IPAM Workshop on Rotating Convection

The origin and dynamics of Jupiter’s polar vortex crystals

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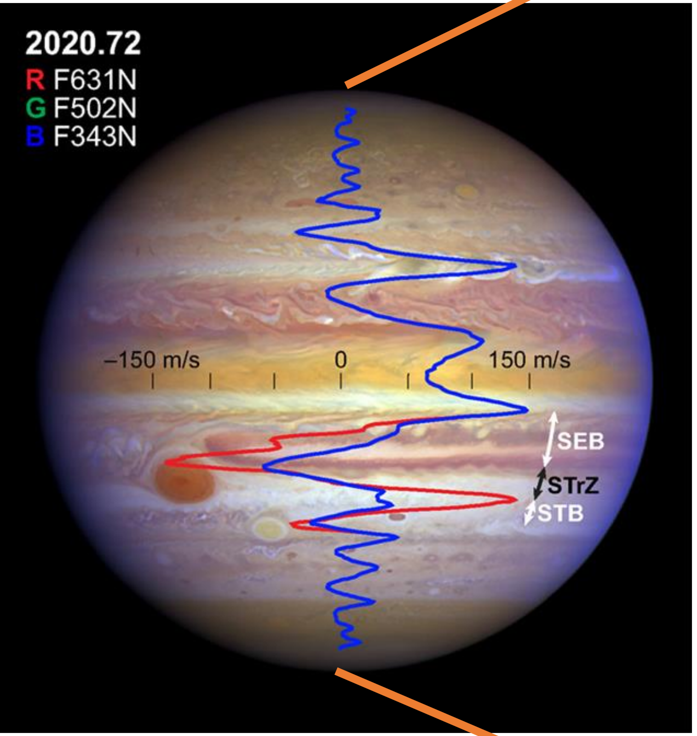
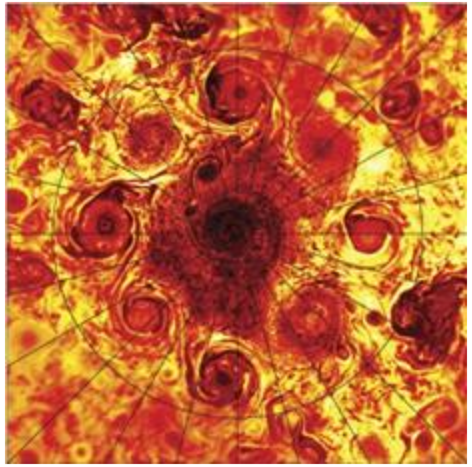
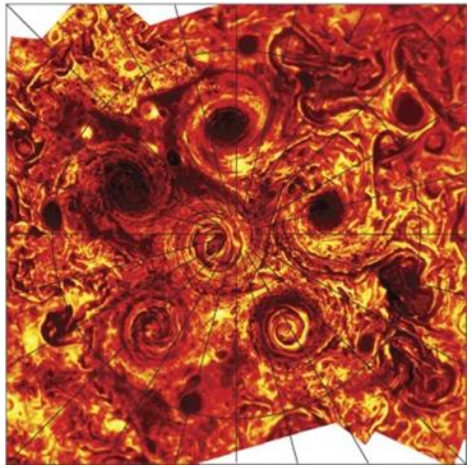






Vortex crystals

- Stable and long lived
- Size: 5000-7000 km
- Spin period: 27.5-60 hr
- Oscillation period: 4-12 mth
- Drift rate: -7.5°/yr (N) and -3°/yr (S)

$$\left\{ \begin{array}{l} \frac{T_{Spin}}{T_{Jup}} \sim O(1-10) \\[2ex] \frac{T_{Oscil}}{T_{Jup}} \sim O(10^2-10^3) \\[2ex] \frac{T_{Drift}}{T_{Jup}} \sim O(10^4 \sim 10^5) \end{array} \right.$$

Adriani et al. ; Mura et al.; Tabataba-Vakili et al.

Questions

- The origin of vortex crystals

  Formation? Depth? Sustainability?

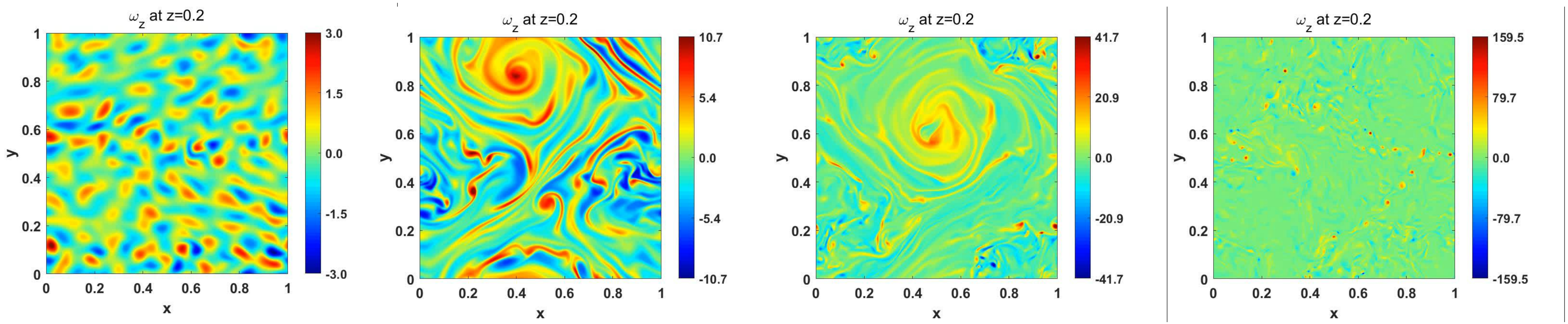
- The dynamics of vortex crystals

  Stability? Drift? Oscillation?

Rotating Rayleigh-Bénard Convection （RRBC）

Cai 2021 ApJ

Rotation Rate



Regime I:
Multiple small-scale vortices

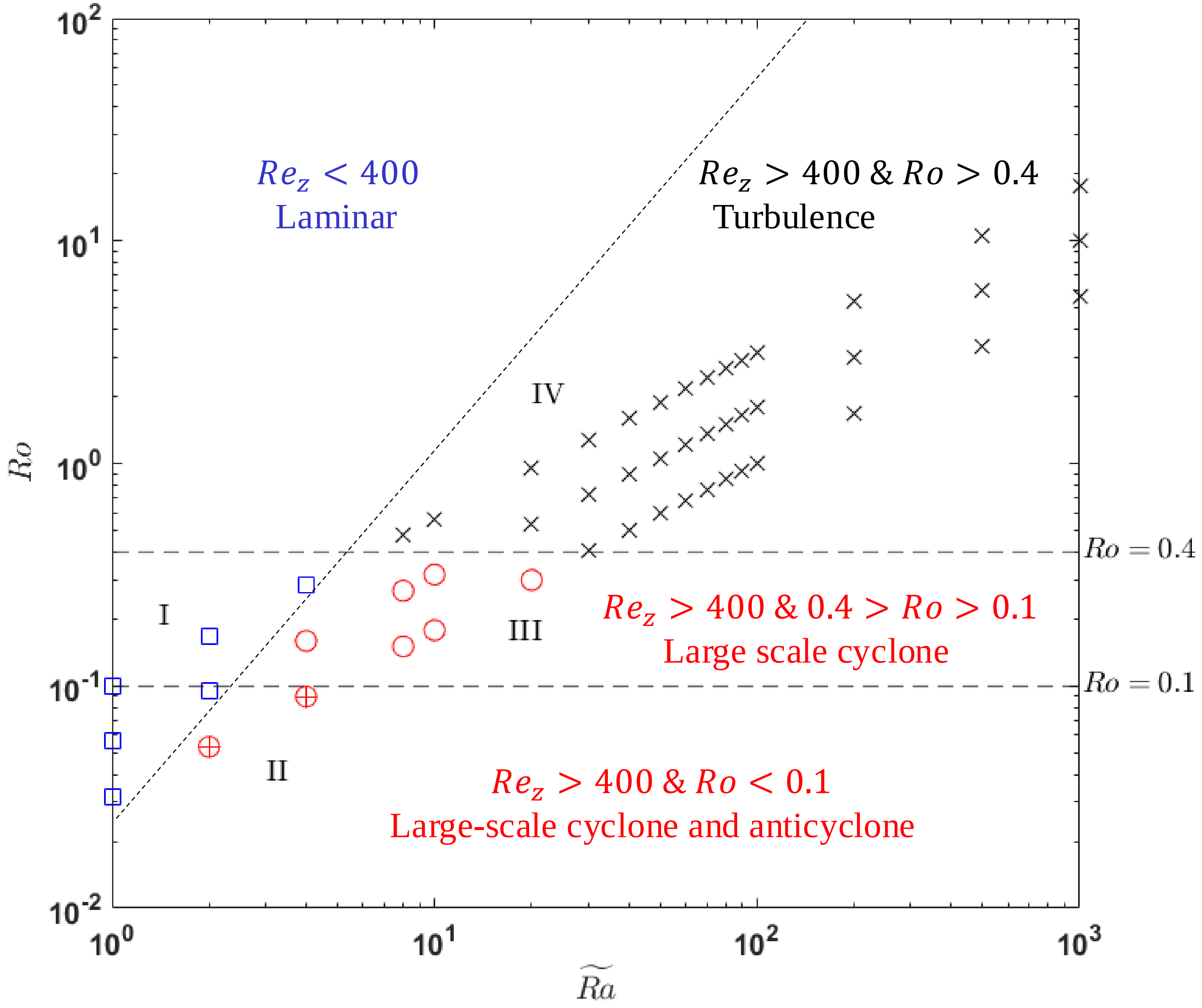
Regime II:
Coexisting large-scale cyclone and anticyclone

Regime III:
Large-scale cyclone

Regime IV:
Geostrophic turbulence

Fast rotation

Turbulent flow



$Ra = 10^{6-8}$ & $Pr = 0.1$ & $\Gamma = 1$

Criterion on generating LSVs:

1. Ro<0.4 (fast rotation)
2. Re_z>400 (strong turbulence)

Cai 2021 ApJ

Compressible flow

$$\partial_t \rho = -\boldsymbol{\nabla} \cdot (\rho \boldsymbol{u})$$

$$\partial_t (\rho \boldsymbol{u}) = -\boldsymbol{\nabla} \cdot (\rho \boldsymbol{u}\boldsymbol{u}) + \boldsymbol{\nabla} \cdot \boldsymbol{\Sigma} - \boldsymbol{\nabla} p + \rho \boldsymbol{g} - 2\rho \boldsymbol{\Omega} \times \boldsymbol{u}$$

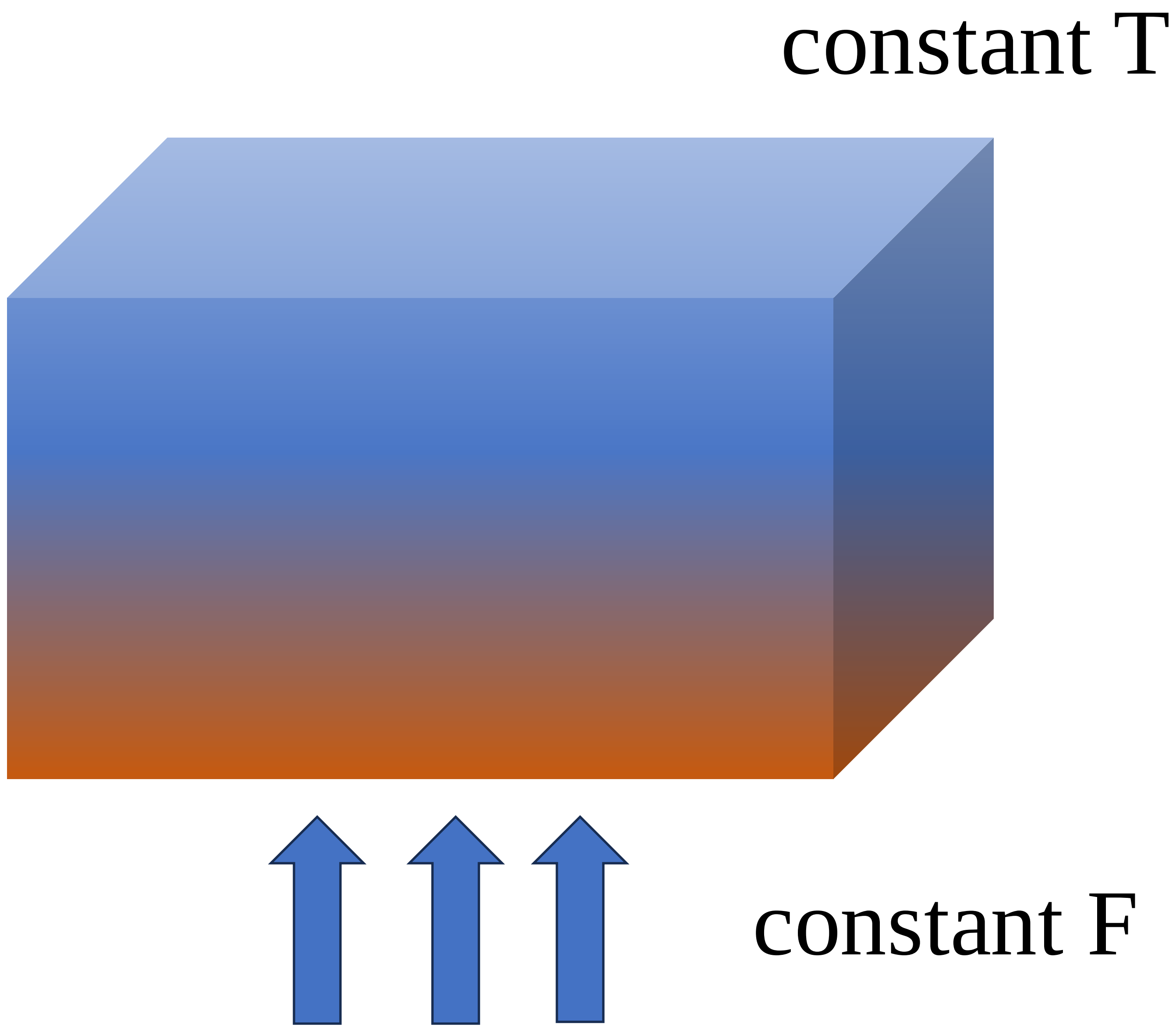
$$\partial_t E = -\boldsymbol{\nabla} \cdot [(E + p)\boldsymbol{u} - \boldsymbol{u} \cdot \boldsymbol{\Sigma} + \boldsymbol{F}] + \rho \boldsymbol{u} \cdot \boldsymbol{g}$$

