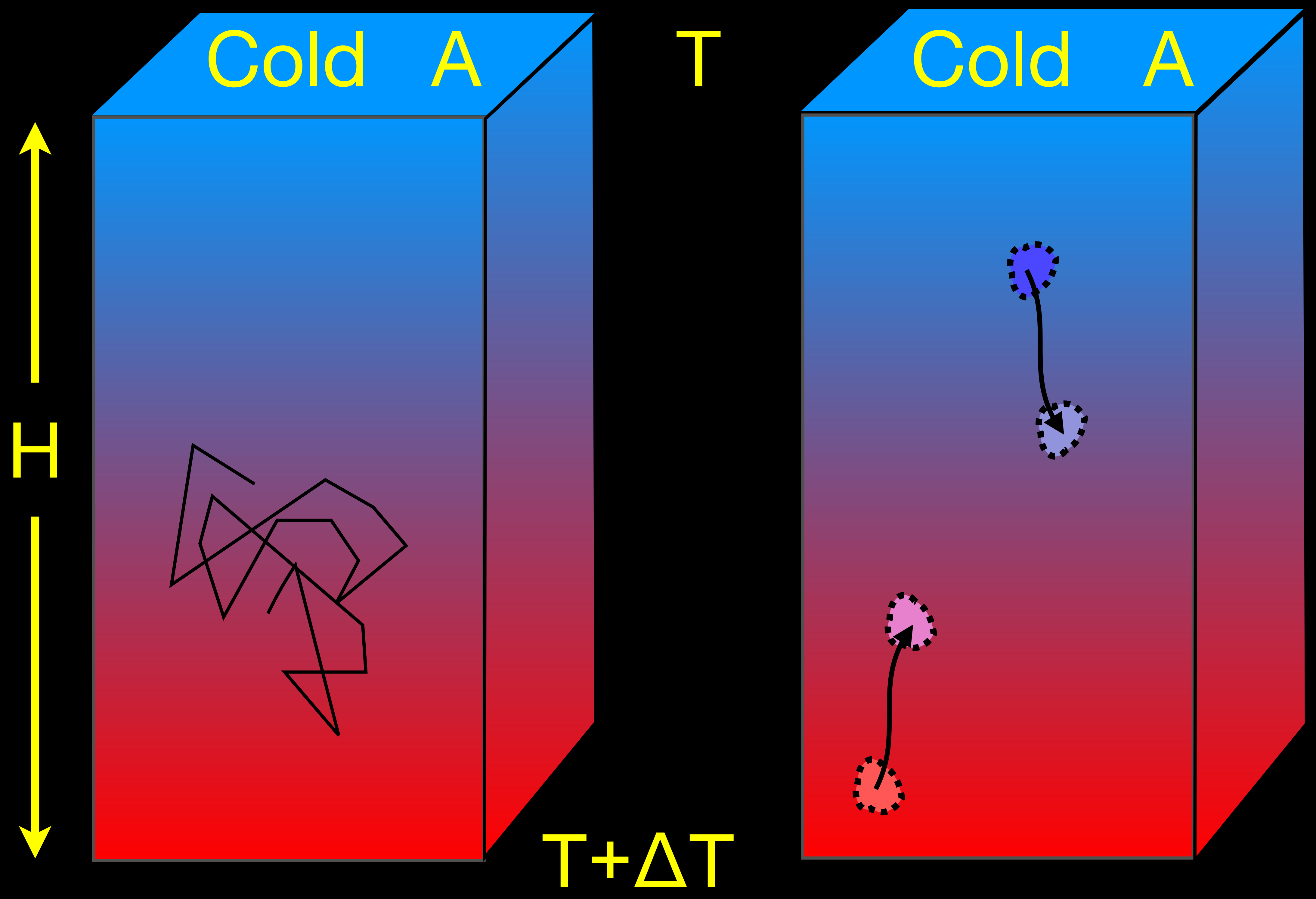
Fourier

1822

Théorie analytique de la chaleur

Solid

Fluid



Hot

Hot

slow

faster

no motion

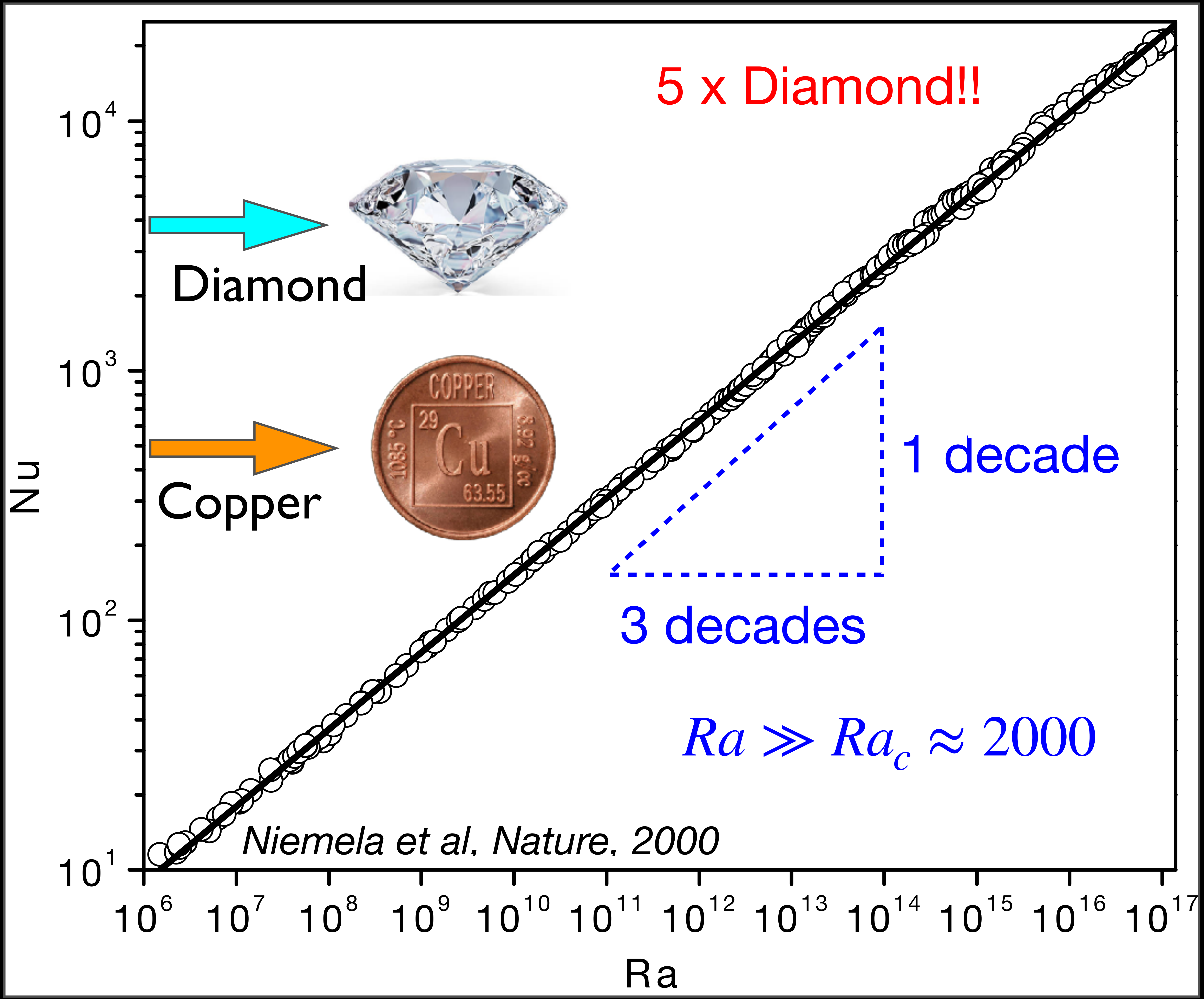
$$Q_{solid} = k_{diffusion} \frac{Area}{height} \Delta T$$

motion

$$Q_{fluid} = k_{advection} \frac{Area}{height} \Delta T$$

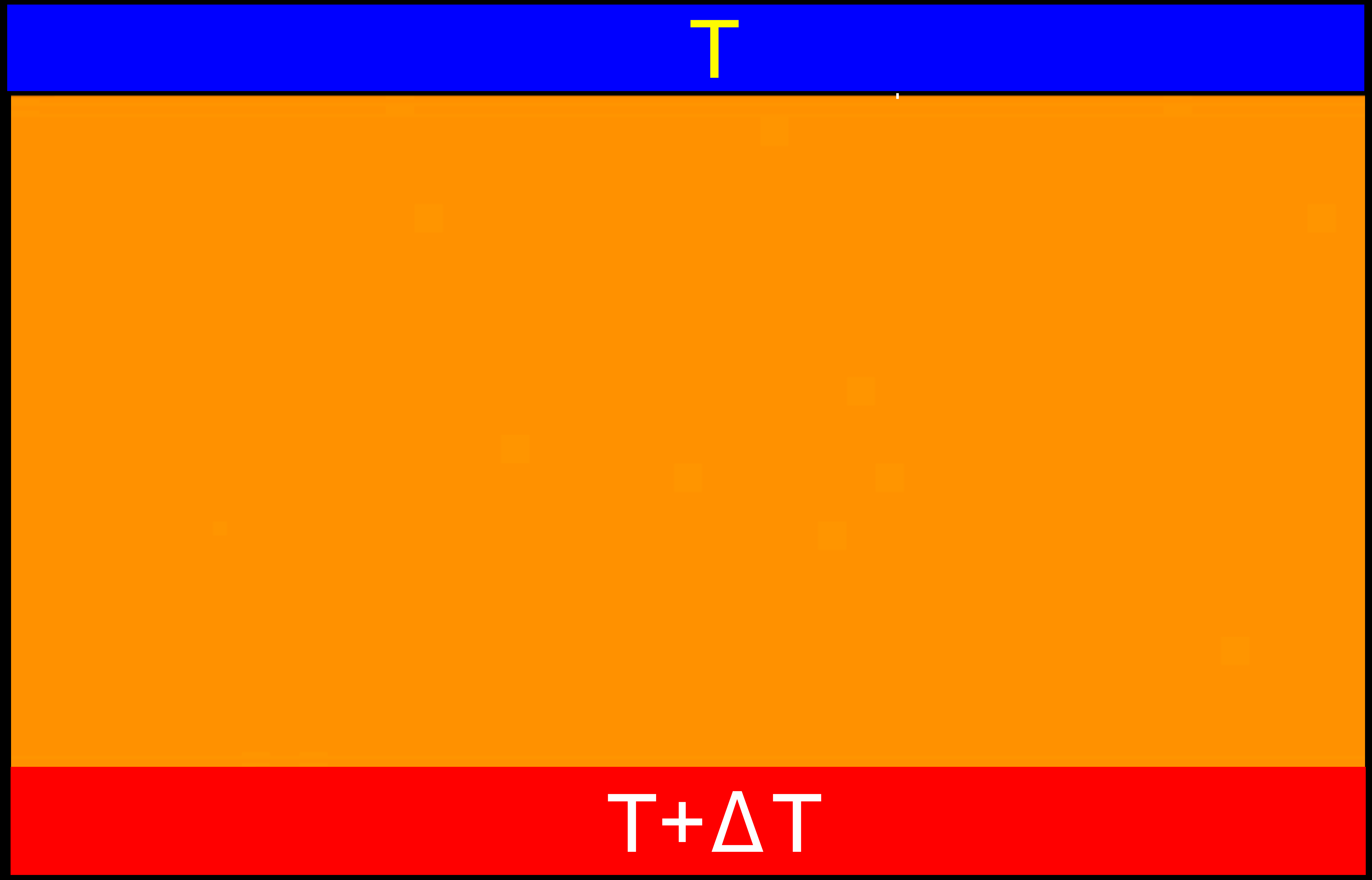
$$Nu = \frac{k_{advection}}{k_{diffusion}} \geq 1$$

Heat Transport - Convection rocks!



- Makes radiant home heating possible.
- Makes the Earth livable and creates weather
- Generates the Earth's magnetic field
- Controls how fast water boils
- And so much more!

What does turbulent convection look like?



