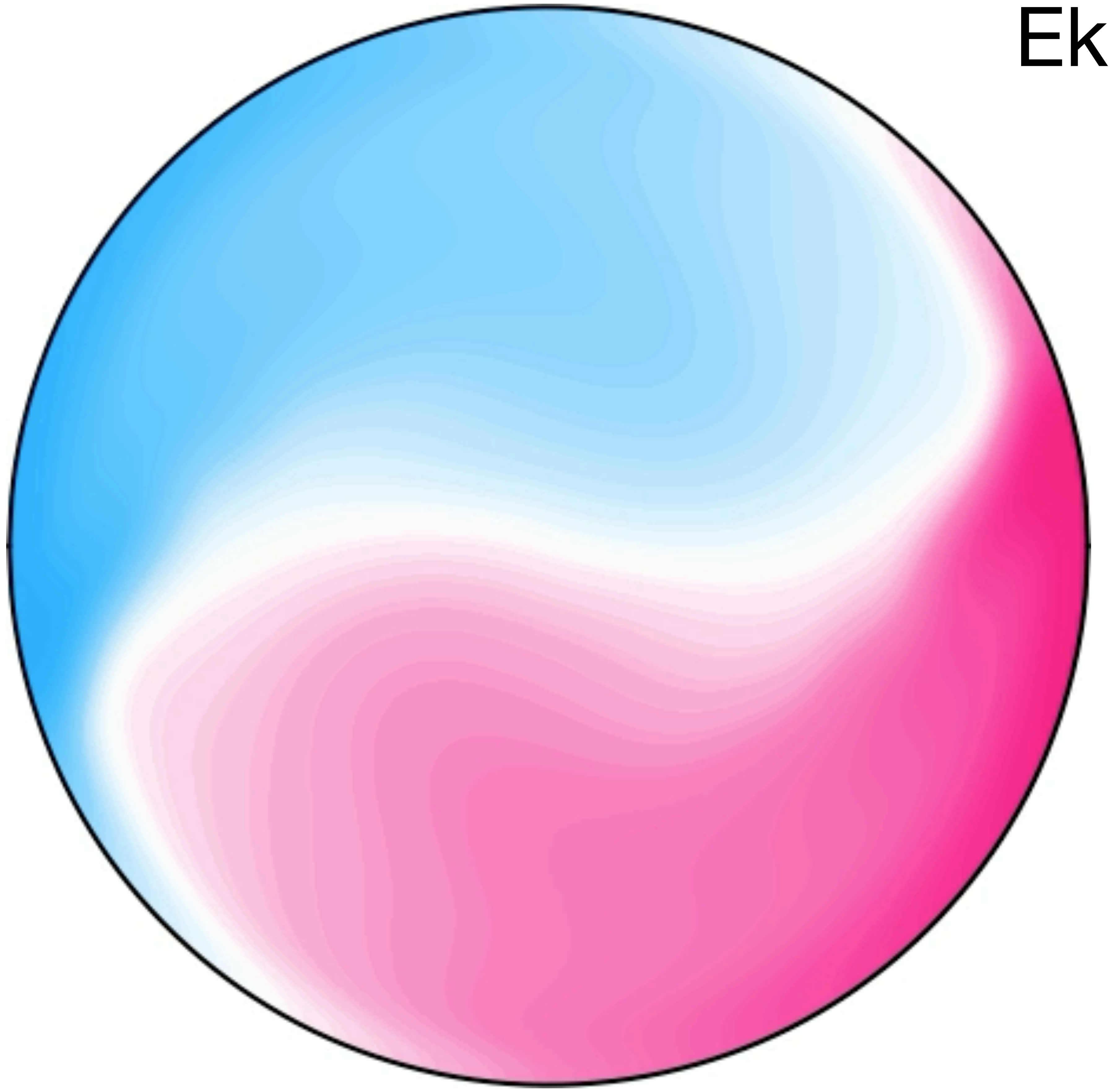
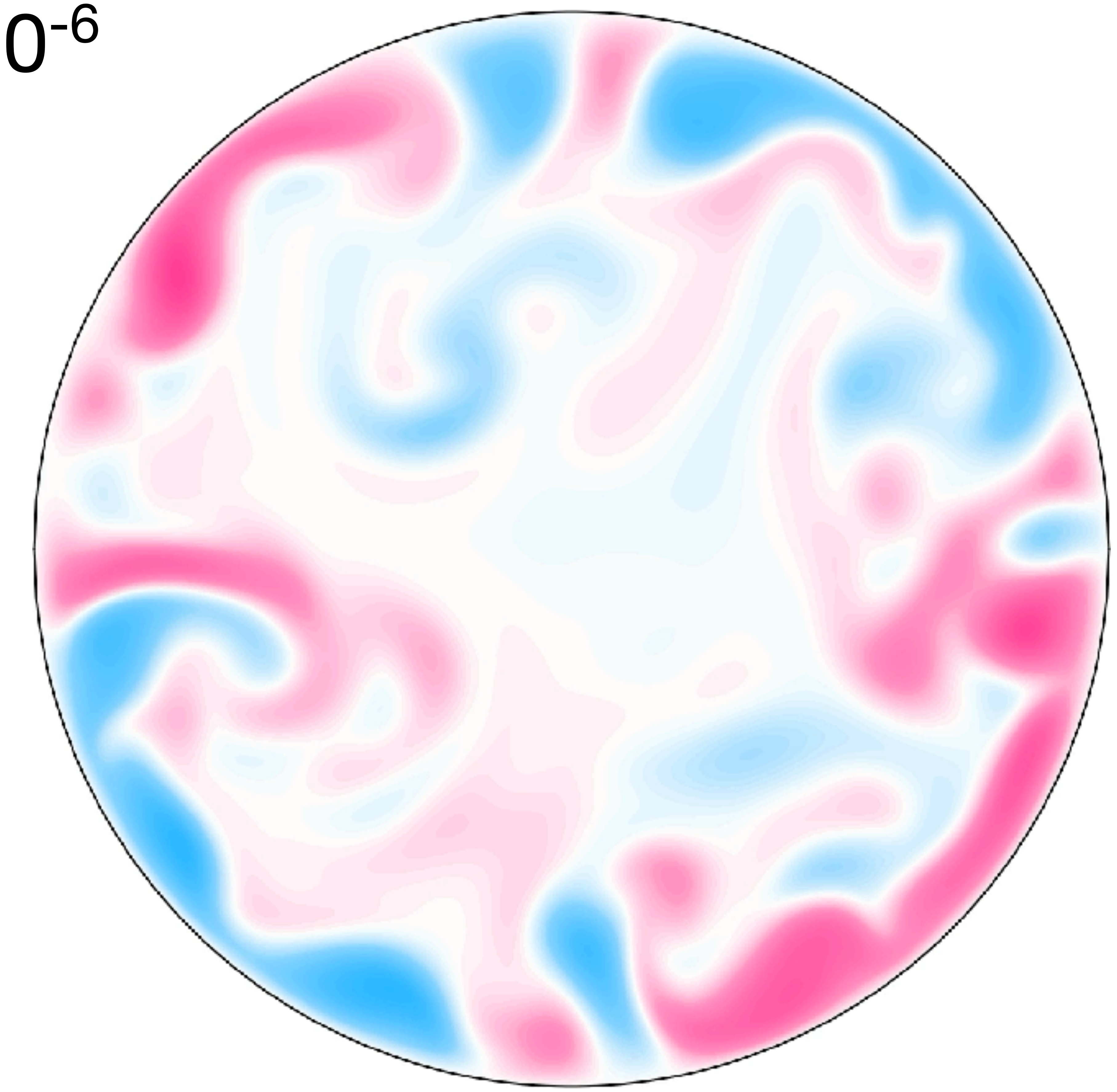
Everything you wanted (?) to know about wall modes

$Ra = 5 \times 10^8$

$Ek = 10^{-6}$



T



Insulating SW

Conducting SW

Heat Transport and Visualization - 1993

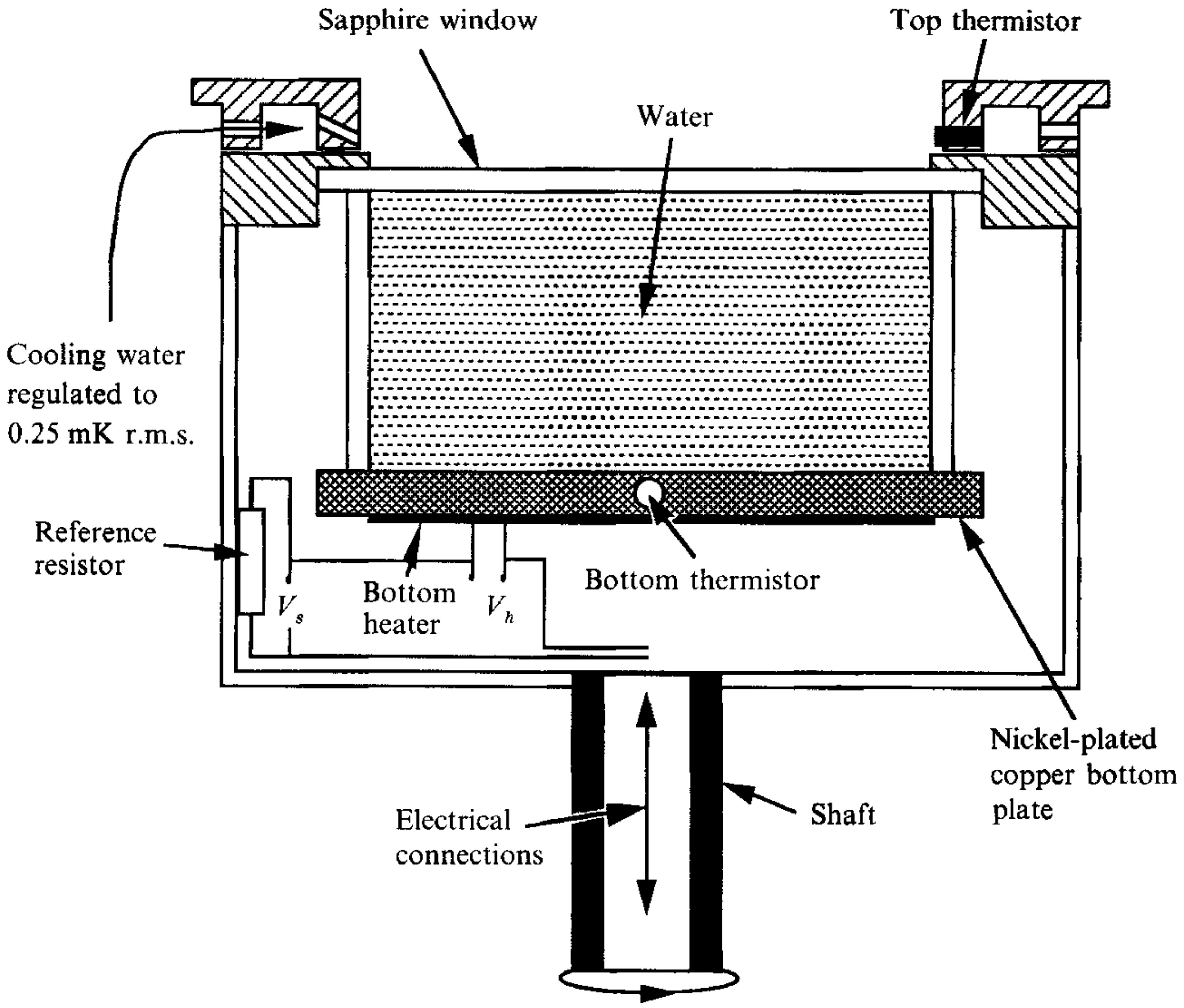
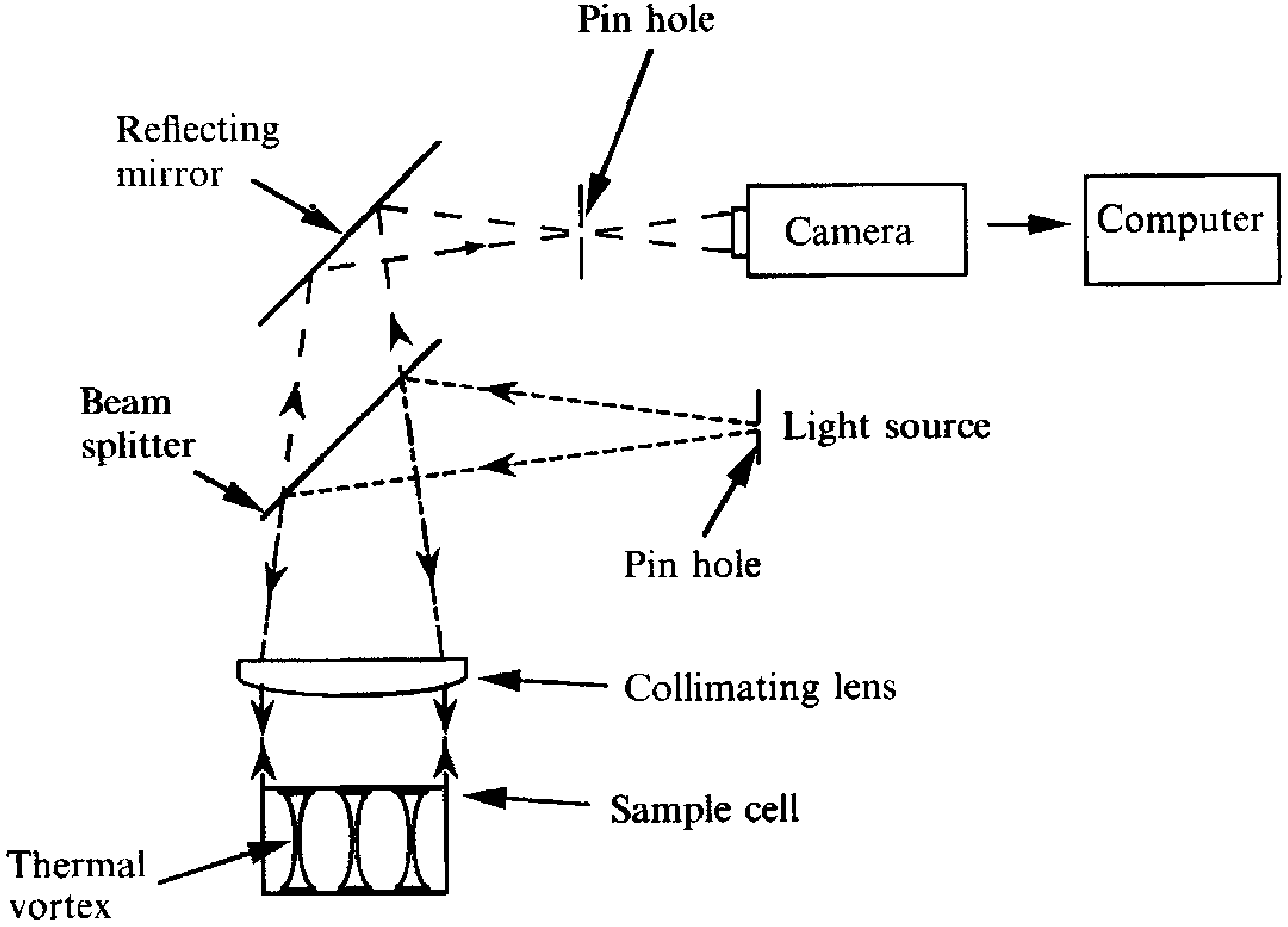


FIGURE 1. Illustration of the Rayleigh–Bénard convection cell.

Γ=D/H=1

Water: Pr = 6.4

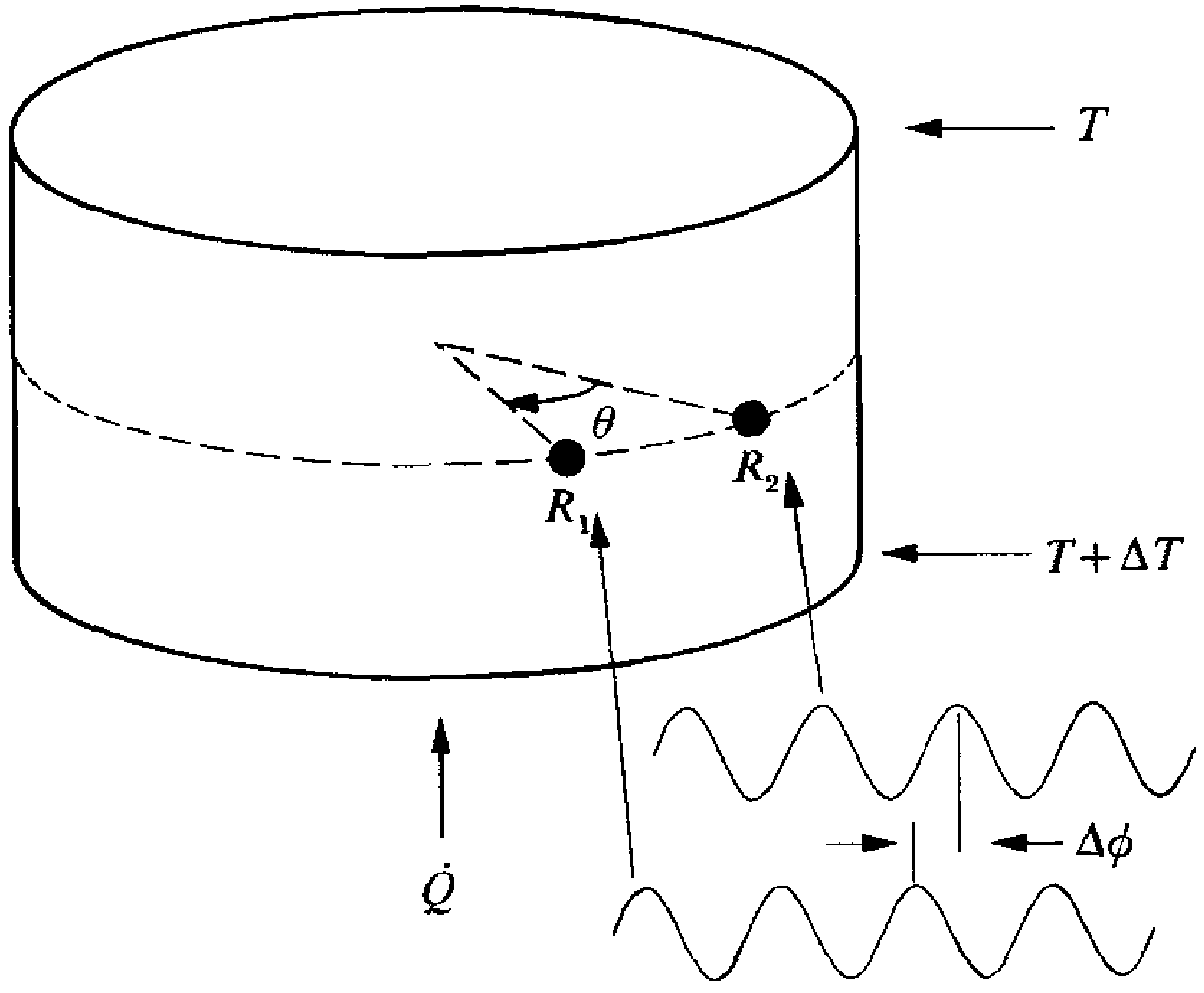


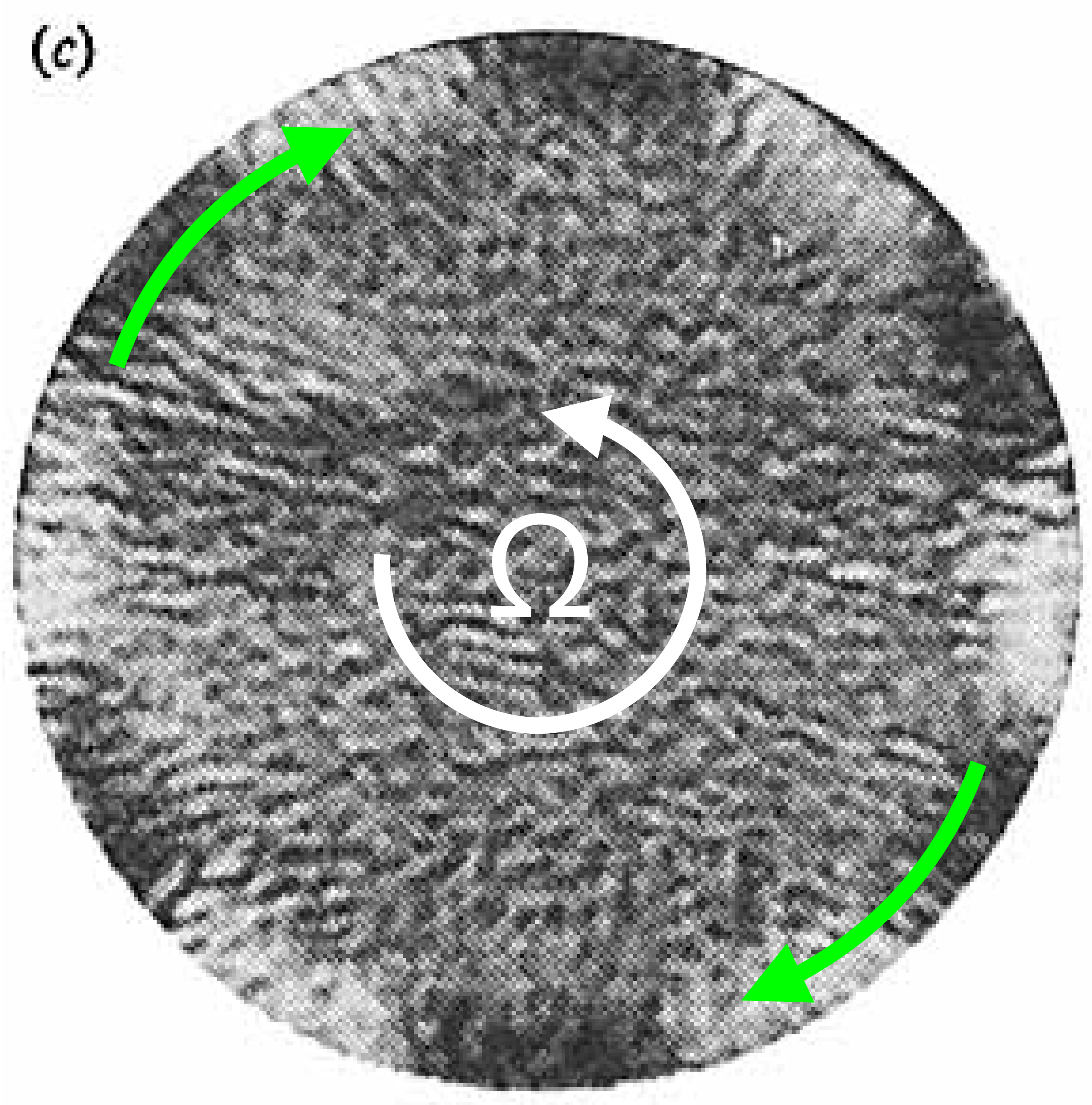
Heat Transport

Shadowgraph

Zhong, Ecke, Steinberg: PRL 1991, JFM 1993

$Ta = 1.8 \times 10^{7}$ $Ek = 2.3 \times 10^{-4}$





$Ra = 1.5 \times 10^{5}$

$Ro = 0.04$

retrograde precession

Azimuthal Traveling Wave

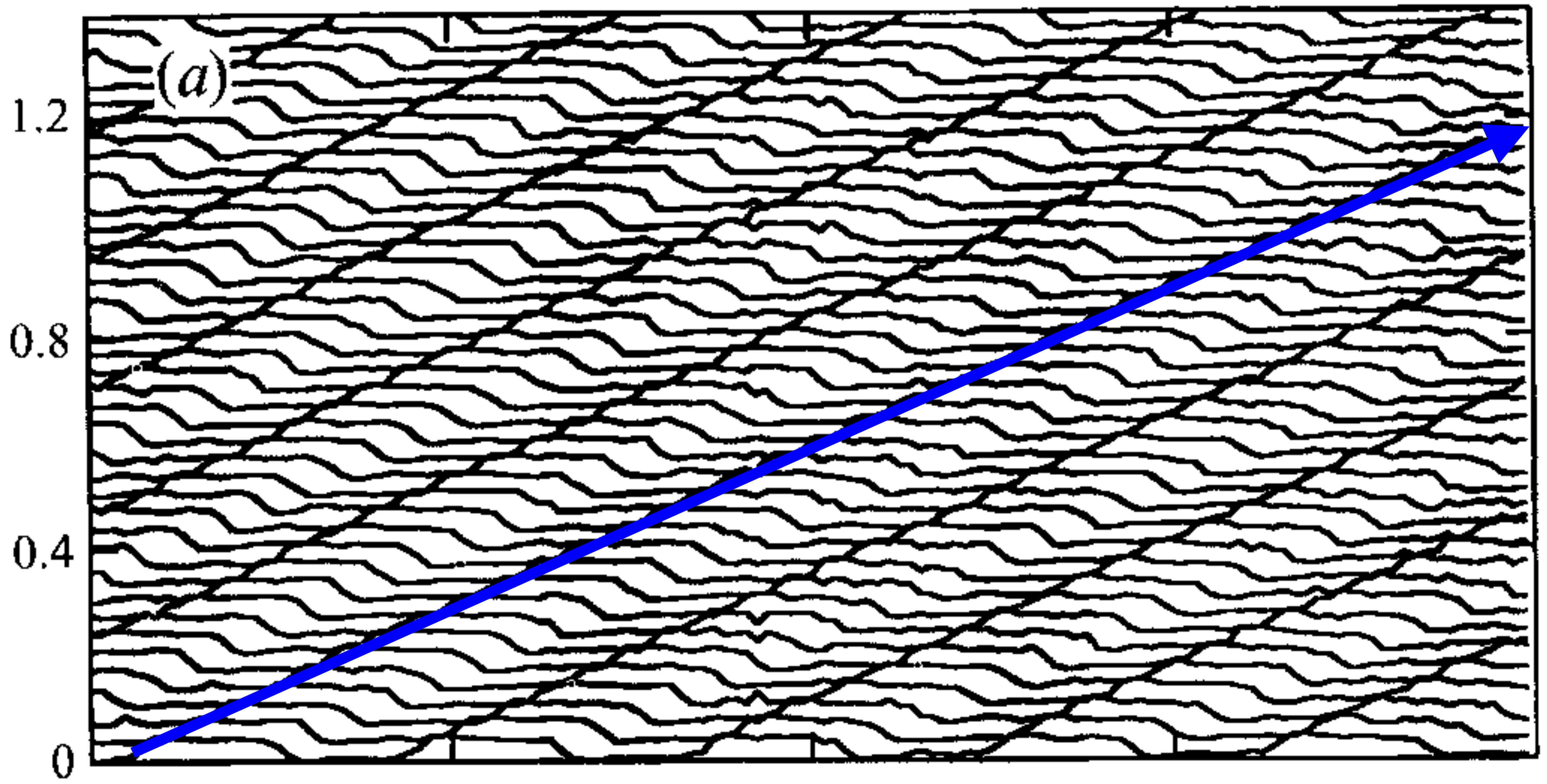
E. Knobloch

$\Omega = 0 \quad \phi \rightarrow -\phi$

$\Omega > 0 \quad \phi \not\rightarrow -\phi$

Ecke et al, EPL 1992

Time



300

Time (Ω^{-1})

