Imaging shapes from seismic data: what wavefronts tell us about percolation induced singularities in the Earth subsurface

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joint work with
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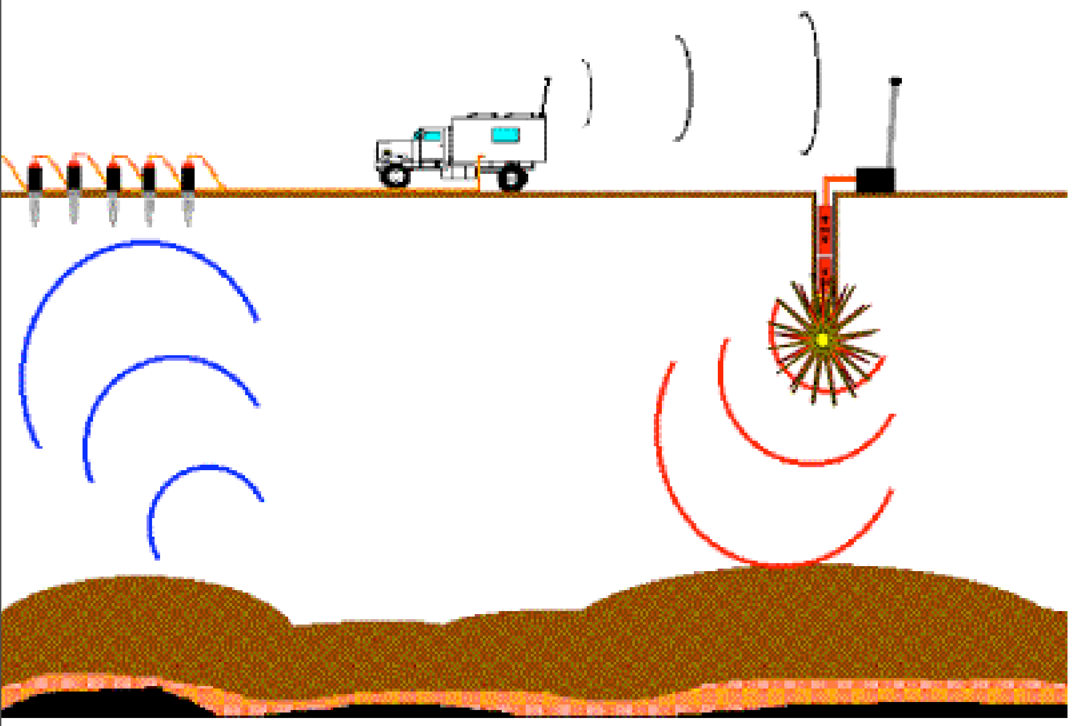
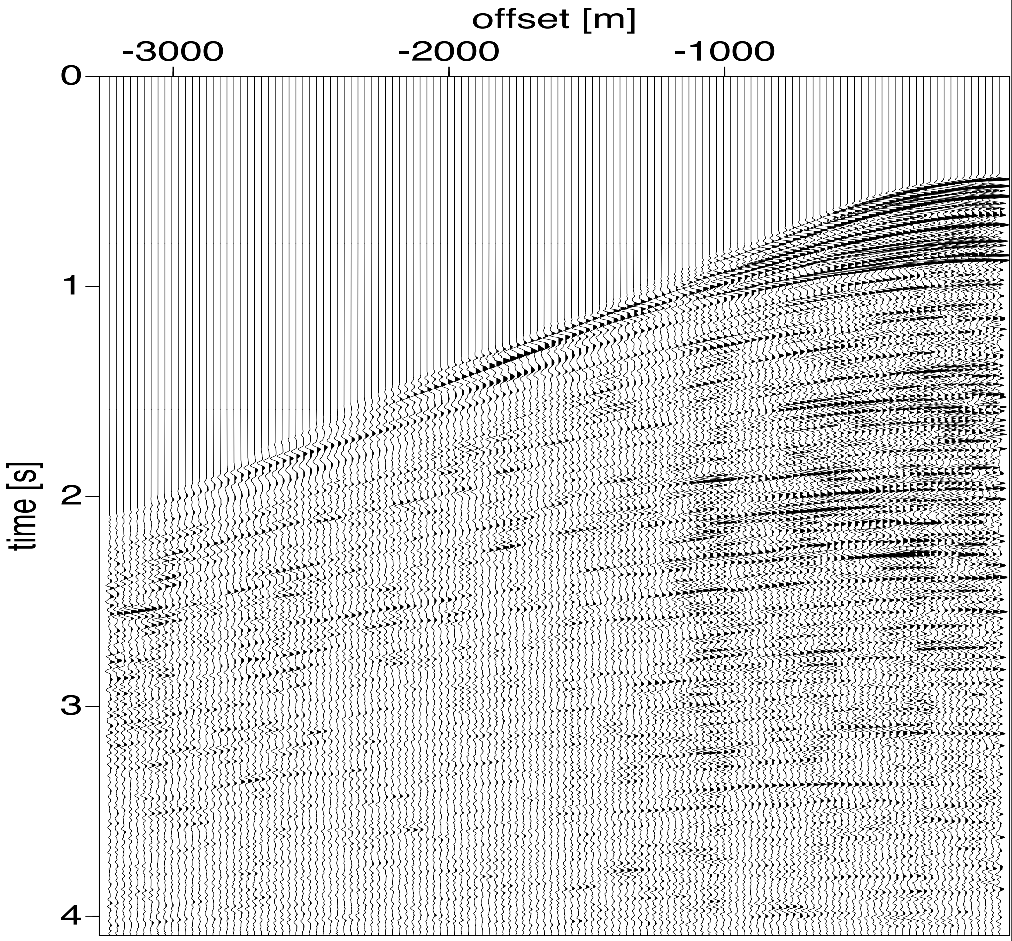
Research interests

- Develop techniques to obtain higher quality images from (incomplete) data **<=>** imaging of shapes
- Characterization of shapes **<=>** estimation of singularity orders of imaged reflectors
- Understand physical processes that generate the observed singular transitions **<=>** Percolation phenomena

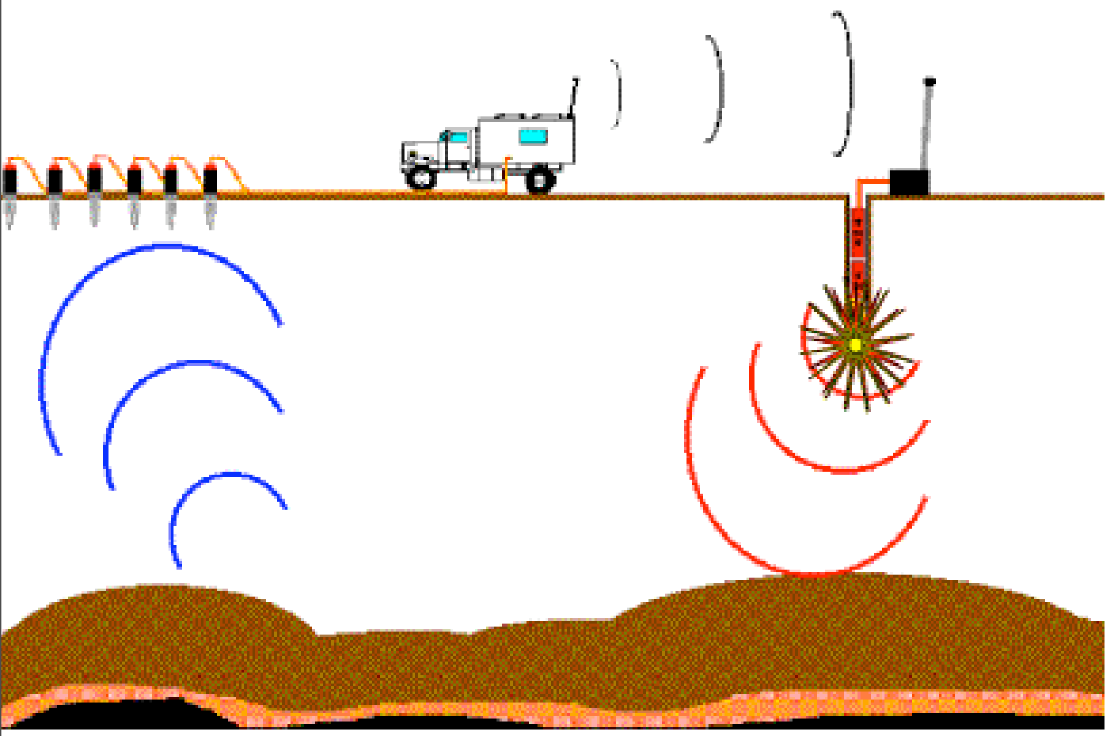
Seismic imaging method & some challenges