thin boundary layer

Well Mixed Interior

thin boundary layer

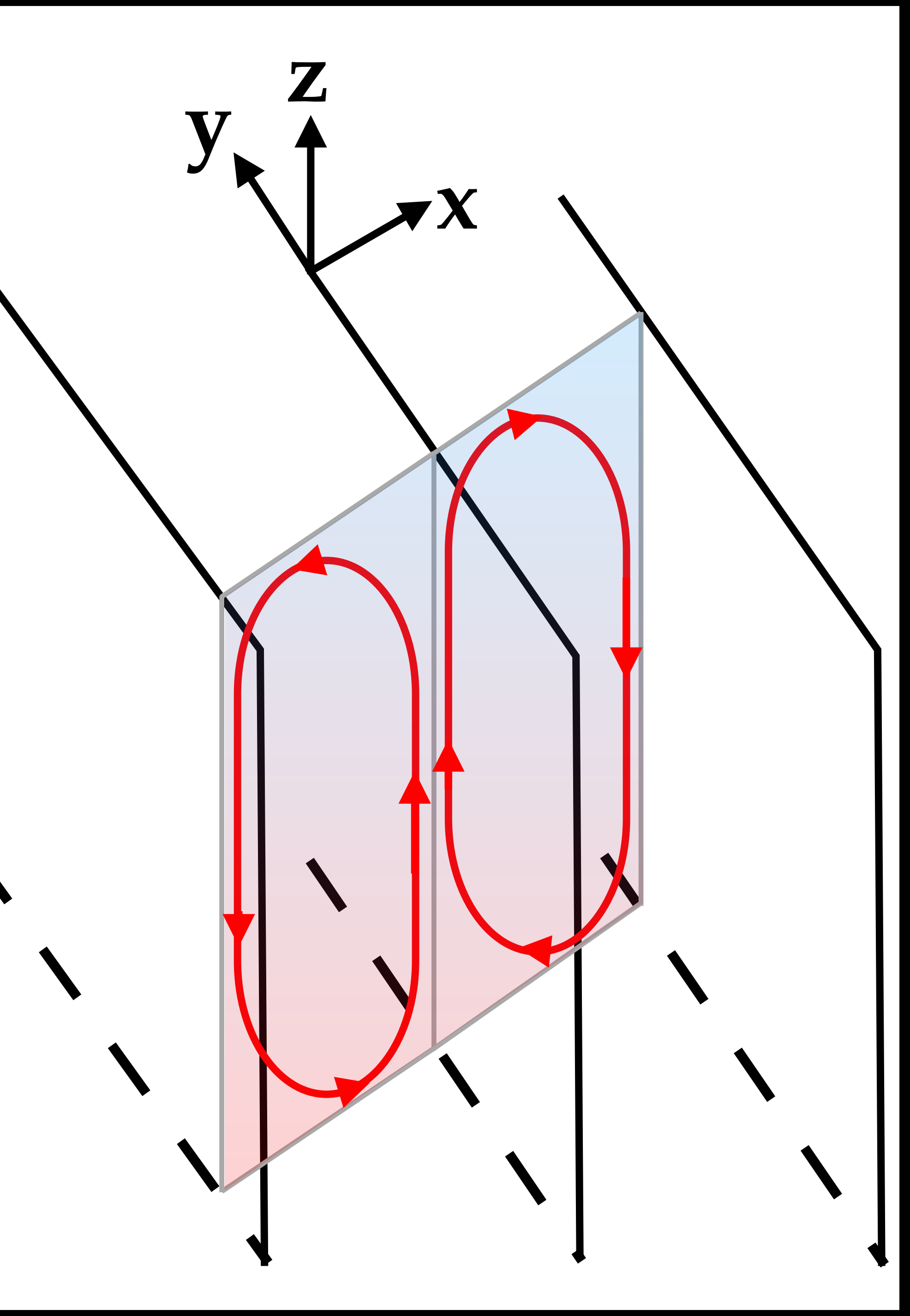
2D Convection: University of Muenster

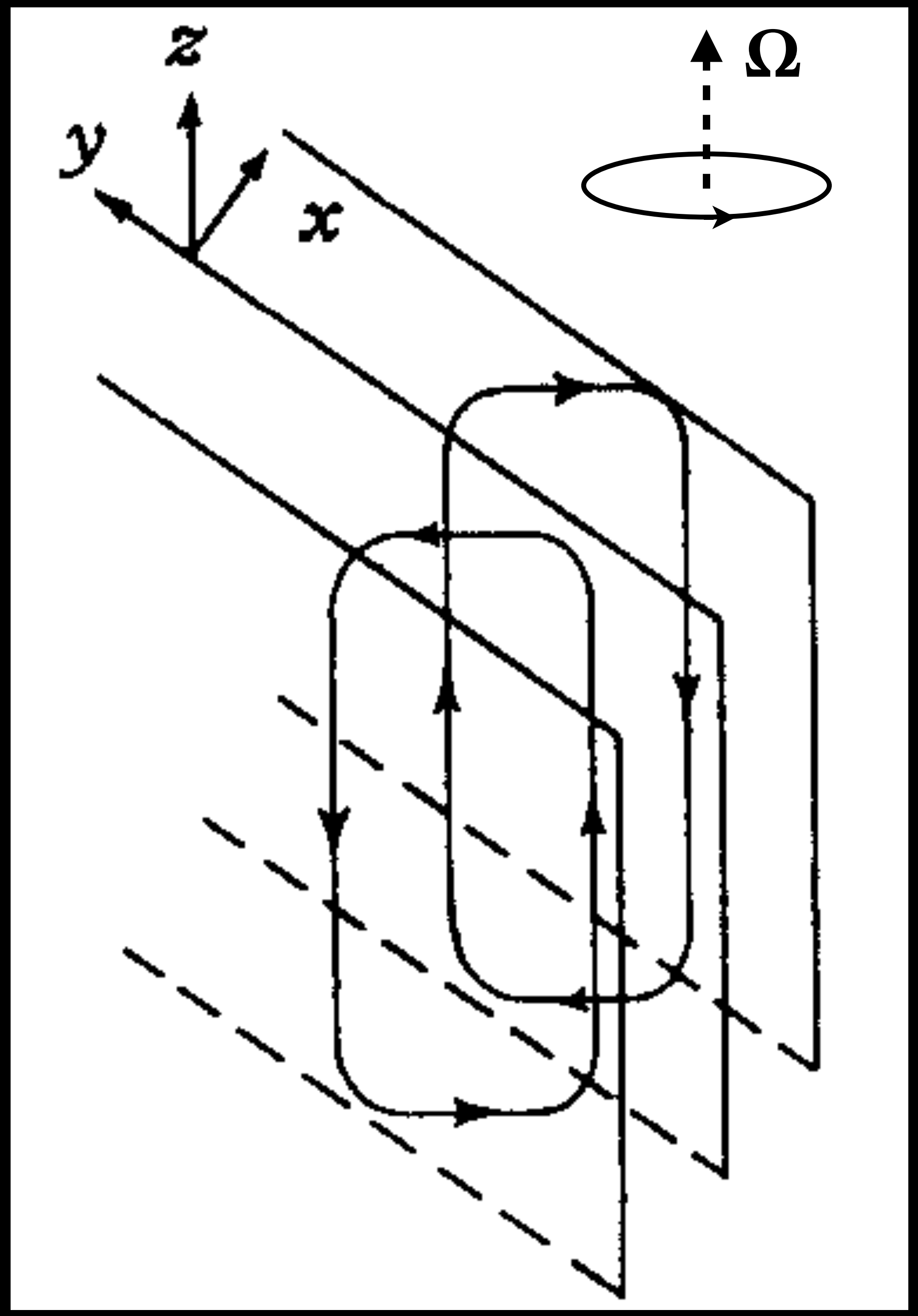
Let's Rotate

No Rotation

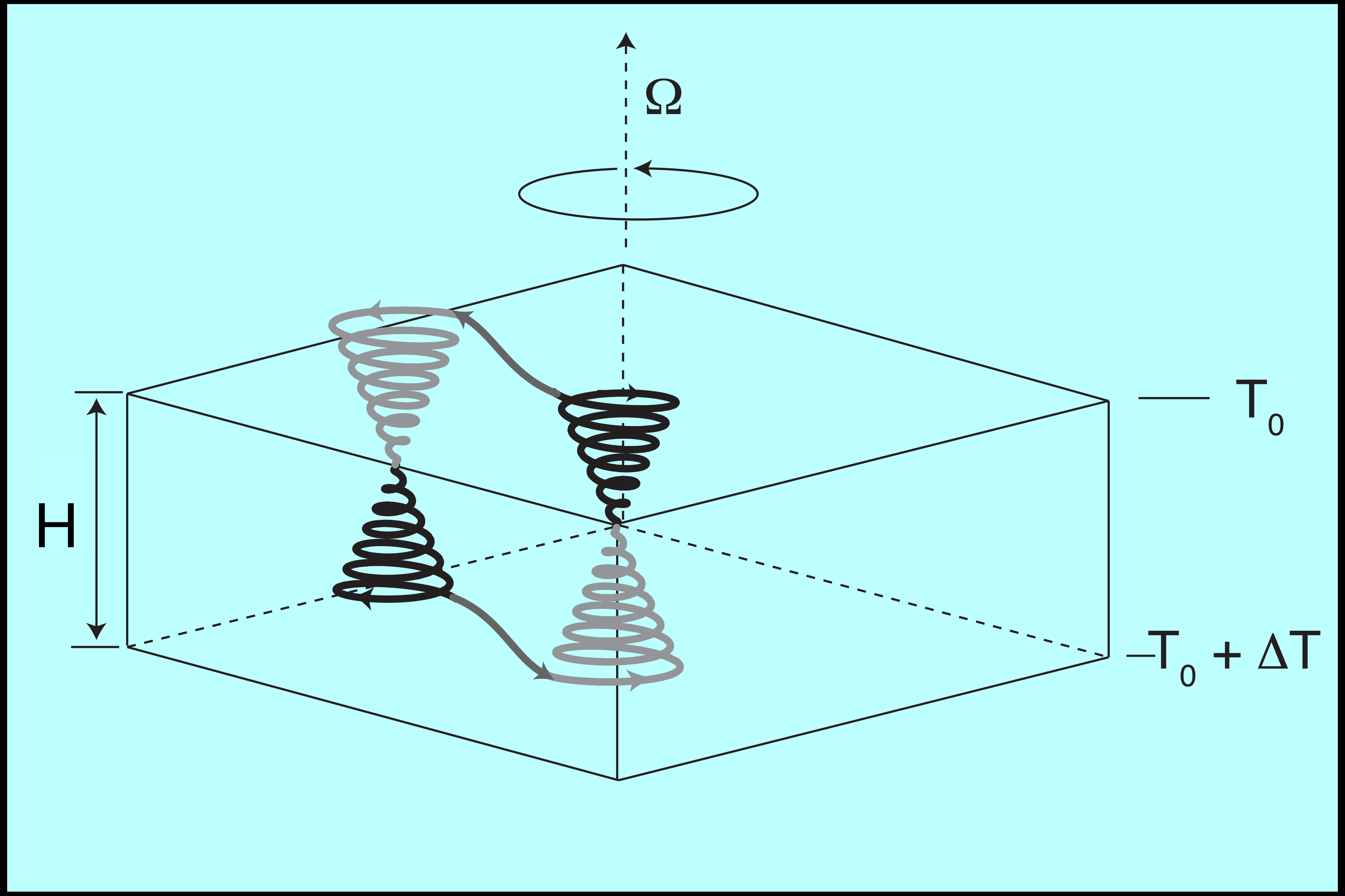
Low Rotation

High Rotation





Lateral flow bends
to the right

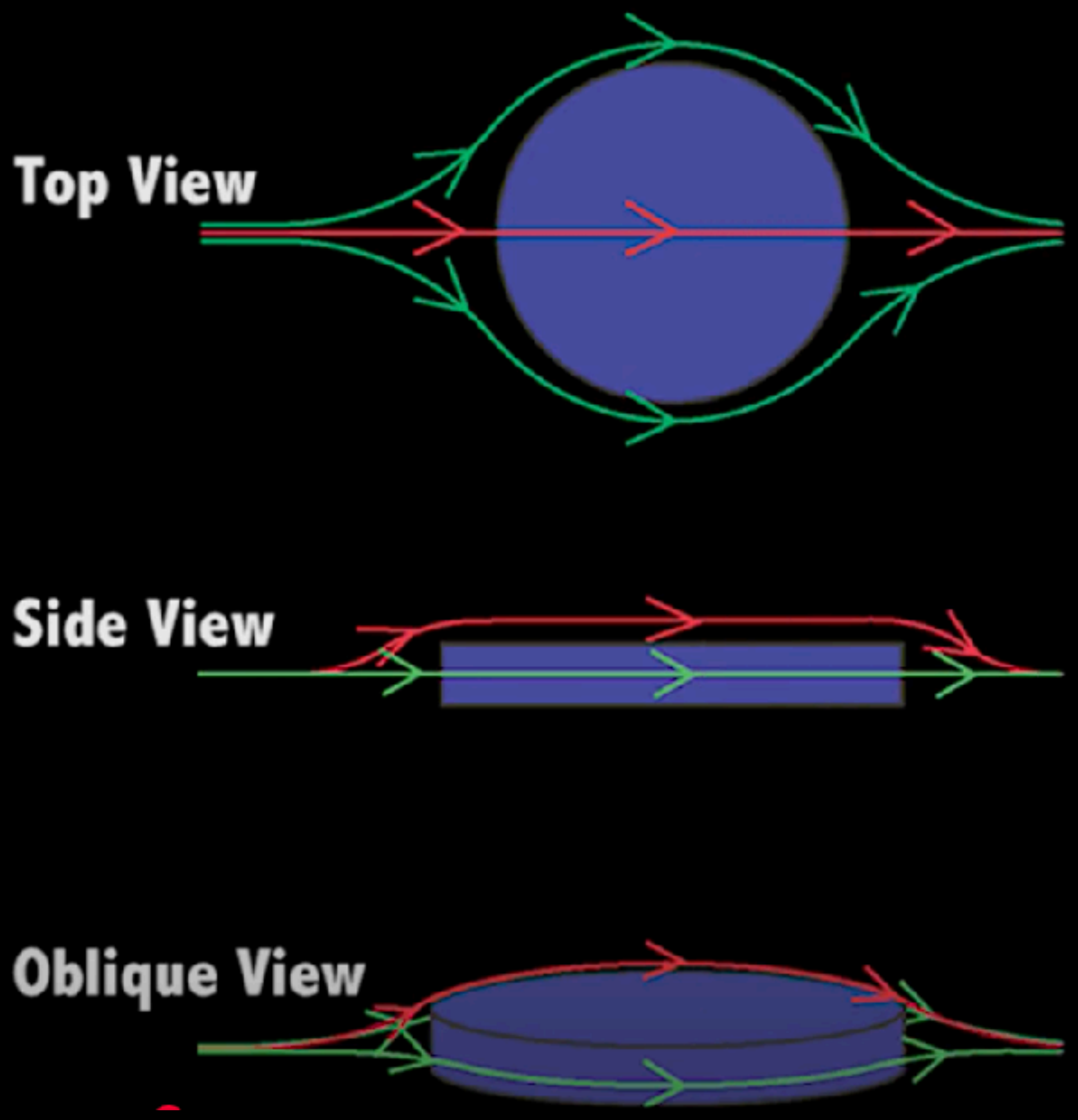


Lateral separations shrink

Veronis JFM 1959

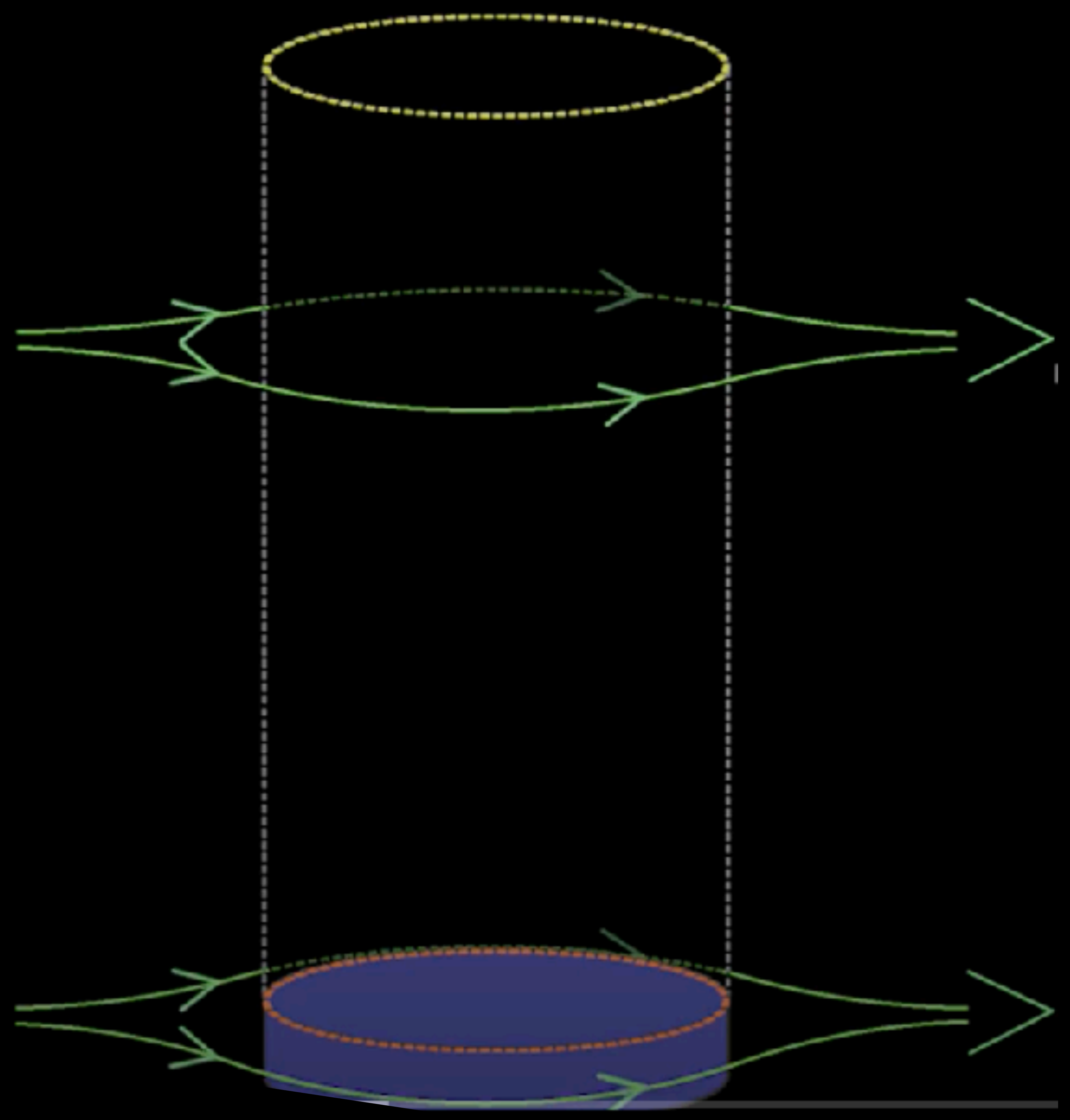
Rotating flow vertical stiffness (Taylor-Proudman 1916)

No Rotation



“Flow forced over an obstacle goes around it in 3D.”

