Scaling laws and force balances in rotating convection: are they consistent?

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Keith Julien, Stefano Maffei

University of Colorado **Boulder**

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Small scale quasigeostrophic convective turbulence at large Rayleigh number

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JFM PAPERS

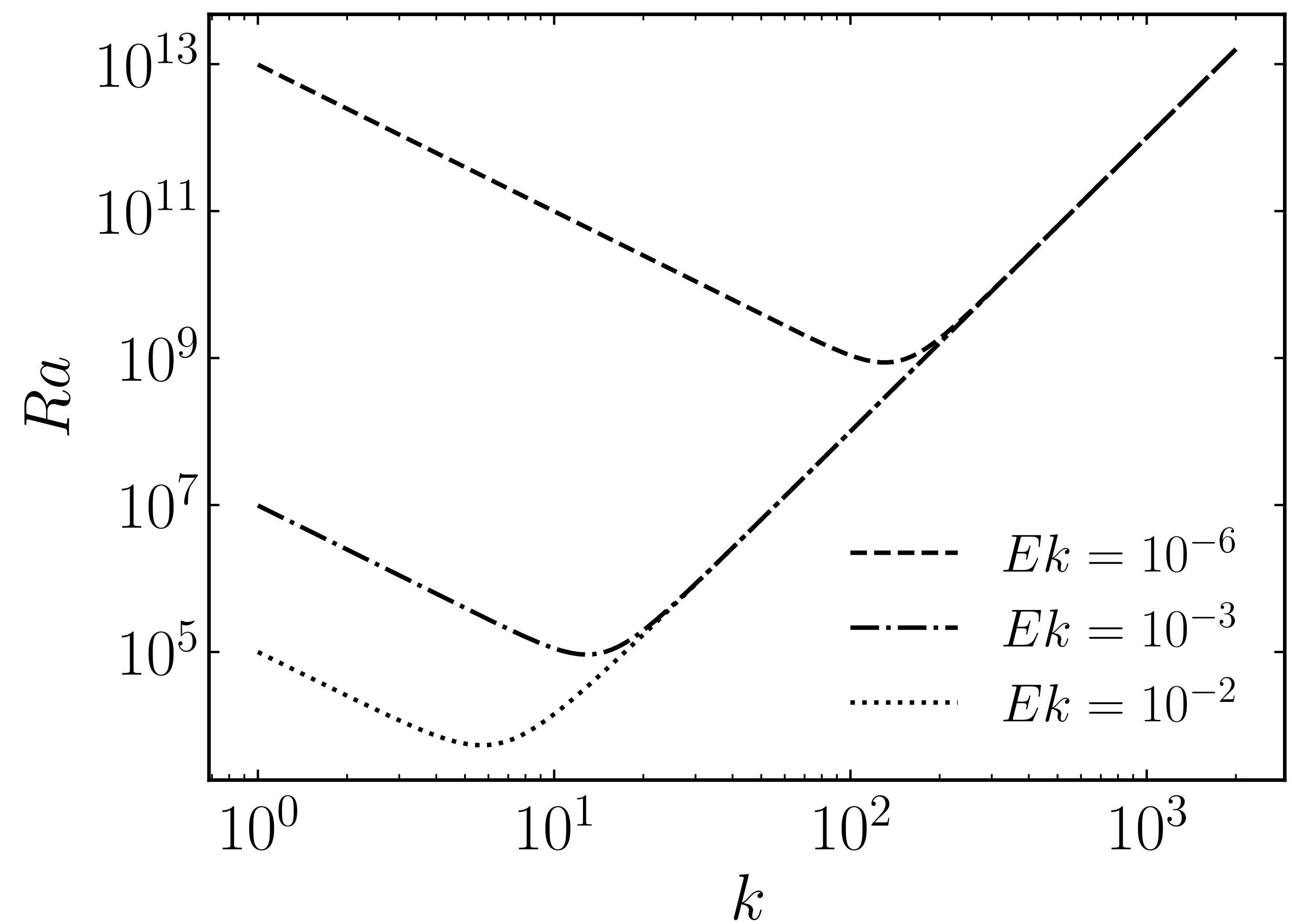
Asymptotic scaling relations for rotating spherical convection with strong zonal flows

Justin A. Nicoski[1], Anne R. O'Connor[1] and Michael A. Calkins[1],†

Linear Theory

- Mapped out in Chandrasekhar (1961):

$$Ra_m = \frac{(\pi^2 + k^2)^3 + \pi^2 Ek^{-2}}{k^2}$$

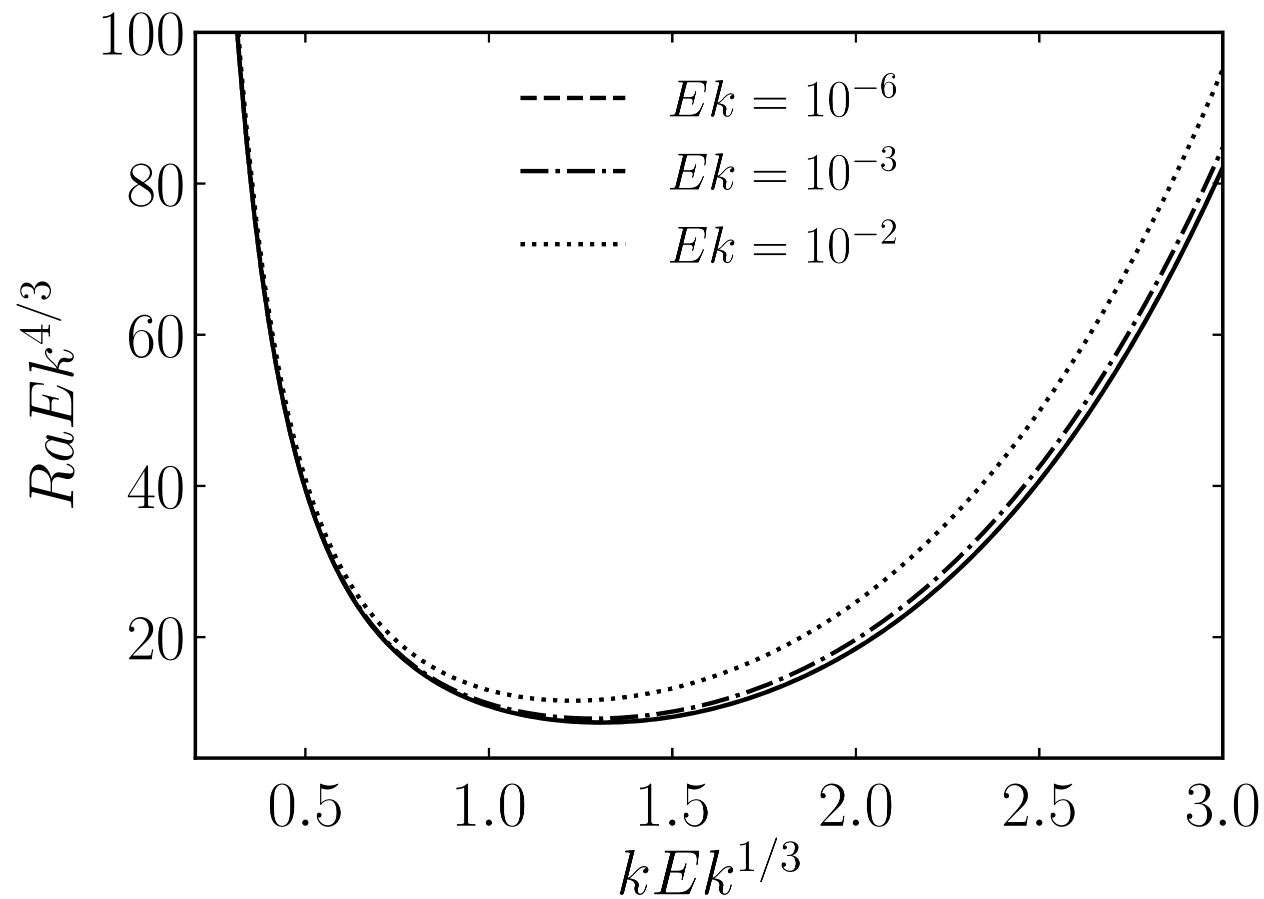


Linear Theory

- Most unstable mode is viscous length scale:

$$k = O(Ek^{-1/3})$$

$$\widetilde{Ra} = RaEk^{4/3}$$



$$\Rightarrow \widetilde{Ra}_m = \tilde{k}^4 + \pi^2 \tilde{k}^{-2}$$

All wavenumbers scale viscously

Nonlinear (QG) Theory

Julien, Knobloch, Werne & collaborators (1998, etc.)

Geostrophy at leading order:

$$\widehat{\mathbf{z}} \times \mathbf{u} \approx -\nabla p$$

Dynamics at next order:

$$D_t \mathbf{u} + \ldots \approx \ldots \frac{\widetilde{Ra}}{Pr}\theta + \nabla_\perp^2 \mathbf{u}$$

$$\Rightarrow \mathbf{u} = O(Ek^{-1/3}) \qquad \theta = O(Ek^{1/3}) \qquad \ell = O(Ek^{1/3})$$

Scalings persist in turbulent regime, independent of geometry

Scaling with Rayleigh: CIA Balance

We really want a dependence on Rayleigh number:

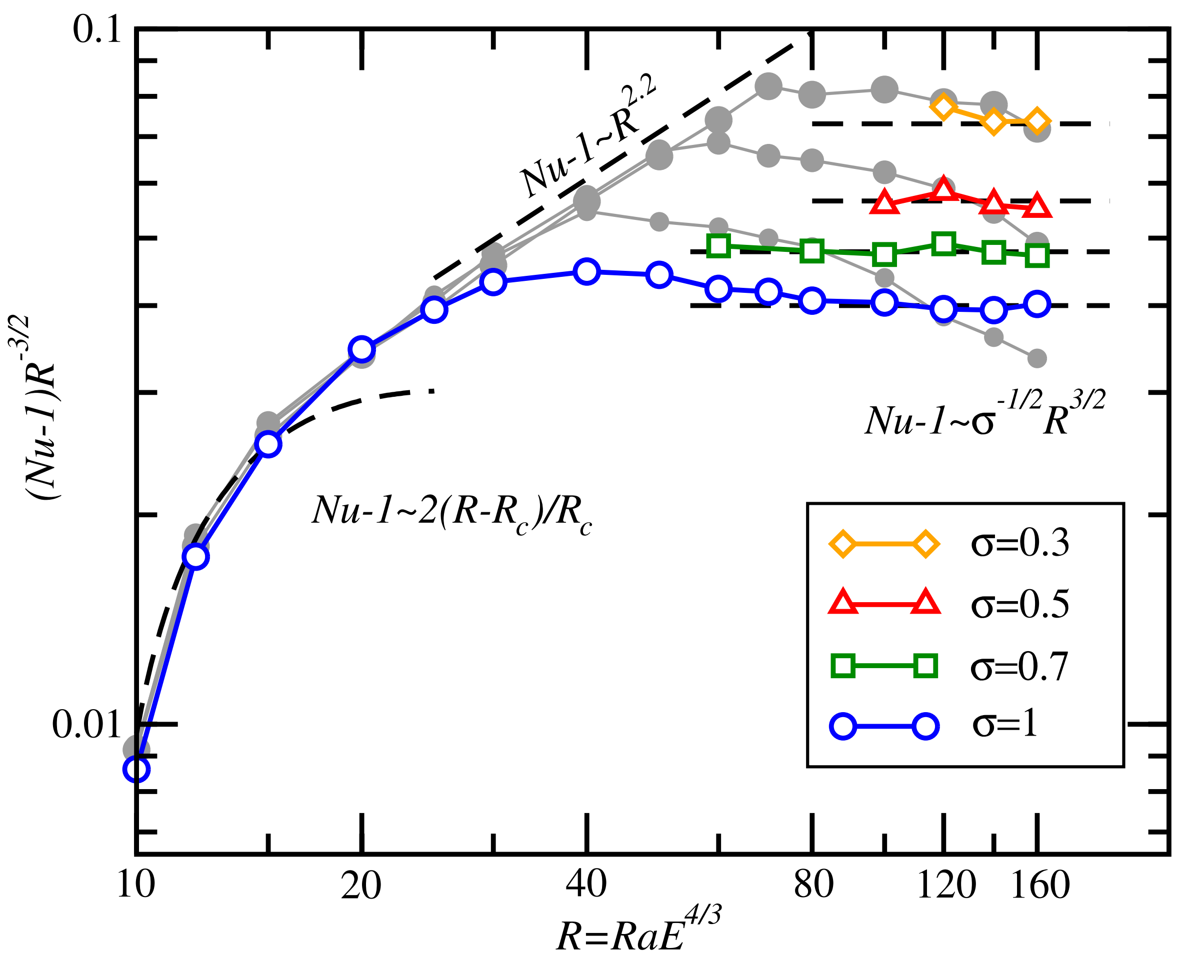
CIA: $u \sim \frac{\widetilde{Ra}}{Pr} Ek^{-1/3}$ $\qquad$ $\ell \sim \sqrt{\frac{\widetilde{Ra}}{Pr}} Ek^{1/3}$

$$\Rightarrow \widetilde{Re} \sim \frac{\widetilde{Ra}}{Pr}$$

Aurnou, Horn & Julien (PRR, 2020)
Oliver et al. (PRF, 2023)
Nicoski, O'Connor & Calkins (JFM, 2024)

QG Simulations

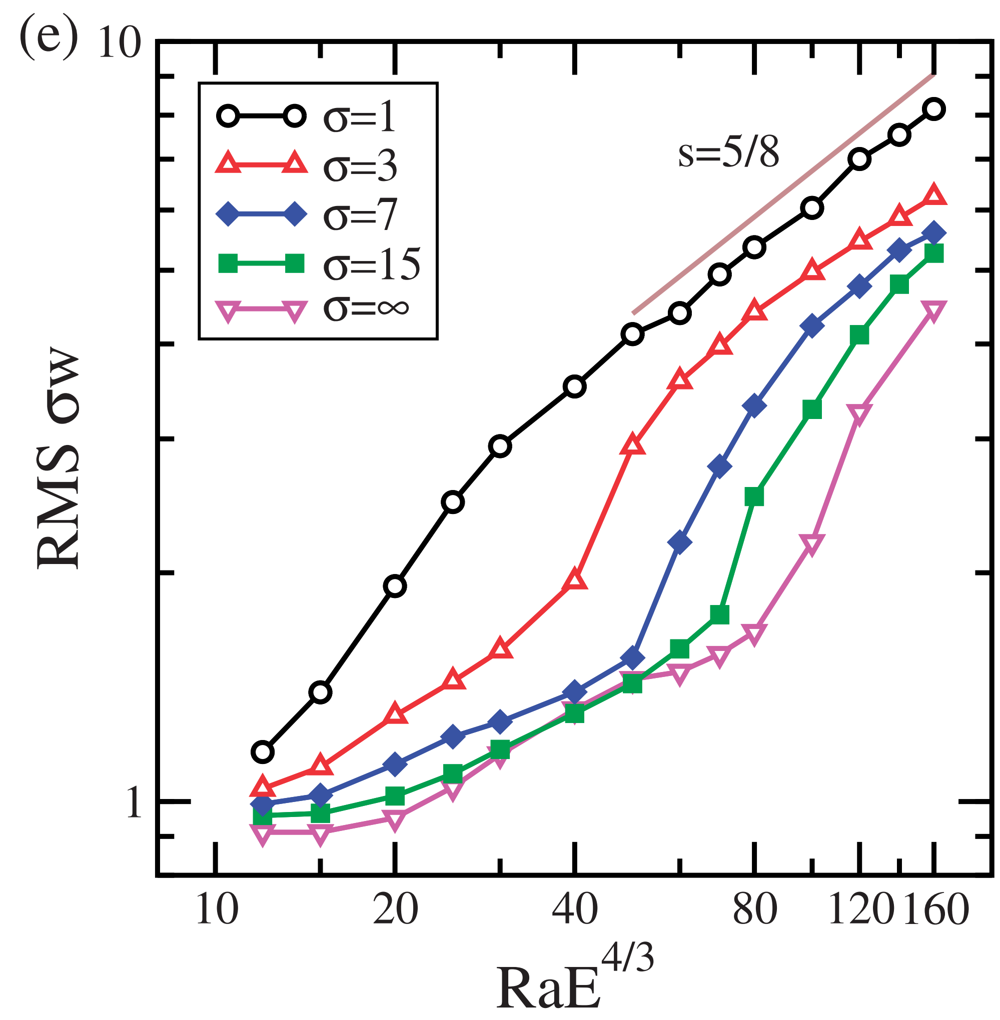
- Diffusion free scaling observed for heat transport



Julien et al. (PRL, 2012)