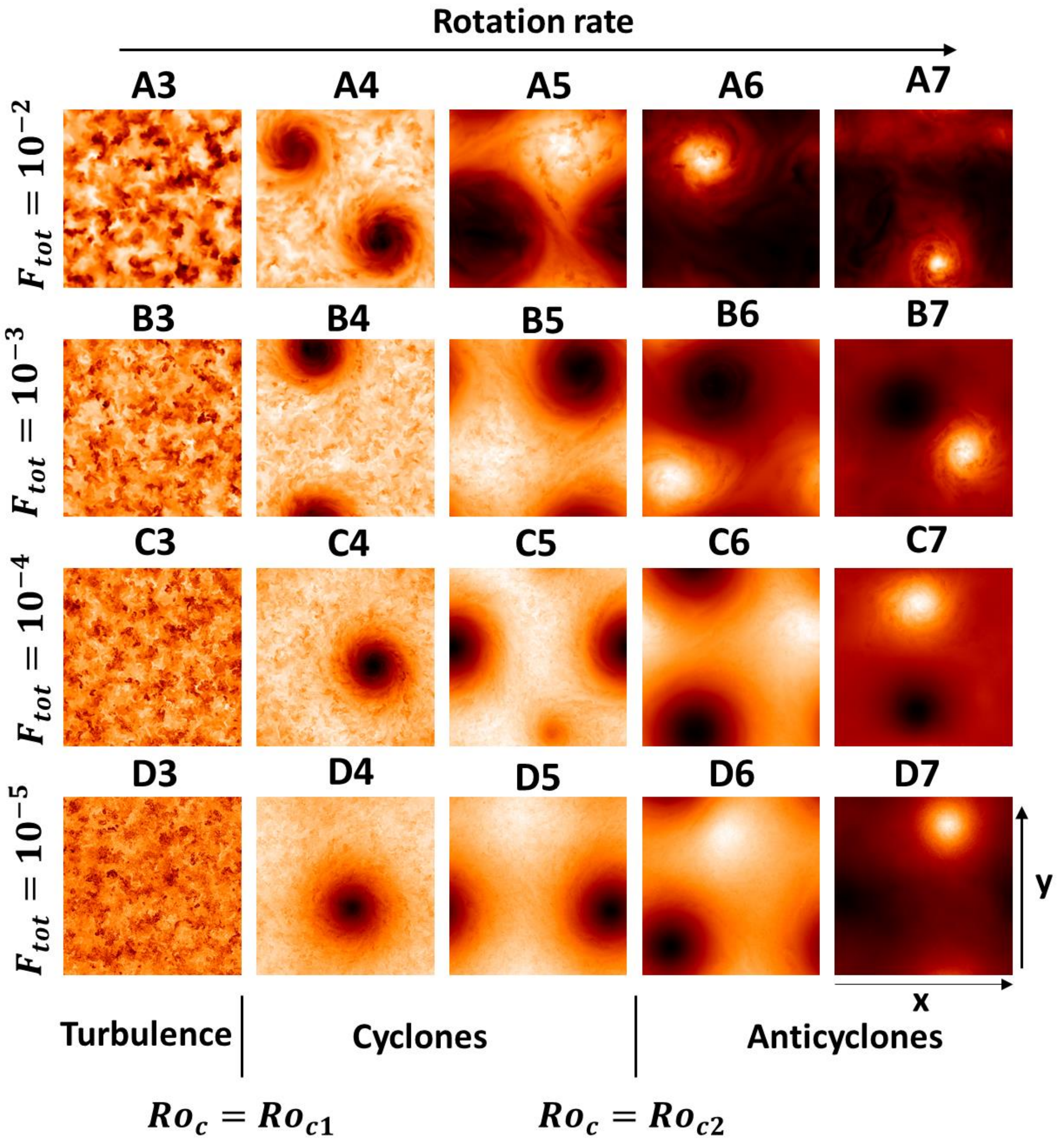
$$Ro \sim \frac{\rho U U / H}{\rho \Omega U} = \frac{U}{2\Omega H} = \frac{1/\Omega}{2H/U} = \frac{\tau_\Omega}{\tau_c}$$

$$F \sim p U \sim \rho U^3 \Rightarrow U \sim \left(\frac{F}{\rho}\right)^{1/3}$$

Mixing length theory

Cai 2016 JCP

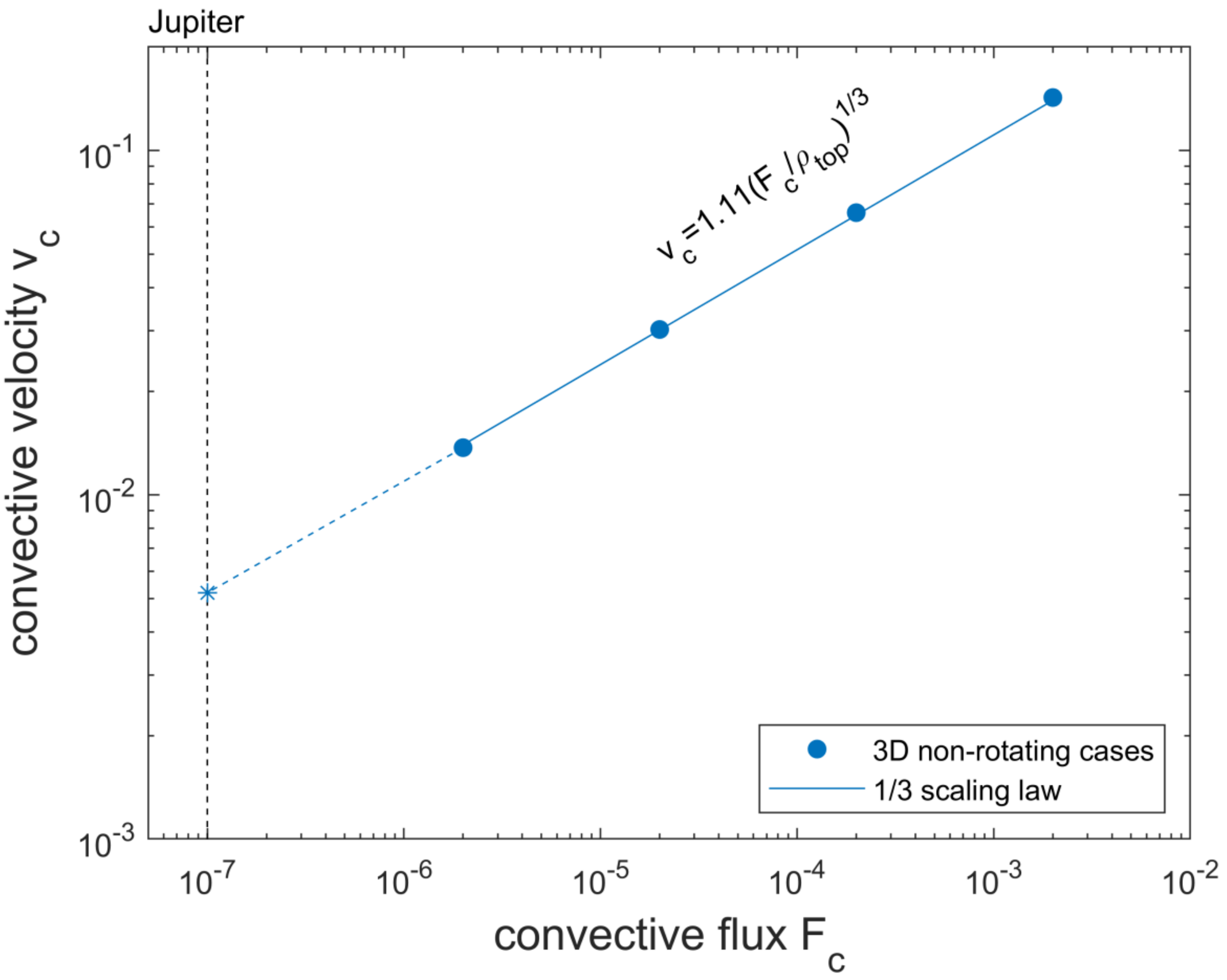

constant T
constant F



$Ro < Ro_{c1} = 0.11$ cyclone

$Ro < Ro_{c2} = 0.023$ anticyclone

Cai, Chan & Chow 2022 ApJ



Mixing length theory

$$F \sim \rho v^3$$

Inference of the depth from the simulations

Critical Rossby numbers:

$Ro < Ro_{c1} = 0.11$ Cyclone

$Ro < Ro_{c2} = 0.023$ Anticyclone

$$U \sim 1.11 \left(\frac{F}{\rho}\right)^{1/3} = 1.11 \times \left(\frac{7.48 \mathrm{W} m^{-2}}{0.167 kg m^{-3}}\right)^{1/3} \approx 4 m s^{-1}$$

for non-rotating Jupiter

$$\Omega \sim \left(\frac{2\pi}{36000 s}\right) = 1.75 \times 10^{-4} \ s^{-1}$$

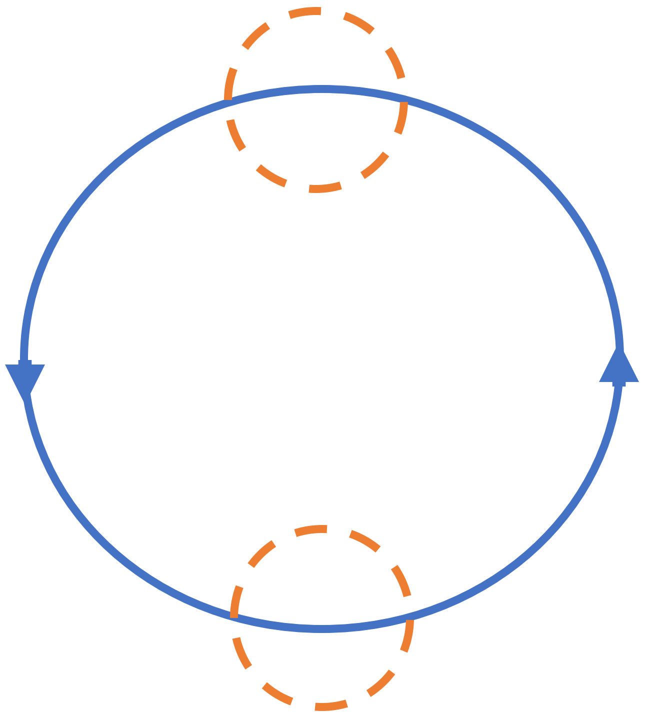
$$Ro = \frac{U}{2\Omega H} < Ro_{c2} \Longrightarrow \ H > \frac{U}{2\Omega Ro_{c2}} \approx 500 km$$