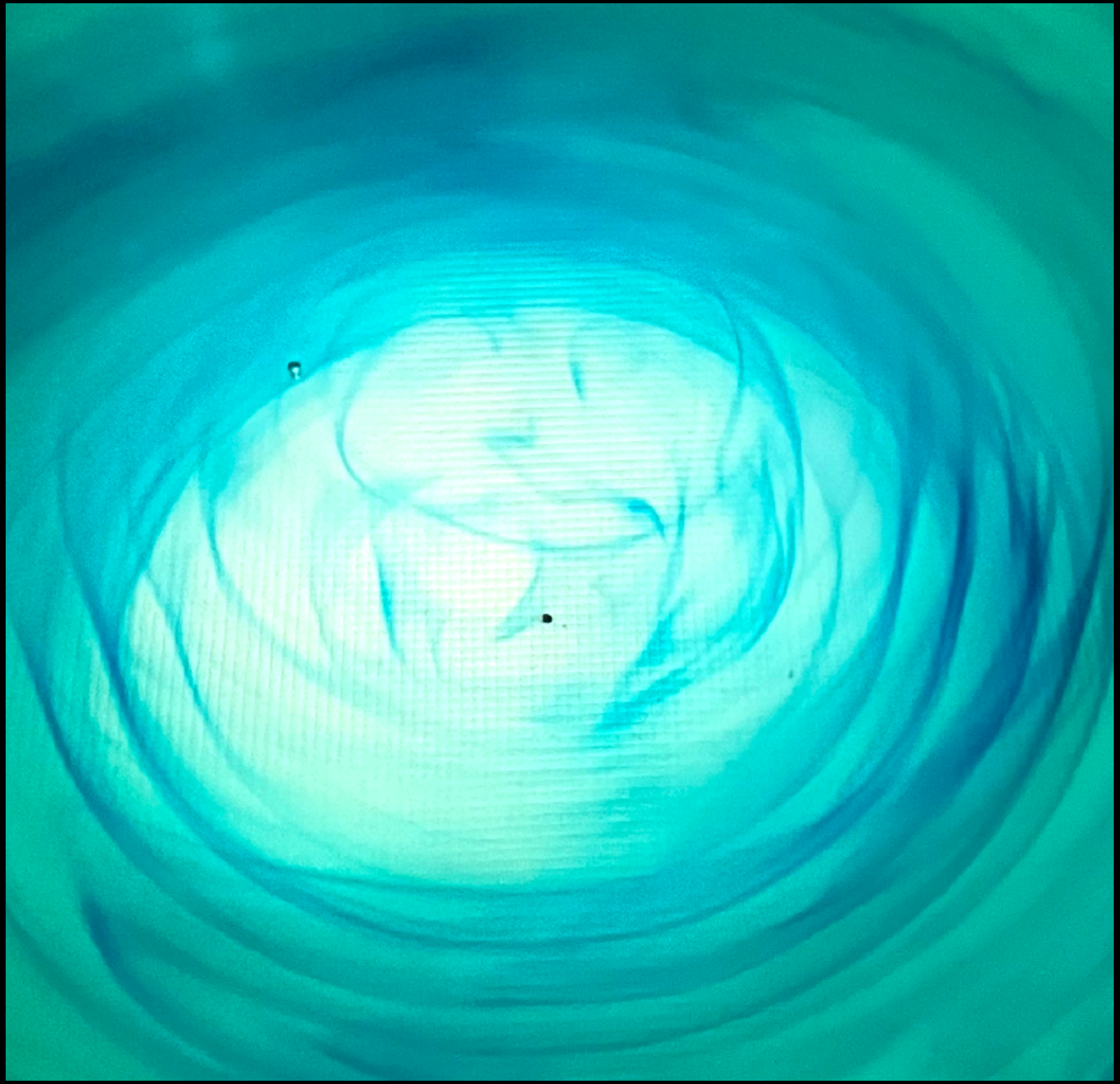
Rapid Rotation



“For rapidly rotating flows, slow flows do not vary with depth.” So they “must go around obstacles at all heights.”

From SpinLabUCLA 2014 (Aurnou) YouTube

Rotating flow vertical stiffness

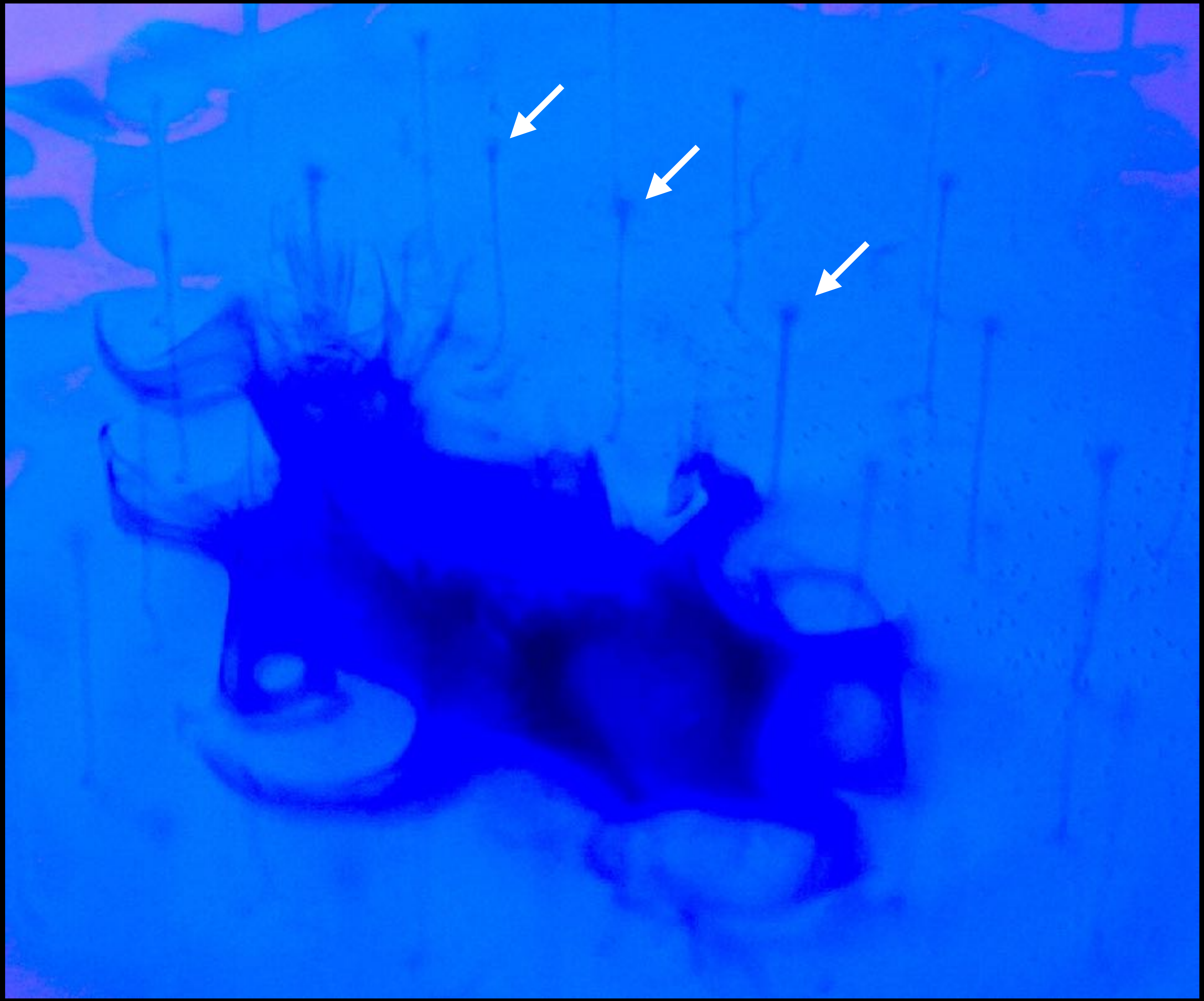


From Jon's House 1/23/2024

The laminations show how rotating flows sustain vertical stiffness

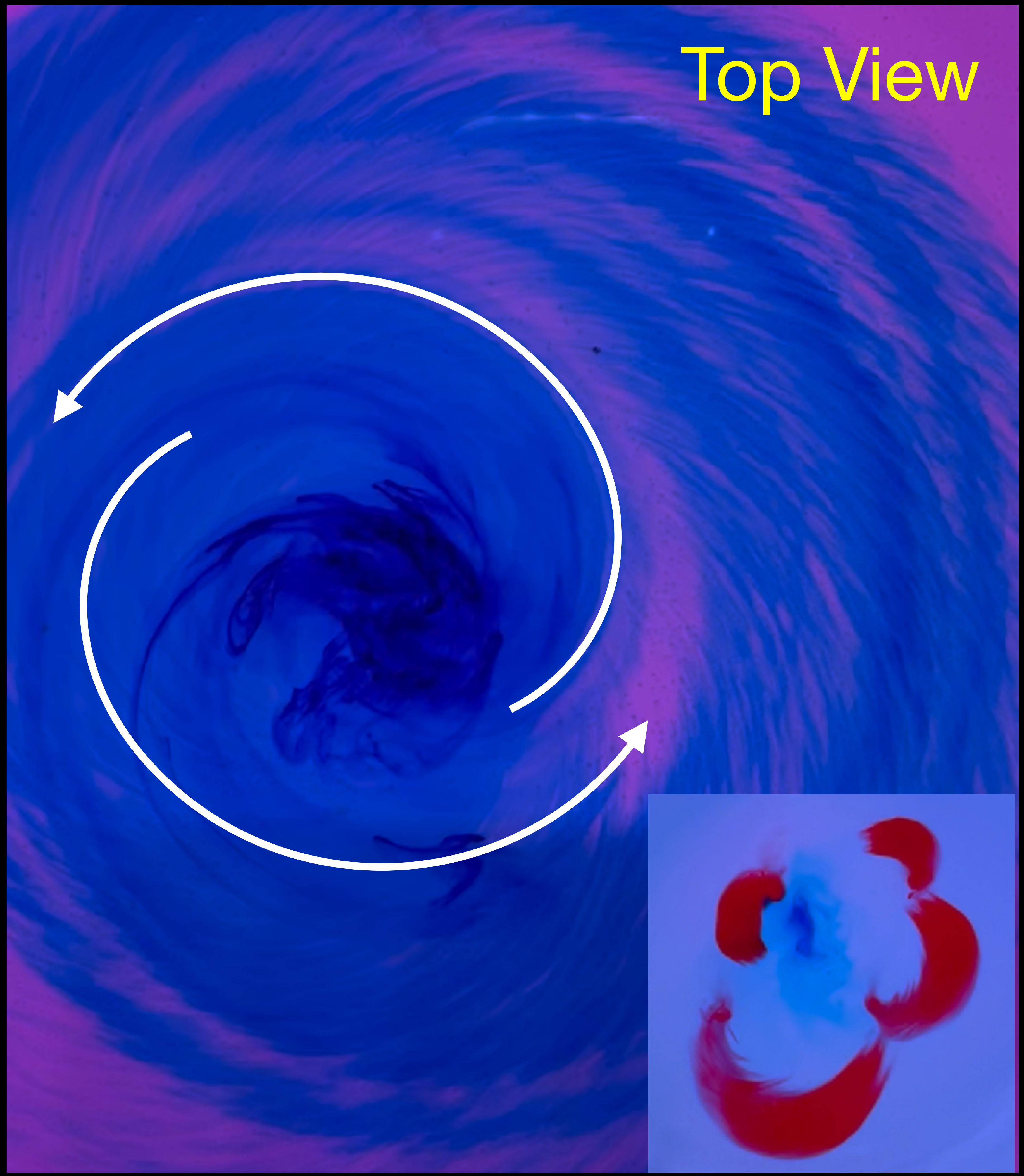
We'll have some real time demos for you in just a little while - they are fascinating. After my talk please stay around and try them for yourself.