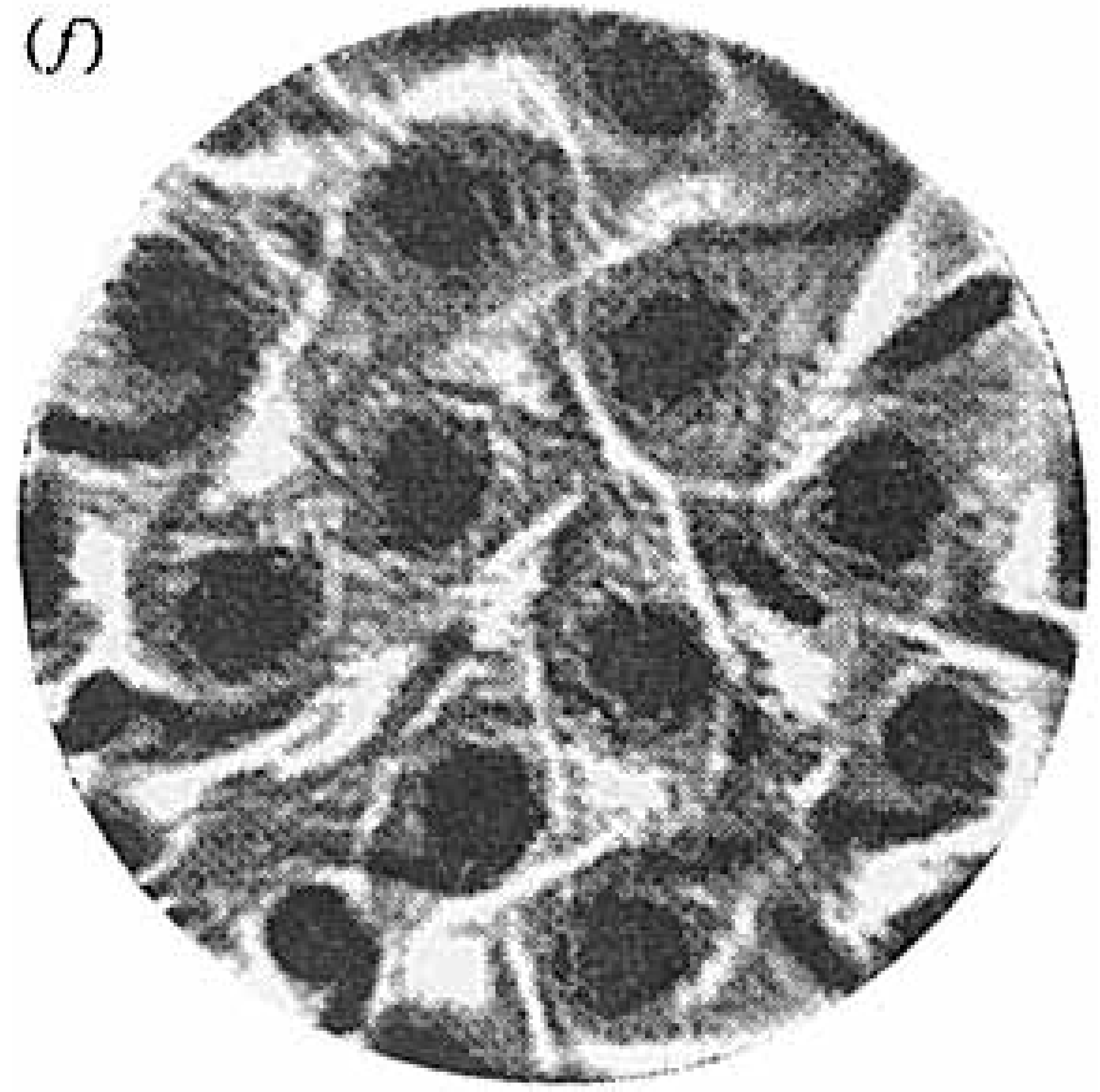
0

0 ϕ 2π

Sidewall + Bulk State

Ra = 5.8 x 10^5

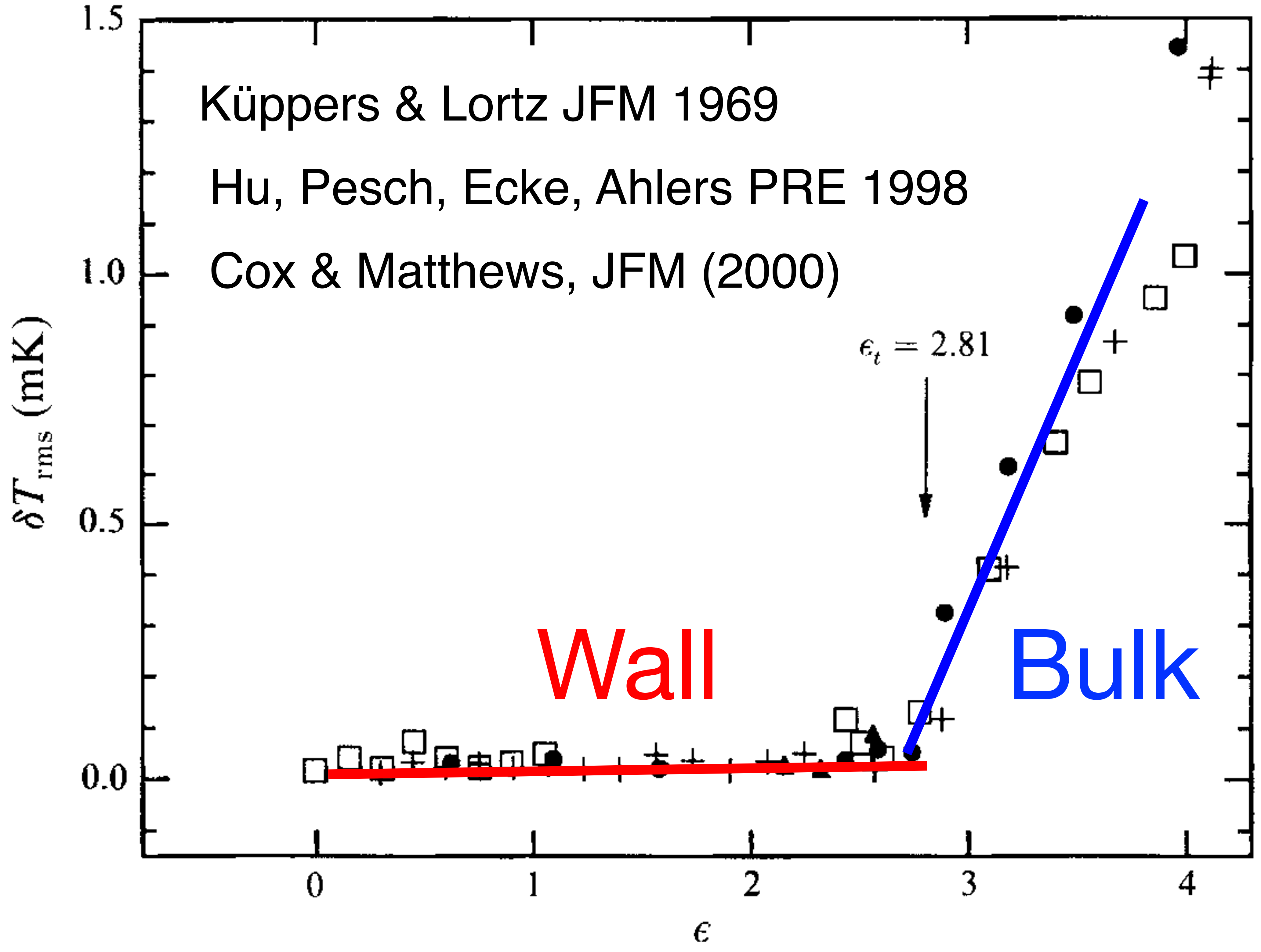
Ro=0.07

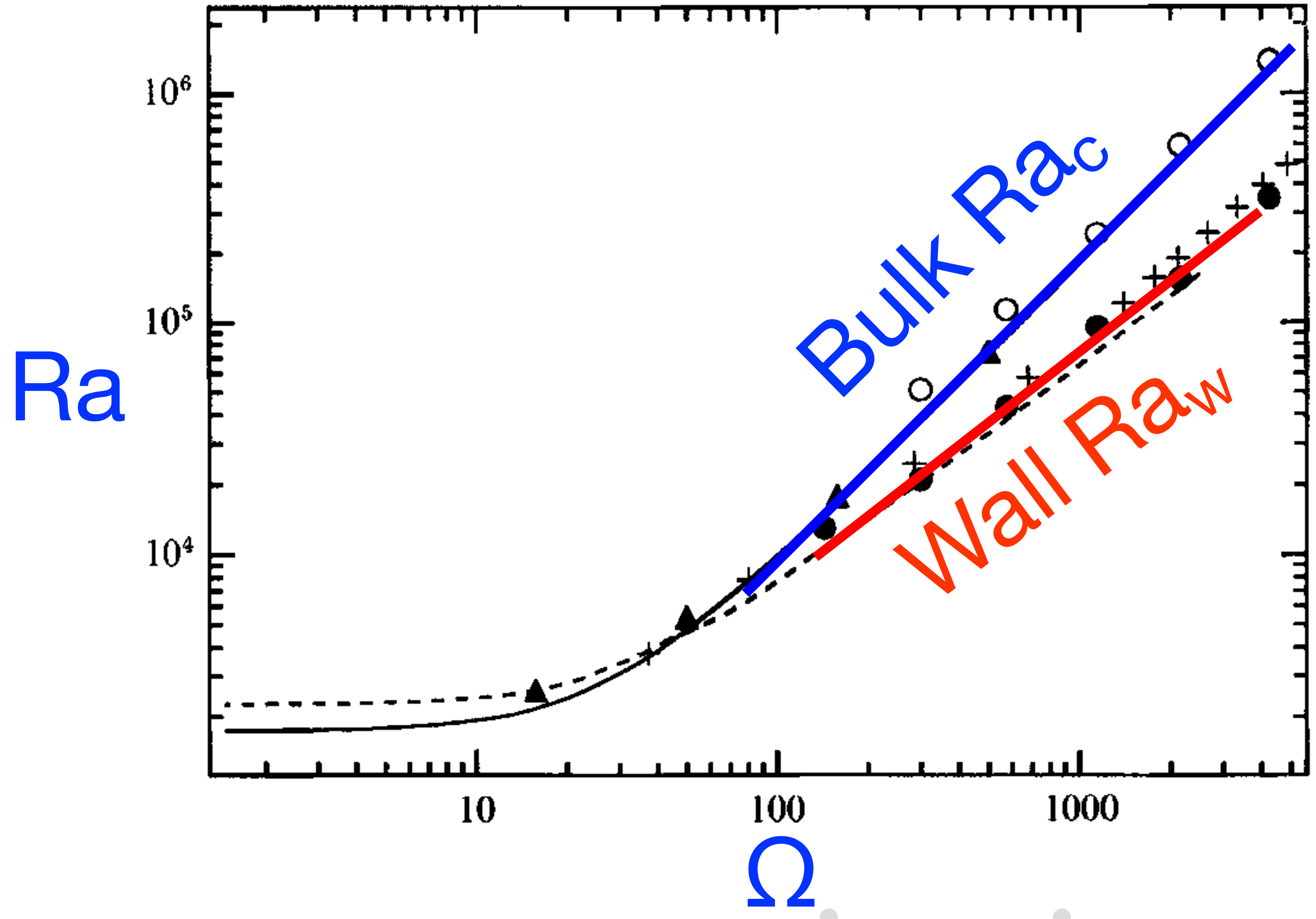


Bulk State

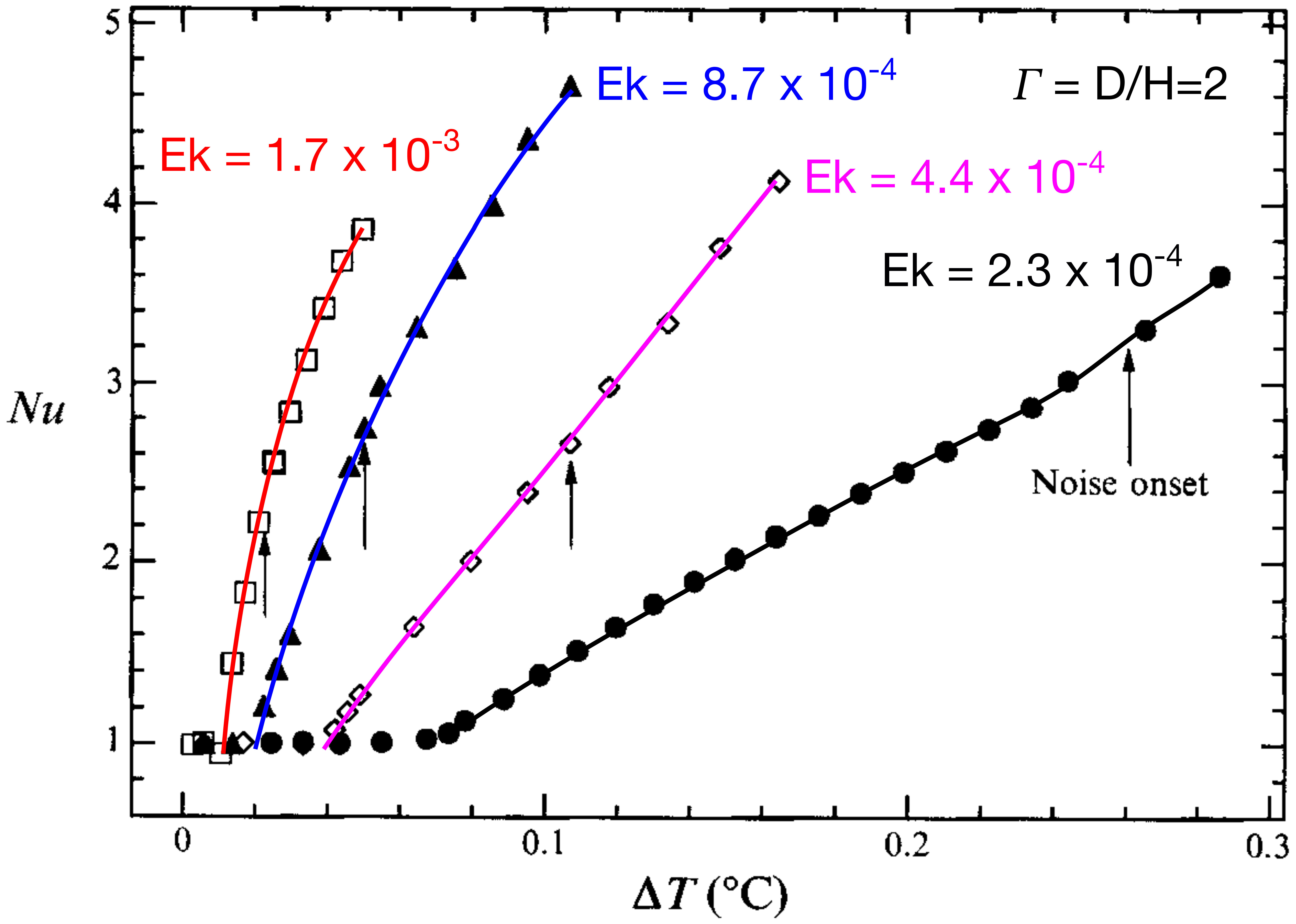
Ra = 2.6 x 10^6

Ro = 0.15





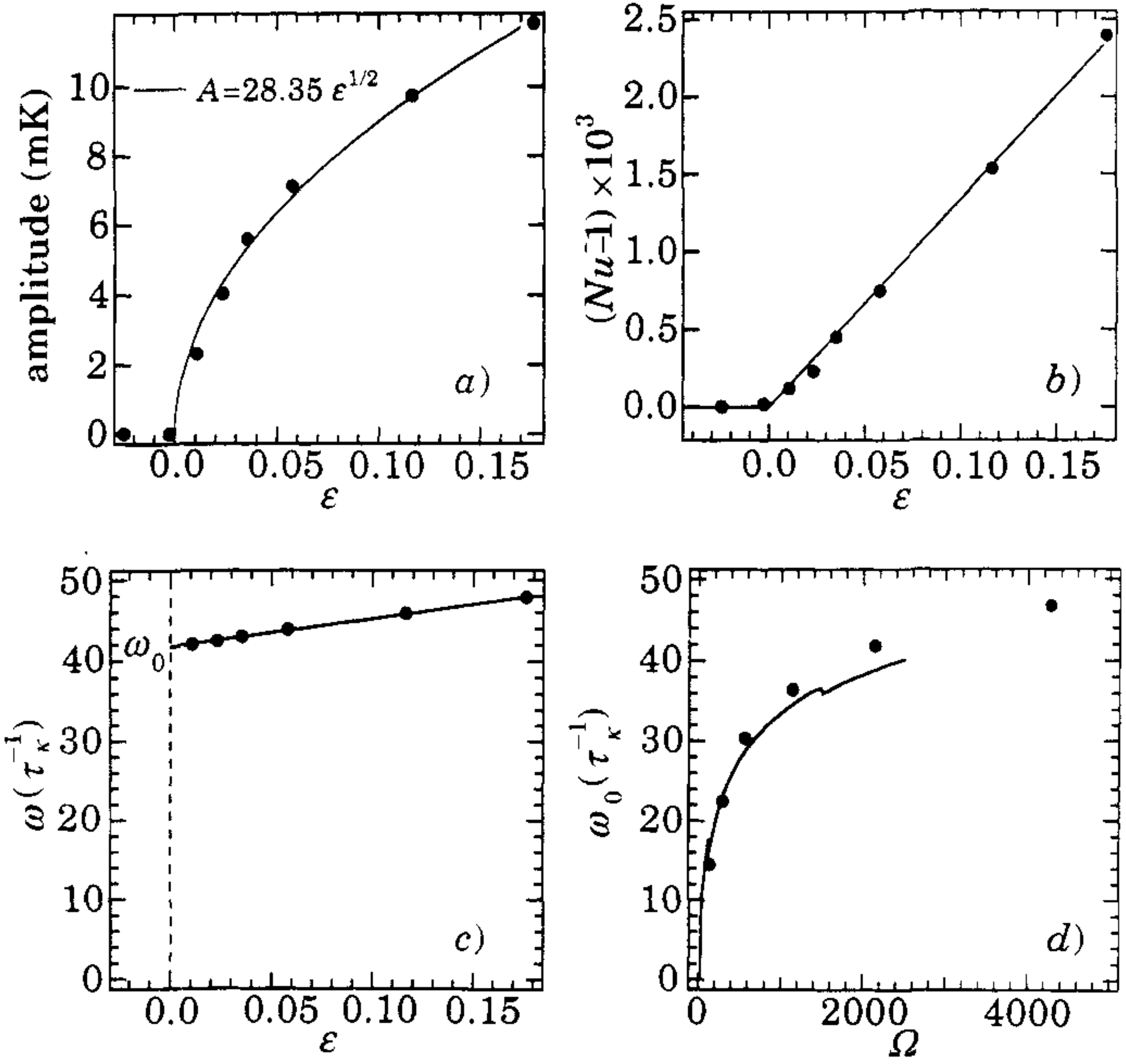
Some things aren't obvious. They depend on Γ



Noise onset indicates bulk state

Bulk state occurs further from the wall mode state as Ω increases

Hard to detect change of Nu slope at onset of bulk state



Non-rotating

$$\phi \to -\phi$$

Stationary bifurcation

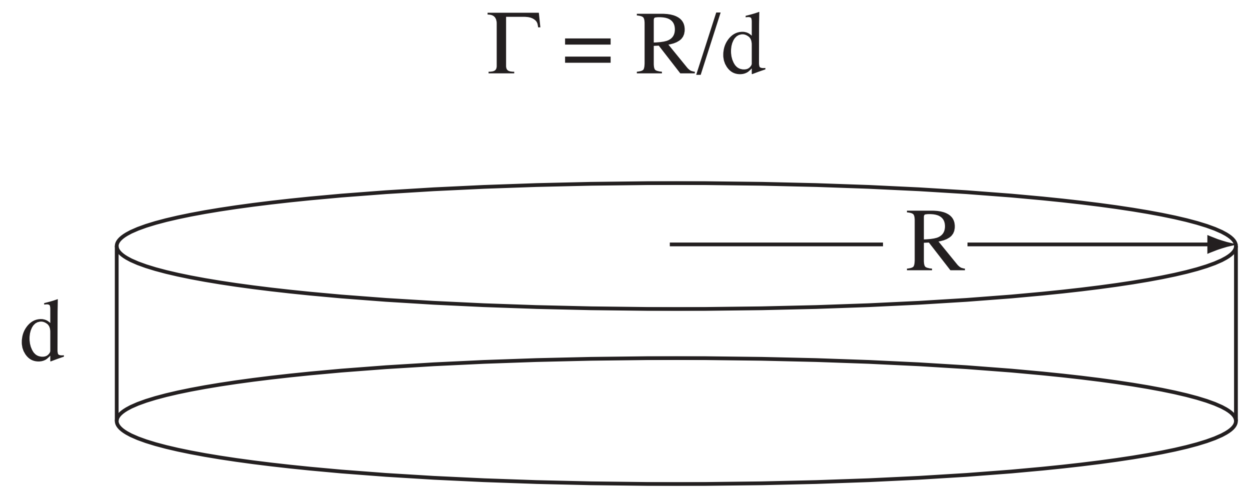
Rotating

$$\phi \not\to -\phi$$

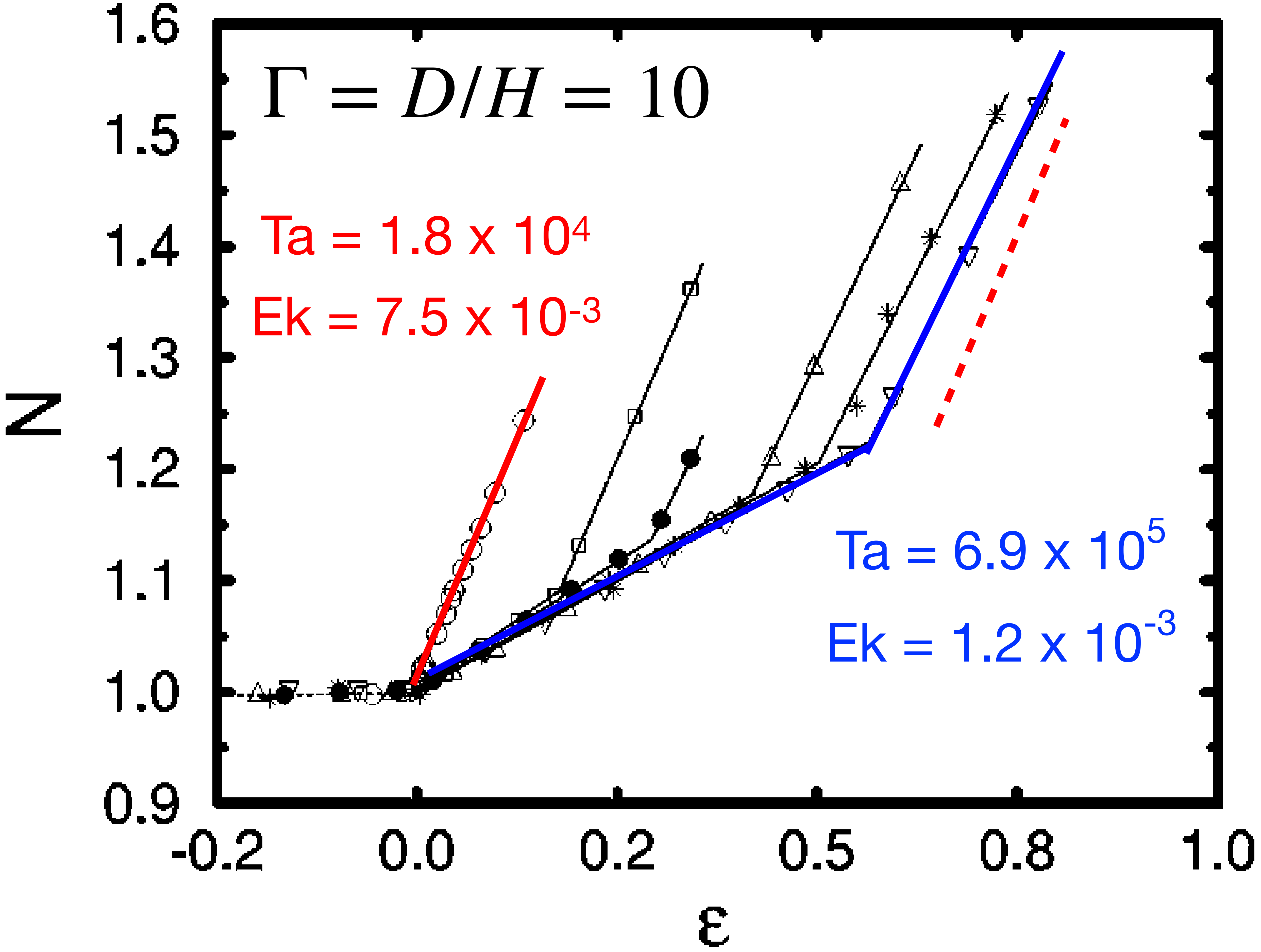
Traveling wave

Ecke, Zhong, Knobloch, EPL (1992)