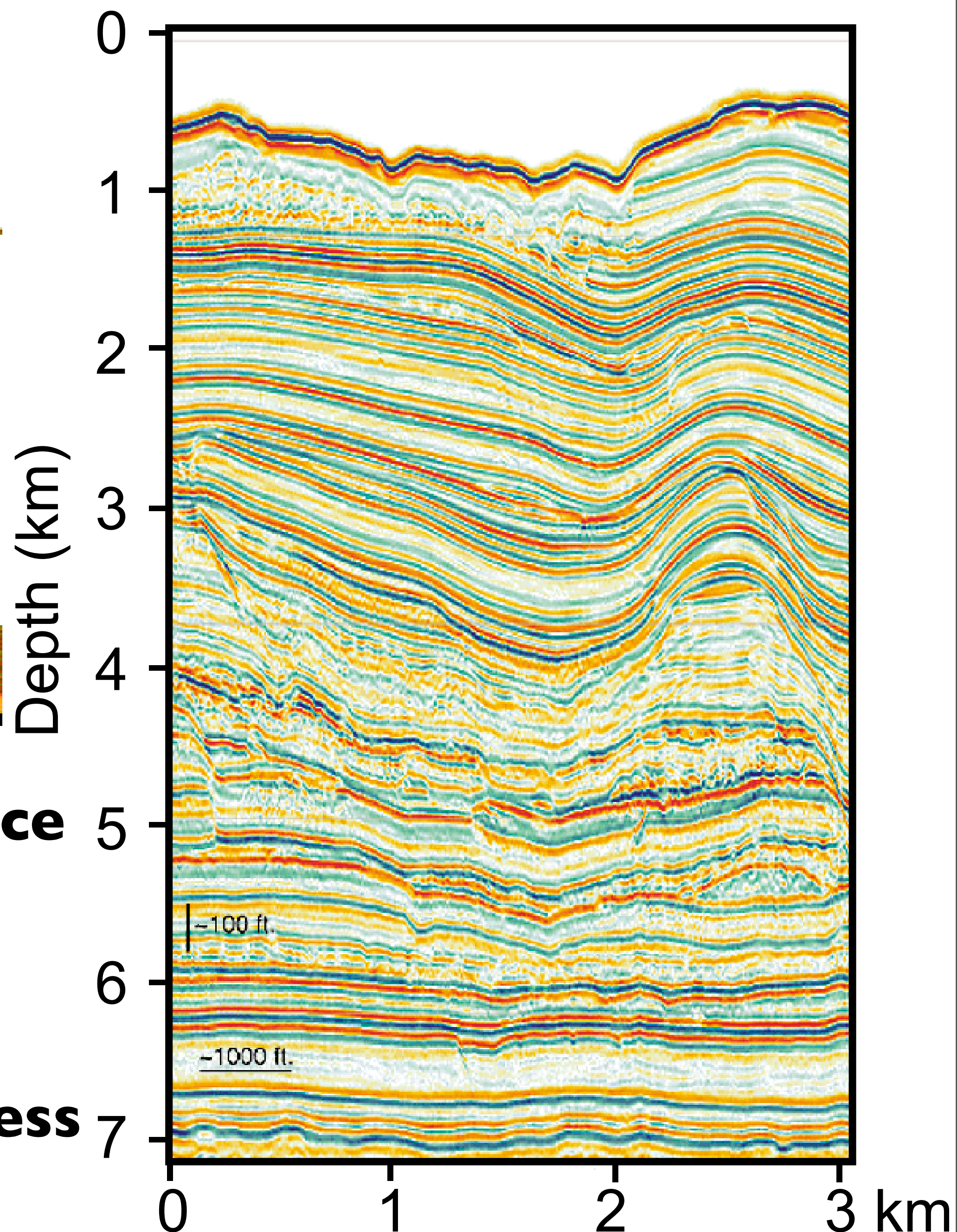
Seismic data acquisition





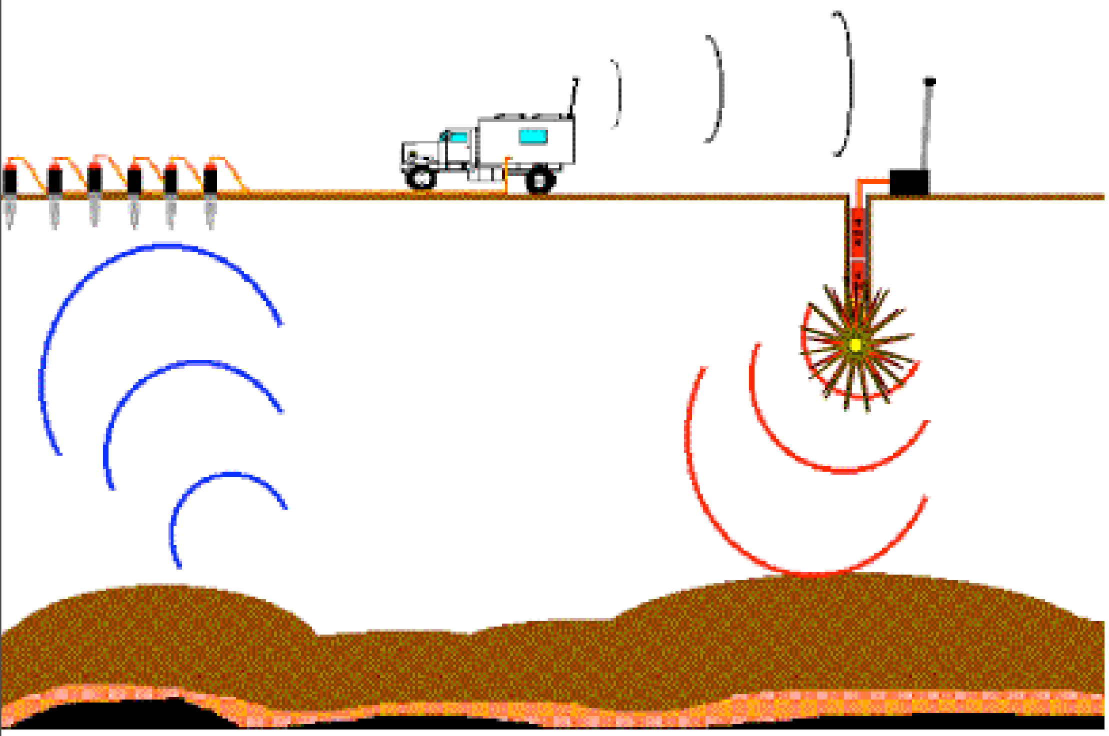
Seismic imaging

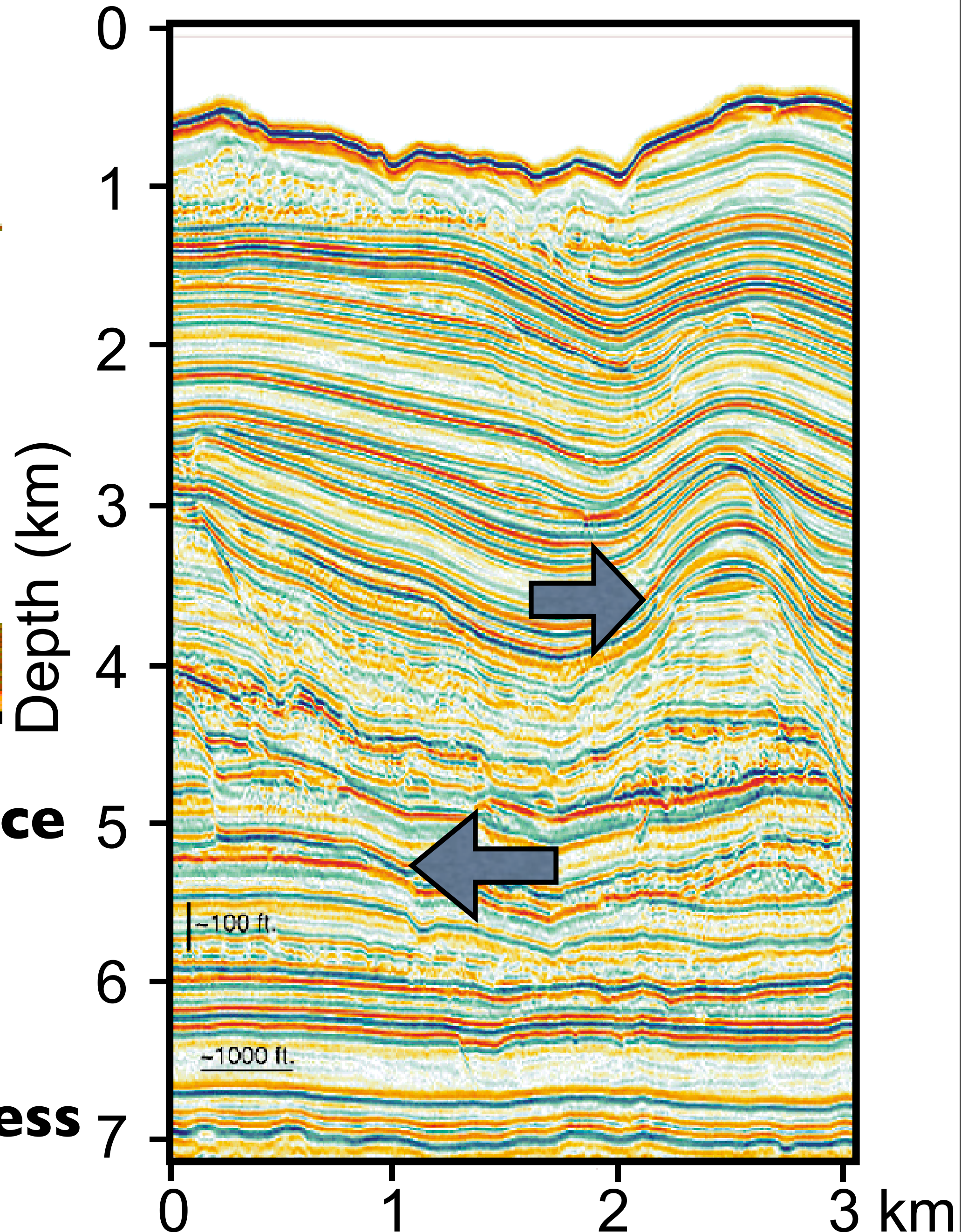




- **create images of the subsurface**
- **need for higher resolution/ deeper**
- **clutter and data incompleteness are problems**