Jupiter: Ro~0.004 Saturn: Ro~0.0008 Sun: Ro~0.02-0.2

Cai, Chan & Chow 2022 ApJ

Beta effect

$$\frac{D}{Dt}\left(\frac{f+\xi}{h}\right)=0$$



Beta effect

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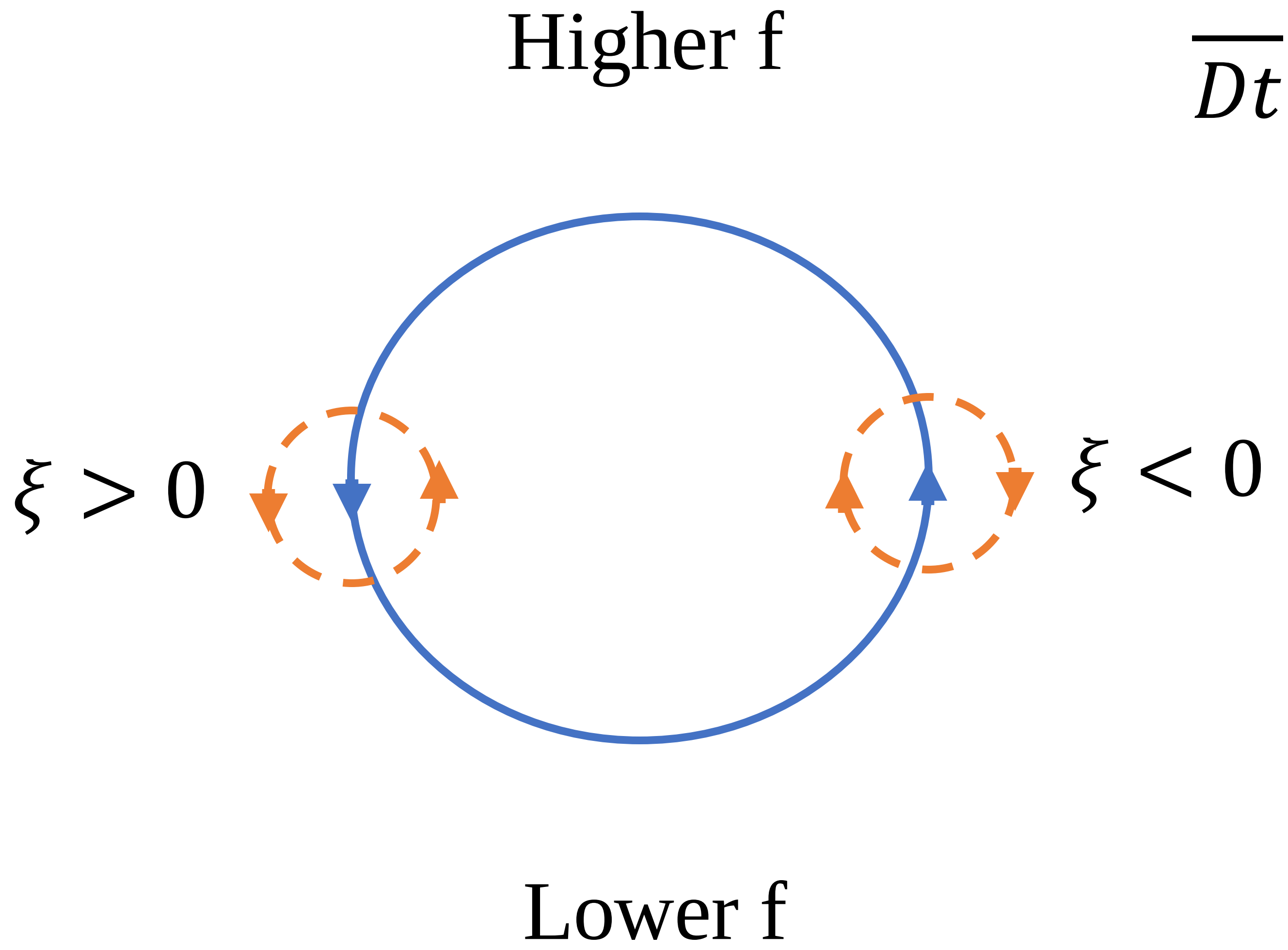
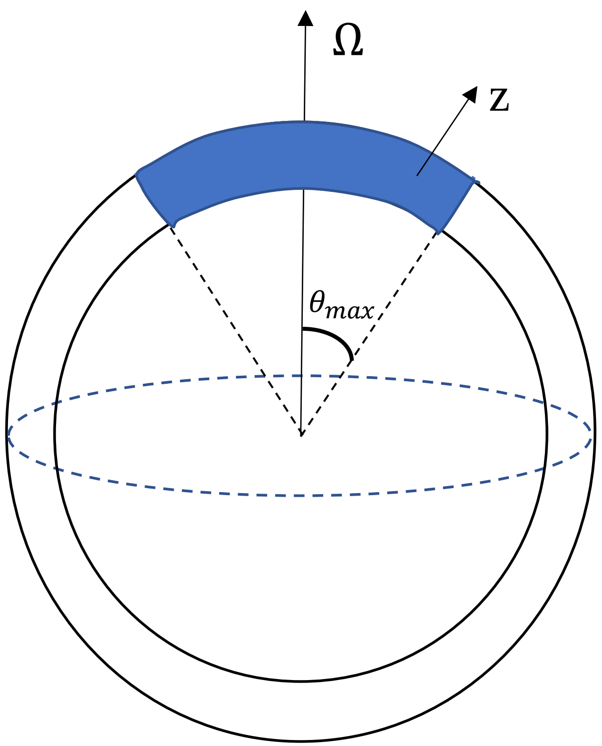
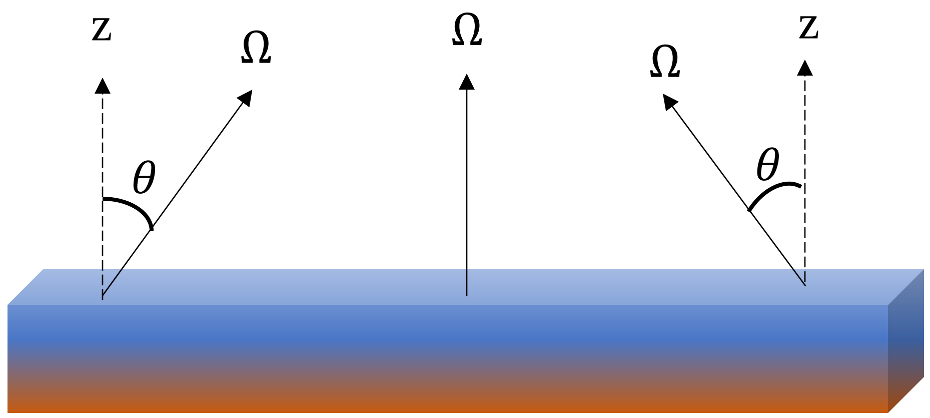