QG Simulations

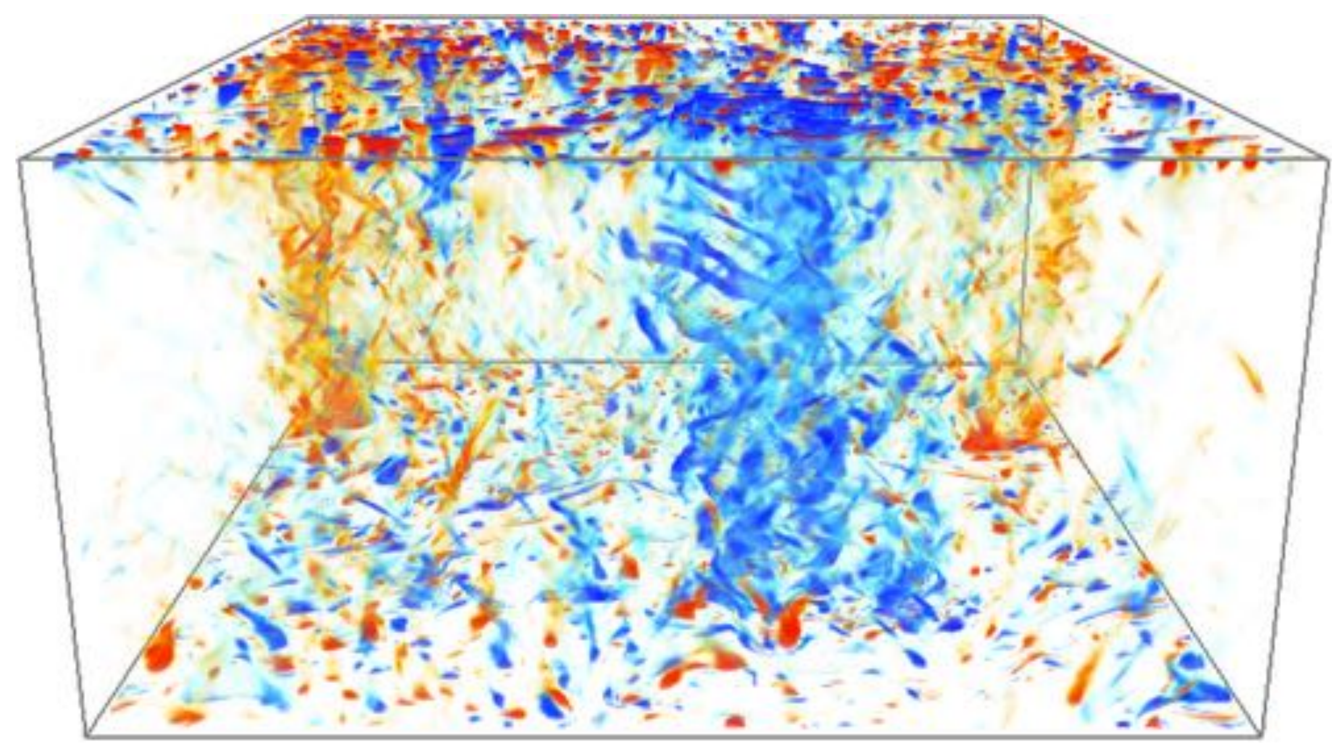
- Somewhat steeper scale observed for flow speeds.
- Prandtl number influences the observed scaling.



Julien et al. (GAFD, 2012)

QG Simulations

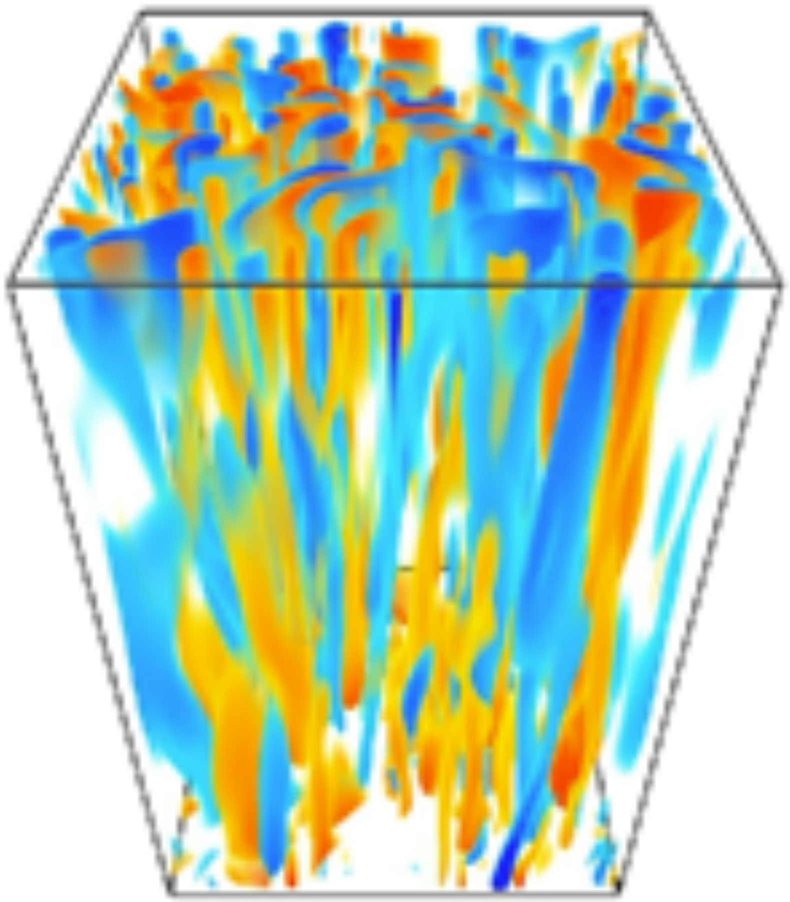
- Inverse cascade leads to large scale vortex (LSV)
- How does LSV influence scaling?

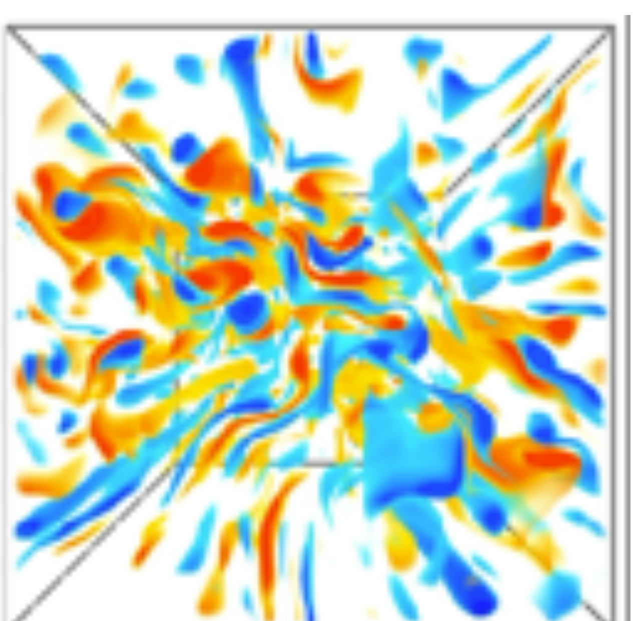


Rubio et al. (PRL, 2014)

QG Simulations

- Inverse cascade leads to large scale vortex (LSV)
- How does LSV influence scaling?



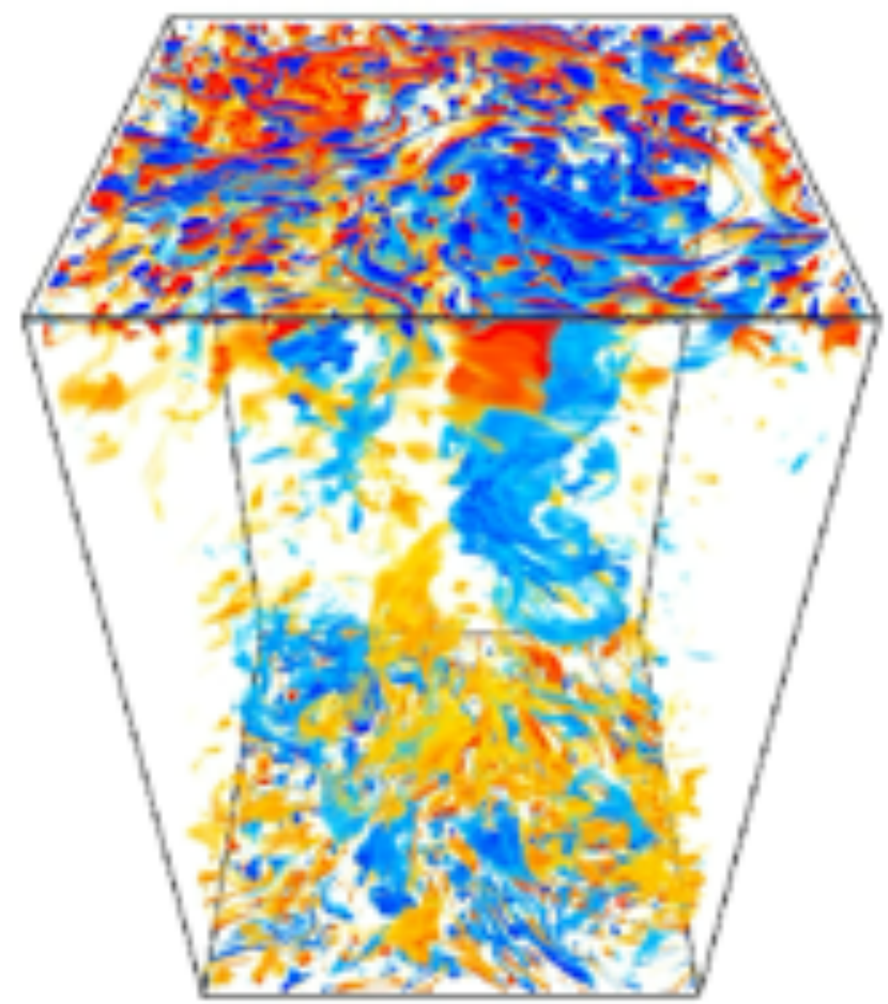


$$\widetilde{Ra} = 40$$

Maffei et al. (JFM, 2021)

QG Simulations

- Inverse cascade leads to large scale vortex (LSV)
- How does LSV influence scaling?

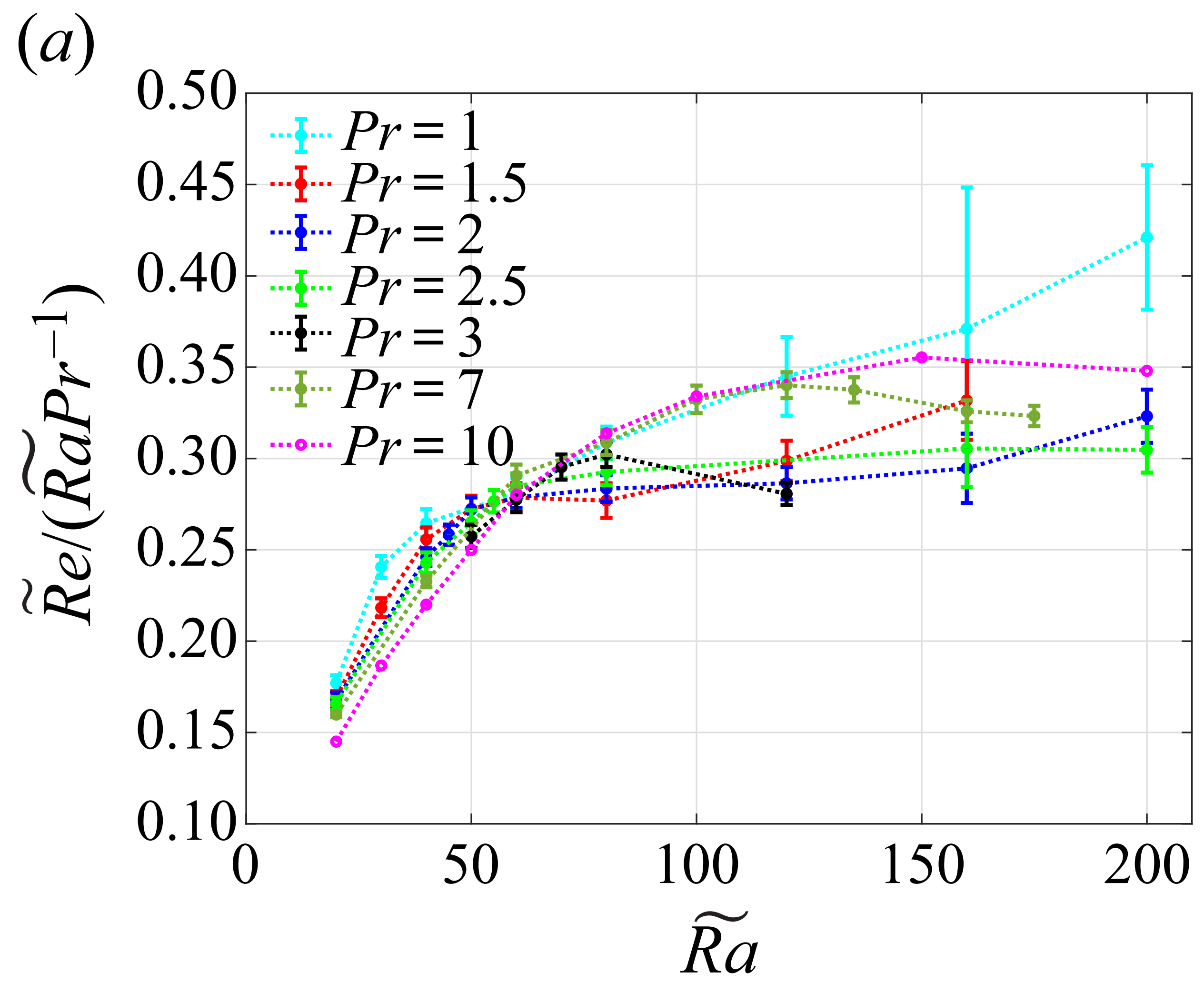


$\widetilde{Ra} = 200$

Maffei et al. (JFM, 2021)

QG Simulations

- Inverse cascade leads to large scale vortex (LSV)
- How does LSV influence scaling?



Maffei et al. (JFM, 2021)

QG Simulations: No LSV

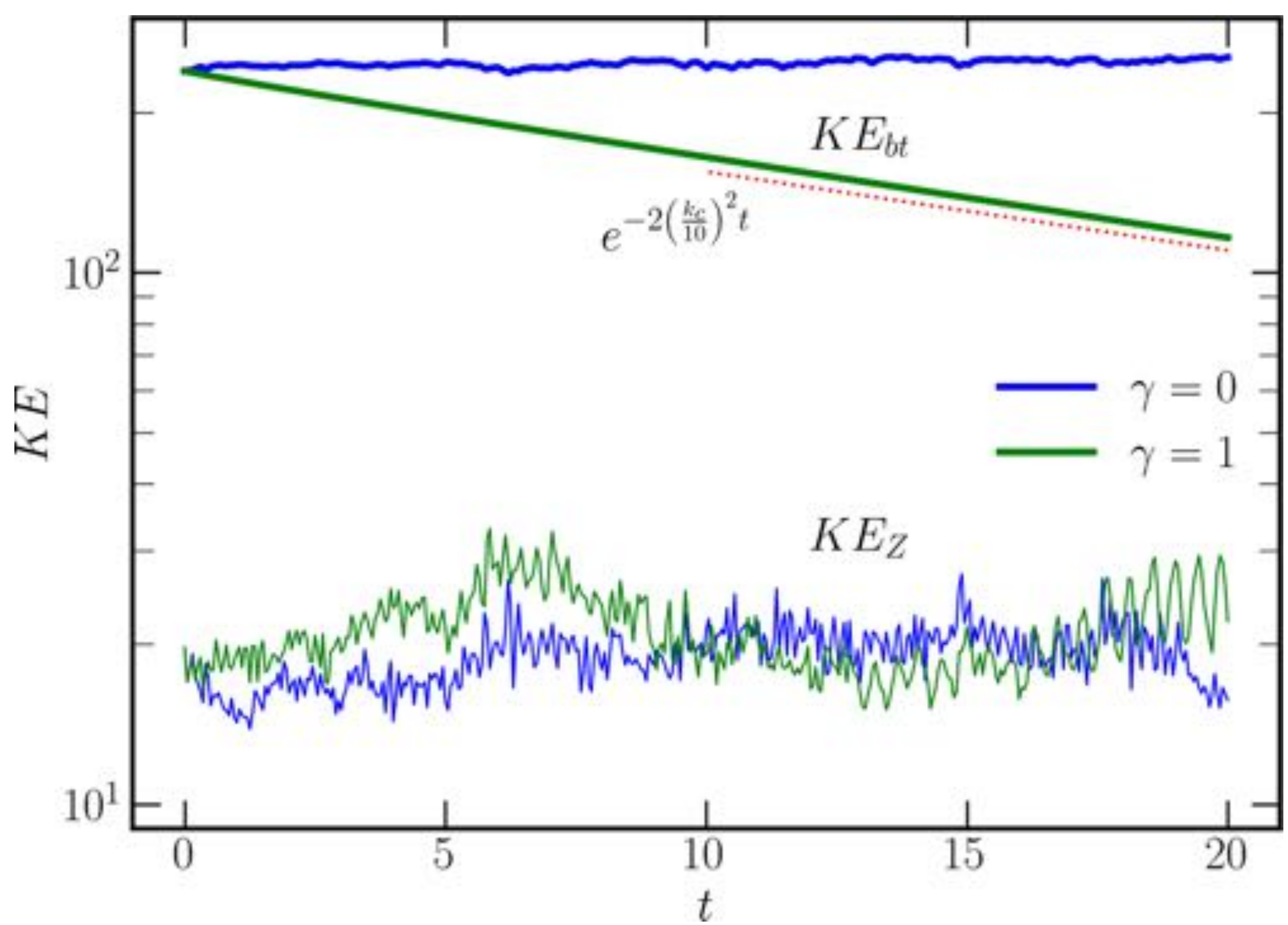
- We can "remove" the influence of the LSV:

$$\partial_t \zeta + J[\psi, \zeta] - \gamma \langle J[\psi, \zeta] \rangle - \partial_Z w = \nabla_\perp^2 \zeta,$$

$$\gamma = 1 \Rightarrow \qquad \partial_t \langle \zeta \rangle = \nabla_\perp^2 \langle \zeta \rangle$$