Seismic imaging





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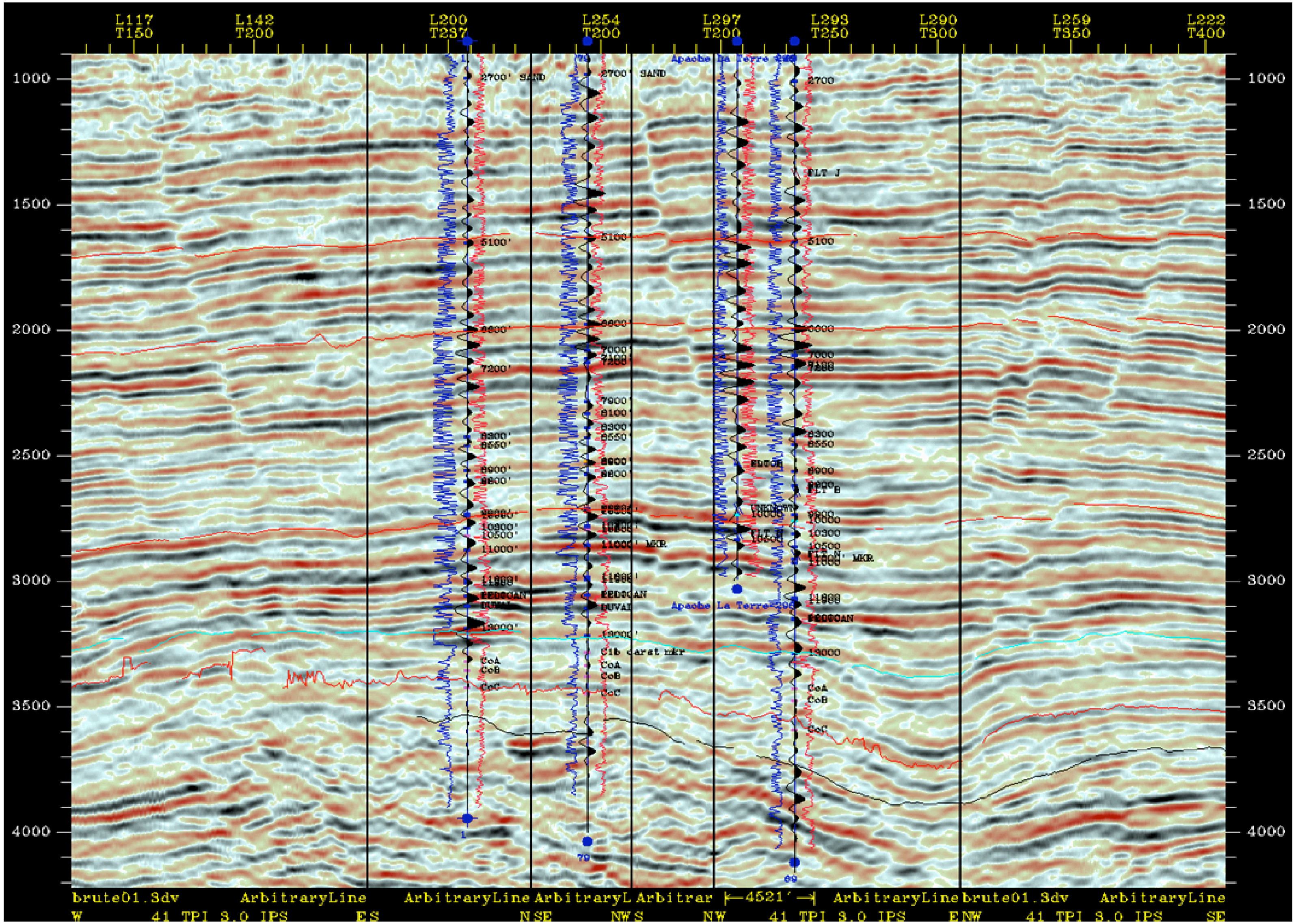
Seismic imaging

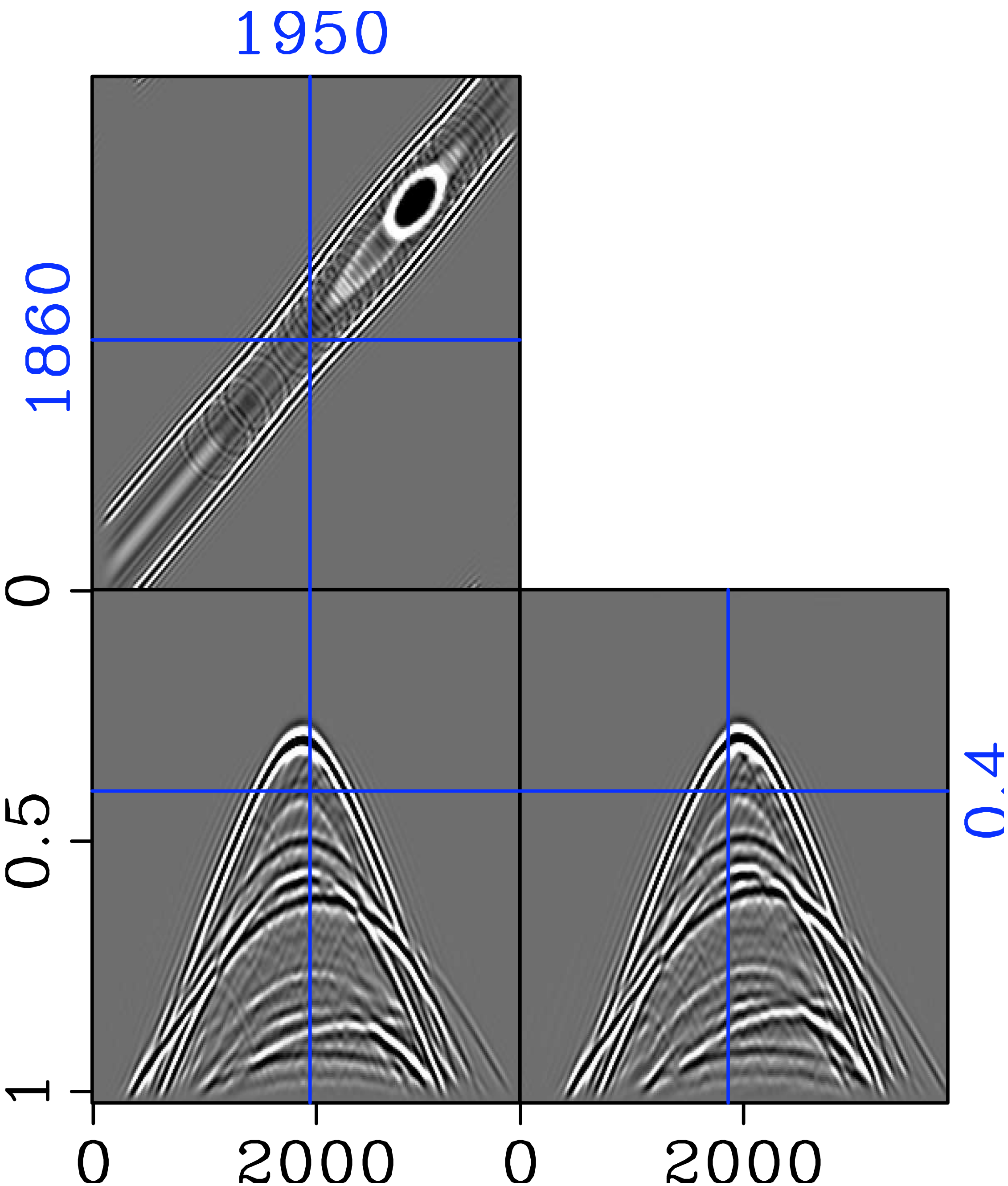


courtesy Apache

Observations

- Earth subsurface is highly heterogeneous, multiscale and intermittent (fractal like).
- Seismic data contain bandwidth limited wave fronts.
- Differences in smoothness delineate "layer" structure.
- Imaged waveforms contain ***coarse-scale*** information on the **fine-structure** of the transitions.
- Reflection seismology lives by virtue of singularities.
- *How can we obtain information on the fine structure?*
- *How is this fine structure related to the underlying physical processes?*