Rotating flow vortex structures



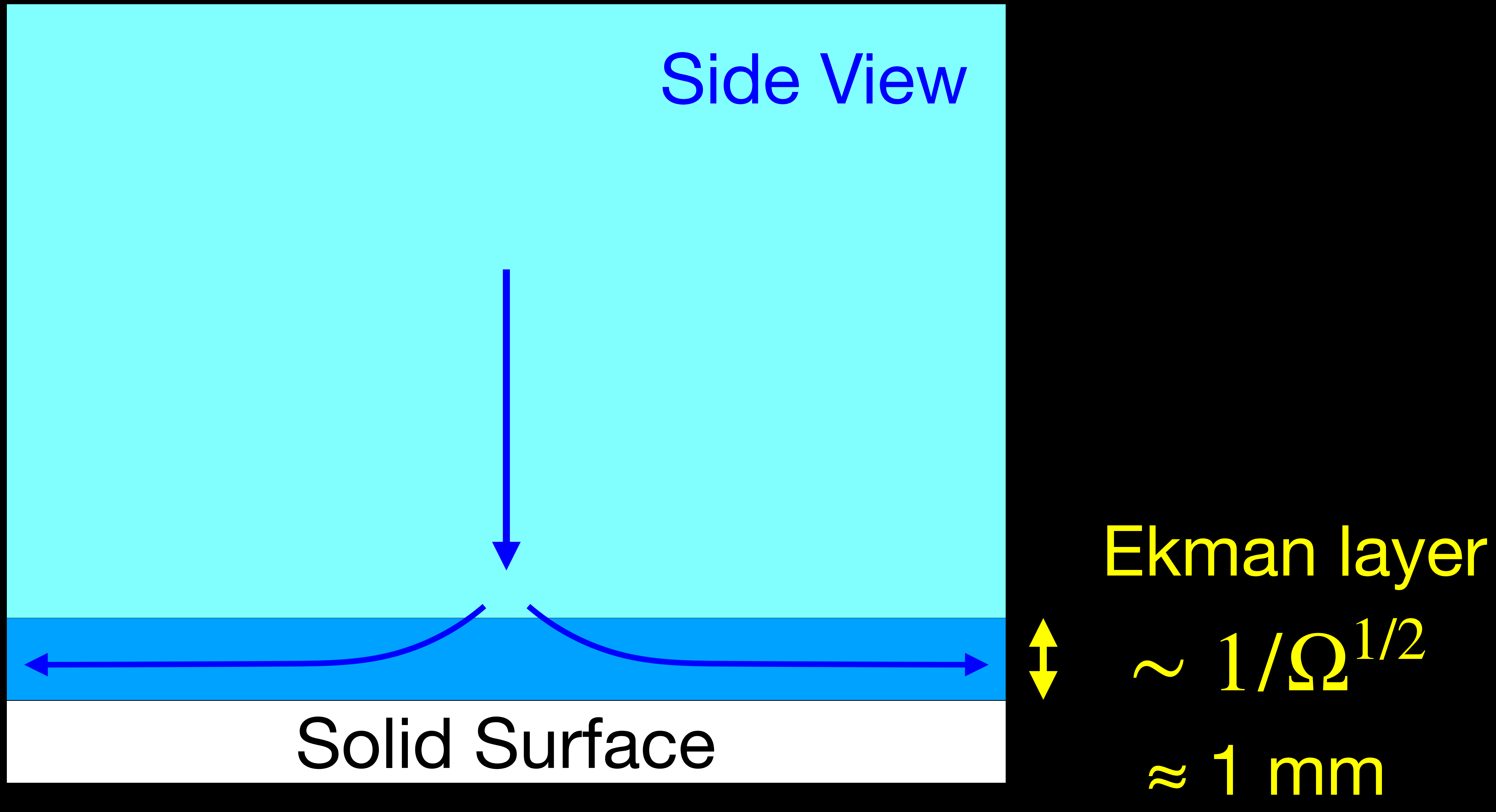
From Jon's Lab 1/24/2024

Rotating flow boundaries matter



From Jon's Lab 1/24/2024

For a rapidly rotating flow, a small change in rotation rate pulls fluid out of the bulk (suction) into a thin layer (an Ekman layer).



Rotating Convection: New Numbers

Buoyancy time ~ Ra^{-1} $Ra \sim \Delta T\, H^3$

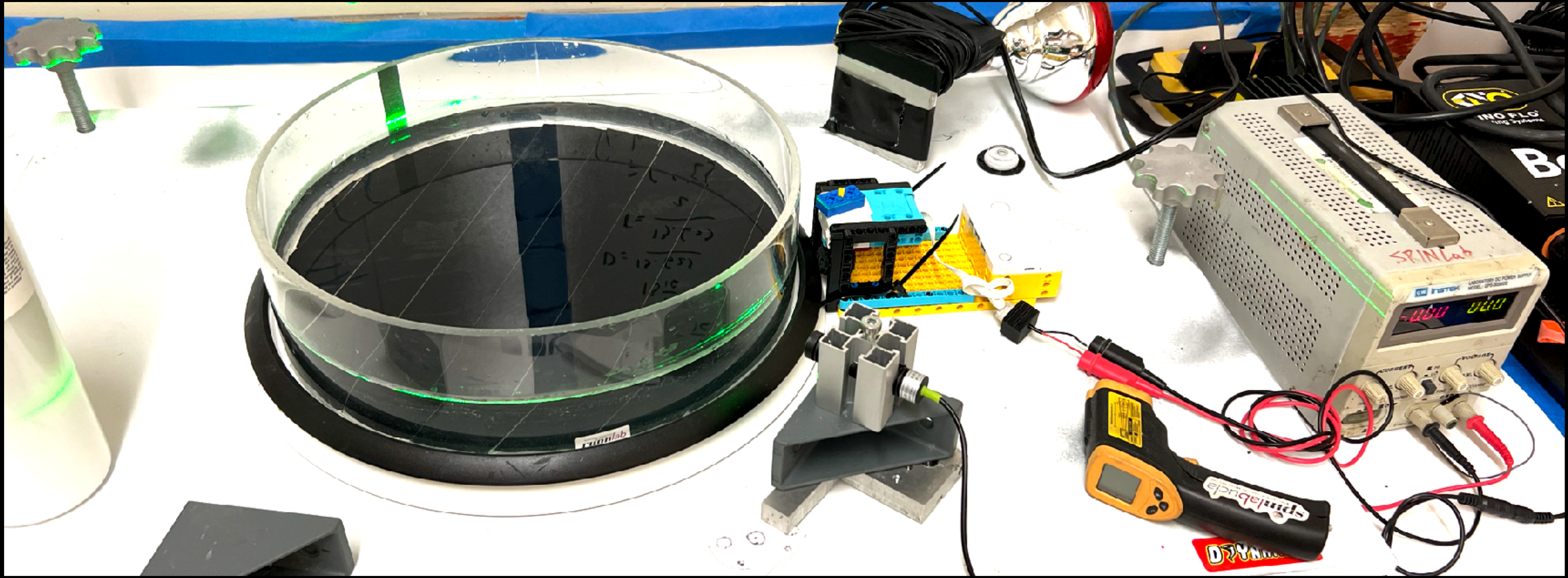
Rotation time *Ekman Number* $Ek = \dfrac{\nu}{2H^2\Omega}$

Rotation time vs buoyancy time *Rossby Number* $Ro = \sqrt{Ra/Pr^*}\ Ek$

- *Ro >>1 buoyancy wins*
- *Ro<<1 rotation wins*
- *Ek<<1 & Ro<<1 rapid rotation*

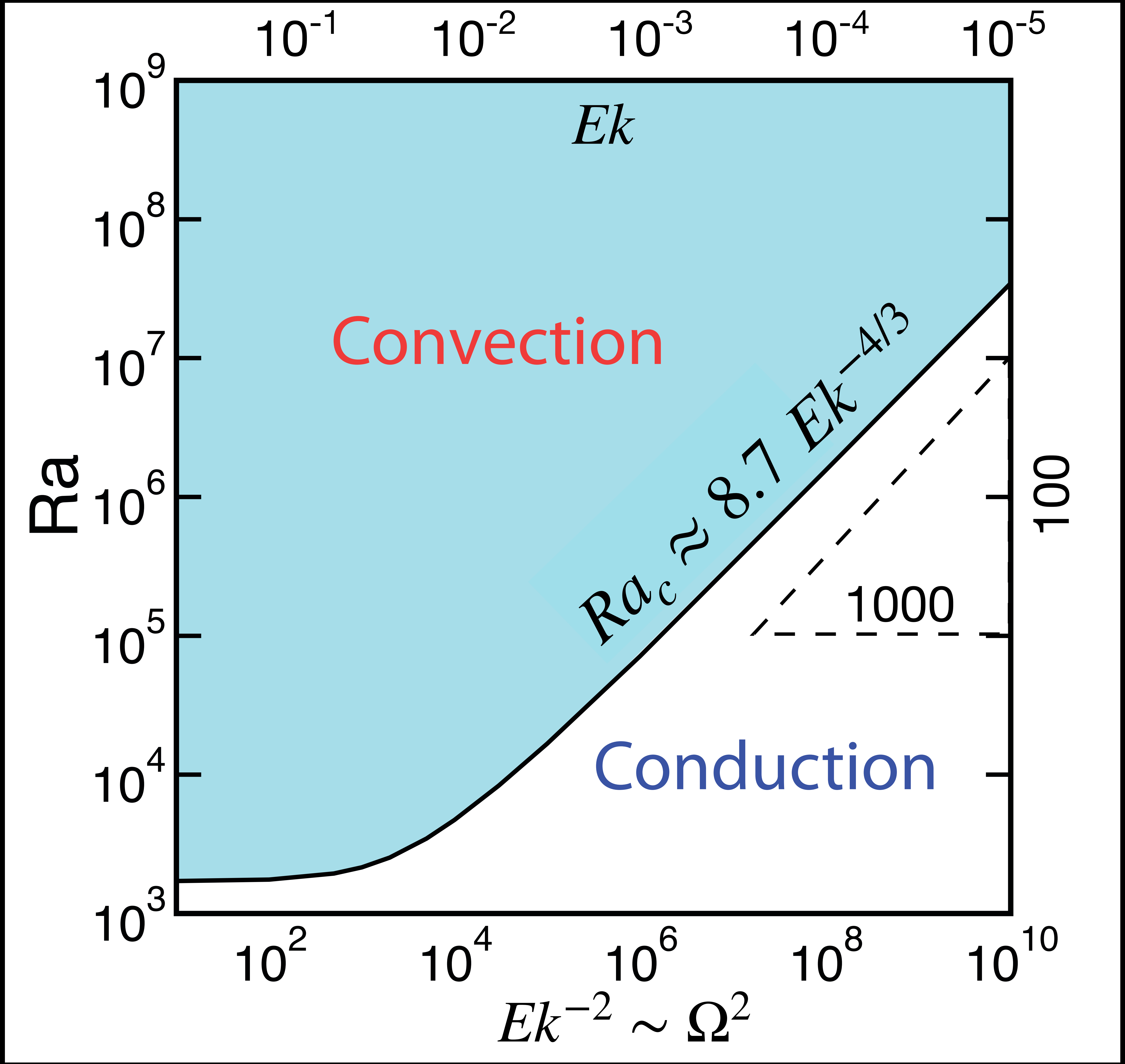
Prandtl Number
$^*Pr = \nu/\kappa$

It's demo time!



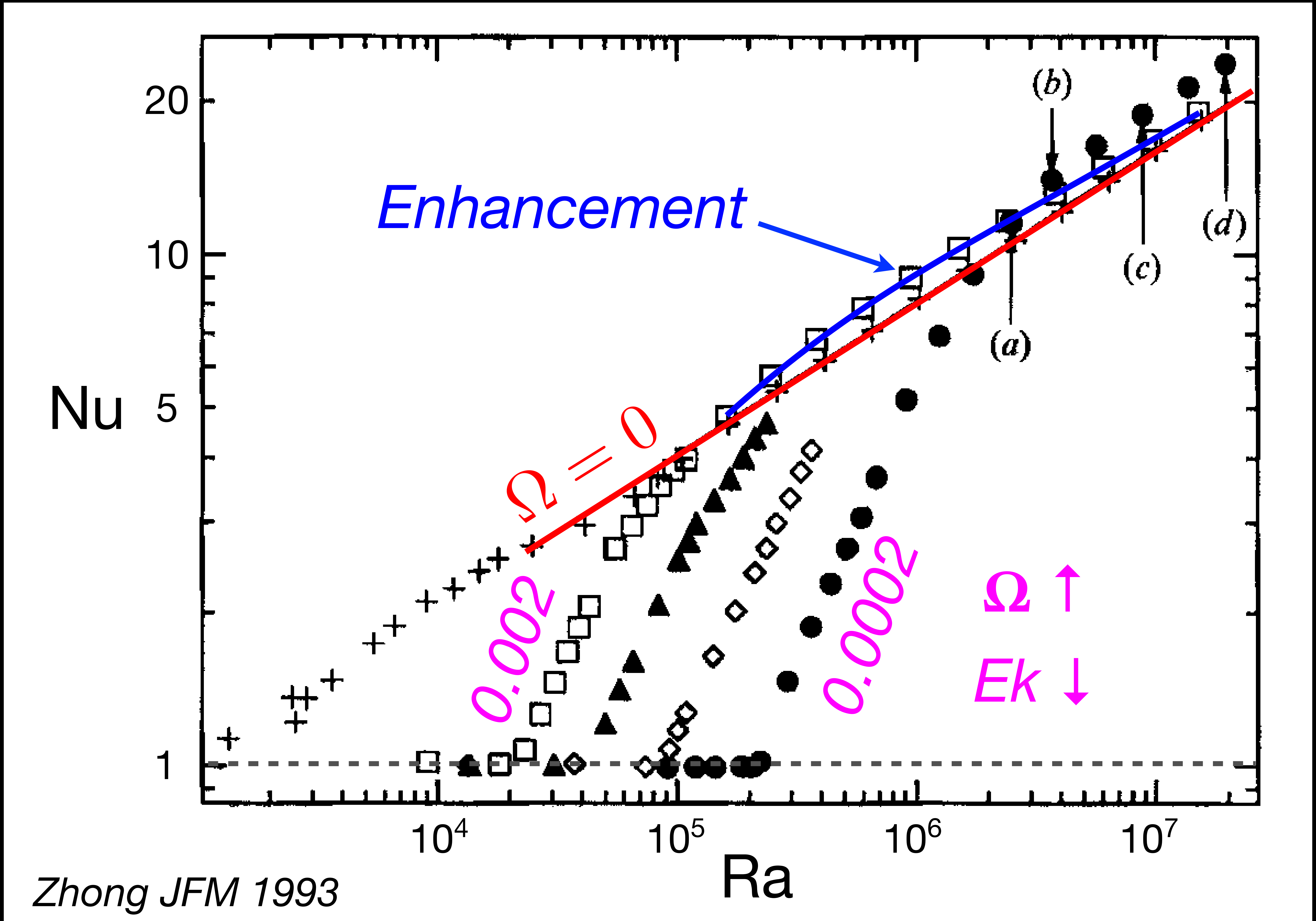
"Wheel of Fortune"