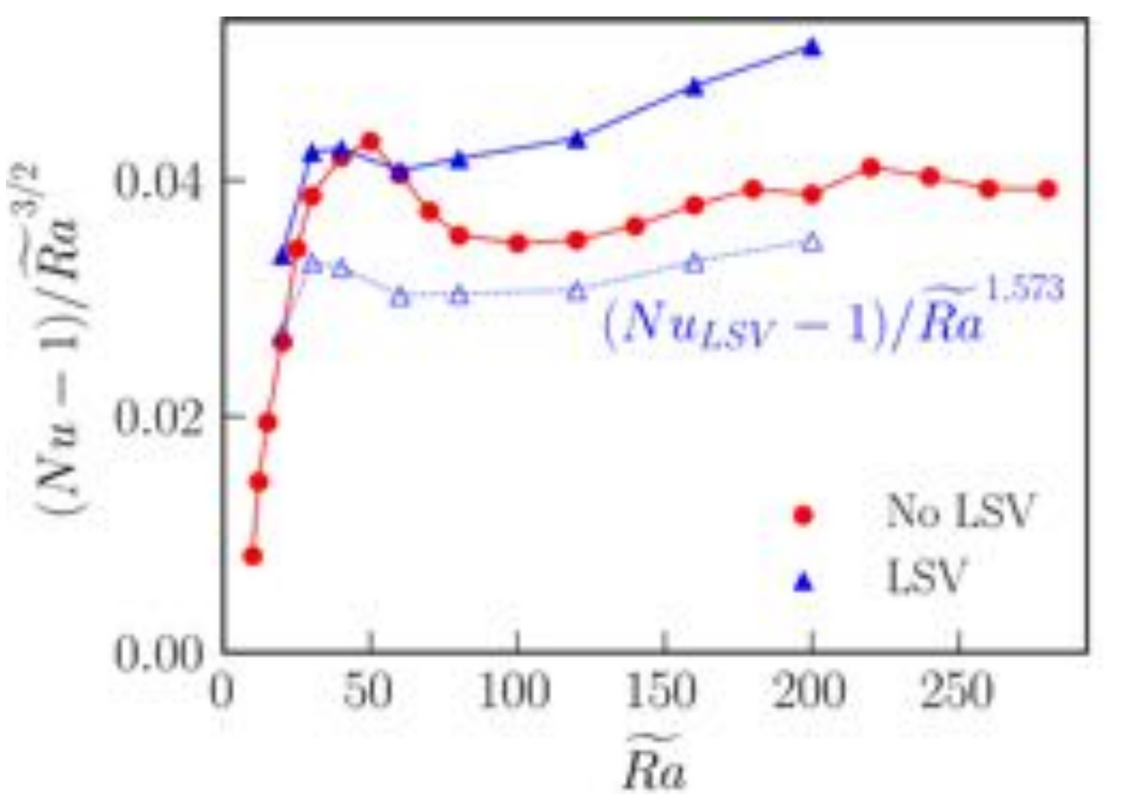
Oliver et al. (PRF, 2023)

QG Simulations: No LSV



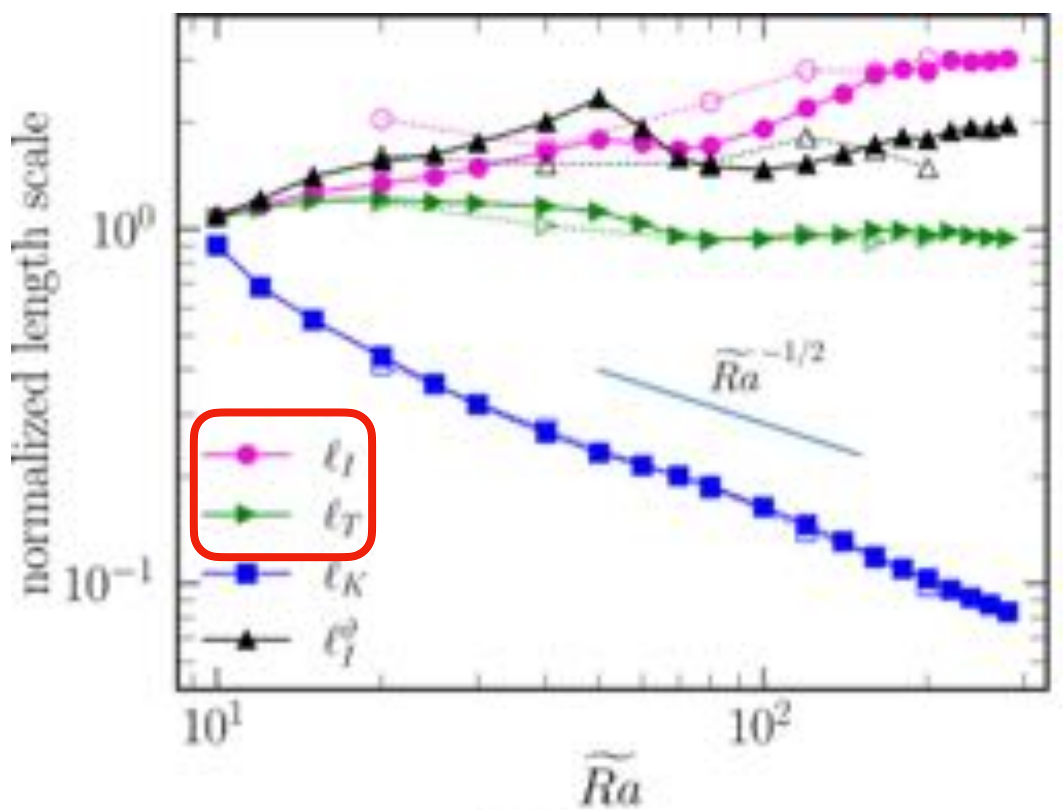
Oliver et al. (PRF, 2023)

Global Transport



Oliver et al. (PRF, 2023)

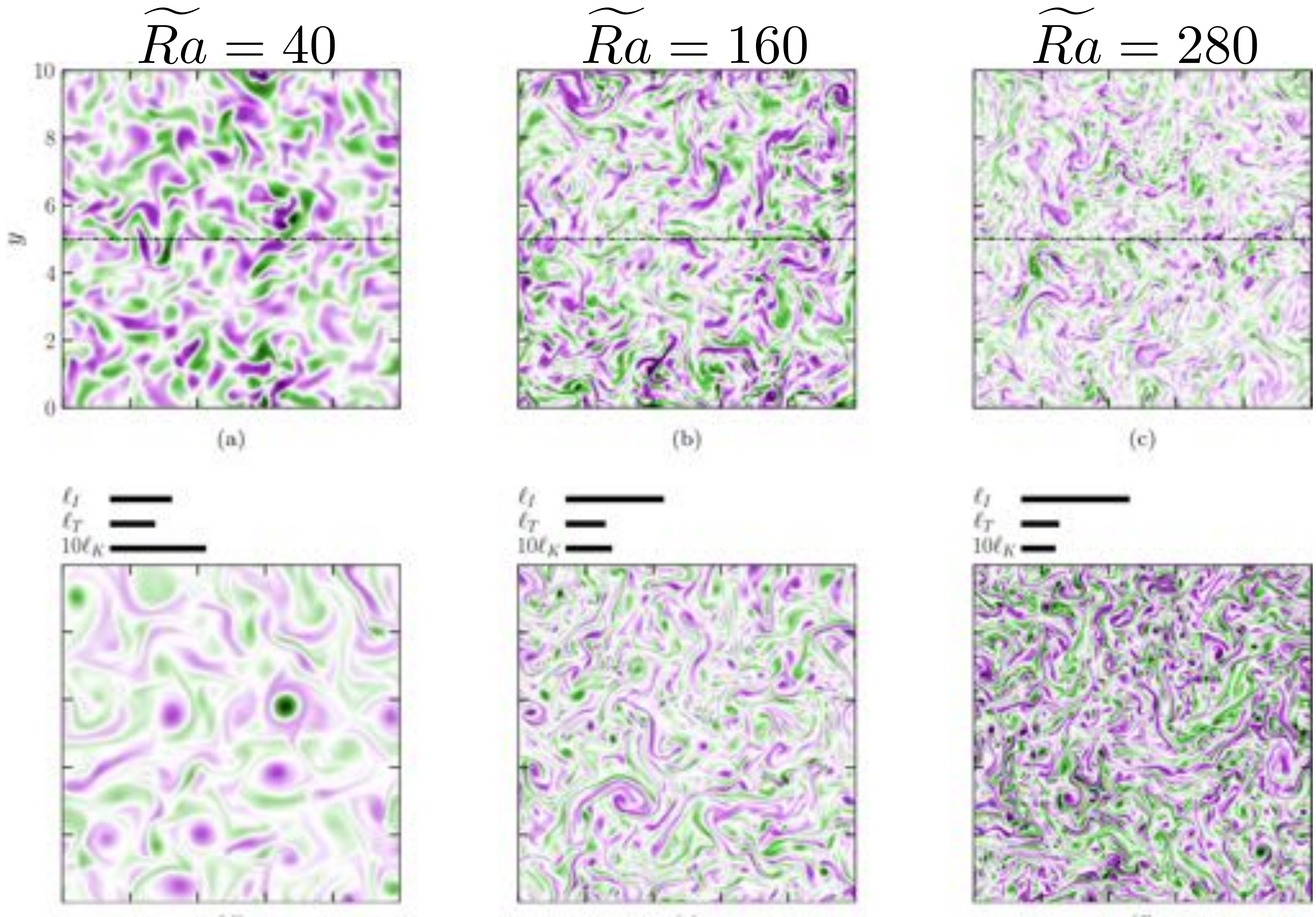
Length Scales



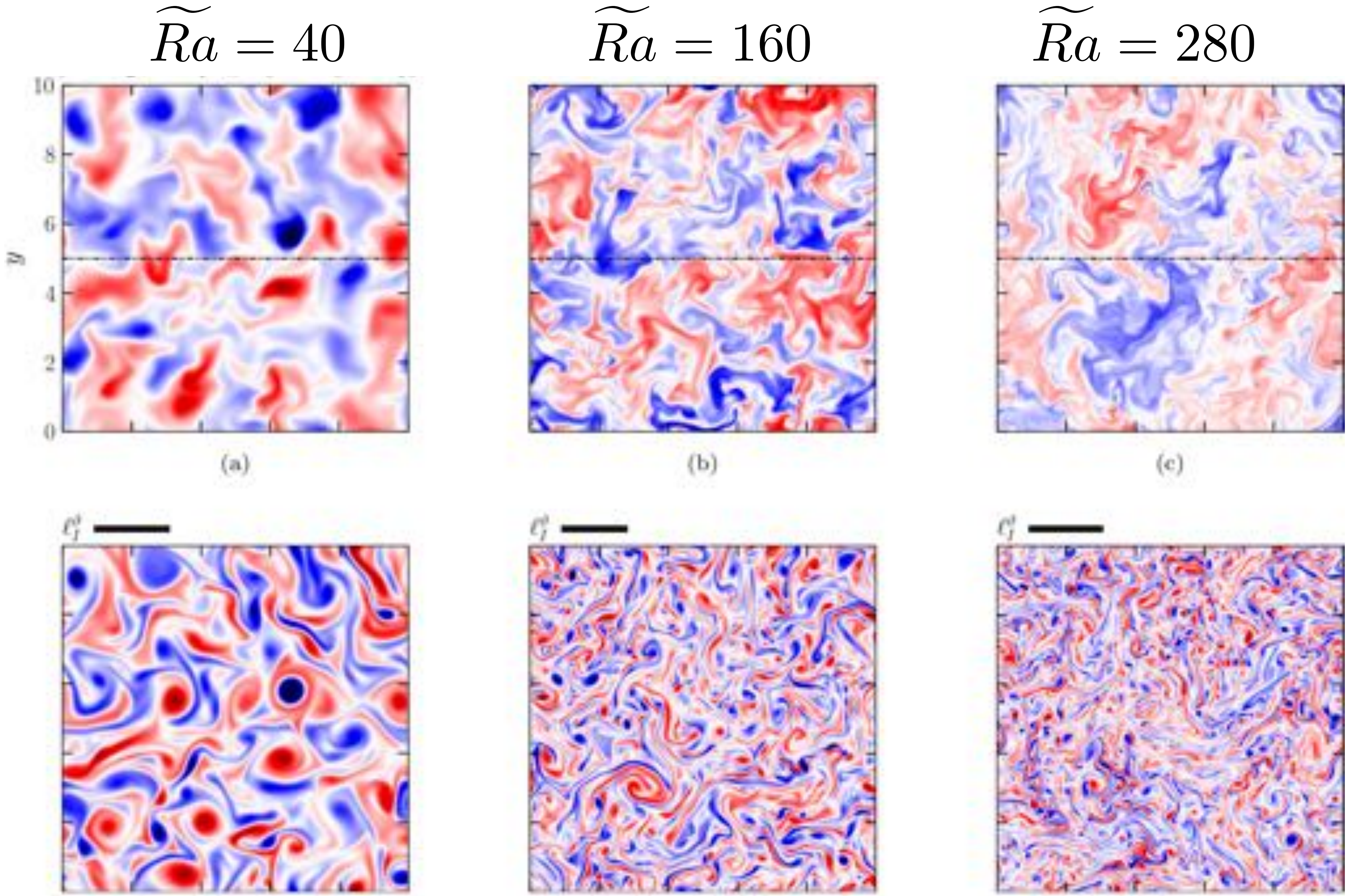
Length scales remain within the viscous range

Oliver et al. (PRF, 2023)