Recovery from incomplete data


1950
1860
0
0.5
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0.4
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2000
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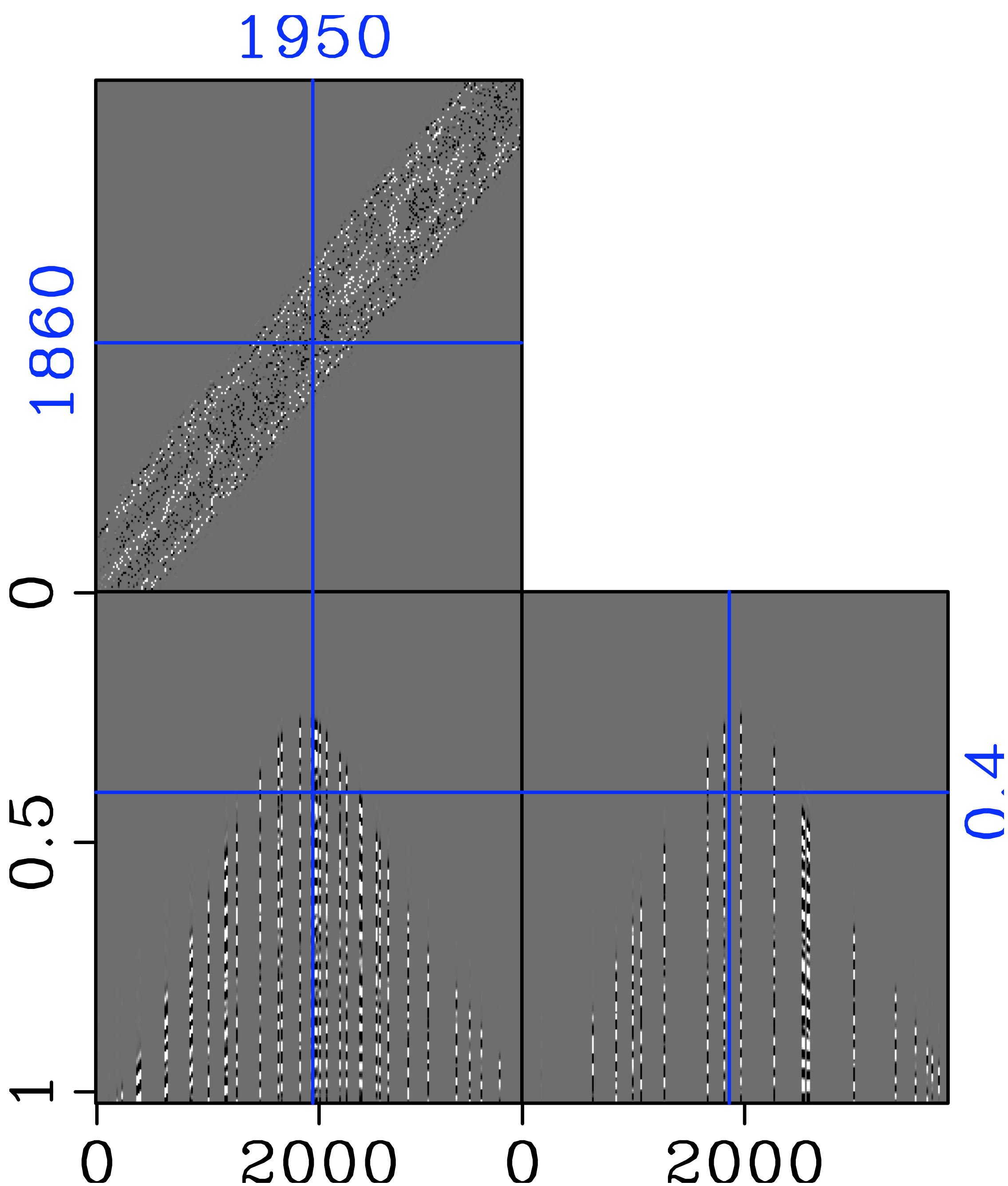
2000



1950
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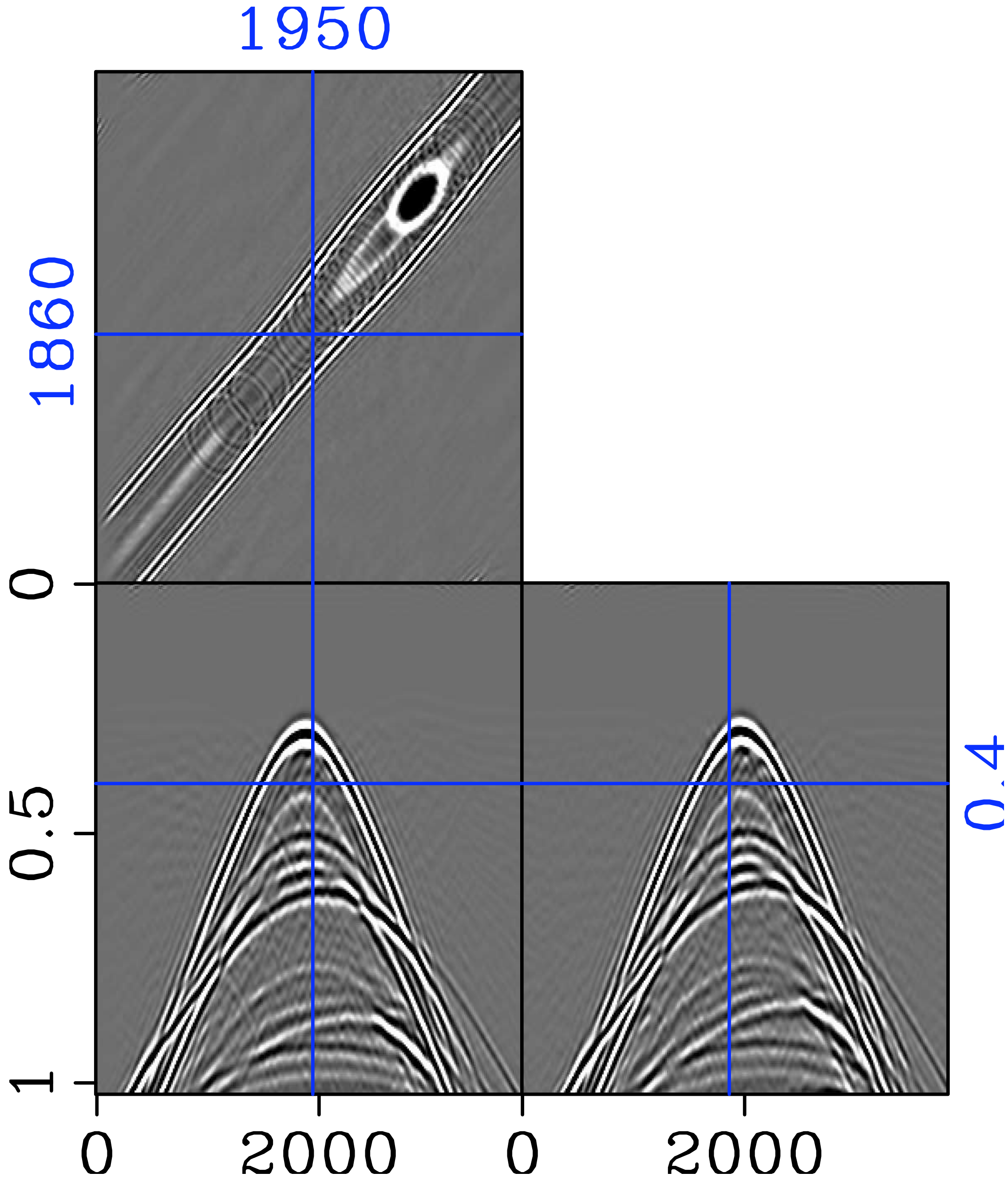
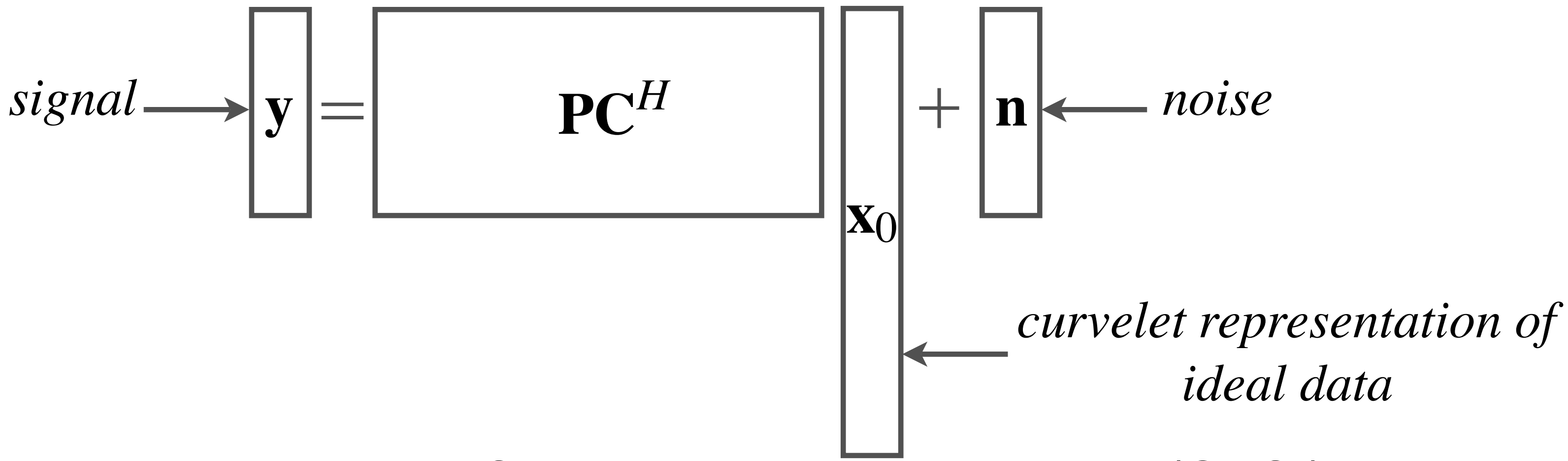


1950
1860
0
0.5
1
0.4
0
2000
0
2000

Sparsity-promoting inversion*

- reformulation of the problem



- Curvelet Reconstruction with Sparsity-promoting Inversion (CRSI)
  - look for the **sparsest/most compressible**, **physical** solution

KEY POINT OF THE RECOVERY

$$
(P_1)\begin{cases}
\tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \overbrace{\|\mathbf{W}\mathbf{x}\|_1}^{\text{sparsity constraint}} \quad \text{s.t.} \quad \overbrace{\|\mathbf{y} - \mathbf{P}\mathbf{C}^H\mathbf{x}\|_2}^{\text{data misfit}} \leq \varepsilon \\
\tilde{\mathbf{f}} = \mathbf{C}^H\tilde{\mathbf{x}}
\end{cases}
$$

* inspired by work on Impainting by Elad et. al., Stable Signal Recovery (SSR) theory by E. Candès, J. Romberg, T. Tao, Compressed sensing by D. Donoho & Fourier Reconstruction with Sparse Inversion (FRSI) by P. Zwartjes

Observations

- Recovery possible for > 80 % data missing
- Works because
  - exploit the high dimensional geometry
  - randomness of sampling that breaks the aliasing
- Uses ideas from compressive sampling.
- Is “impressive” since we “solve” a norm-one problem with 2^30 unknowns.