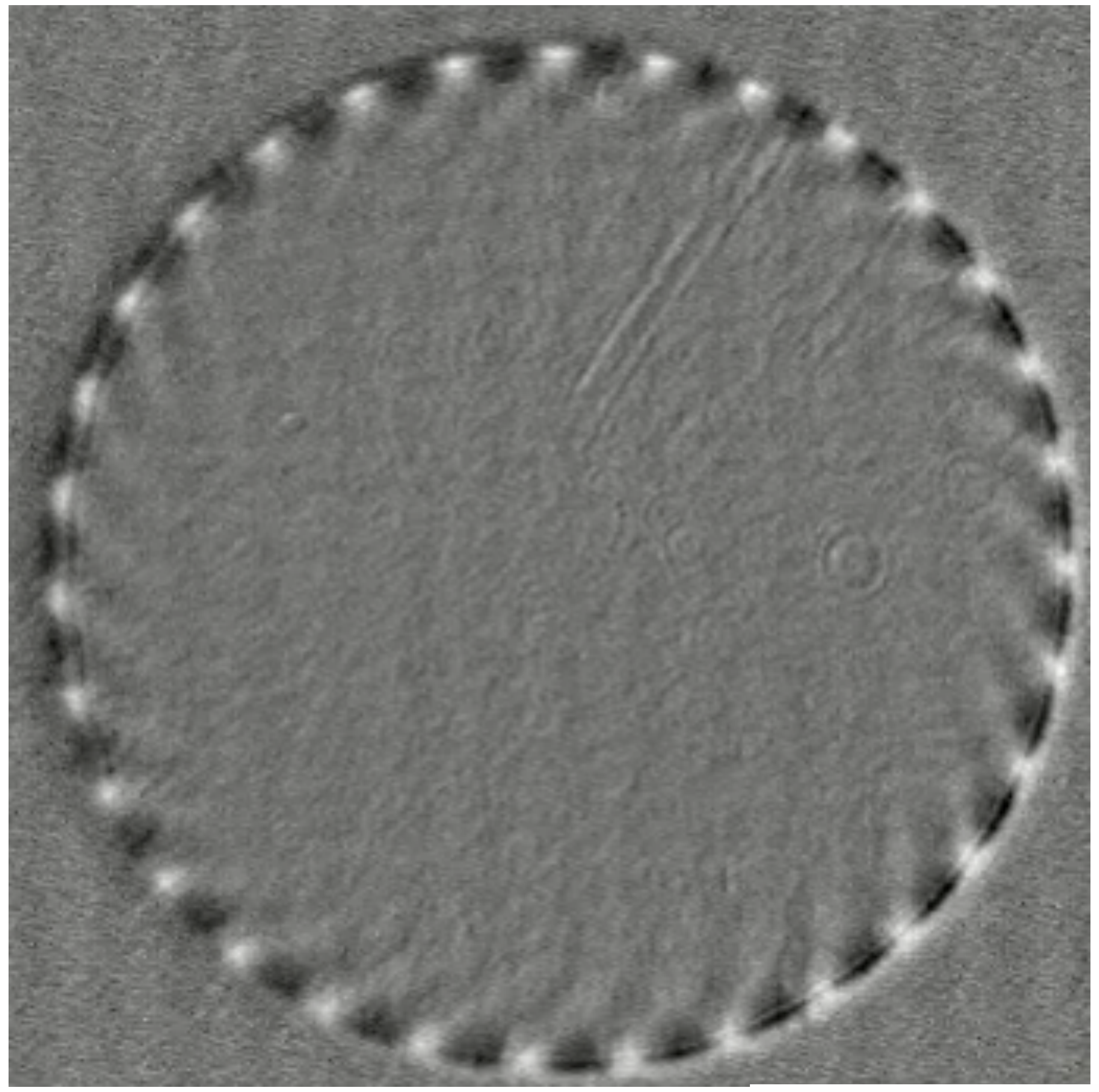
Sidewall States - Clarified



Goldstein et al 1993
Kuo & Cross 1993
Herrmann & Busse 1993

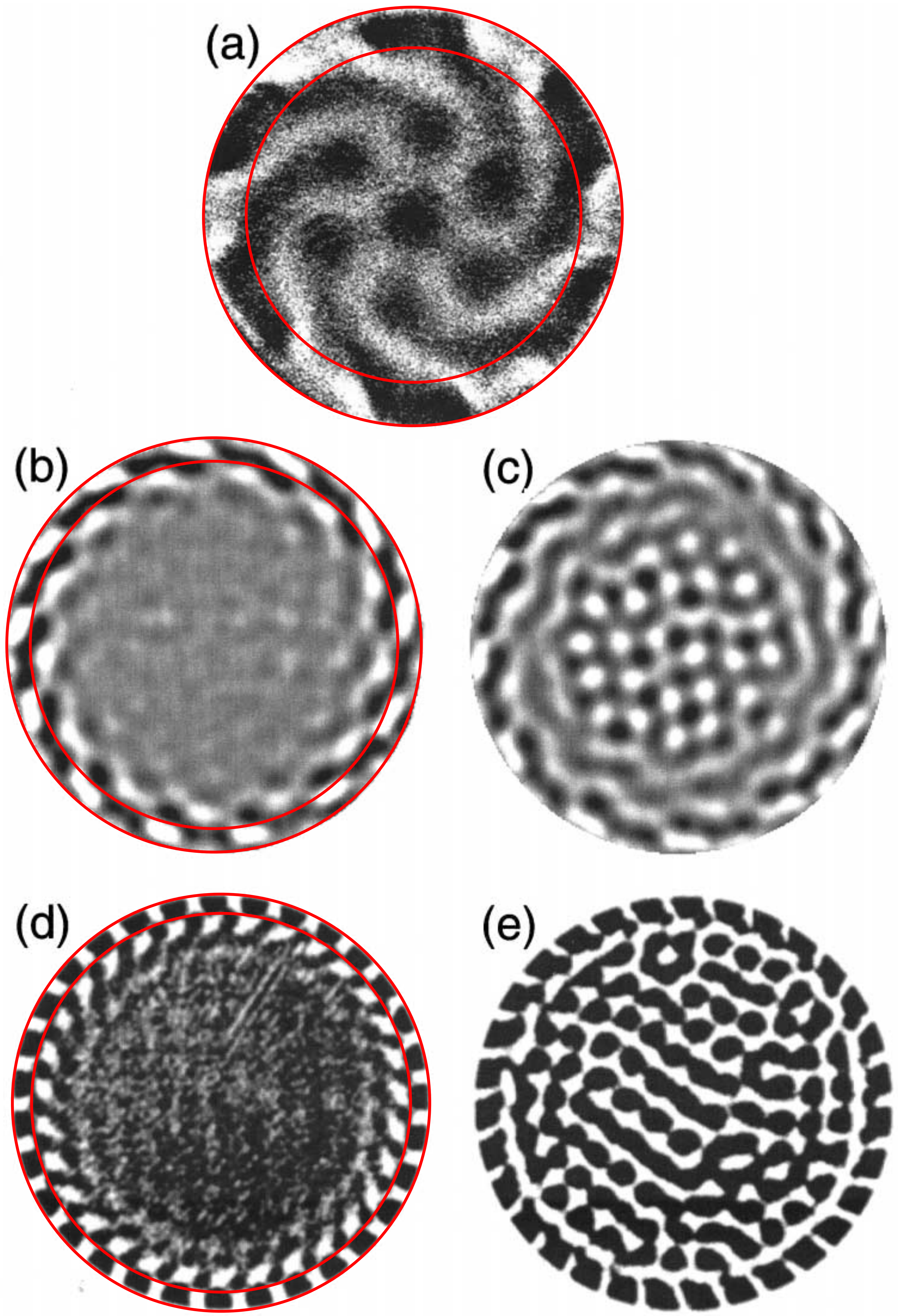


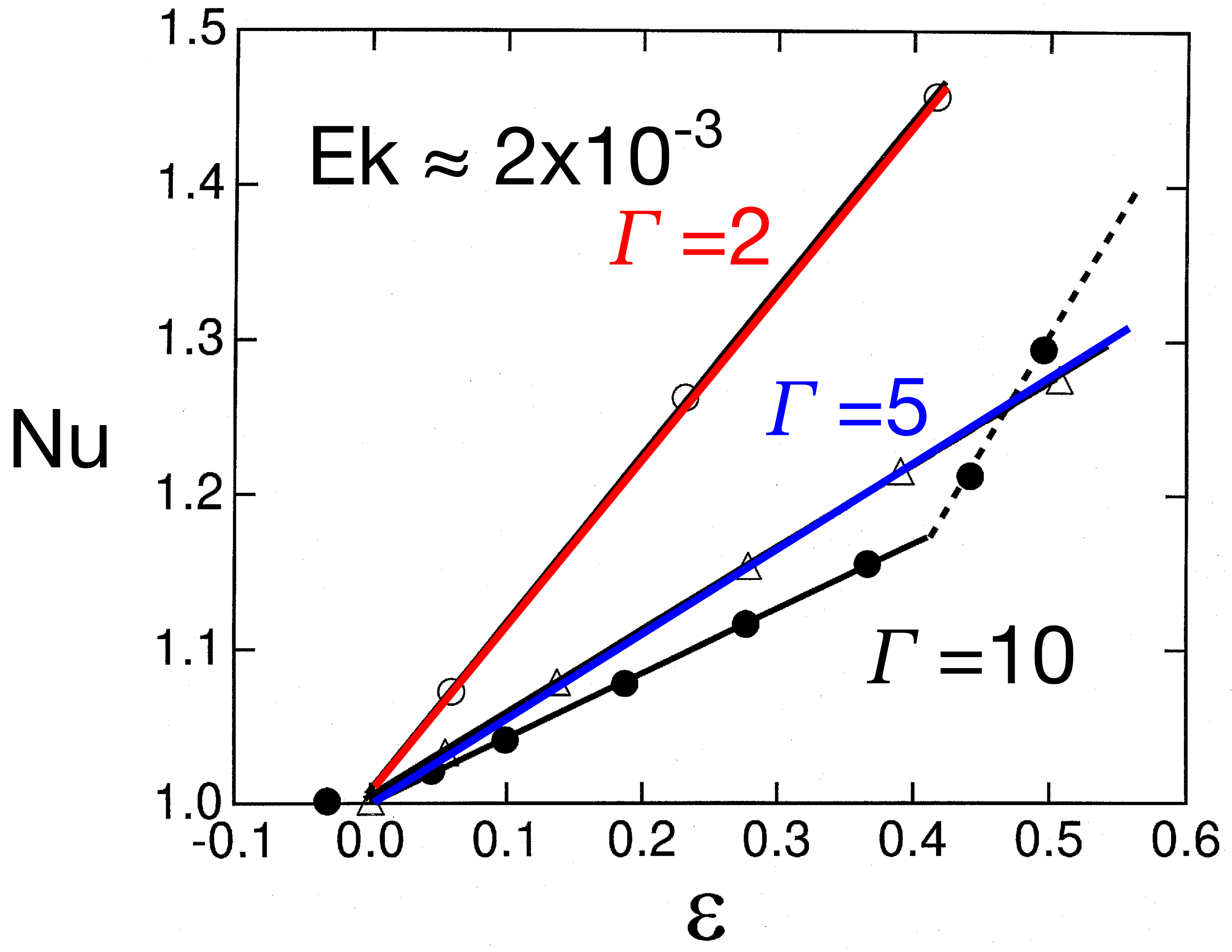
Liu & Ecke: PRL 97 & PRE 99

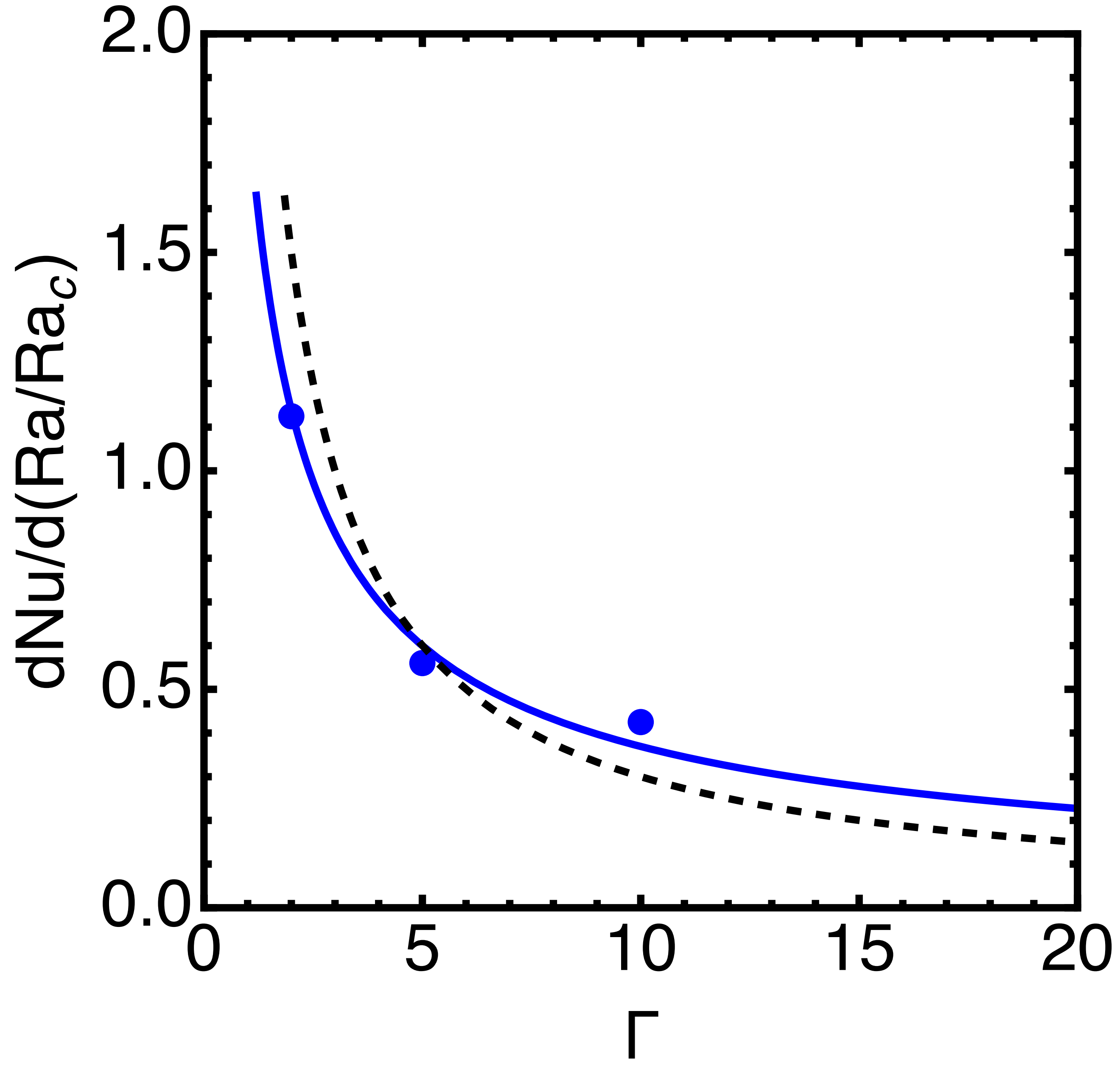


Wall Mode

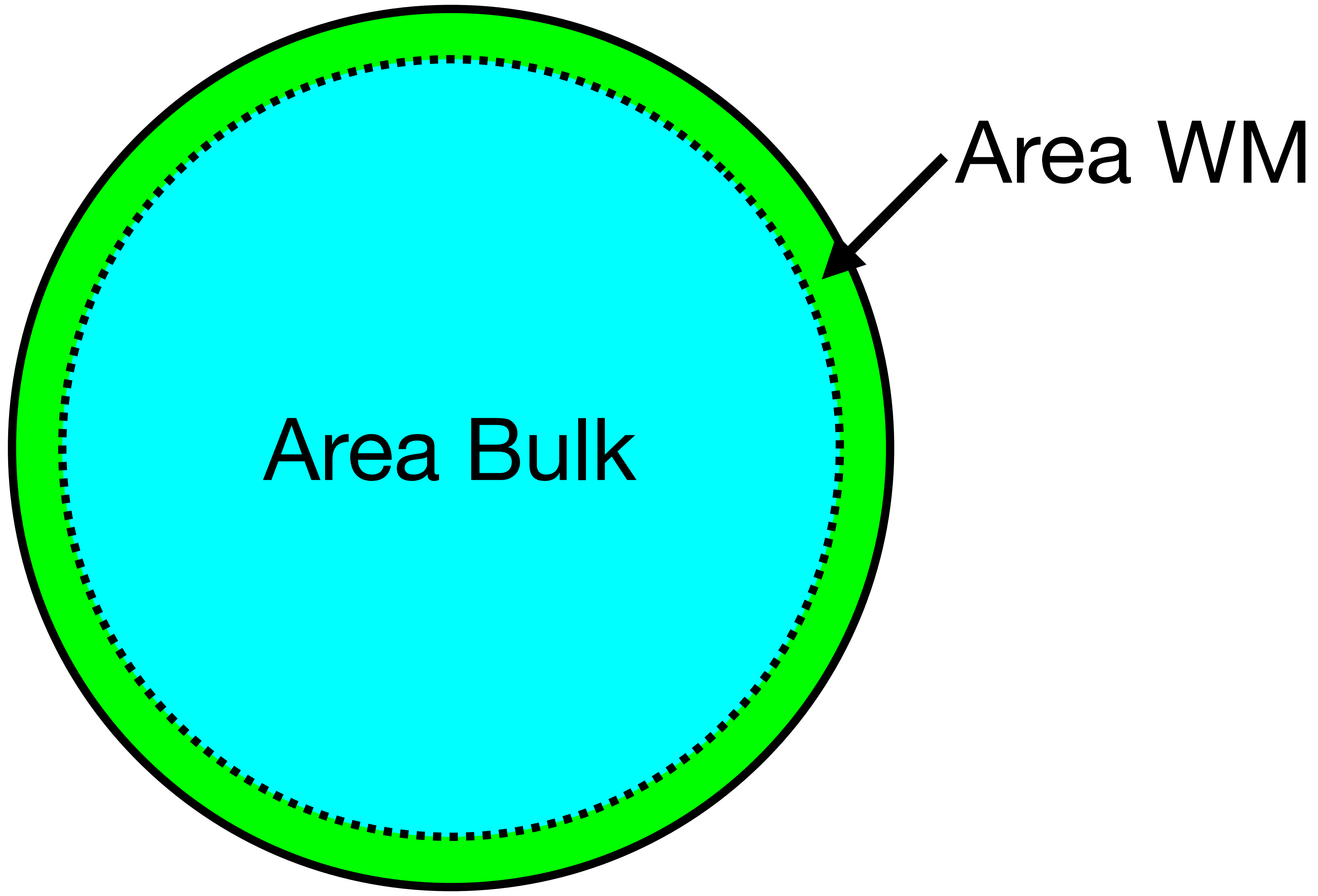
Wall + Bulk

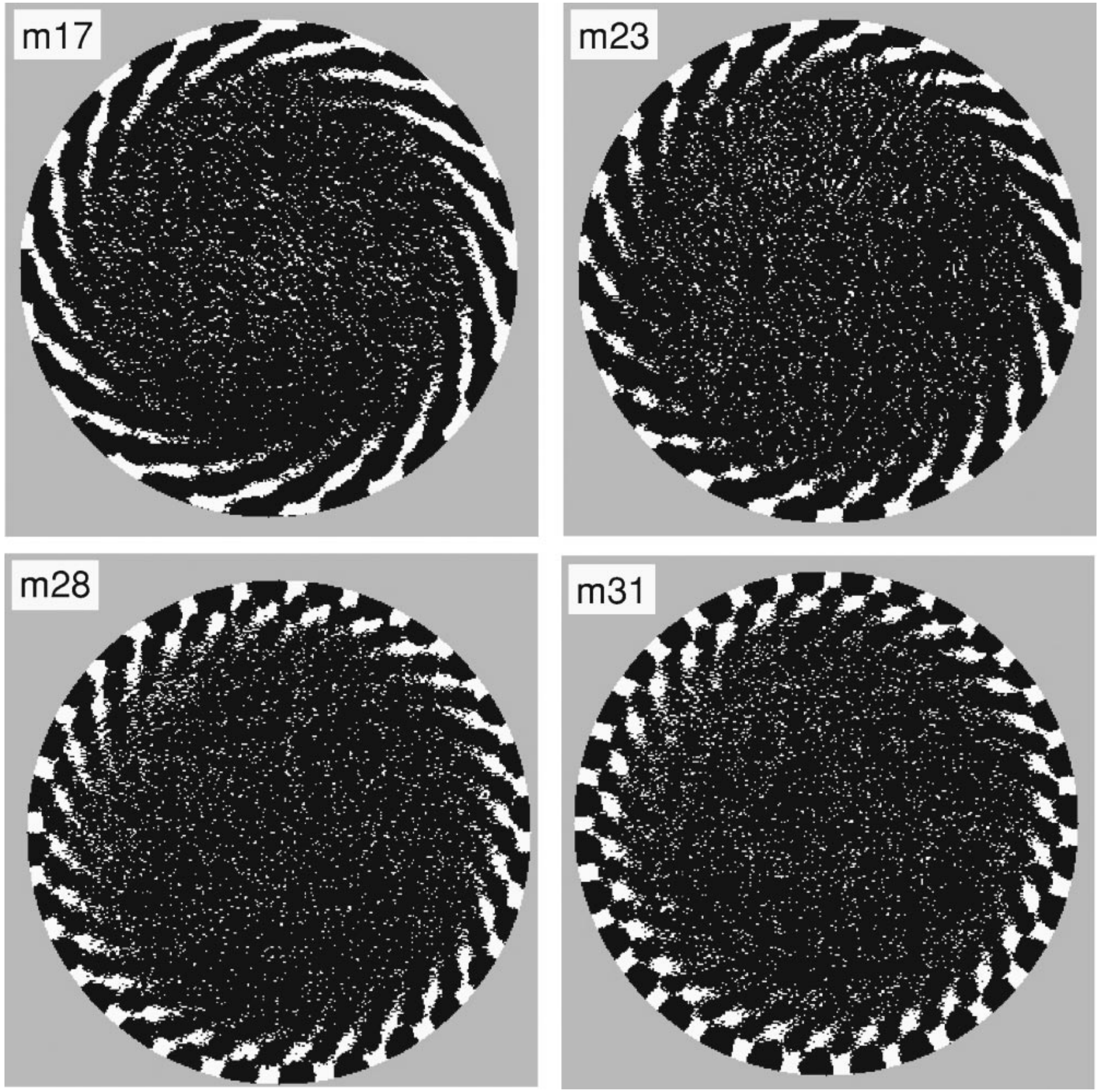






$$Nu_{wm} \sim a/\Gamma - b/\Gamma^2$$



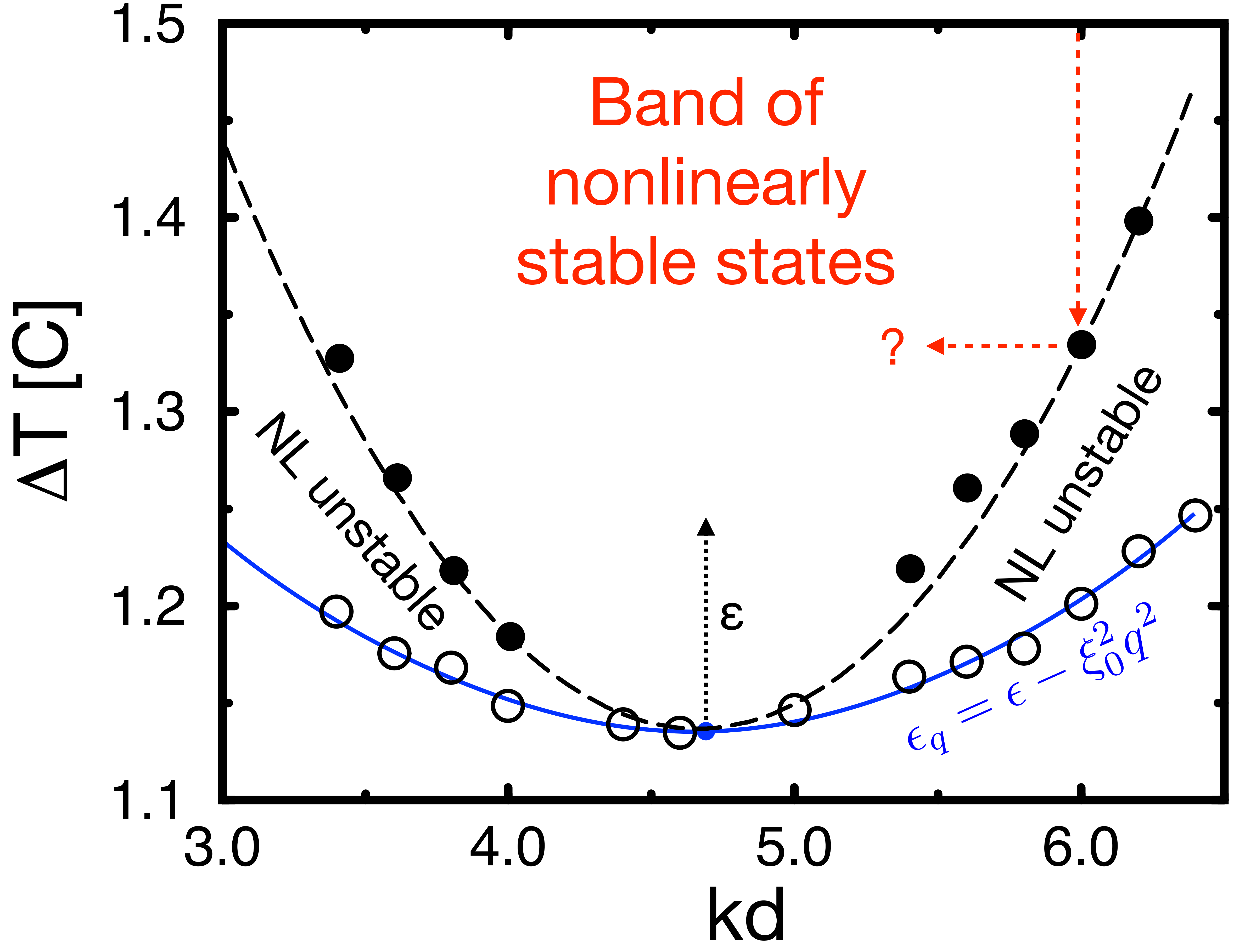


By varying conditions one can prepare large range of m values that are stable.

