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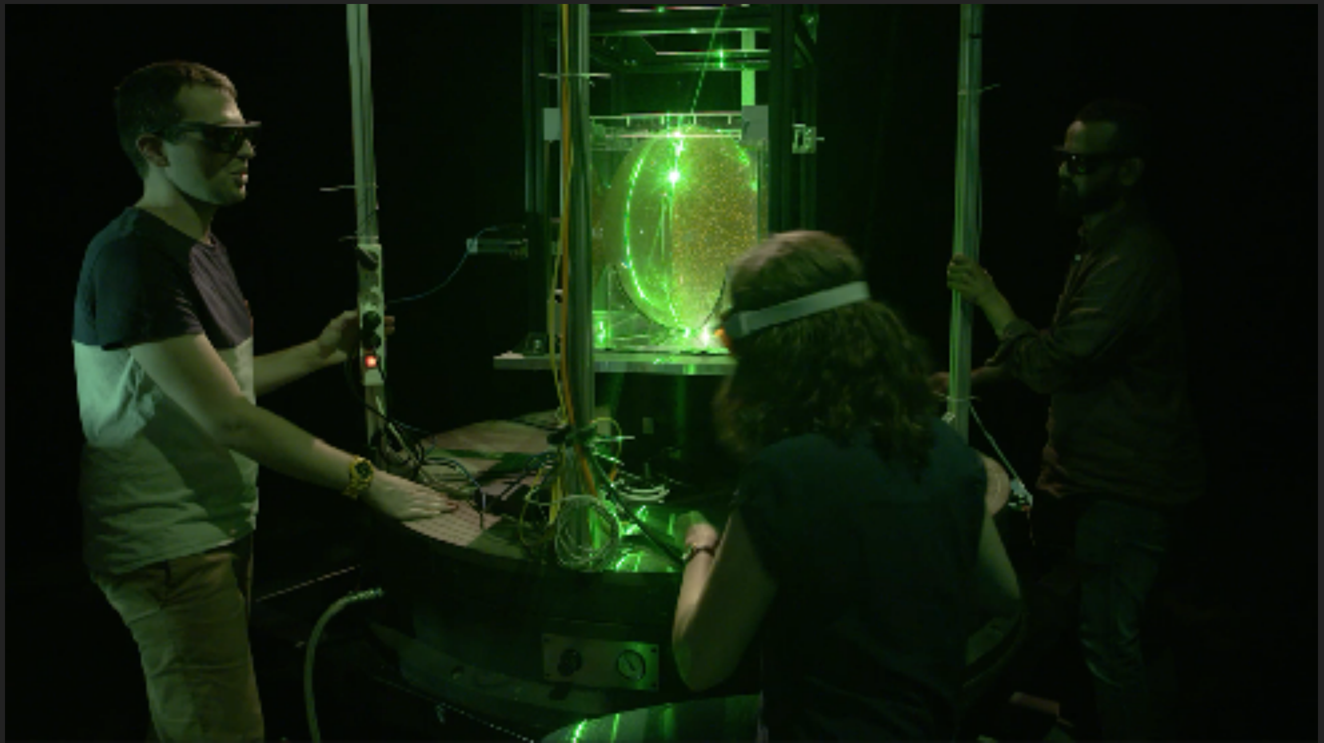
INTERPLAY OF BOUNDARY AND BULK DYNAMICS IN ROTATING TURBULENCE DRIVEN BY LIBRATION

M. LE BARS, B. FAVIER & T. LE REUN



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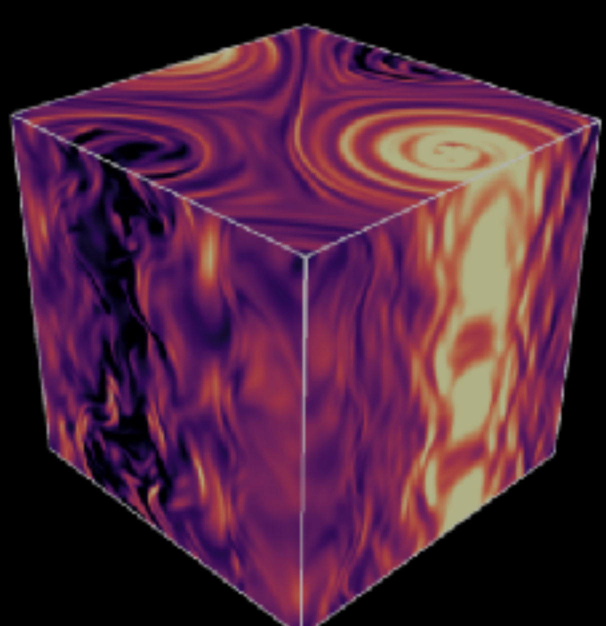
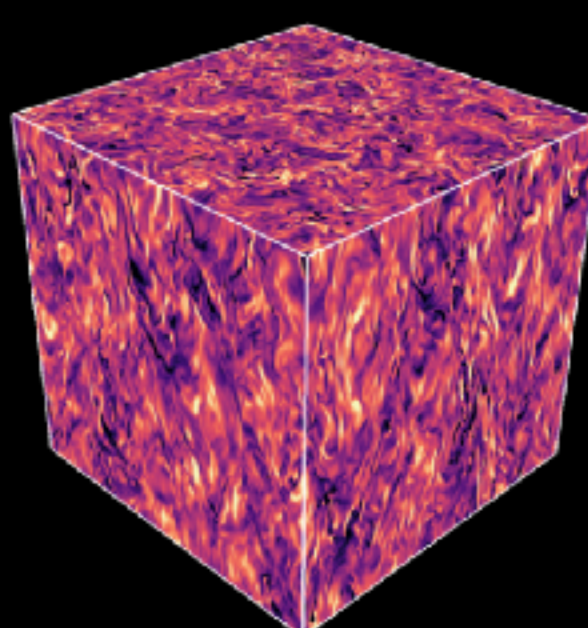


**2D QG vs.
3D inertial wave
turbulence in rotating
fluids**

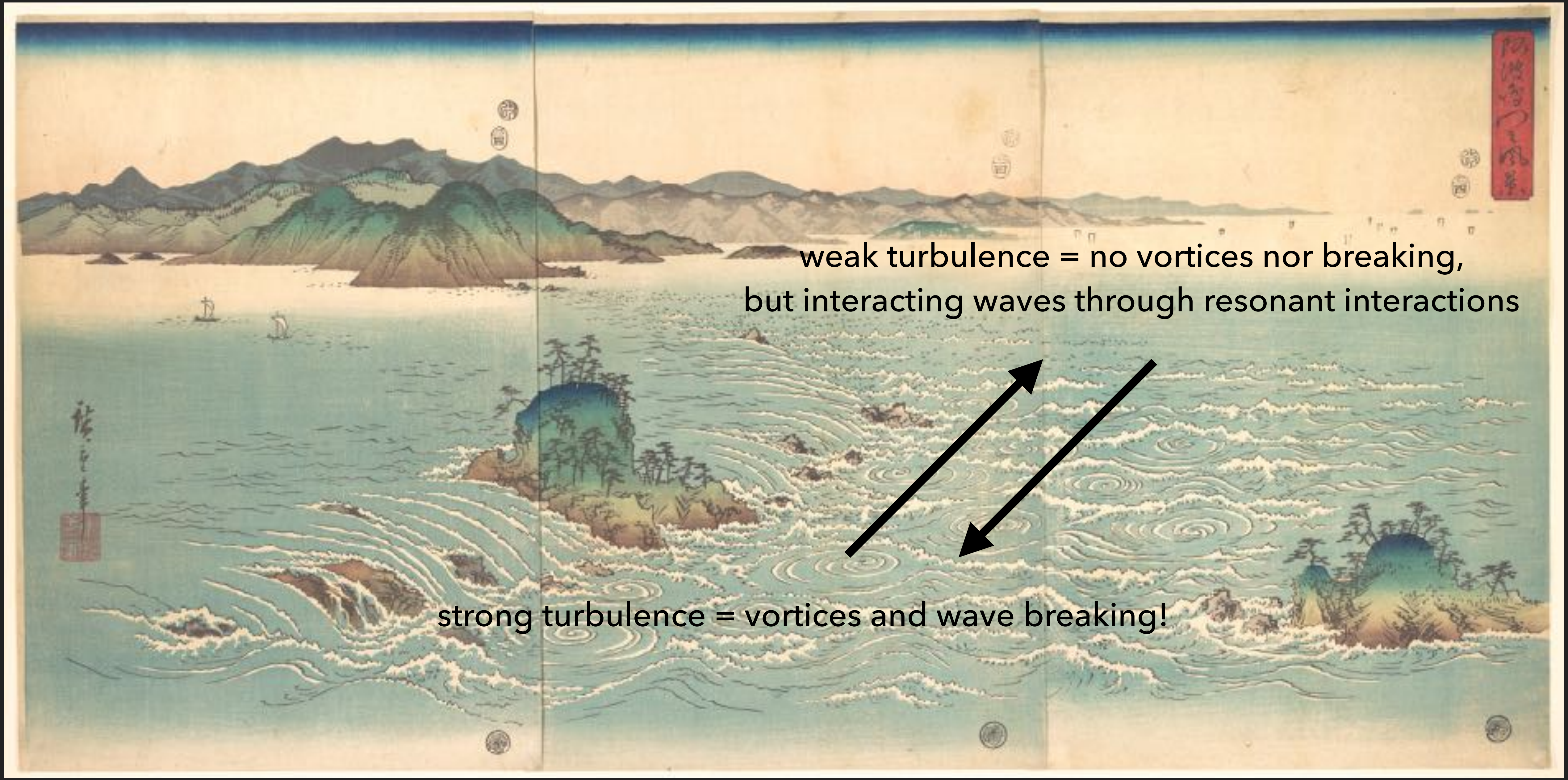


1. STRONG VS. WEAK TURBULENCE

The Whirlpools of Awa by Utagawa Hiroshige (1857), image from The Met collection.

(Le Bars JFM 2023)

1. STRONG VS. WEAK TURBULENCE



WEAK WAVE TURBULENCE = LESS KNOWN, BUT POSSIBLY AS IMPORTANT AND RELEVANT IN NATURE!

(Le Bars JFM 2023)

2. BASIC MODELLING: NON-LINEAR, BUT NOT TOO MUCH

- resonant interactions in a world with waves

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\nabla P + \rho \mathbf{g} + \boldsymbol{F}$$

sustained wave 1: $\mathbf{u_1} = \mathbf{a_1^+} e^{i\omega_1 t}$ through NL interactions: $\pm 2\omega_1 t$ and 0…

now if another wave 2: $\mathbf{u_2} = \mathbf{a_2^+} e^{i\omega_2 t}$ through NL interactions: $\pm\omega_1 \pm \omega_2$

so possible energy transfer from wave 2 to a third wave 3 provided e.g. $\omega_3 = \omega_1 + \omega_2$ (resonance condition, and same with the wave vectors)

- wave turbulence = spreads quantities over larger ranges of time and length scales by such resonant interactions of waves
- weak turbulence = the amplitude remains small otherwise the waves break and generate vorticity
- interactions between 3 waves, but sometimes non constructive… then need more than 3!

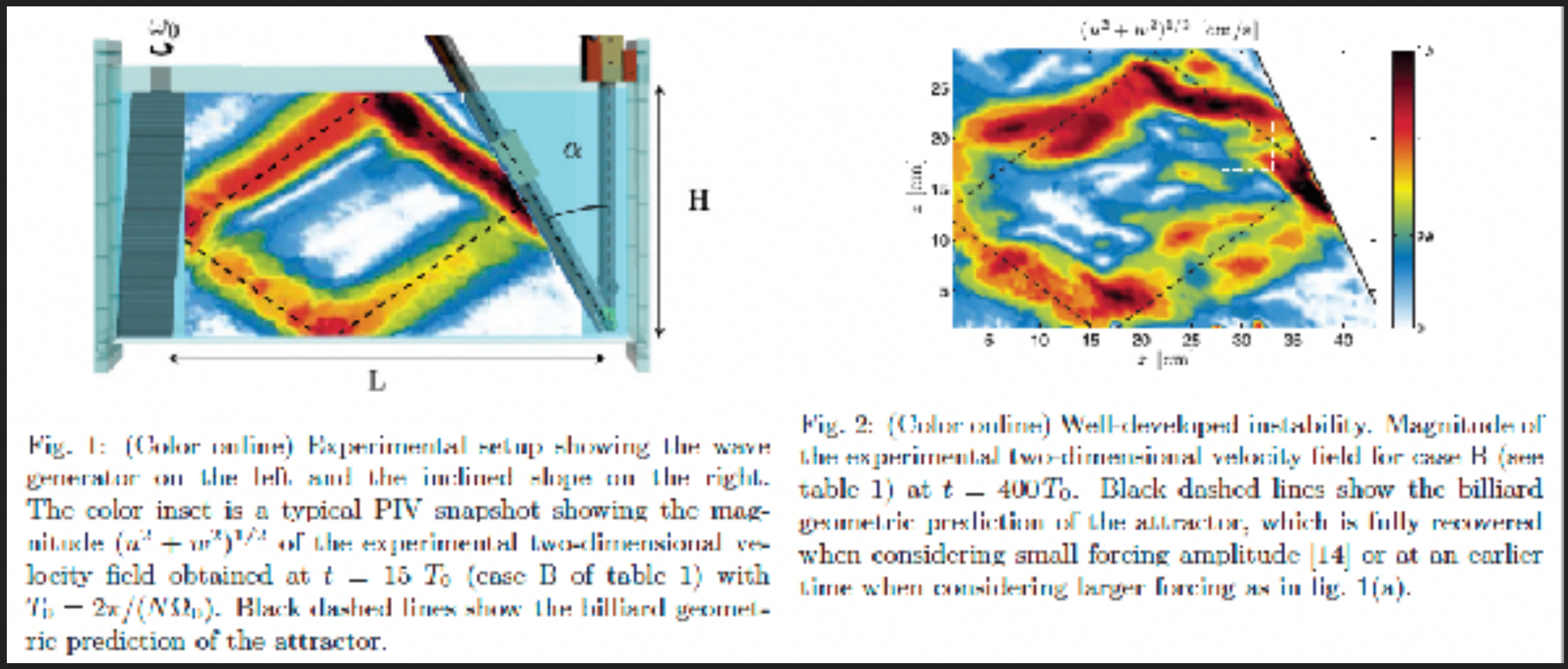
3. WAVE TURBULENCE IN THE REAL WORLD

- playground for mathematicians: there is a small parameter
- exists in a large variety of systems: e.g. in vibrating plates (e.g. Cobelli et al. 2009), in optics (e.g. Picozzi et al. 2014), in cosmology with gravitational waves (e.g. Galtier & Nazarenko 2017)…
- of great relevance for geo- and astro-physics!

surface waves

3-waves resonant interactions occur for capillary waves, but they are forbidden for pure gravity waves where 4-waves interactions must be considered (Falcon & Mordant 2022)

internal gravity waves



single frequency forcing -> large spectrum

3-waves resonant interactions (Brouzet et al. 2016)

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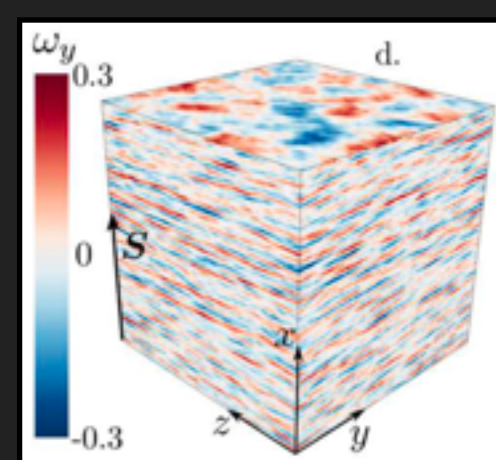
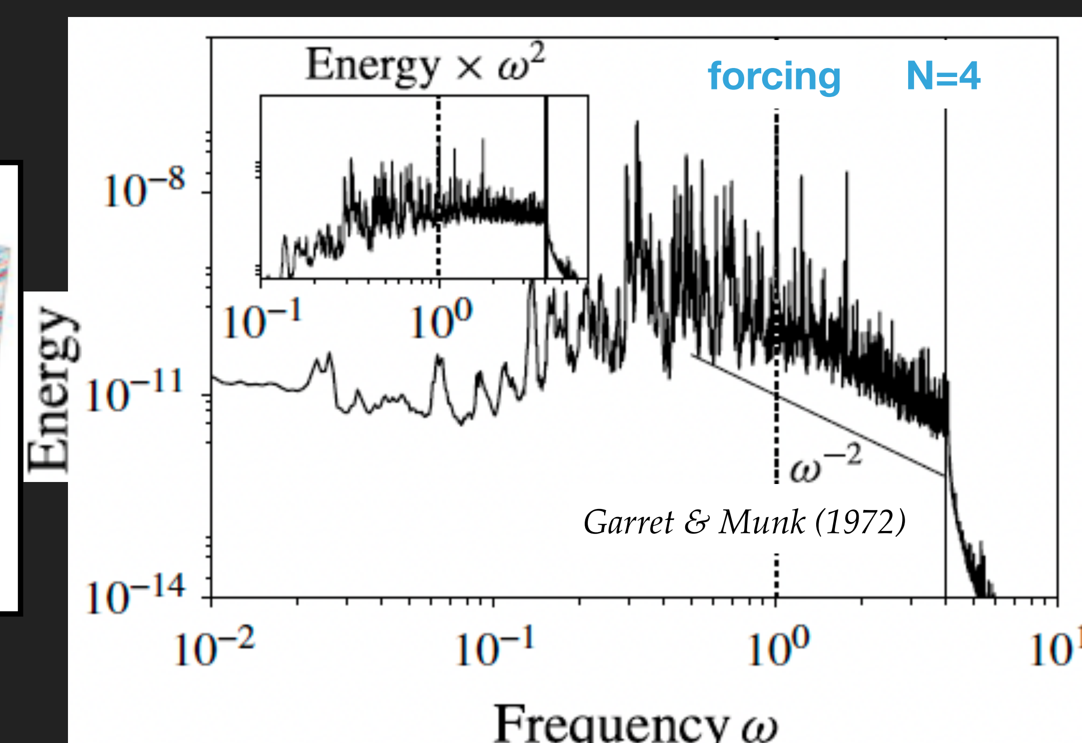




internal wave turbulence as a possible source of Garret & Munk spectrum (Le Reun et al. 2018)

1. COMPETITION BETWEEN QG AND WAVE TURBULENCE

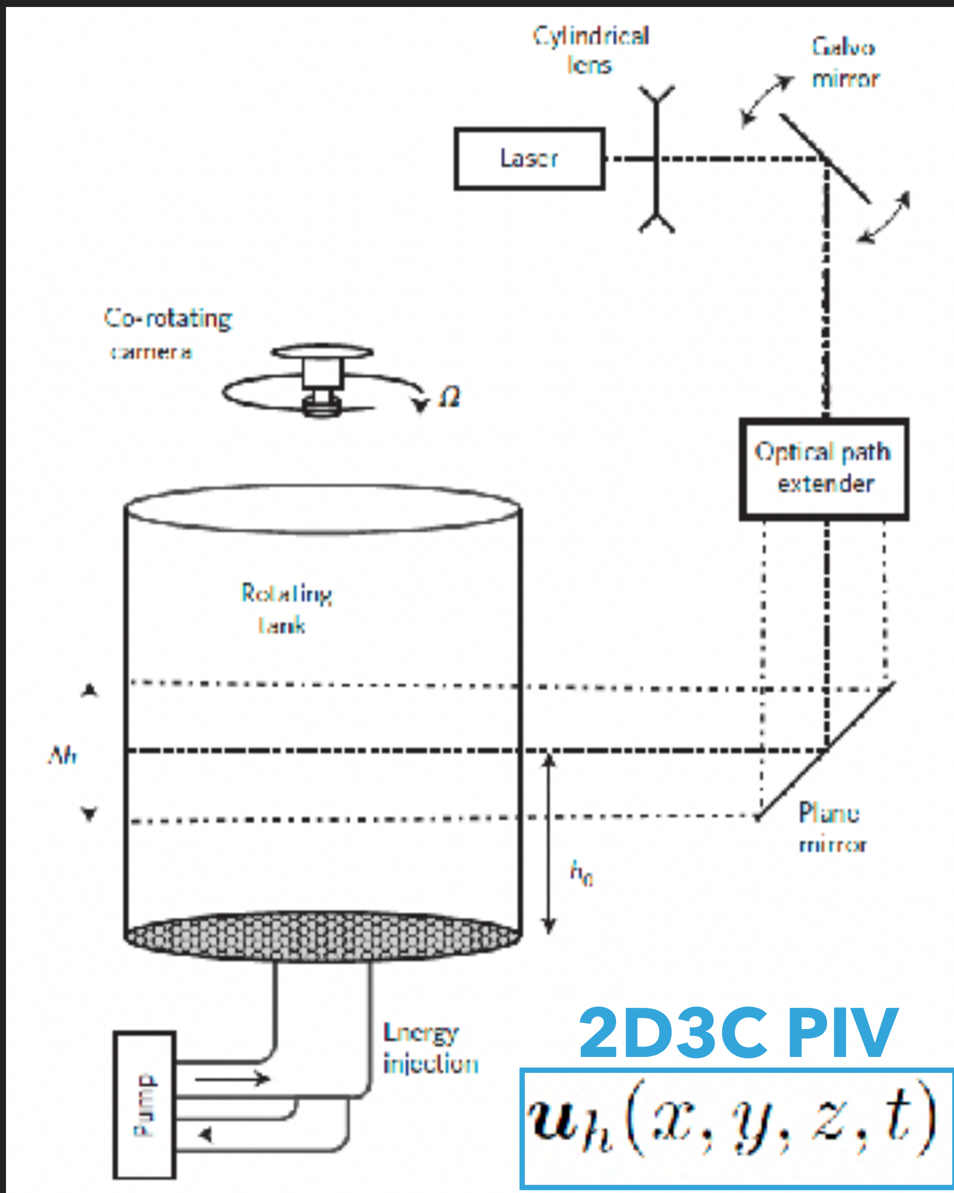
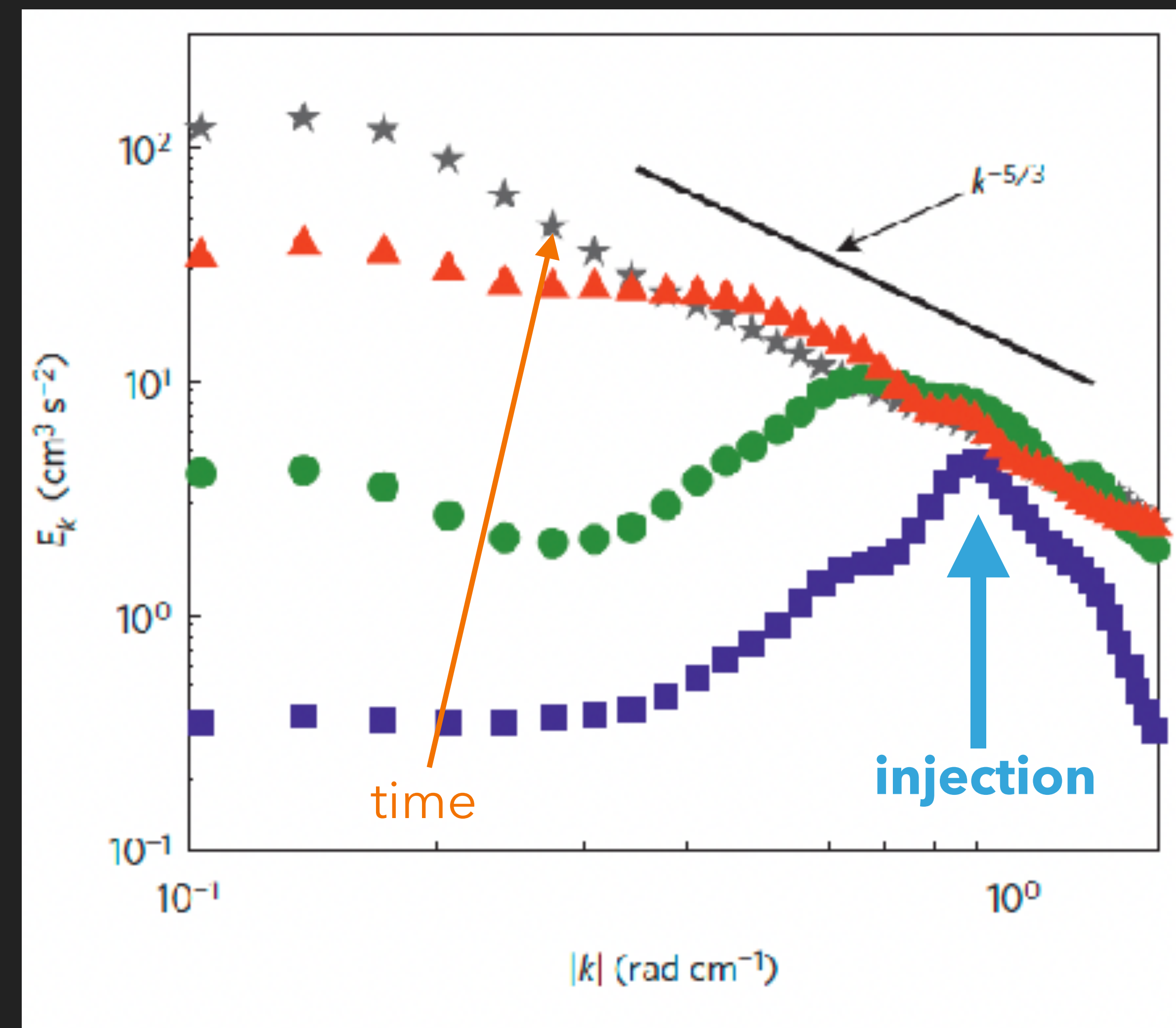


Figure 1 | Experimental set-up. A laser sheet illuminates a rotating Plexiglas cylinder filled with water and seeded with tracer particles. Using a galvo mirror, the sheet is repeatedly swept vertically through 30 horizontal planes, in the range $\Delta h = 25.6$ cm around height h_0. A co-rotating camera (~750 frames per second) images the light scattered from the tracer particles.

1m, 120rpm -> Ek~10^{-6} and Ro~0.006



seminal study by Yarom & Sharon (2014)

2. INVERSE CASCADE, A 2D DYNAMICS

- both inertial ranges are separated

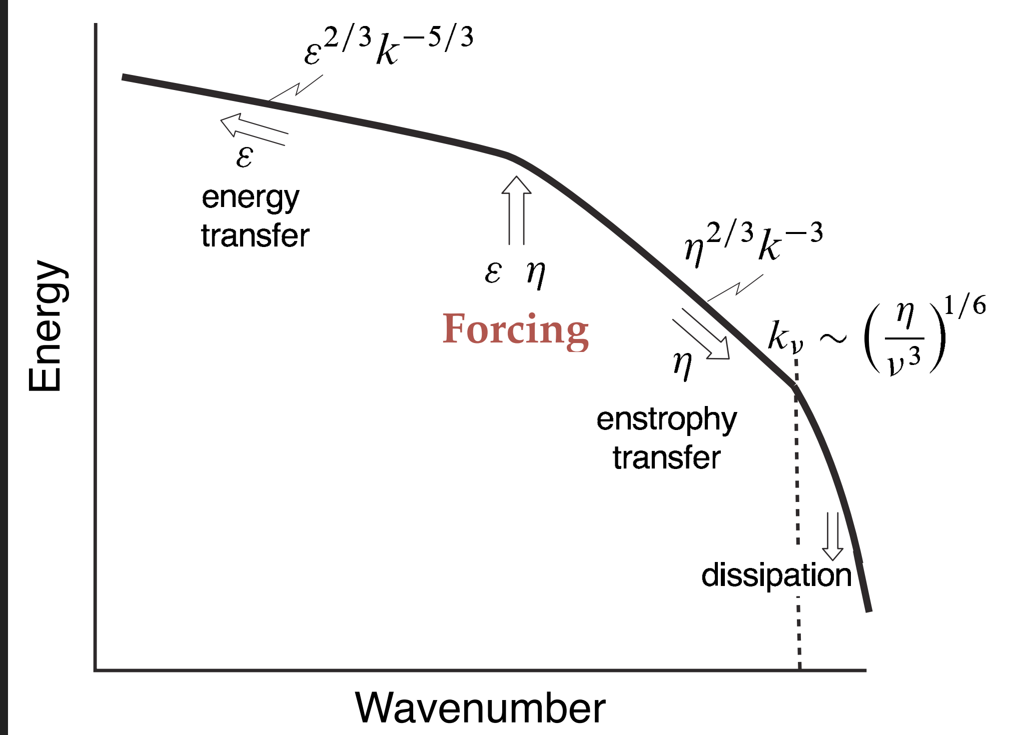


Figure 8.7 The energy spectrum of two-dimensional turbulence. (Compare with Fig. 8.3.) Energy supplied at some rate ε is transferred to large scales, whereas enstrophy supplied at some rate η is transferred to small scales, where it may be dissipated by viscosity. If the forcing is localized at a scale k_f^{-1} then $\eta \approx k_f^2 \varepsilon$.