Selection of the sparsity representation

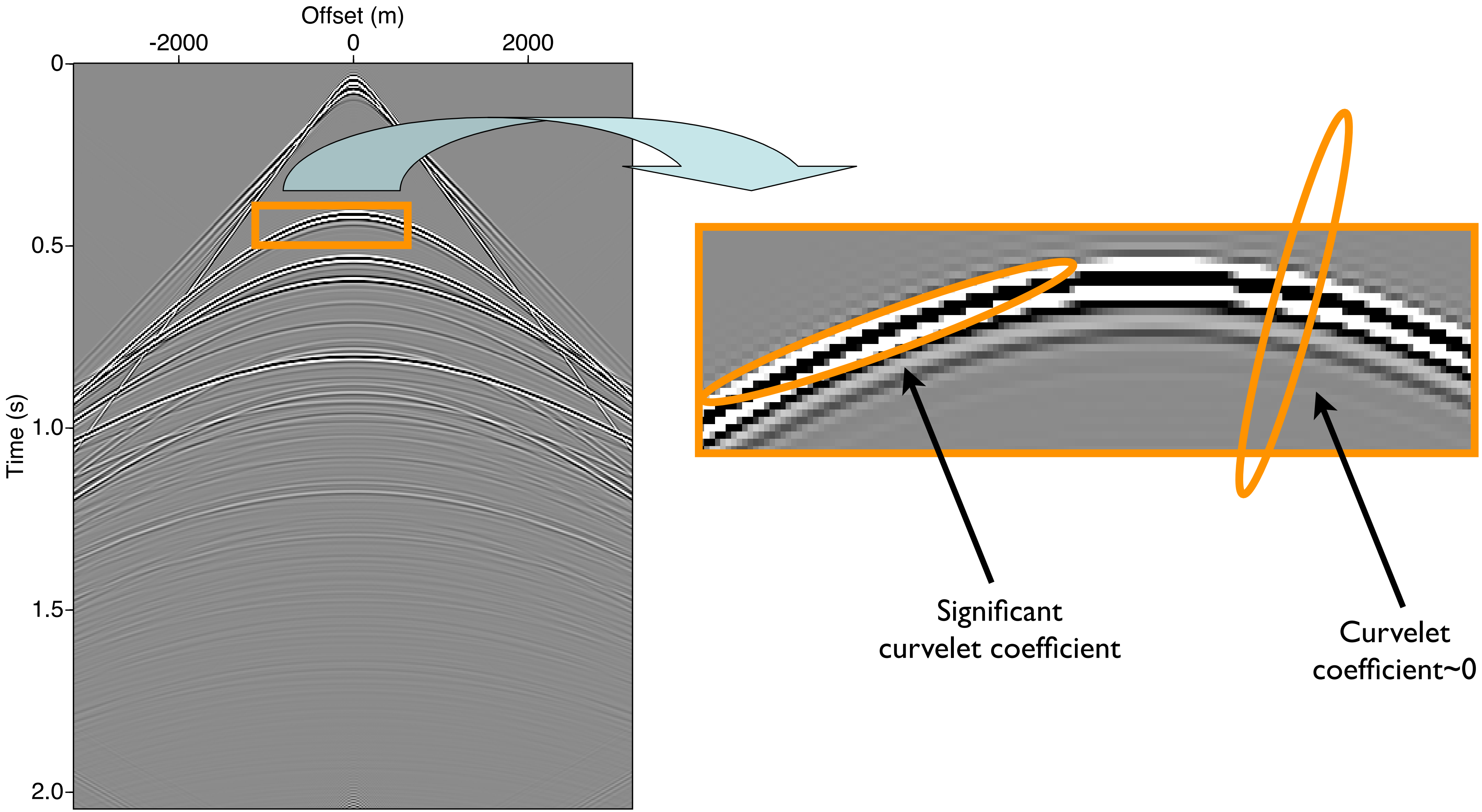
Problem

Find a representation that is compressible for seismic data & images

- multiscale & multidirectional
- intermittent regularity (caustics and pinch outs)
- certain invariance properties

Contains wavefronts that are smooth in the tangential direction and oscillatory in the normal direction.

Wavefront detection



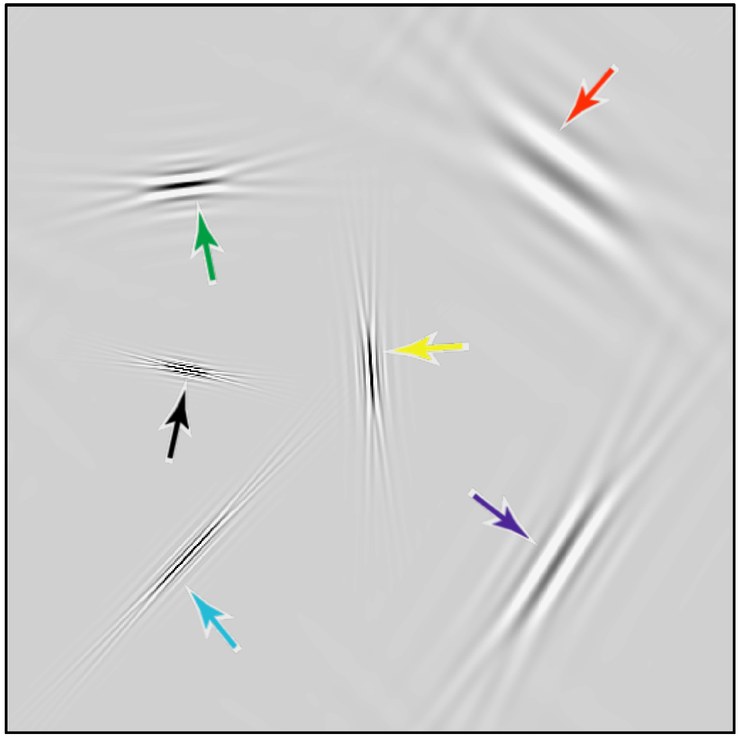
Curvelets

[Candes & Donoho '02-'05, Do '02, Demanet '05, Ying '05]

Partitioning example

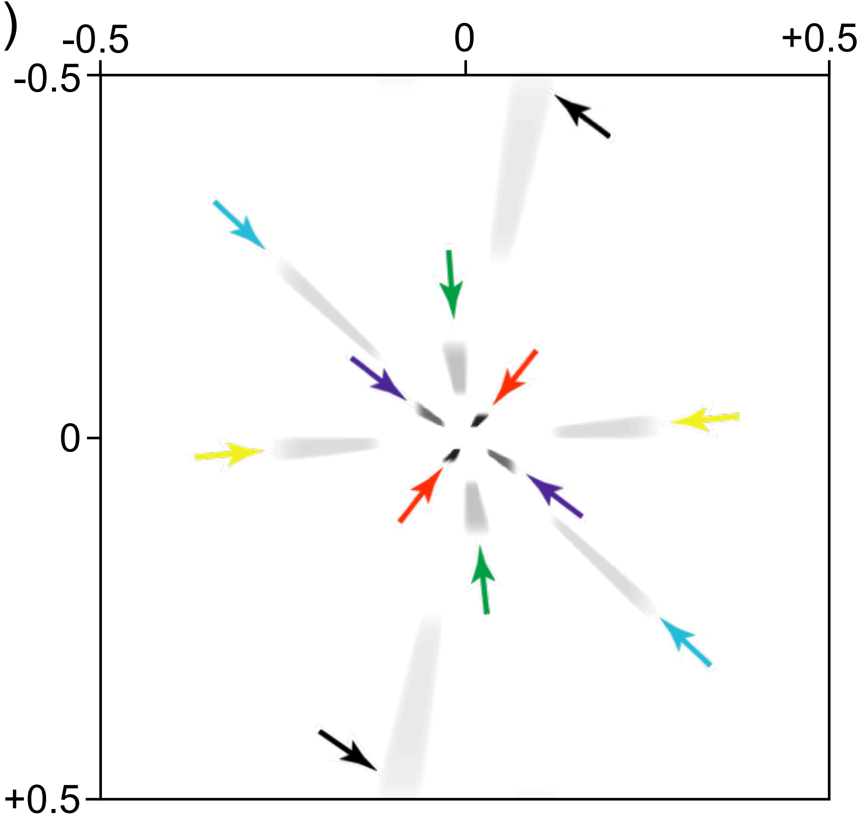
Curvelet in the space domain

(a)



Curvelet in the Fourier domain

(b)



Micro-local correspondence

Curvelets

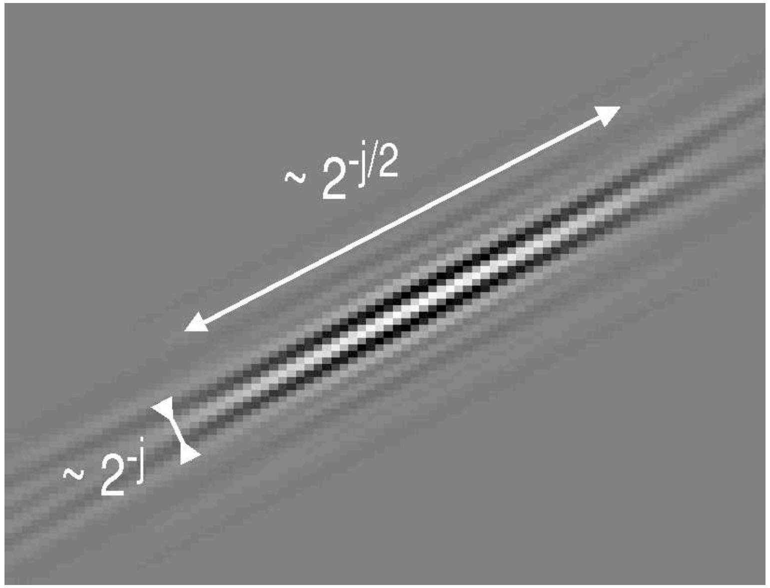
[Candes & Donoho '02-'05, Do '02, Demanet '05, Ying '05]

Collection of wave packets $\varphi_\mu(x)$, $x \in \mathbb{R}^2$, indexed by the quadruple of integers $\mu = (j, k_1, k_2, \ell)$.

$$\varphi_\mu(x) \simeq 2^{3j/4} \varphi(D_j R_{\theta_\ell} x - k),$$

$$D_j = \begin{pmatrix} 2^j & 0 \\ 0 & 2^{j/2} \end{pmatrix},$$

$$\theta_\ell \simeq \ell \cdot 2^{-\lfloor j/2 \rfloor}.$$



Tight frame: $f = \sum_\mu \langle f, \varphi_\mu \rangle \varphi_\mu$.