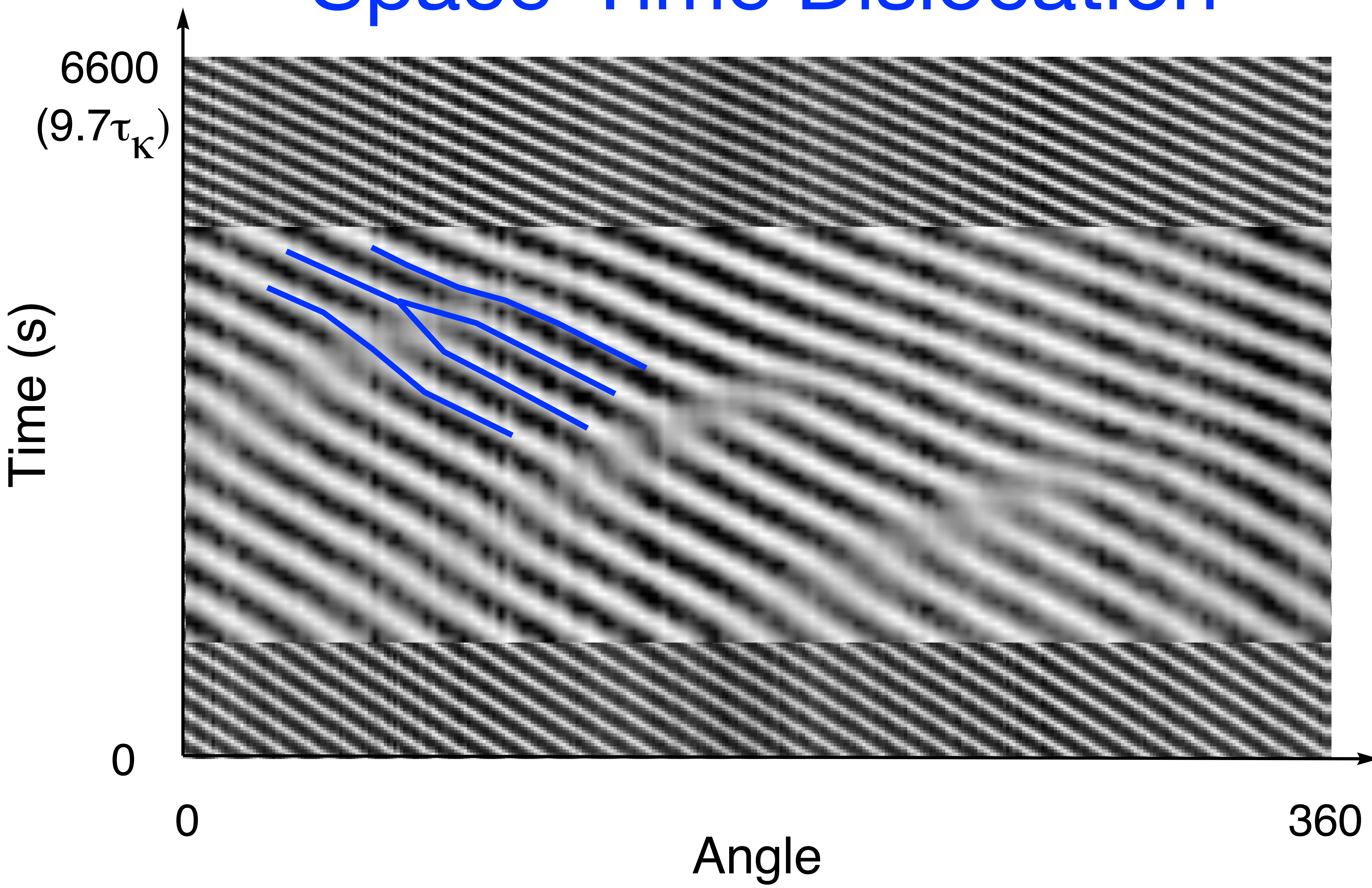
Slow or rapid changes in Ra

Slow or rapid changes in rotation rate at Ra > Ra_c

1-D Pattern Formation



Space-Time Dislocation



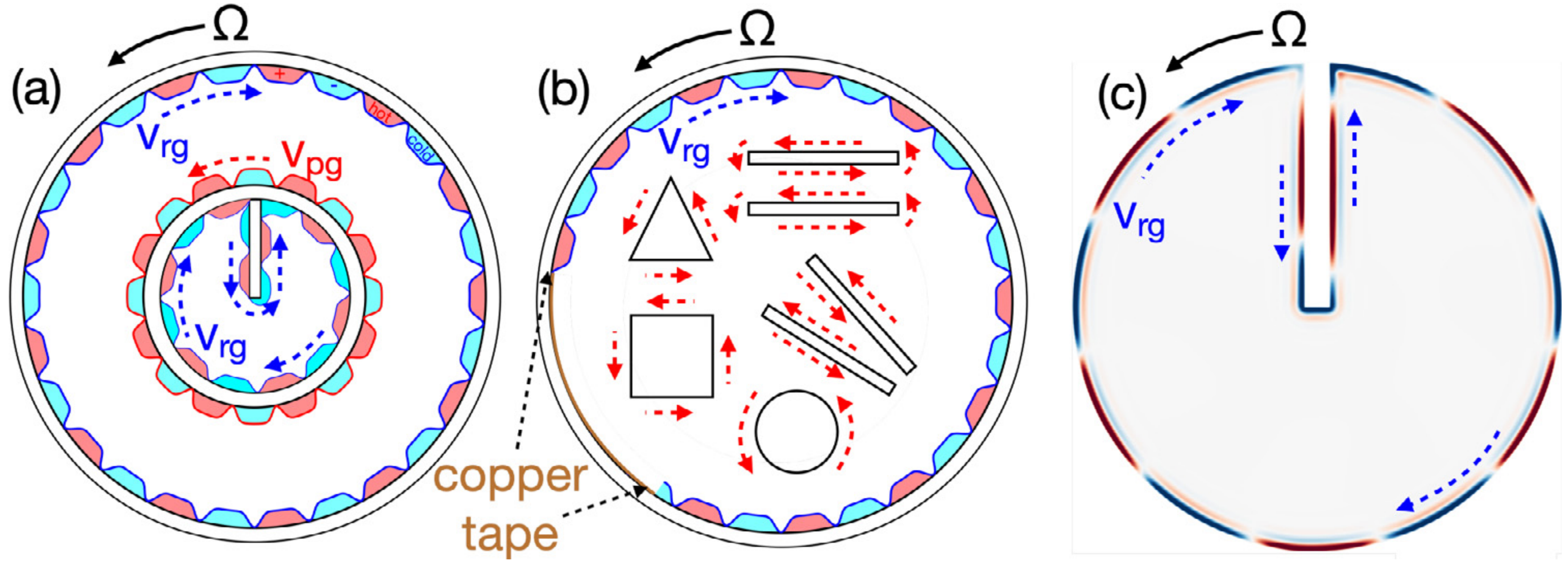
$$\tau_0 \left(\partial_t A + s\partial_x A\right) = \epsilon \left(1 + ic_0\right) A + \xi_0^2 \left(1 + ic_1\right) \partial_{xx} A$$

$$-g \left(1 + ic_3\right) |A|^2 A$$

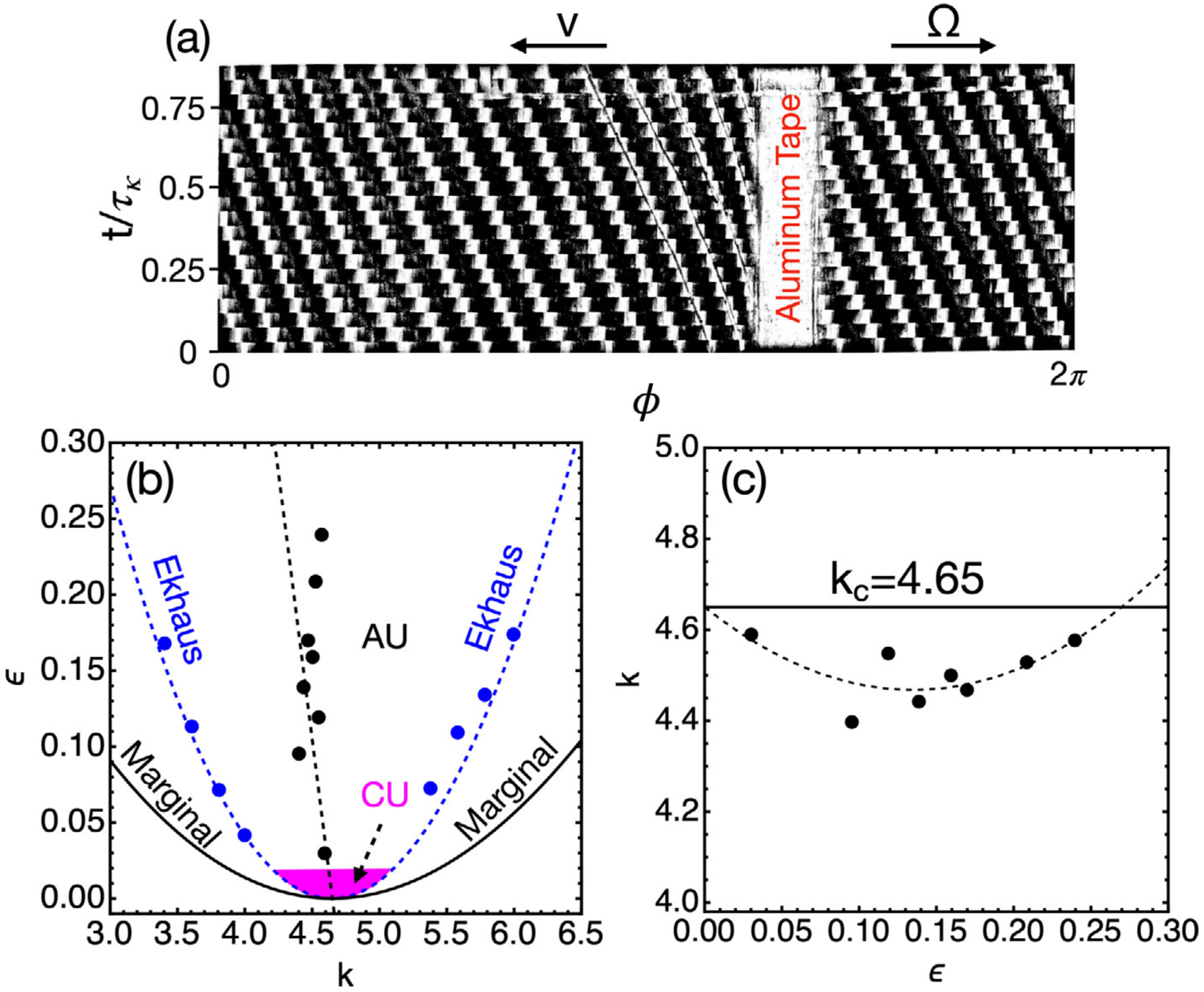
Complex Ginzburg-Landau Equation

Liu & Ecke: PRL 97 & PRE 99

Some useful but incomplete information about Wall Modes



Use conducting tape to break azimuthal periodicity



Wall modes don't care about geometry

Ning Li

Ecke, Phys. D (2023)

Where have all the wall modes gone?

2010: Some new experiments with more rapid rotation rates

2014: Me: “But you have to include the wall mode contribution to the heat transport.” Jon: “We don’t see any wall modes!” Hmmm. I wondered why?

2018: Confusing results from Göttingen SF6 rotating convection experiment Γ=1/2 - bimodal distribution.

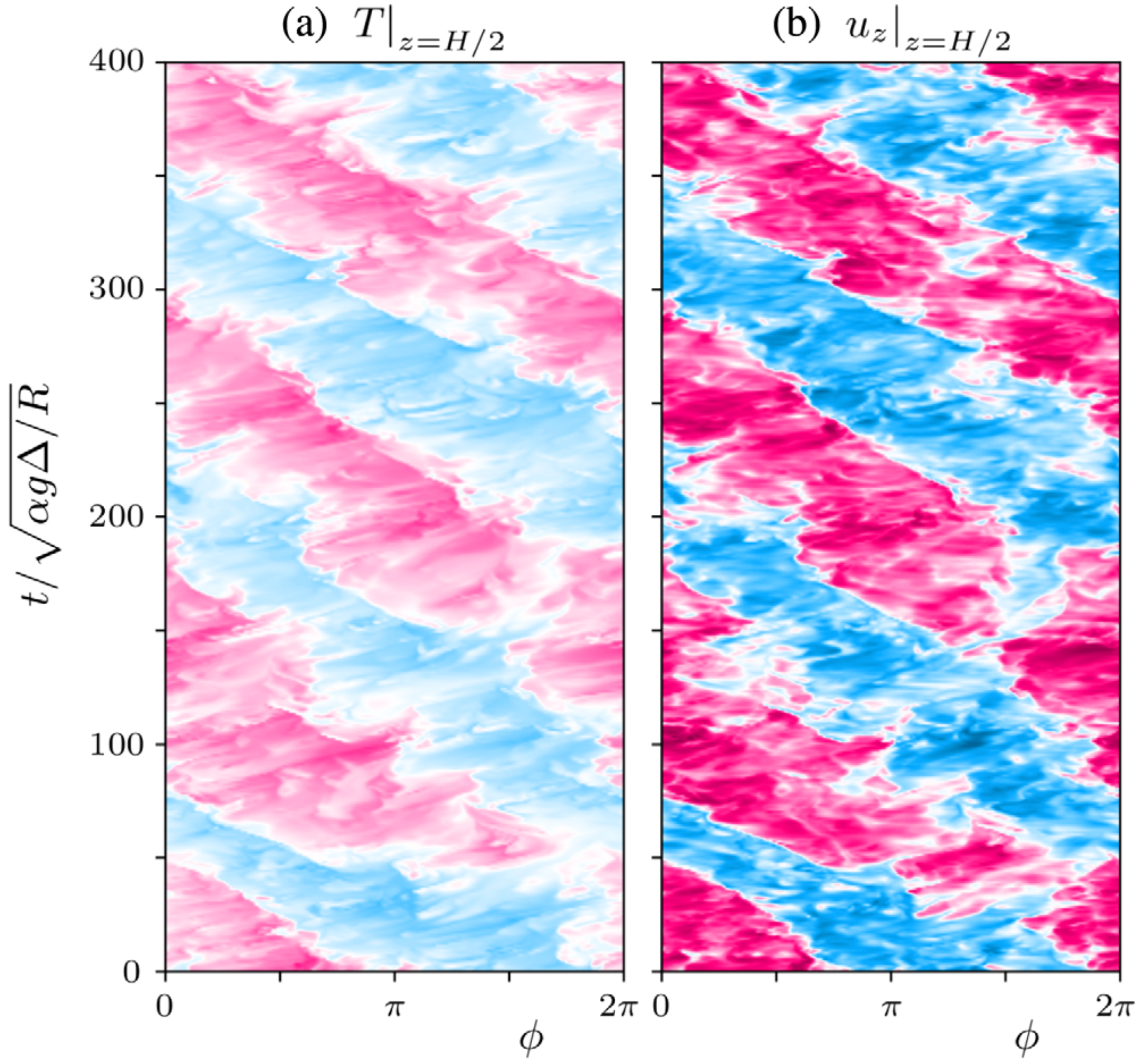
DNS by Zhang and Shishkina.

PHYSICAL REVIEW LETTERS **124,** 084505 (2020)

Boundary Zonal Flow in Rotating Turbulent Rayleigh-Bénard Convection

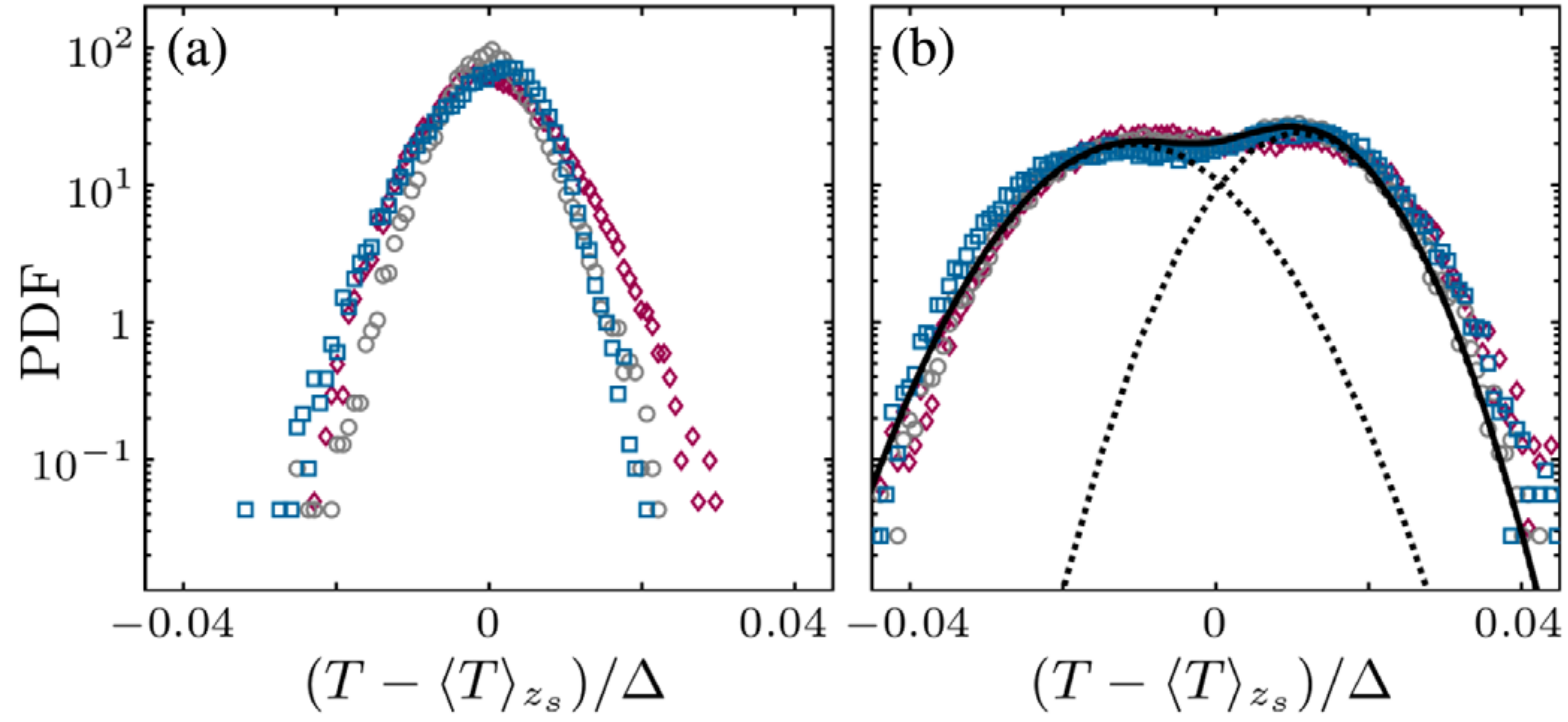
Xuan Zhang,[1] Dennis P. M. van Gils,[1,2] Susanne Horn,[1,3,4] Marcel Wedi,[1] Lukas Zwirner,[1] Guenter Ahlers,[1,5] Robert E. Ecke,[1,6] Stephan Weiss,[1,7] Eberhard Bodenschatz,[1,8,9] and Olga Shishkina[1,*]
[1]Max Planck Institute for Dynamics and Self-Organization, 37077 Göttingen, Germany
[2]Physics of Fluids Group, J.M. Burgers Center for Fluid Dynamics, University of Twente, P.O. Box 217, 7500 AE Enschede, Netherlands
[3]Department of Earth, Planetary, and Space Sciences, University of California, Los Angeles, California 90095, USA
[4]Centre for Fluid and Complex Systems, Coventry University, Coventry CV1 5FB, United Kingdom
[5]Department of Physics, University of California, Santa Barbara, California 93106, USA
[6]Center for Nonlinear Studies, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA
[7]Max Planck—University of Twente Center for Complex Fluid Dynamics
[8]Institute for the Dynamics of Complex Systems, Georg-August-University Göttingen, 37073 Göttingen, Germany
[9]Laboratory of Atomic and Solid-State Physics and Sibley School of Mechanical and Aerospace Engineering, Cornell University, Ithaca, New York 14853, USA