Vallis 2017

THE ENERGY—ENSTROPHY DOUBLE CASCADE IN 2D

$$\mathrm{K} = \int E(k)\mathrm{d}k \qquad \mathrm{Z} = \int k^2 E(k)\mathrm{d}k$$

turbulence broadens the spectra i.e. it spreads quantities over a larger range of k…
the only way to conserve K & Z given the different k weights: more of the energy is transferred toward larger scales while more of the enstrophy is transferred toward smaller scales!

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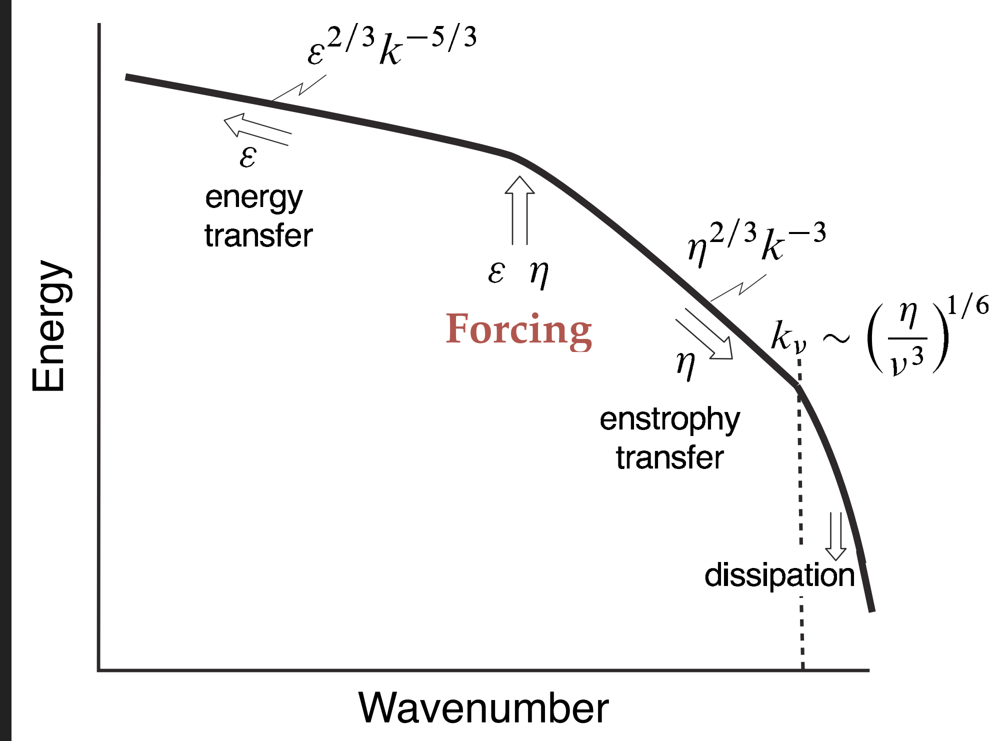

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Vallis 2017

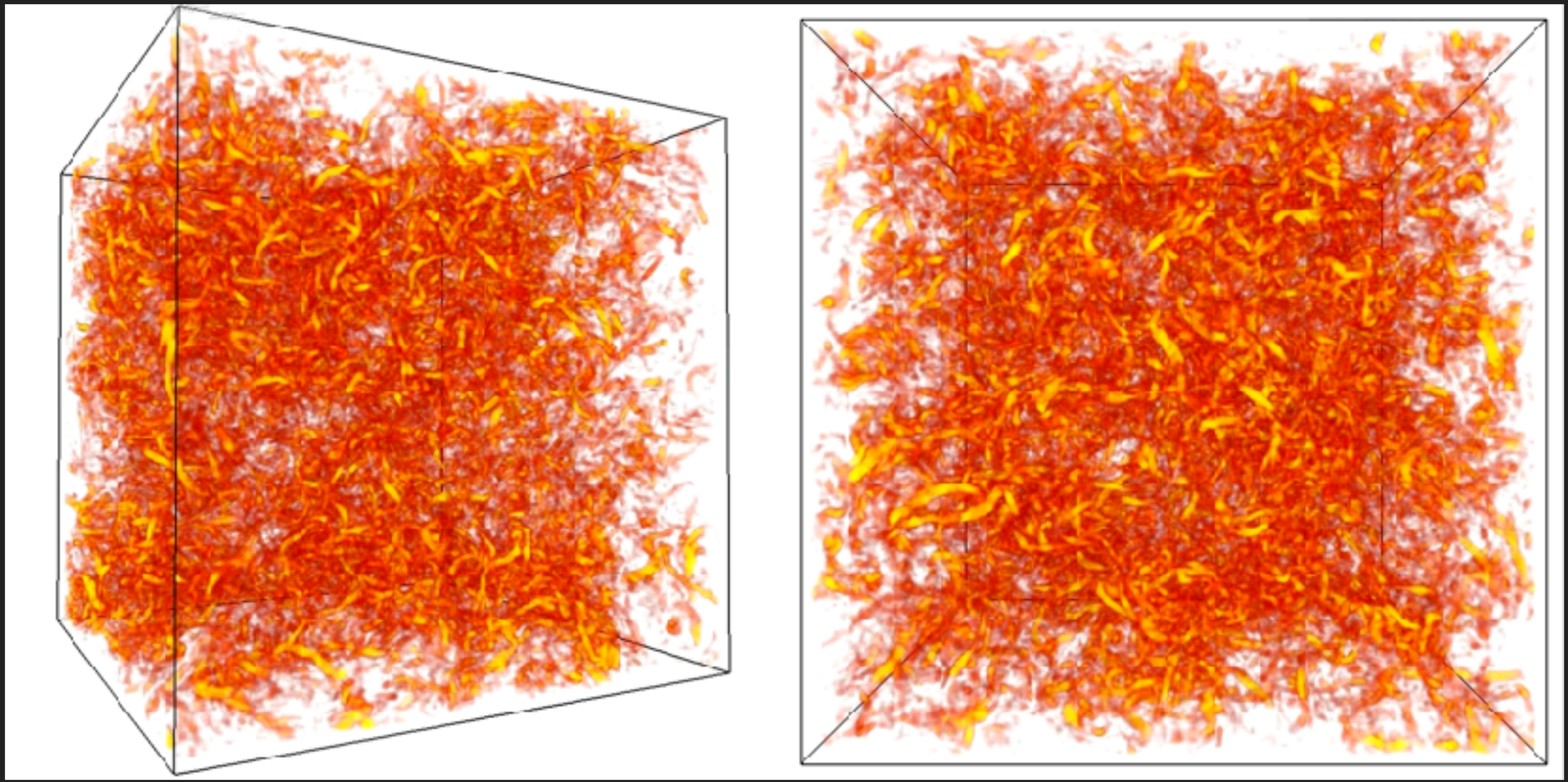
THE ENERGY—ENSTROPHY DOUBLE CASCADE IN 2D

- quasi 2D in geo- and astrophysical flows
  - geometrical confinement
  - rapid rotation (Taylor Proudman)

2. INVERSE CASCADE, A 2D DYNAMICS

- with strong forcing, rotating turbulence naturally tends to transfer energy to the kz= 0 plane

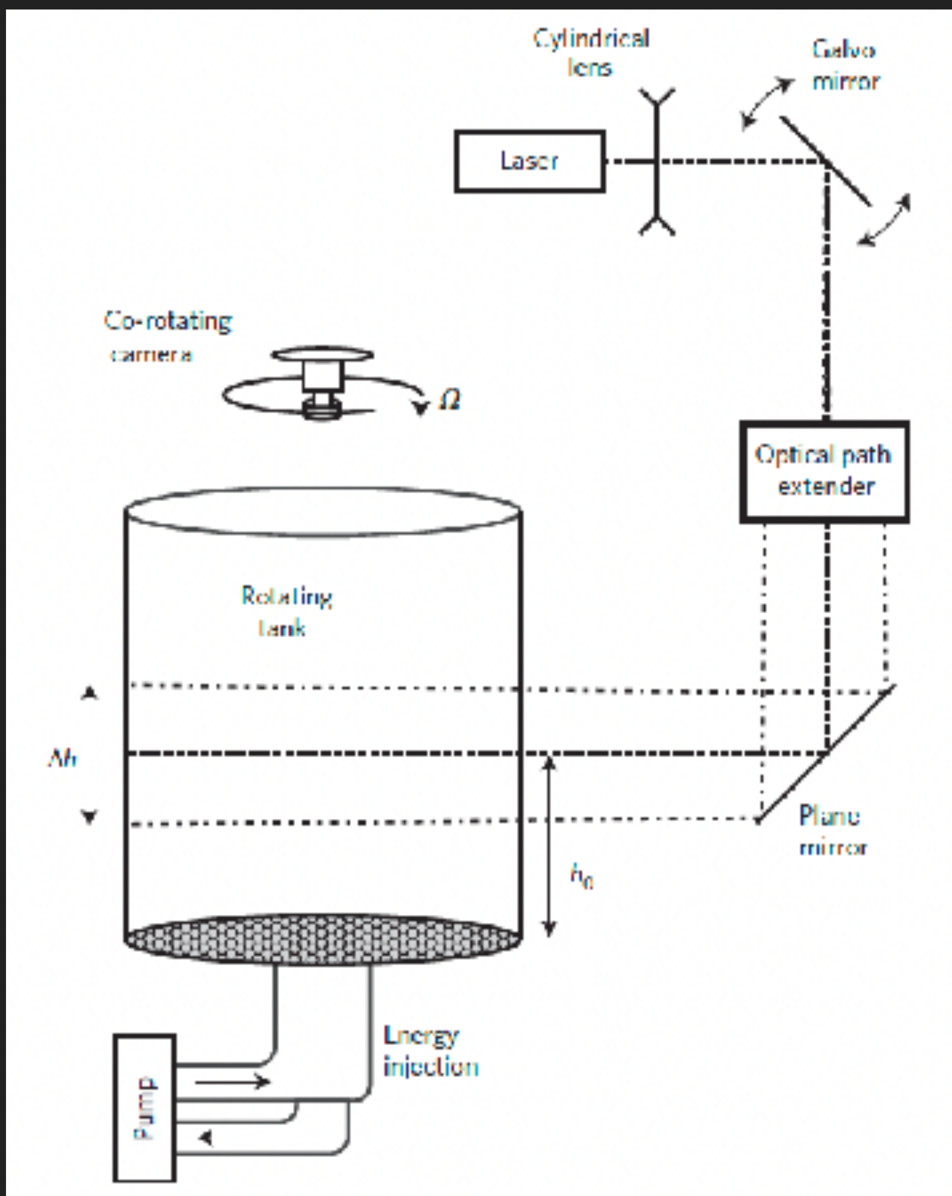
side view



top view

Favier 2009

3. 3D WAVE DYNAMICS



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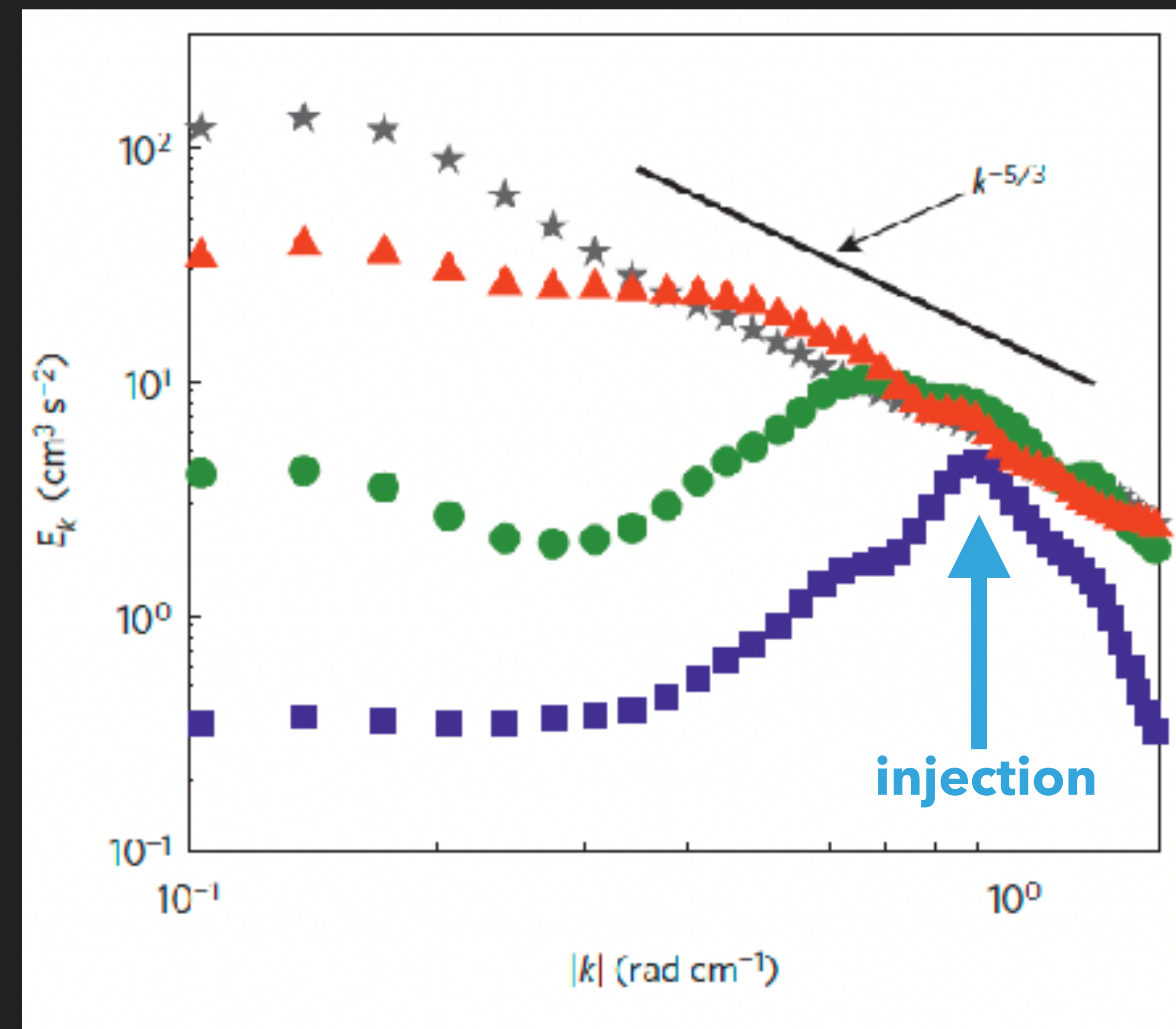


now focusing on large wavenumbers…

seminal study by Yarom & Sharon (2014)

3. 3D WAVE DYNAMICS

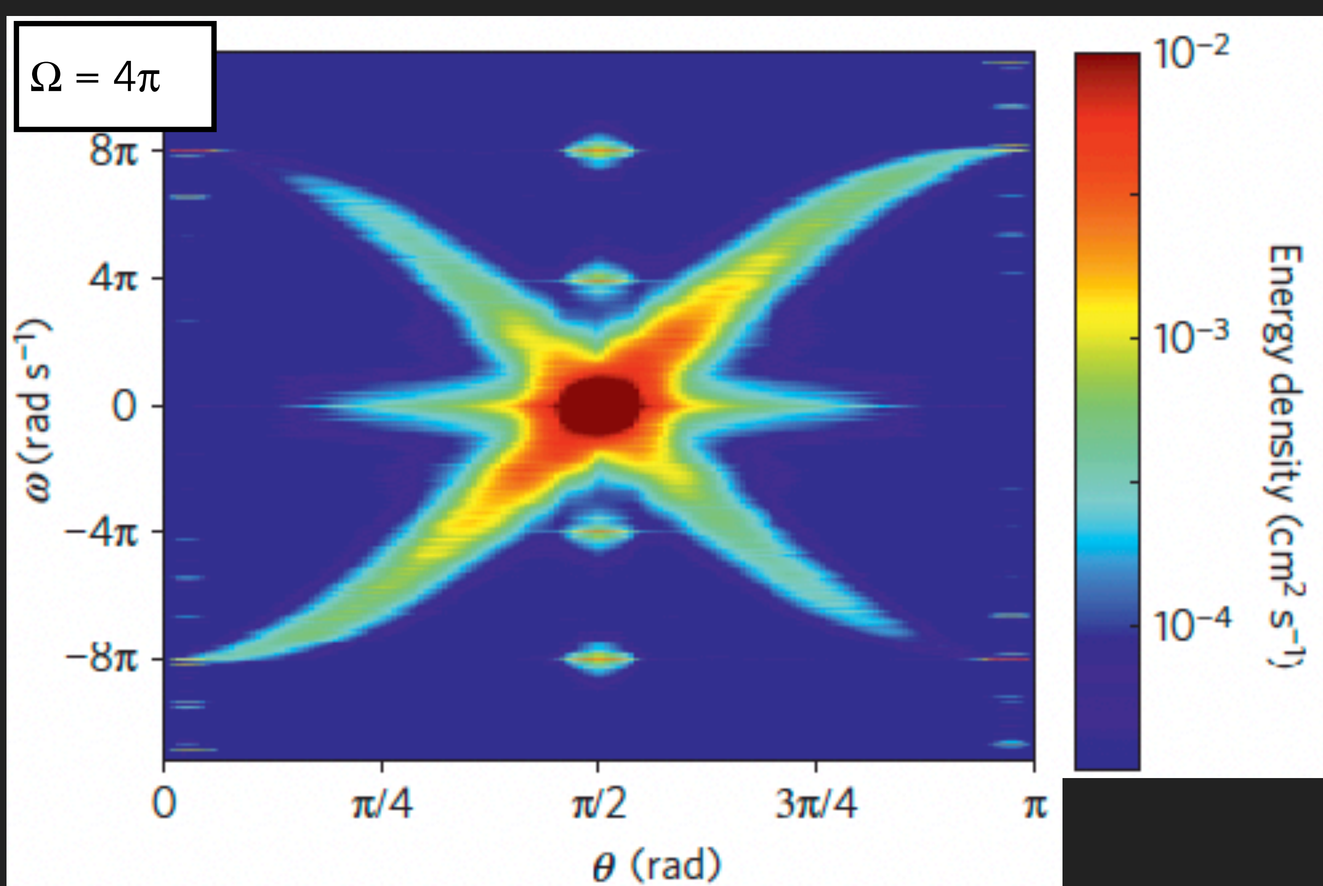
- a smart, adhoc decomposition to highlight inertial waves

2D3C PIV

$$\boldsymbol{u}_h(x, y, z, t) \quad \rightarrow \quad \hat{\boldsymbol{u}}_h(k_x, k_y, k_z, \omega)$$

$$E(k_x, k_y, k_z, \omega) \quad \rightarrow \quad E(k, \theta, \omega)$$

wave vector angle vs. rotation axis



focusing on large wavenumbers, here 1.42-2.03 rad/cm

seminal study by Yarom & Sharon (2014)

3. 3D WAVE DYNAMICS

- inertial waves

Navier–Stokes equations in a rotating frame

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u} + \underbrace{2\boldsymbol{\Omega}\times\boldsymbol{u}}_{\text{Coriolis}} = -\nabla\Pi + \nu\nabla^2\boldsymbol{u} + \boldsymbol{F}$$

$$\nabla\cdot\boldsymbol{u} = 0$$

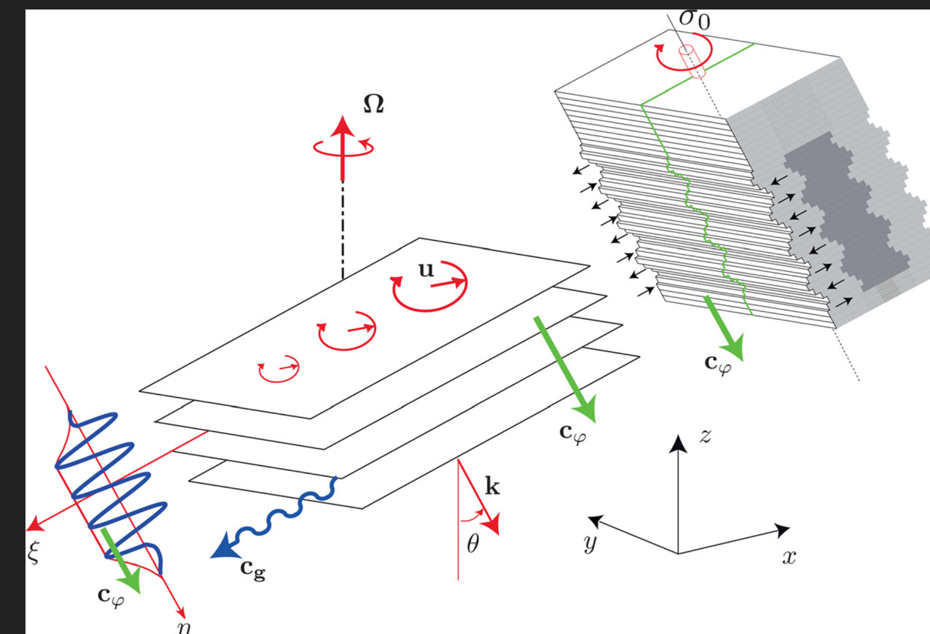
Poincaré equation (linear inviscid limit)

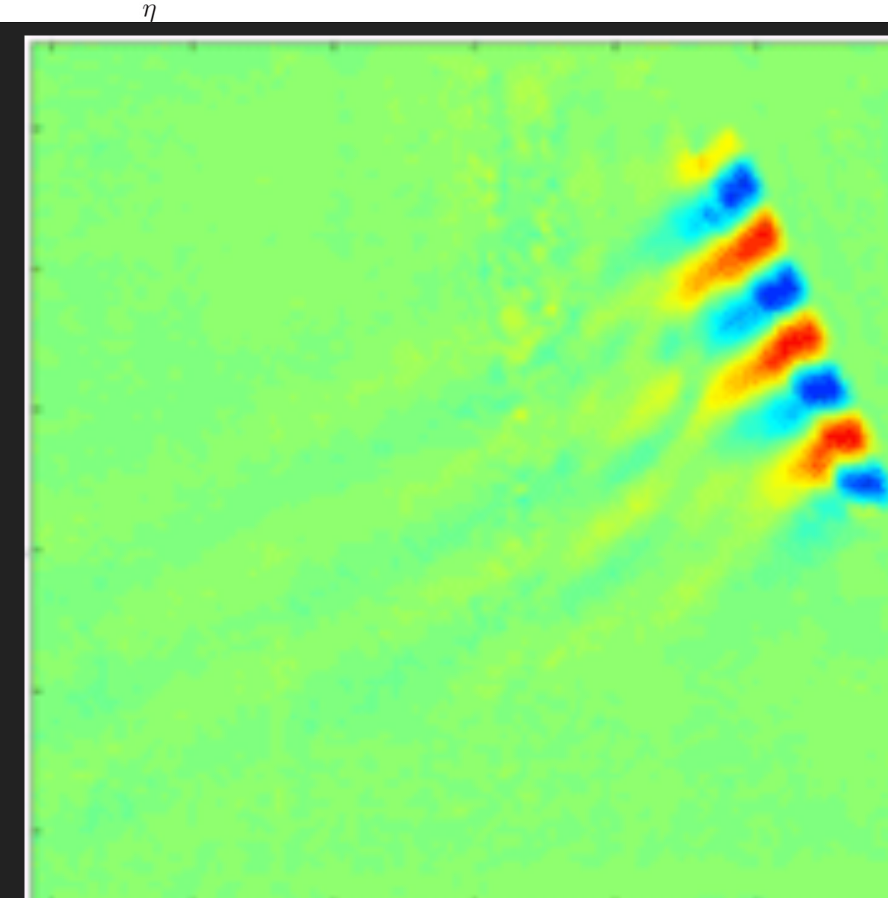
$$\frac{\partial^2\nabla^2\boldsymbol{u}}{\partial t^2} + 4\Omega^2\frac{\partial^2\boldsymbol{u}}{\partial z^2} = 0$$

Dispersion relation of inertial waves

$$\omega = \pm 2\Omega\frac{k_z}{k} = \pm 2\Omega\cos\theta$$

Inertial waves frequency is bounded by 2Ω so that they are expected to dominate to low-frequency part of the spectrum.