Rotating Convection: What happens where



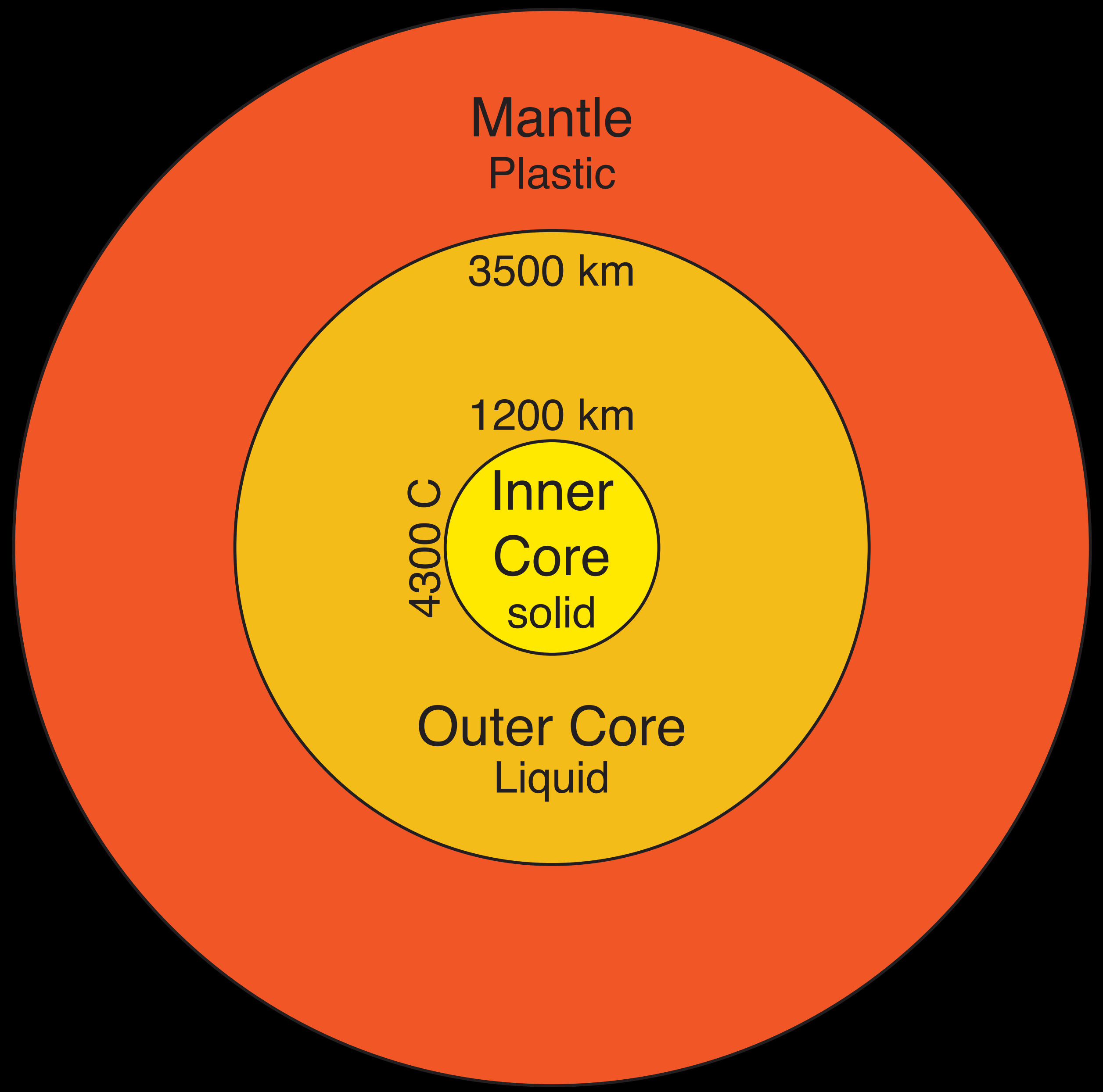
- *Rotation produces vertical stiffness and suppresses convection*
- *Bucket of water 12″ high convects for $\Delta T > 0.0000001$ C*
- *Bucket of water 12″ high rotating at 1 RPS convects for $\Delta T > 1$ C*
- *1.5 cm spacing of rotating structures*

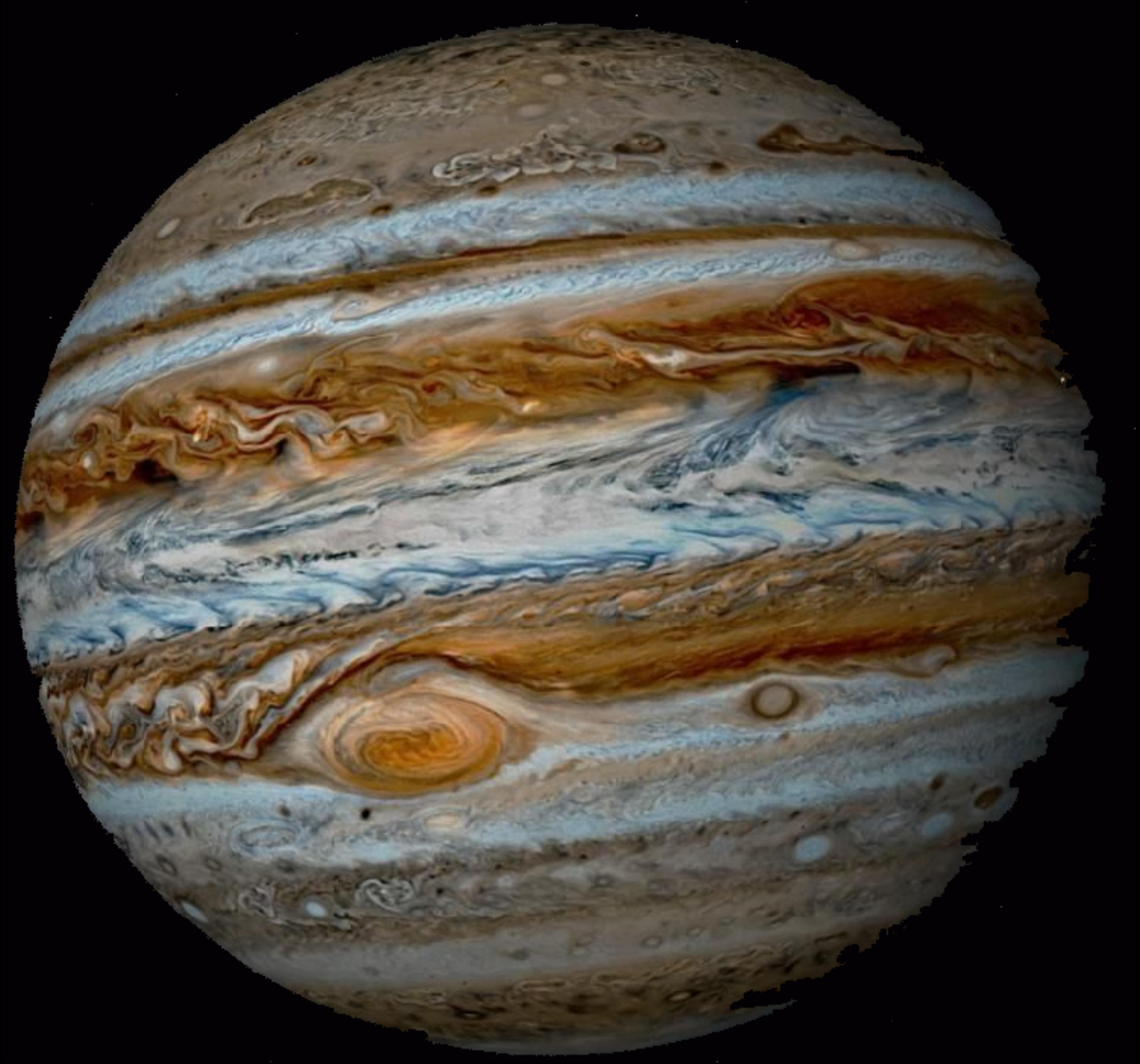
Rotating Convection: Heat Transport

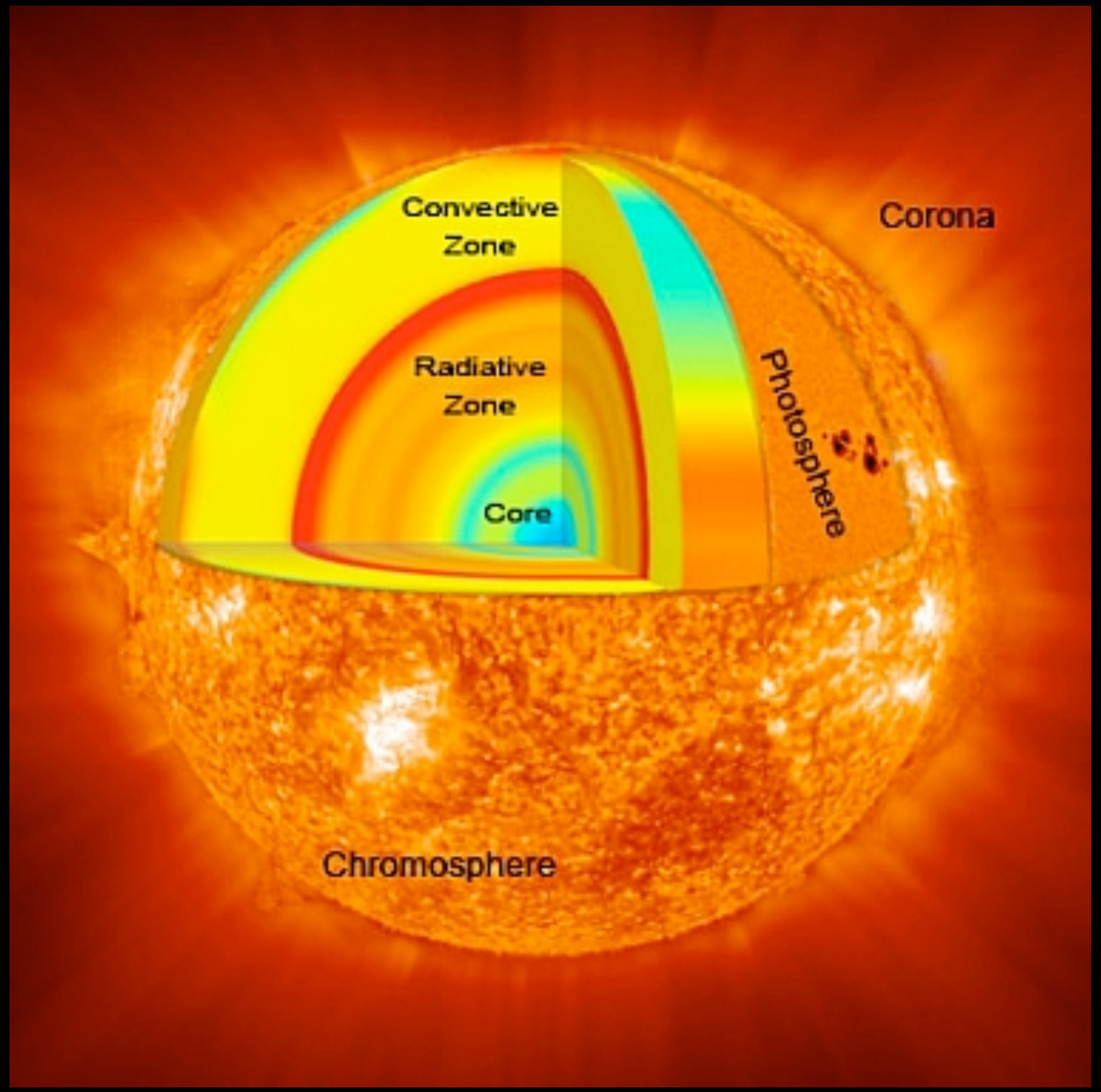


- *Two main regions of flow*
- *Region of heat transport enhancement*
- *What happens when Ra → ∞ and Ek → 0?*