Compression

Interested in functions discontinuous along a piecewise smooth (C^2) interface, and otherwise smooth (C^2).

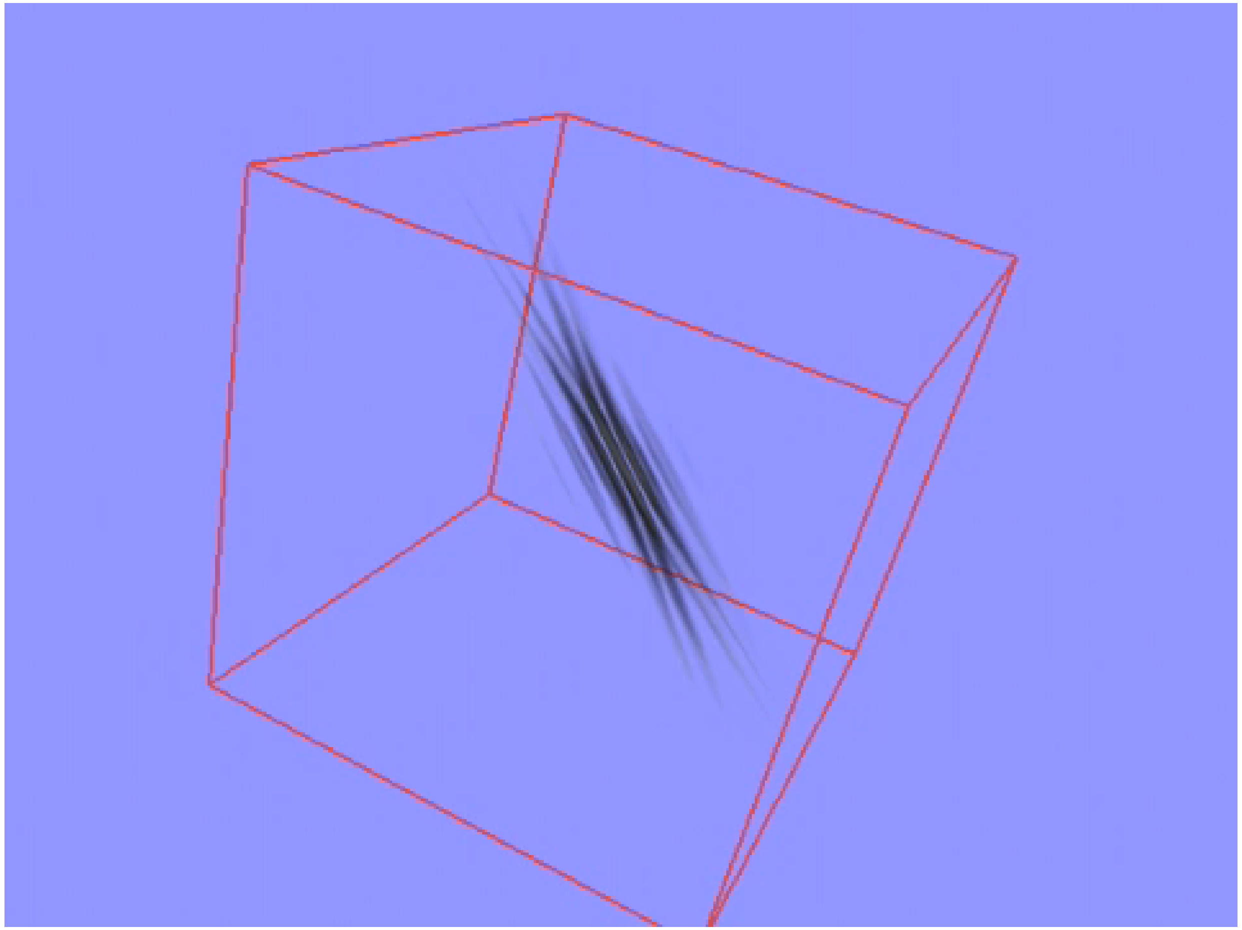
Theorem (Candès, Donoho). For such a model f, the best m-term curvelet expansion f_m obeys

$$||f - f_m||^2 \leq C m^{-2} (\log m)^3.$$

Note: wavelets would give $O(m^{-1})$, so do ridgelets (Candès).

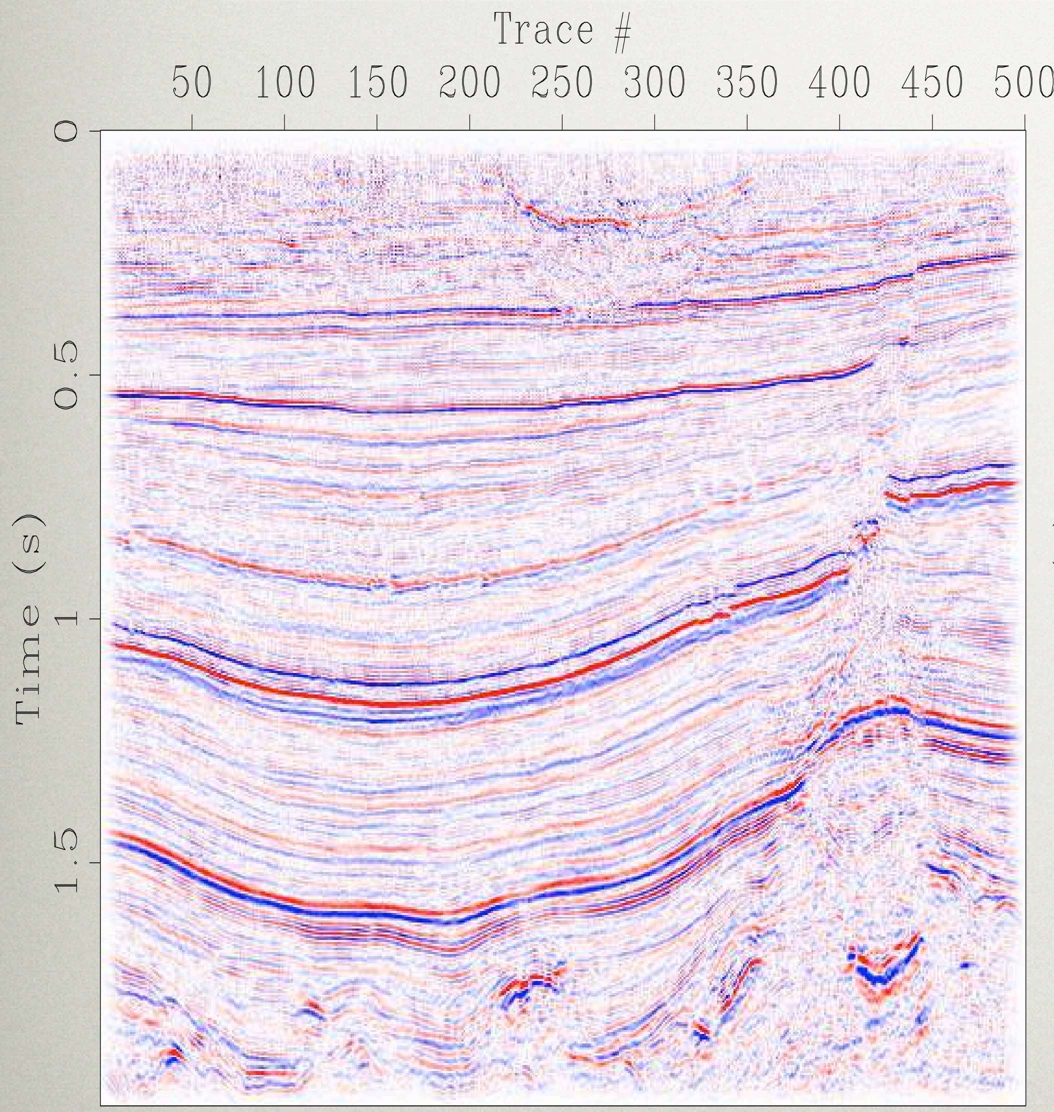
[From Demanet '05]