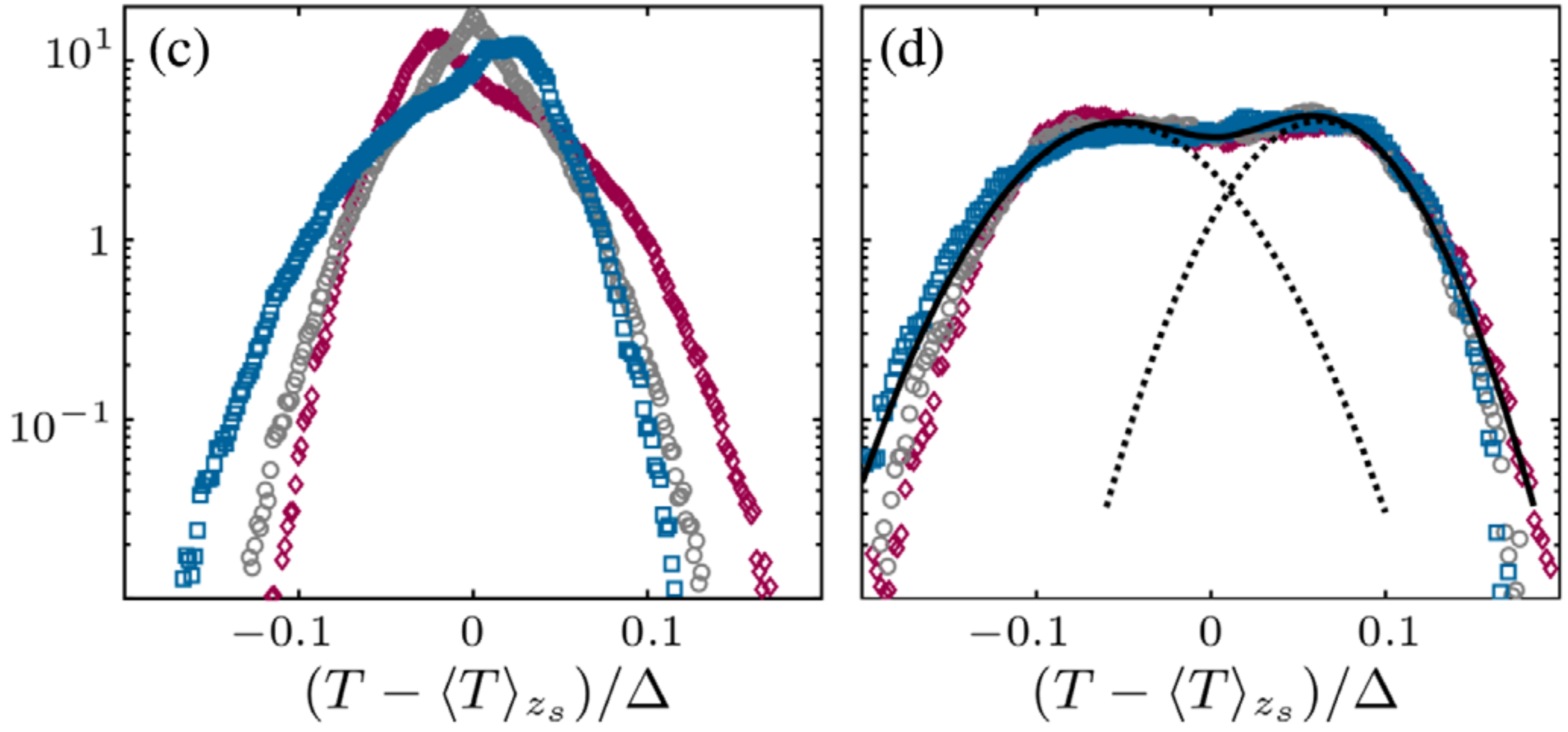
m=1



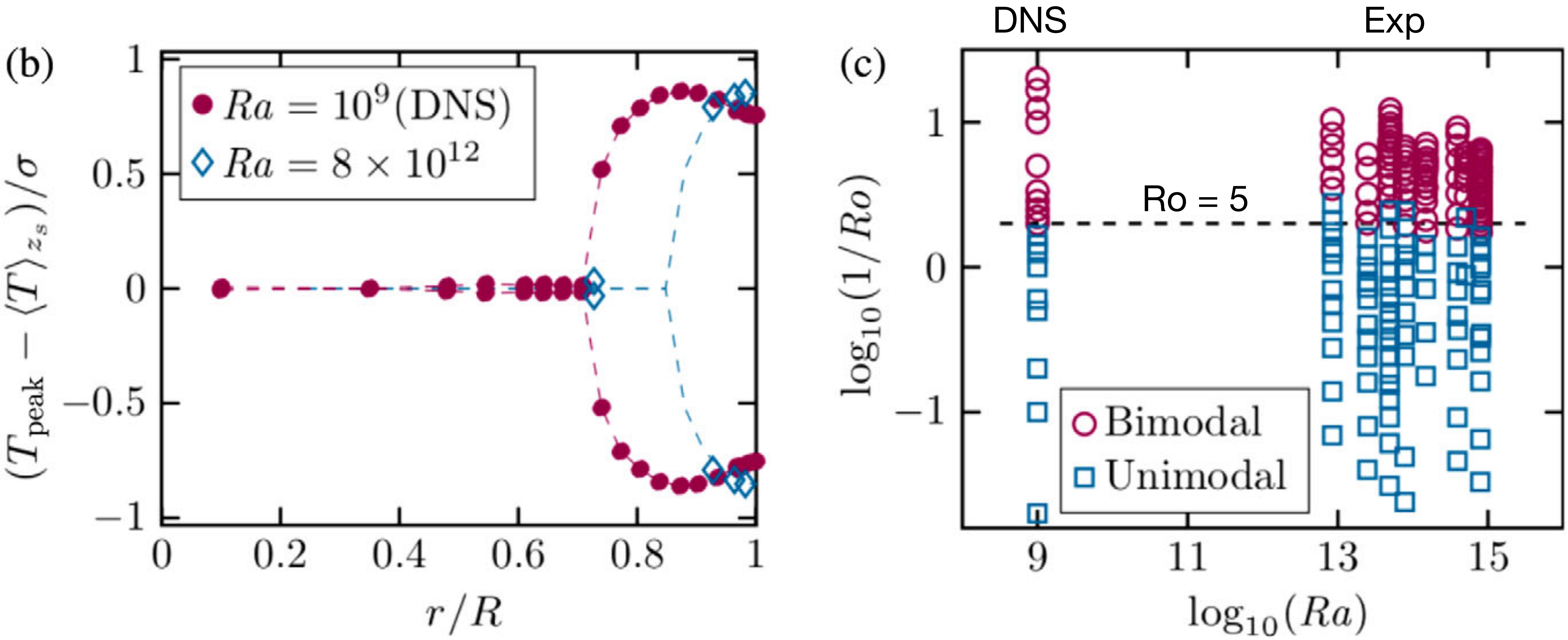
Experiment

DNS

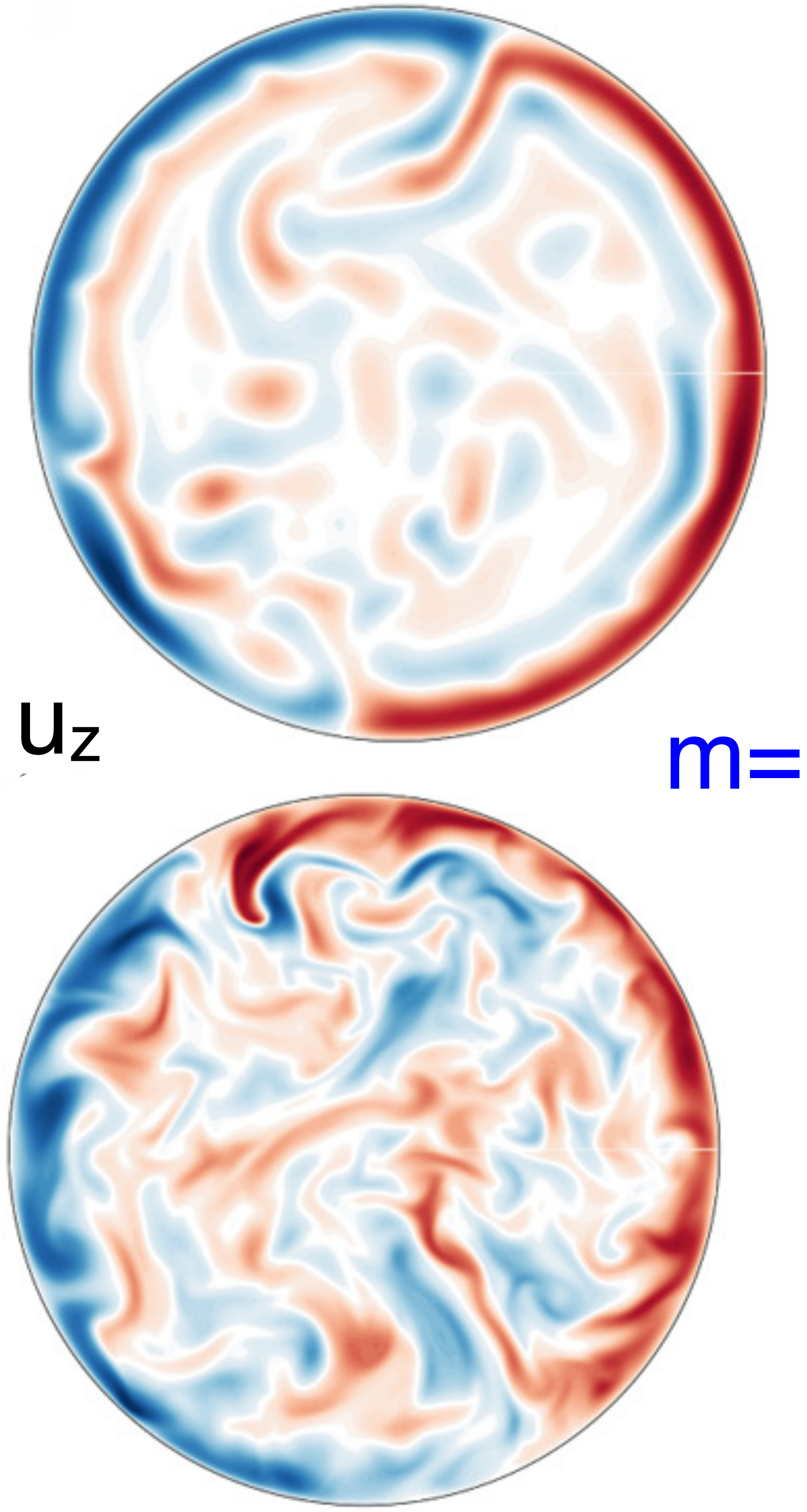




Pr = 0.8

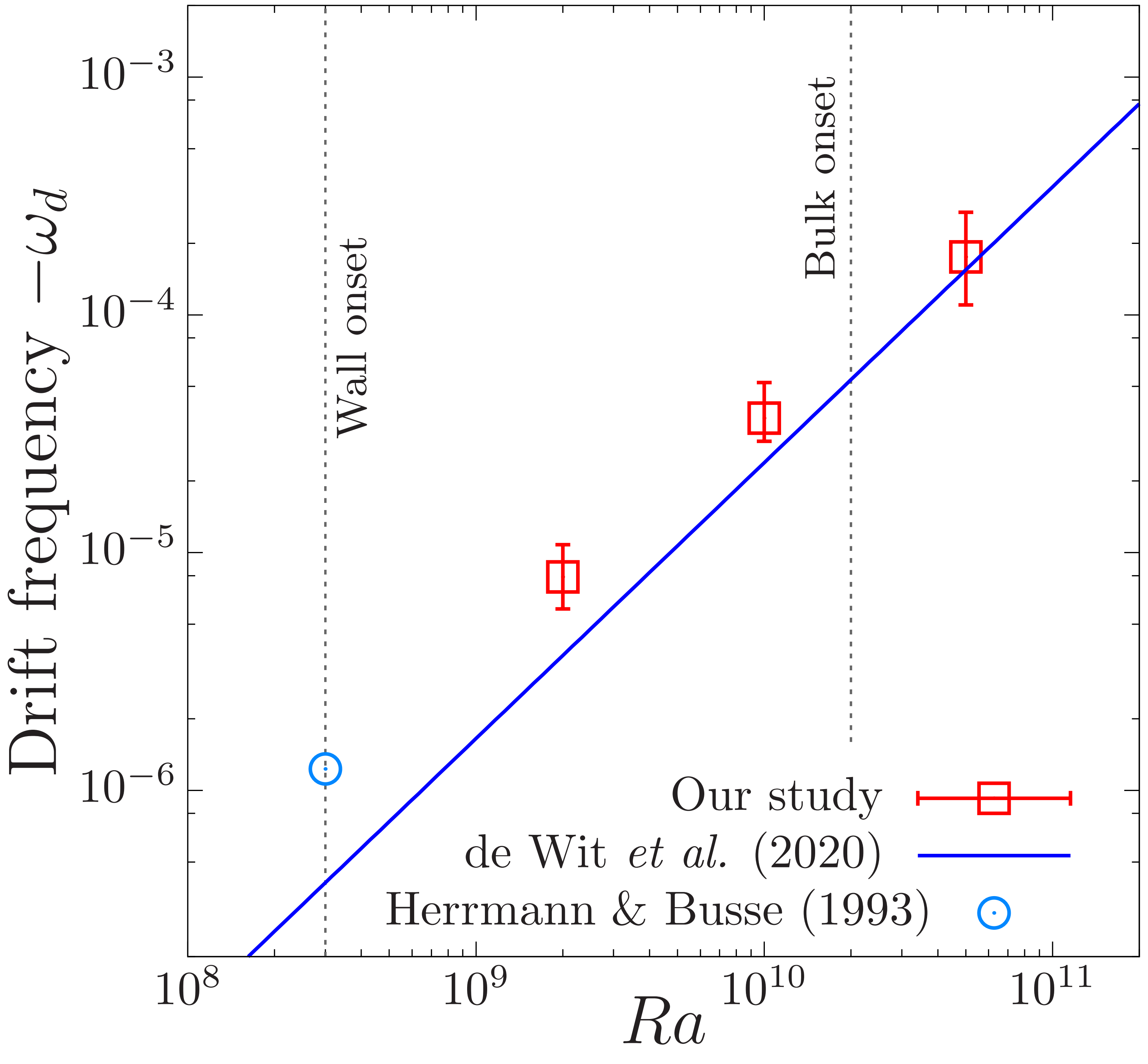

DNS
Exp
(b)
(c)
$Ra = 10^9$(DNS)
$Ra = 8 \times 10^{12}$
$(T_{\mathrm{peak}} - \langle T \rangle_{z_s})/\sigma$
r/R
Ro = 5
Bimodal
Unimodal
$\log_{10}(1/Ro)$
$\log_{10}(Ra)$

Pr = 5.2 Ek = 10^{-7} Γ = 1/5



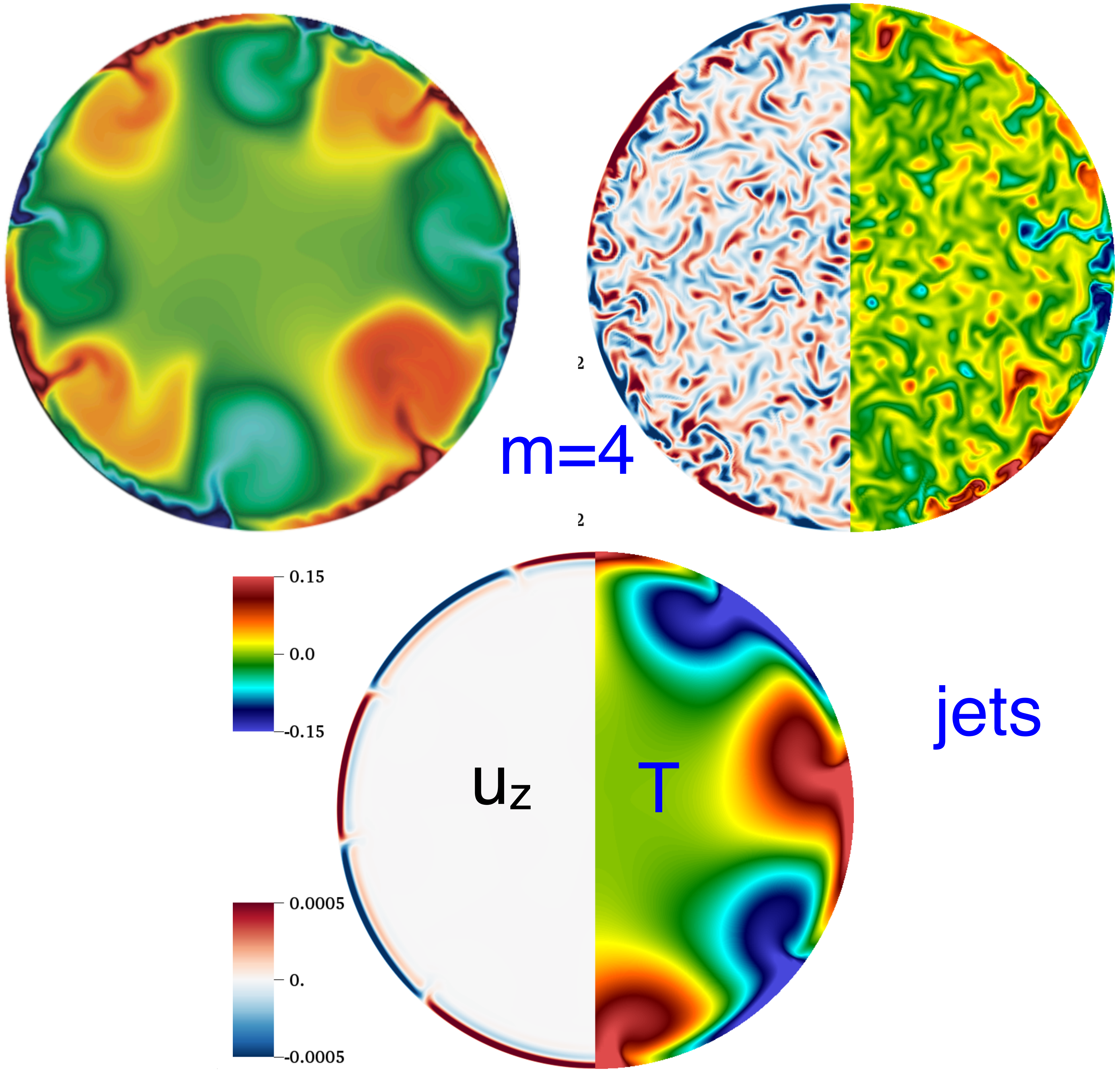
u_z

m=1



de Wit et al, Phys. Rev. Fluids (2020)

Pr = 1 Ek = 10^{-6} Γ = 3/2

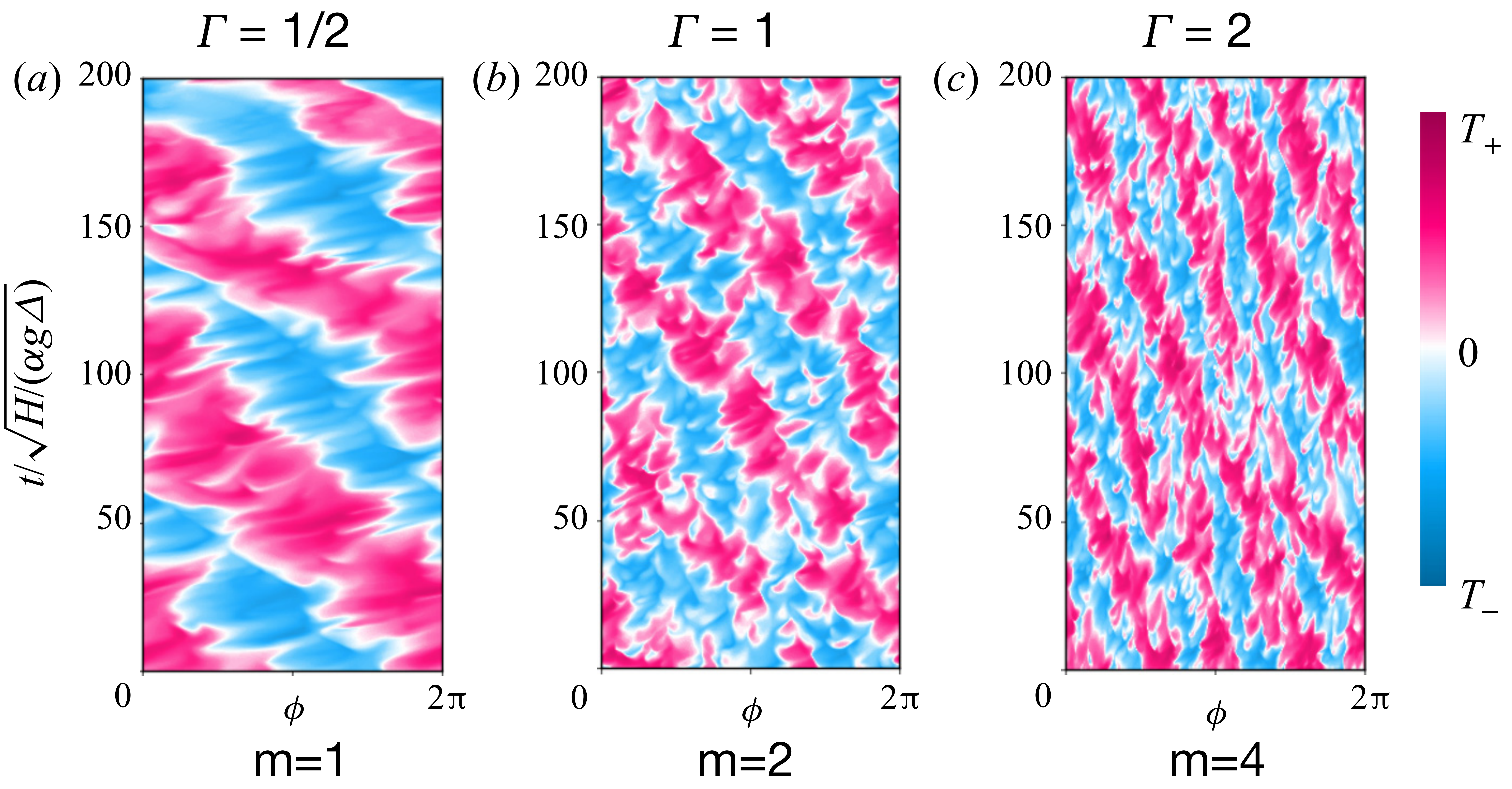


jets

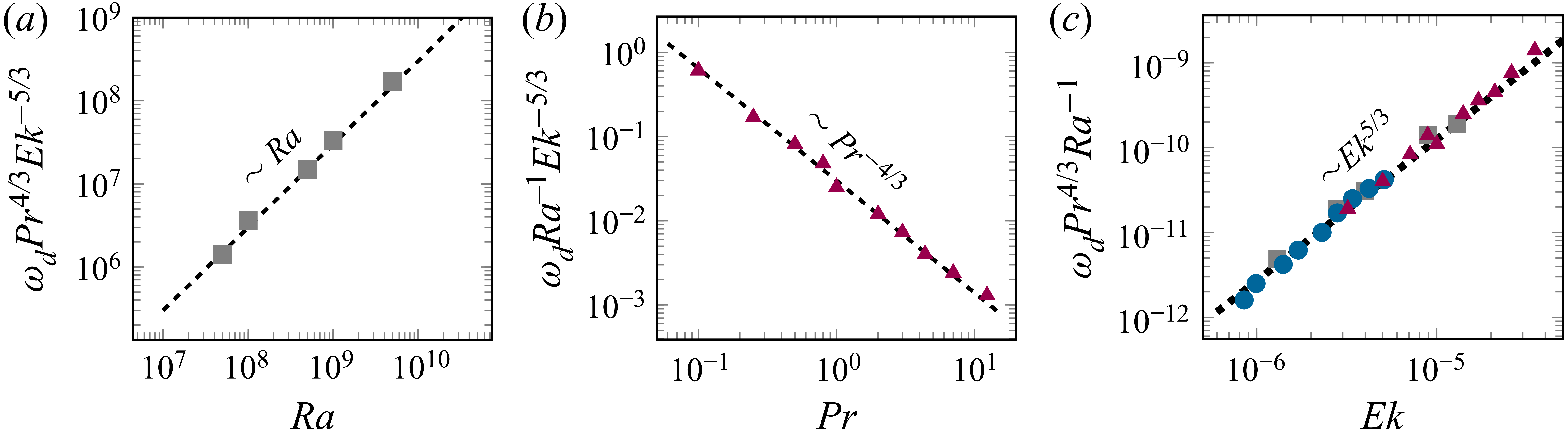
Wall modes ↔ BZF Robust!