Rotating Convection: The extremes in nature







<u>Earth's Outer Core</u>

$Ek \sim 10^{-15}$

$Ra \sim 10^{25}$

<u>Jupiter</u>

$Ek \sim 10^{-12}$

$Ra \sim 10^{24}$

<u>Solar Convection</u>

$Ek \sim 10^{-12}$

$Ra \sim 10^{20}$

Mathematics to the rescue

Keith Julien

Edgar Knobloch

Many collaborators

very large buoyancy

$$Ra \to \infty$$

very rapid rotation

$$Ek \to 0$$

$$\widetilde{Ra} = RaEk^{4/3} = A \ Ra_c$$

Vertical stiffness ~ H

Lateral Scale $Ek^{1/3}$ $H \ll H$

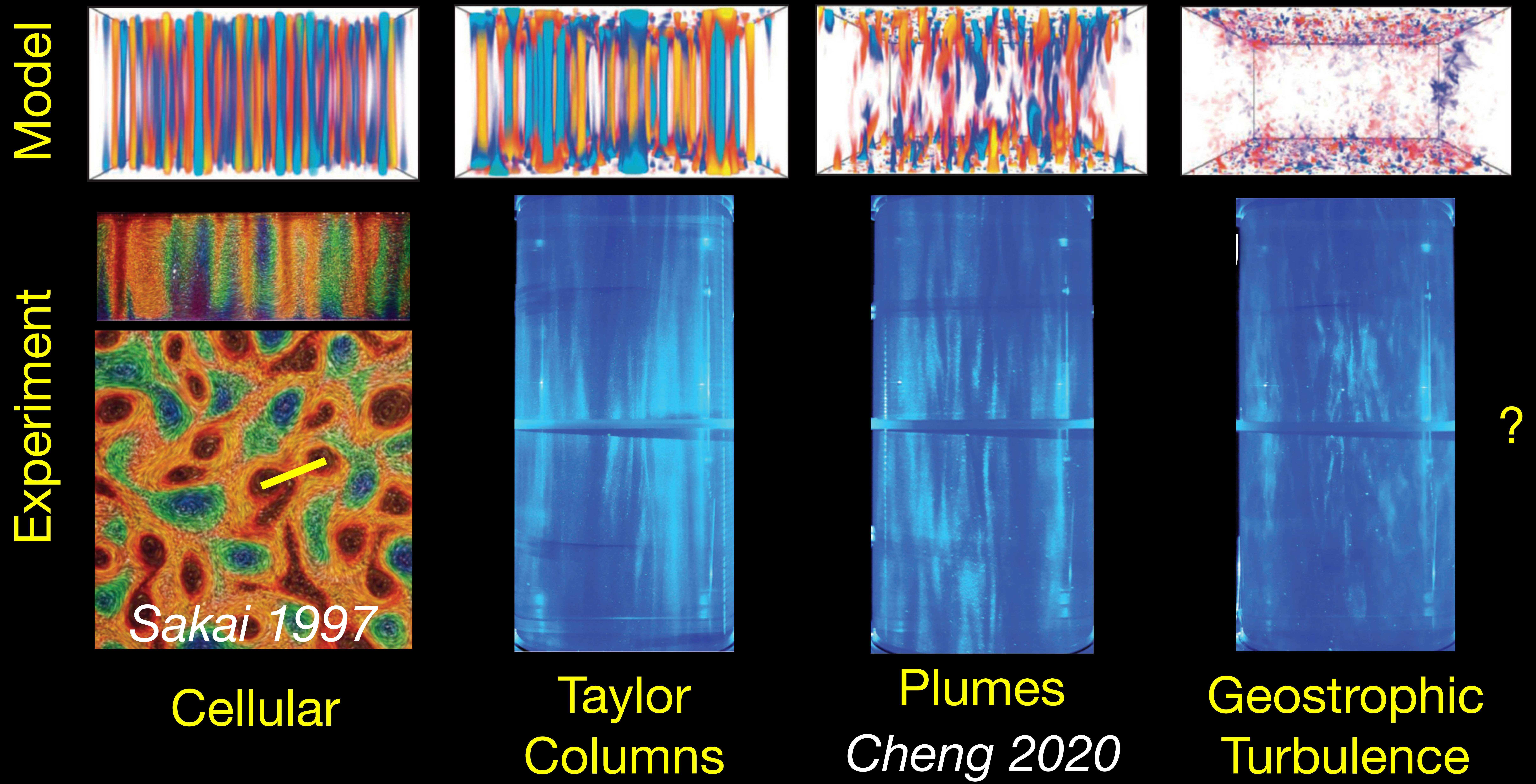
No Ekman layer

No fast waves

No non-rotating state

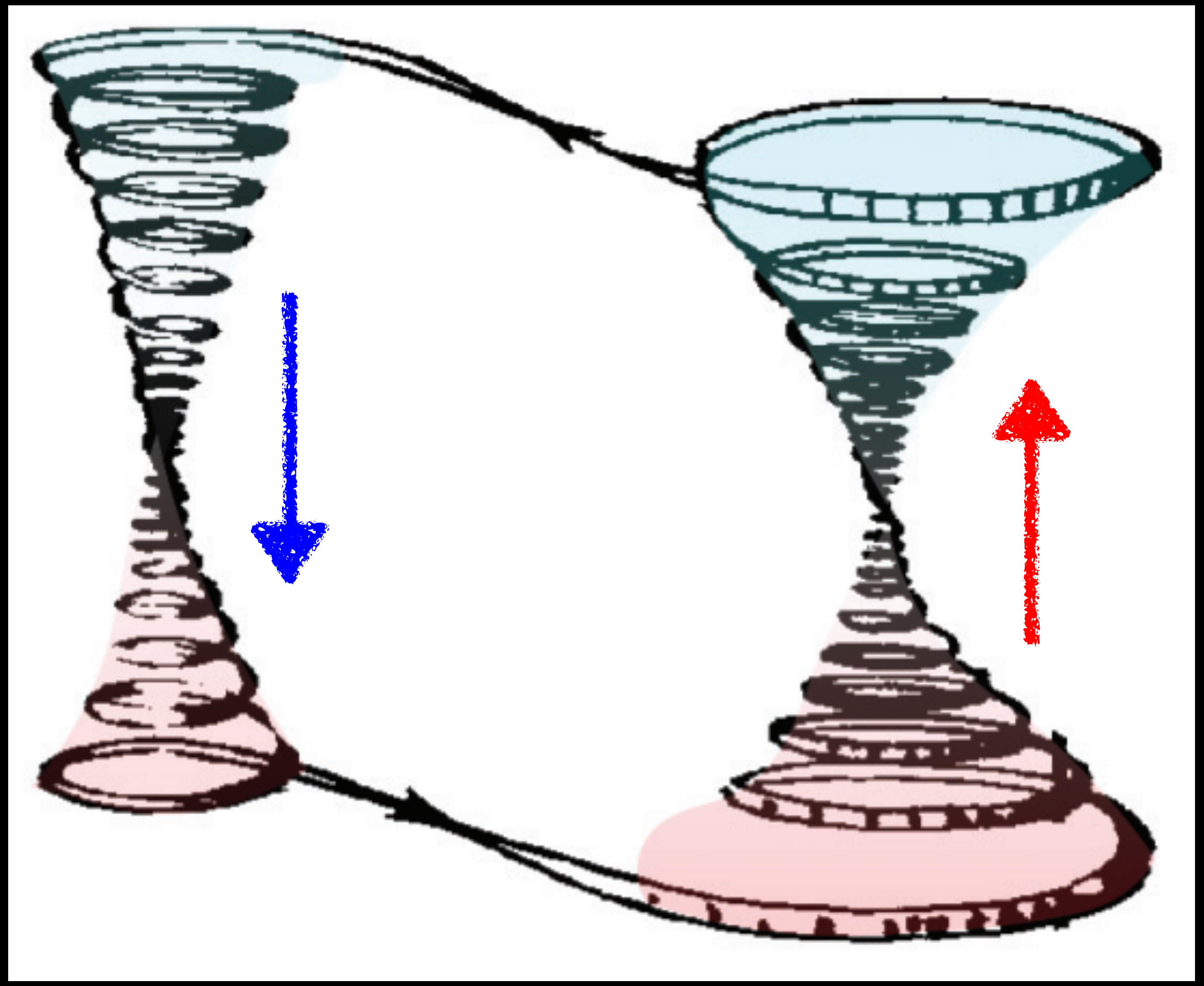
But does it work!?