Bordes et al. 2012

3. 3D WAVE DYNAMICS

- a smart, adhoc decomposition to highlight inertial waves

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$$\boldsymbol{u}_h(x, y, z, t) \quad \rightarrow \quad \hat{\boldsymbol{u}}_h(k_x, k_y, k_z, \omega)$$

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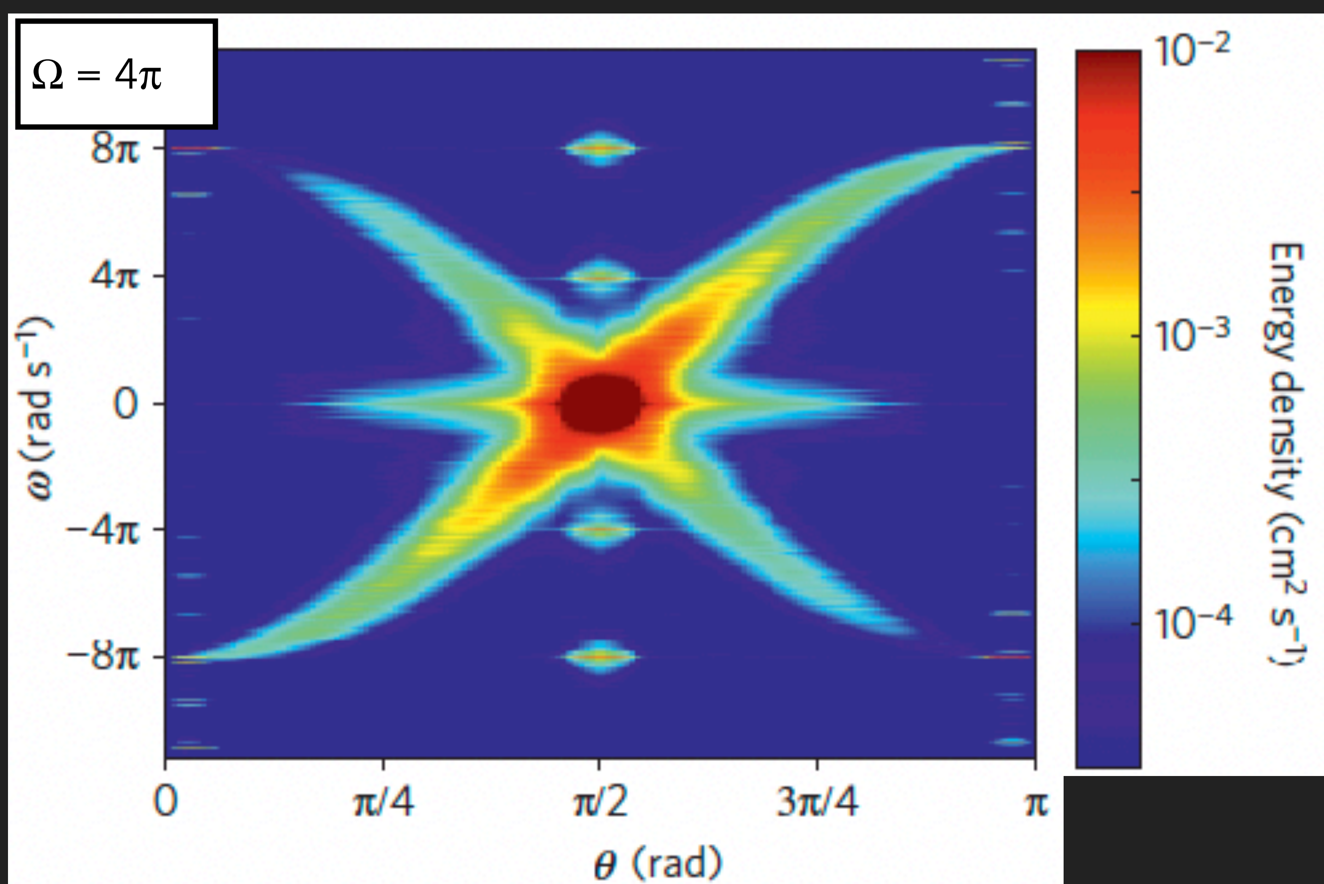
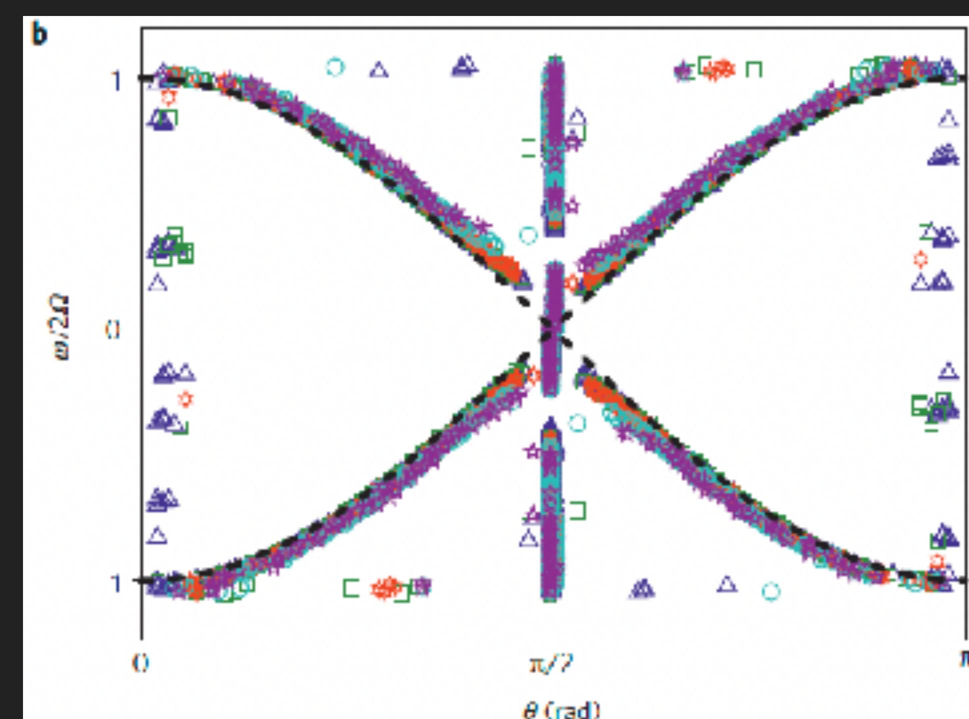


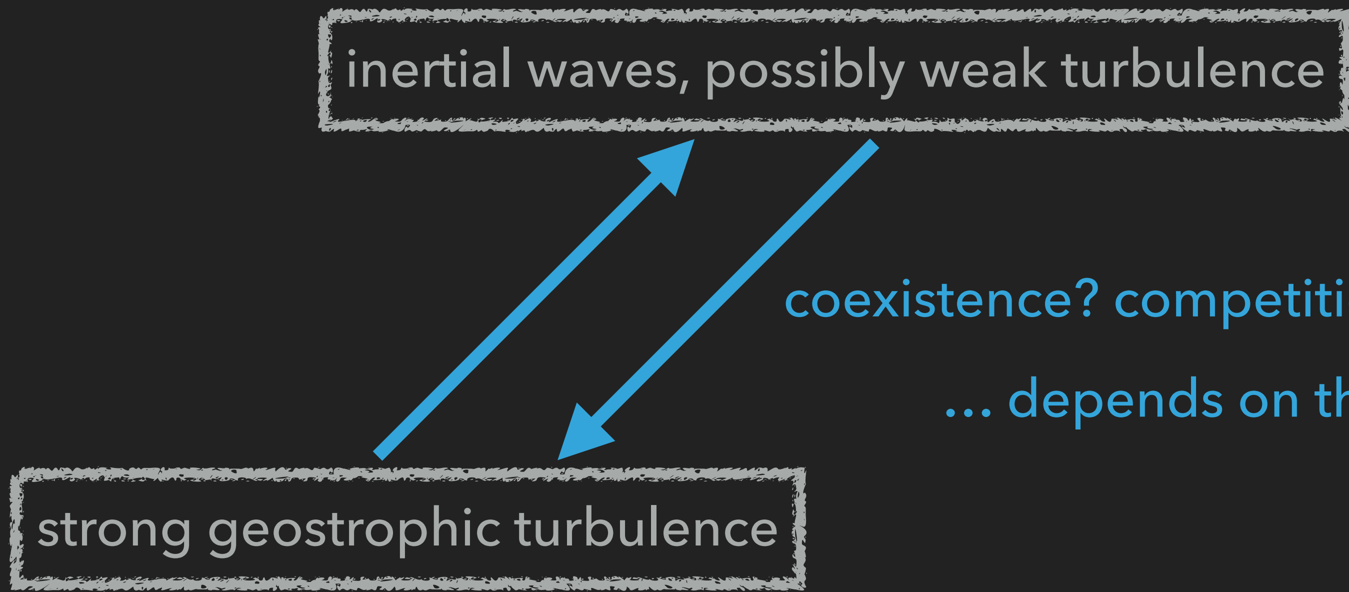
focusing on large wavenumbers, here 1.42-2.03 rad/cm



energy peaks for all ω<2Ω with integration across the full range of wavenumbers for different rotation rates Ω (in rad/s): 1.6π (purple); 2.2π (cyan); 2.8π (red); 3.4π (green); and 4 (blue).

seminal study by Yarom & Sharon (2014)

4. VARIOUS ASPECTS OF ROTATING TURBULENCE



coexistence? competition? coupling?

… depends on the shape and strength of the forcing…

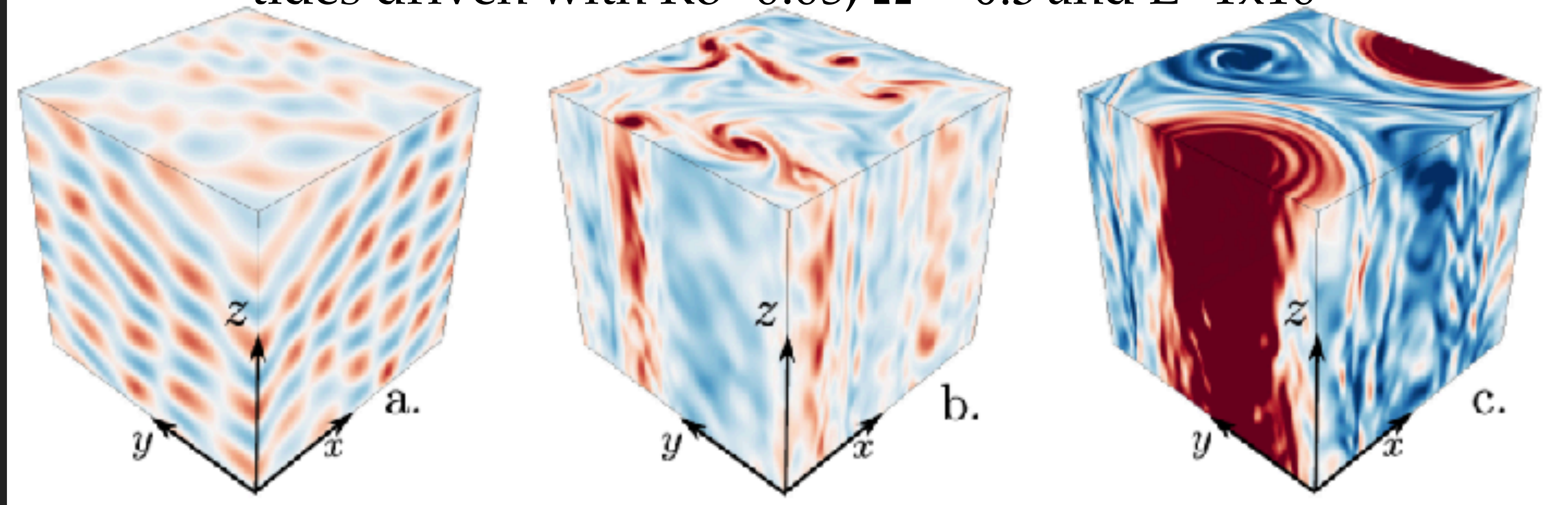
- strong forcing, or some forcing on the geostrophic modes -> strong turbulence, with some subdominant waves
  - rotating turbulence naturally tends to transfer energy to the kz= 0 plane
  - geostrophic turbulent patterns radiate waves
  - inertial waves might spread through triadic interactions

- weak forcing on waves only: inertial wave turbulence with no geostrophic component?

… a completely different state of rotating turbulence…

5. PURE INERTIAL WAVE TURBULENCE?

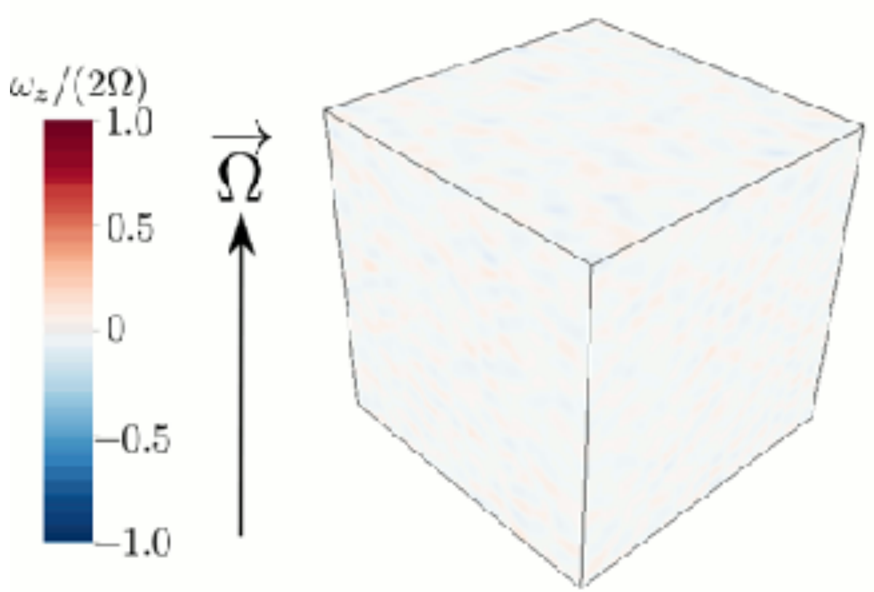
tides driven with Ro=0.05, Ω=–0.5 and E=1x10^{-5}

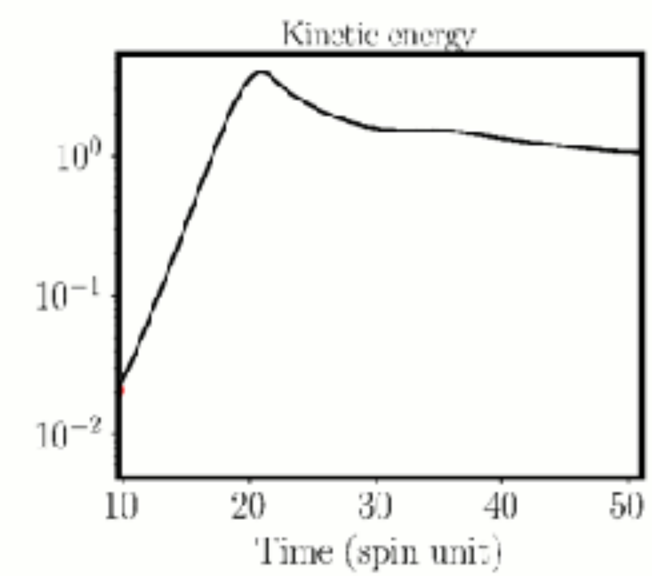


… in the small dissipation / small forcing limit: another type of turbulence…

Ro=0.005, Ω=–0.5 and E=1x10^{-7}

wave turbulence

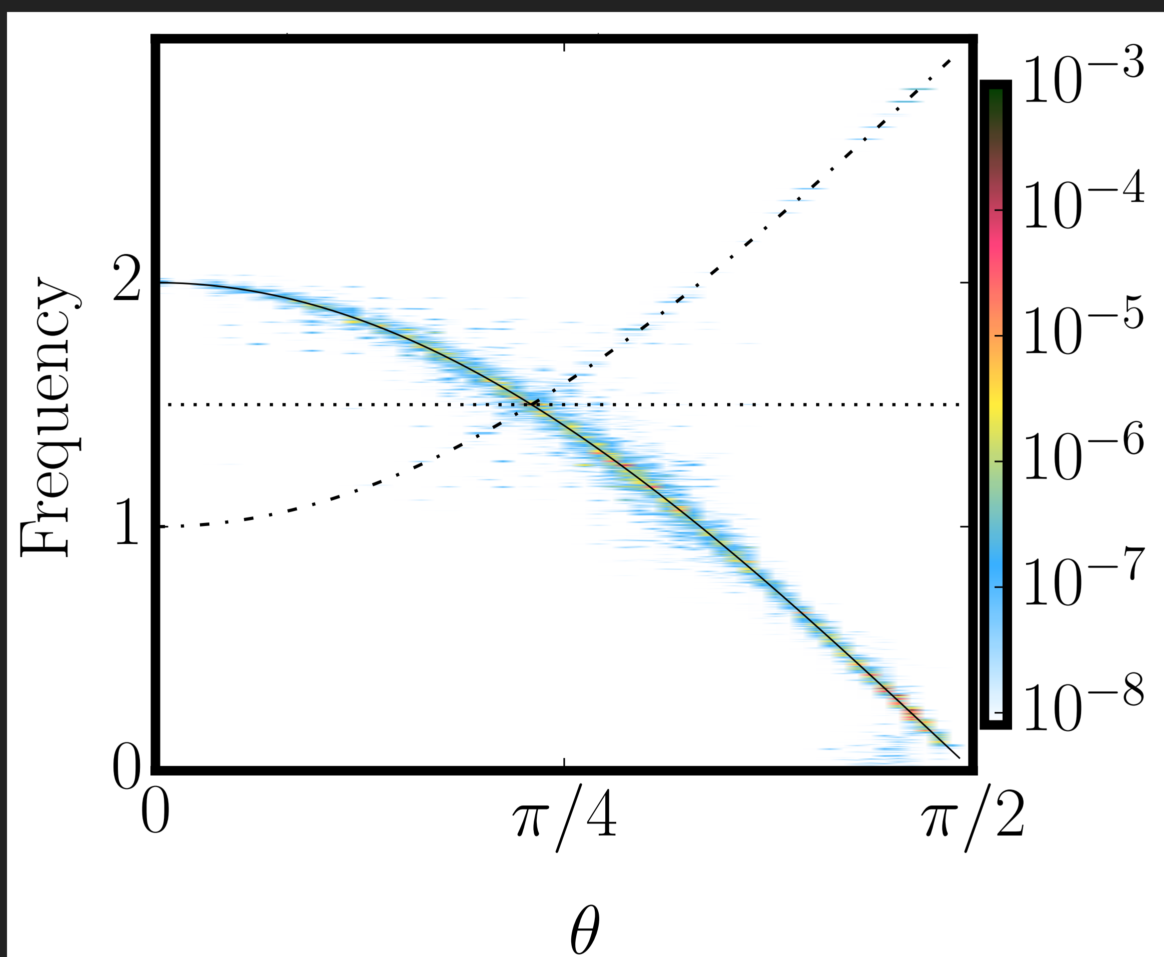


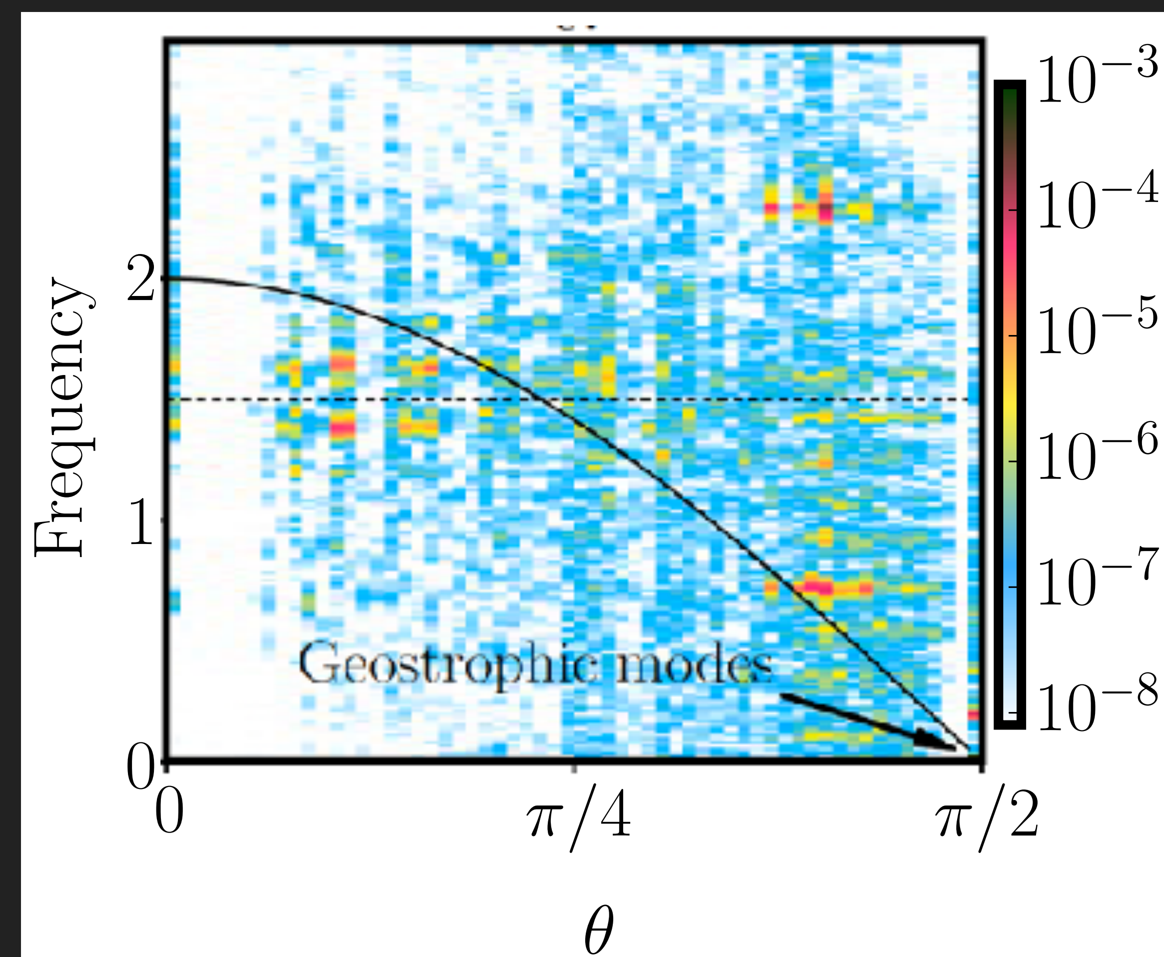


Le Reun et al. (2017)

5. PURE INERTIAL WAVE TURBULENCE

OK at small enough Ro



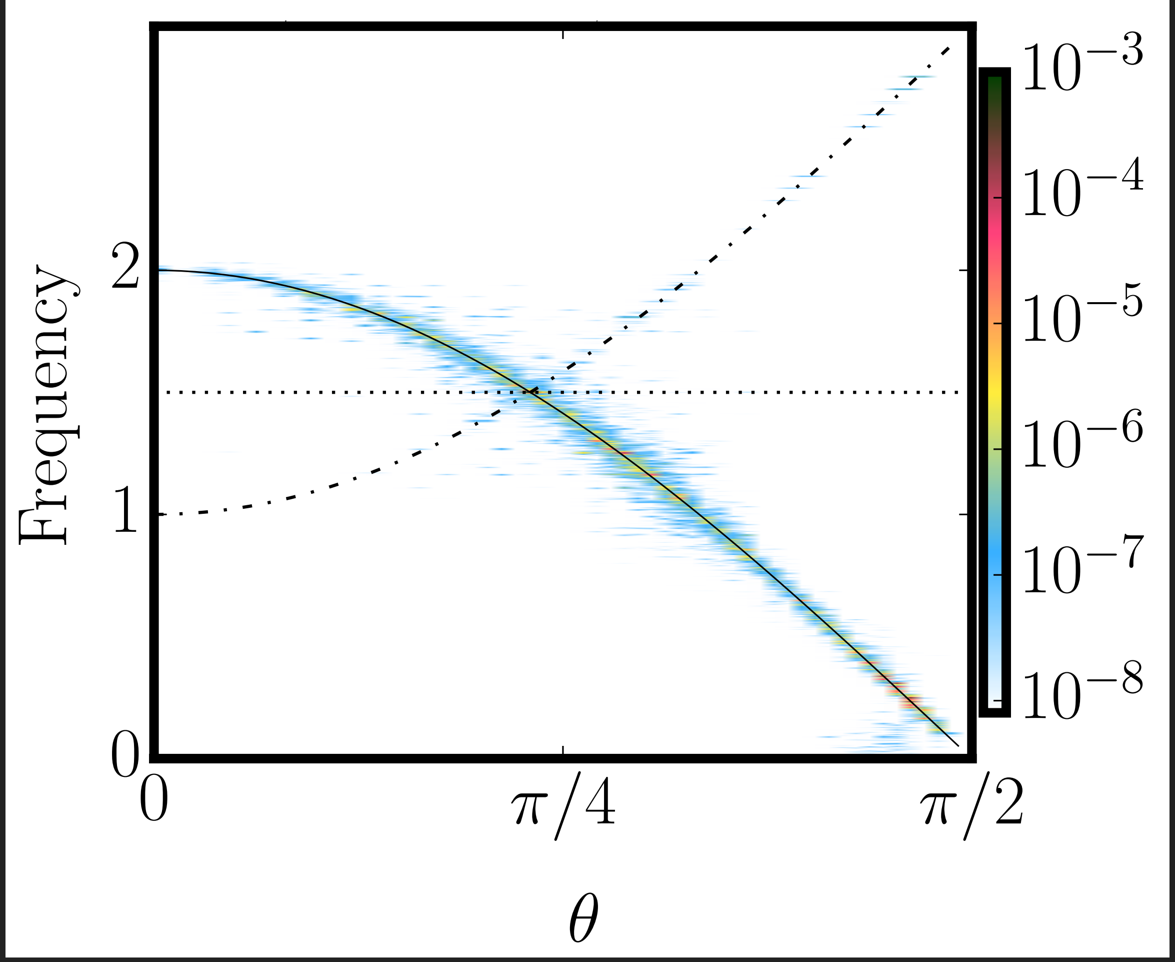


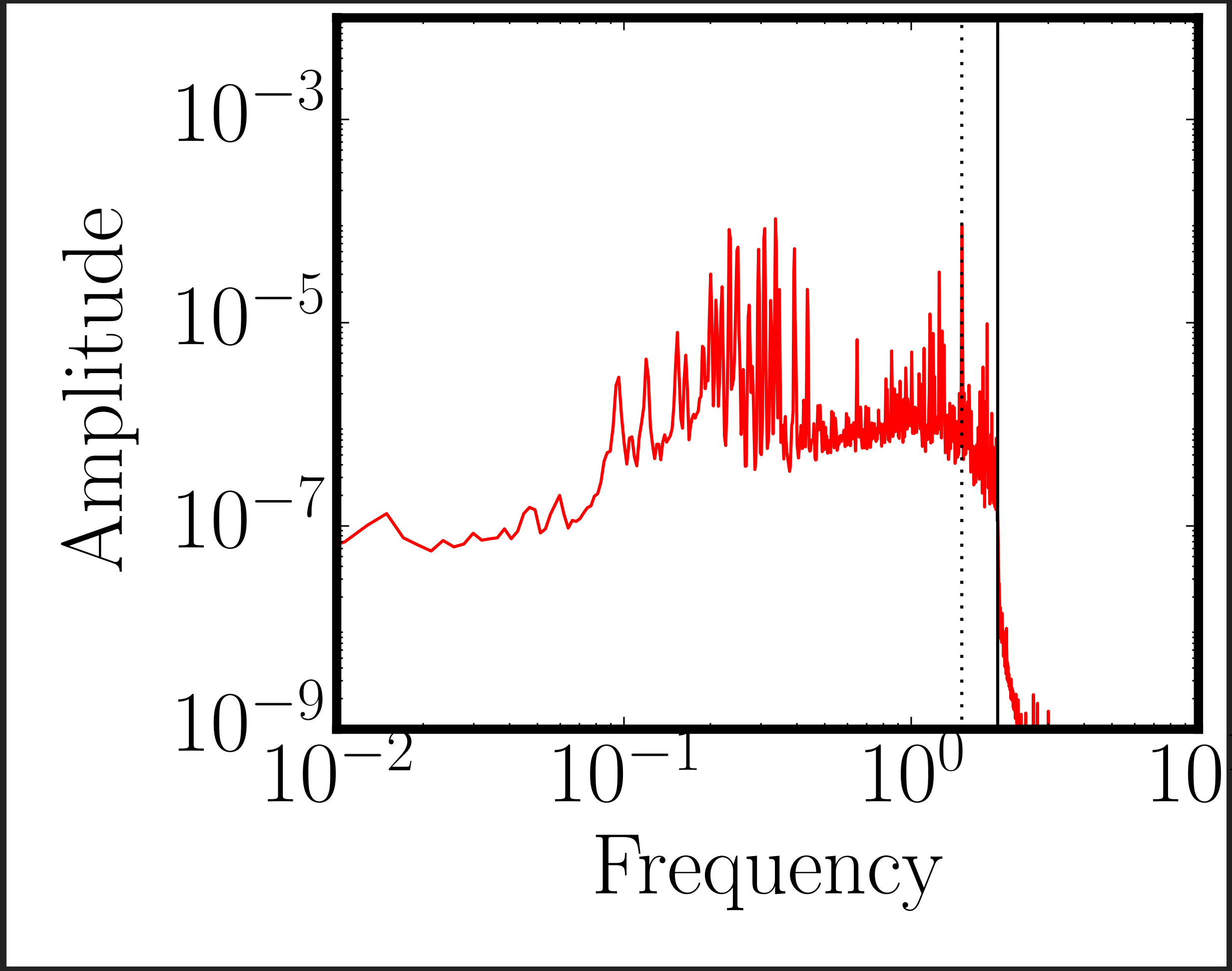
case 1…

Le Reun et al. (2017)

5. PURE INERTIAL WAVE TURBULENCE

OK at small enough Ro





Le Reun et al. (2017)

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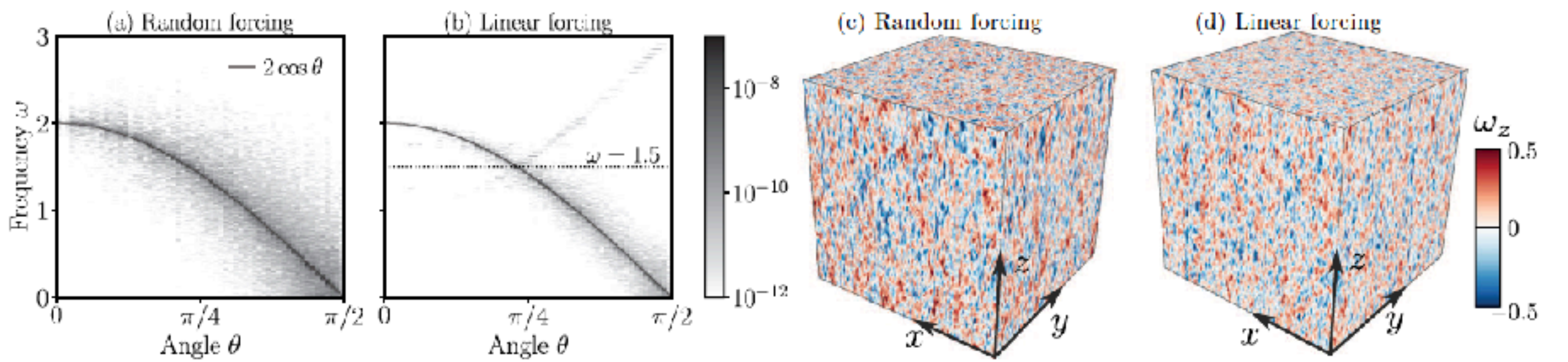