cyclone moves poleward and anticyclone moves equatorward

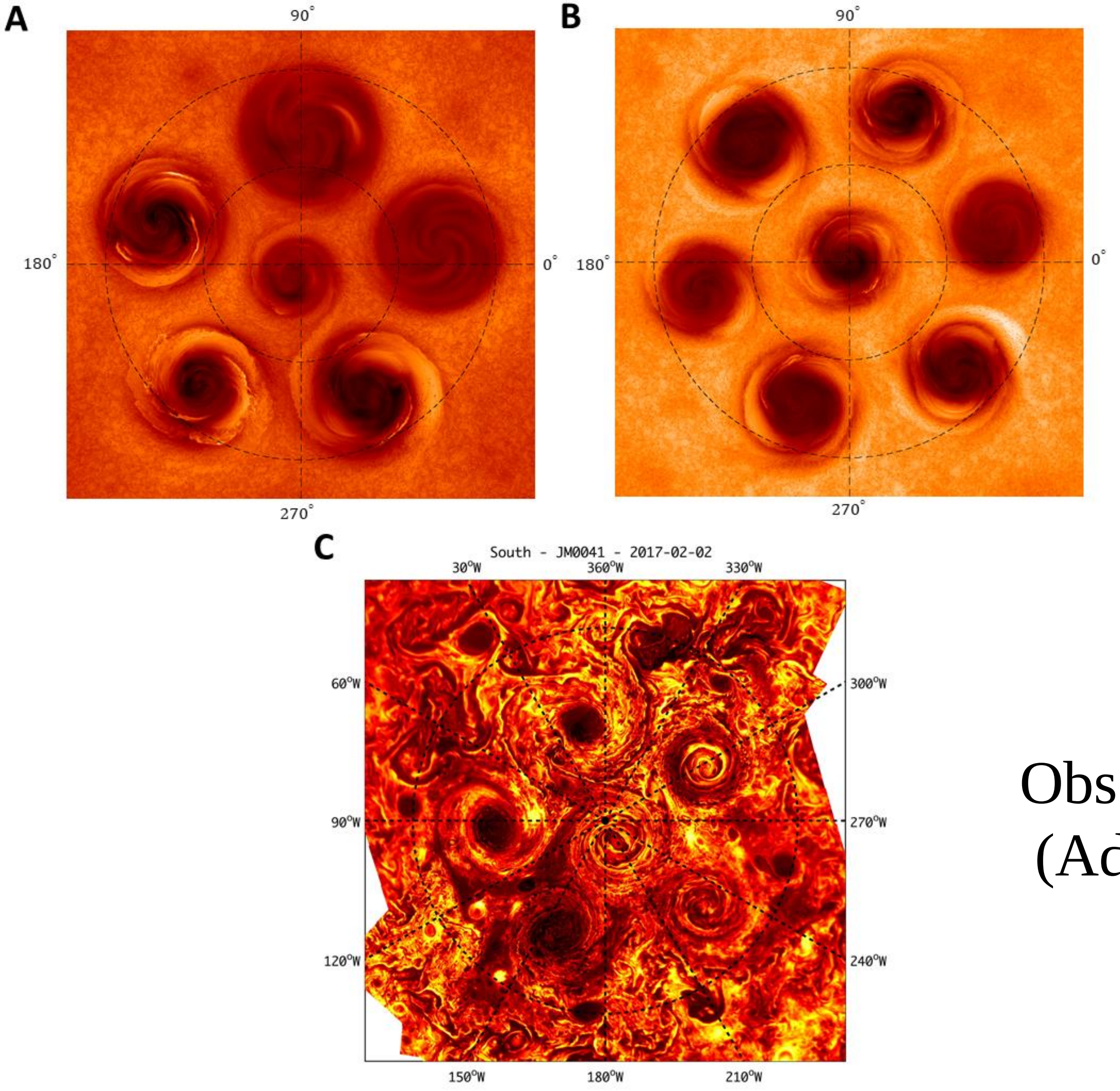
Polar gamma box model





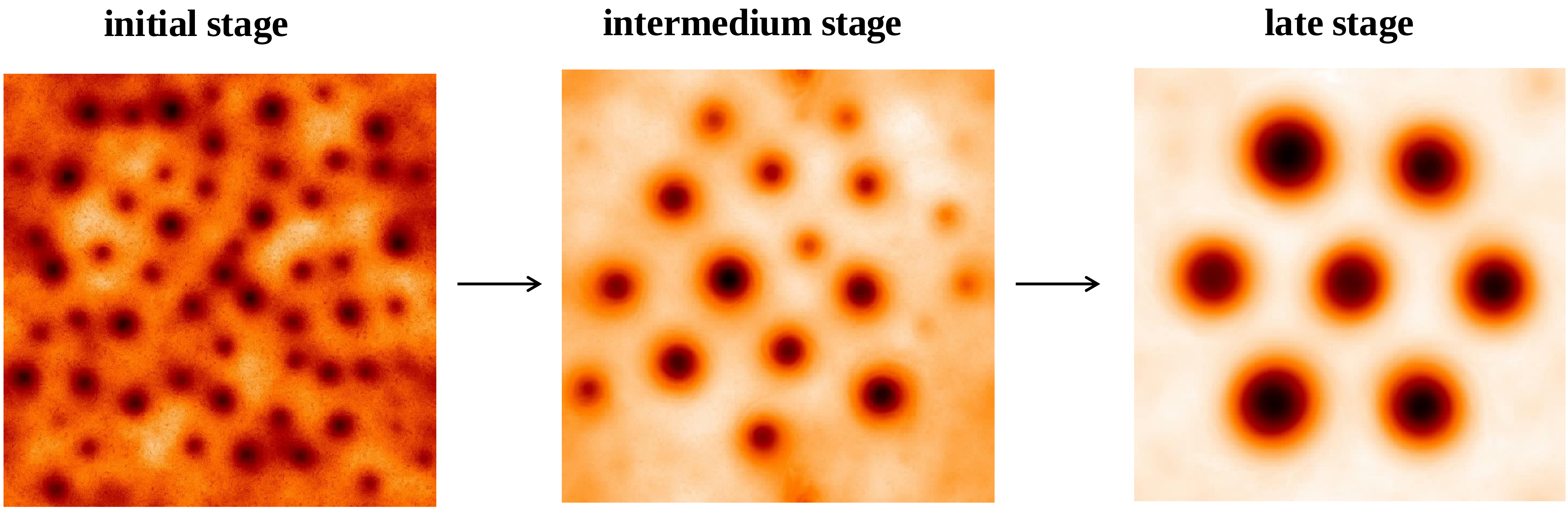
Polar gamma plane

Cai, Chan & Mayr 2021 PSJ

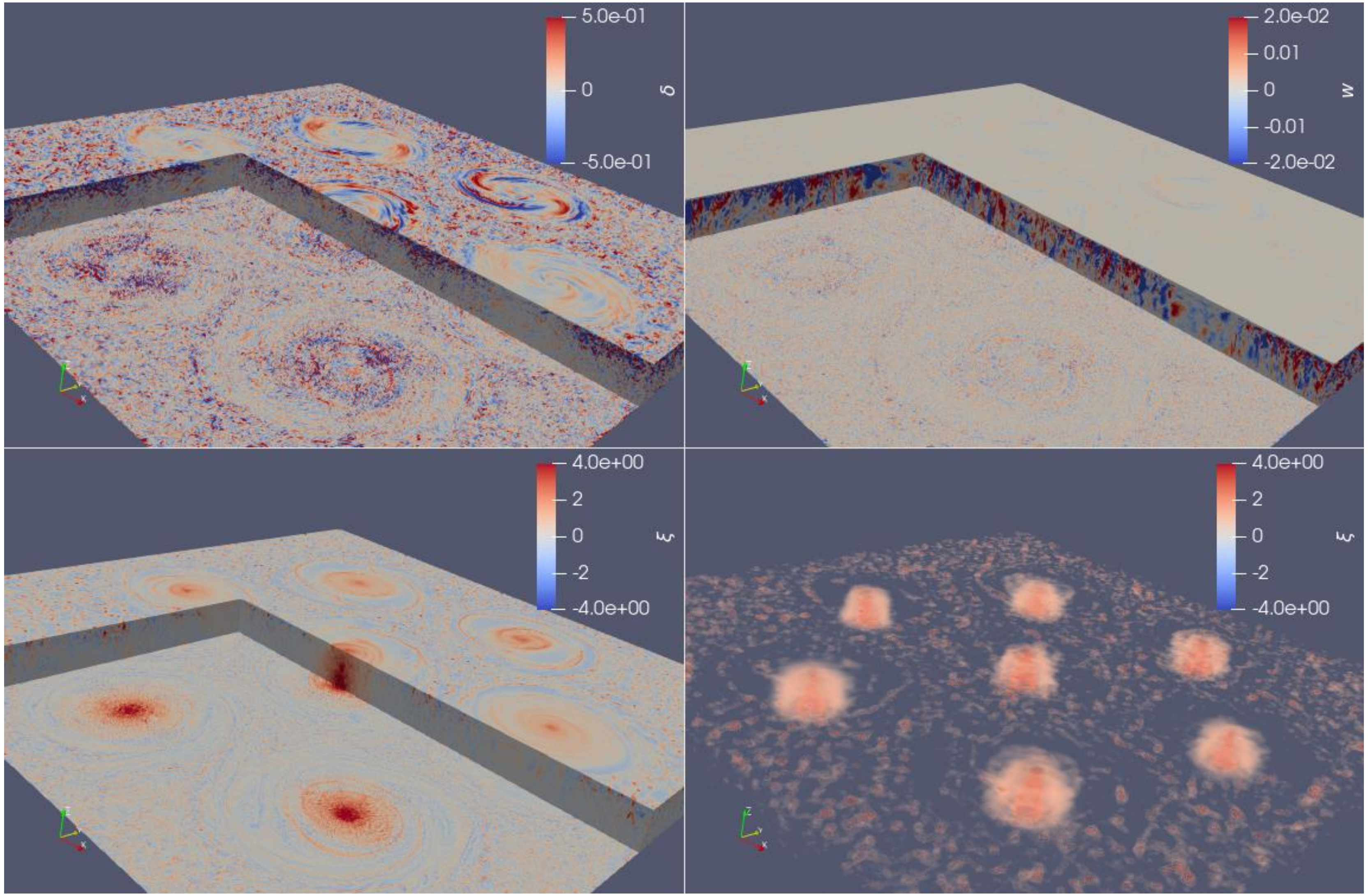


Simulation
(Cai, Chan & Mayr 2021 PSJ)

Observation
(Adriani et al. 2018 Nature)



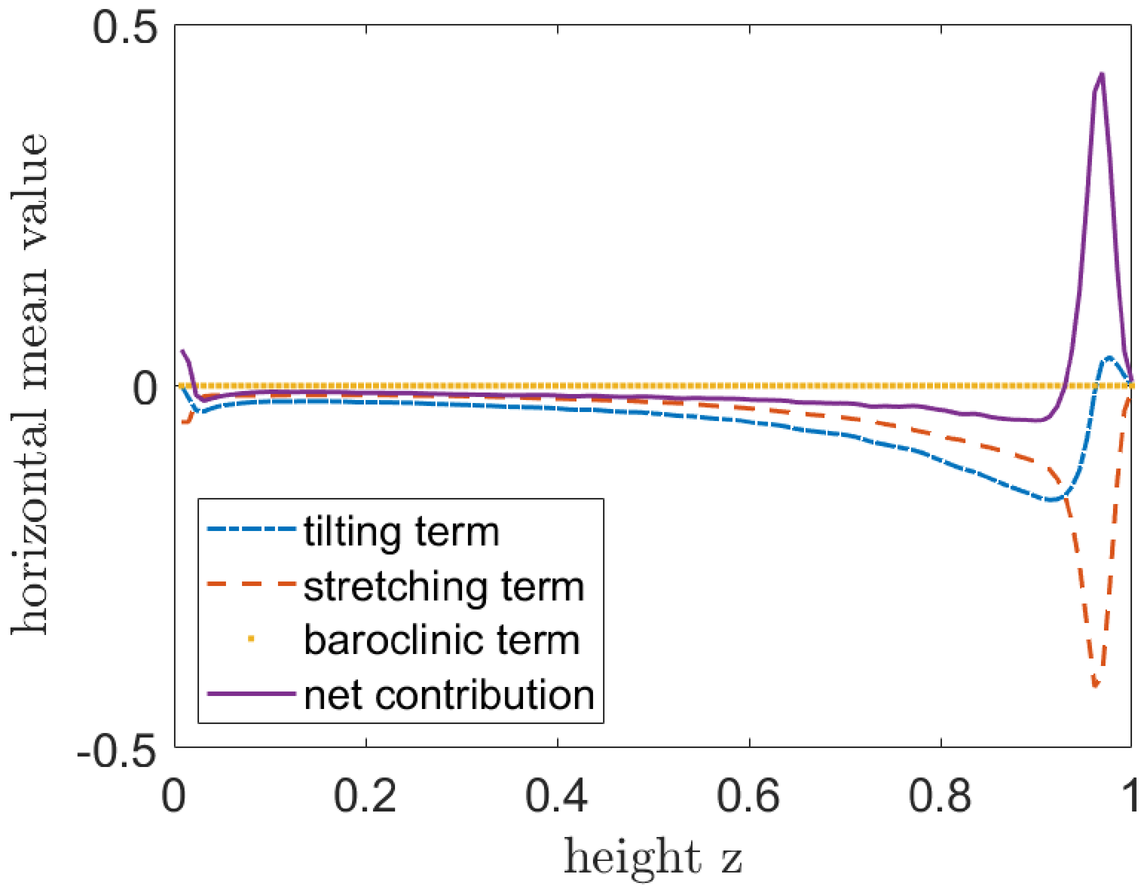
initial stage
intermedium stage
late stage



Cai 2024 JGR Planets

How to maintain the LSVs in the stable layer?

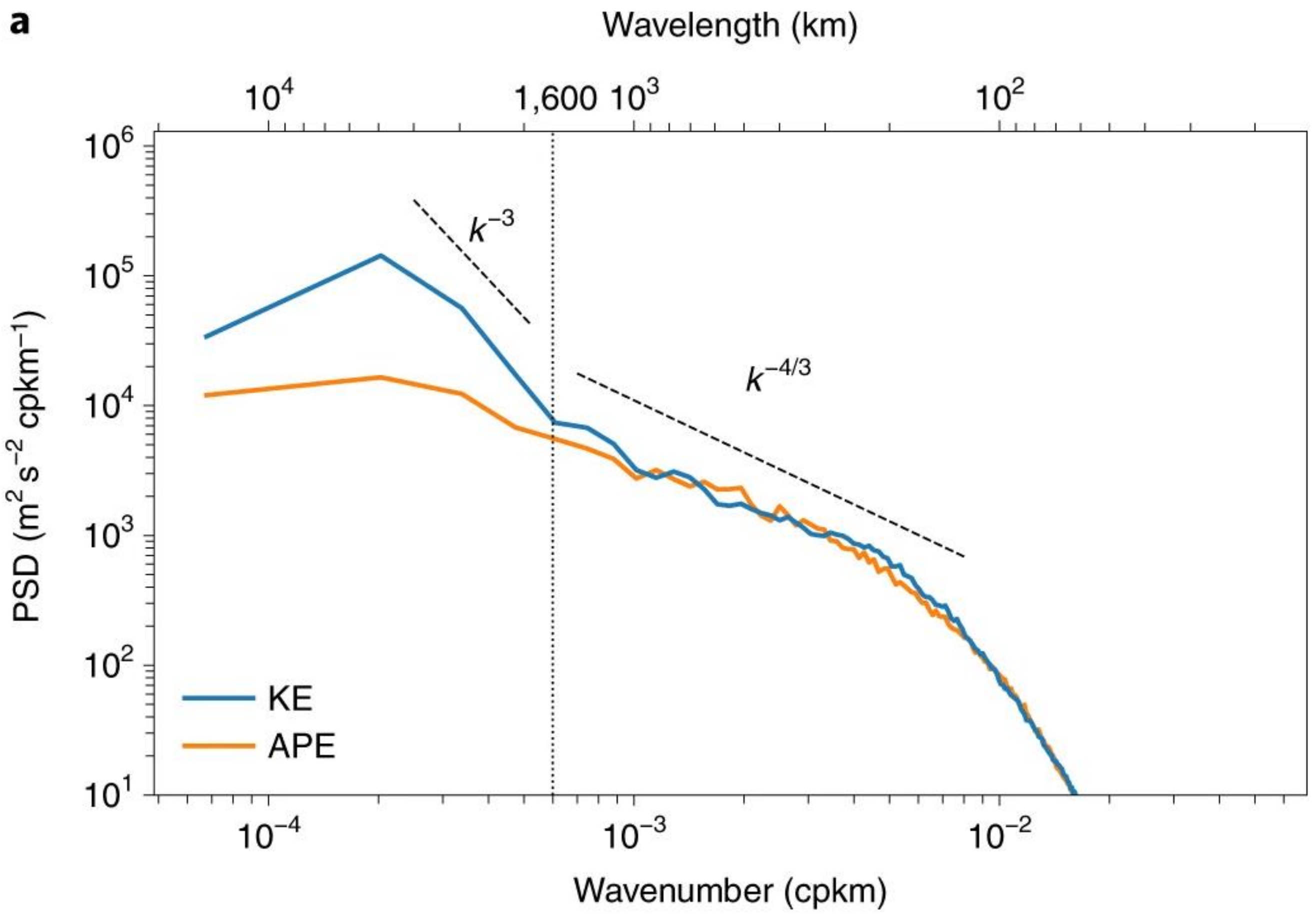
$$\overline{D_t\xi} = \underbrace{\overline{(\boldsymbol{\xi}_h \cdot \boldsymbol{\nabla}_h)w}}_{\text{tilting term}} - \underbrace{\overline{\xi\delta}}_{\text{stretching term}} + \underbrace{\overline{\rho^{-2}\boldsymbol{\nabla}_h\rho \times \boldsymbol{\nabla}_h p}}_{\text{baroclinic term}} .$$



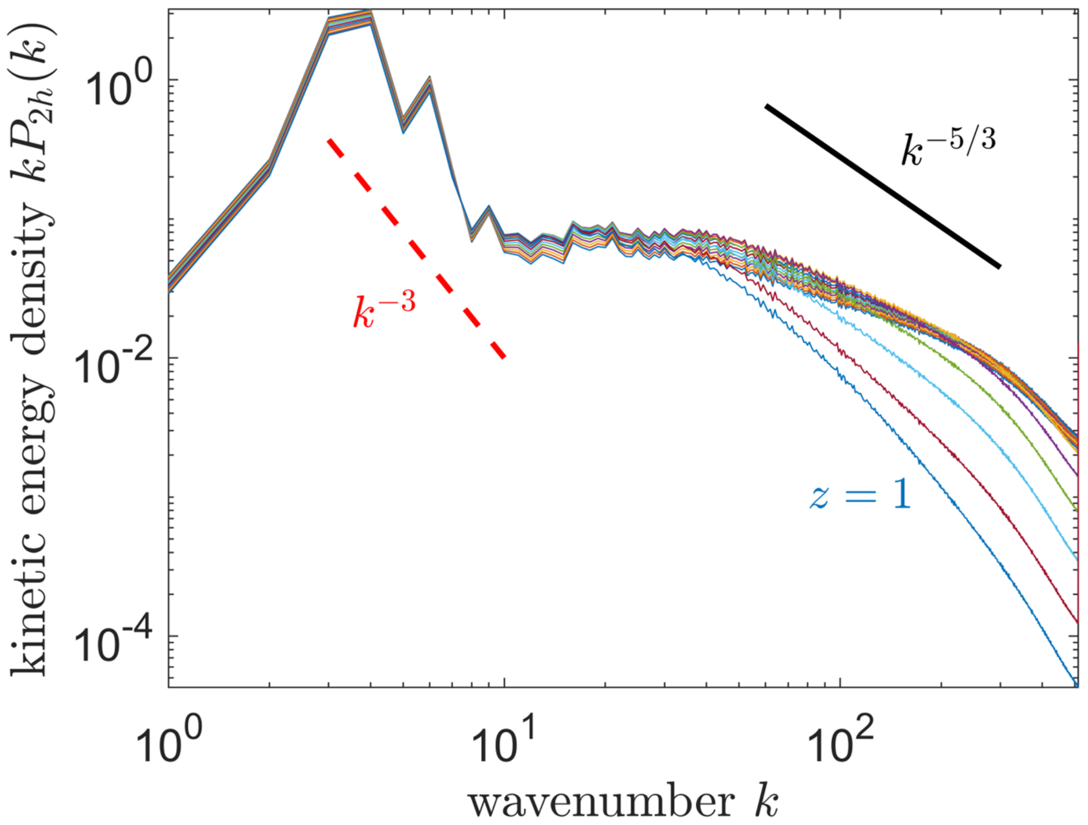
- Tilting effect dominates in the convective layer
- Stretching effect dominates in the radiative layer
- Vorticity is transferred from convective layer to radiative layer

Cai 2024 JGR Planets

Energy spectrum



Siegelman 2022 Nat. Phys.