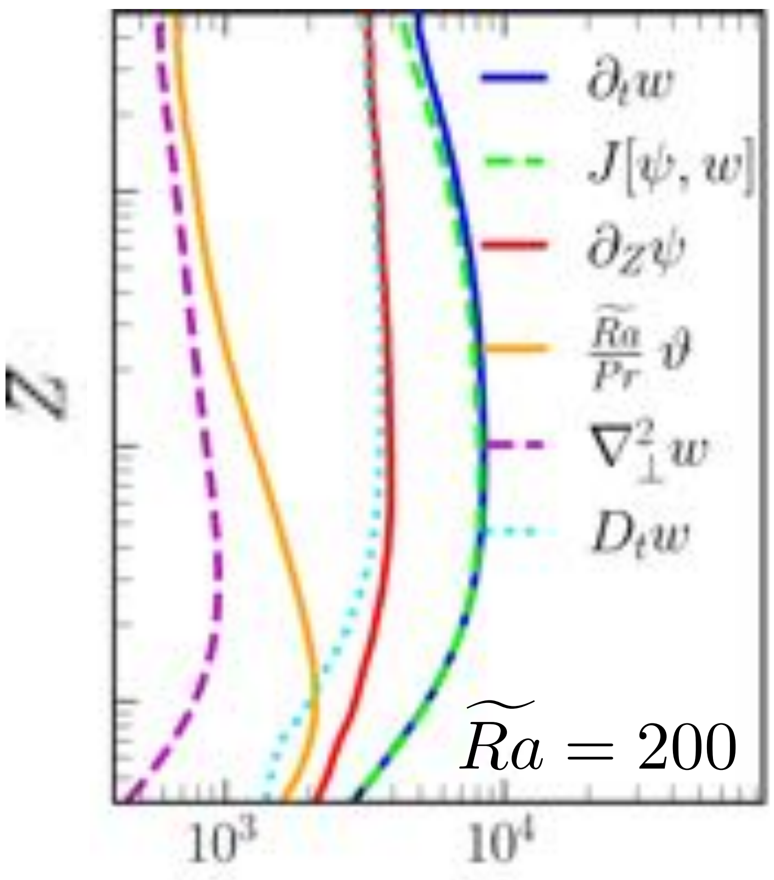
Flow Structure: vorticity

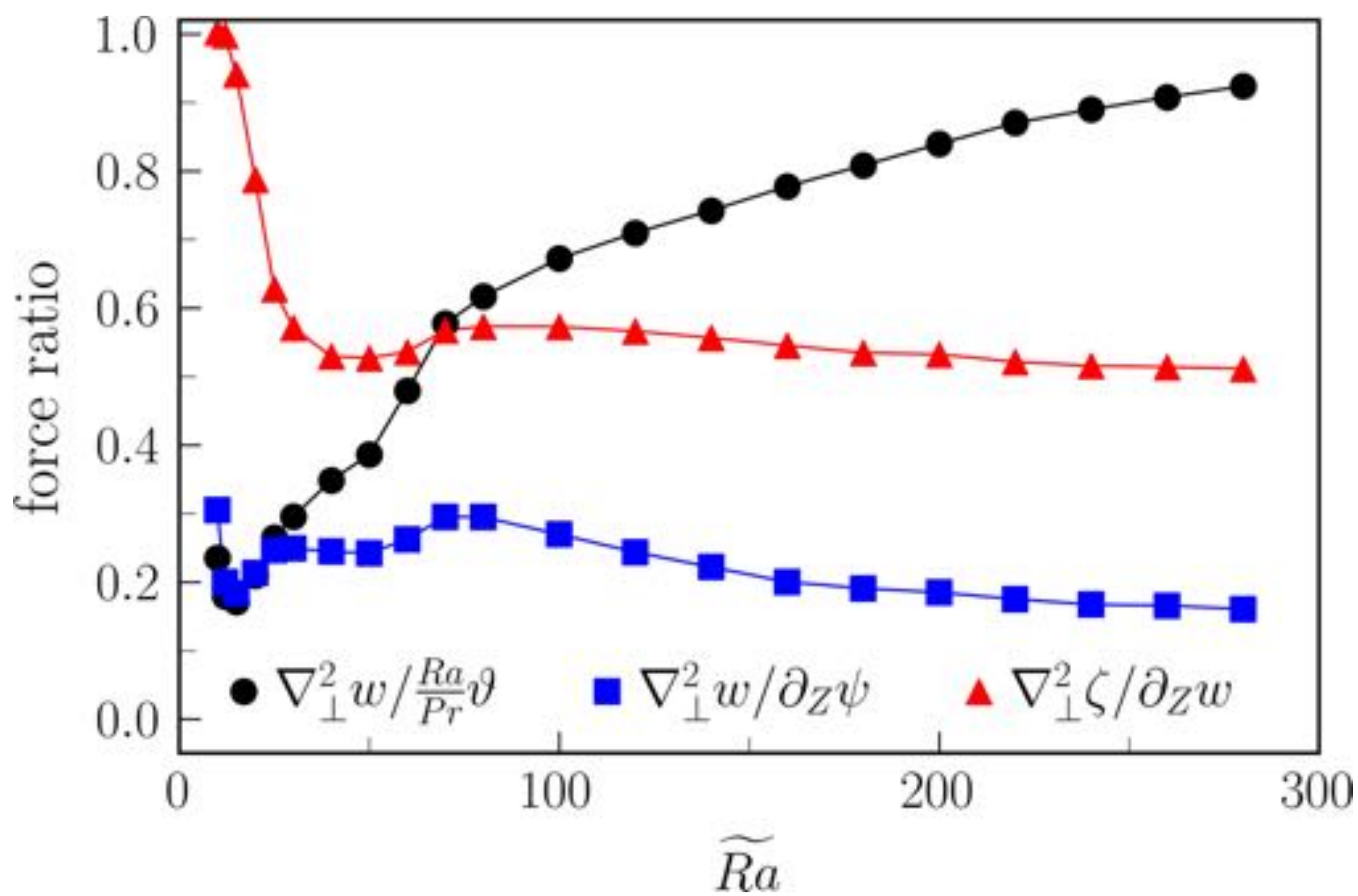


Flow Structure: temperature



QG Balances

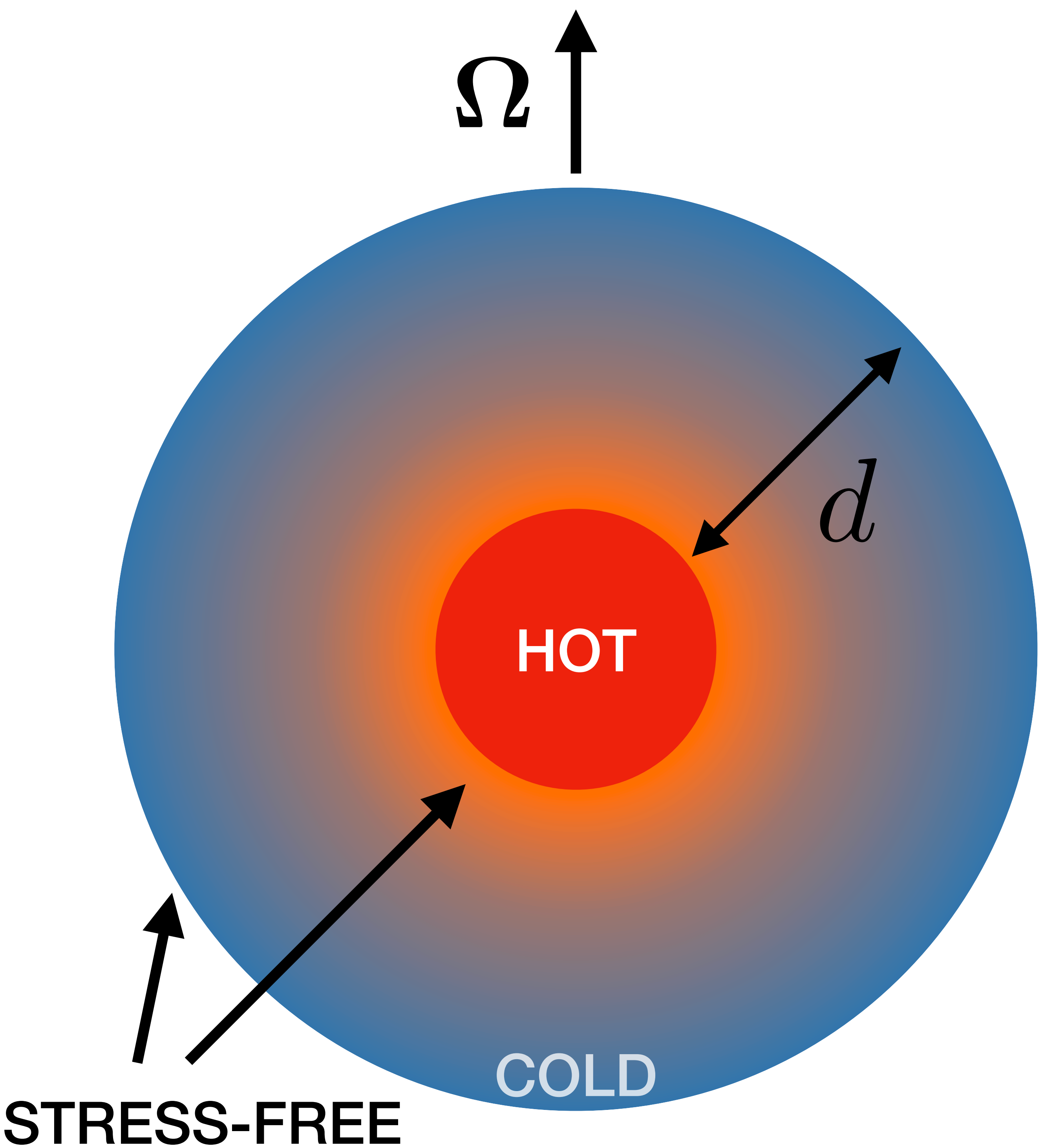




Balances & length scales are not consistent with CIA

Similar in spherical geometries?

Spherical Convection



Ekman number: Ek

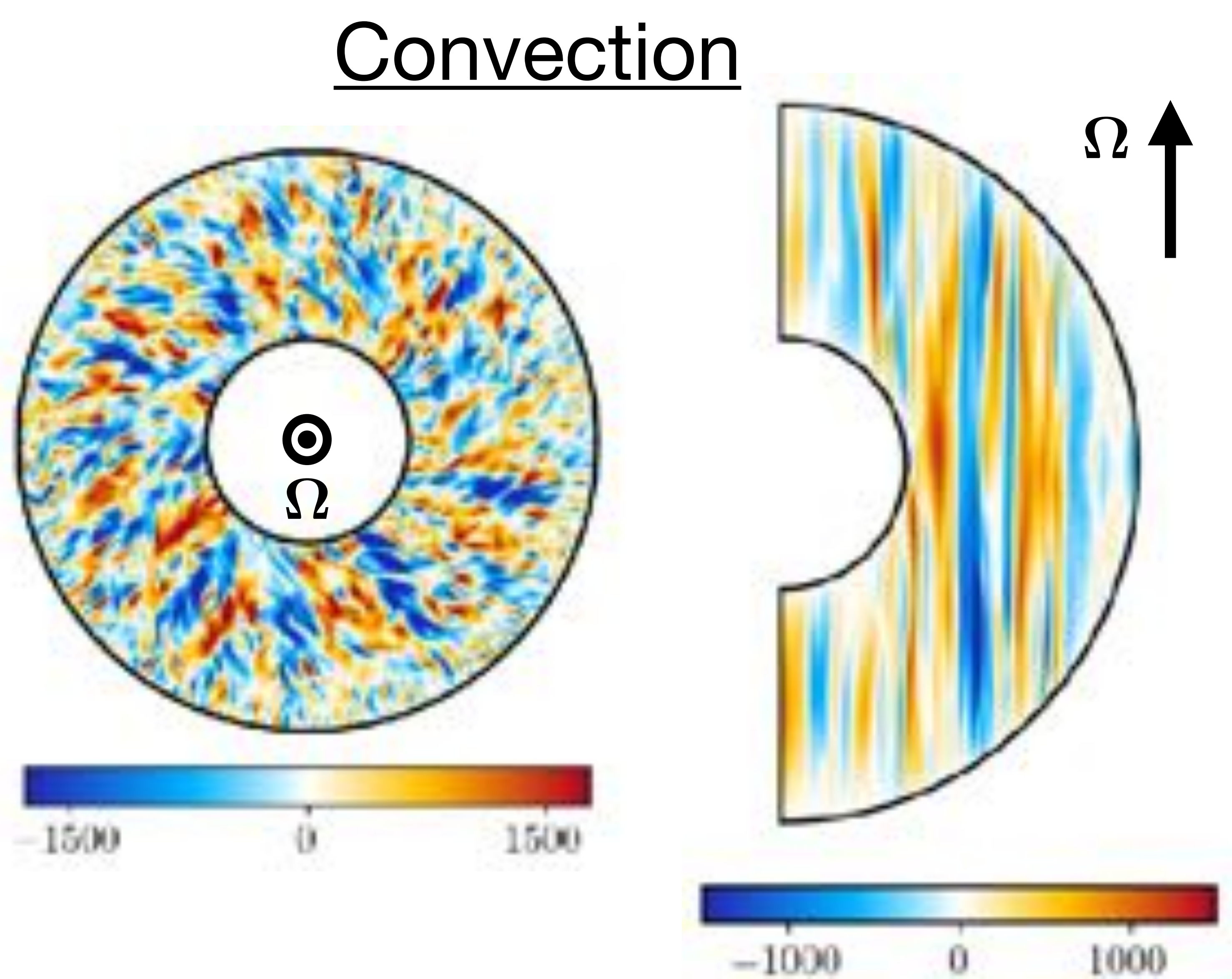
Reduced Rayleigh number:

$$\widetilde{Ra} = Ra Ek^{4/3}$$

DNS: $10^{-6} \ll Ek \ll 10^{-3}$

Nicoski, O'Connor & Calkins (JFM, 2024)

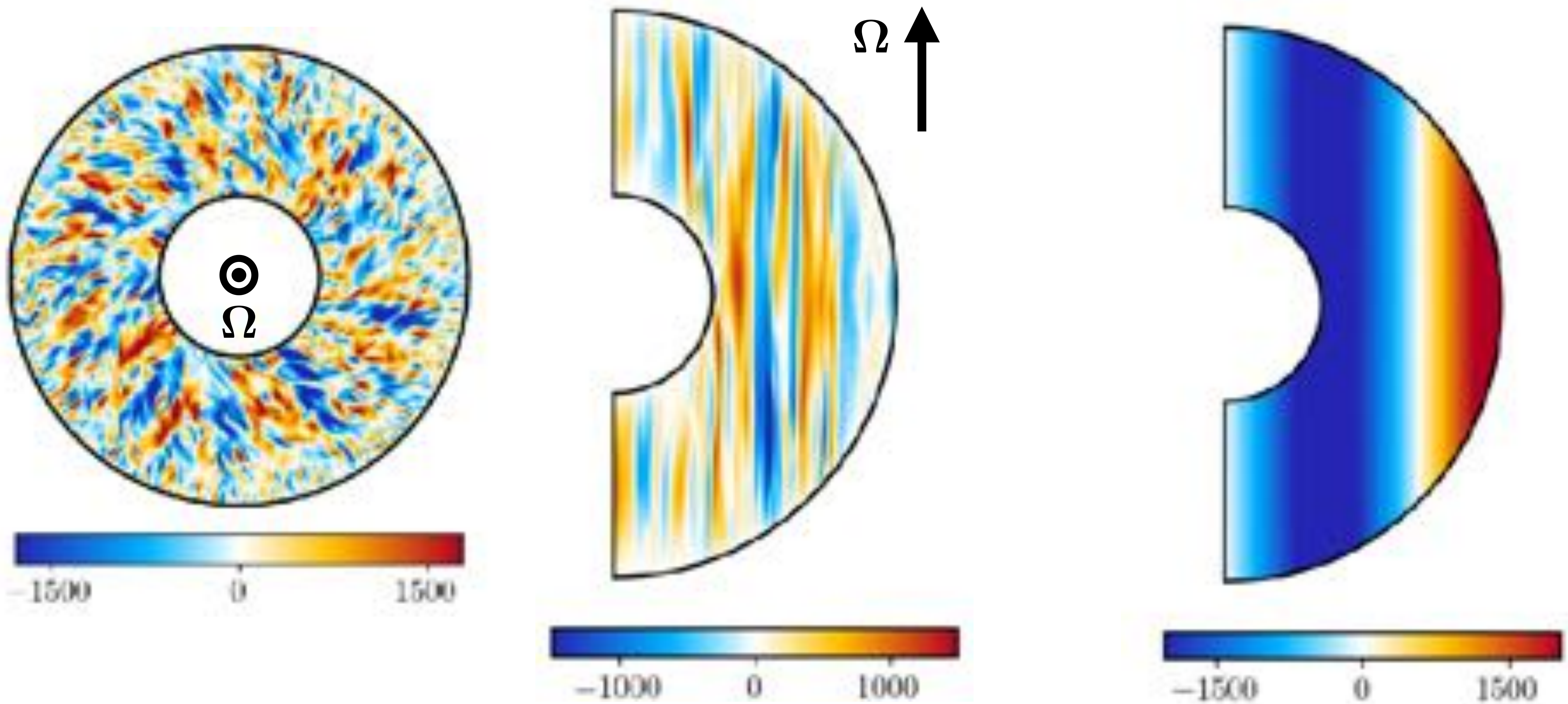
Flow Decomposition



$$\mathbf{u}(r,\theta,\phi,t) = \overline{\mathbf{u}}(r,\theta,t) + \mathbf{u}'(r,\theta,\phi,t)$$