Fig. 1: (a) and (b): spatio-temporal spectra $\mathcal{E}(\theta, \omega)$ for the two most extreme simulations ($E = 10^{-7.5}$, resolution 512^3) using random (a) and linear (b) forcing. The continuous line in panel (a) shows the dispersion relation of inertial wave (4). The value and the location of the forcing frequency is specified in panel (b). In panel (b), the secondary locations of energy mirroring the dispersion relation are bound waves due to non-linear, non-resonant interaction between inertial waves and the forcing flow. (c) and (d): snapshots of the vertical vorticity ω_z taken from simulations using random (c) and linear (d) forcing. *Ro=7.5e-3*

- random forcing = noise that is δ-correlated in time and applied to the modes k located in a spherical shell
- linear forcing = tidal forcing at a given frequency

Le Reun et al. (2020)

6. SPECTRAL DESCRIPTION OF INERTIAL WAVE TURBULENCE

- anisotropic spectrum $k_\perp$ vs. $k_\parallel$ & $u_\ell^2 \sim E(k_\perp, k_\parallel) k_\perp k_\parallel$

inertial waves dispersion relation $\omega = 2\Omega \frac{k_\parallel}{k}$

- time decoupling between the fast wave time $\tau_\omega = 1/\omega$, the intermediate non-linear time $\tau_{NL} = 1/ku_l$, and the long transfer time τ_{tr} due to NL wave-wave interactions -> small parameter $\chi = \frac{\tau_\omega}{\tau_{NL}}$ and since 2 waves interactions, assume $\frac{\tau_\omega}{\tau_{tr}} \simeq O(\chi^2)$ so $\tau_{tr} \simeq \frac{\tau_{NL}^2}{\tau_\omega}$ $\sim \frac{\Omega k_\parallel}{k^3 u_\ell^2}$
- then as usual, $\varepsilon \sim \frac{E(k_\perp, k_\parallel) k_\perp k_\parallel}{\tau_{tr}} \sim \frac{E^2(k_\perp, k_\parallel) k_\perp^5 k_\parallel}{\Omega}$ assuming $k_\parallel \ll k_\perp$

(ε = rate of dissipation of turbulence kinetic energy)

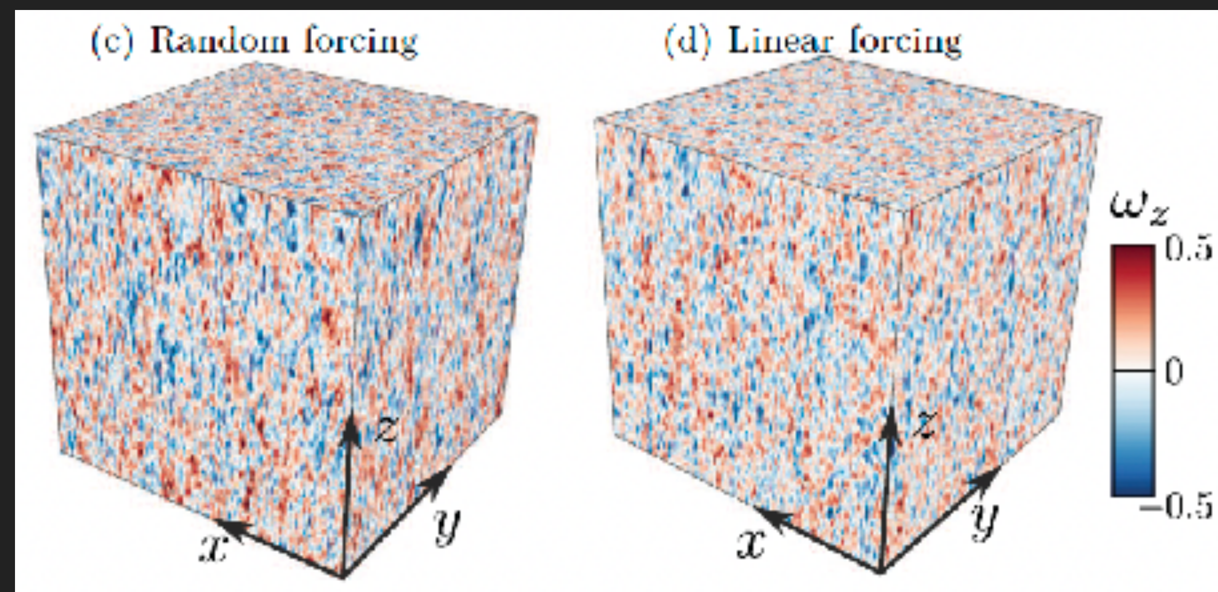
- leading to the final spectral prediction

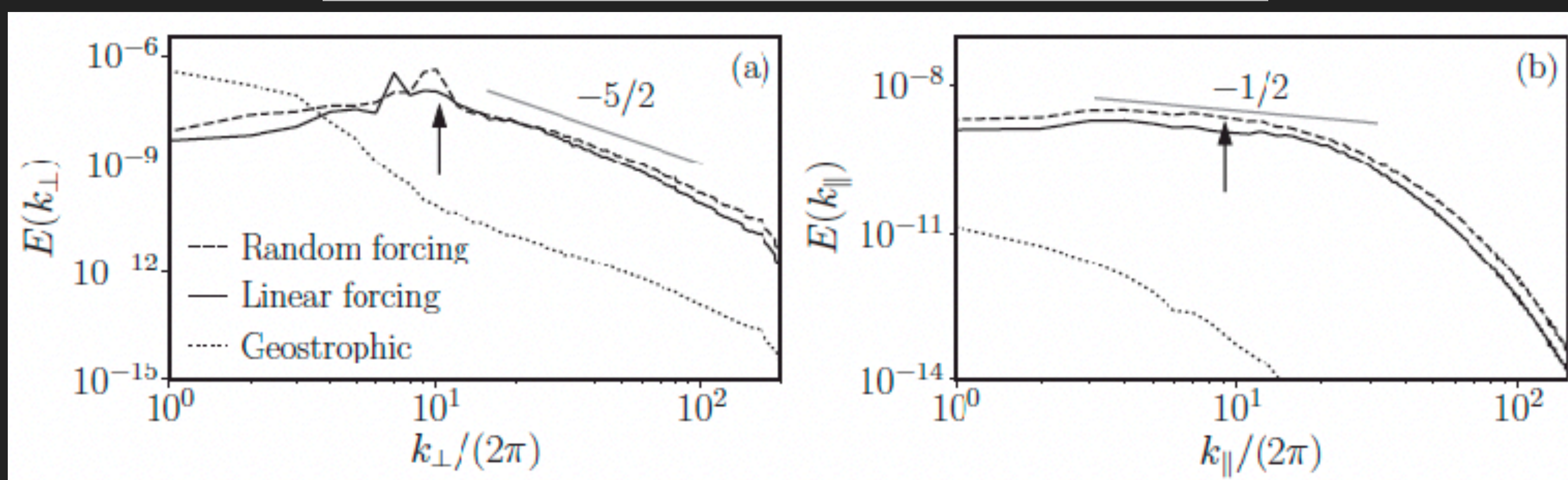
$$E(k_\perp, k_\parallel) \sim \sqrt{\varepsilon \Omega}\, k_\perp^{-5/2} k_\parallel^{-1/2}$$

the Zakharov-Kolmogorov spectrum

Galtier (2003, 2023)

6. SPECTRAL DESCRIPTION OF INERTIAL WAVE TURBULENCE





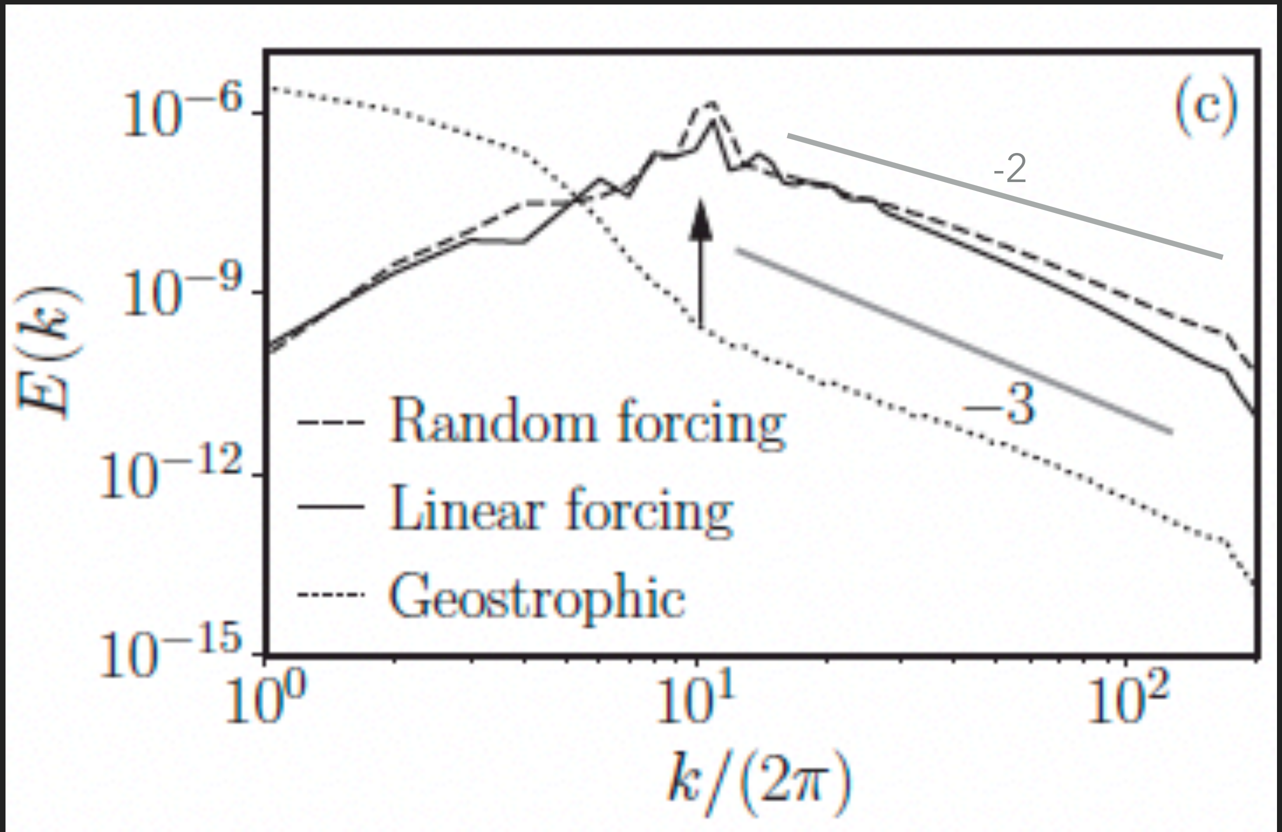
$$E(k_\perp, k_\parallel) \sim \sqrt{\varepsilon\Omega}\, k_\perp^{-5/2} k_\parallel^{-1/2}$$

the Zakharov-Kolmogorov spectrum

Le Reun et al. (2020)

6. SPECTRAL DESCRIPTION OF INERTIAL WAVE TURBULENCE

- isotropic spectrum (or θ~π/2) $E(k) \sim (\epsilon\Omega)^{1/2} k^{-2}$
- while 2D spectrum $E(k) \sim \eta^{2/3} k^{-3}$



$$E(k_\perp, k_\parallel) \sim \sqrt{\varepsilon\Omega}\, k_\perp^{-5/2} k_\parallel^{-1/2}$$

the Zakharov-Kolmogorov spectrum

Le Reun et al. (2020)

7. IN THE REAL WORLD?

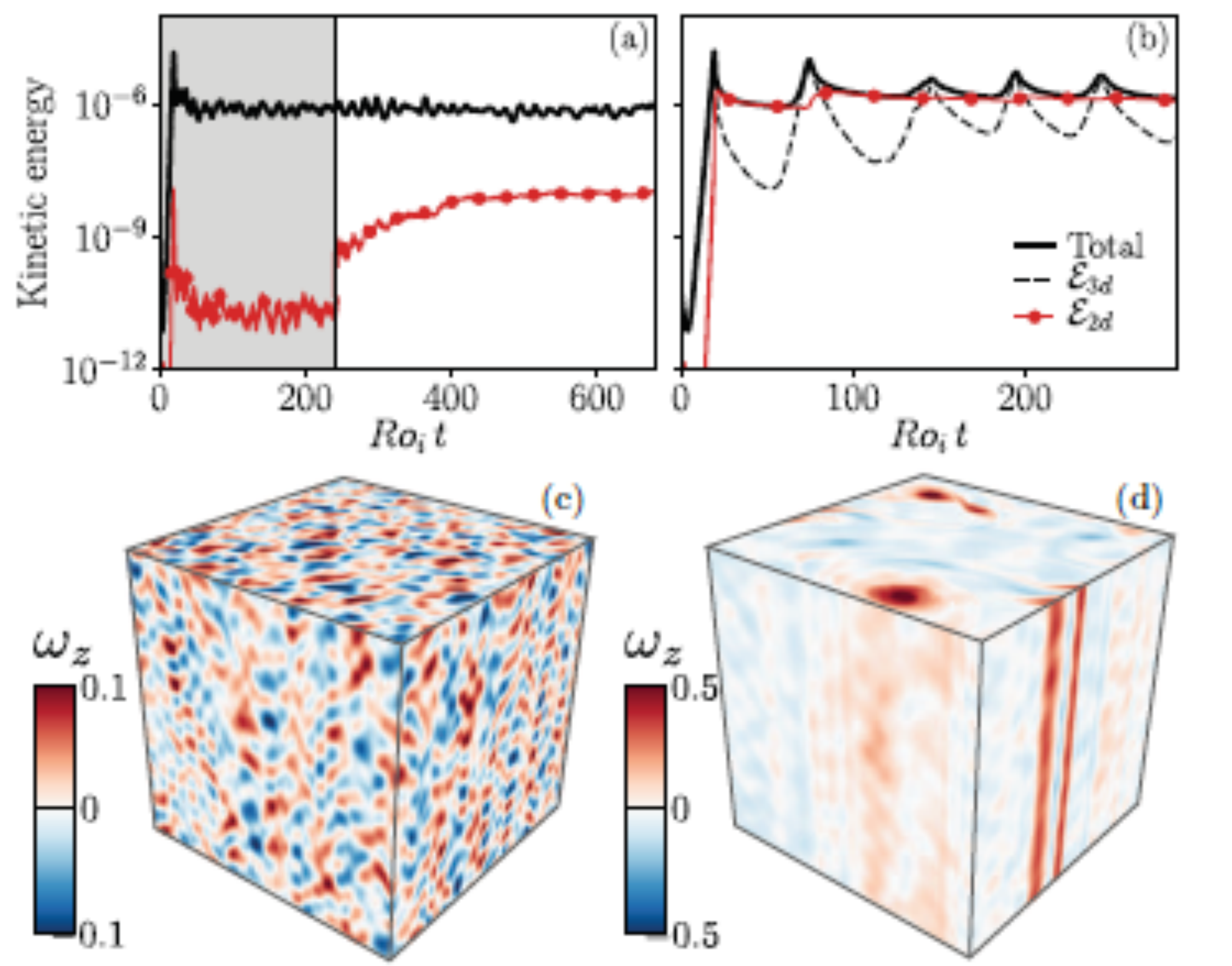


Fig. 4: (a) and (b): times series of the kinetic energy for two simulations using linear forcing both carried out at $Ro_i = 7.5\times 10^{-3}$ and $E = 10^{-6.5}$. In the first simulation (a), geostrophic friction is first applied (grey area) and then released, whereas in the second (b), no friction is applied. The corresponding snapshots of the vertical vorticity are shown in panels (c) and (d) and taken at times $Ro_i t = 500$ and $Ro_i t = 90$, respectively.

but there is a trick: friction specific to geostrophic modes to force them to remain of small amplitude…

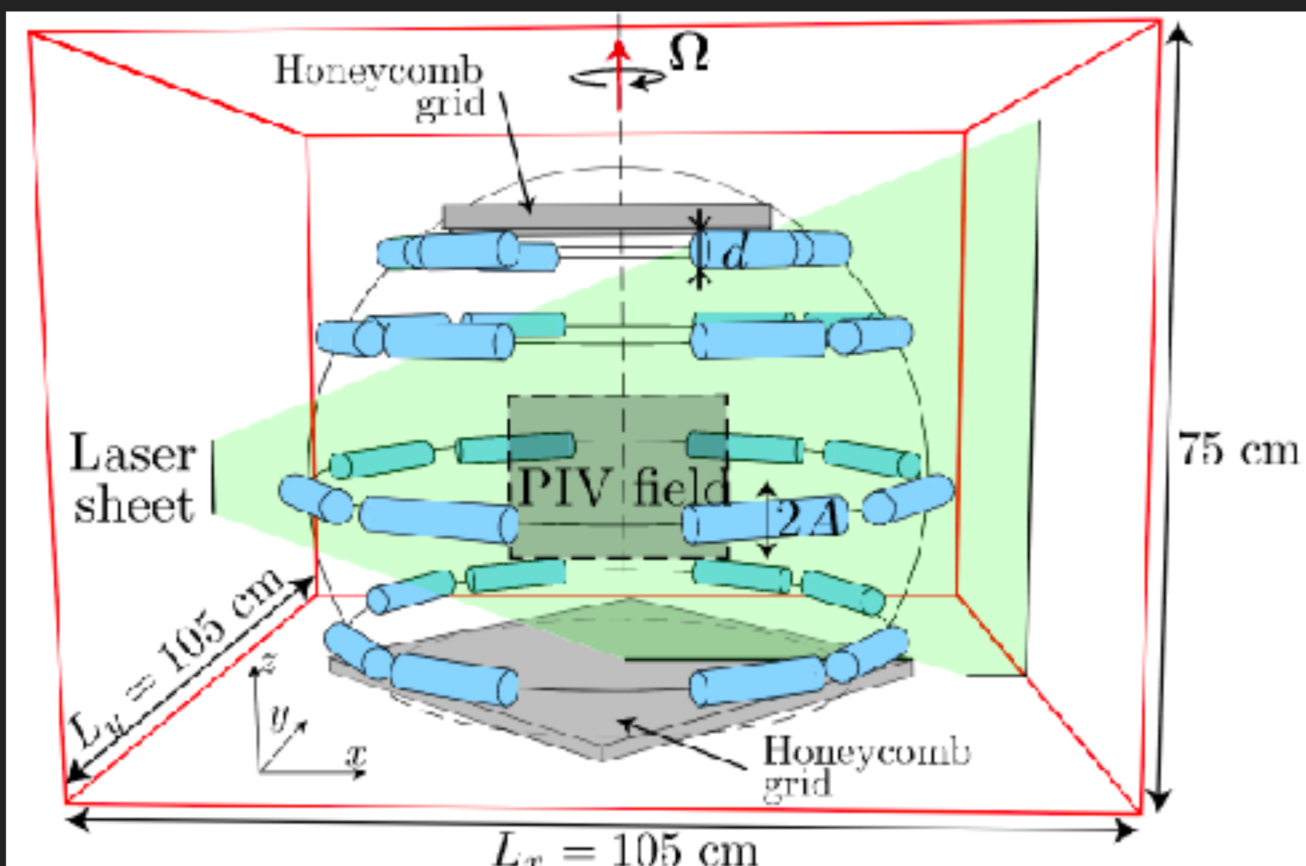
- finite Ek / Ro effect?
- realistic effect in close domains?

Bi-stability of rotating turbulence?

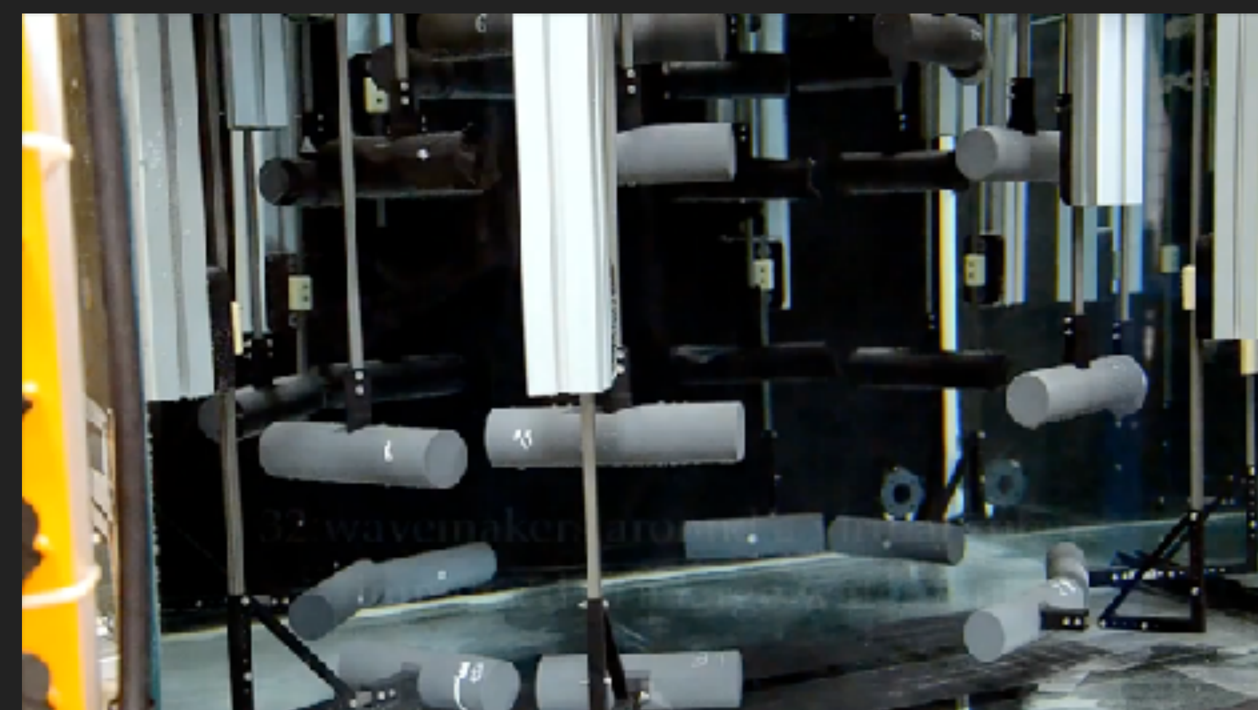
Le Reun et al. (2020)

7. EXPERIMENTAL REALIZATION OF THE INERTIAL WAVE TURBULENCE

- weak enough forcing on the waves only -> inertial wave turbulence



honeycomb grids to help dissipate geostrophic motions



size ~80cm, rotation up to 20rpm -> $E=10^{-6}$

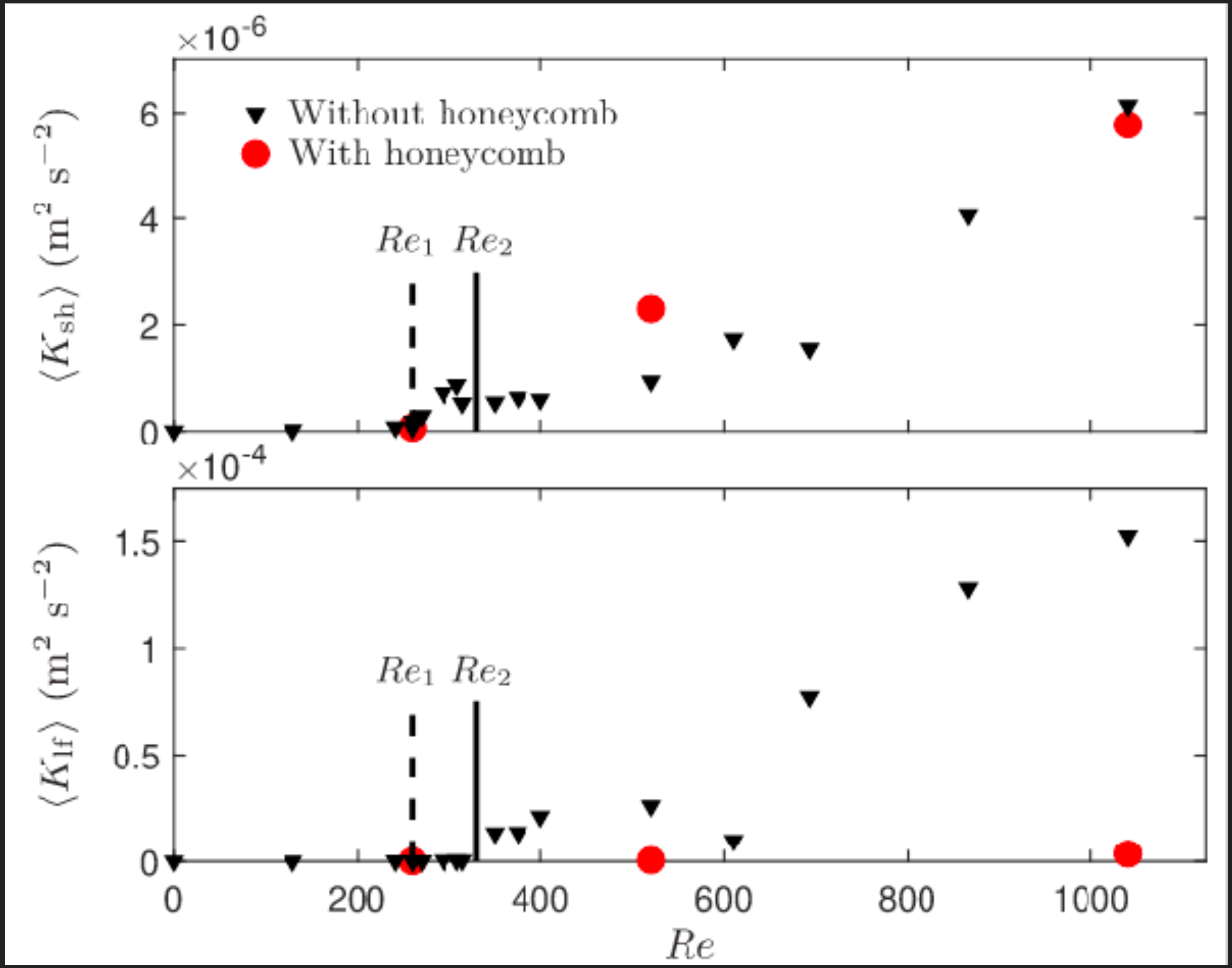
Brunet et al. (2020)
Monsalve et al. (2020)

7. EXPERIMENTAL REALIZATION OF THE INERTIAL WAVE TURBULENCE

- weak enough forcing on the waves only -> inertial wave turbulence

kinetic energy in the waves

kinetic energy in the geostrophic modes



$Re = Ro/Ek$

Brunet et al. (2020)
Monsalve et al. (2020)

7. EXPERIMENTAL REALIZATION OF THE INERTIAL WAVE TURBULENCE

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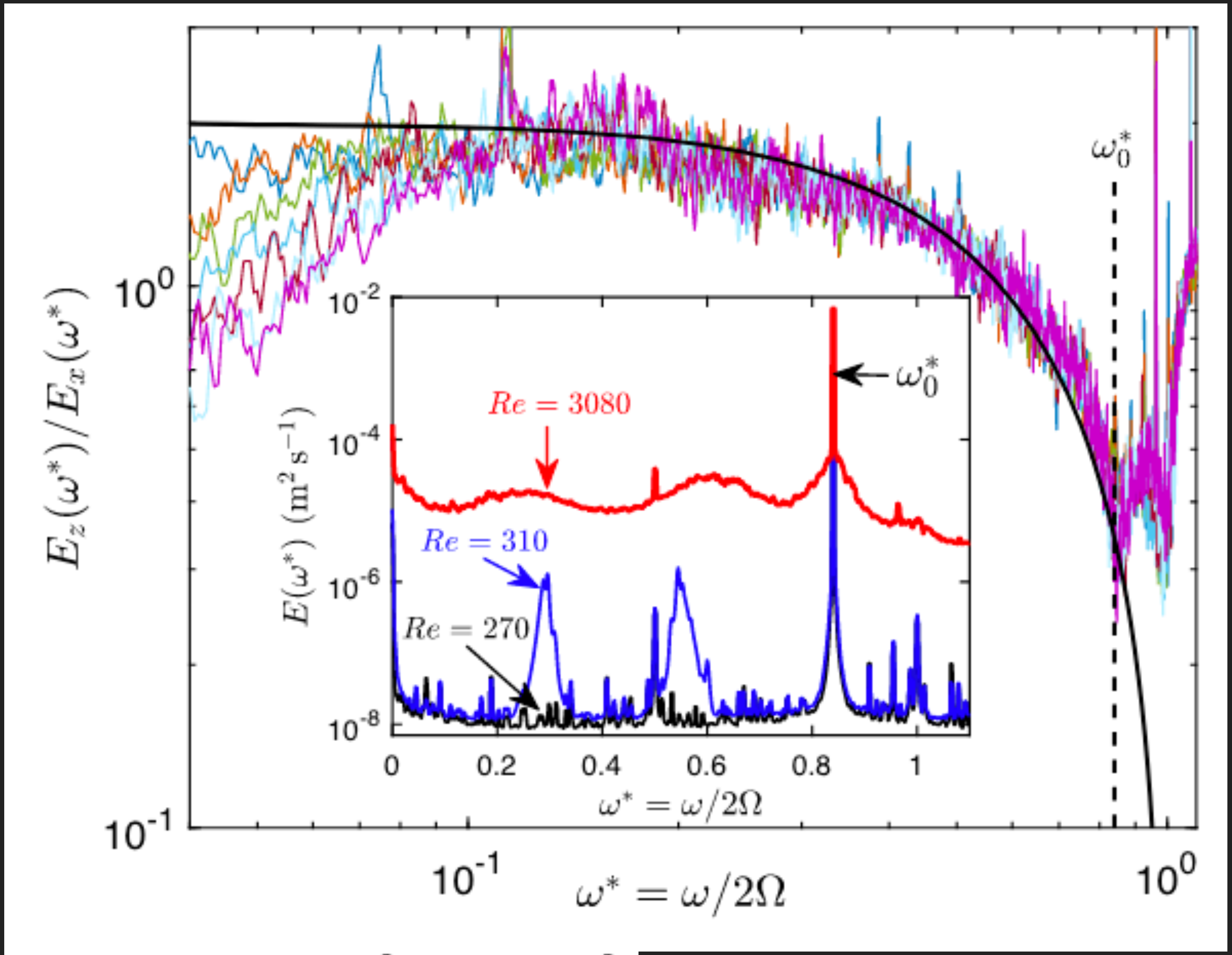


$$E_z/E_x = 2(1-\omega^{*2})/(1+\omega^{*2})$$

Brunet et al. (2020)
Monsalve et al. (2020)

8. A THEORETICAL PARADOX

- weak enough forcing on the waves only -> inertial wave turbulence

in the numerics and experiments, weak forcing of waves only, and very small dissipation

Greenspan theorem (1969): triadic resonance cannot account for wave-to-geostrophic transfers in the asymptotic limit of vanishing velocity amplitude (Ro) and dissipation (Ek)…

but we do see geostrophic modes growing above some Ro<<1!

why? and where does the threshold in Ro come from?