Cai 2024 JGR Planets

The origin of vortex crystals

- Formation?

  It can be generated naturally in rapidly rotating convection

- Depth?

  It can deeper than 500km

- Sustainability?

  Vorticity is transferred from convection zone to radiation zone

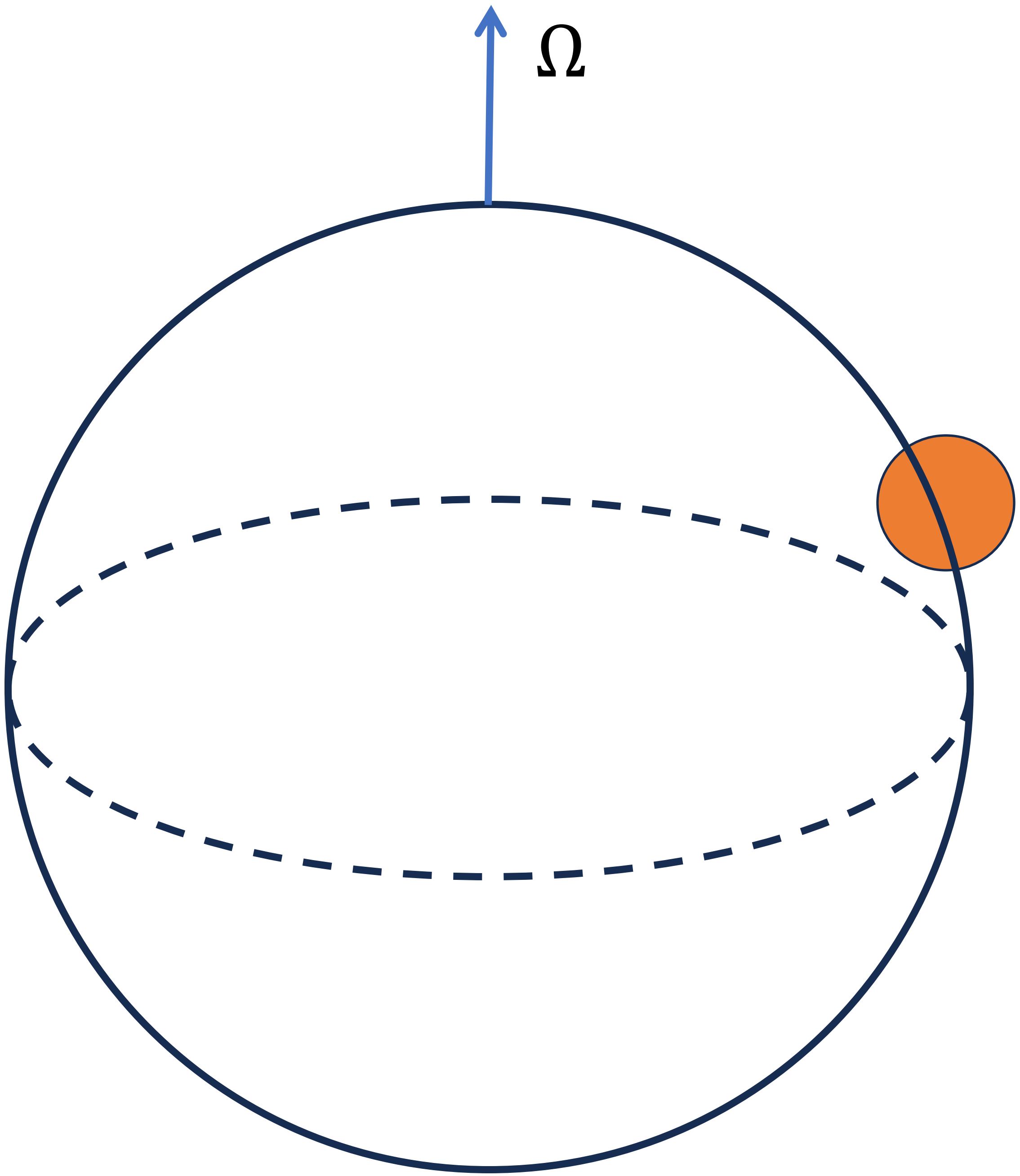
How to remain stable?

Empirical model

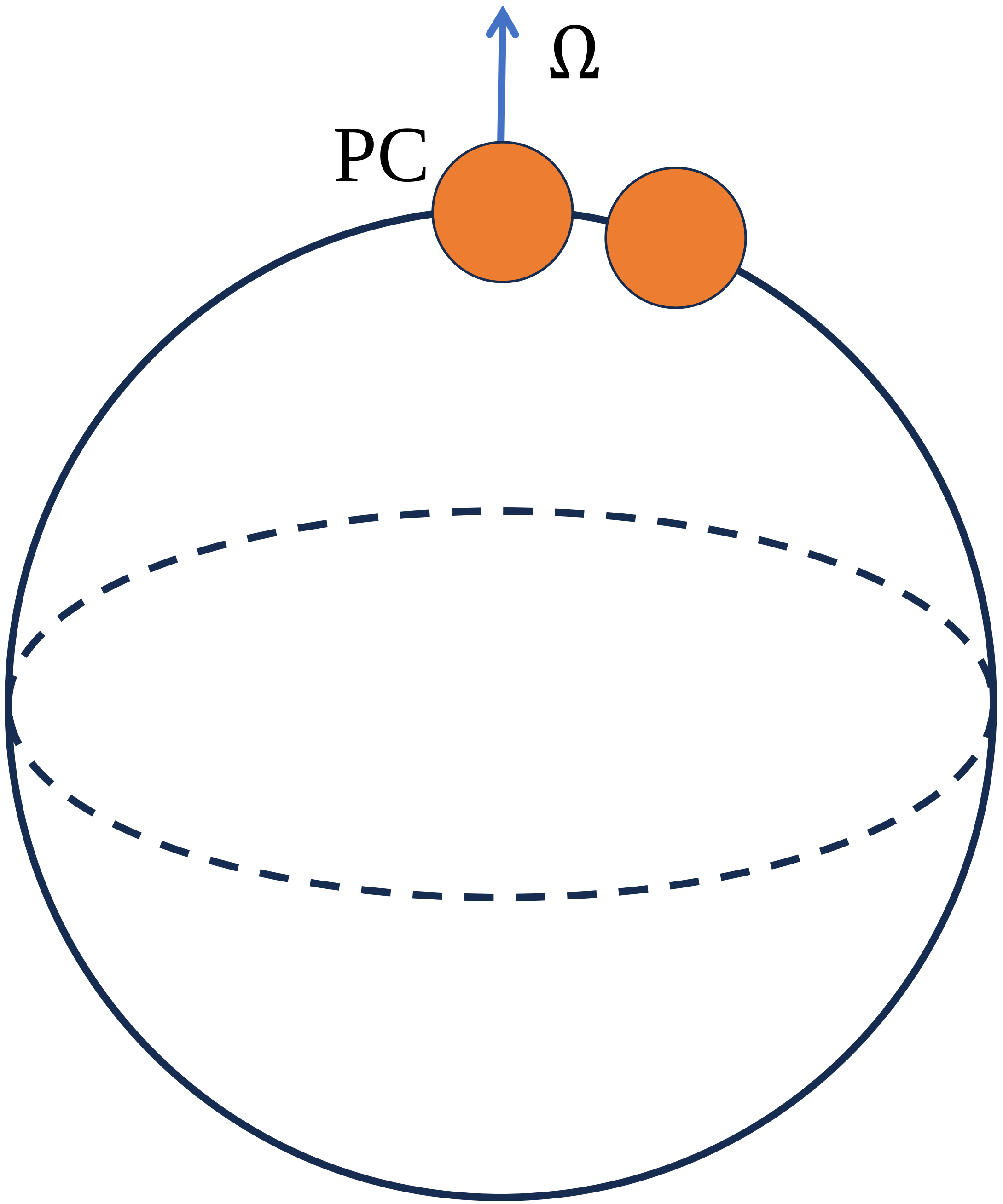
- Anticyclonic shear (Li et al. 2020)
- Large Coriolis effect (Cai et al. 2021)

Analytical model

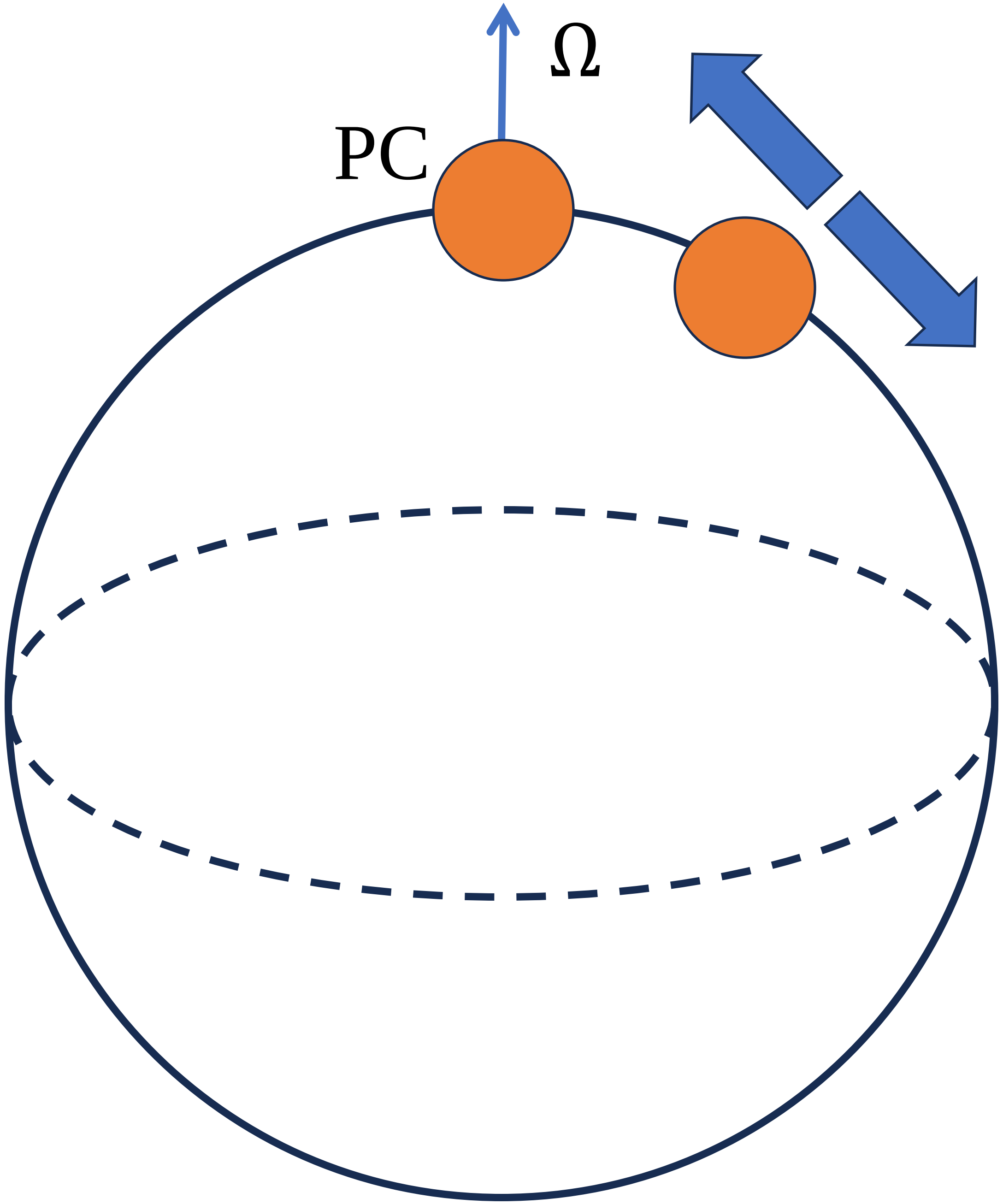
- Force balance between push force from PC and pull force from beta effect (Gavriel & Kaspi 2021)