Favier & Knobloch, J. Fluid Mech. Rapids (2021)

Constant Ra

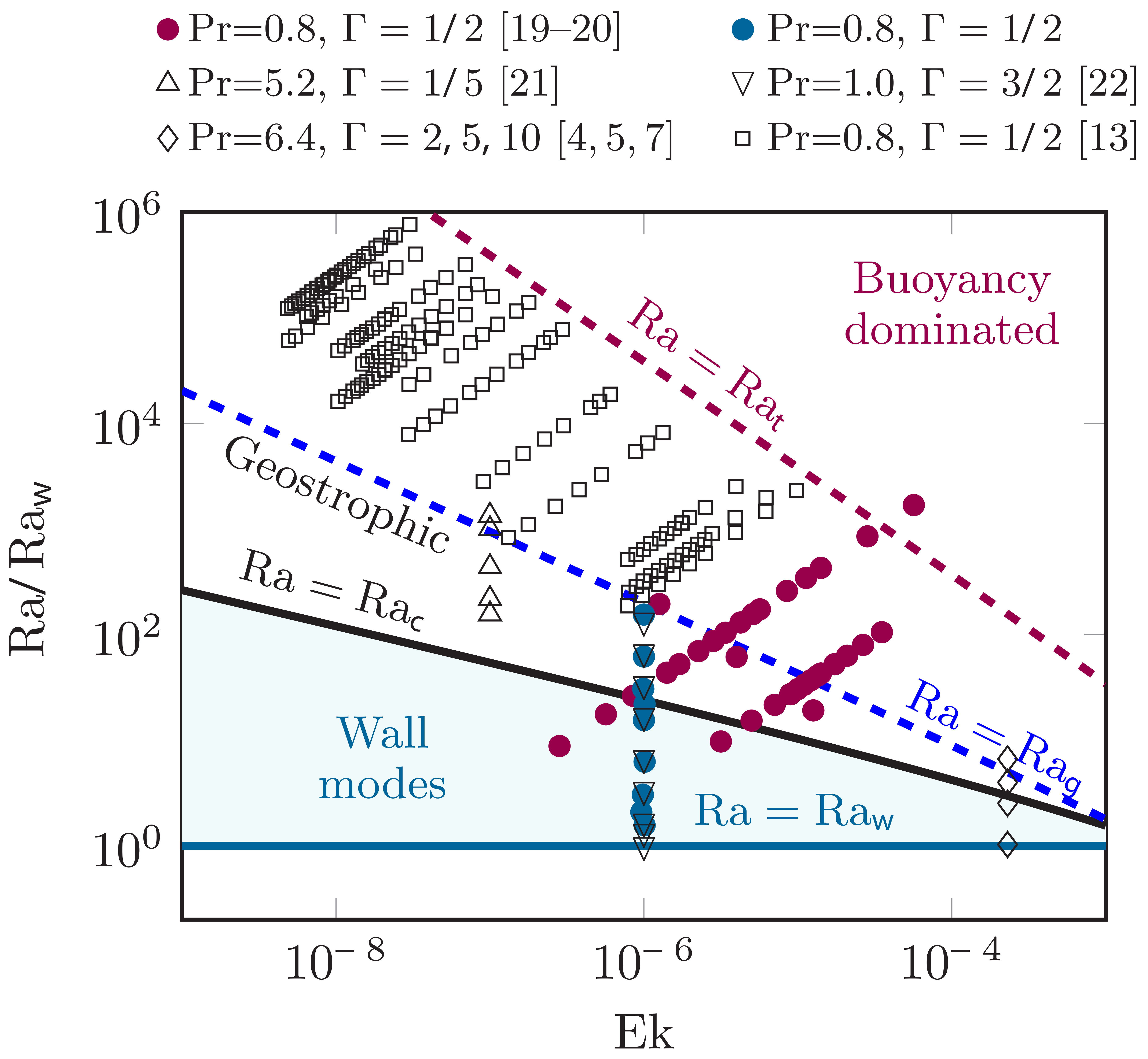


Zhang, JFM (2021)

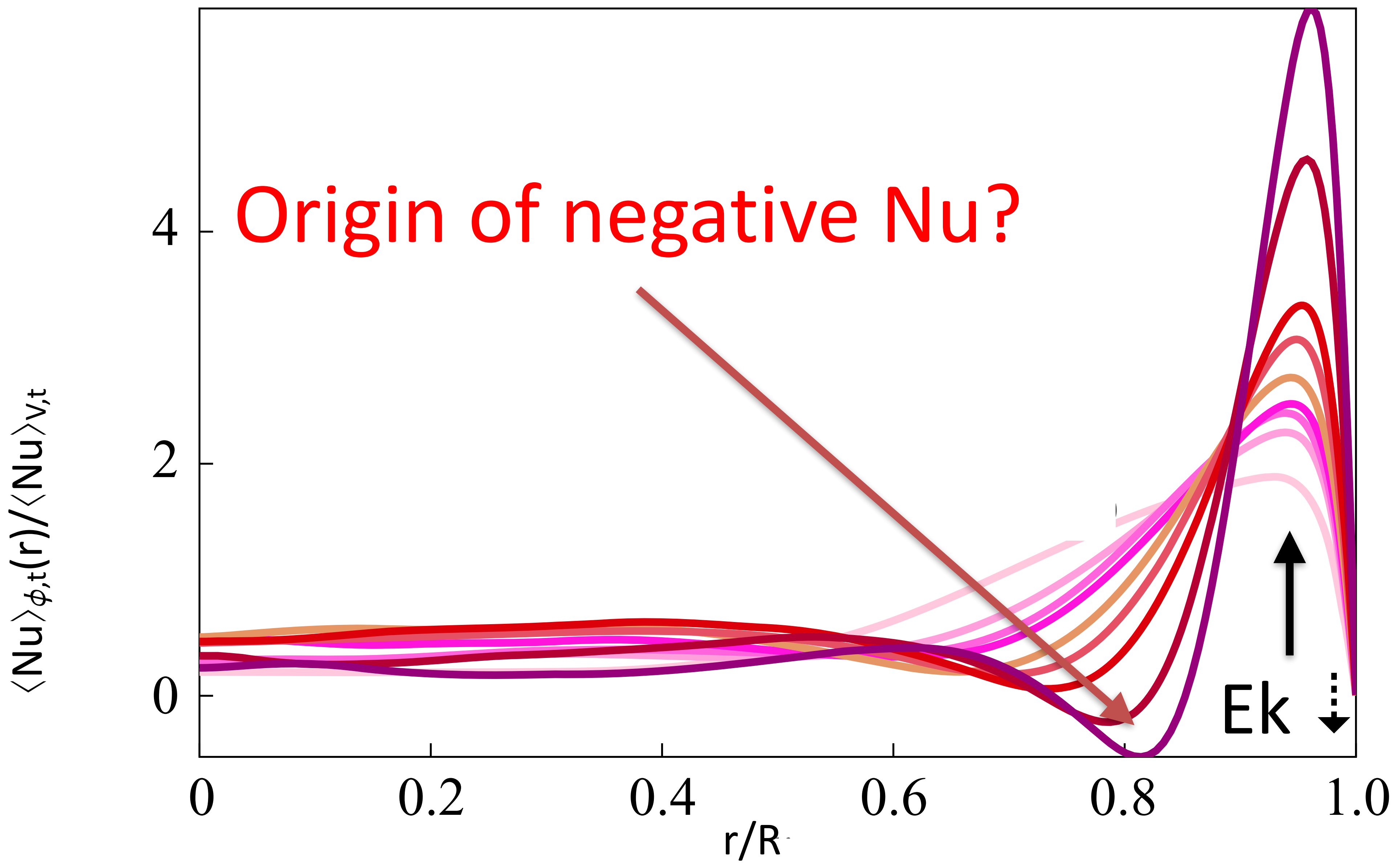


$$\omega_d/\Omega = 0.03\ \Gamma^0\ Pr^{-4/3} Ek^{5/3} Ra$$

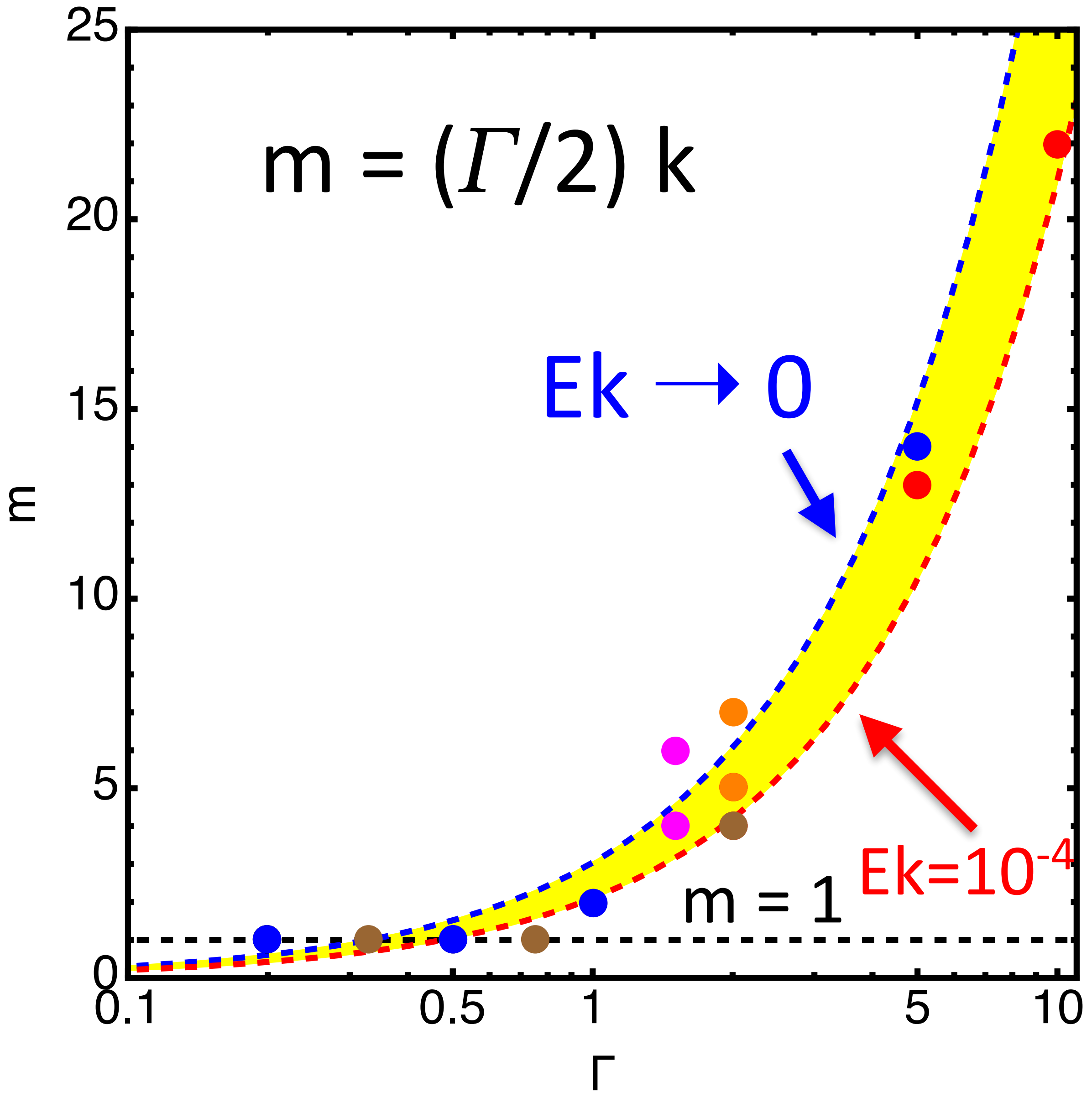
$$\omega_\kappa = \omega H^2/\kappa = 0.015\ \Gamma^0\ Pr^{-1/3} Ek^{2/3} Ra$$



Constant Ra ?

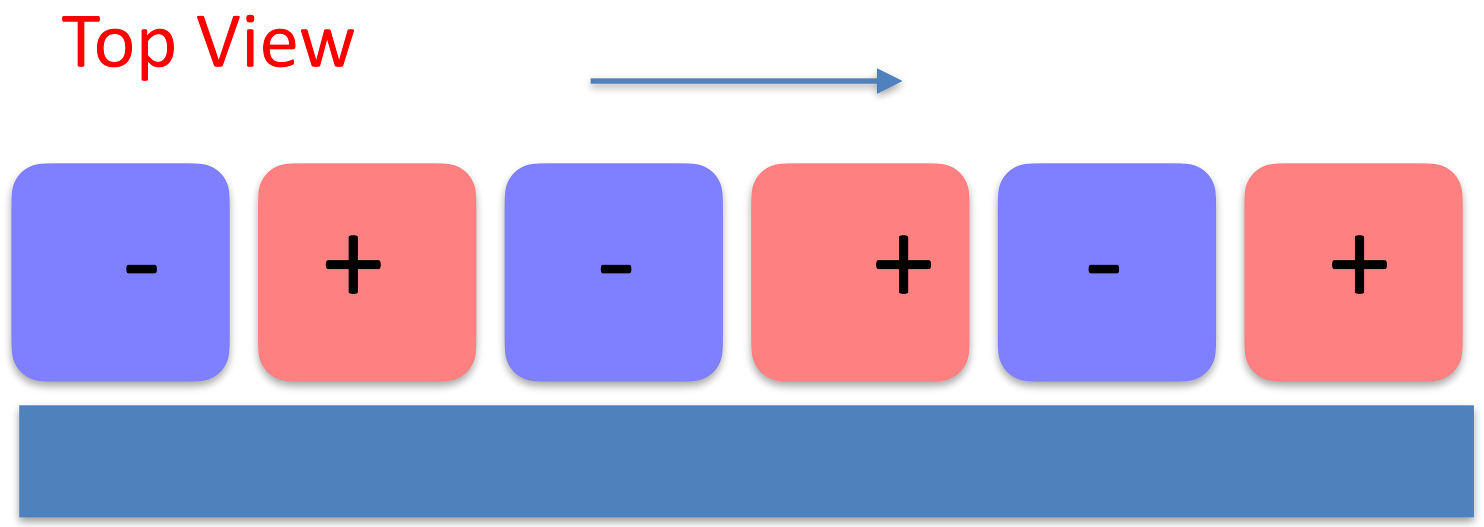


Have to see constant Ek to understand wall modes



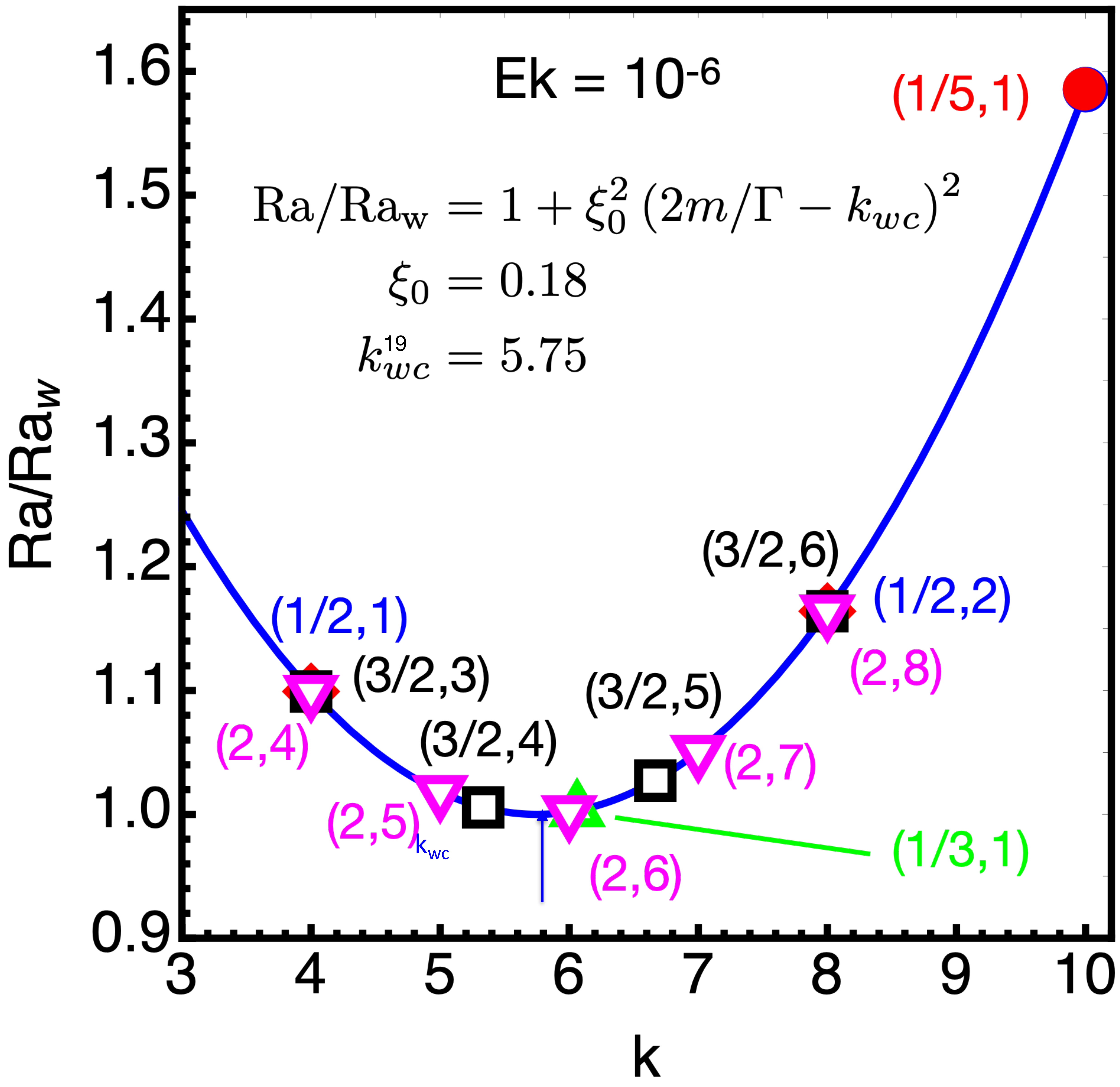
0.8 < Pr < 7

$10^{-7} < Ek < 10^{-4}$



$k_{wc}H = 2\pi H/\lambda_{wc} = 6.07 - 35.0\ Ek^{1/3}$

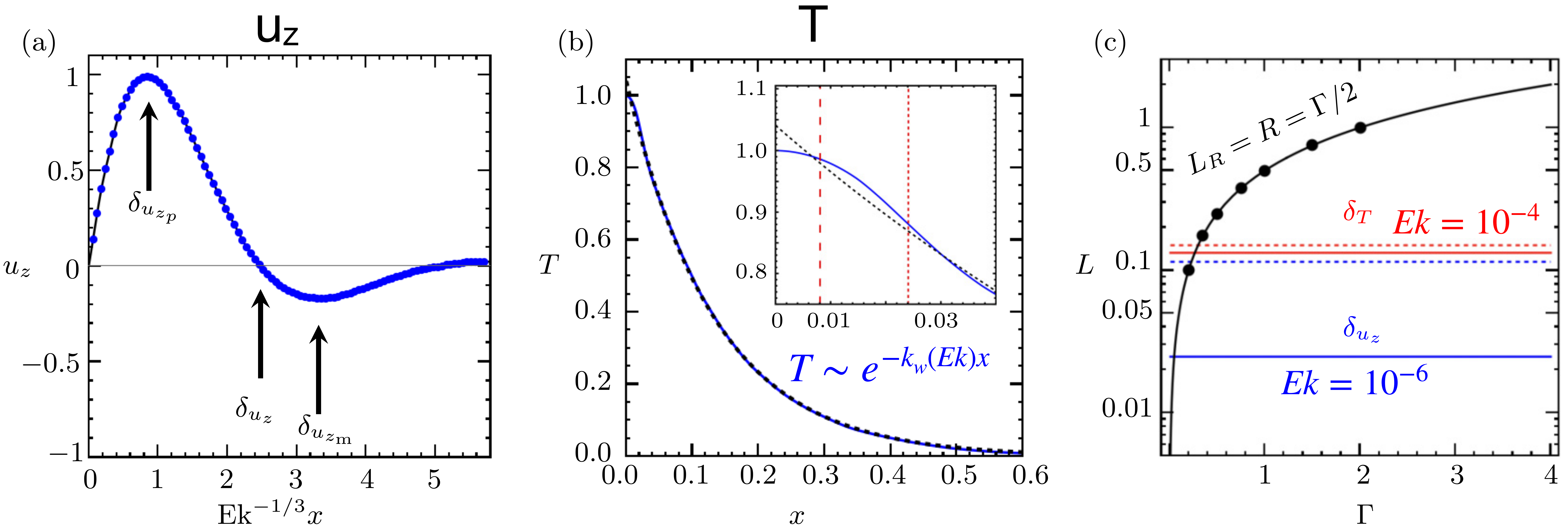
Herrmann & Busse, JFM 1993



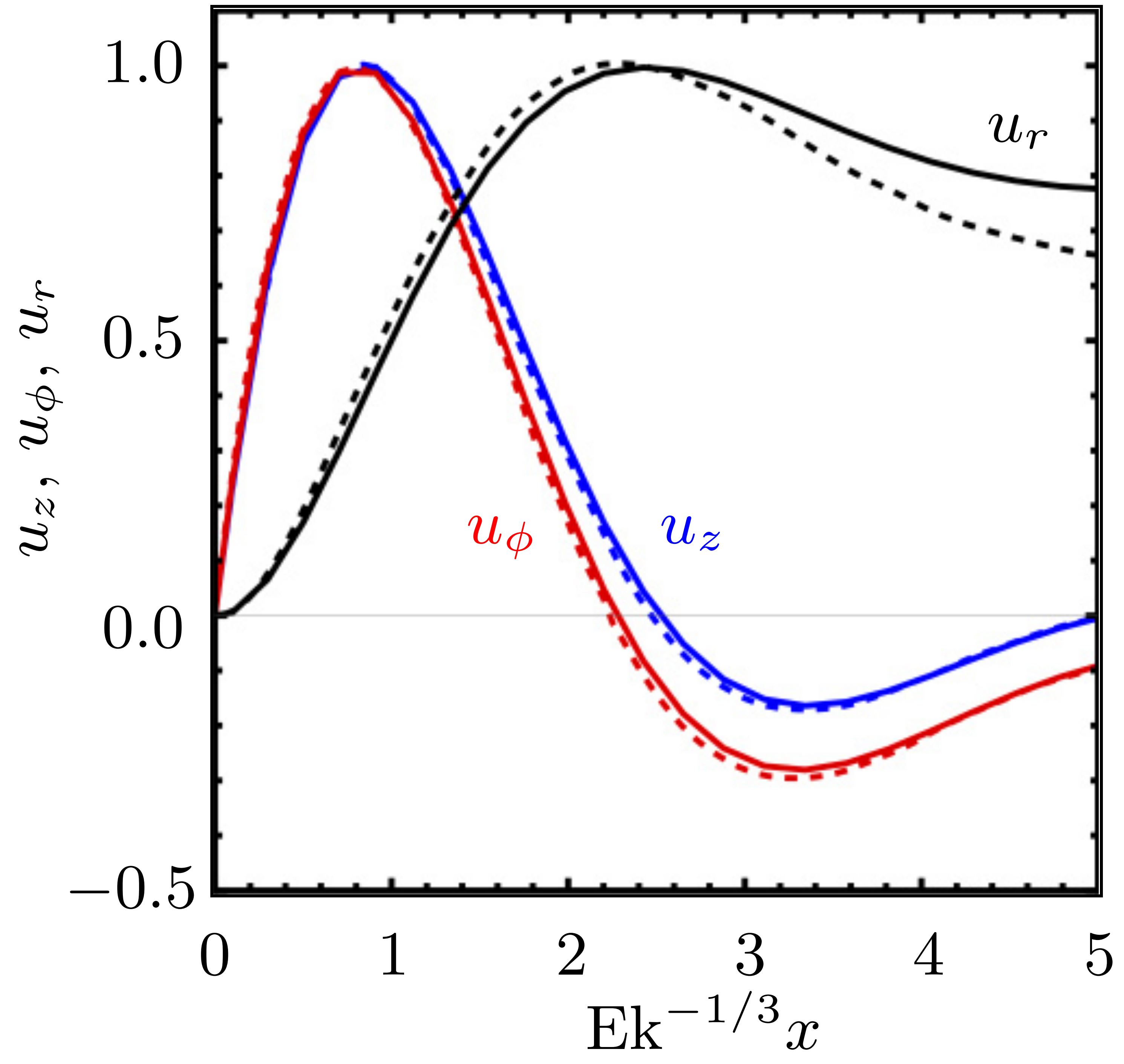
Ecke, Zhang, Shishkina, PRF (2022)

Zhang, Shishkina, Ecke PRF (2023)

Eigenfunctions of u_z and T (HB93)