8. A THEORETICAL PARADOX

near-resonant O(kRo)² triads of inertial waves, involving one geostrophic mode

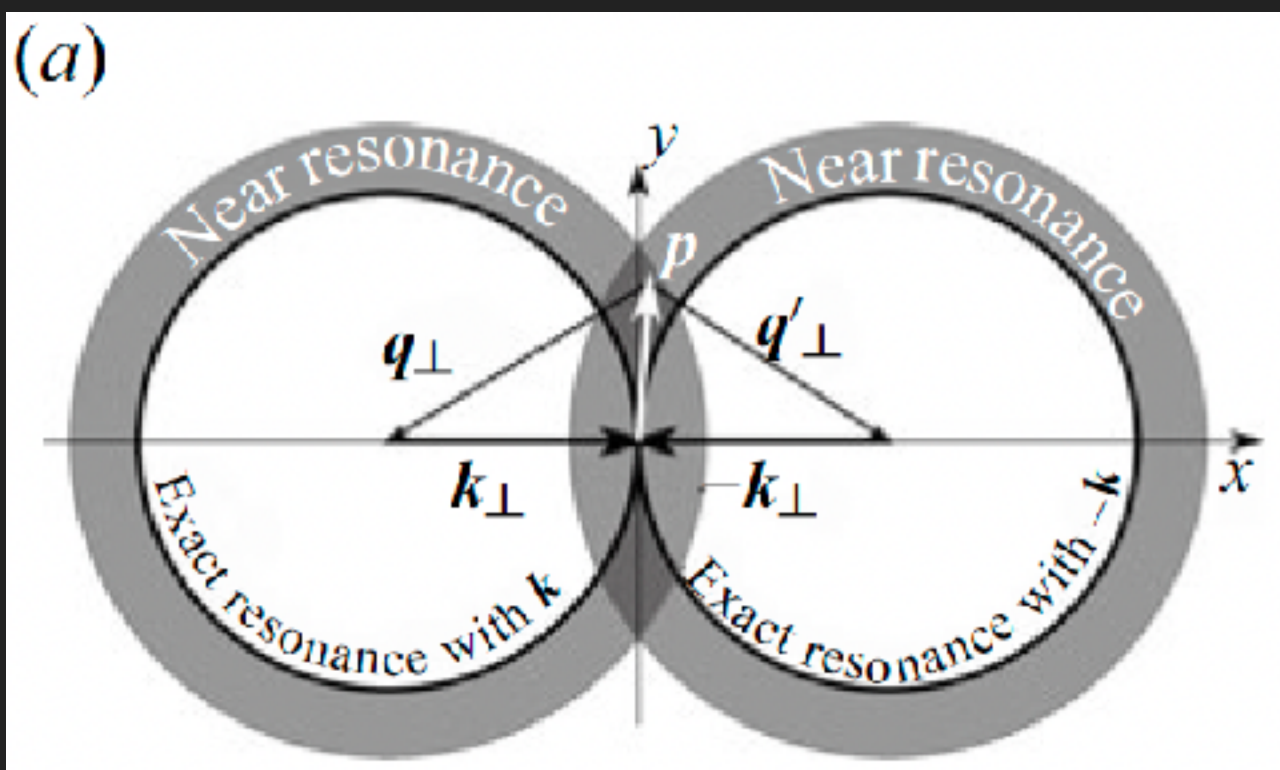


FIGURE 4. (a) Schematic cartoon of a geostrophic mode p in near resonance with both imposed modes +\-k at the same time

Le Reun et al. (2020)

resonant quartets of inertial waves, involving one geostrophic mode

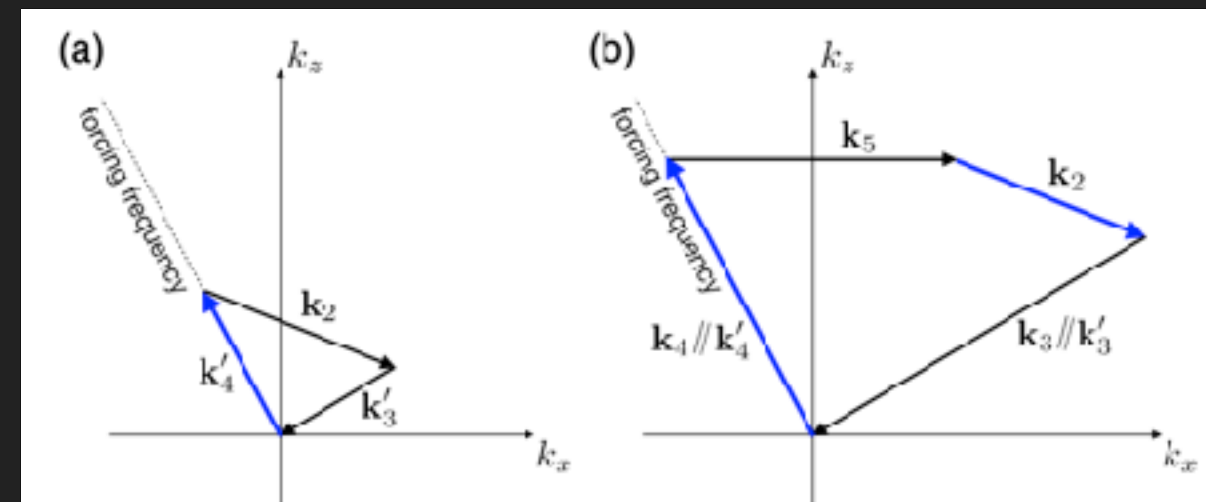


FIG. 4. An illustrative example of the quartetic secondary instability. (a) The triadic instability transfers energy from the mode $\mathbf{k}'_4$ (at the forcing frequency) to modes at $\mathbf{k}_2$ and $\mathbf{k}'_3$. (b) One can build a resonant quartet by keeping $\mathbf{k}_2$, inserting a horizontal wave number $\mathbf{k}_5$, and closing the quartet with two wave vectors $\mathbf{k}_3$ and $\mathbf{k}_4$ parallel to $\mathbf{k}'_3$ and $\mathbf{k}'_4$, respectively. $\mathbf{k}_4$ is energized by the forcing, while $\mathbf{k}_2$ has been energized at step (a). Through a resonant quartet instability, the geostrophic mode $\mathbf{k}_5$ then spontaneously emerges, together with $\mathbf{k}_3$.

Brunet et al. (2020)

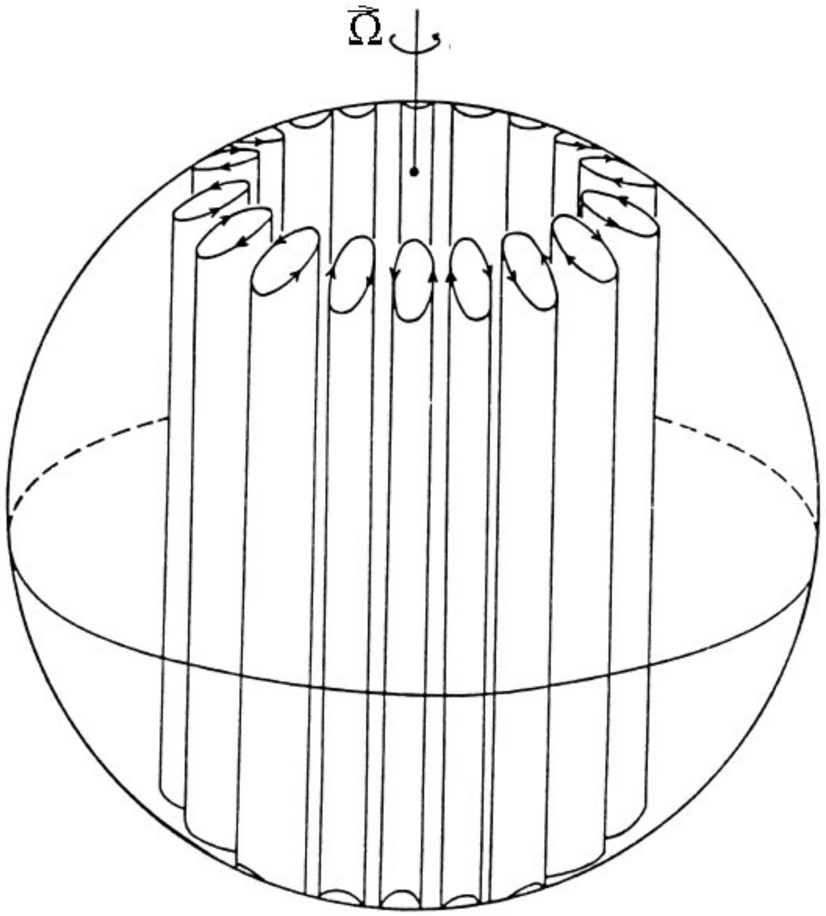
Mechanism for geostropic mode excitation still under discussion, but in both cases, threshold $Ro \propto E^{1/4}$

CAN ANY OF THIS APPLY TO PLANETARY CORES AND SUBSURFACE OCEANS ?

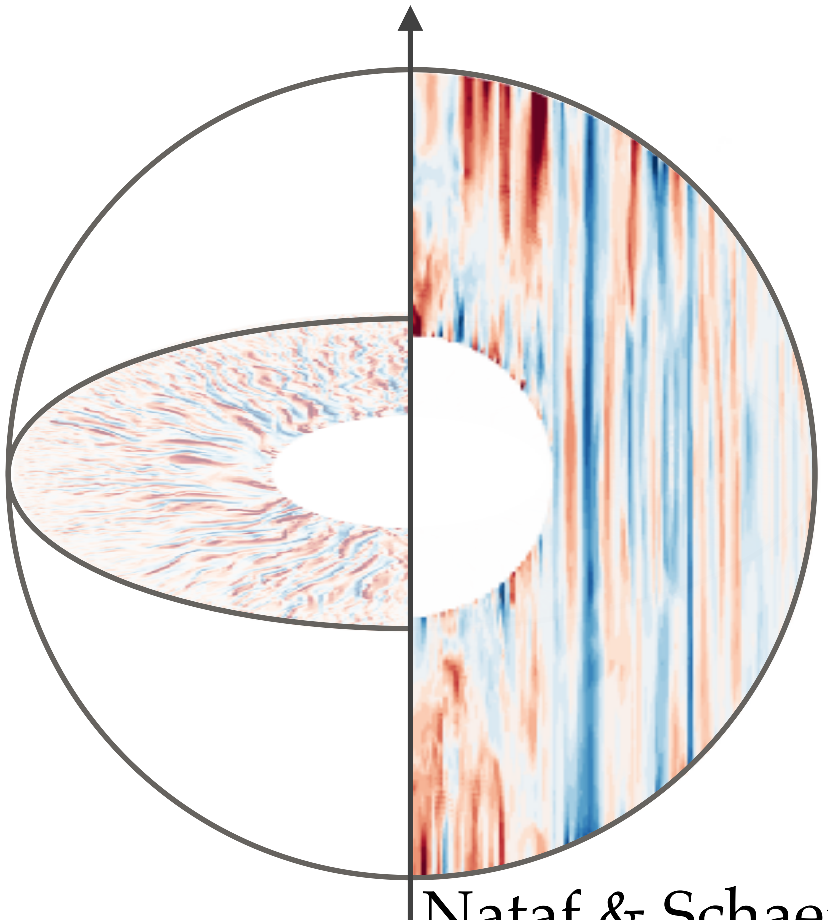
if yes, completely change our estimates for, e.g., energy dissipation, mixing, induction and dynamo, etc.

HOW TO EXCITE INTERNAL WAVES IN PLANETARY FLUID LAYERS?

Standard model of planetary fluid layers



Busse 1974

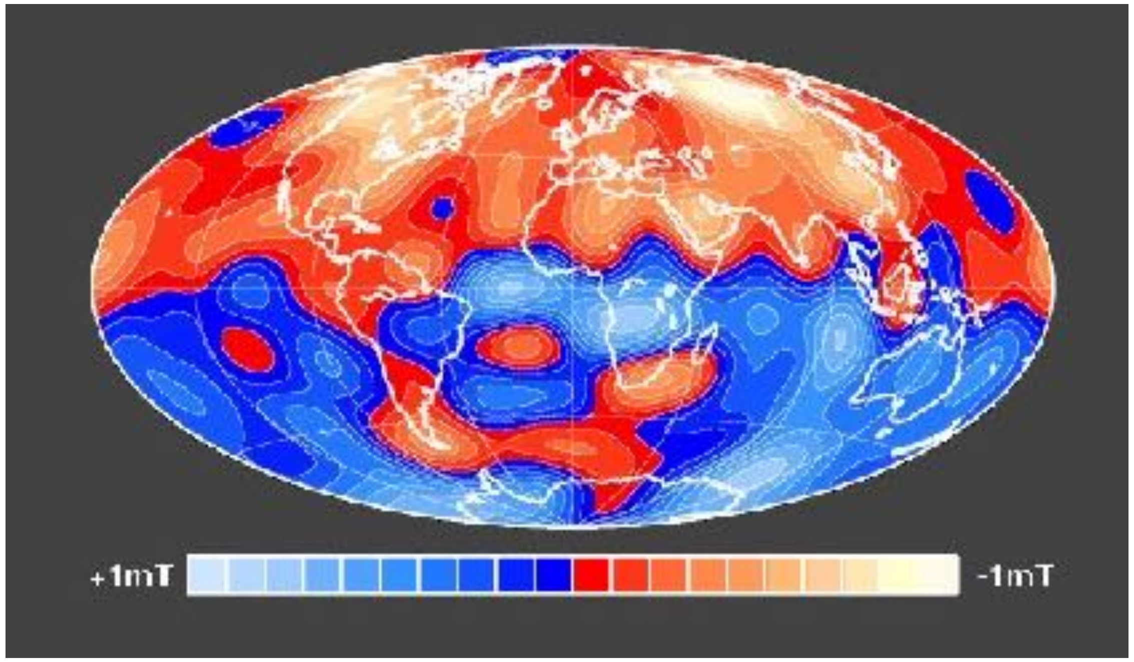


Nataf & Schaeffer 2015

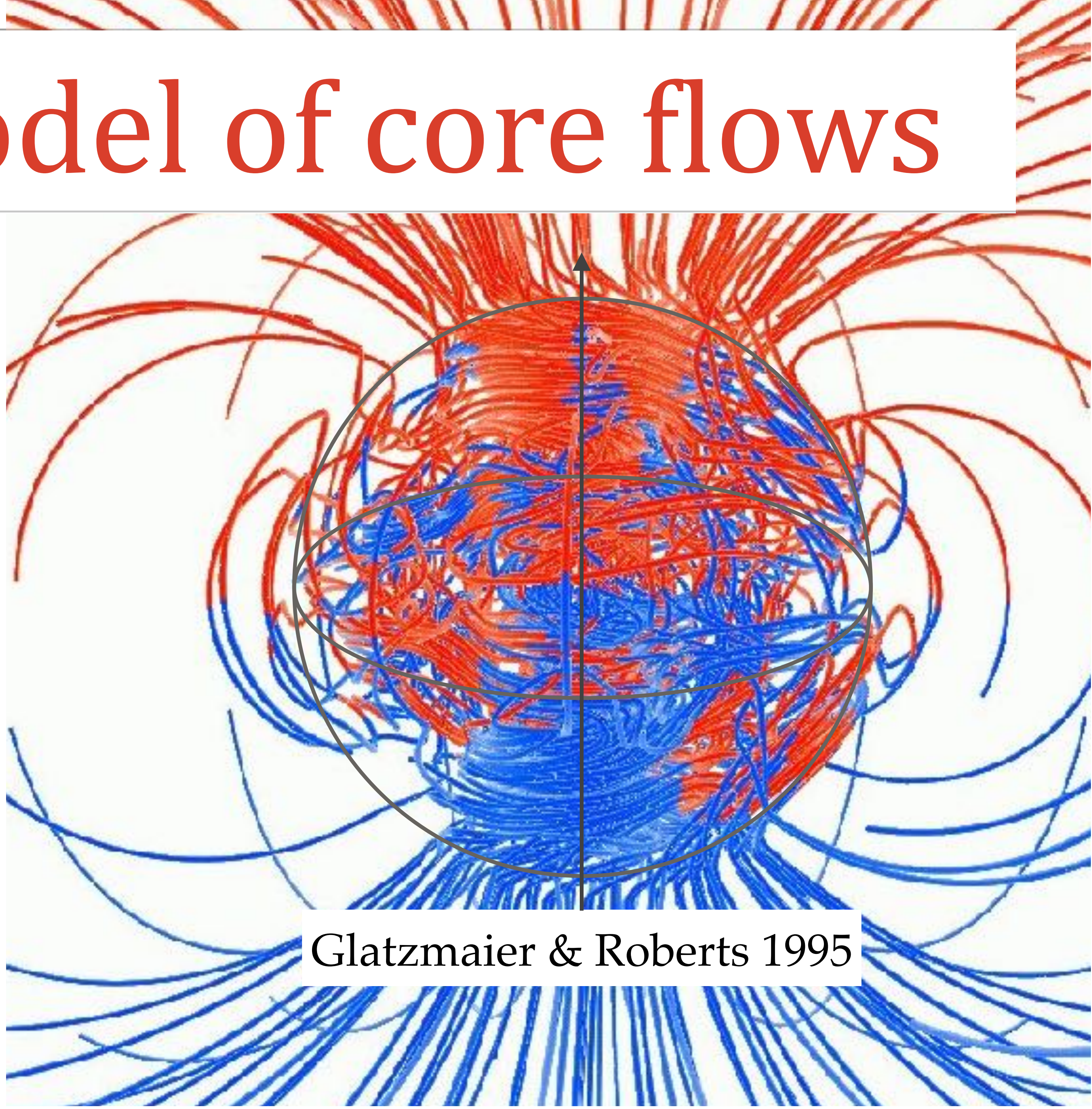
Cylindrical radial velocity component
DNS at $E = 10^{-7}$, $Pr = 1$, $Ra = 2.4 \times 10^{13}$

- buoyancy driven flows: thermal and / or compositional convection

Standard model of core flows



R. Holme, Liverpool

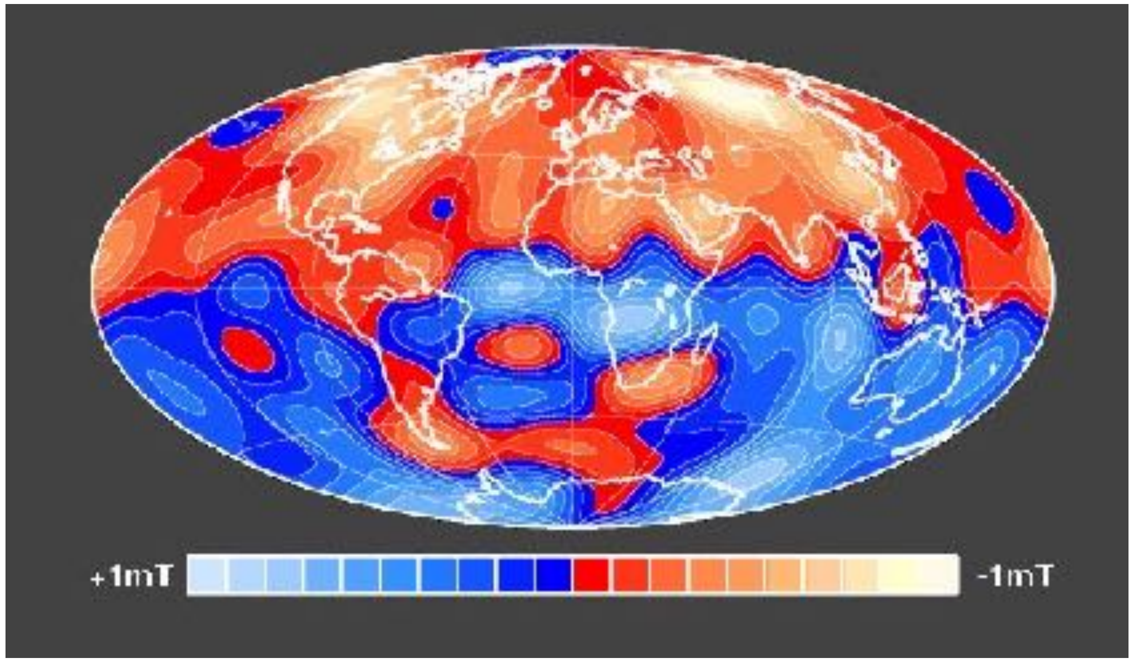


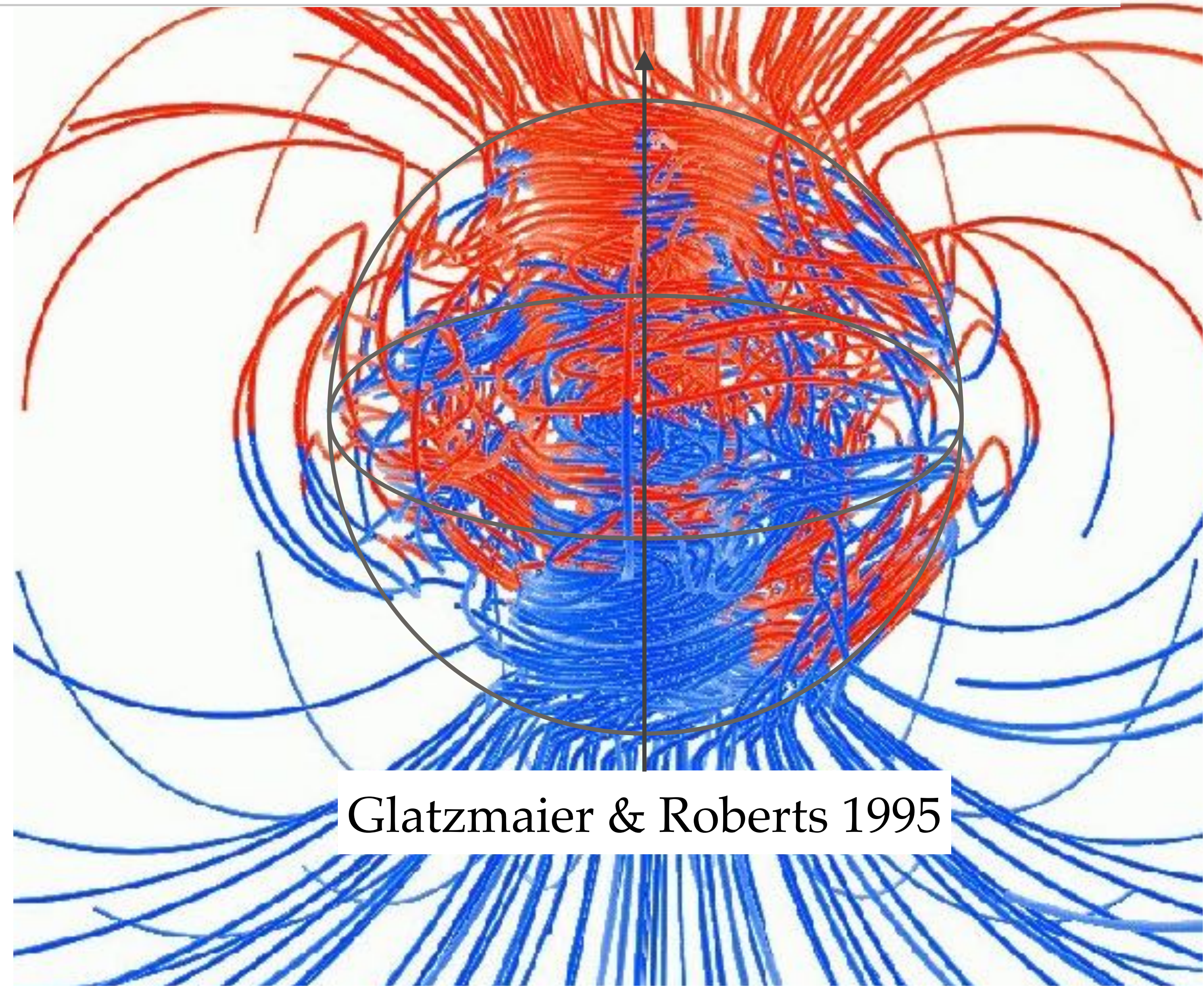
Glatzmaier & Roberts 1995

- buoyancy driven flows: thermal and/or compositional convection
- dynamo capable, with reversals and "Earth-like" patterns

… ok for the Earth today…

Standard model of core flows





Glatzmaier & Roberts 1995

- but **tight energy budget** with lots of uncertainties: radiogenic heating, temperature contrast , thermal conductivity, age of the inner core, …
- what about other bodies, and especially **small bodies ?**
- **subsurface oceans stably stratified ?**

Alternative routes to turbulence

- **gigantic reservoir of energy: rotation**

e.g. on Earth:

- rotational energy 2x10^{29} J (lower bound…)
- necessary for present day dynamo 0.1 - 2 TW (Buffet 2002)

hence, sufficient to power Earth’s dynamo during 3 - 63 Gy…

but how?