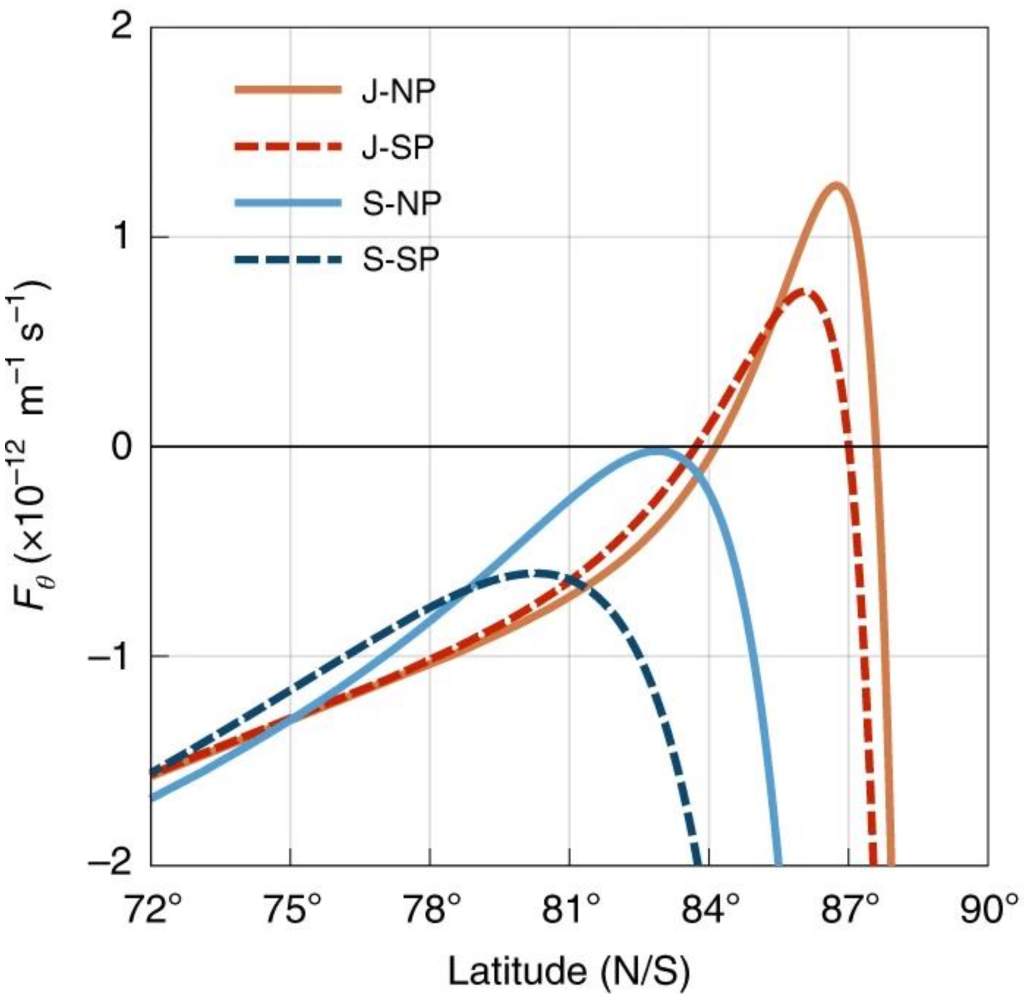
The beta effect tends to pull cyclone poleward



The polar cyclone tends to push cyclone equatorward

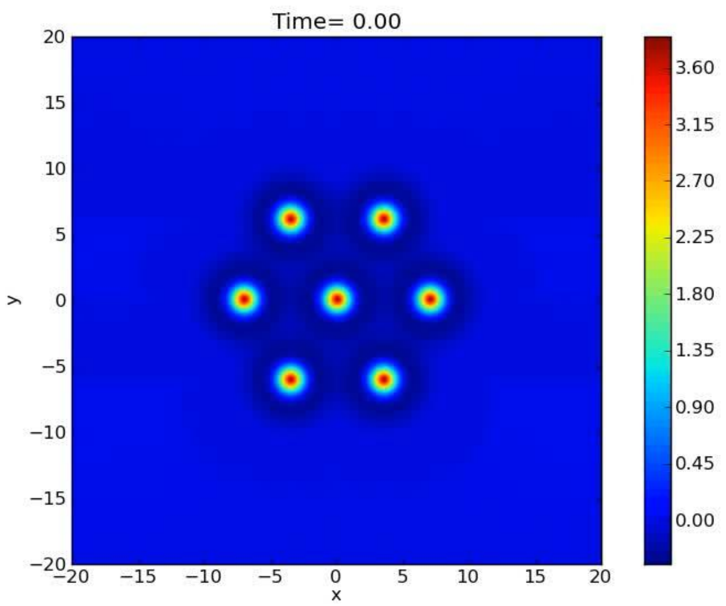


Equilibrium:
pull force = push force

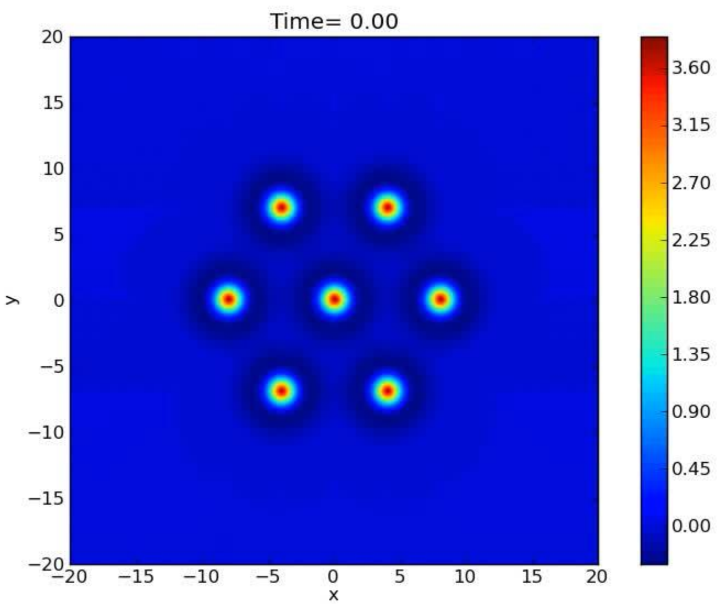


Gavriel & Kaspi 2021

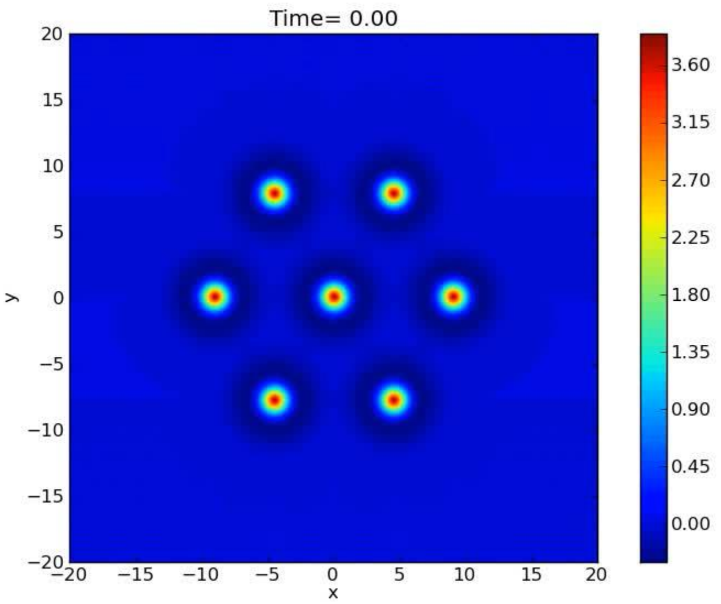
Only one stable equilibrium point?



L/R=7



L/R=8

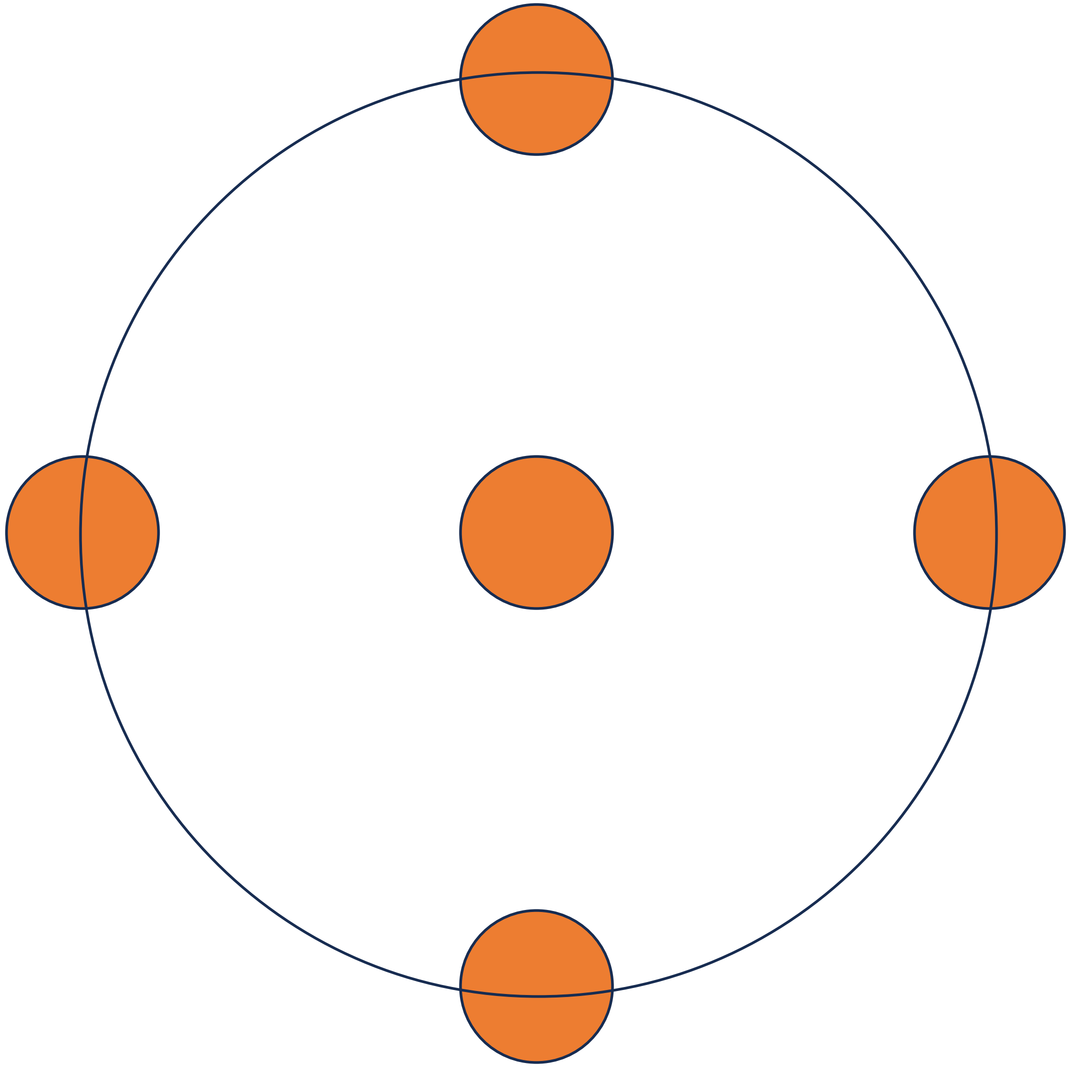


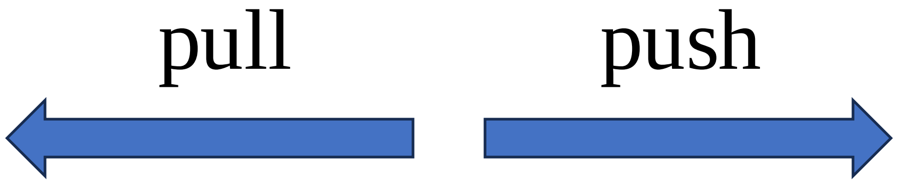
L/R=9

The stable equilibrium point is nonunique!

Why?