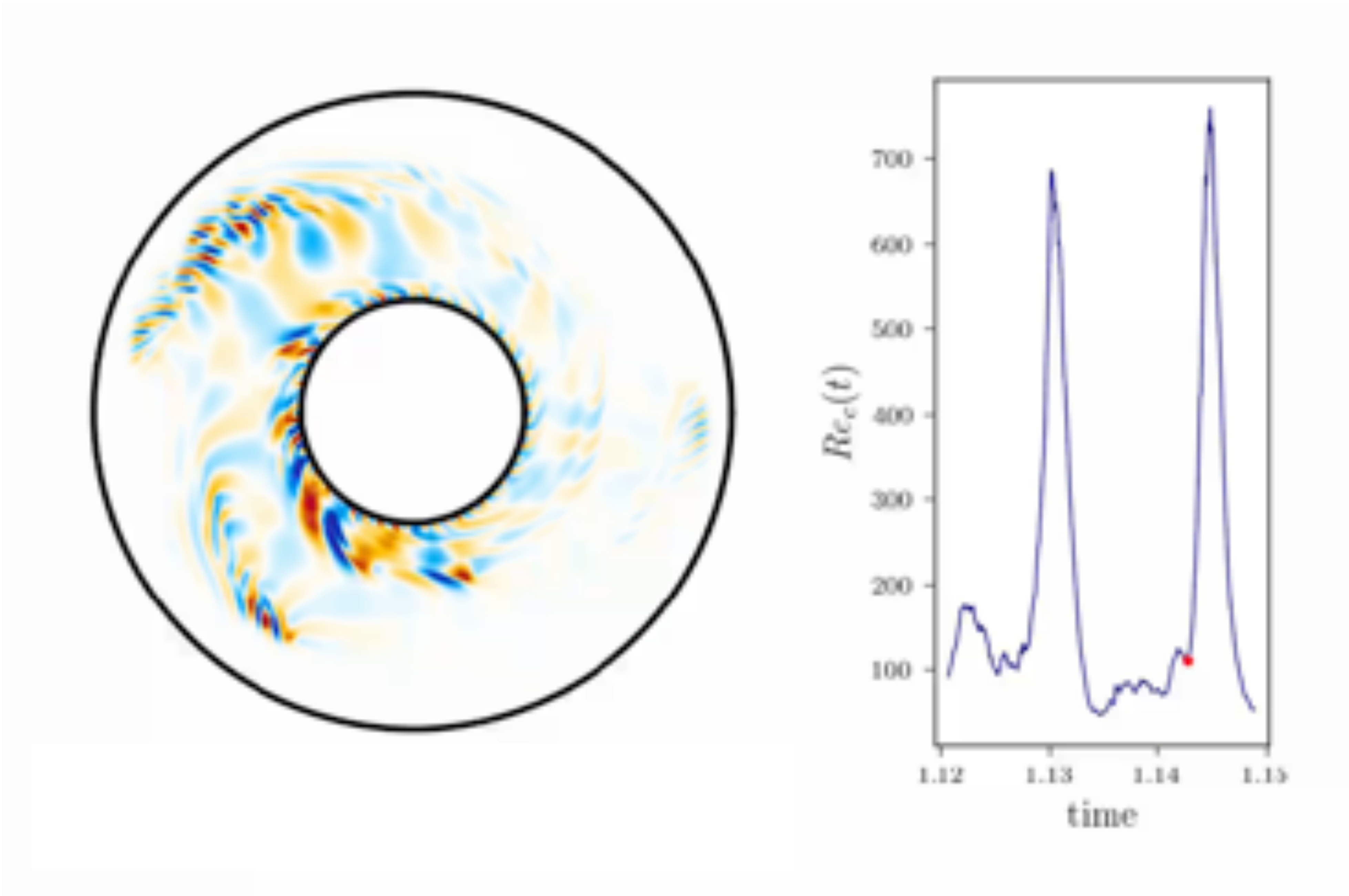
Flow Decomposition

Convection — Zonal Flow



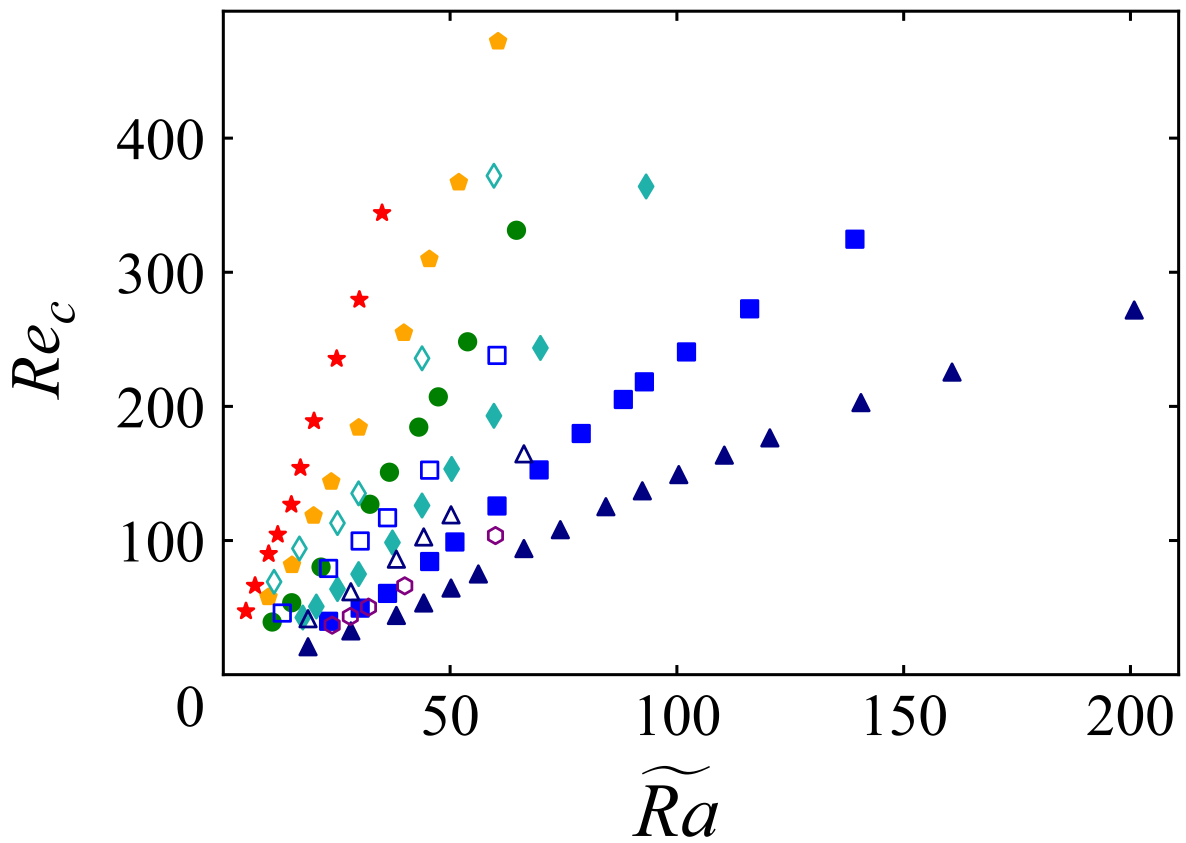
$$\mathbf{u}(r,\theta,\phi,t) = \boxed{\overline{\mathbf{u}}(r,\theta,t)} + \mathbf{u}'(r,\theta,\phi,t)$$

Typical Dynamics



$$\mathbf{u}(r,\theta,\phi,t) = \overline{\mathbf{u}}(r,\theta,t) + \boxed{\mathbf{u}'(r,\theta,\phi,t)}$$

Convective Speeds

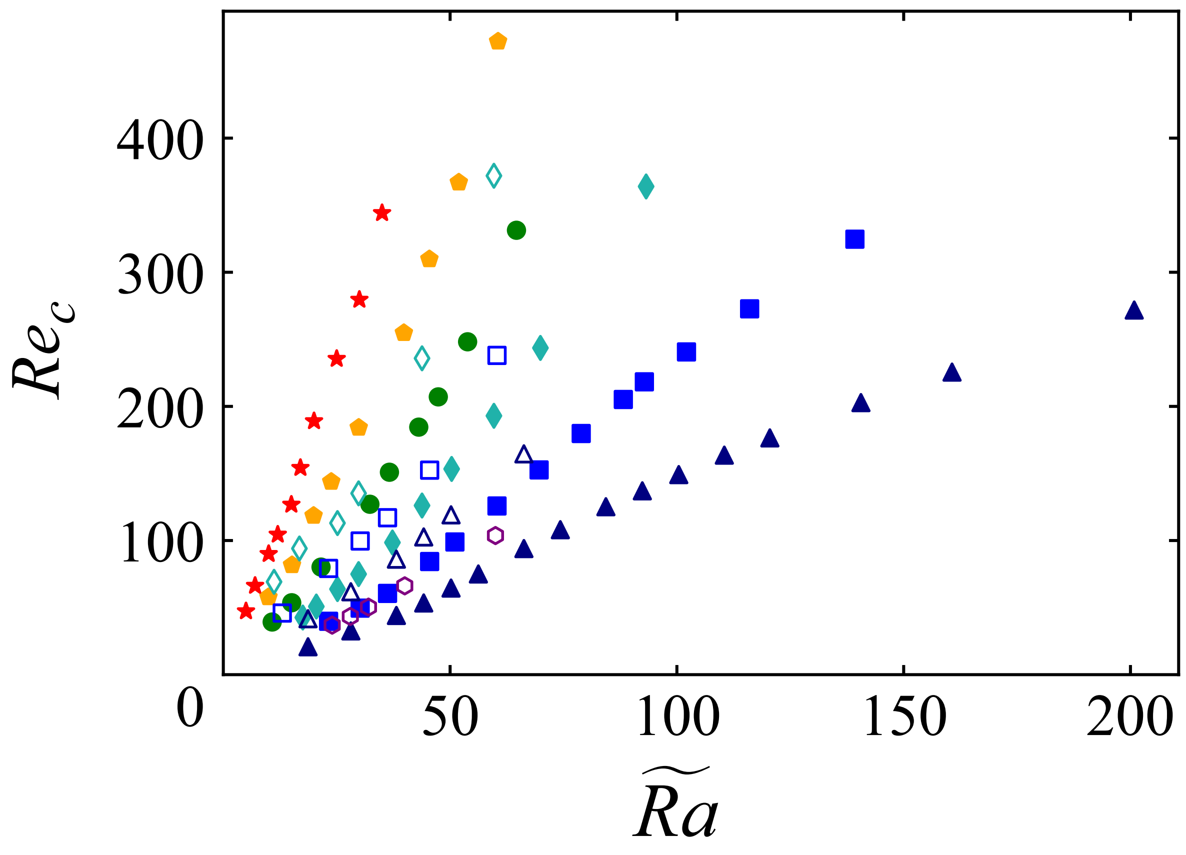


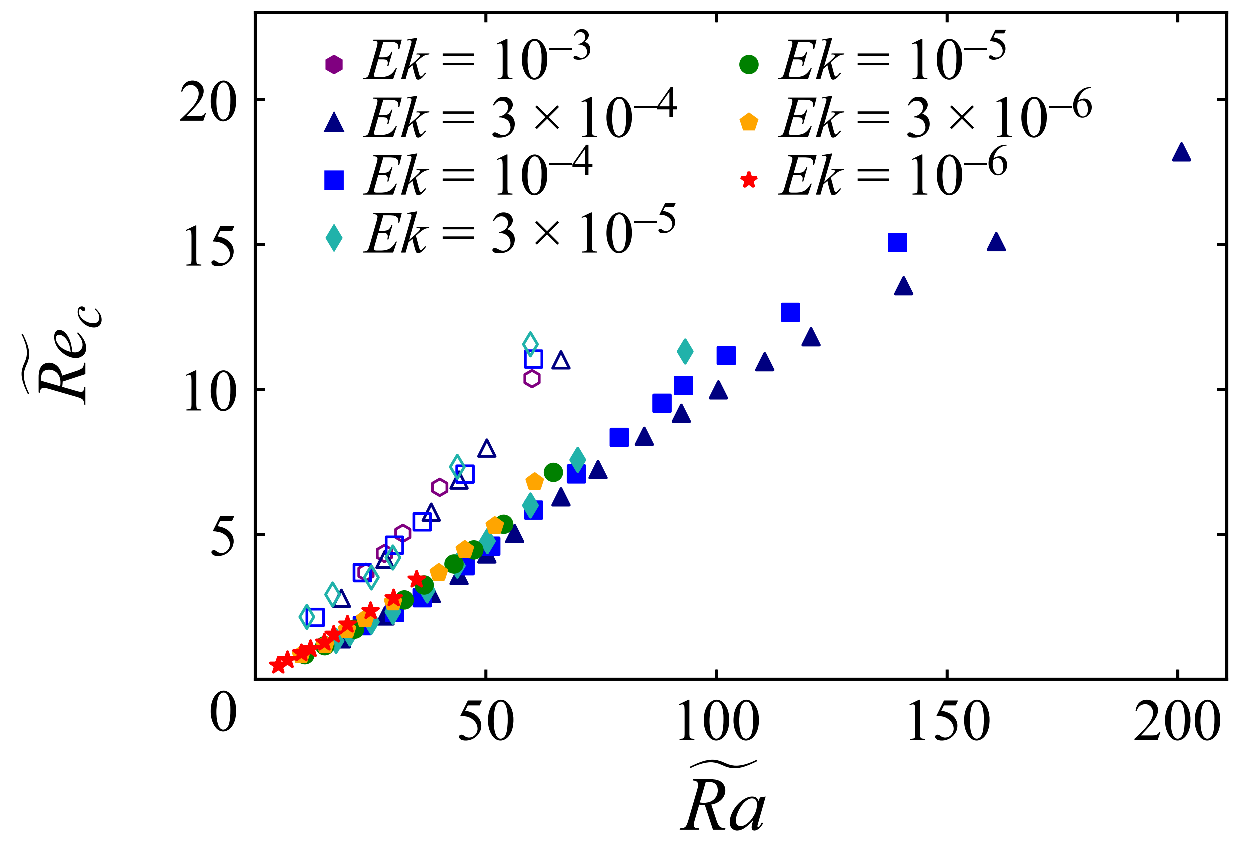
Geostrophy on viscous length scale:

$$Re_c = O\left(Ek^{-1/3}\right)$$

$$\mathbf{u}(r,\theta,\phi,t) = \overline{\mathbf{u}}(r,\theta,t) + \boxed{\mathbf{u}'(r,\theta,\phi,t)}$$

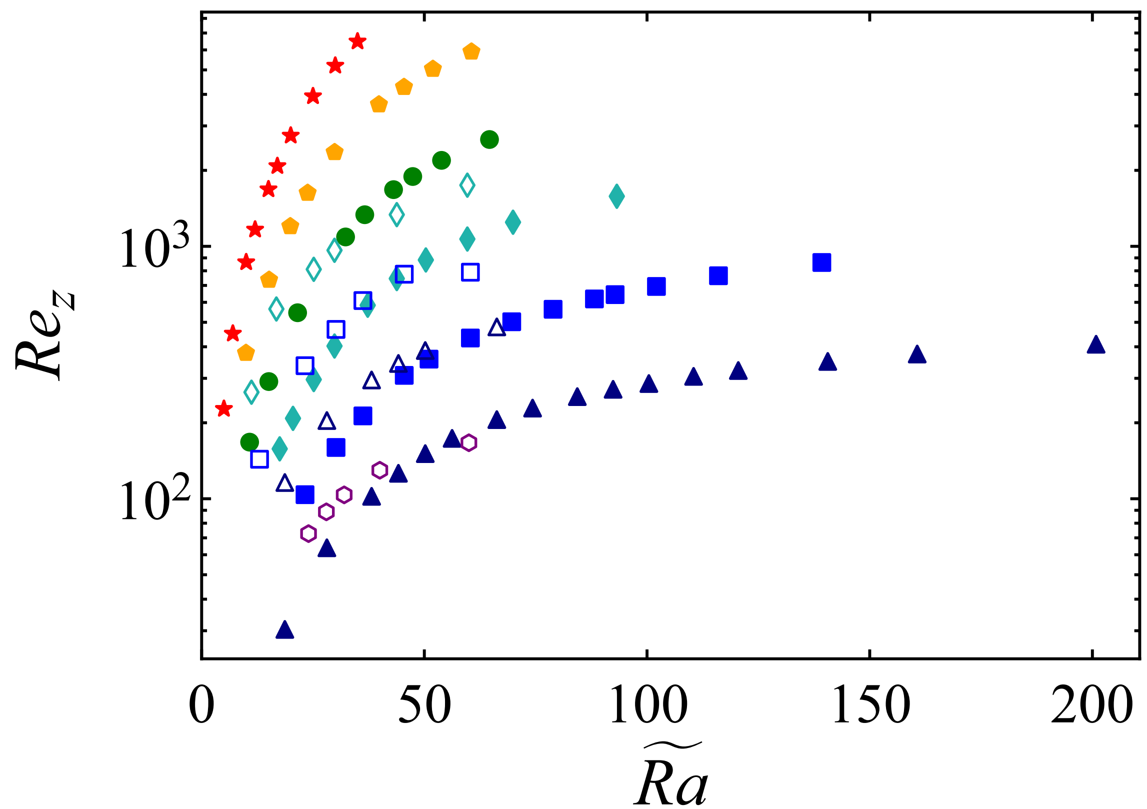
Convective Speeds





$$\mathbf{u}(r, \theta, \phi, t) = \overline{\mathbf{u}}(r, \theta, t) + \mathbf{u}'(r, \theta, \phi, t)$$