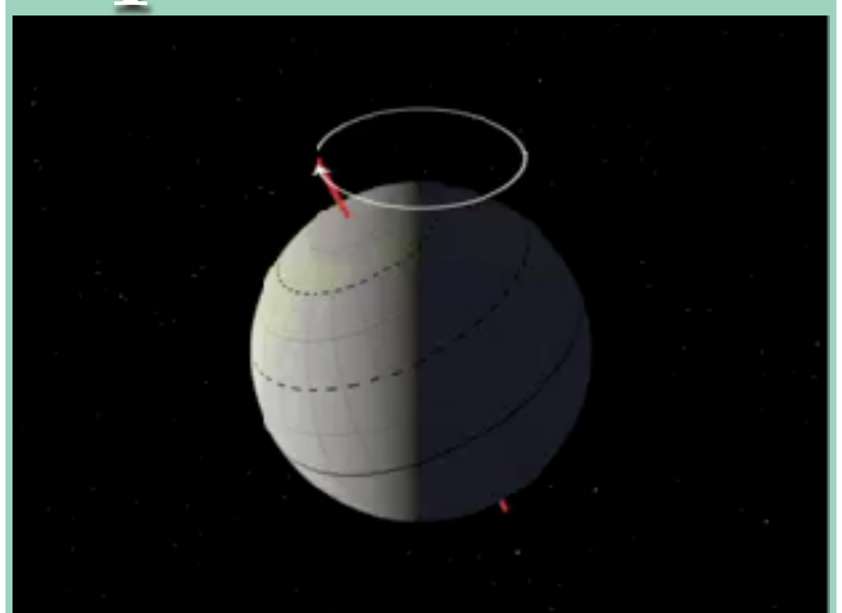
Alternative routes to turbulence

- **gigantic reservoir of energy: rotation**
- if rigid container and constant rotation: solid body
  but **gravitational interactions = small perturbations**

libration

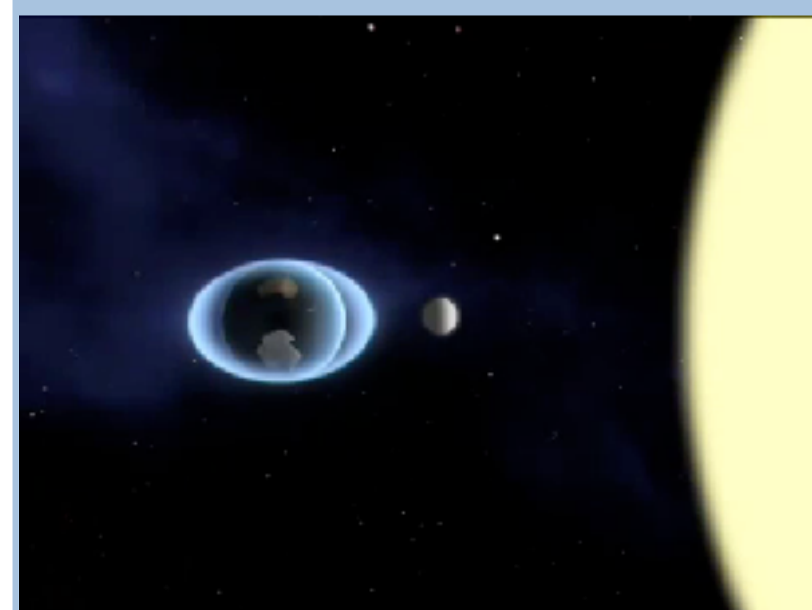
periodic perturbation
of the rotation rate

precession



periodic perturbation
of the rotation direction

tides



periodic perturbation
of the shape

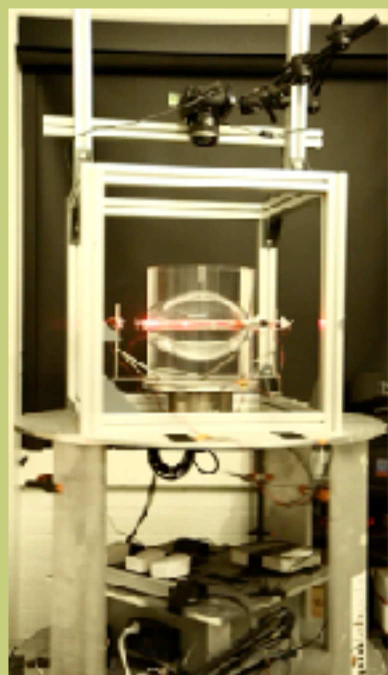
“harmonic or mechanical forcings”

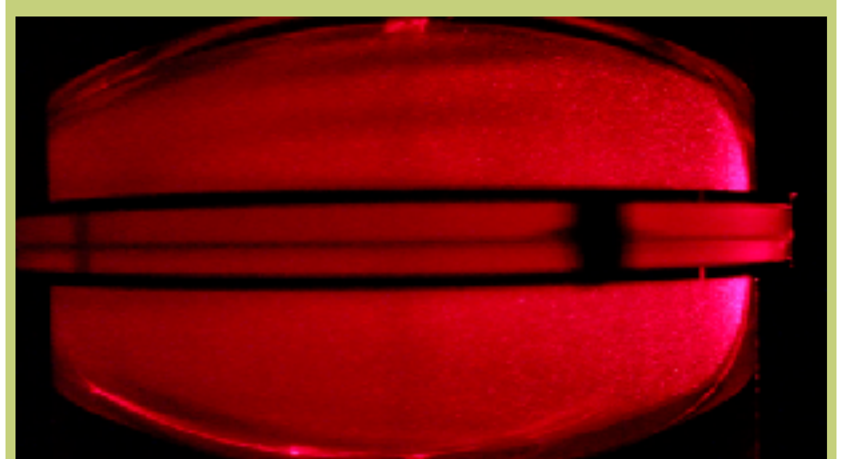
(animations
from NASA)

Small forcing but large consequences

libration

Grannan et al. (2014)



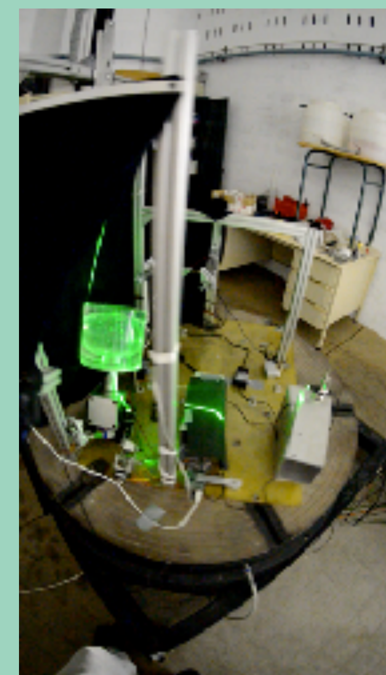


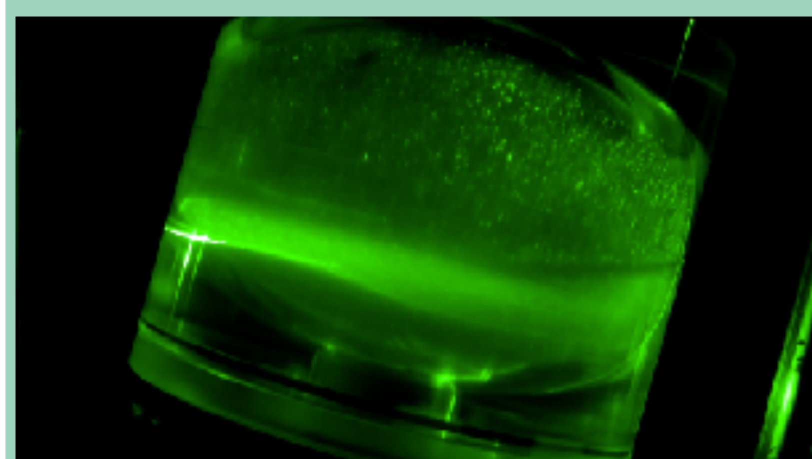
dimensionless parameters
β=0.34, f=4, ε=0.68 and E=2x10^{-5}

periodic perturbation
of the rotation rate

precession

Nobili et al. (2019)



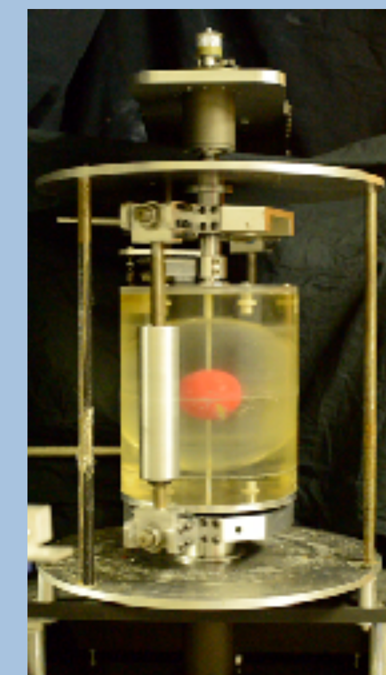


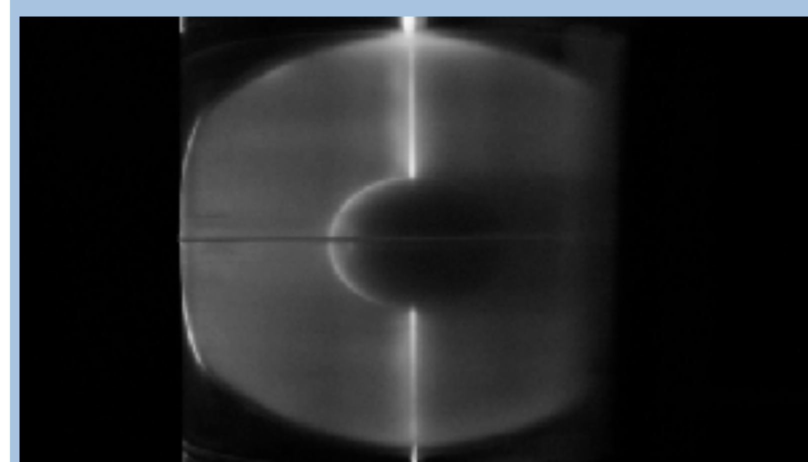
dimensionless parameters
α=15º, Ω=−0.44 and E=2.2x10^{-5}

periodic perturbation
of the rotation direction

tides

Grannan et al. (2016)





dimensionless parameters
β=0.07, Ω=−1 and E=1.5x10^{-5}

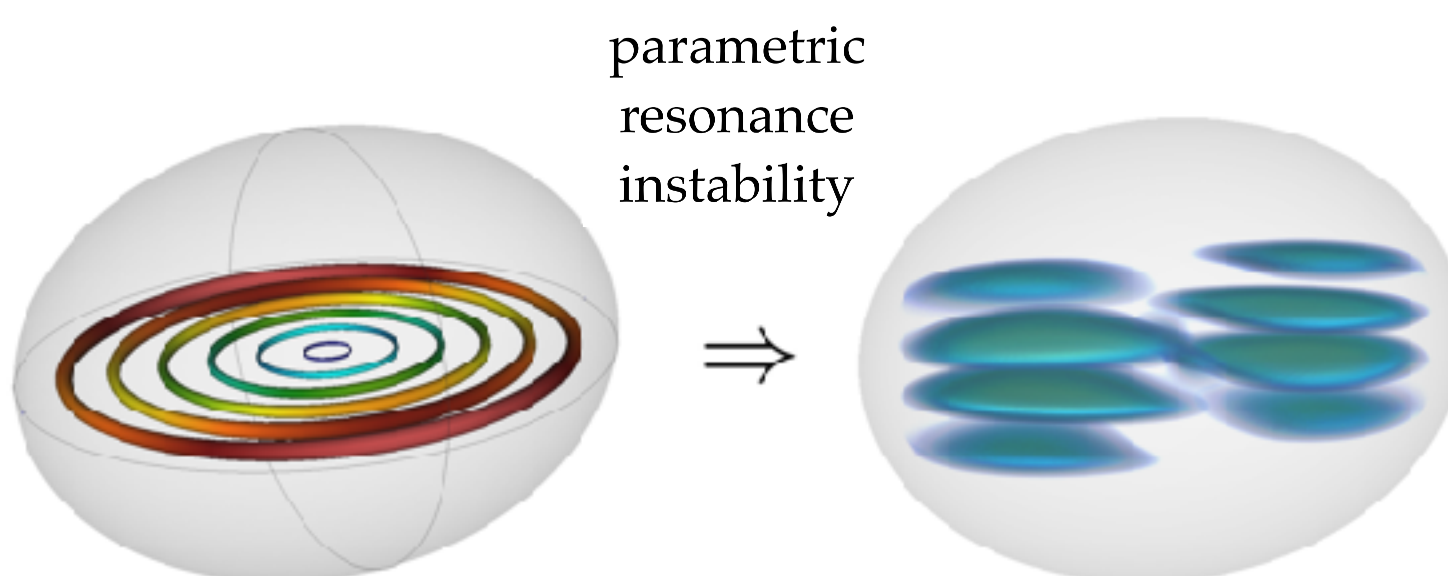
periodic perturbation
of the shape

Bulk injection by bulk instability

pioneer work by Malkus (1963, 1968, 1989)

Key points:

- small, but regular forcing
- natural vibrational states in any rotating fluid = the inertial modes
- fluid parametric resonance instability involving the base flow & 2 inertial waves



Base flow
tides, libration, precession

2 resonant inertial modes

Bulk injection by bulk instability

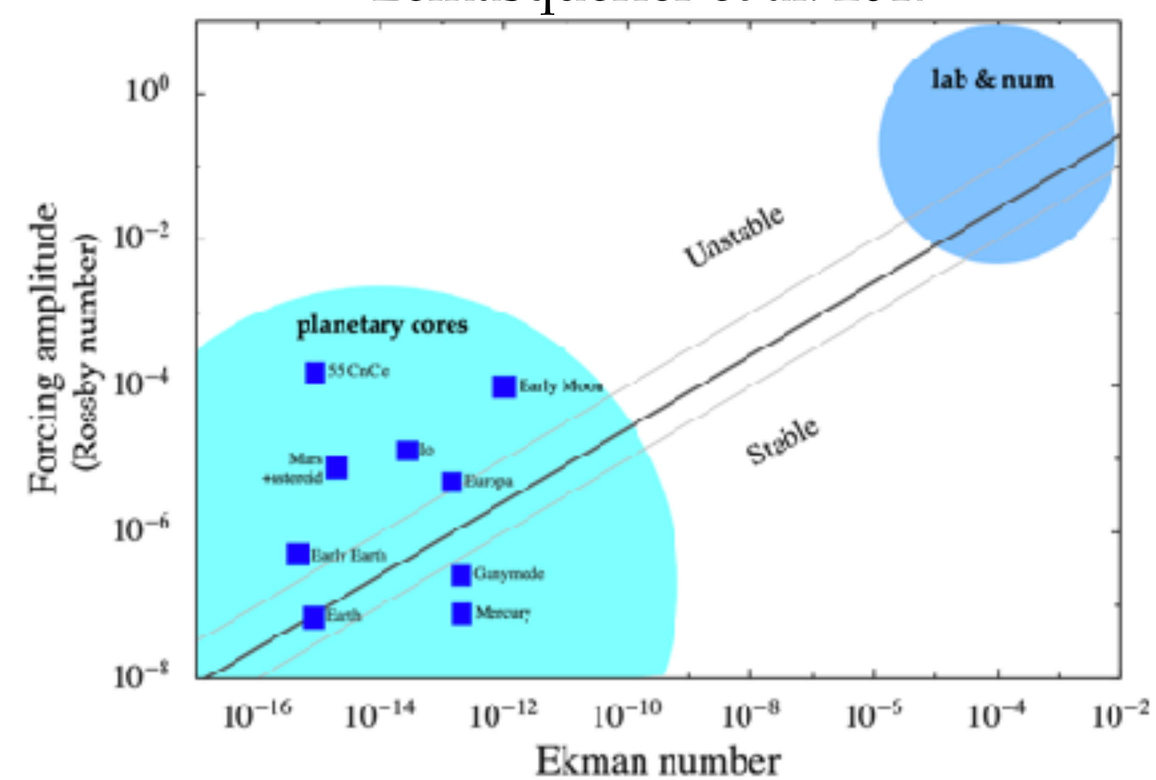
pioneer work by Malkus (1963, 1968, 1989)

Key points:

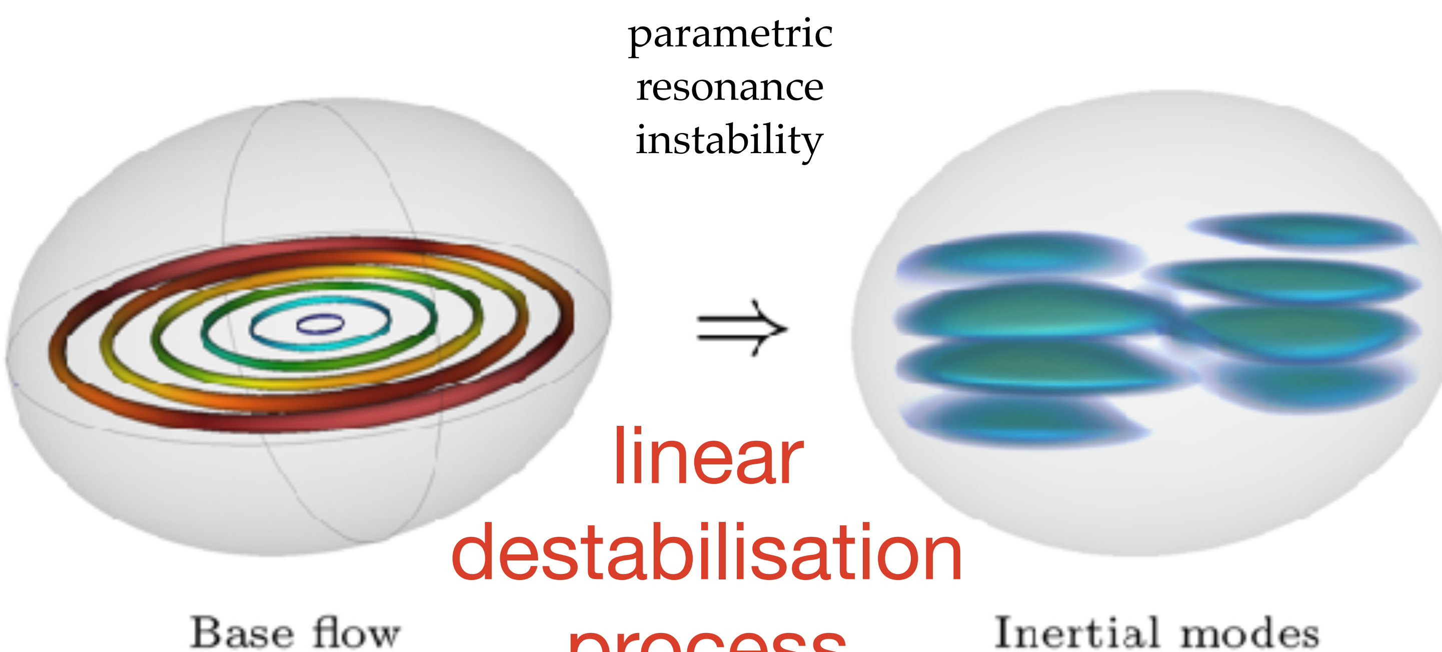
- small, but regular forcing
- natural vibrational states in any rotating fluid = the inertial modes
- fluid parametric resonance instability involving the base flow & 2 inertial waves

- the mechanisms and thresholds of instabilities are well known (e.g. Le Bars et al. ARFM 2015)
- extrapolation towards planets
- dynamo capable (e.g. Reddy et al. 2018; Cebron et al. 2019)

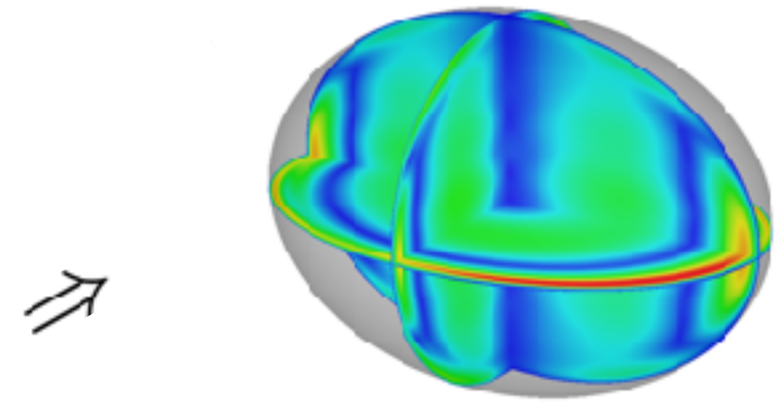
Lemasquerier et al. 2017



the nonlinear fate of the resonance instability

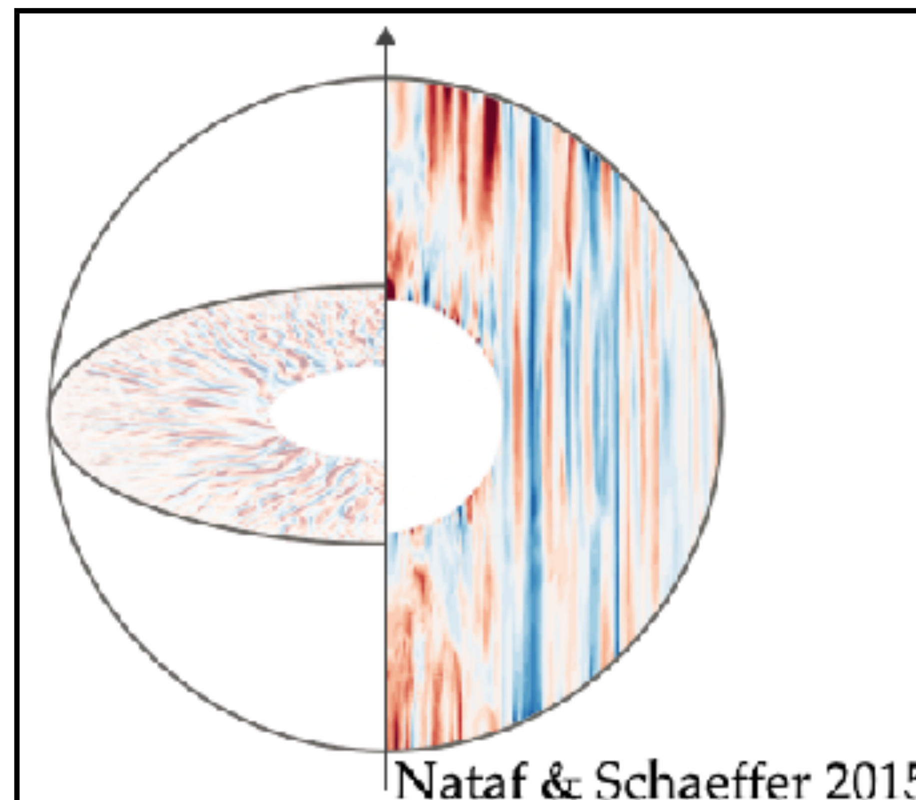


linear destabilisation process



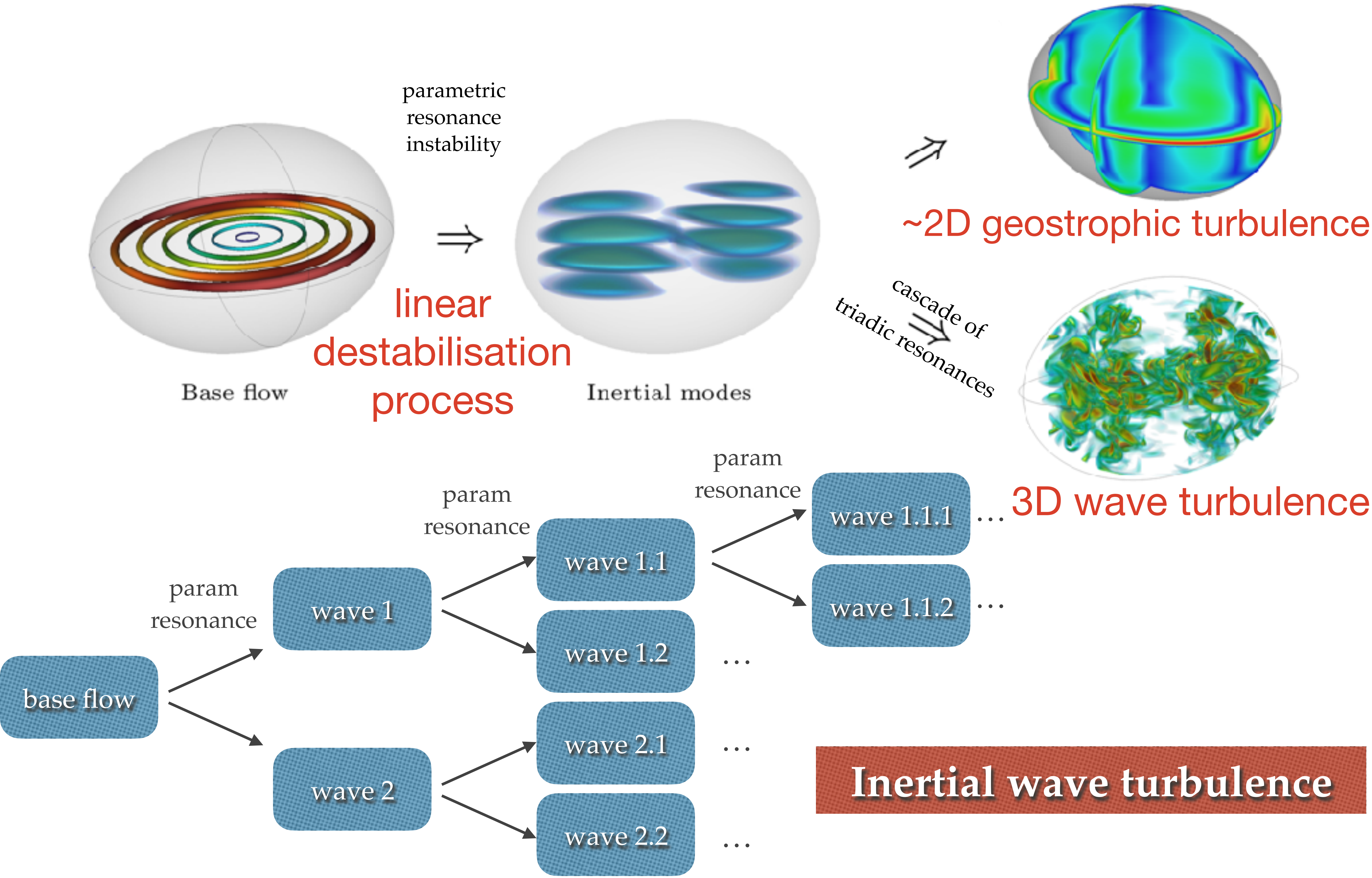
~2D geostrophic turbulence

- paradigm of turbulence in rotation
- « convective-like » dynamics
- case studied in dynamo DNS