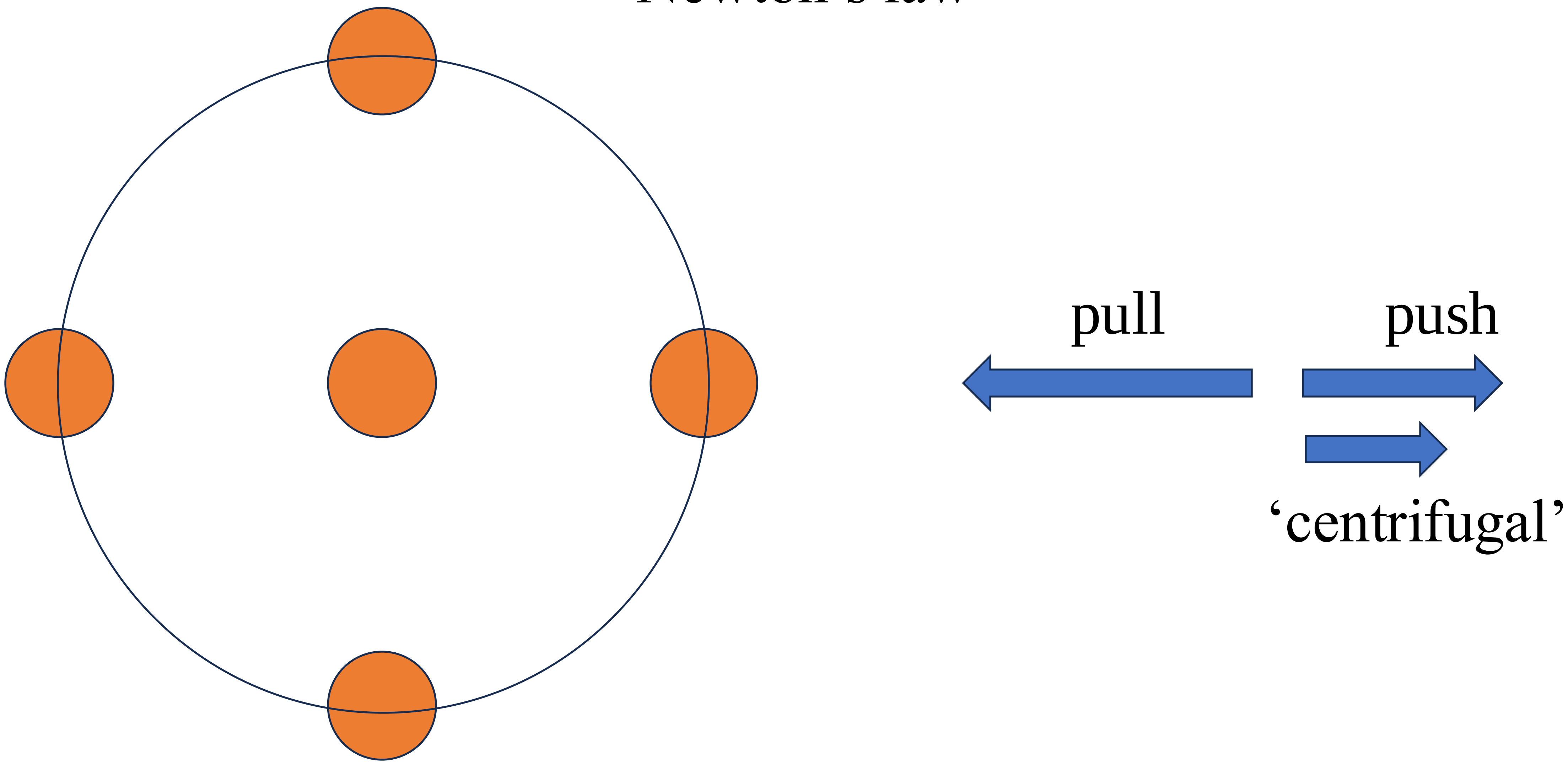
Newton’s law





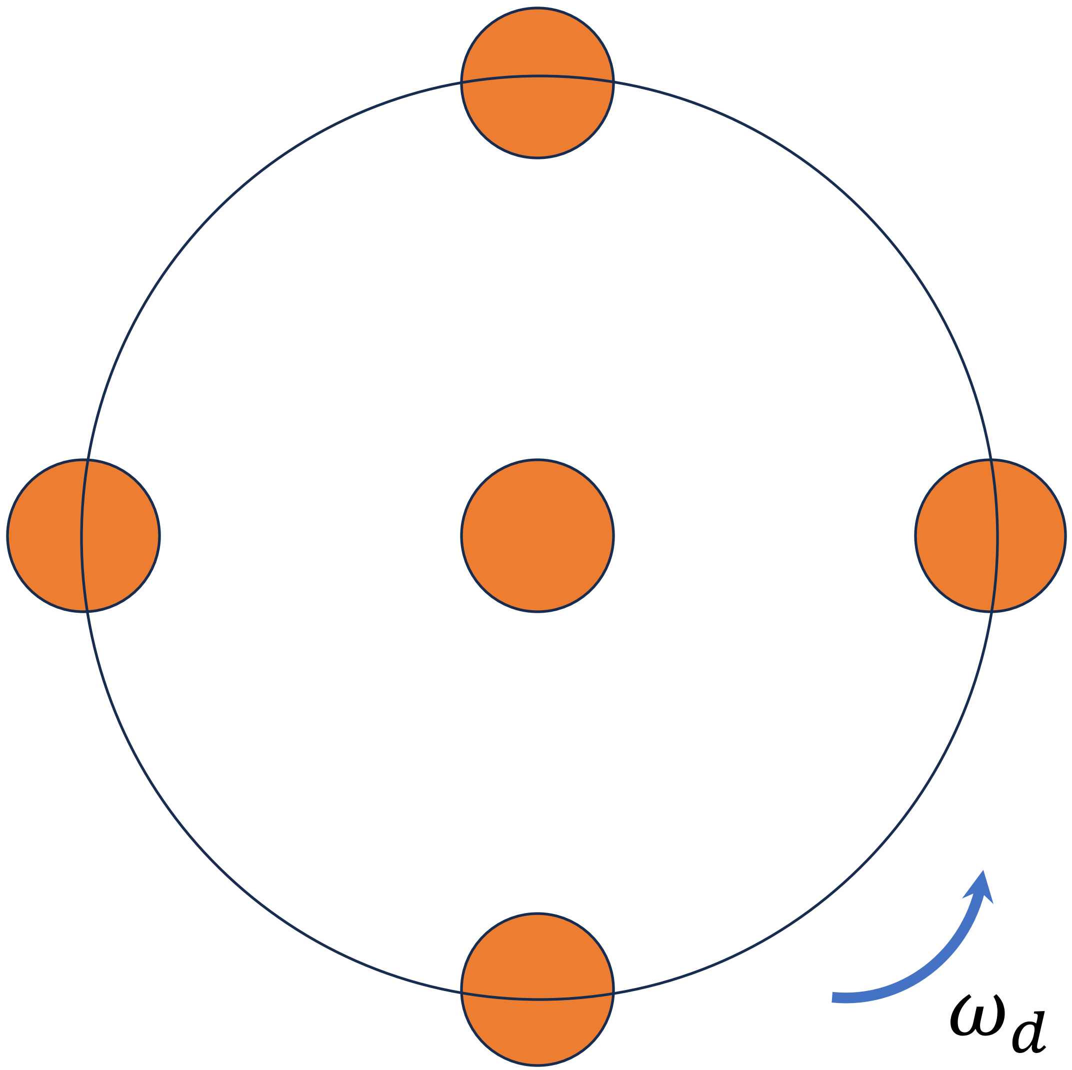
Static equilibrium

Newton’s law



Dynamic equilibrium

N-body dynamic model



$$d\boldsymbol{x_i}^2/dt^2 = \boldsymbol{a_i}(\boldsymbol{x_1}, \cdots, \boldsymbol{x_N}, \omega_d)$$

Equilibrium:
$$\boldsymbol{a_i}(\boldsymbol{x_1}, \cdots, \boldsymbol{x_N}, \omega_d) = 0$$

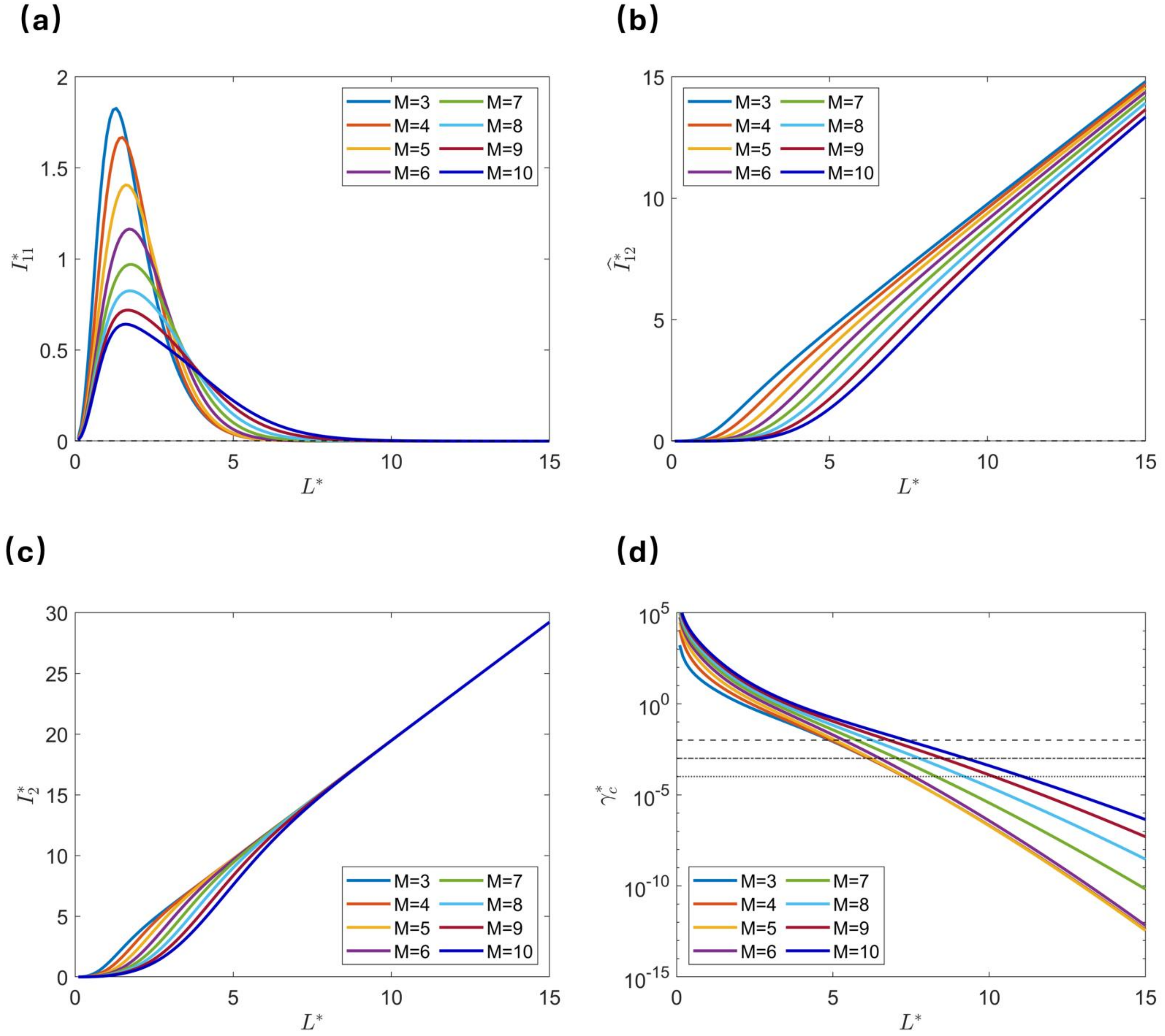
ω_d is nonunique!

Interactive effect

$$\omega_d I_2 = I_{11} - \gamma \widehat{I_{12}}$$

Omega effect

Beta effect



Vortex crystal can be prograde or retrograde

Stability and oscillation?

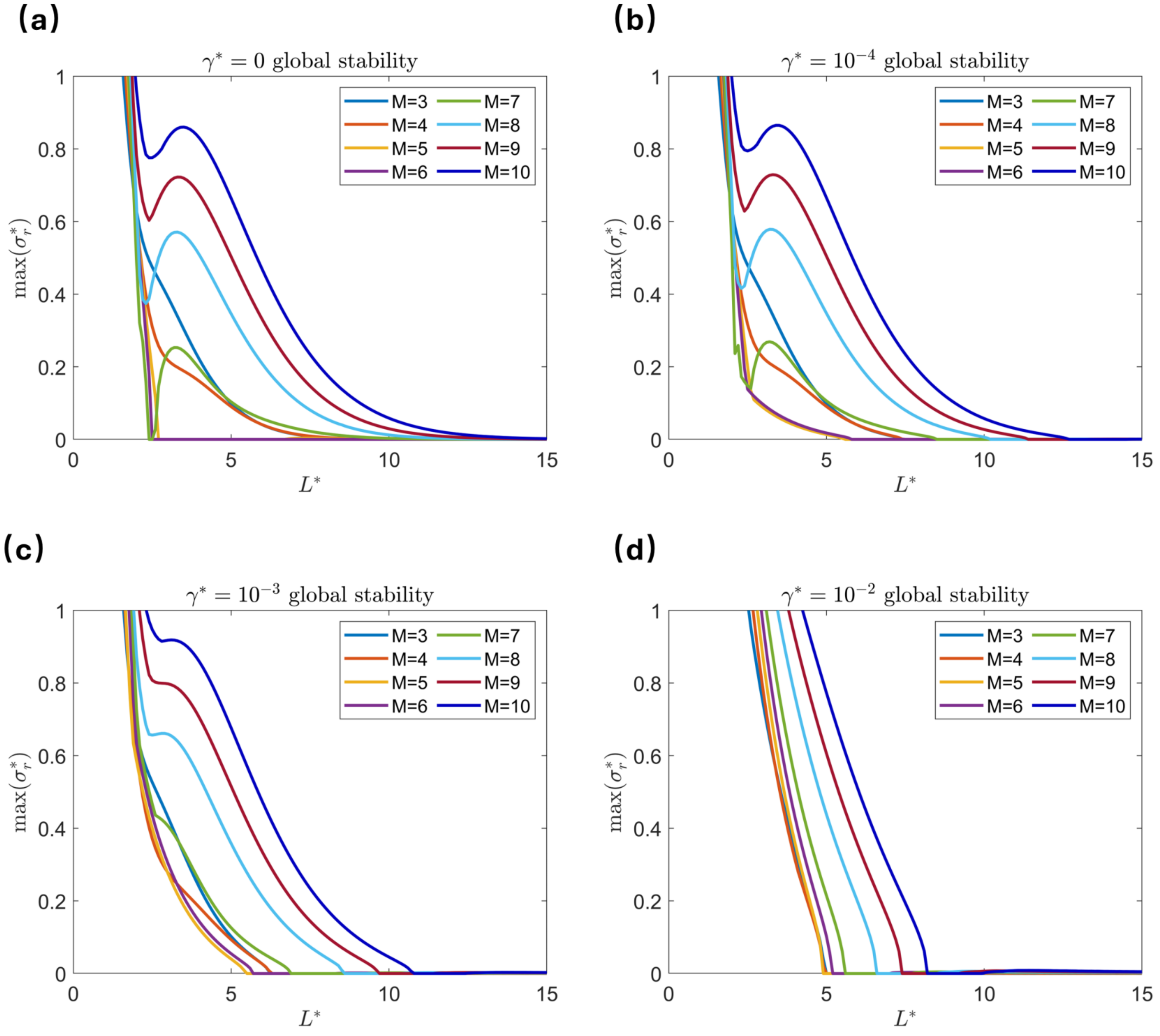
$$d\boldsymbol{x_i}^2/dt^2 = \boldsymbol{a_i}(\boldsymbol{x_1}, \cdots, \boldsymbol{x_N}, \omega_d)$$