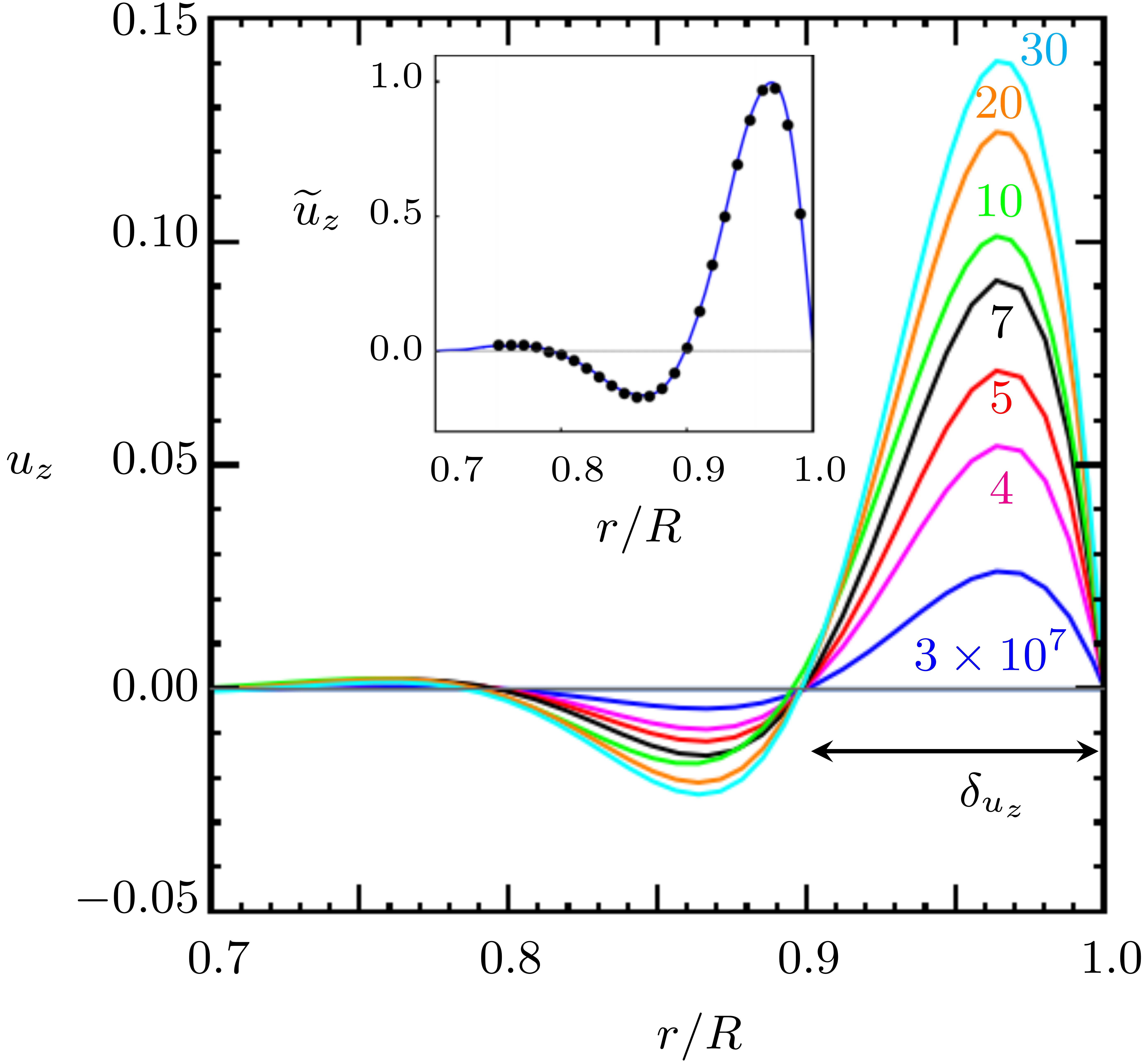


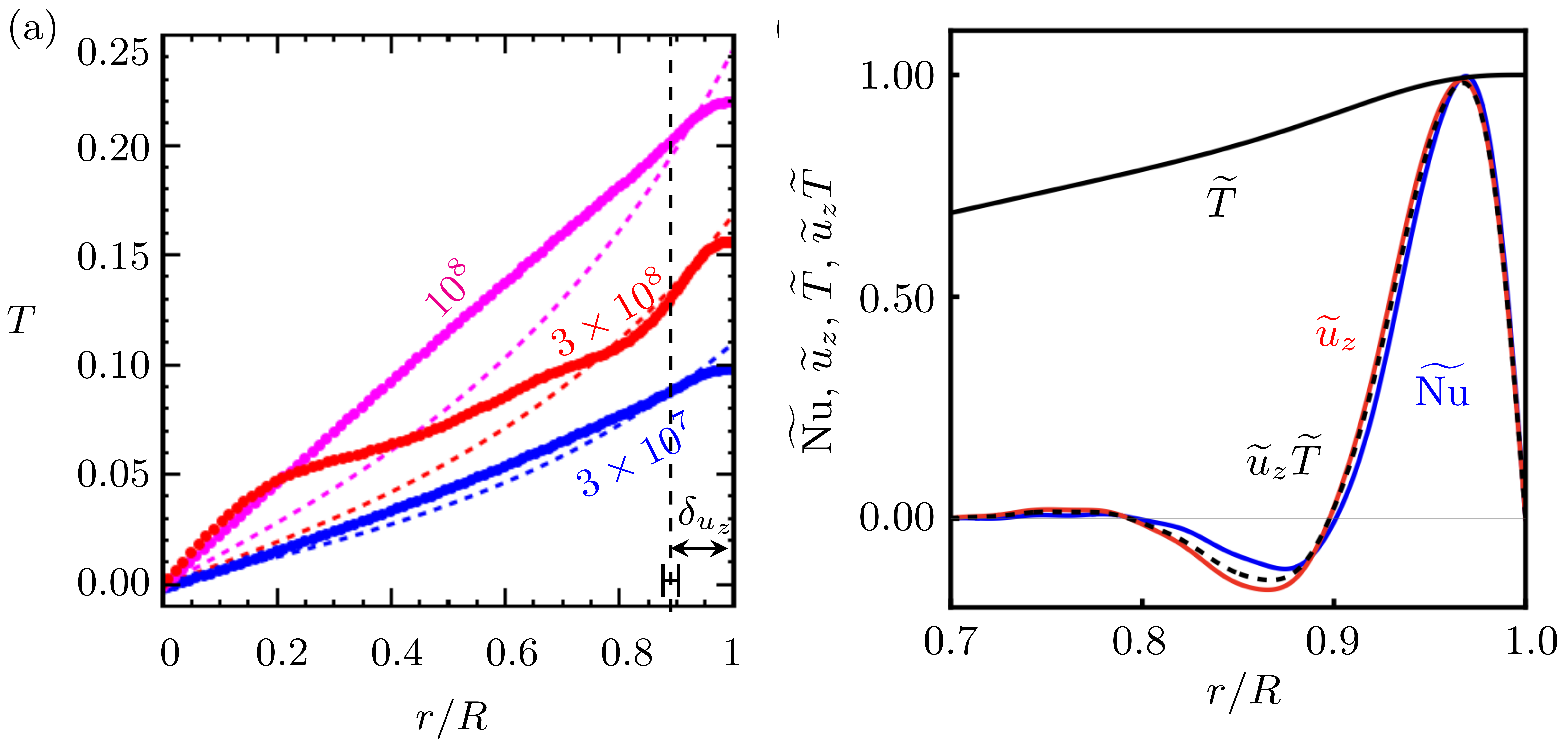
Expect these to work near onset



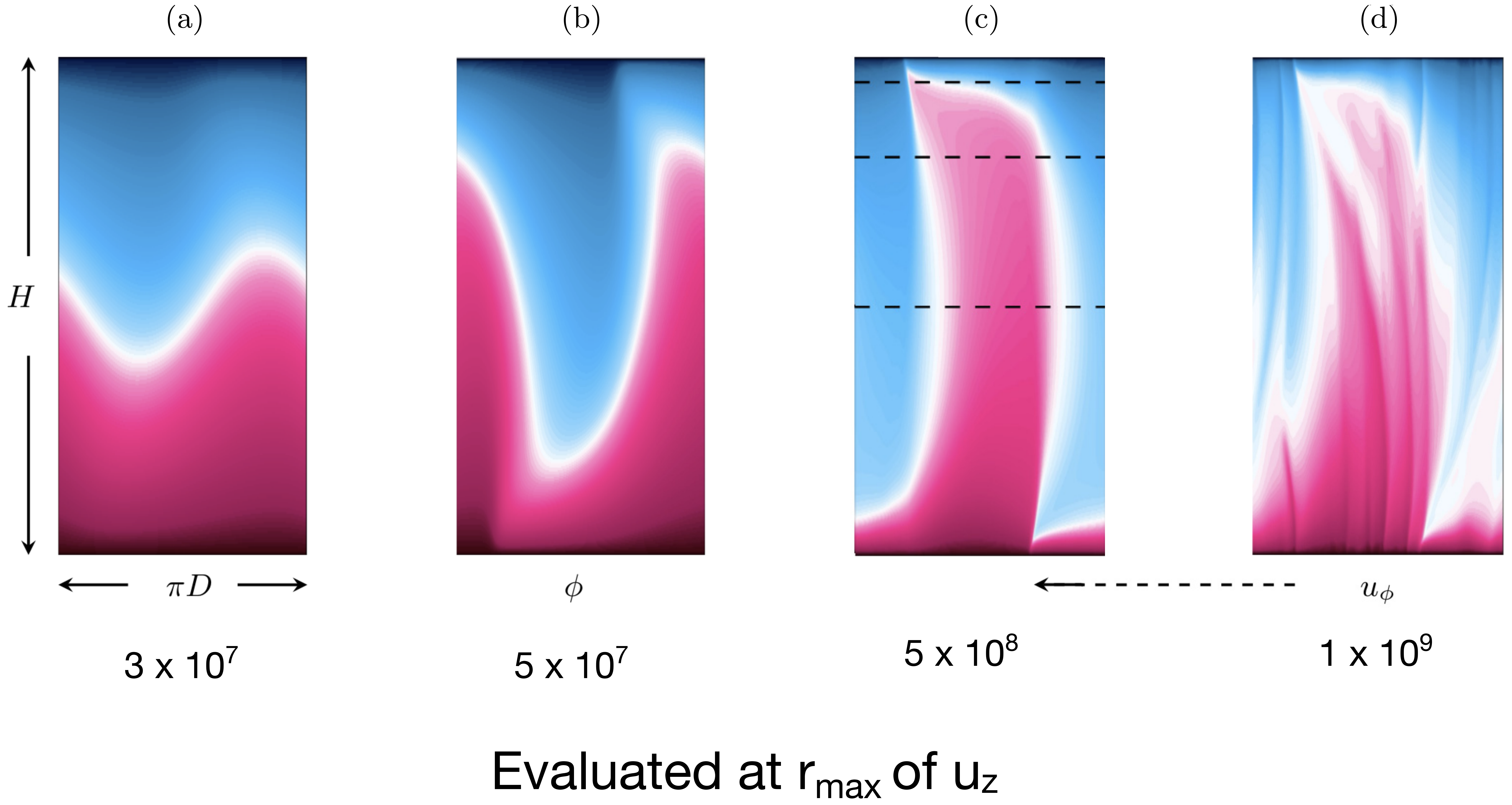
$Ra = 3 \times 10^7 \approx Ra_w$

$\mathrm{Ra}/\mathrm{Ra}_\mathrm{w} = 10$



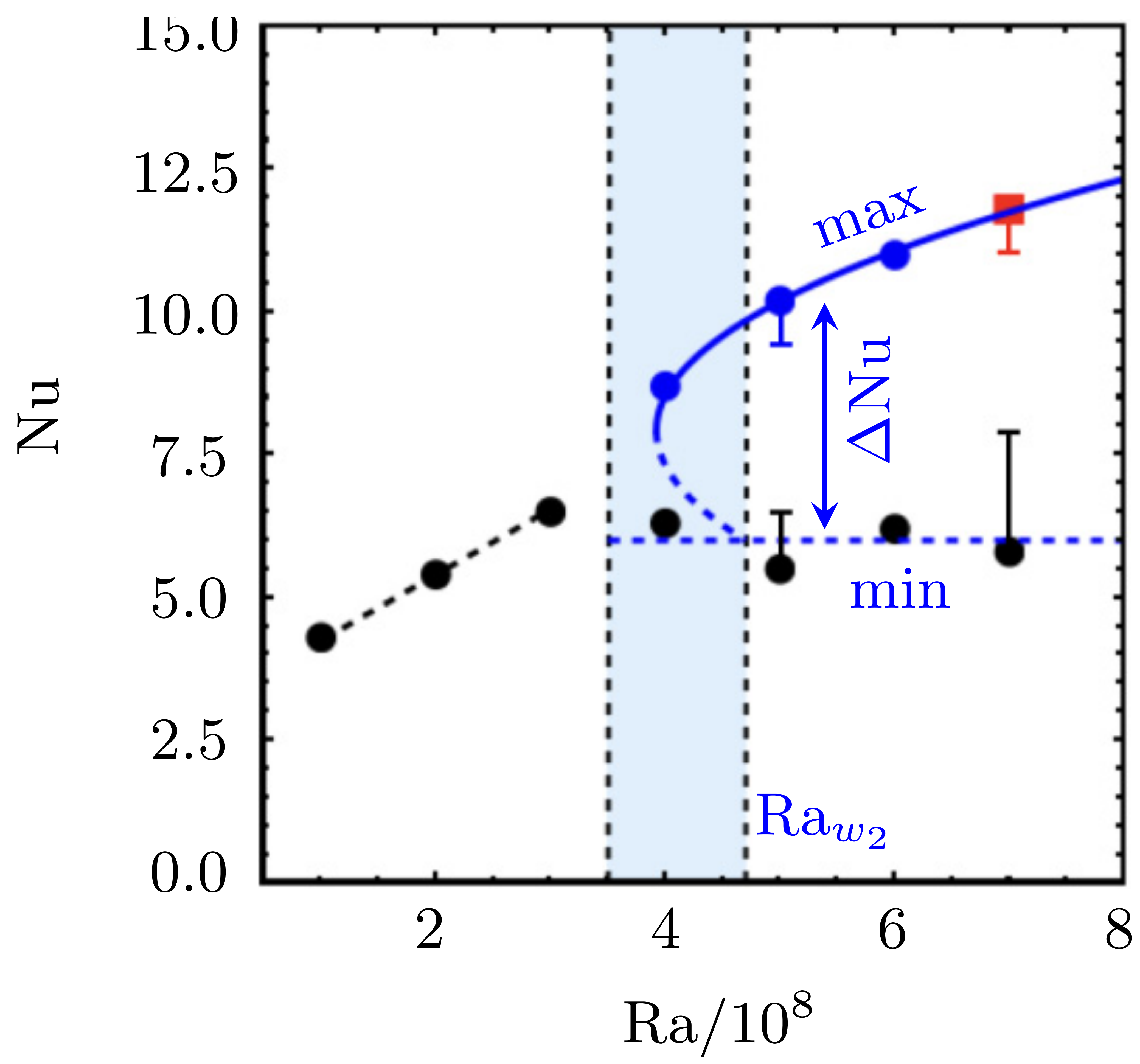


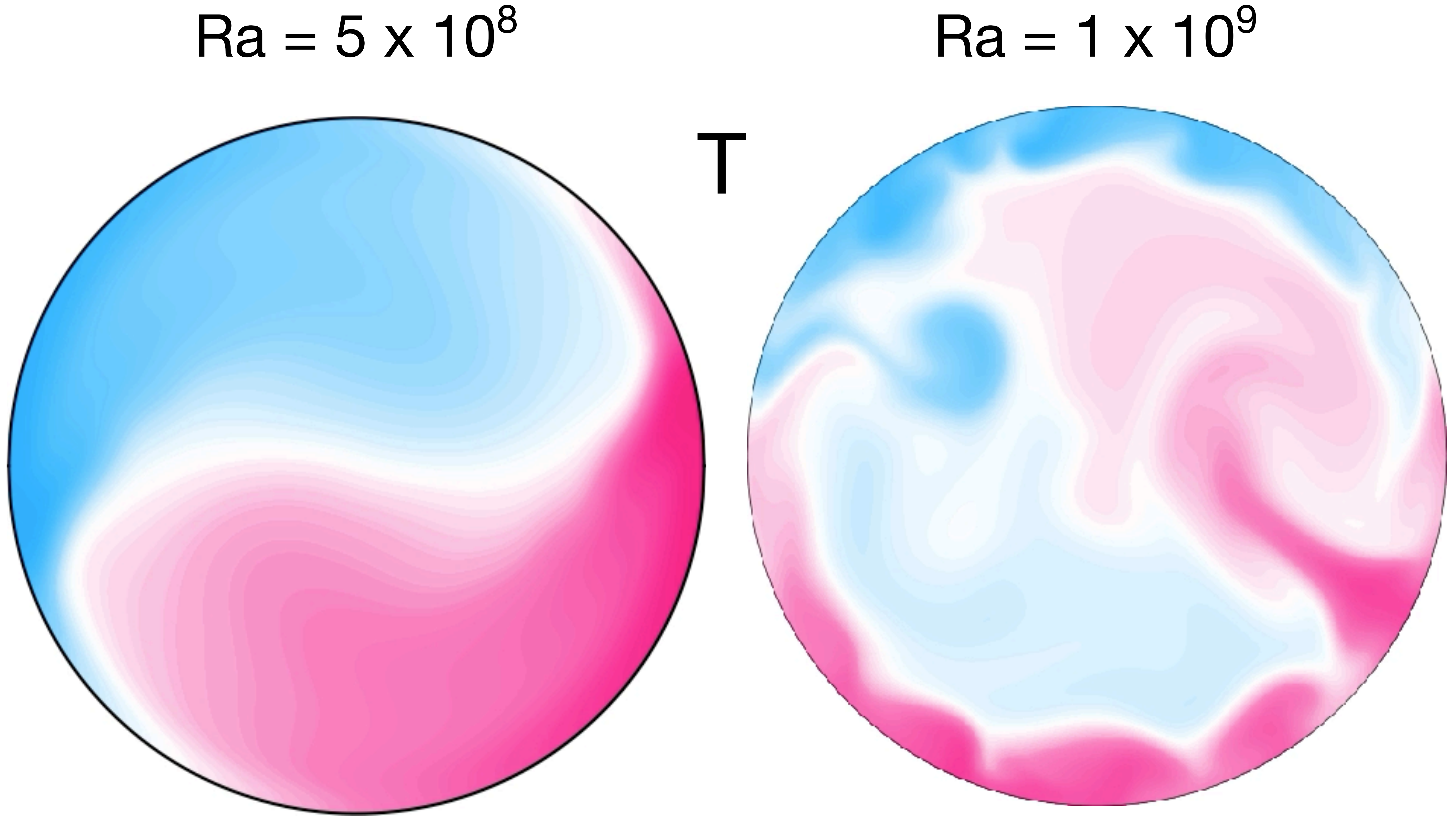
(a)
0.25
0.20
0.15
0.10
0.05
0.00
T
10^8
3×10^8
3×10^7
δ_{u_z}
0
0.2
0.4
0.6
0.8
1
r/R
1.00
0.50
0.00
$\widetilde{\mathrm{Nu}}, \widetilde{u}_z, \widetilde{T}, \widetilde{u}_z\widetilde{T}$
$\widetilde{T}$
$\widetilde{u}_z$
$\widetilde{\mathrm{Nu}}$
$\widetilde{u}_z\widetilde{T}$
0.7
0.8
0.9
1.0
r/R



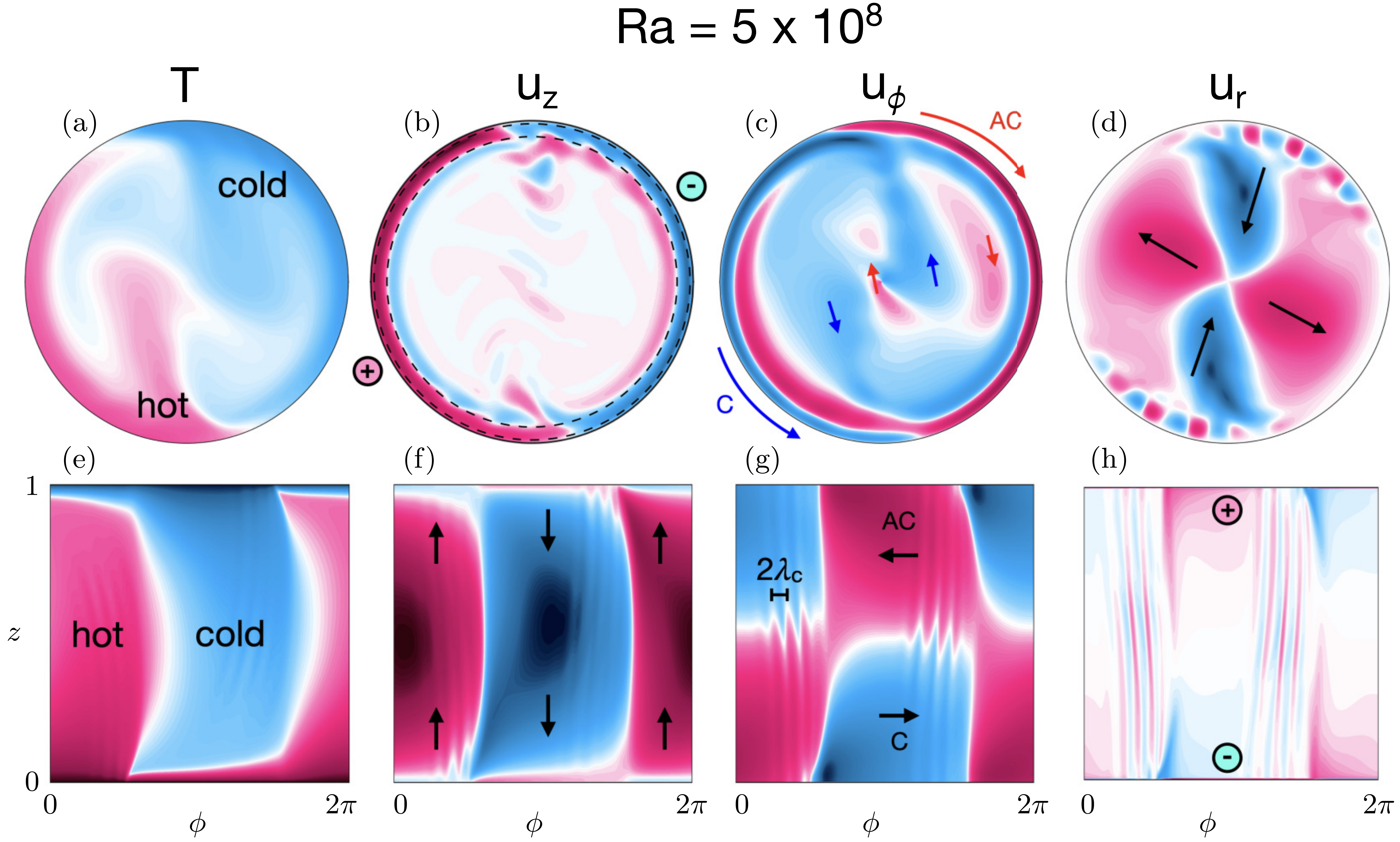
(a)
(b)
(c)
(d)
H
πD
ϕ
u_ϕ
3 x 10^7
5 x 10^7
5 x 10^8
1 x 10^9
Evaluated at r_{max} of u_z

The jet instability is subcritical

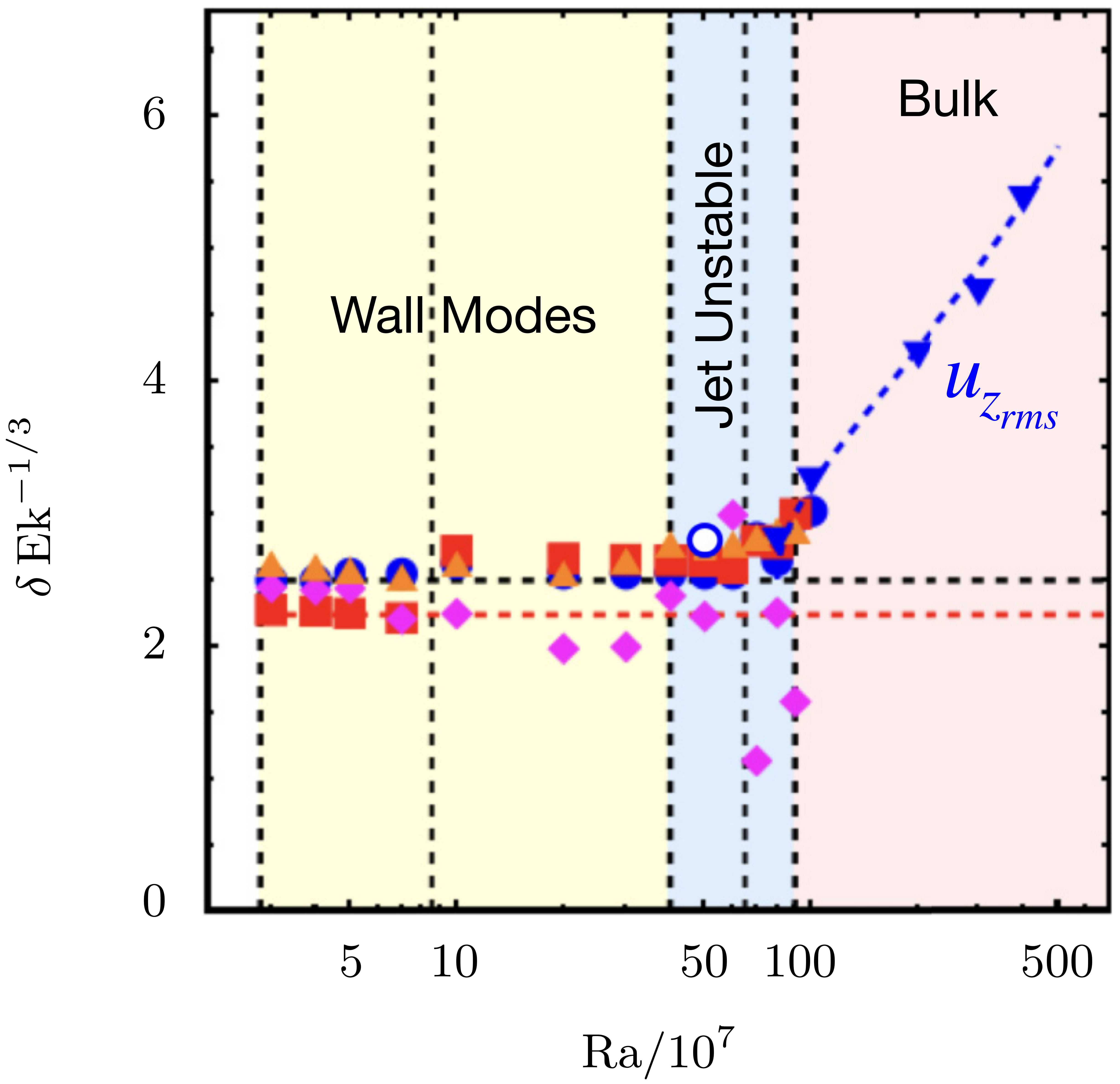




jets invade bulk



Ra = 5 x 10^8
T
u_z
u_ϕ
u_r
(a)
(b)
(c)
(d)
cold
hot
AC
C
(e)
(f)
(g)
(h)
1
0
z
hot
cold
AC
2λc
C
0
2π
ϕ