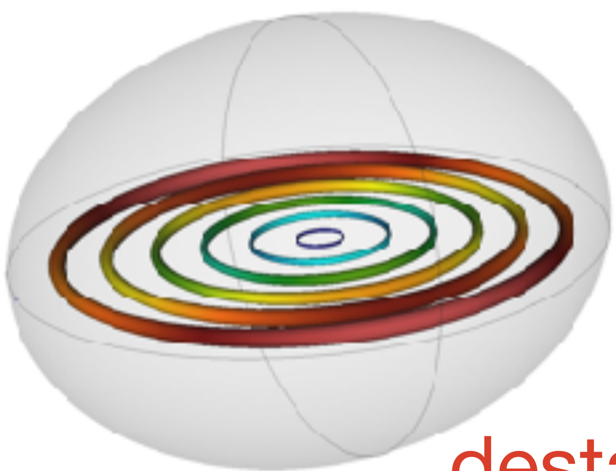


Cylindrical radial velocity component
DNS at E = 10^{-7}, Pr = 1, Ra = 2.4×10^{13}

the nonlinear fate of the resonance instability



the nonlinear fate of the resonance instability

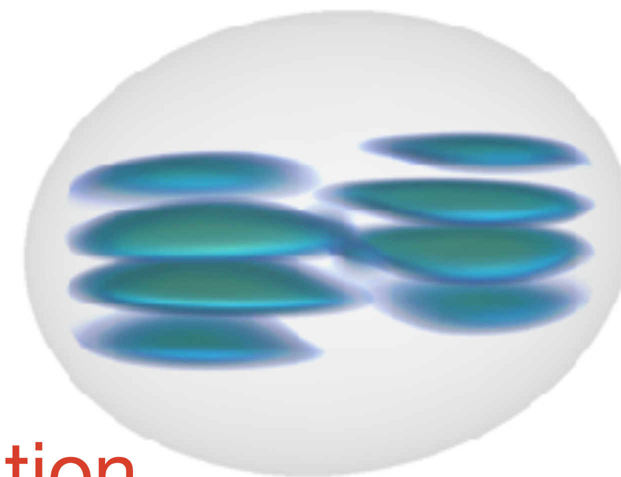


Base flow

parametric
resonance
instability

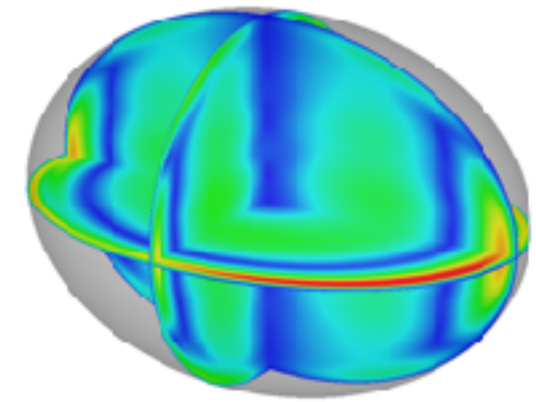
⇒

linear
destabilisation
process



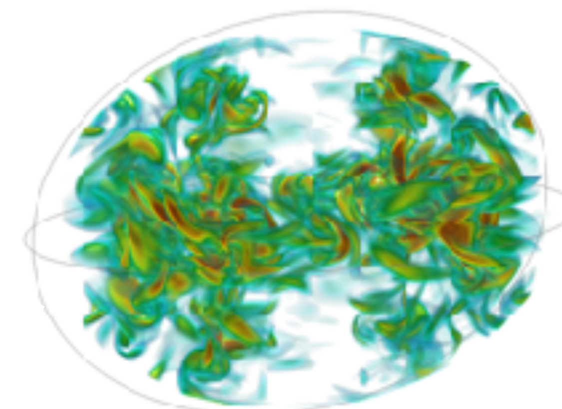
Inertial modes

⇗



~2D geostrophic turbulence

cascade of
triadic resonances
⇘



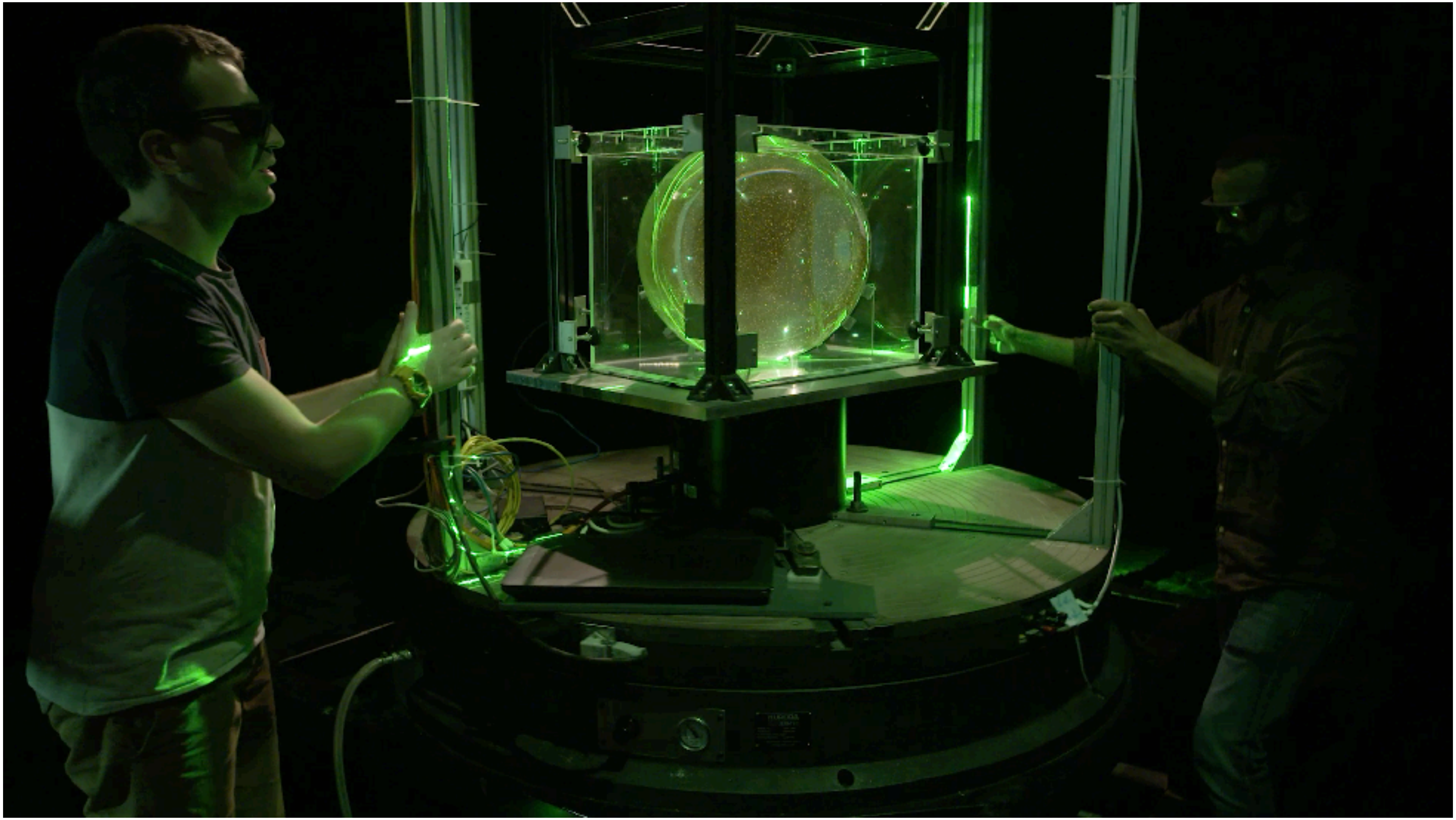
3D wave turbulence

compensate large E
by strong forcing
amplitude (Ro)…
OK for linear stability

not for NL effects!

**need to study small forcing &
small dissipation regimes…**

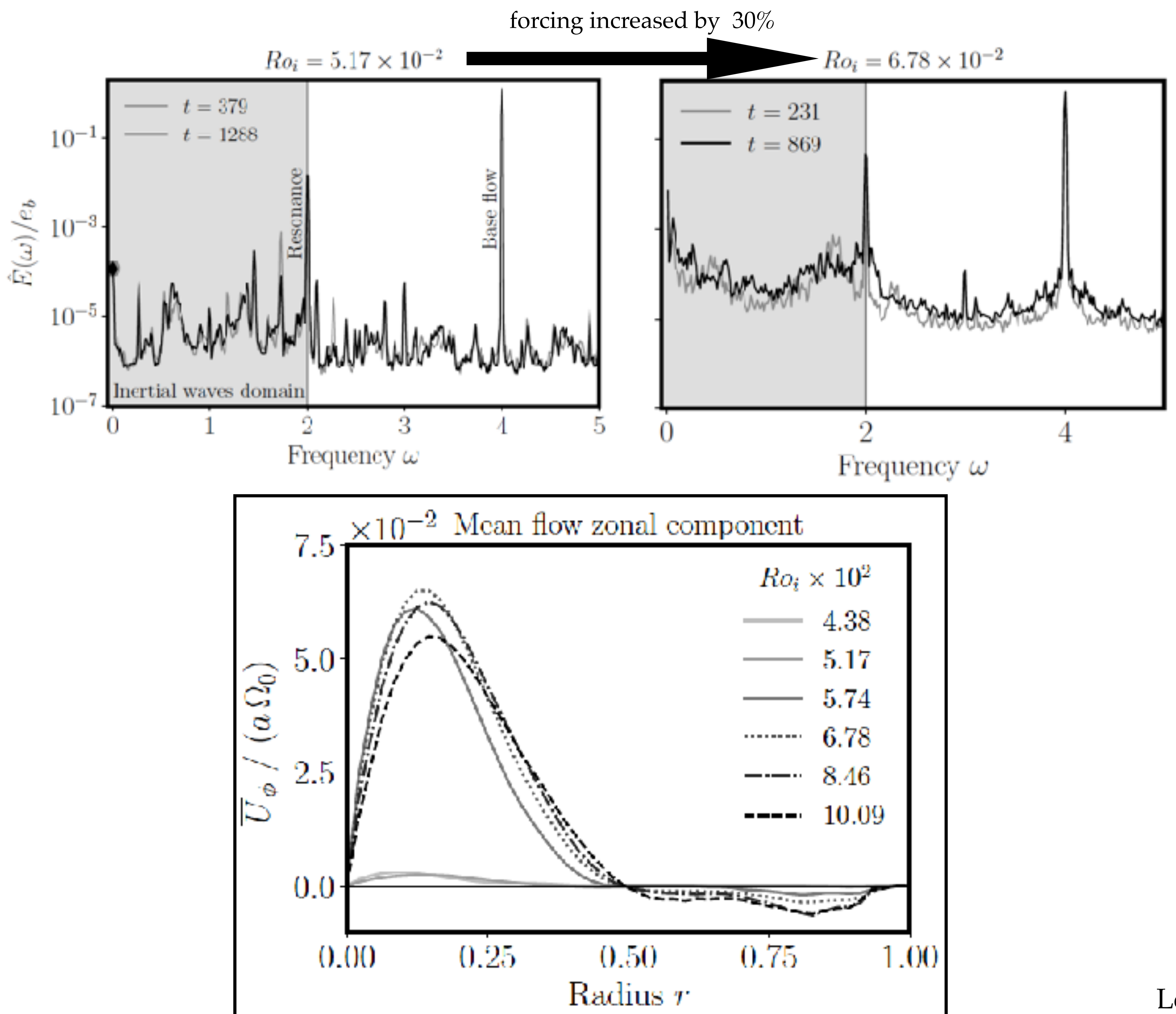
QG/IWT competition confirmed by lab experiments…



size ~50cm, rotation up to 90rpm -> $E=5\times10^{-6}$

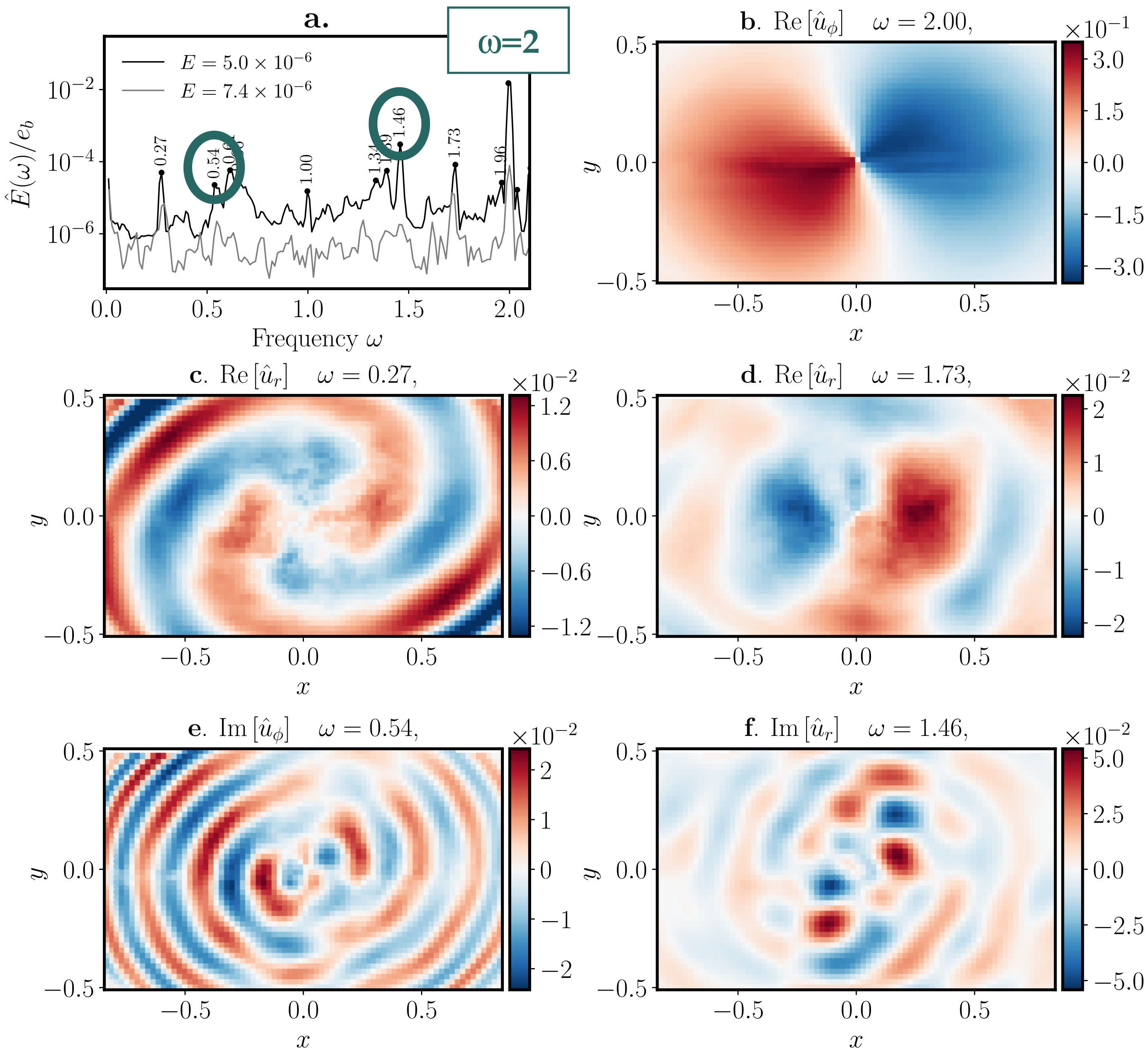
Le Reun et al. 2019

QG/IWT competition confirmed by lab experiments…



Le Reun et al. 2019

Discrete wave turbulence



libration forcing at ω_f=4

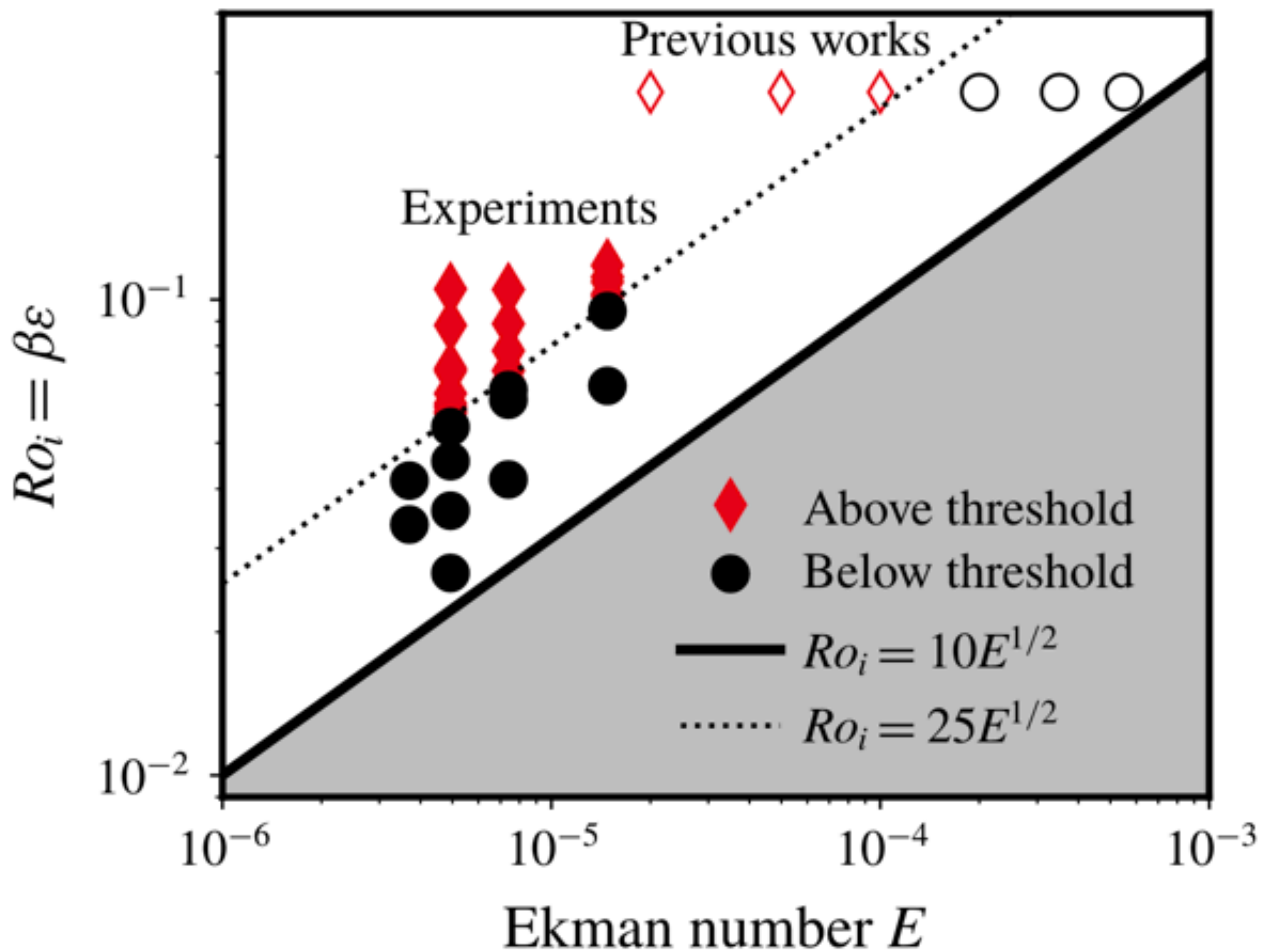
triadic resonance with the 1st resonant waves $m_0=\pm1$ & $\omega_0=2$

$m_1=2$ and $m_2=1$

$m_1=2$ and $m_2=3$

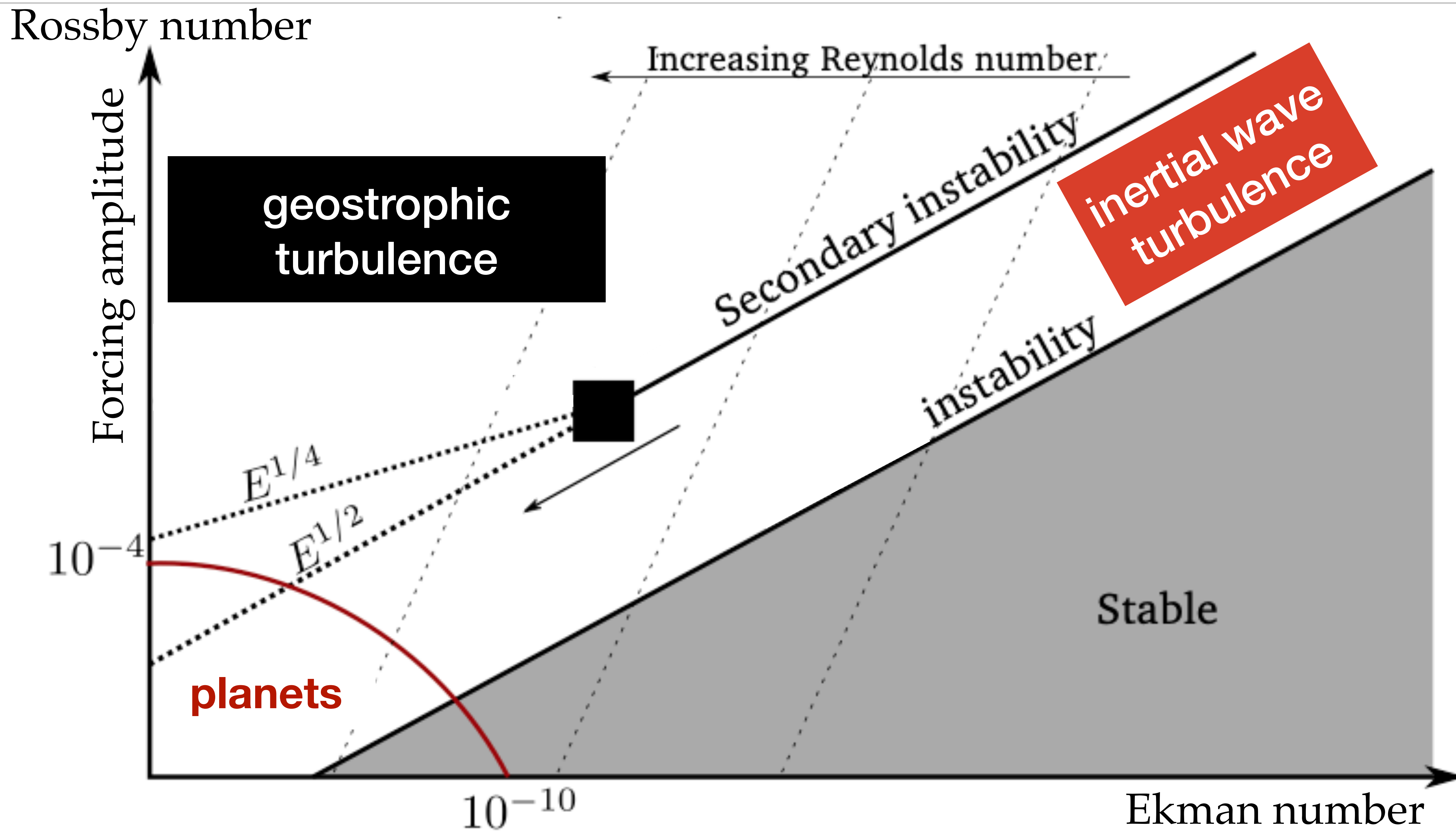
Le Reun et al. 2019

Discrete wave turbulence



Le Reun et al. 2019

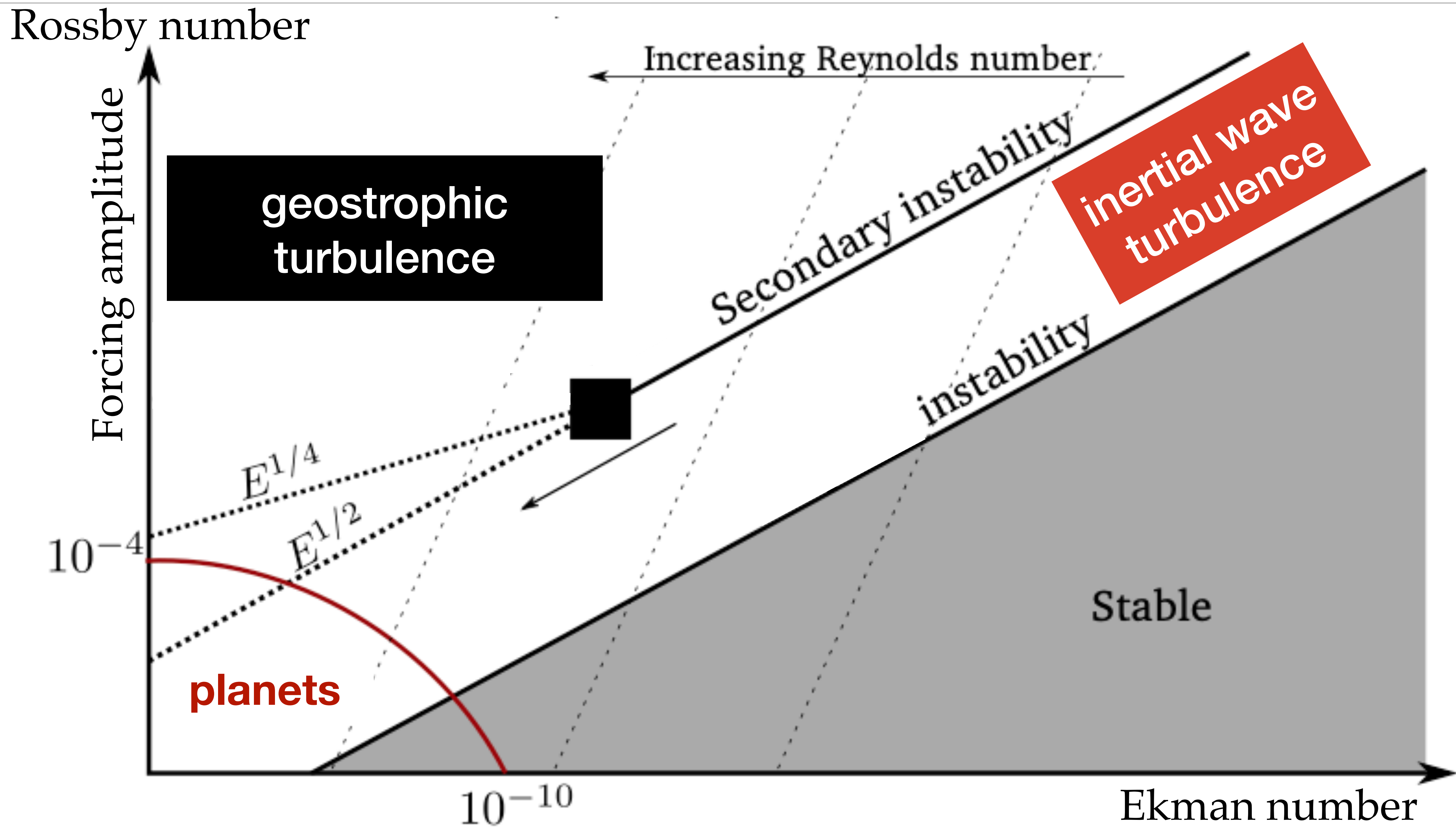
extrapolation towards planetary cores



Mechanism for geostropic mode excitation still under discussion:

- Le Reun et al. 2020: near-resonant triad involving a geostrophic mode -> threshold $\propto E^{1/4}$ transiting towards $\propto E^{1/2}$ at moderate forcing (OK with our experiments)
- Brunet et al. 2020: resonant quartet, including a geostrophic mode -> threshold $\propto E^{1/4}$