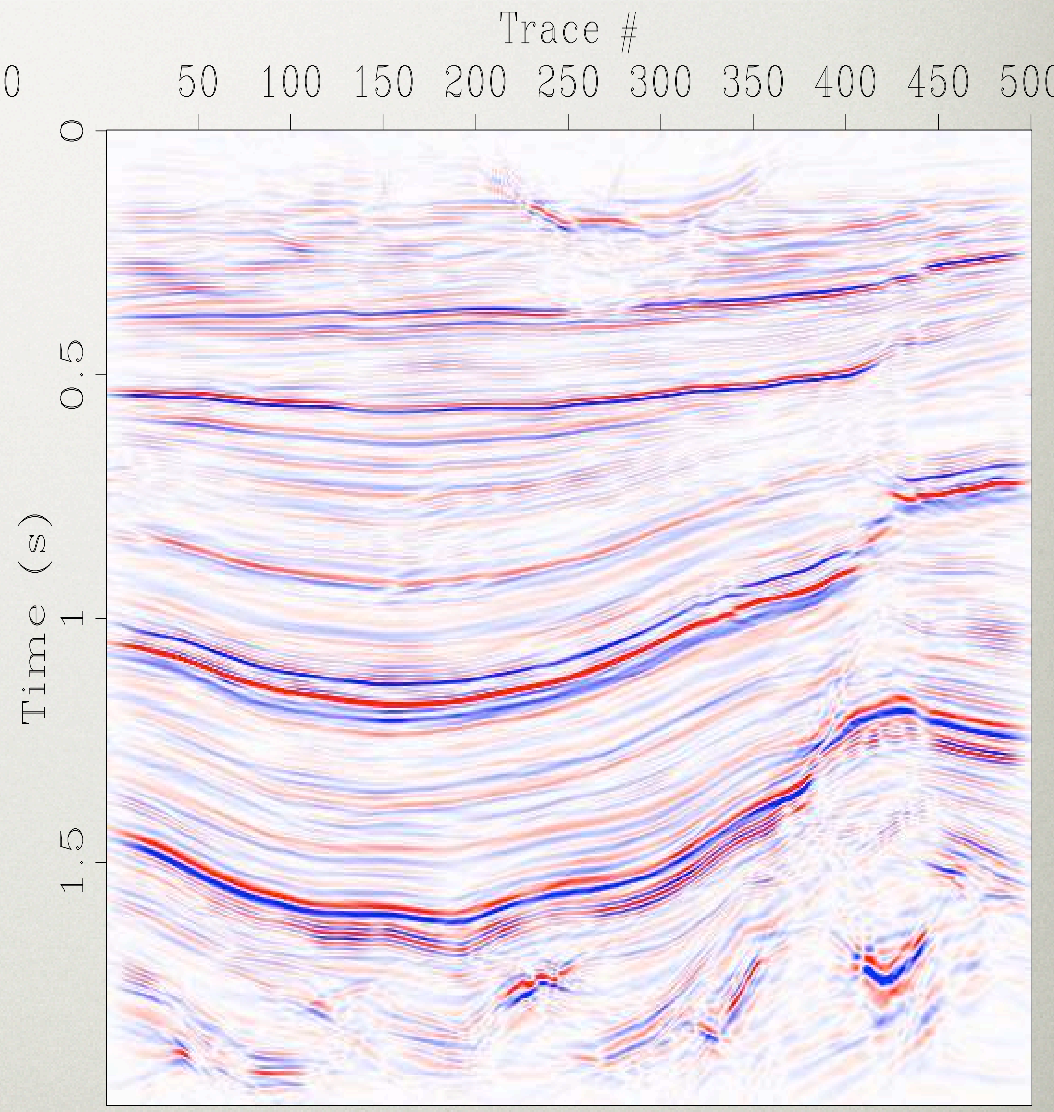
3-D curvelets



Curvelets live in wedges in the 3 D Fourier plane...

Partial Reconstruction
Curvelets (1% largest coefficients)





SNR = 6.0 dB

Observations

- **Curvelets:**
  - are multiscale, multi-angular & anisotropic
  - detect the ‘wavefront set’
  - “invariant” under wave propagation
- **Ideal representation for seismology**

Imaging singularities

- An optimal true-amplitude least-squares prestack depth-migration operator [Chavent & Plessix, 99]
- Frequency-domain finite difference amplitude preserving migration [Plessix & Mulder, 99]
- A microlocal analysis of migration [ten Kroode, Verdel & Smit, 98]
- TR 06-18: Reverse time migration with optimal checkpointing [Symes 2007]
- TR 06-19: Optimal Scaling for Reverse Time Migration [Symes 2007]
- The Curvelet Representation of Wave Propagators is Optimally Sparses [Demanet and Candes 2005]

Forward problem

$$F[c]u := \left(\frac{1}{c^2(x)} \cdot \frac{\partial^2}{\partial t^2} - \sum_{i=1}^{d} \frac{\partial^2}{\partial x_1^2} \right) \mathbf{u}(x,t) = f(x,t)$$

- second order hyperbolic PDE
- interested in the singularities of

$$m = c - \bar{c}$$

Inverse problem

Minimization:

$$\widetilde{m} = \arg\min_m \|d - F[m]\|_2^2$$

After linearization (Born app.) forward model with noise:

$$d(x_s, x_r, t) = \big(Km\big)(x_s, x_r, t) + n(x_s, x_r, t)$$

Conventional imaging:

$$\begin{aligned} \big(K^T d\big)(x) &= \big(K^T K m\big)(x) + \big(K^T n\big)(x) \\ y(x) &= \big(\mathbf{\Psi} m\big)(x) + e(x) \end{aligned}$$

Approximation

So let $\mathbf{\Psi} = \mathbf{\Psi}(x, D)$ be a pseudodifferential operator of order 0, with homogeneous principal symbol $a(x, \xi)$.

$$K \mapsto K\,(-\Delta)^{-1/2} \quad \textbf{or} \quad K \mapsto \partial_t^{-1/2} K$$

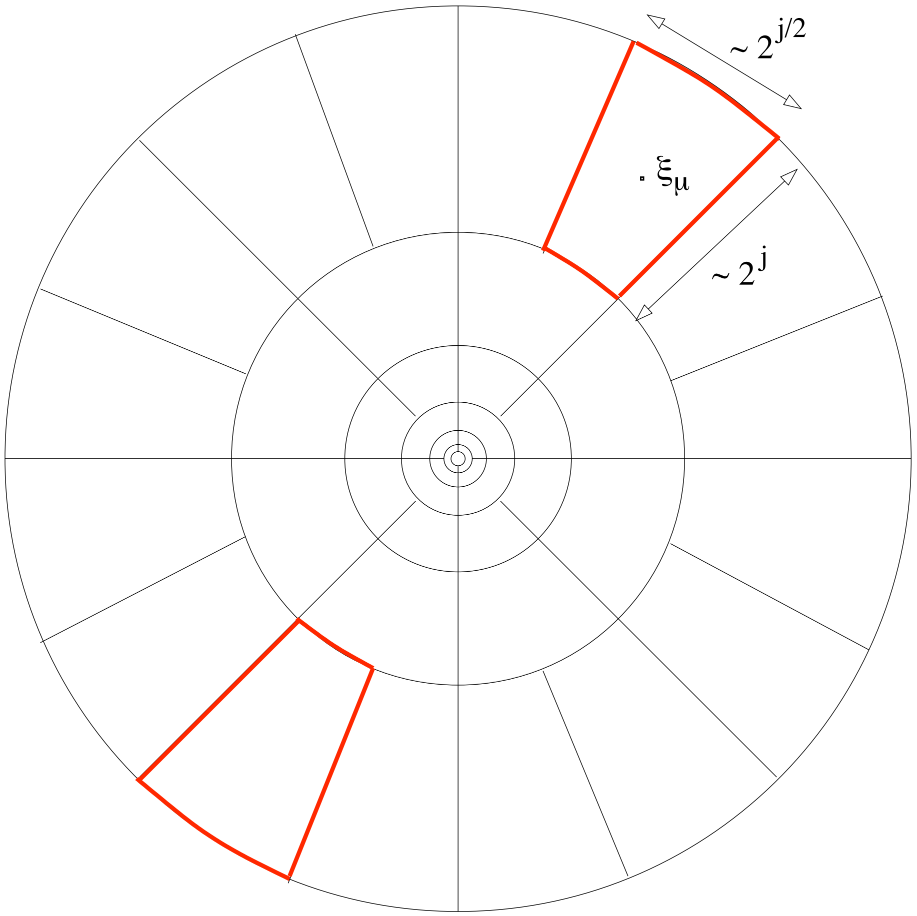
$$m \mapsto (-\Delta)^{1/2}\, m \quad \textbf{with} \quad ((-\Delta)^{\alpha} f)^{\wedge}(\xi) = |\xi|^{2\alpha} \cdot \hat{f}(\xi).$$

Lemma 1. *With C' some constant, the following holds*

$$\|(\Psi(x, D) - a(x_\nu, \xi_\nu))\varphi_\nu\|_{L^2(\mathbb{R}^n)} \leq C' 2^{-|\nu|/2}. \tag{14}$$

To approximate Ψ, we define the sequence $\mathbf{u} := (u_\mu)_{\mu \in \mathcal{M}} = a(x_\mu, \xi_\mu)$. Let $\mathbf{D}_\Psi$ be the diagonal matrix with entries given by $\mathbf{u}$. Next we state our result on the approximation of Ψ by $C^T \mathbf{D}_\Psi C$.

Tiling the ξ space



Approximation

Theorem 1. *The following estimate for the error holds*

$$\|(\Psi(x,D) - C^T\mathbf{D}_\Psi C)\varphi_\mu\|_{L^2(\mathbb{R}^n)} \leq C''2^{-|\mu|/2},$$

where C'' *is a constant depending on* Ψ*.*

Allows for the decomposition

$$\begin{aligned}\big(\Psi\varphi_\mu\big)(x) &\simeq \big(C^T\mathbf{D}_\Psi C\varphi_\mu\big)(x)\\ &= \big(AA^T\varphi_\mu\big)(x)\end{aligned}$$

with $A := \sqrt{\mathbf{D}_\Psi}C$ and $A^T := C^T\sqrt{\mathbf{D}_\Psi}$.