$$\Big\downarrow \; x_i \sim e^{\sigma t}$$

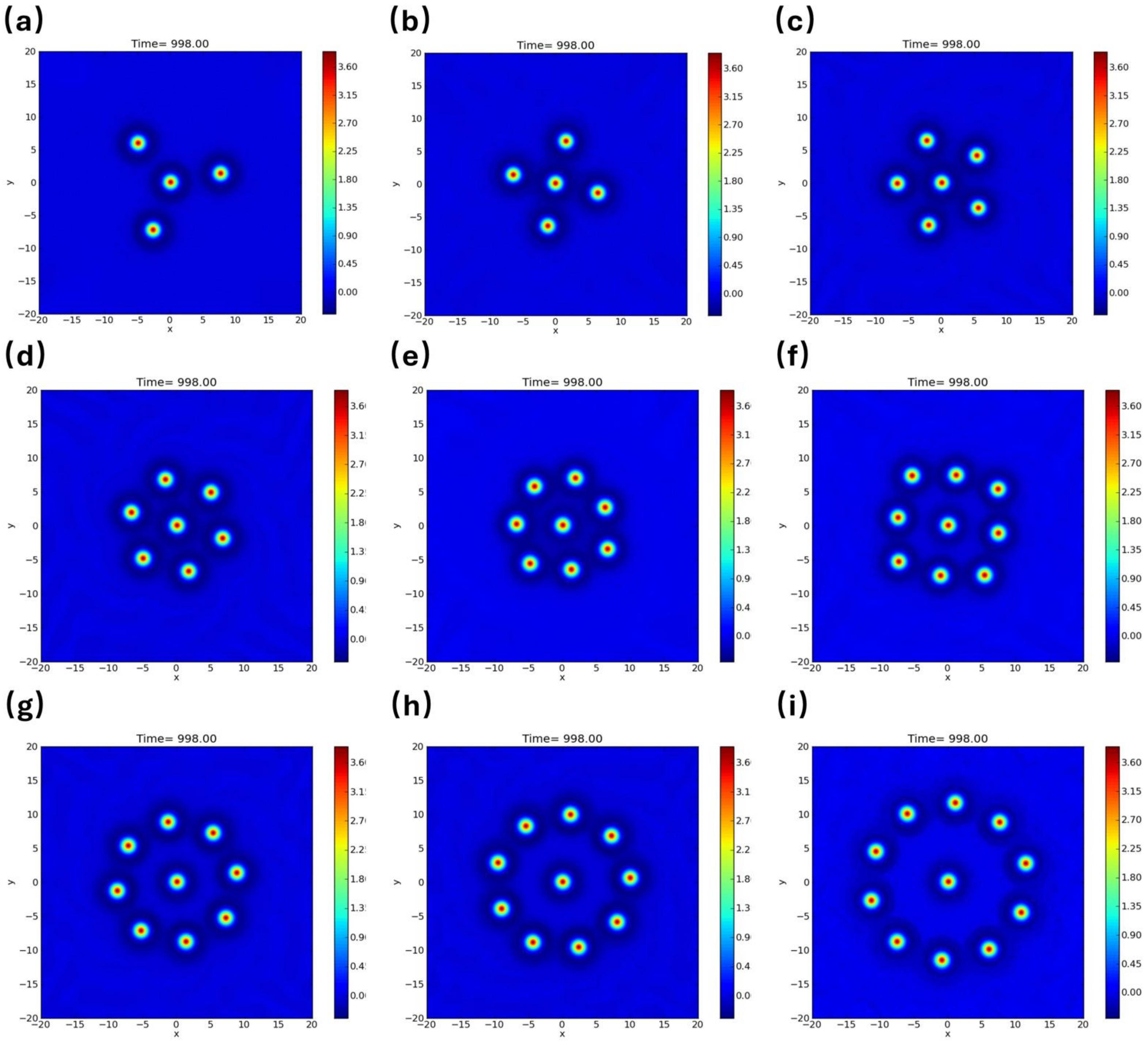
$$\sigma^2 x = Ax$$

Stability condition: $Re(\sigma) \leq 0$

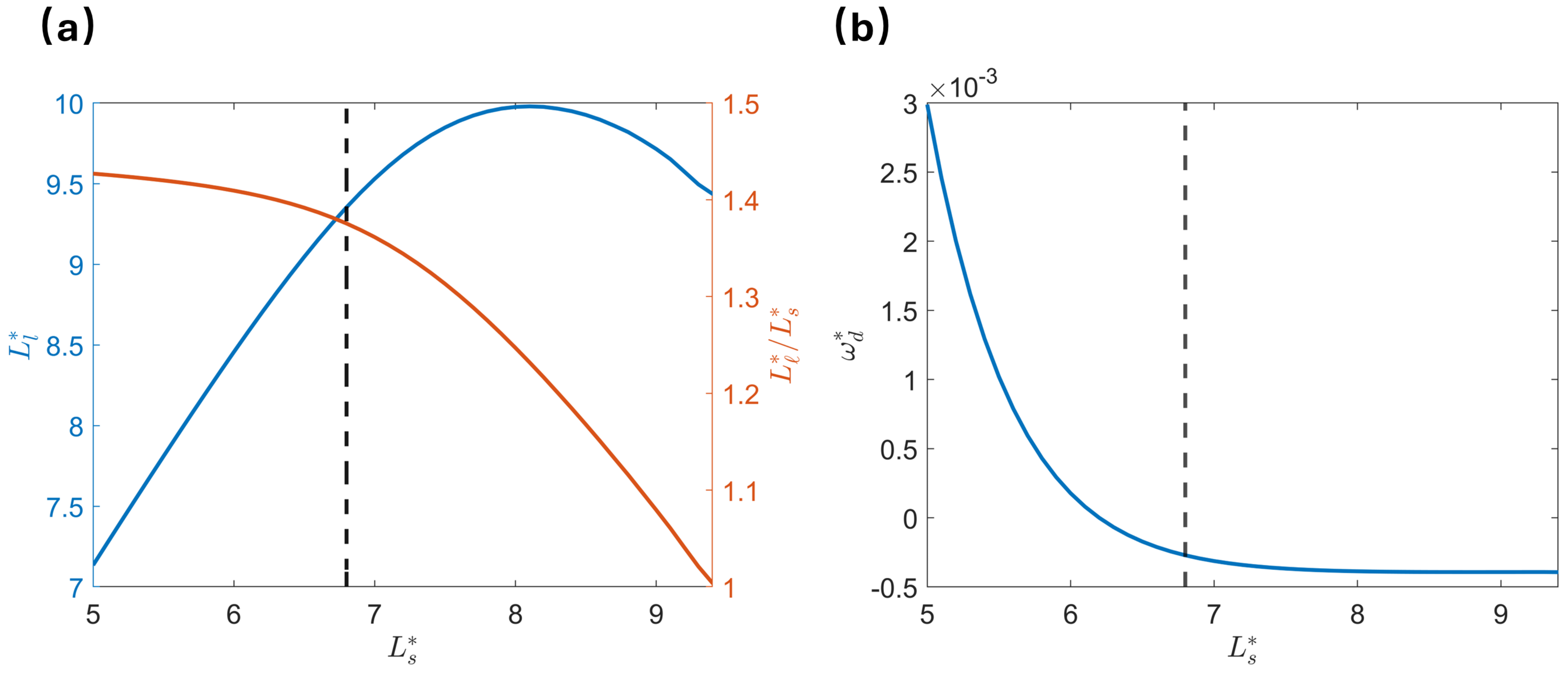
Oscillation frequency: $Im(\sigma)$



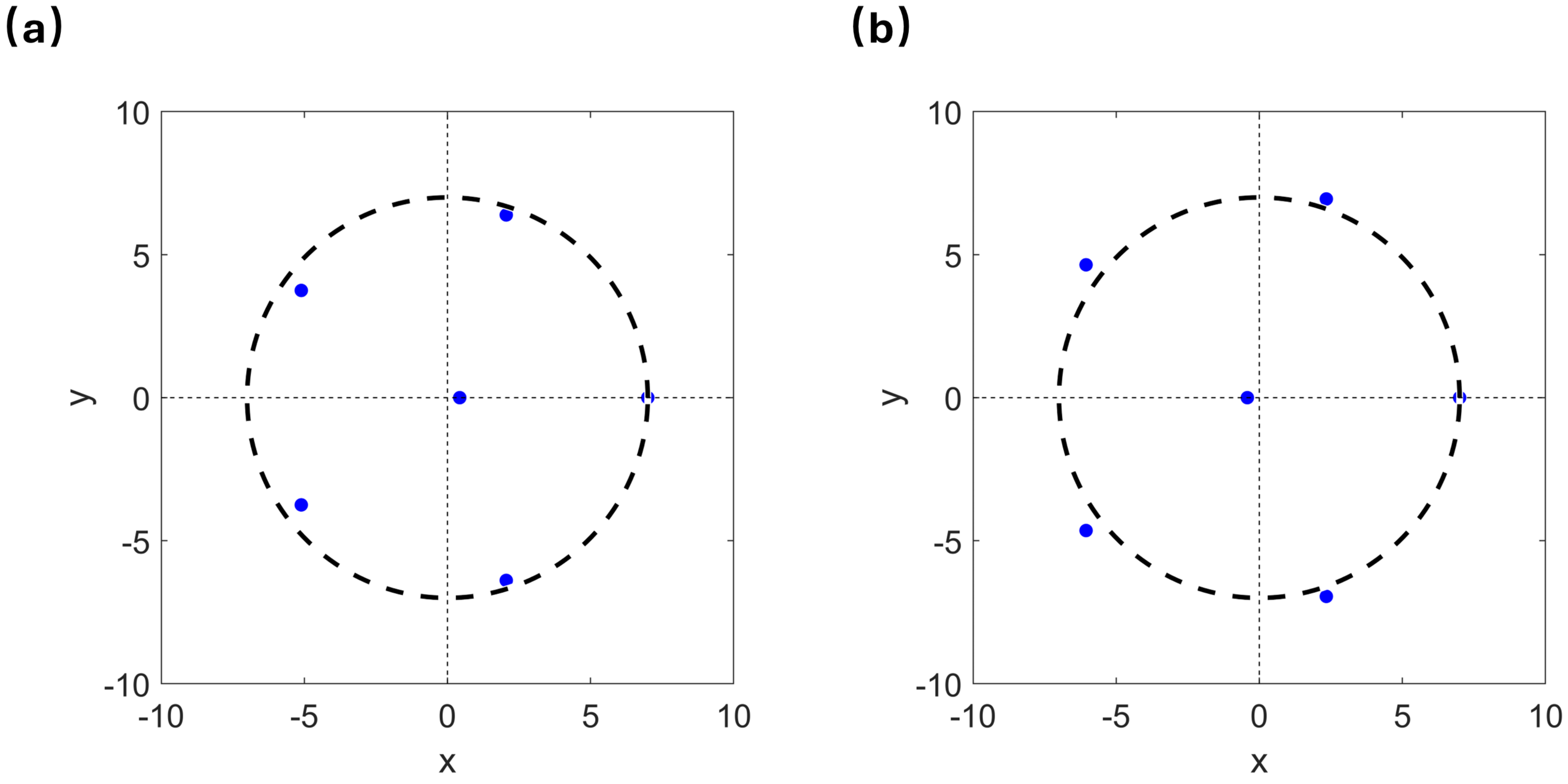
Vortex crystal can be stable when separation distance is large enough



Stable M-fold vortex crystals



8-fold vortex crystal can be stable as a vortex Sudoku



vortex crystal can be shifted from the pole when the vortices have varying strengths

	R_j (km)	V_j (ms^{-1})	L (km)	γ $(m^{-2}s^{-1})$	P_s $(hours)$	P_d $(years)$	P_o $(months)$
North pole	(1016,1199)	75	(7600,8467)	7.1×10^{-20}	(24,28)	97	(9.08, 9.08, 1.04, 0.99, 0.94, 0.87, 0.87, 0.53, 0.40, 0.40, 0.34, 0.29, 0.29, 0.24, 0.2, 0.18, 0.18, 0.17)
South pole	1102	(80,56)	7400	7.1×10^{-20}	(24,34)	47	(3.17, 0.91, 0.91, 0.7, 0.65, 0.5, 0.42, 0.33, 0.31, 0.28, 0.19, 0.15)

The dynamics of vortex crystals

- Stability?

  The equilibrium is non-unqiue. It can be stable when the separation distance is large.

- Drift?

  It can drift prograde or retrograde, depending on the relative strength of interactive effect to beta effect.

- Oscillation?

  It can oscillate at multiple frequencies.

Thank you for attention!

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