Zonal Flow Speeds

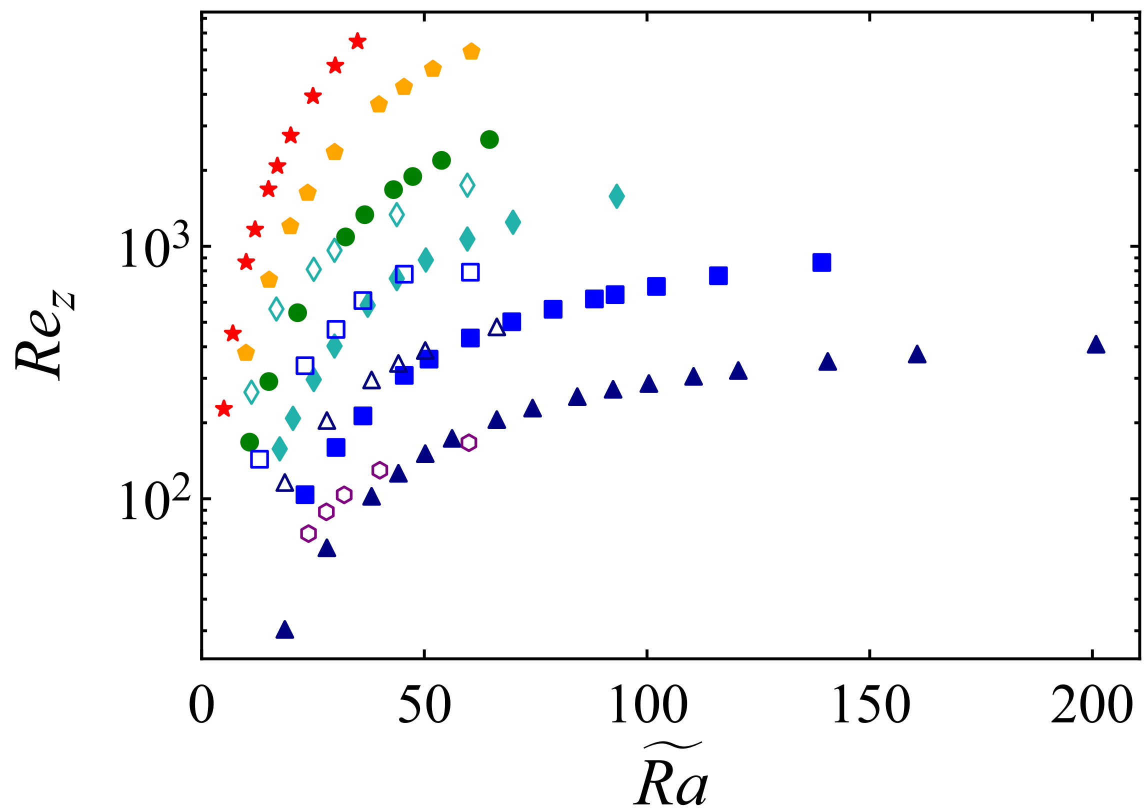


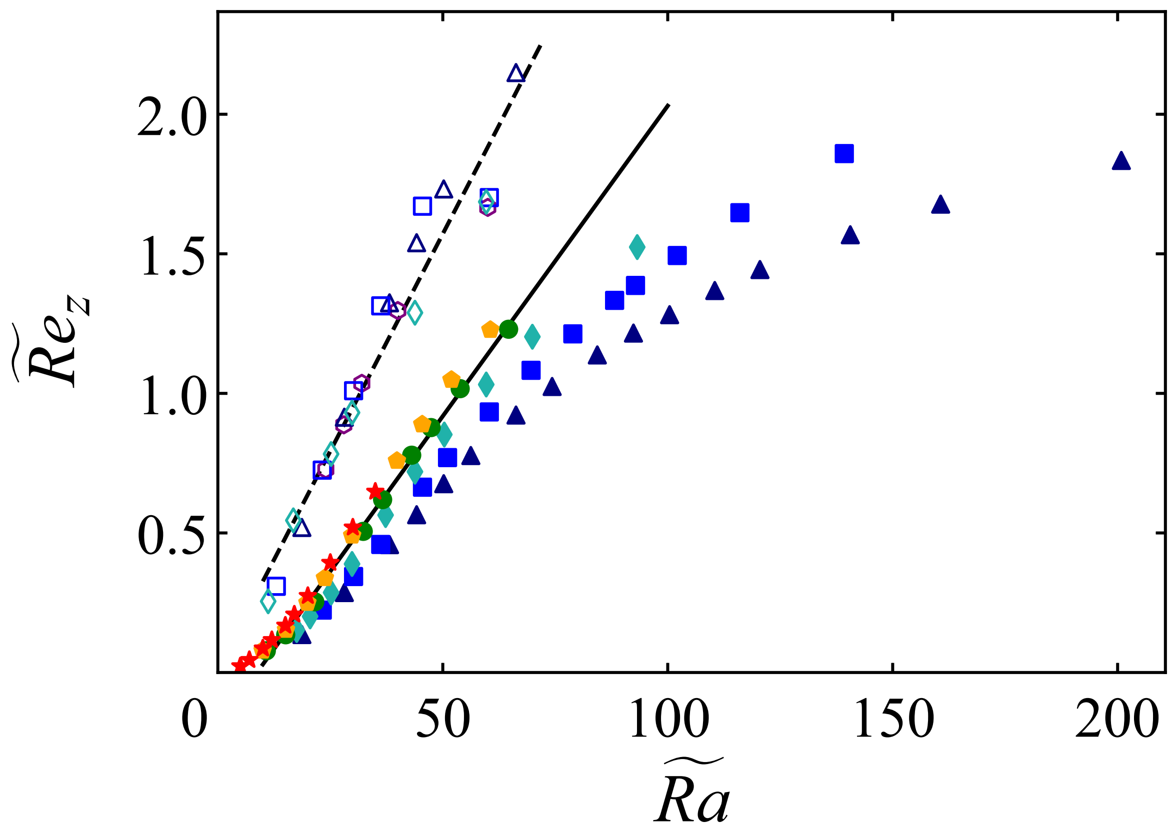
Viscosity/Reynolds stress balance:

$$Re_z = O\left(Ek^{-2/3}\right)$$

$$\mathbf{u}(r,\theta,\phi,t) = \boxed{\overline{\mathbf{u}}(r,\theta,t)} + \mathbf{u}'(r,\theta,\phi,t)$$

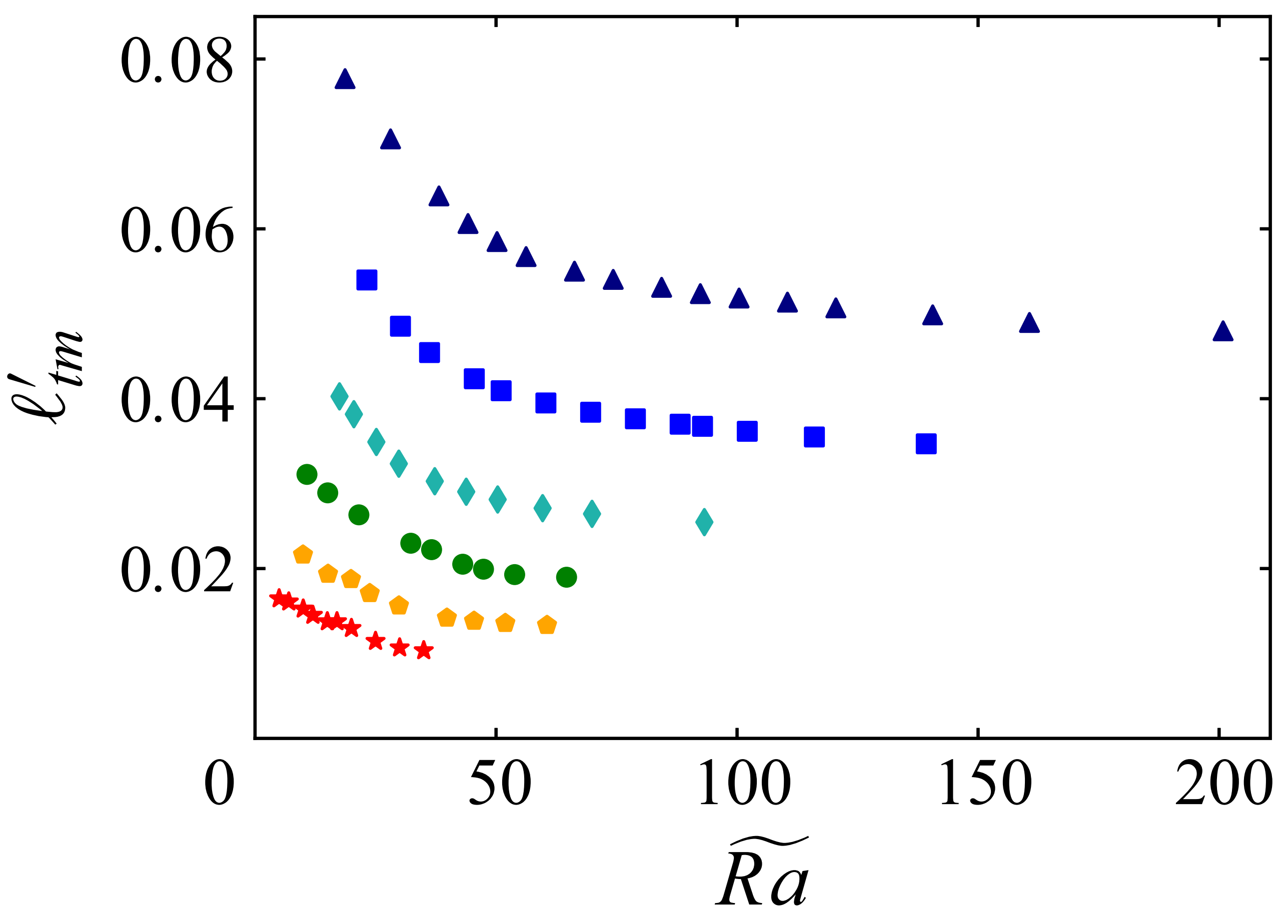
Zonal Flow Speeds

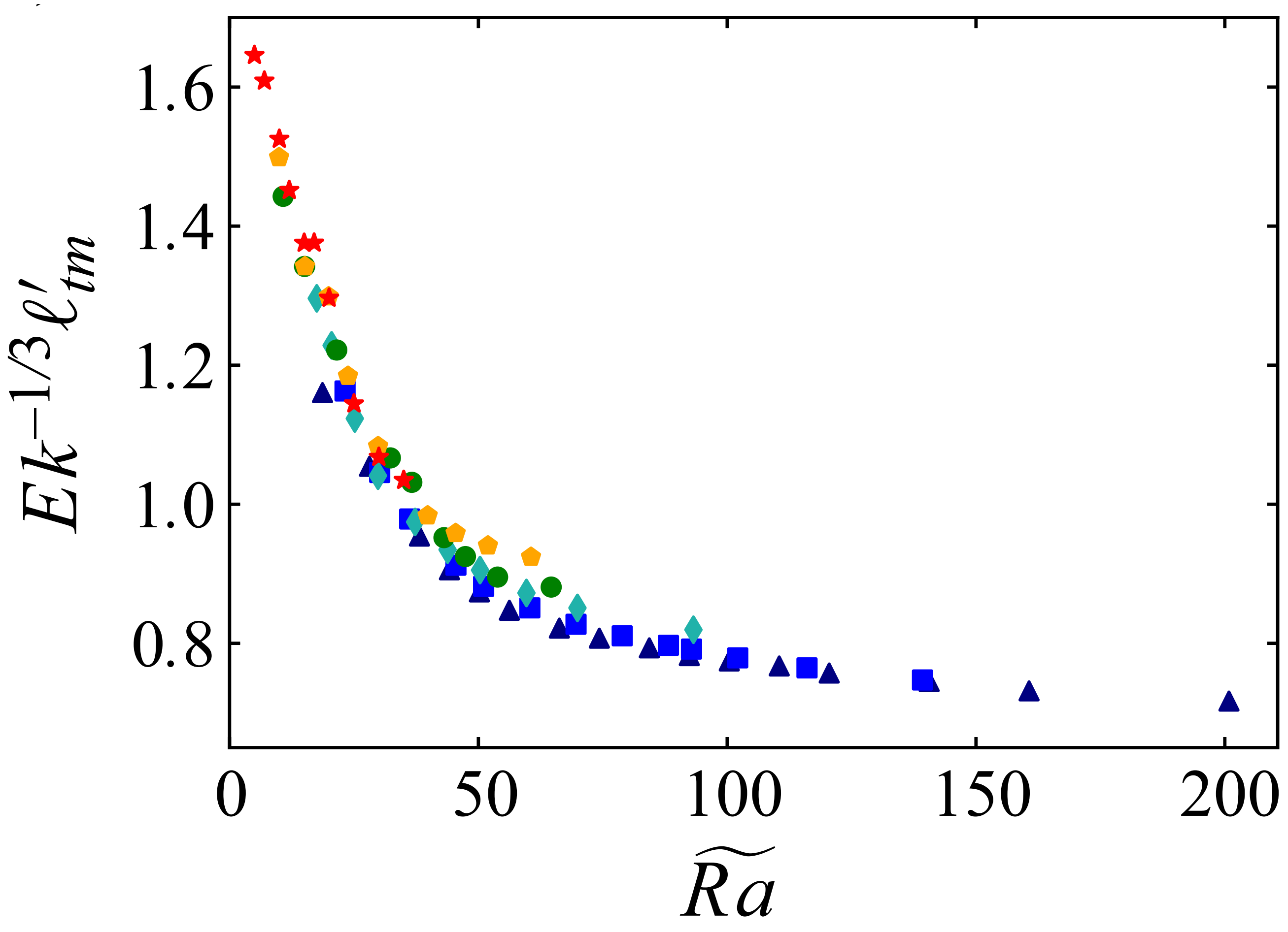




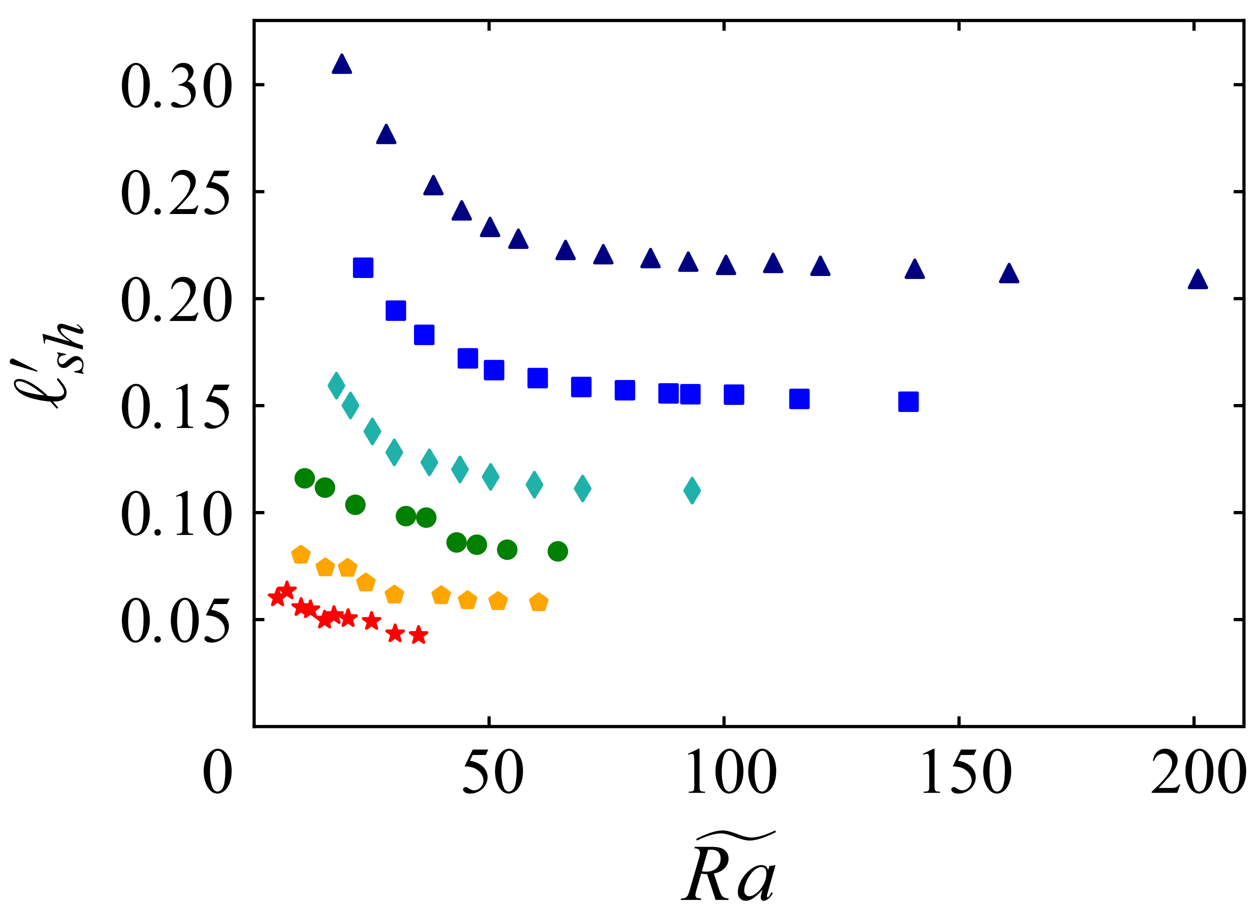
$$\mathbf{u}(r,\theta,\phi,t) = \boxed{\overline{\mathbf{u}}(r,\theta,t)} + \mathbf{u}'(r,\theta,\phi,t)$$