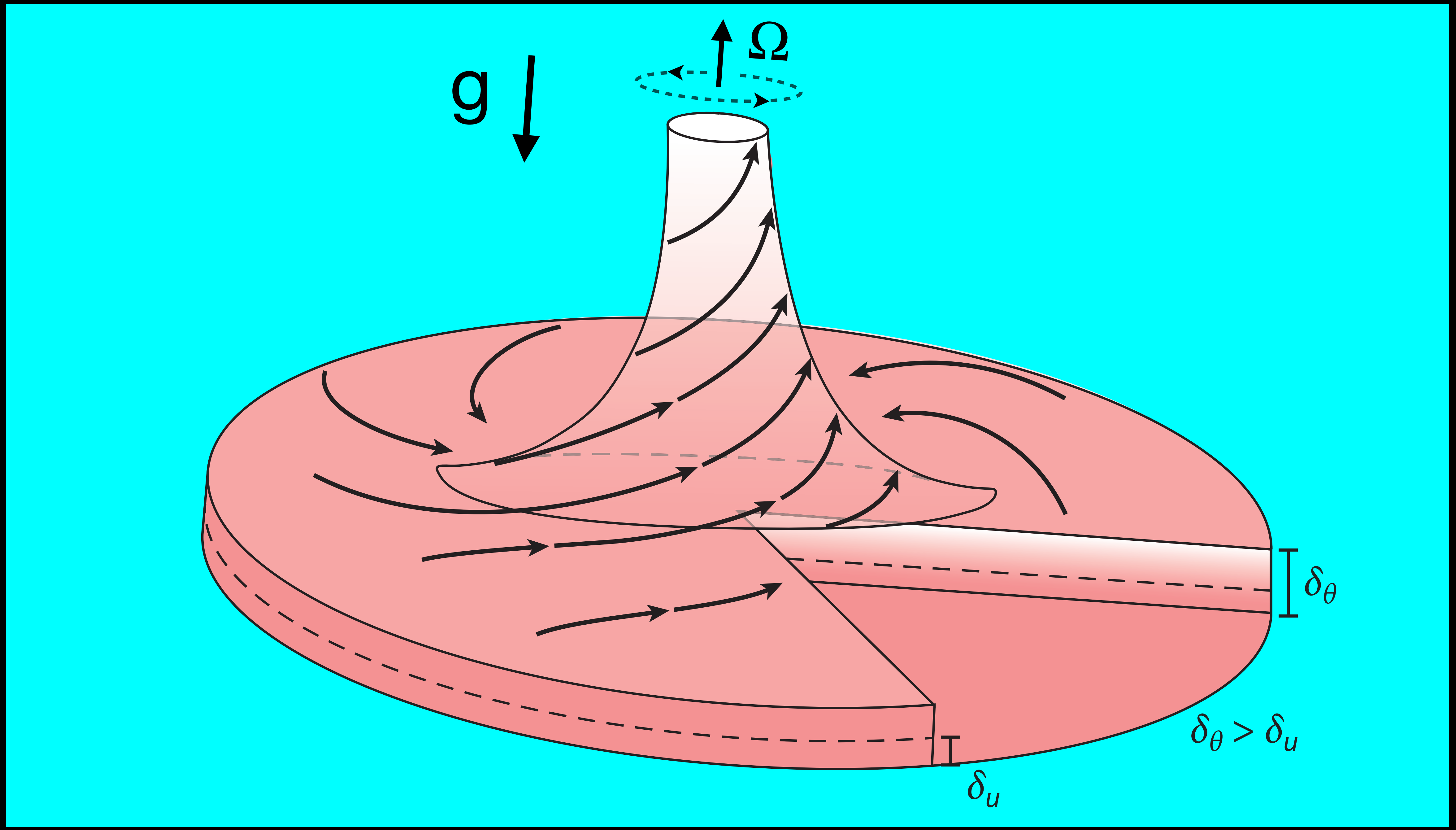
Regimes of Rapidly Rotating Convection



Why do rotating flows make vortices

No Rotation

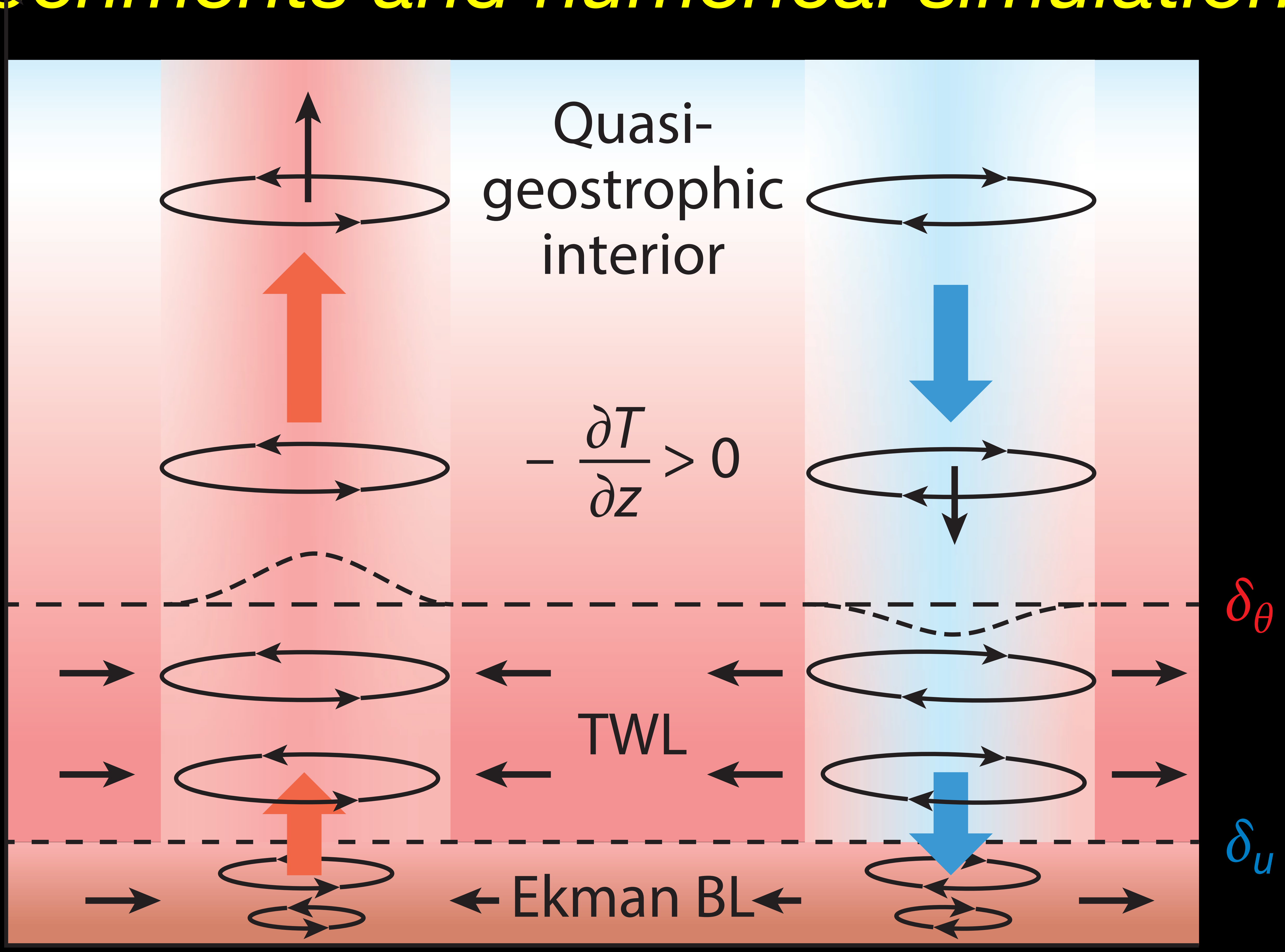




Rapid Rotation

How about Ekman layers

Ekman layers matter for experiments and numerical simulation

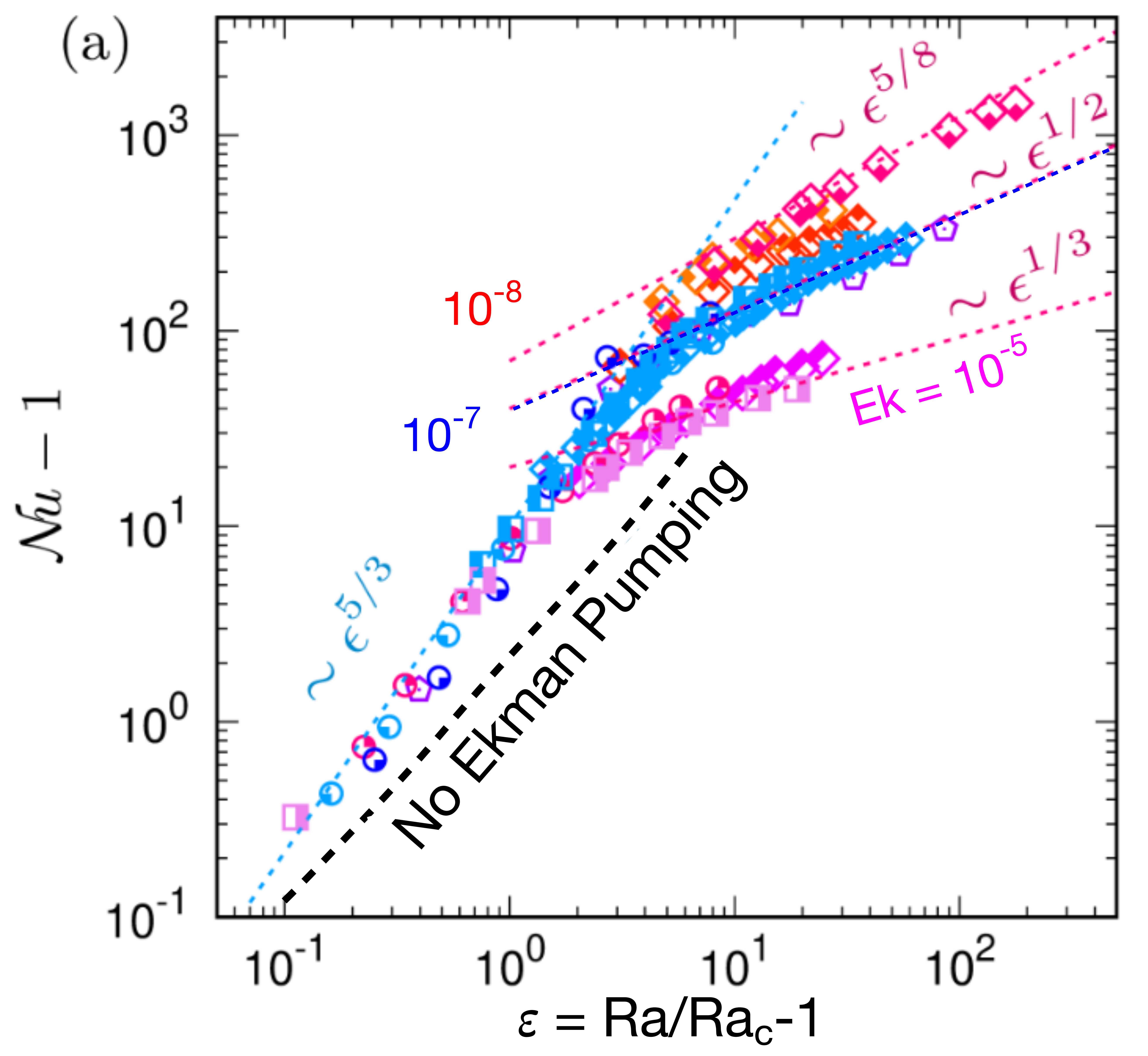


Remember how change in $\boldsymbol{\Omega}$ caused thin layer that pulled fluid from the bulk

works for differential rotation as well

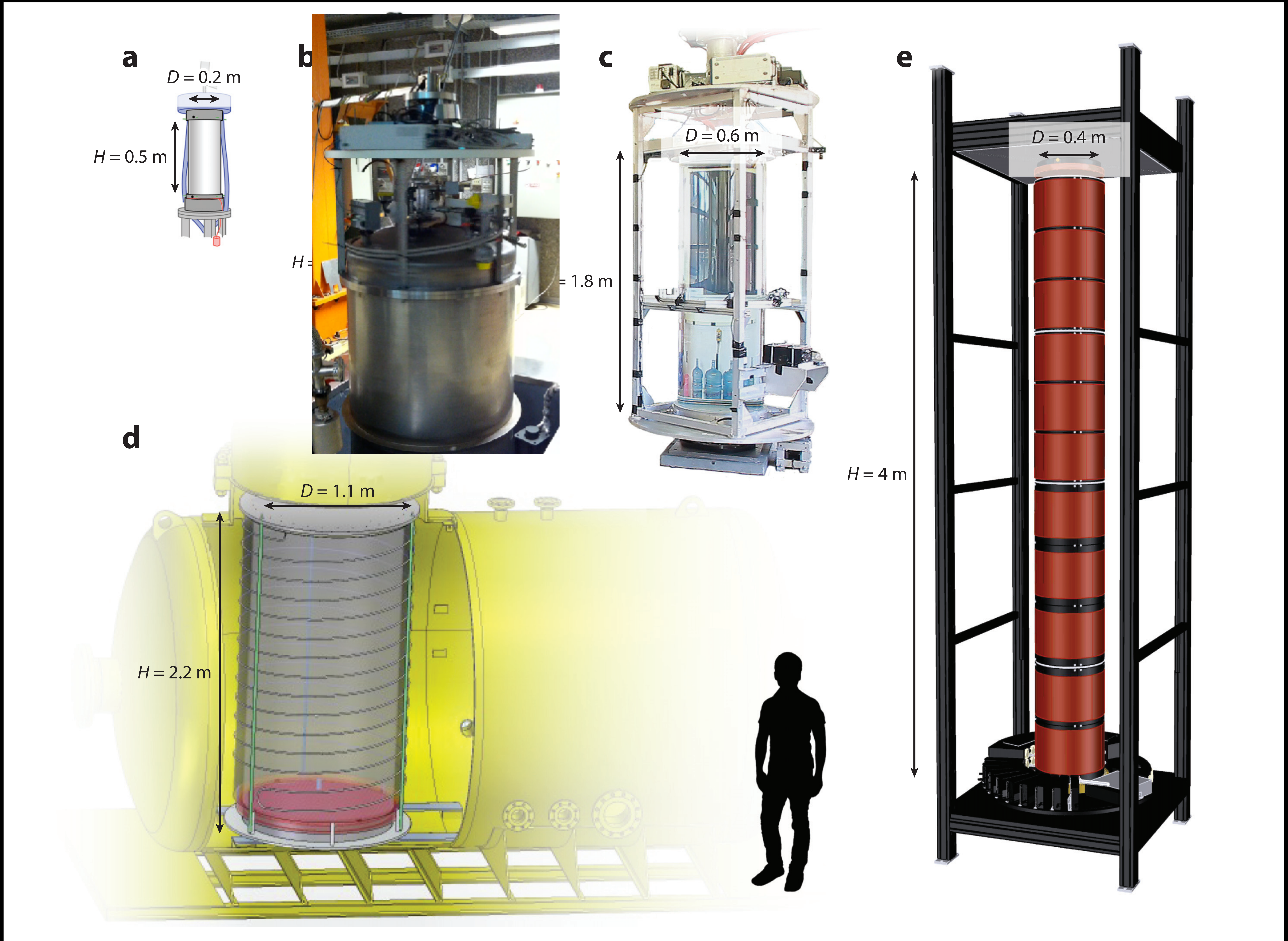
$w \sim Ek\ \omega_z \to 0$ but not very fast!

Why does this matter? It's the heat transport



- *Ekman pumping having big effect*
- *Still running into crossover to non-rotating limit*

Experiments and simulations are tall and thin



More buoyancy

$$Ra \sim H^3$$

More rotation

$$Ek \sim H^{-2}$$

Vortex spacing ~ $H^{-2/3}$

H=4 m D = 0.4 m
tank water ~ 1/2 ton

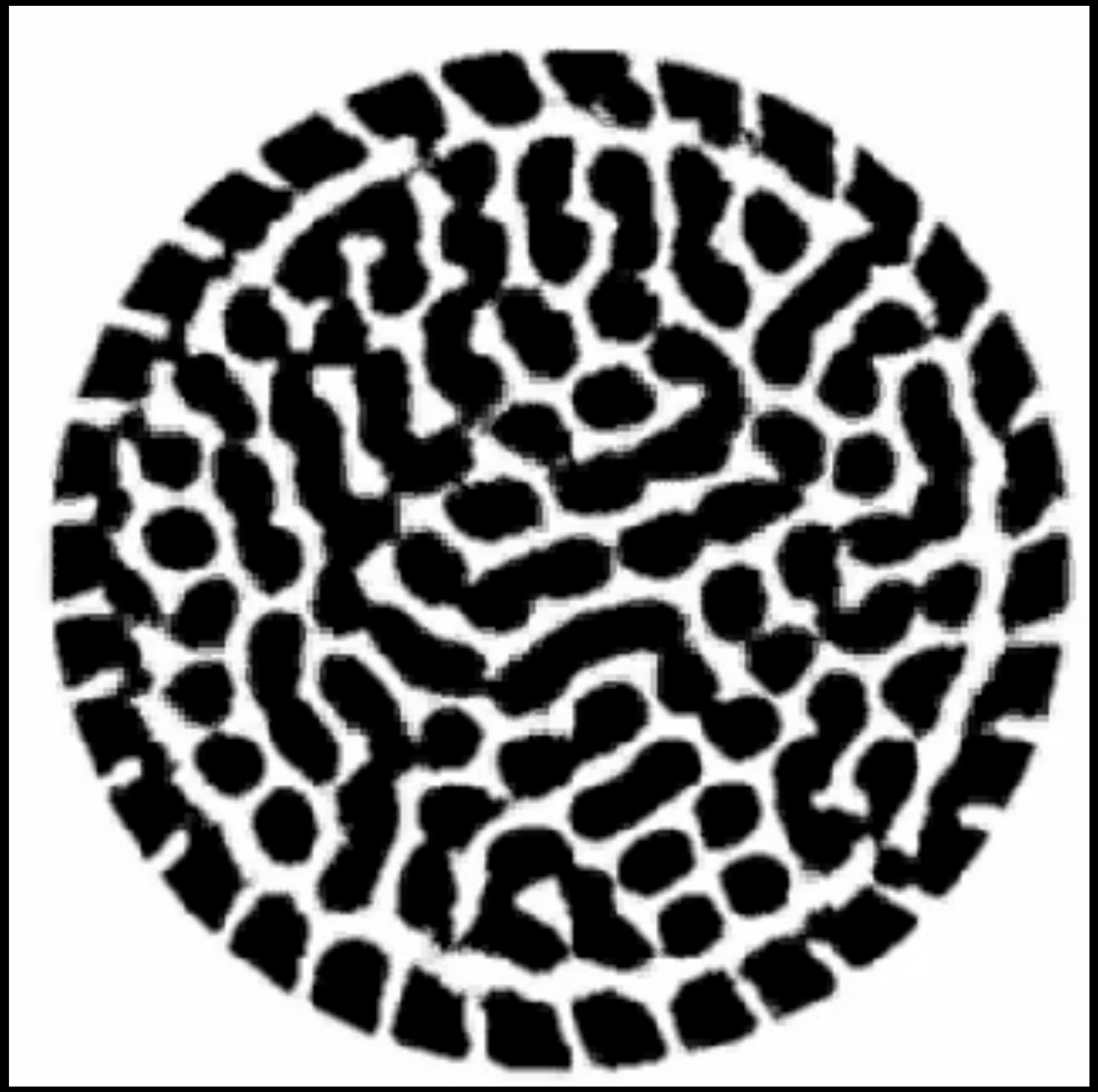
H=4 m D = 4 m
tank water ~ 50 ton!

Are we happy yet?