Measured in the precessing frame

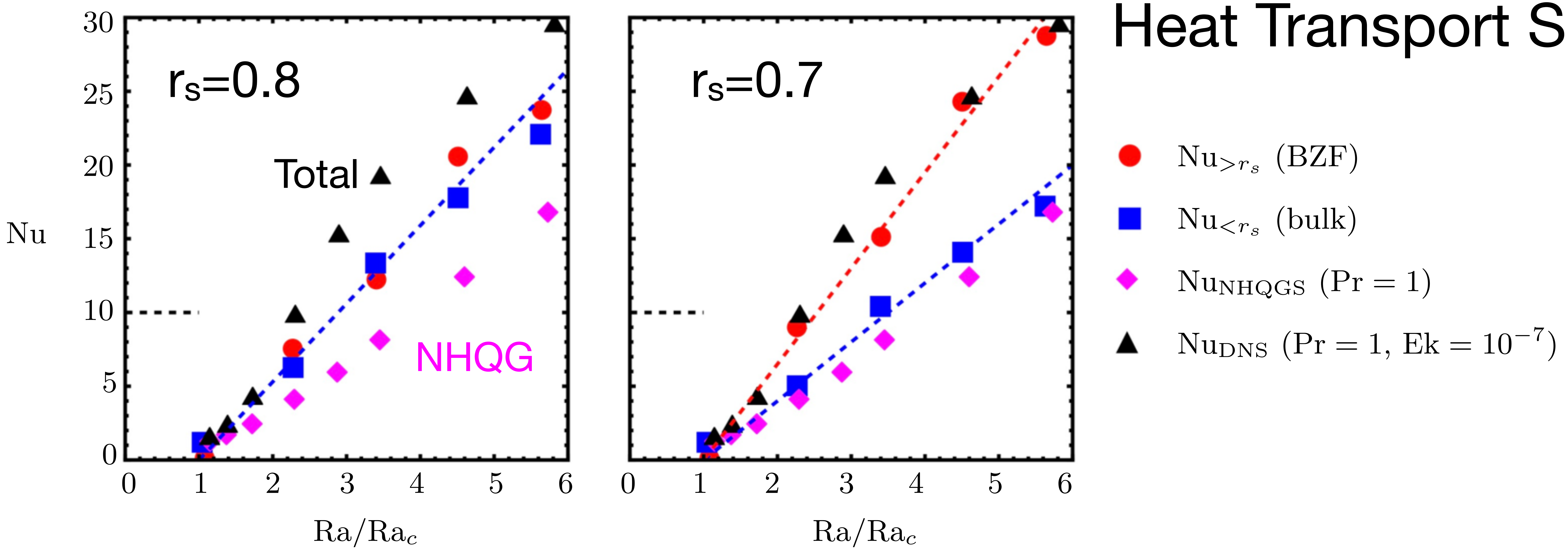
Defined by first zero crossing

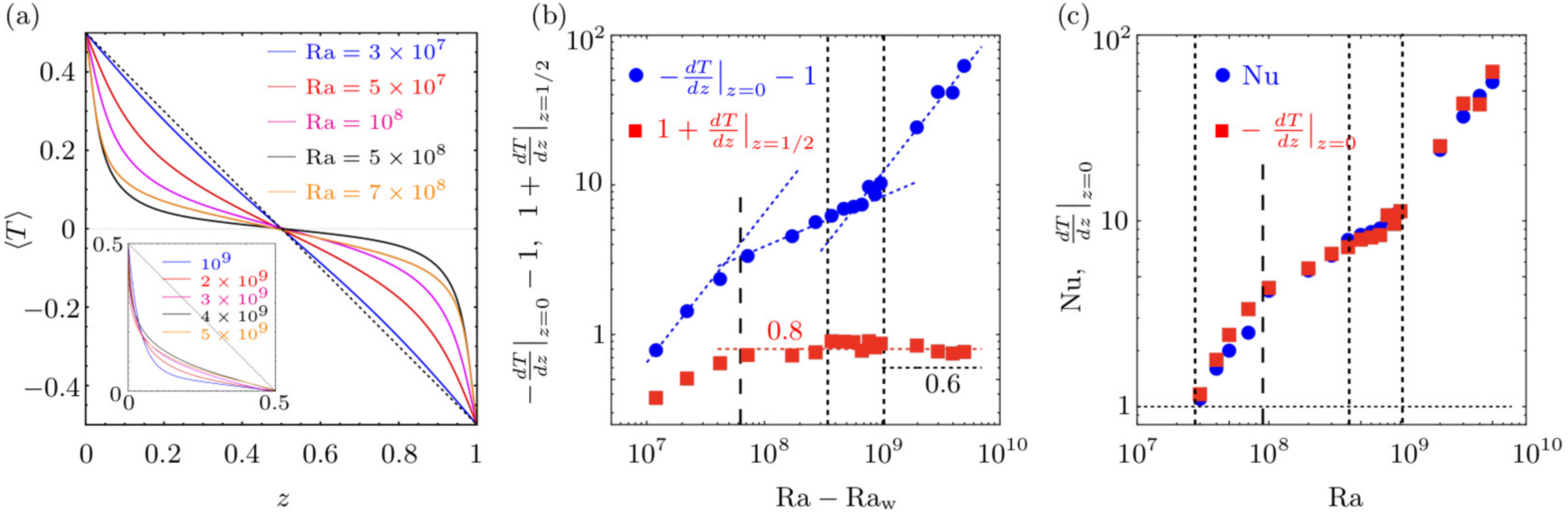
$u_z\ u_\phi\ u_r\ \omega_z$

For u_r first maximum

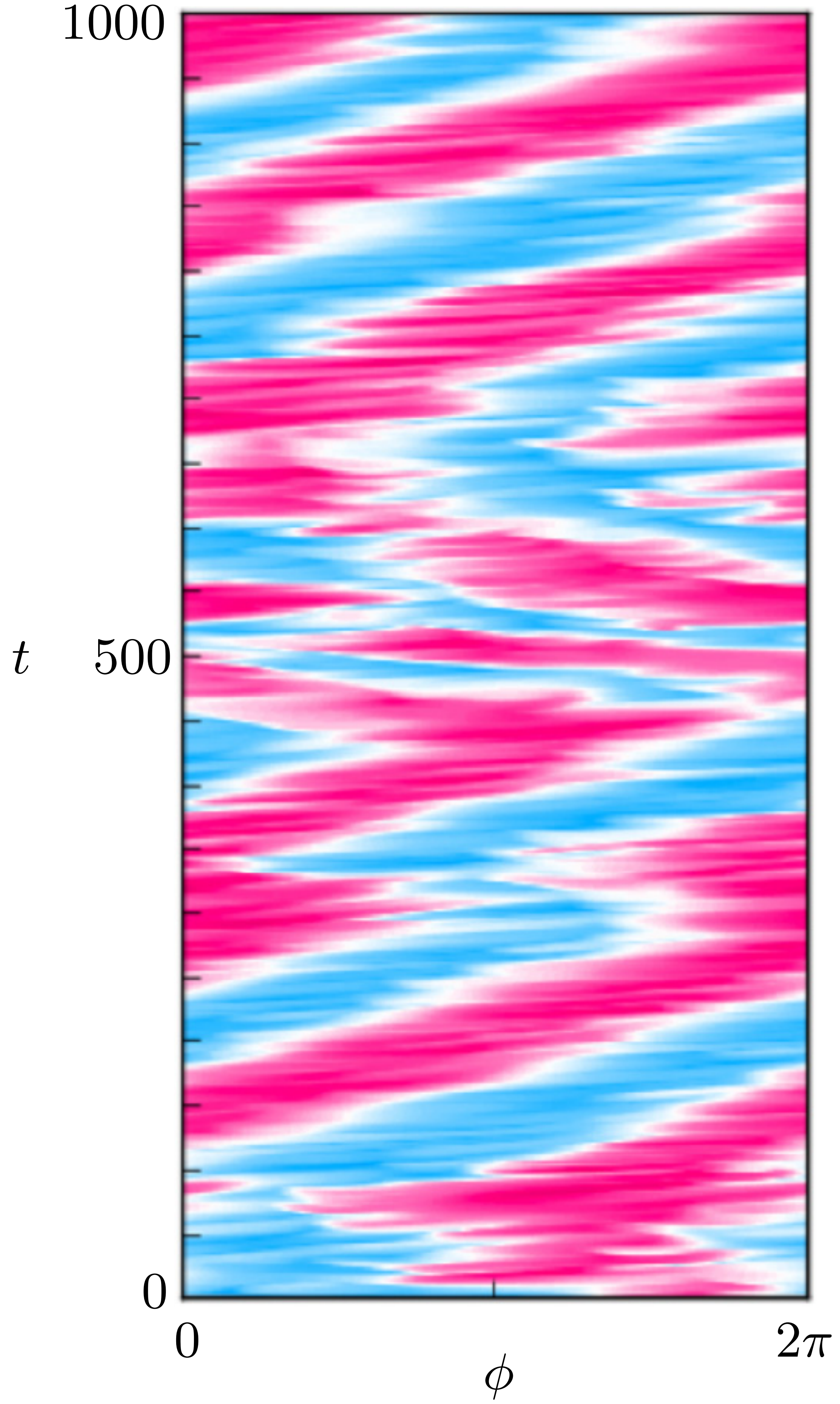
Remarkably insensitive to $Ra<Ra_c$

Heat Transport Separation(?)





“Hovmöller Plot”

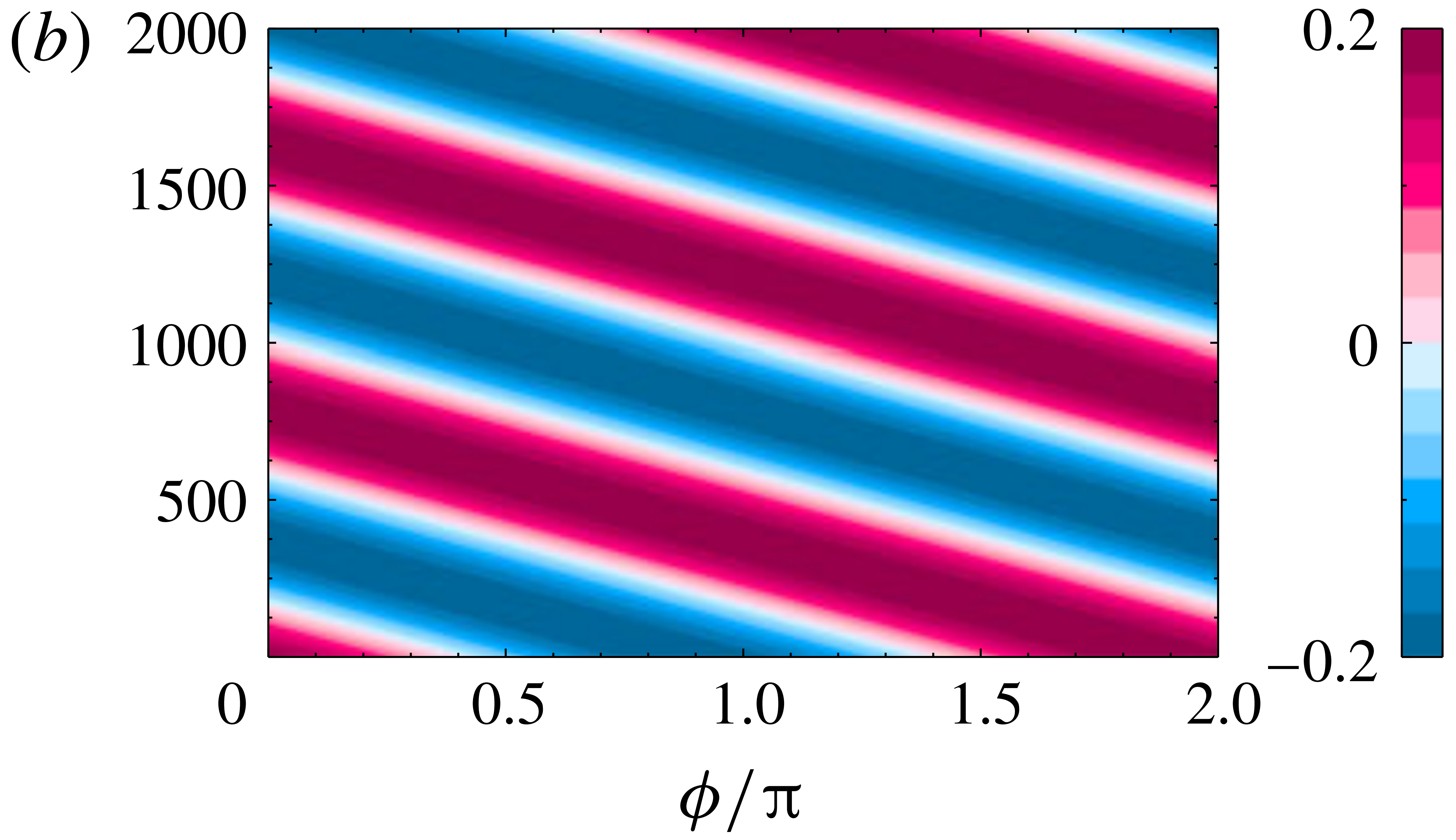


Γ= 1/3, Pr = 0.8,

Ek = 1.1 x 10^{-5},

Ro = 0.1, Ra = 10^{8}

Prograde wall mode precession



Γ= 1/2, Pr = 0.8, Ek = 1.3 x 10^{-3}, Ra = 10^{5}, Ro_C = 0.5

Horn & Schmidt JFM 2017

They do exist

Conducting Sidewall Wall Modes

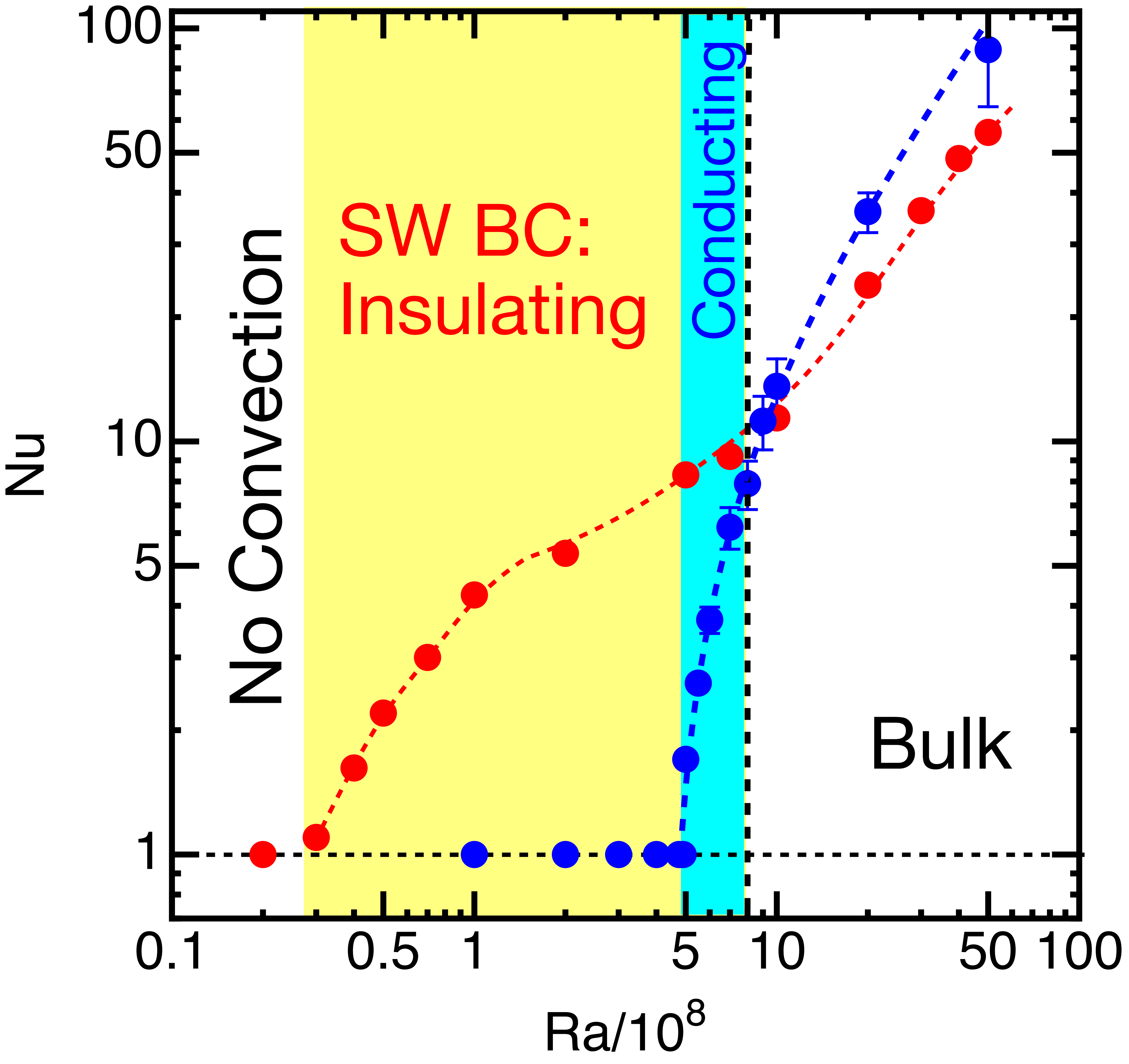
Scalings of Wall Modes and Bulk

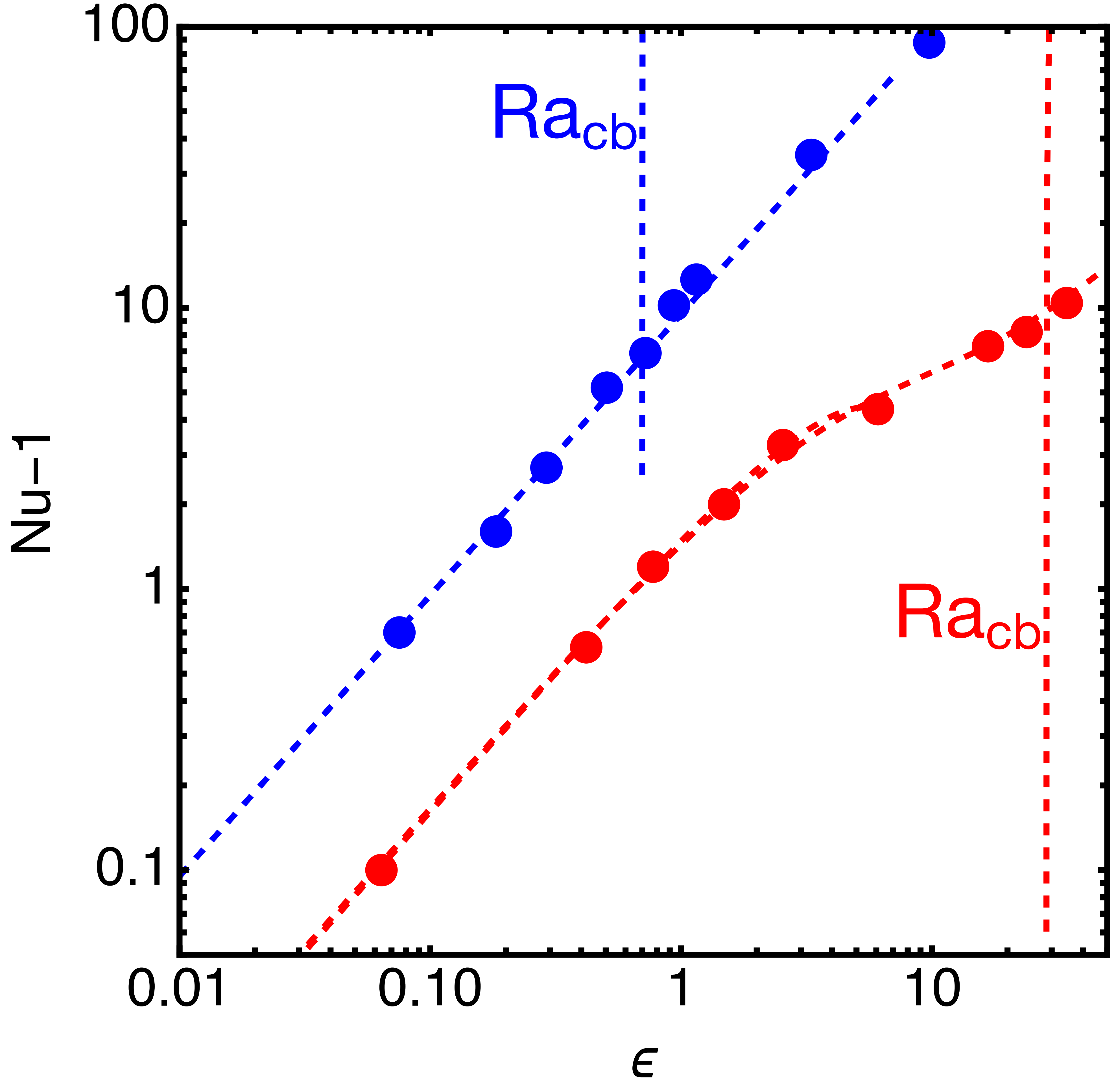
	Insulating	Conducting	Bulk
Ra_c	Ek^{-1}	$Ek^{-4/3}$ ⟷	$Ek^{-4/3}$
k_c	$O(1)$	$Ek^{-1/6}$	$Ek^{-1/3}$
ω_c	$O(1)$	$Ek^{-1/3}$	NA

Conducting Sidewall BC *Herrmann & Busse JFM 1993*

5x10^8 m=3
T_max
r/R
0.04
0.03
0.02
0.01
0.00
−0.01
0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
δr_T ≈ δr_uz
T
u_z
5.5 x 10^8
T
u_z
m=3
T
6 x 10^8
T
u_z
m=2
1 x 10^9

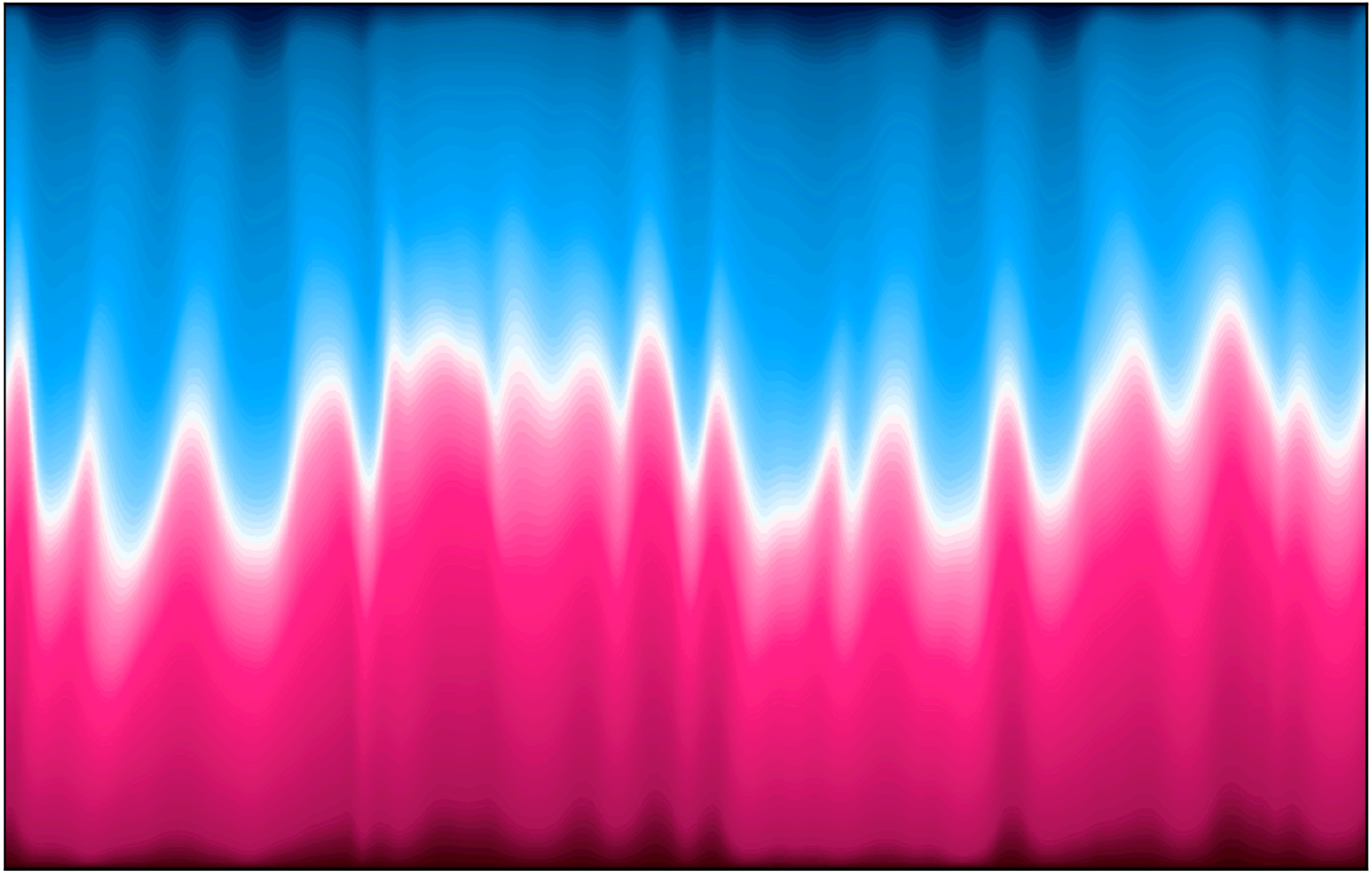
Wall Modes





- Insulating SW BC: Lower Ra_{cI}, slow increase.
- Insulating Nu: Clear bulk onset.
- $Nu_{insu}(Ra_{cb}) \approx Nu_{cond}(Ra_{cb})$! Why?

- Conducting SW BC: Higher Ra_{cc}, rapid increase.
- Conducting: No apparent change in Nu at Ra_{cb}.
- $Nu_{cond} > Nu_{insul}$ $Ra > Ra_{cb}$



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