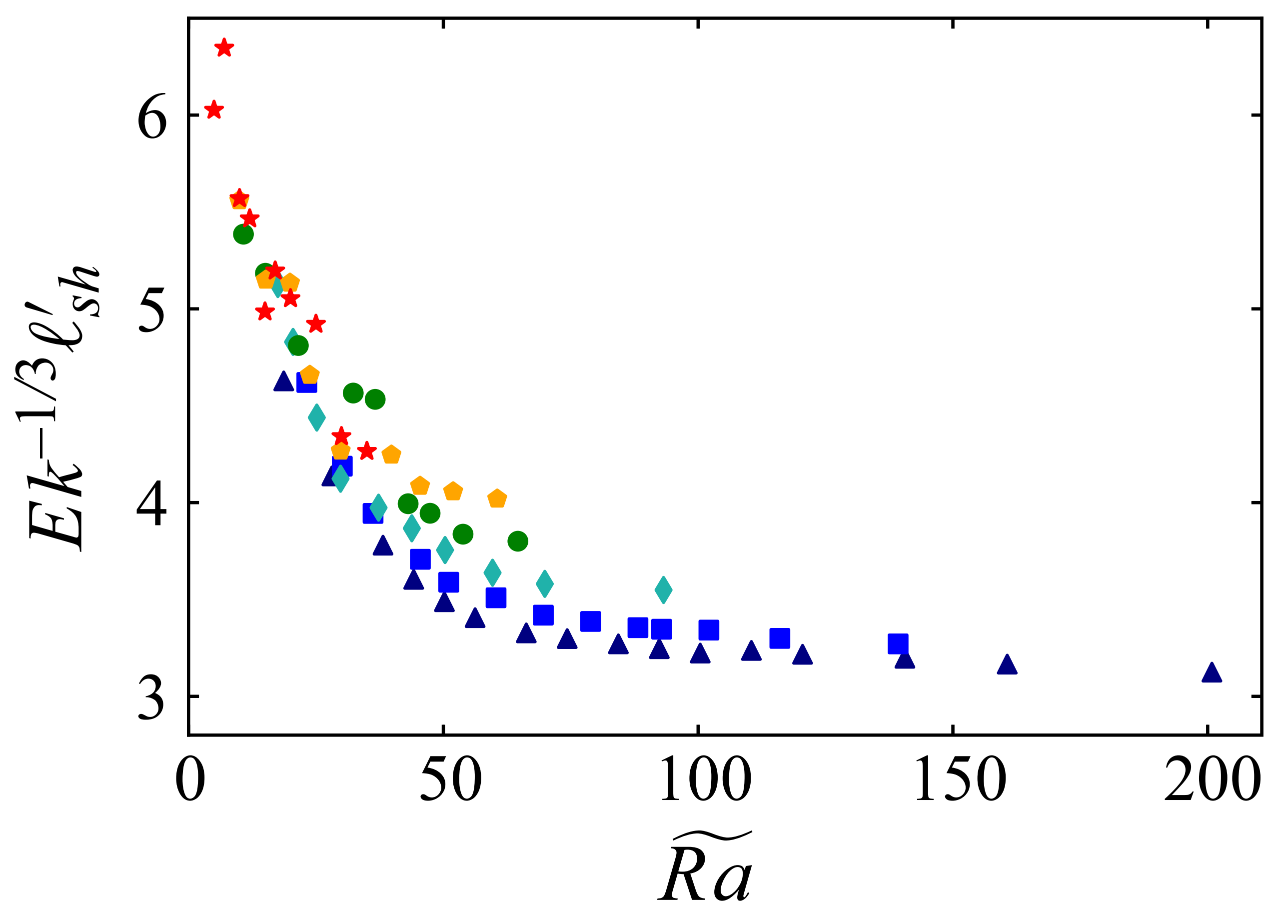
Taylor Microscale





Integral Length Scale

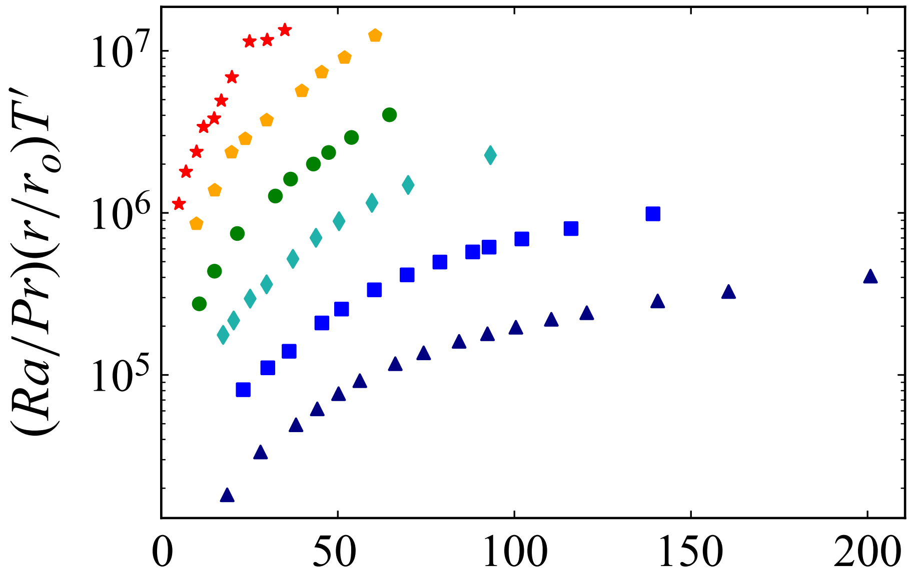




Energetically-dominant length scales: $\ell = O\left(Ek^{1/3}\right)$

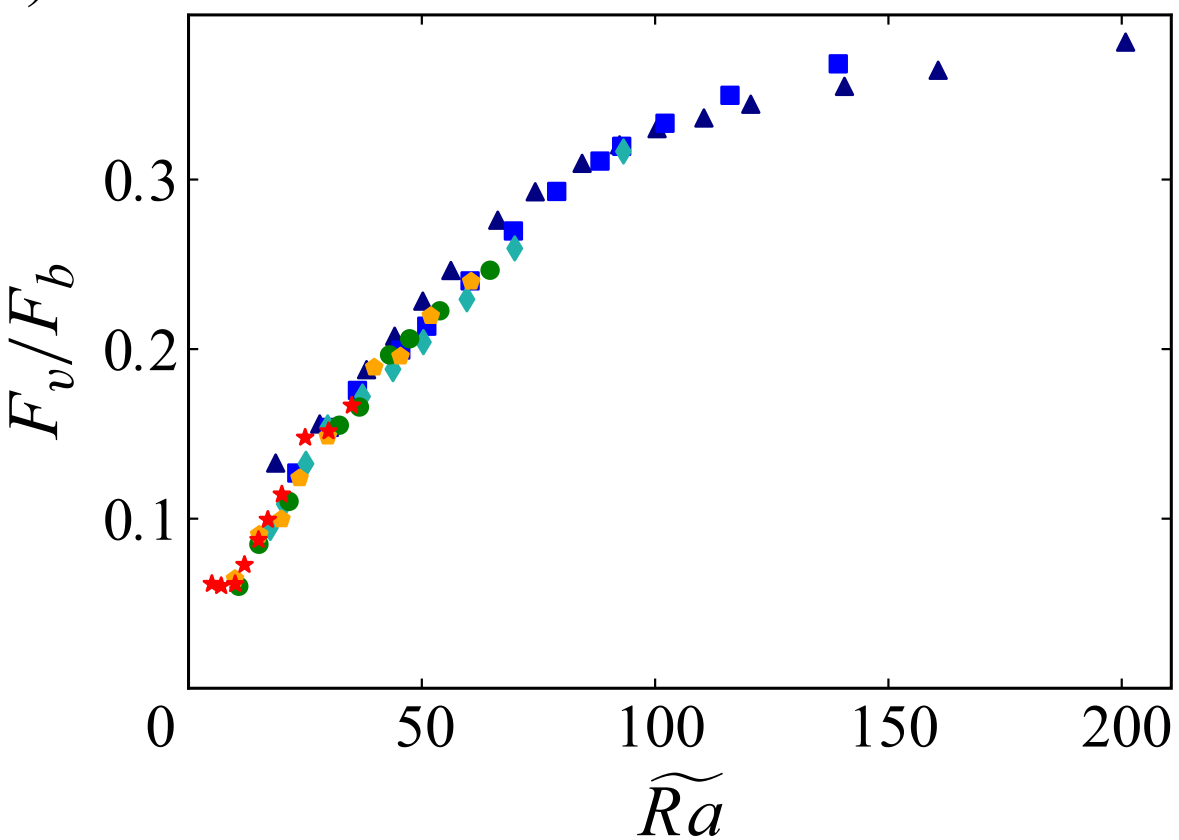
Force Scalings

(c)



Force Scalings

(*a*)



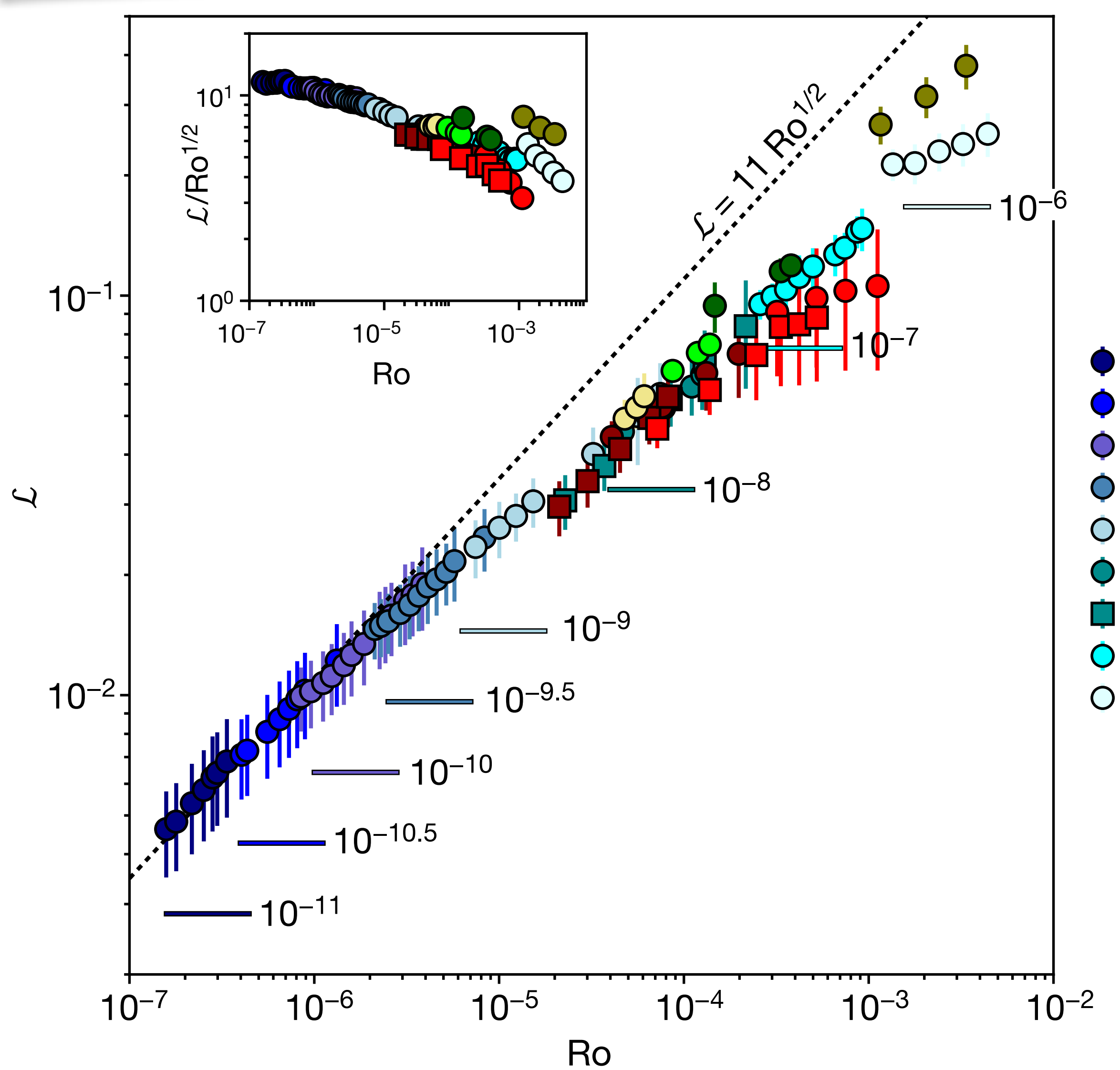
Related Work

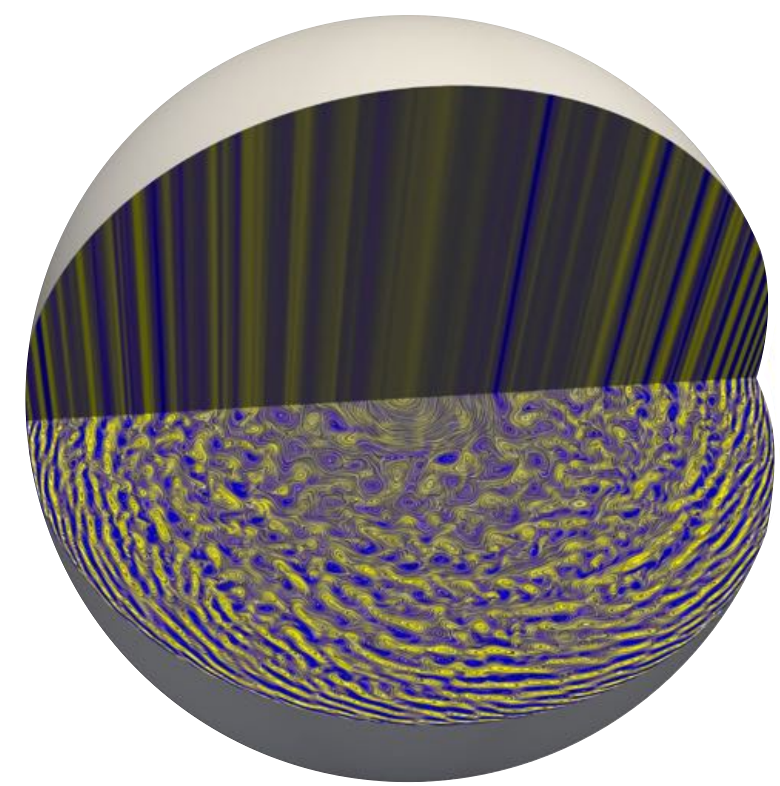

LETTER

https://doi.org/10.1038/s41586-019-1301-5

Turbulent convective length scale in planetary cores

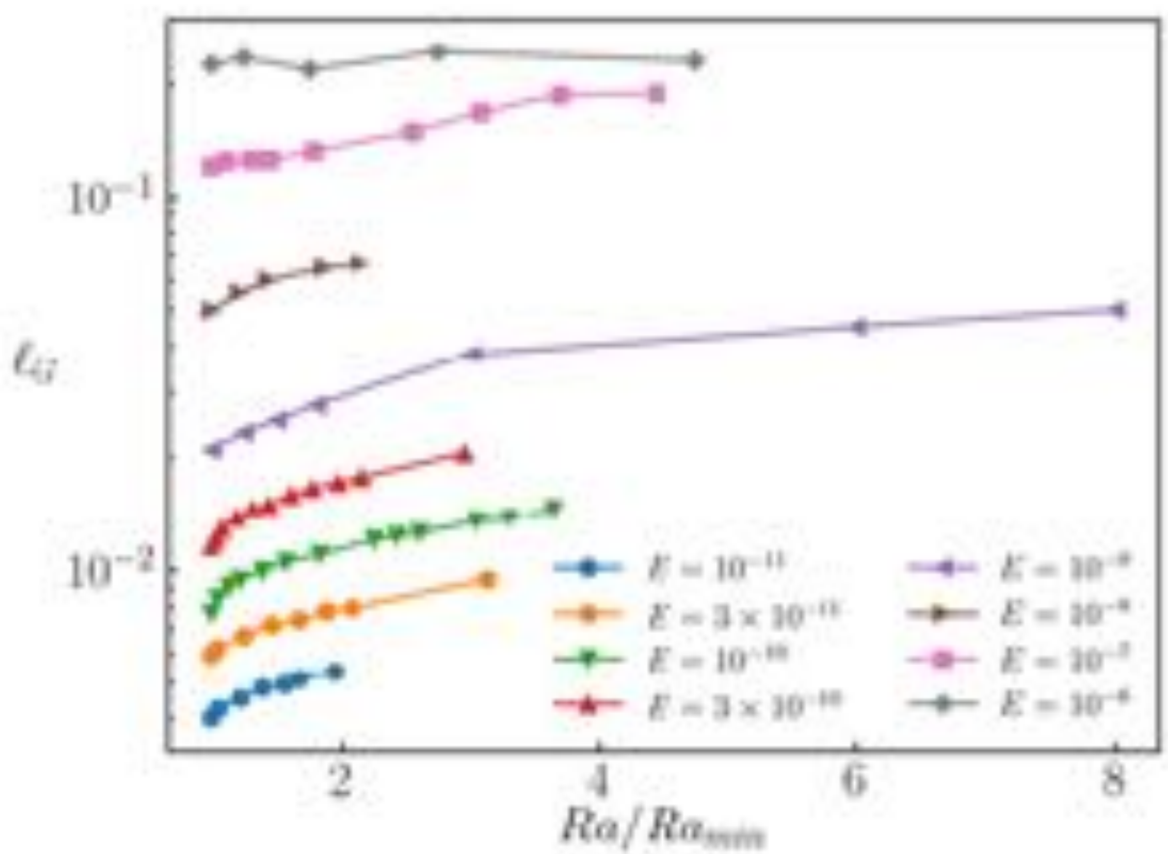
Céline Guervilly[1]*, Philippe Cardin[2] & Nathanaël Schaeffer[2]