There is another important state that has been ignored: Wall Modes

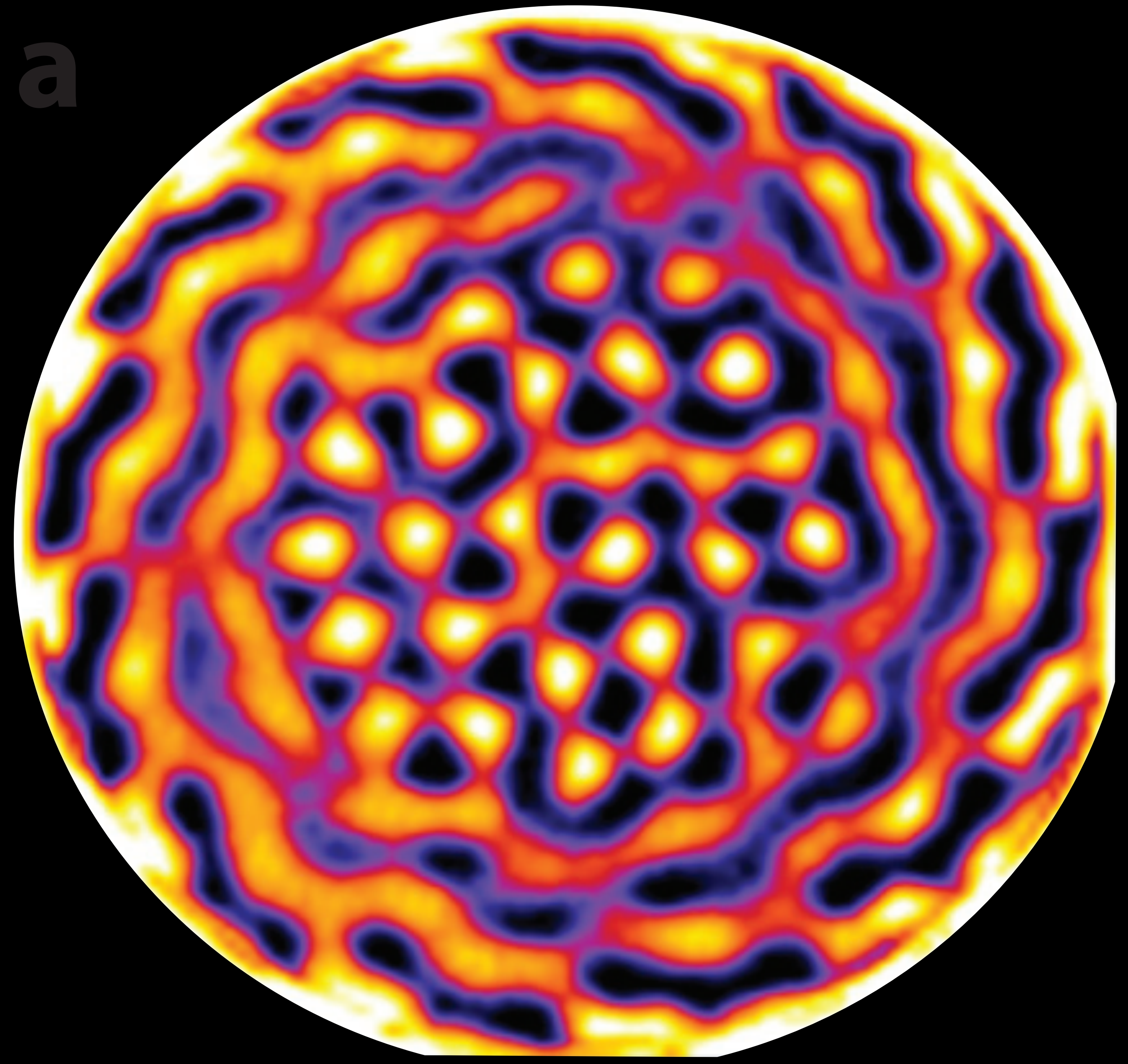
They appear at lower Ra than the bulk state and add to the heat transport

Experiment



D/H = 10

Experiment



D/H = 5

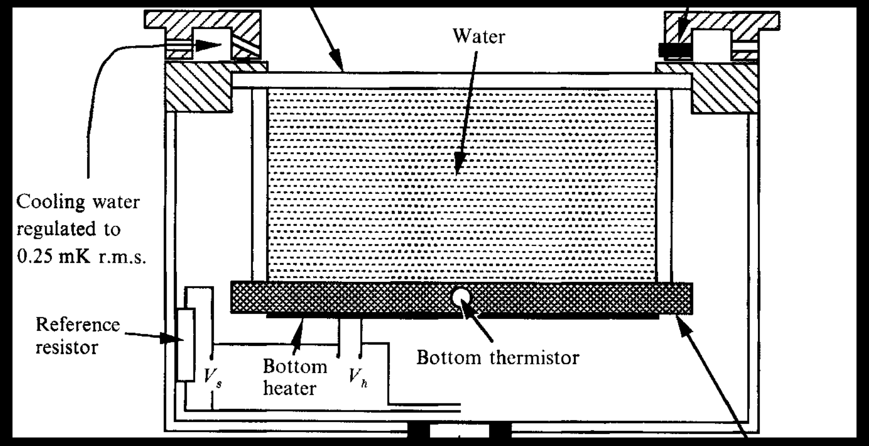
Simulation

D/H = 1/2

Where are we now?

nature

~1990

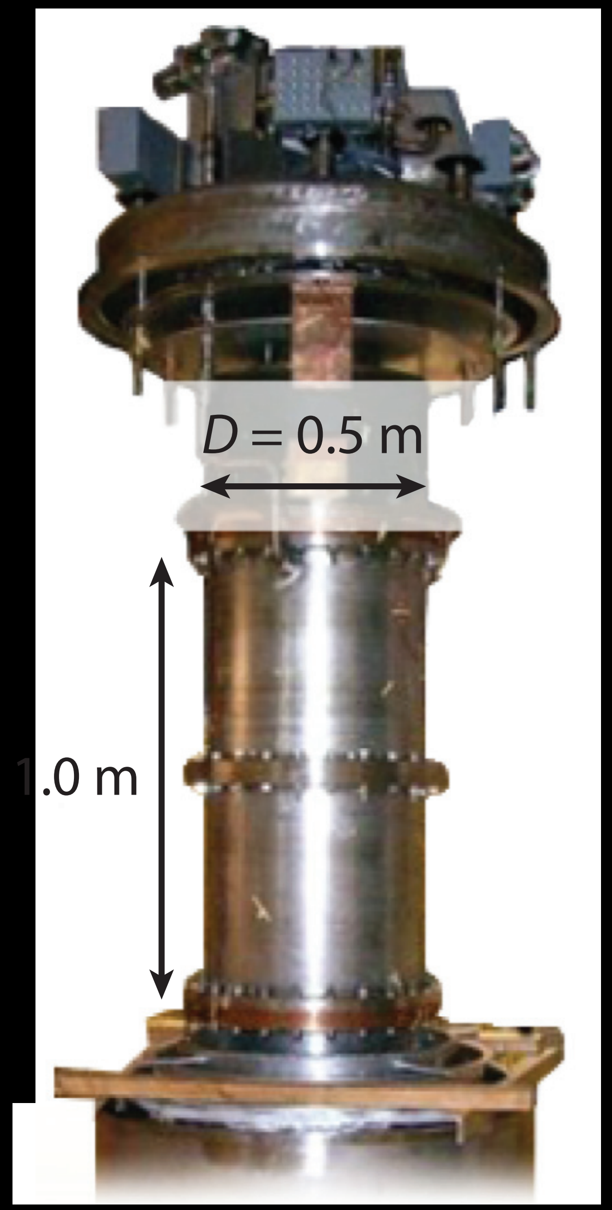


$Ek \approx 10^{-4}$

~2010

$Ek \approx 10^{-7}$

~2015

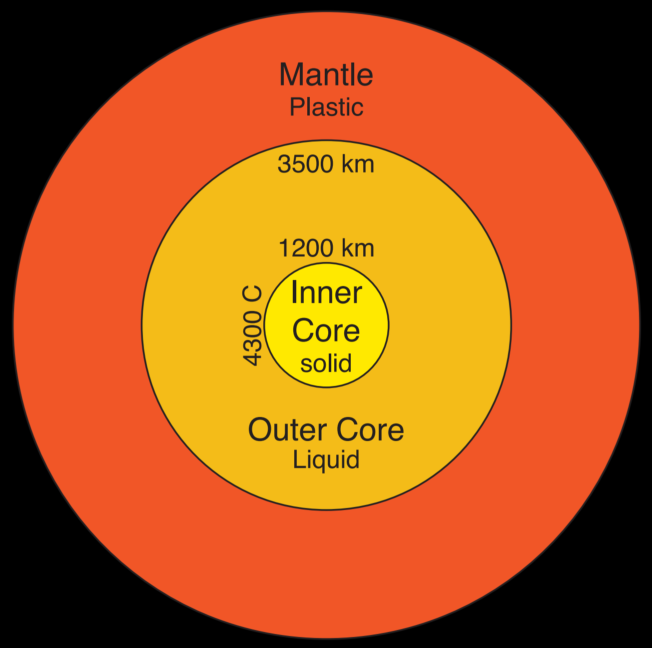


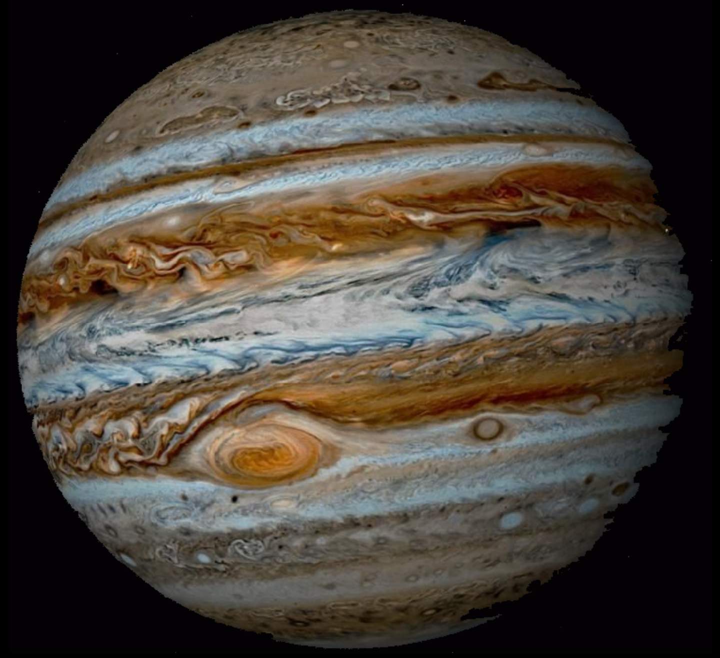
~2020

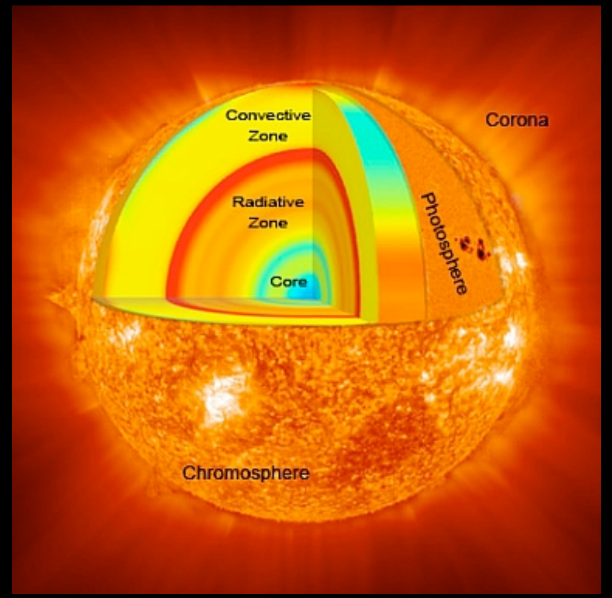
$Ek \approx 5 \times 10^{-9}$

??

Mathematical models help connect







$Ek < 5 \times 10^{-12}$

Experiment

Simulation

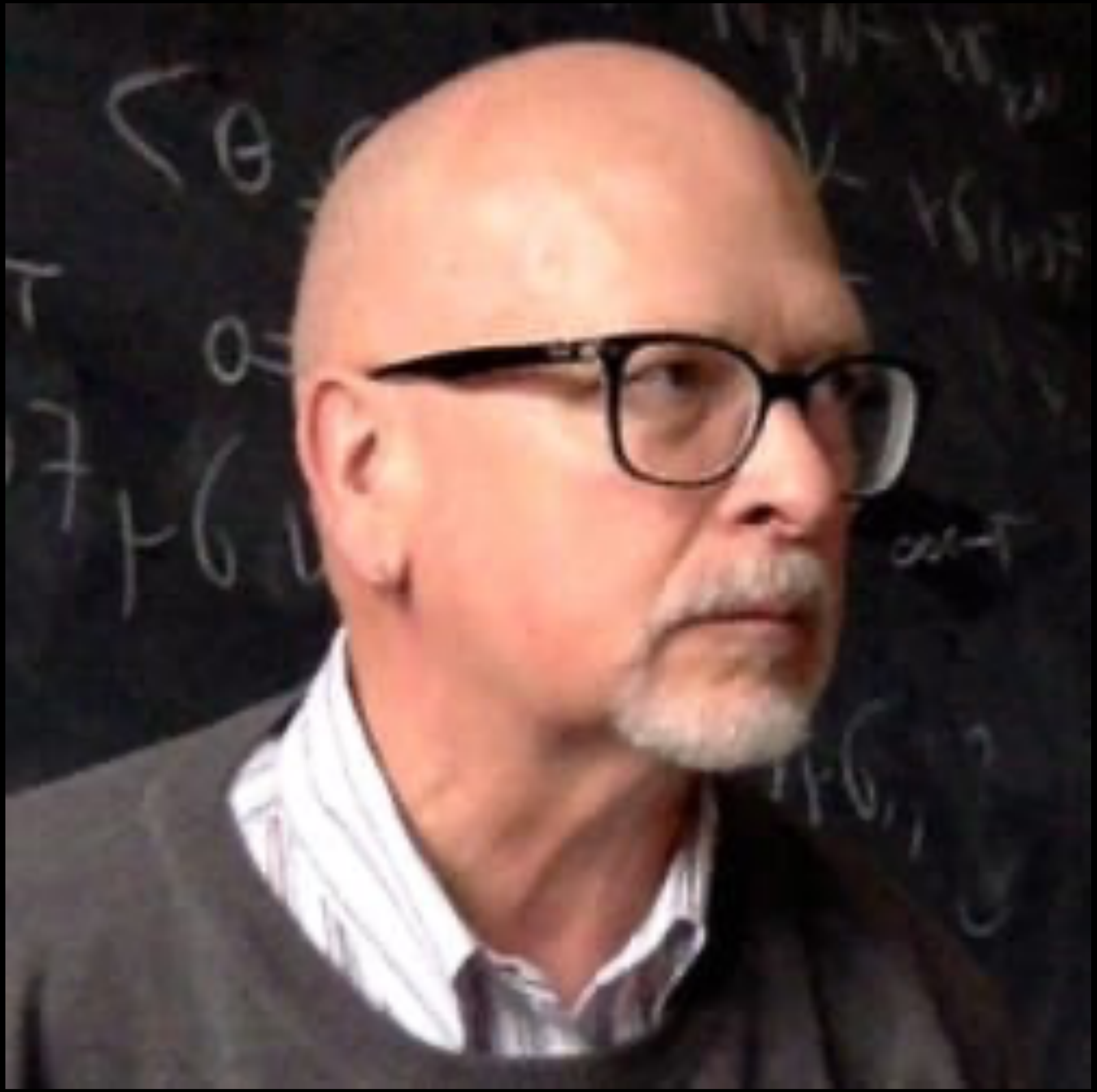
10^{-4} 10^{-9} 10^{-12}

Ek

Kunnen J. Turb. 2021

Ecke & Shishkina Annu. Rev. Fluid Mech 2023

Dedicated to



Charlie Doering 1956-2021

Keith Julien 1965-2024