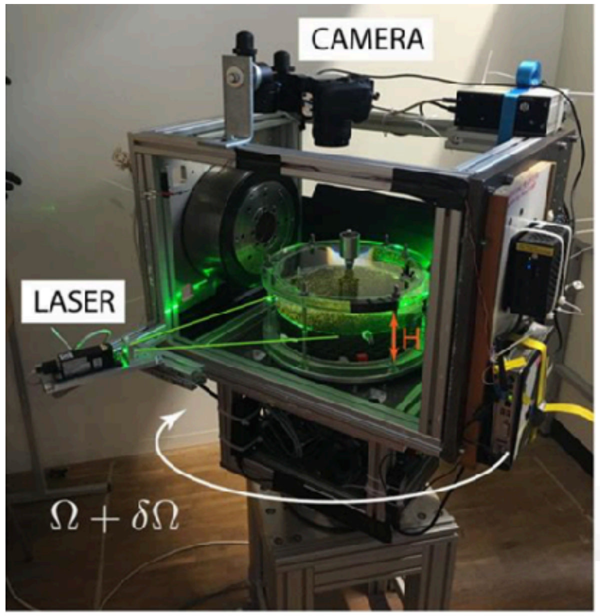
extrapolation towards planetary cores

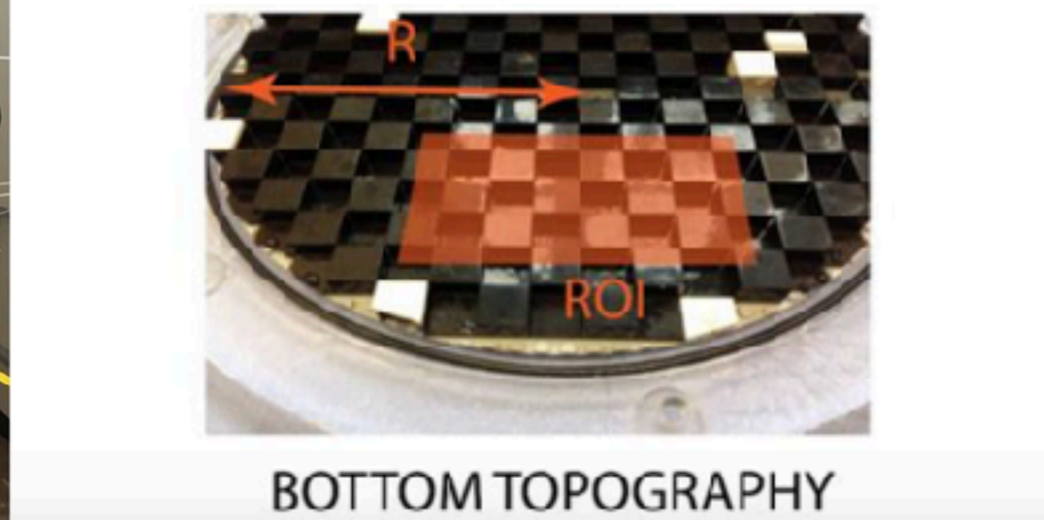


in any case, wave turbulence may dominate in planetary limit

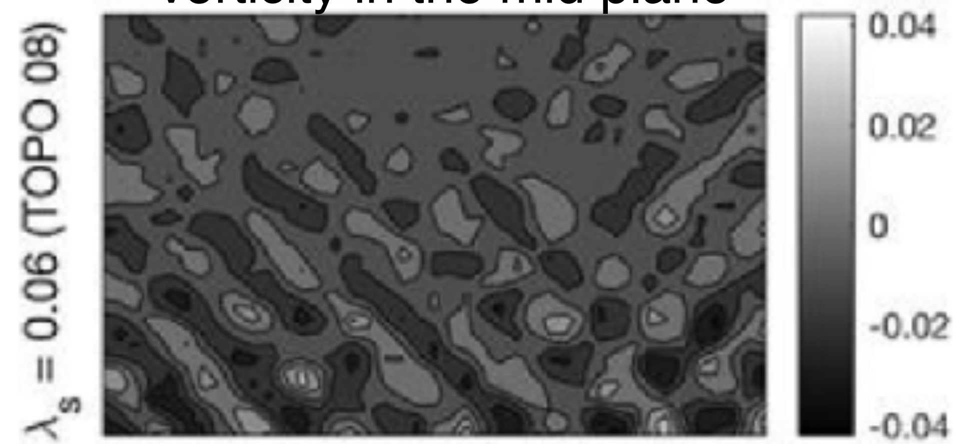
Other possible sources of inertial wave turbulence

- flow over topography





vorticity in the mid plane

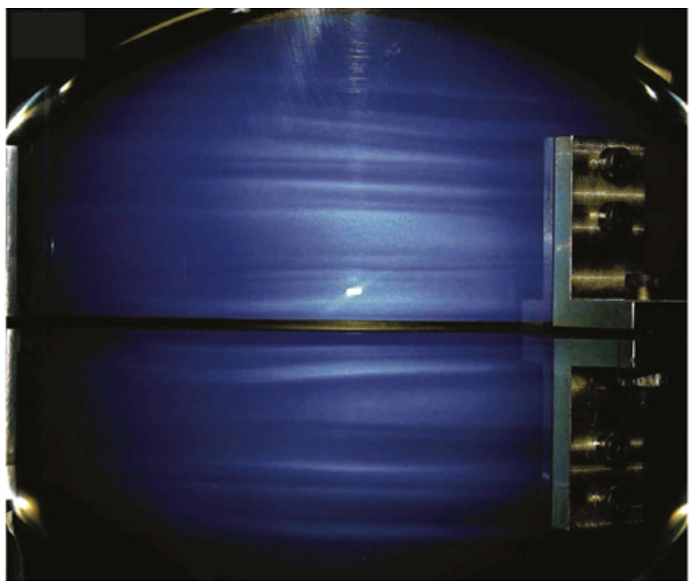


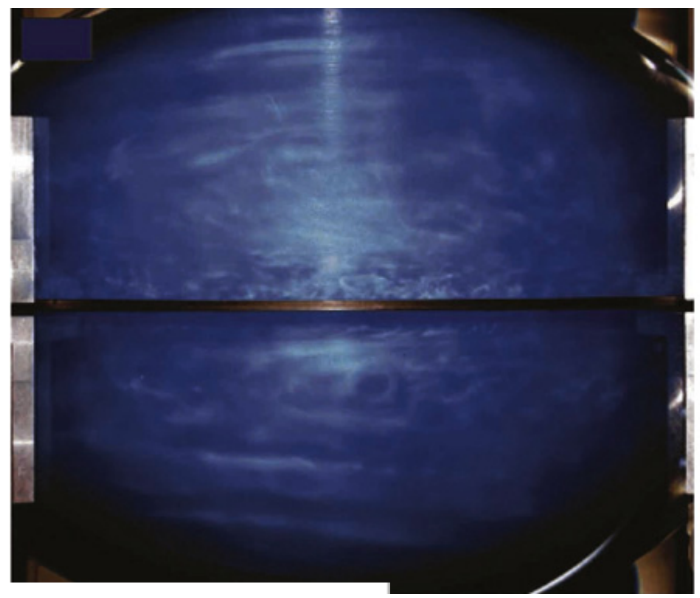
Burmann & Noir 2018

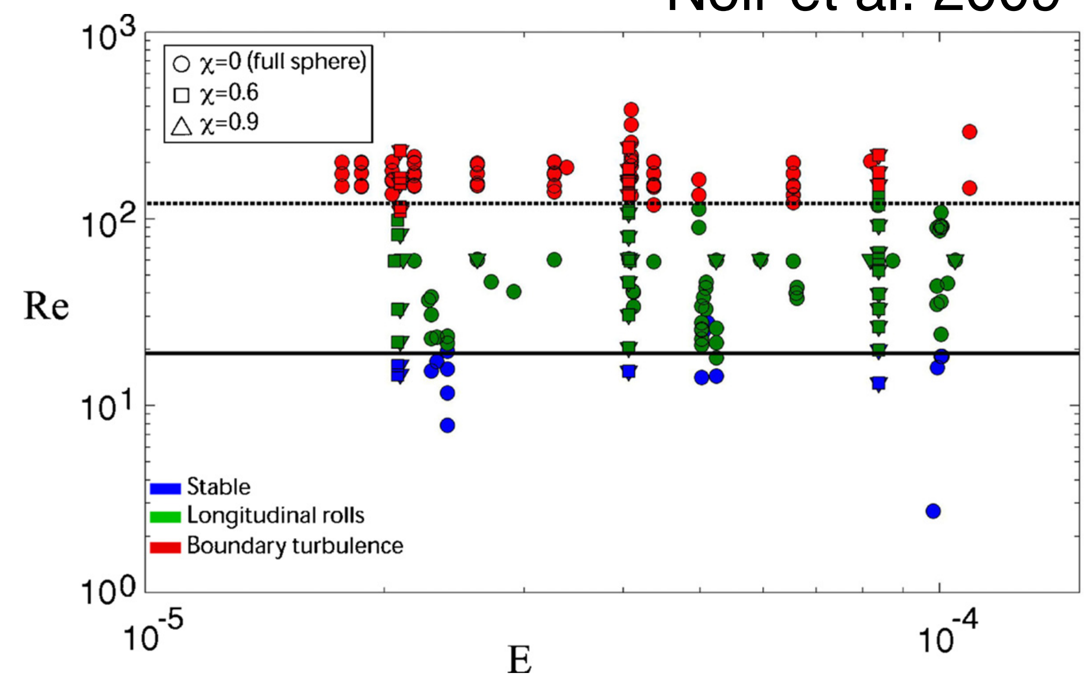
Other possible sources of inertial wave turbulence

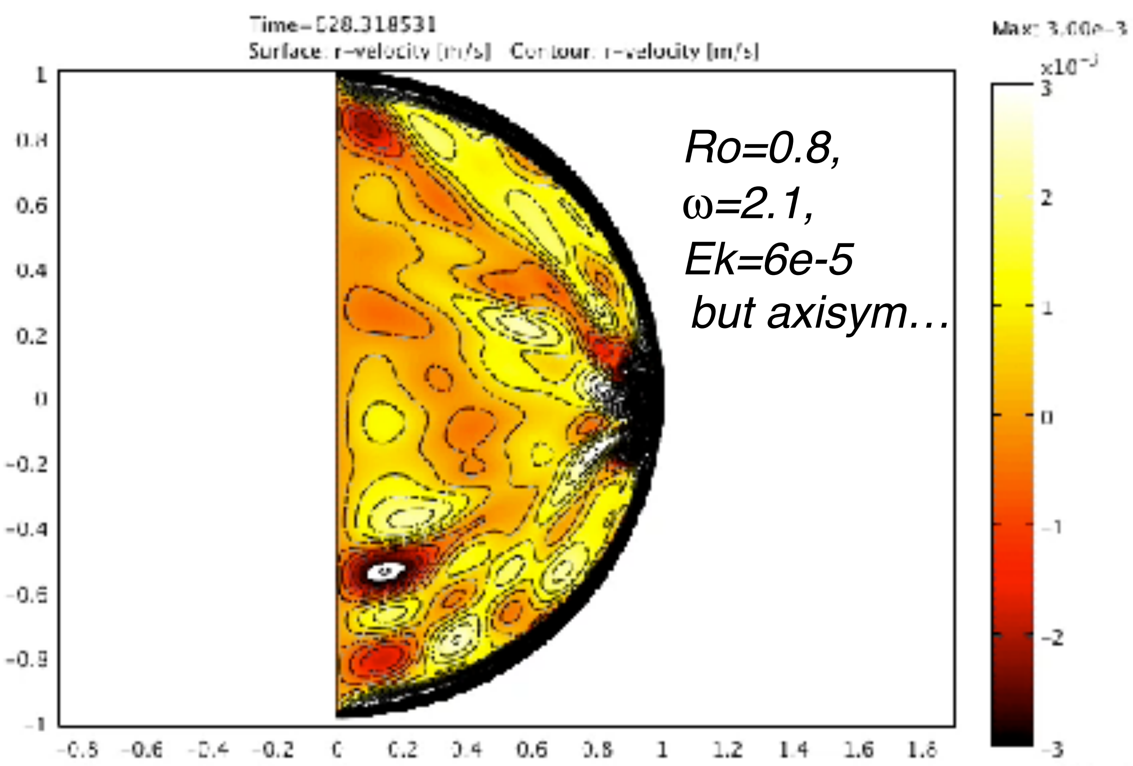
- flow over topography
- boundary turbulence

Noir et al. 2009

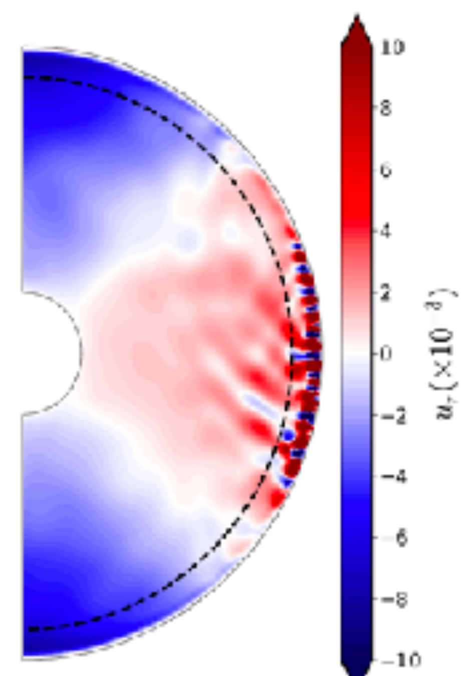


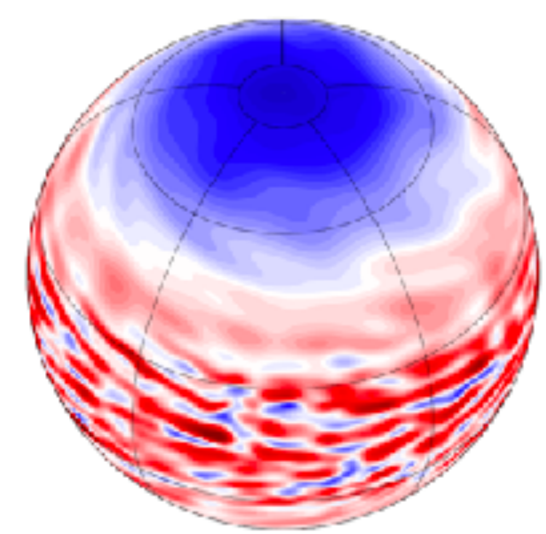






Sauret et al. 2013

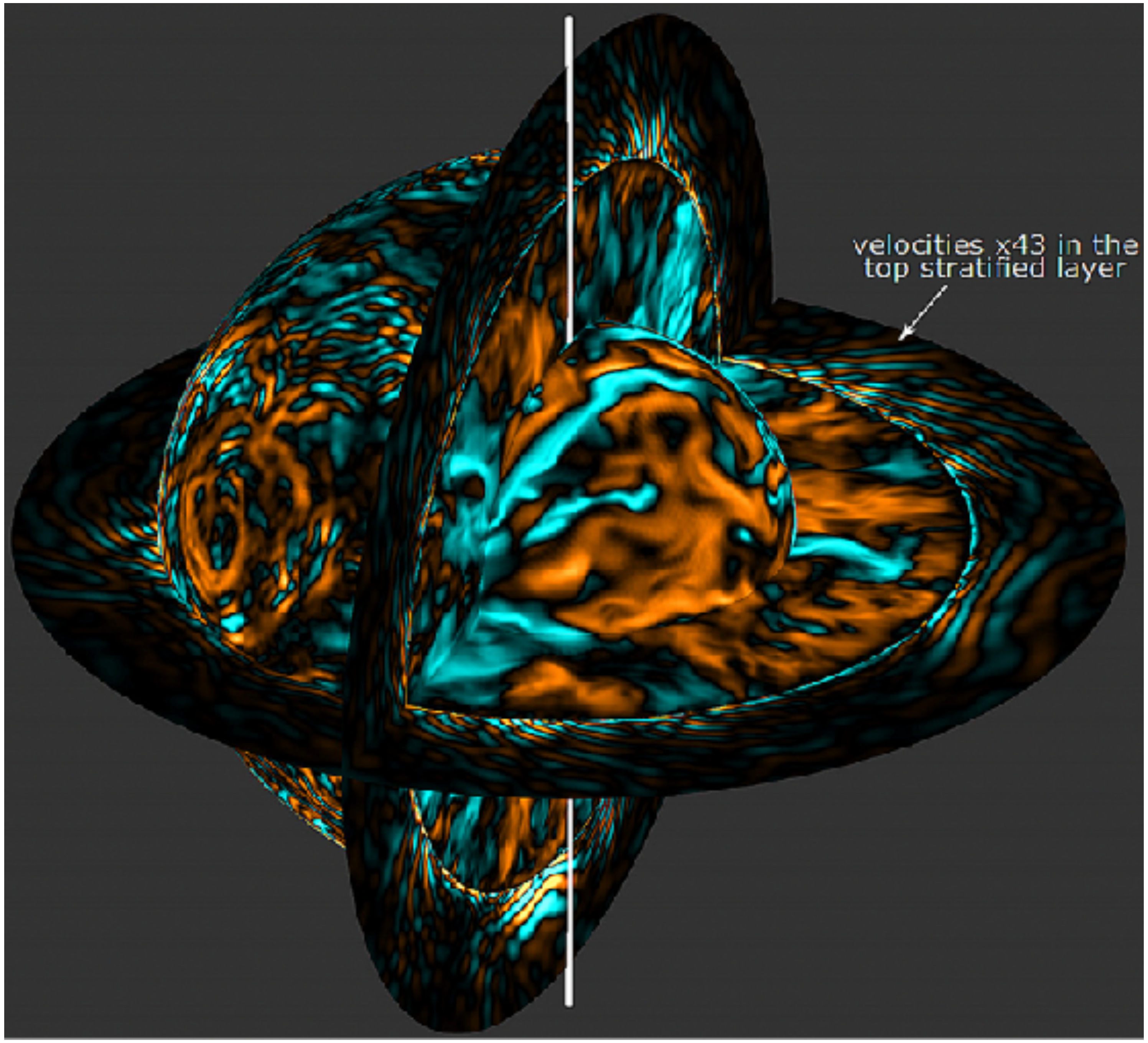




ongoing work with Ankit Barik (Johns Hopkins)

Other possible sources of inertial wave turbulence

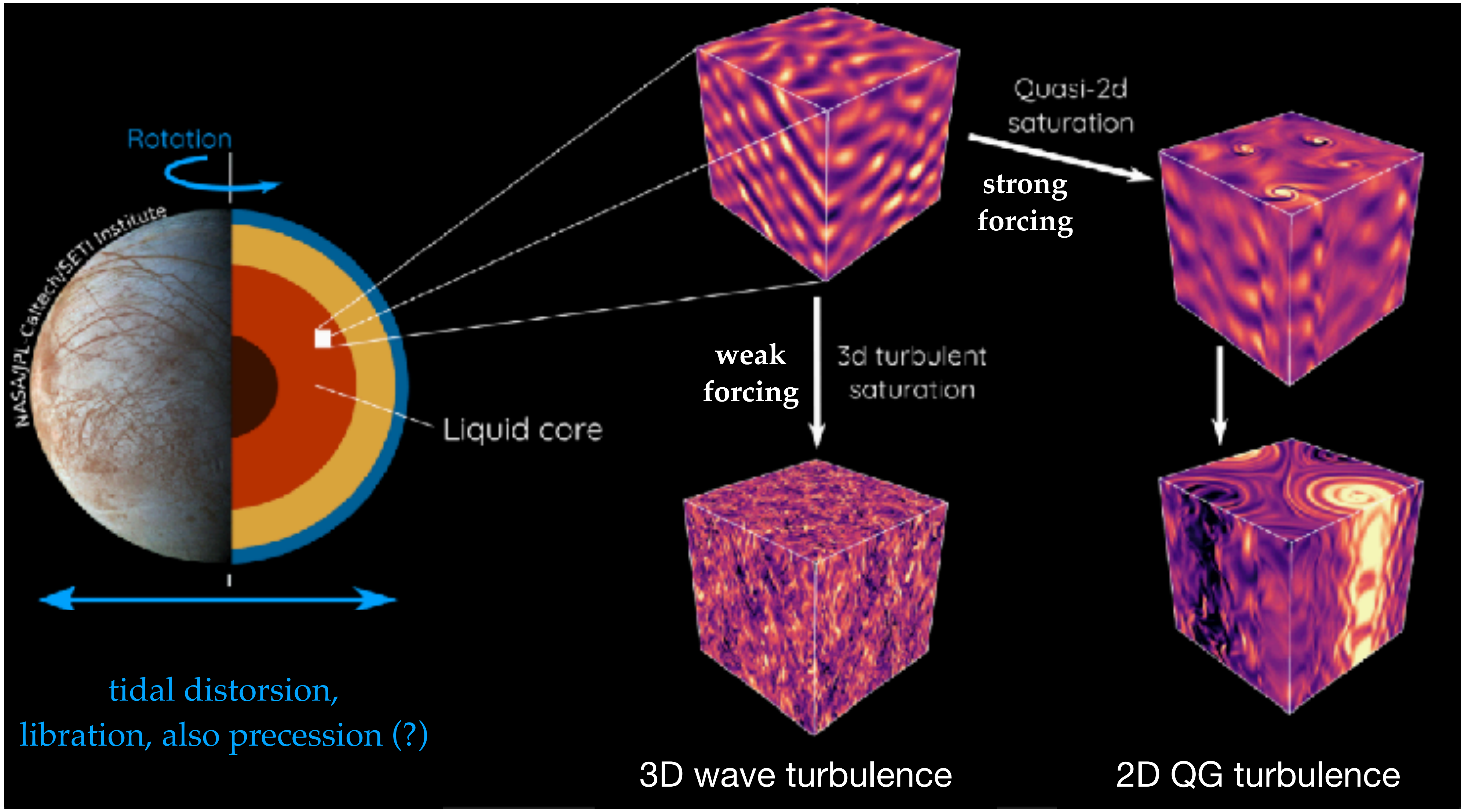
- flow over topography
- boundary turbulence
- emission from an adjacent convective layer



Bouffard et al. 2022

Conclusion & future works

- two possible turbulence regimes

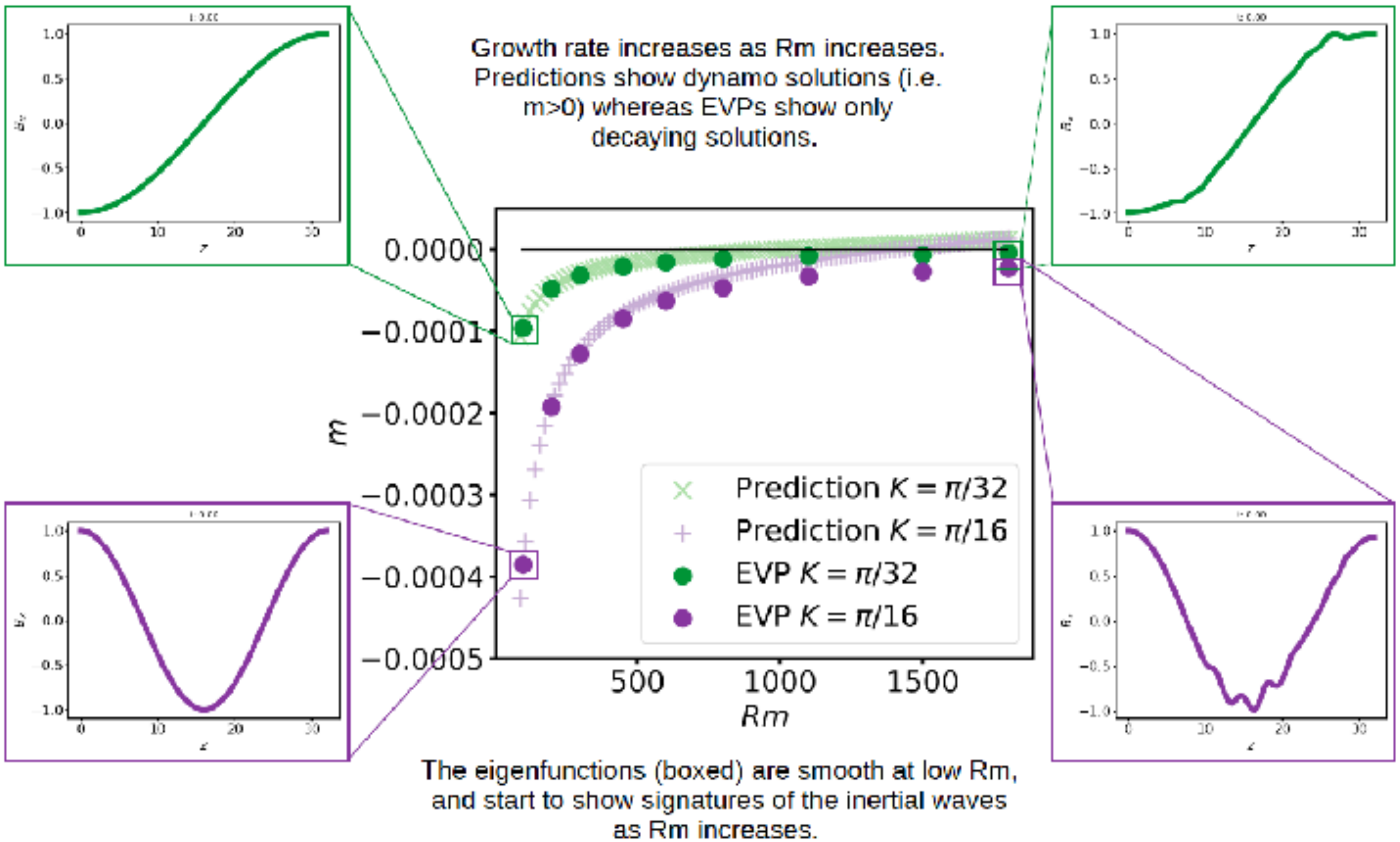


✓wave turbulence => dynamo possible according to Moffatt (1970)… mean field alpha approach assuming a packet of wave with helicity symmetry breaking and space decoupling, but validation with instability & shape, intensity, etc.?

✓QG turbulence => « convective like » dynamo (see e.g. Reddy et al. 2018)

Conclusion & future works

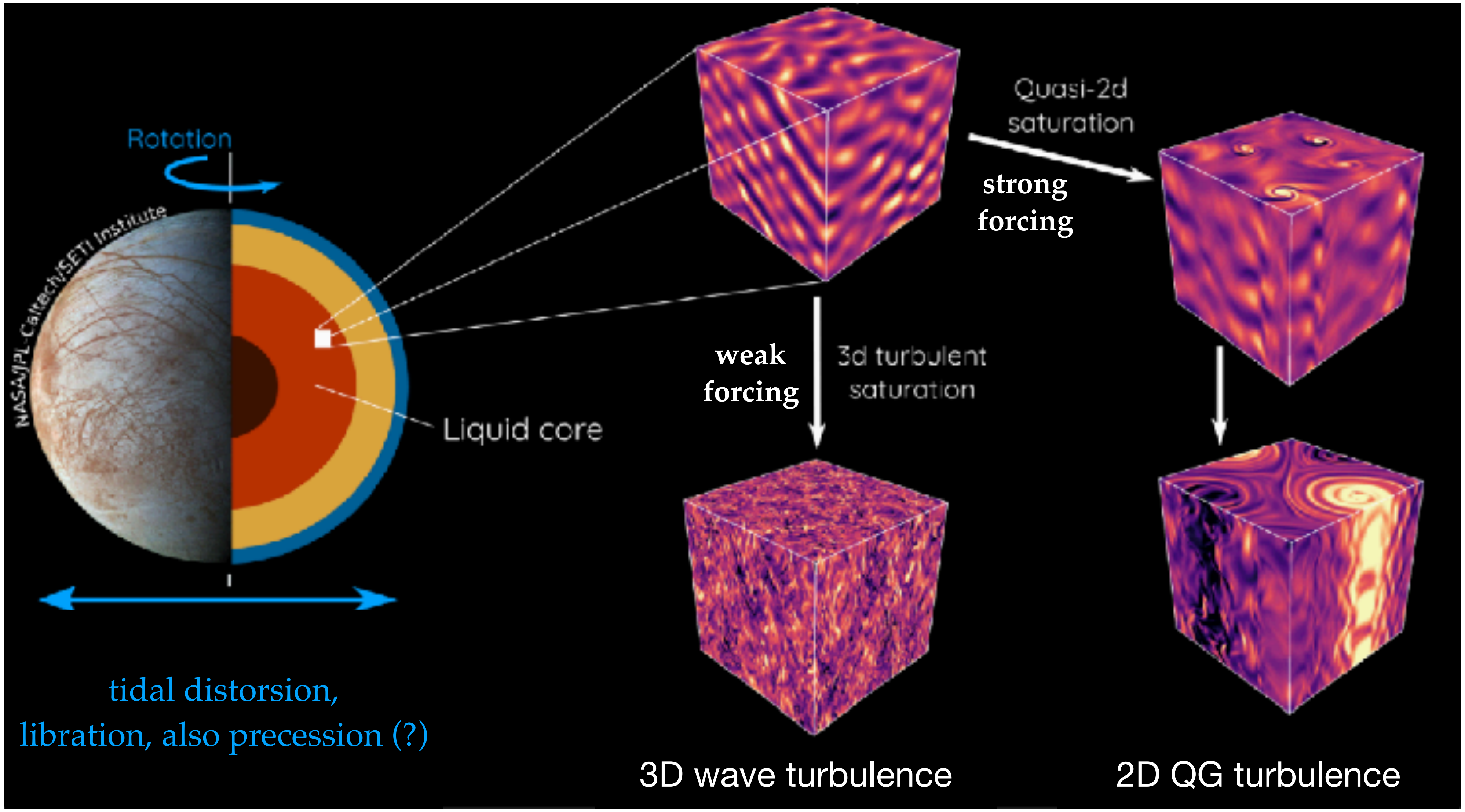
- Moffatt's wave turbulence dynamo? ongoing work with Emma Kauffman and Daniel Lecoanet (Northwestern)



2 inertial waves with equal frequencies and wavenumber magnitude but differing kx and ky, Floquet theory -> 2D eigenvalue problem in which the magnetic fields scale like some periodic function times an exponential of the growth rate times t

Conclusion & future works

- two possible turbulence regimes



- Moffatt's wave turbulence dynamo in planetary cores

- dissipation, heat and chemical transport, magnetic induction in subsurface oceans?