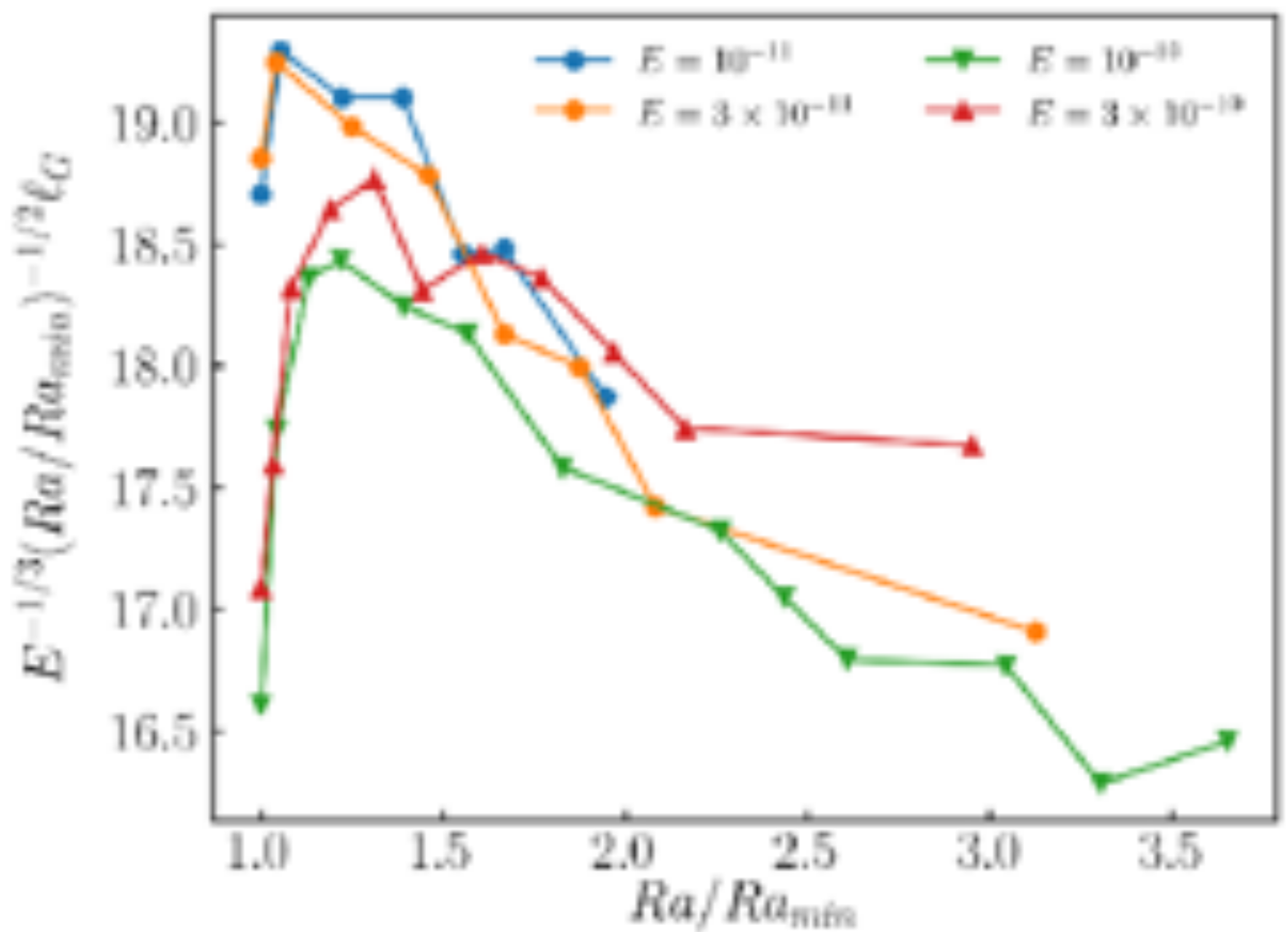




Guervilly, Cardin & Schaeffer (Nat, 2019)

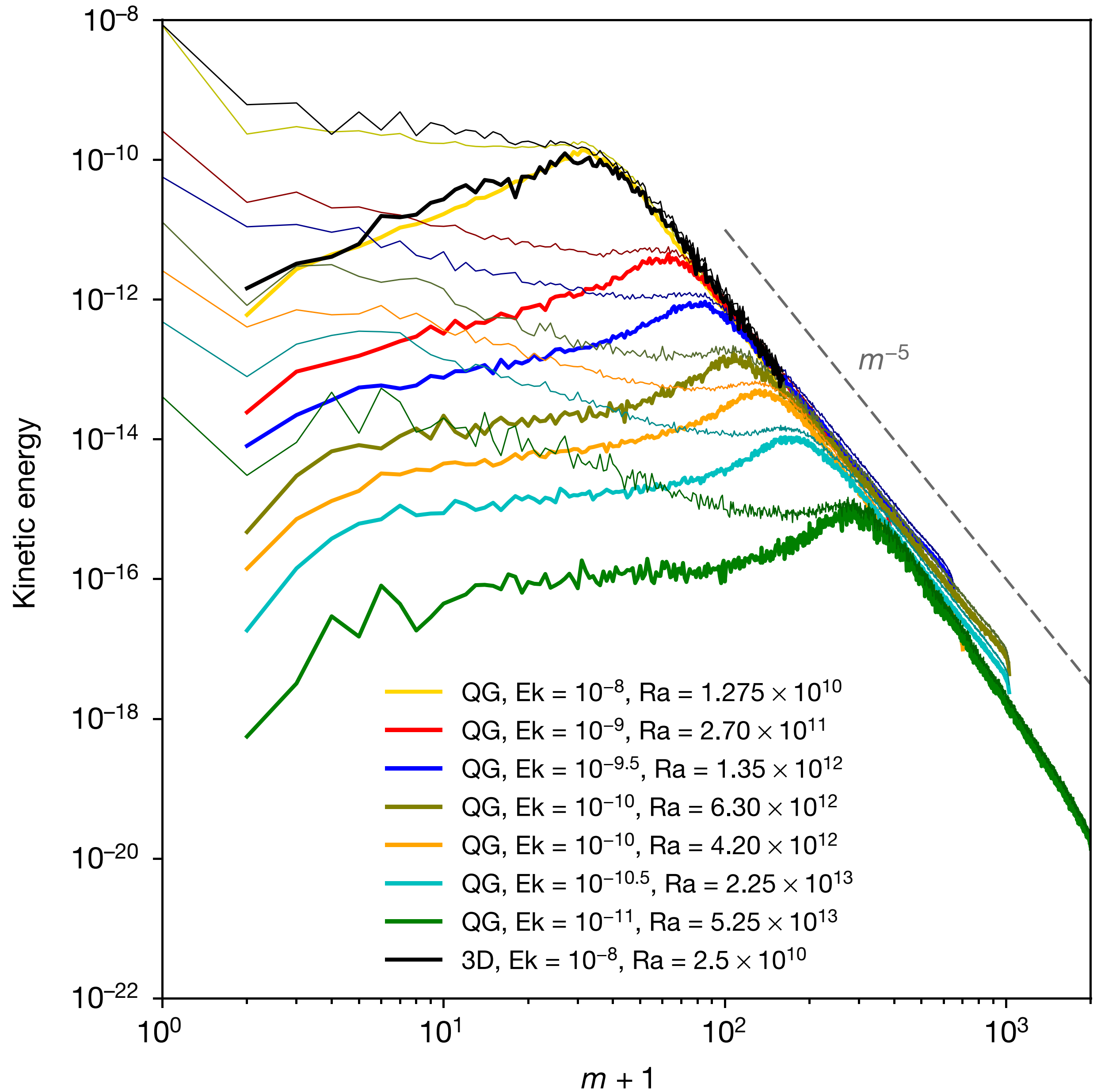
Related Work





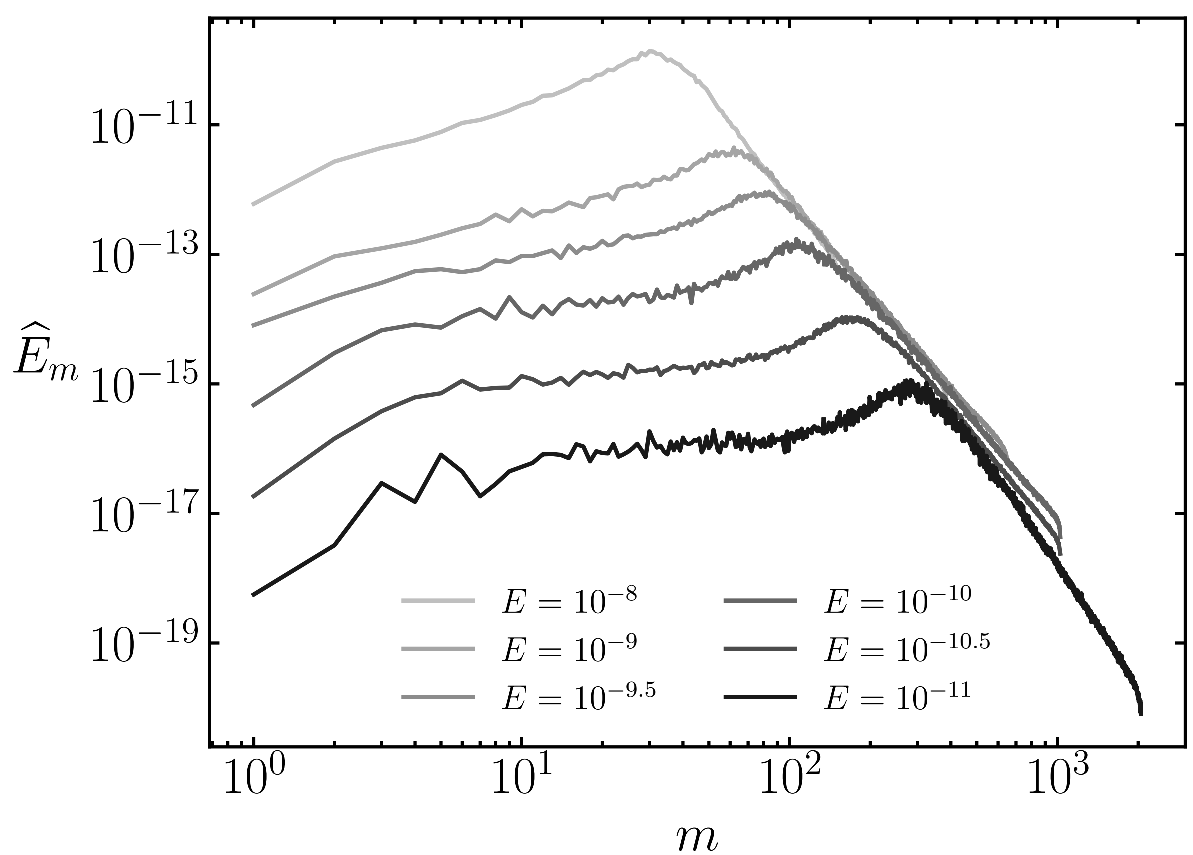
Data from: Guervilly, Cardin & Schaeffer (Nat, 2019)

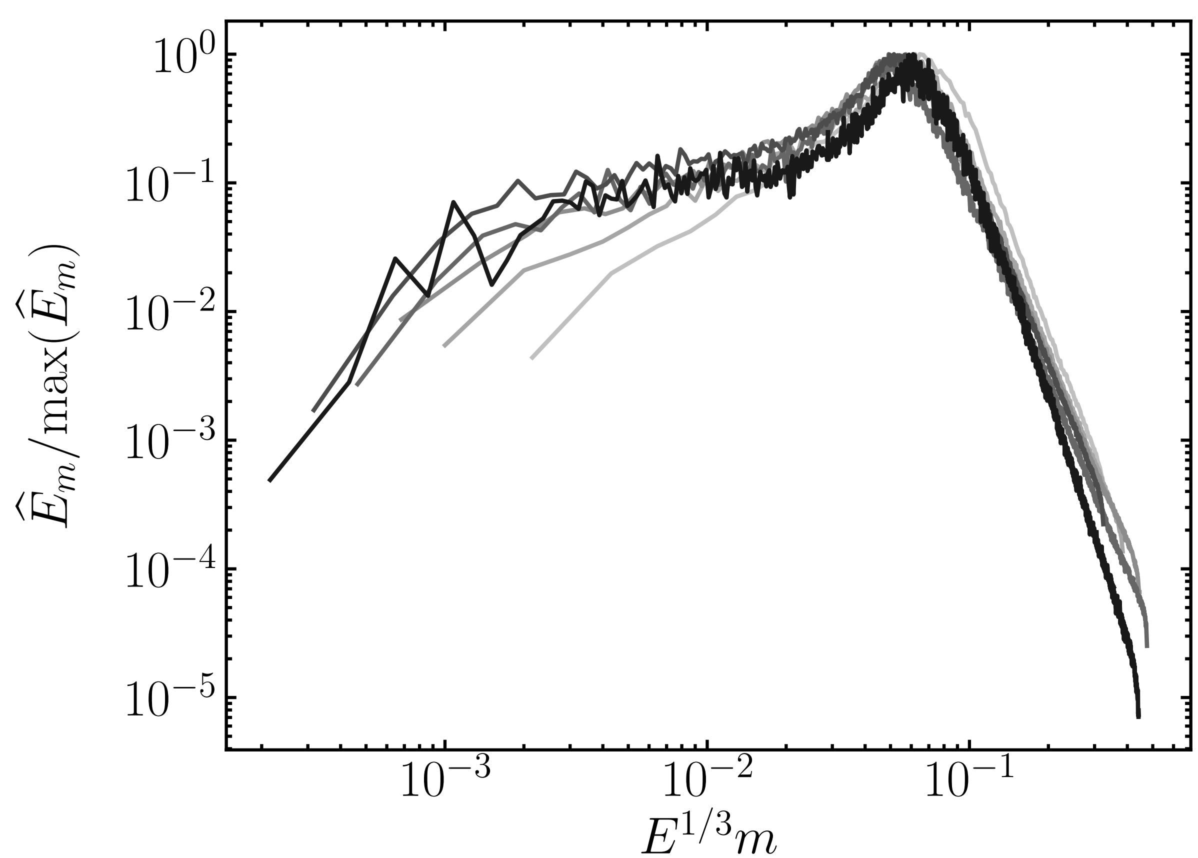
Related Work



Guervilly, Cardin & Schaeffer (Nat, 2019)

Related Work





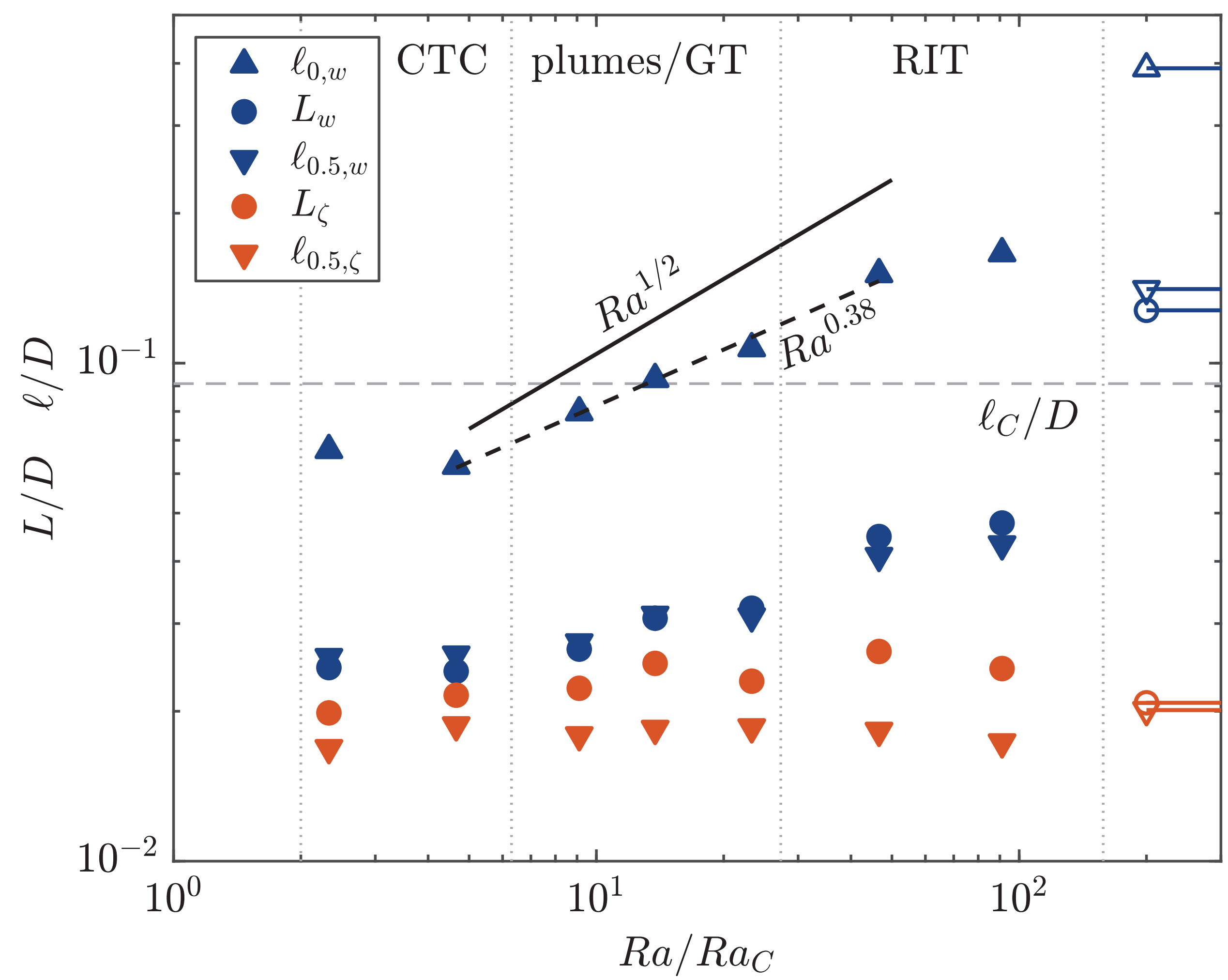
Data from: Guervilly, Cardin & Schaeffer (Nat, 2019)

Related Work

Velocimetry in rapidly rotating convection: Spatial correlations, flow structures and length scales[a]

Matteo Madonia, Andrés J. Aguirre Guzmán, Herman J. H. Clercx and Rudie P. J. Kunnen[b]

- Length scale grows less quickly than CIA.
- Taylor microscale is approximately constant.



Madonia et al. (EPL, 2021)

Summary

- The viscous length scale persists in the turbulent regime of rotating convection, independent of geometry.

- While diffusion-free scalings may approximately describe data, the reason for this is unclear.

    - The importance of the viscous force is likely tied to saturation of interior temperature gradient.

- Spherical data: scalings are all consistent with plane layer asymptotic behavior, zonal flow flows exhibit asymptotic dependence on Ekman number.