Recovery

Final form

$$\mathbf{y} = \mathbf{A}\mathbf{x}_0 + \boldsymbol{\varepsilon}$$

with $\mathbf{x}_0 = \mathbf{\Gamma C m}$ and $\boldsymbol{\epsilon} = \mathbf{A e}$.

Solve

$$\mathbf{P}: \quad \begin{cases} \min_{\mathbf{x}} J(\mathbf{x}) \quad \text{subject to} \quad \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2 \le \epsilon \\ \\ \tilde{\mathbf{m}} = (\mathbf{A}^{\mathbf{H}})^{\dagger}\tilde{\mathbf{x}} \end{cases}$$

with

$$J(\mathbf{x}) = \overbrace{\alpha\|\mathbf{x}\|_1}^{\text{sparsity}} + \beta \underbrace{\|\mathbf{\Lambda}^{1/2}\left(\mathbf{A}^{H}\right)^{\dagger}\mathbf{x}\|_p}_{\text{continuity}}.$$

Image recovery
anisotropic diffusion

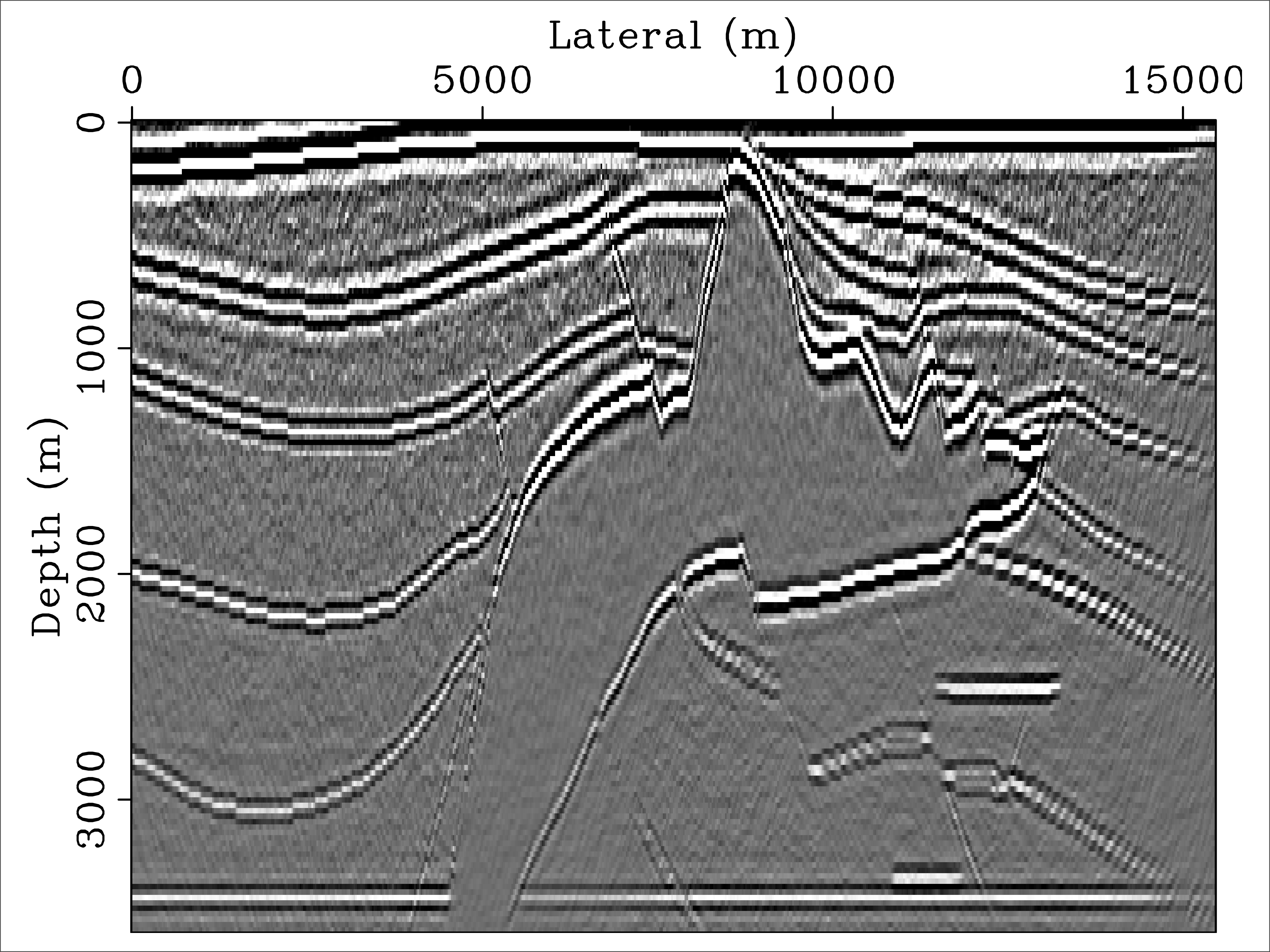
[Black et. al ’98, Fehmers et. al. ’03 and Shertzer ‘03]

Define

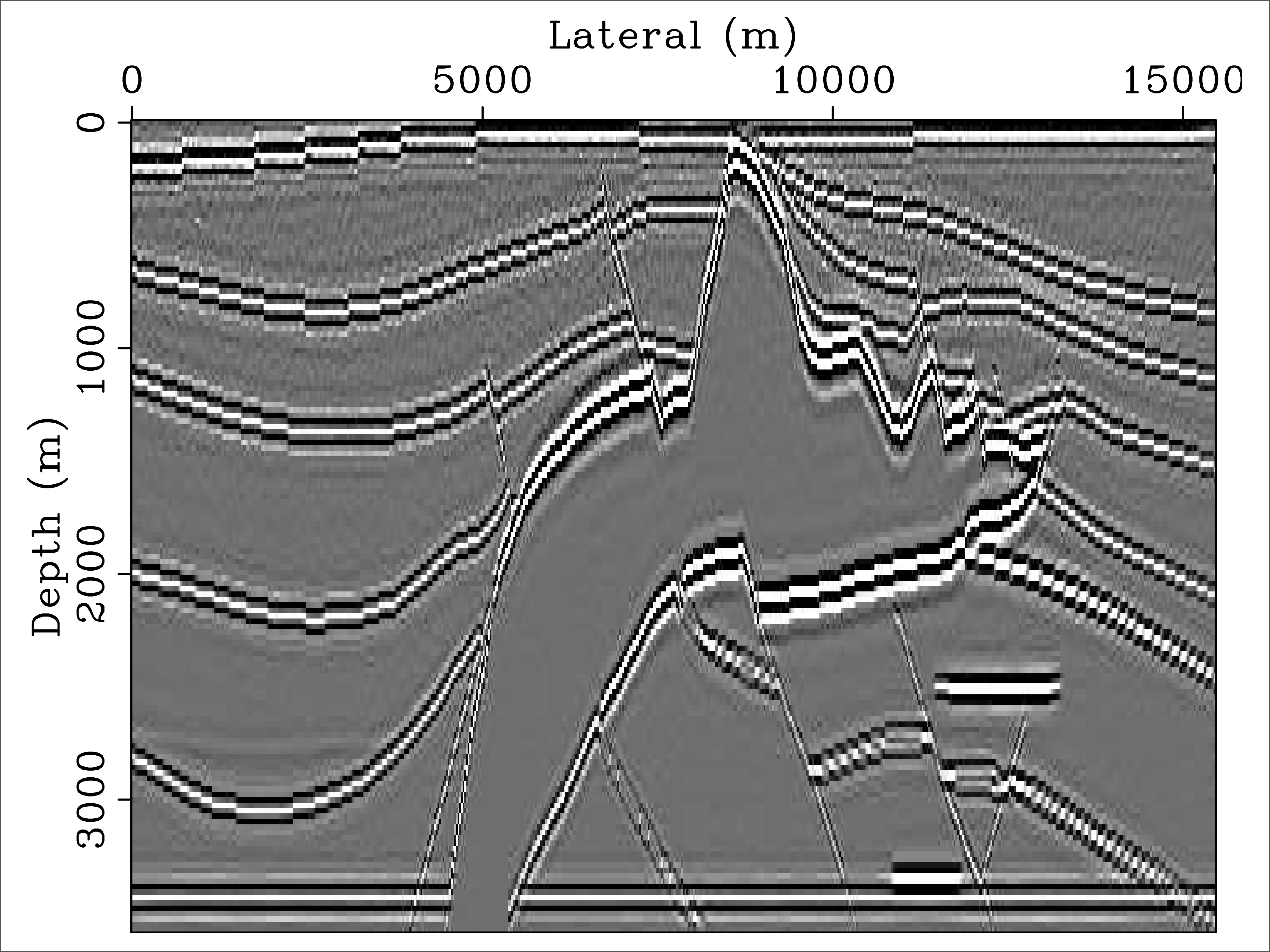
$$J_c(\mathbf{m}) = \|\boldsymbol{\Lambda}^{1/2}\boldsymbol{\nabla}\mathbf{m}\|_p$$

with p=2

$$\boldsymbol{\Lambda}[\mathbf{r}] = \frac{1}{\|\boldsymbol{\nabla}\mathbf{r}\|_2^2 + 2\upsilon}\left\{\begin{pmatrix} +\mathbf{D}_2\mathbf{r} \\ \\ -\mathbf{D}_1\mathbf{r}\end{pmatrix}\begin{pmatrix} +\mathbf{D}_2\mathbf{r} & -\mathbf{D}_1\mathbf{r}\end{pmatrix} + \upsilon\mathbf{Id}\right\}$$



Lateral (m)
0
5000
10000
15000
Depth (m)
0
1000
2000
3000



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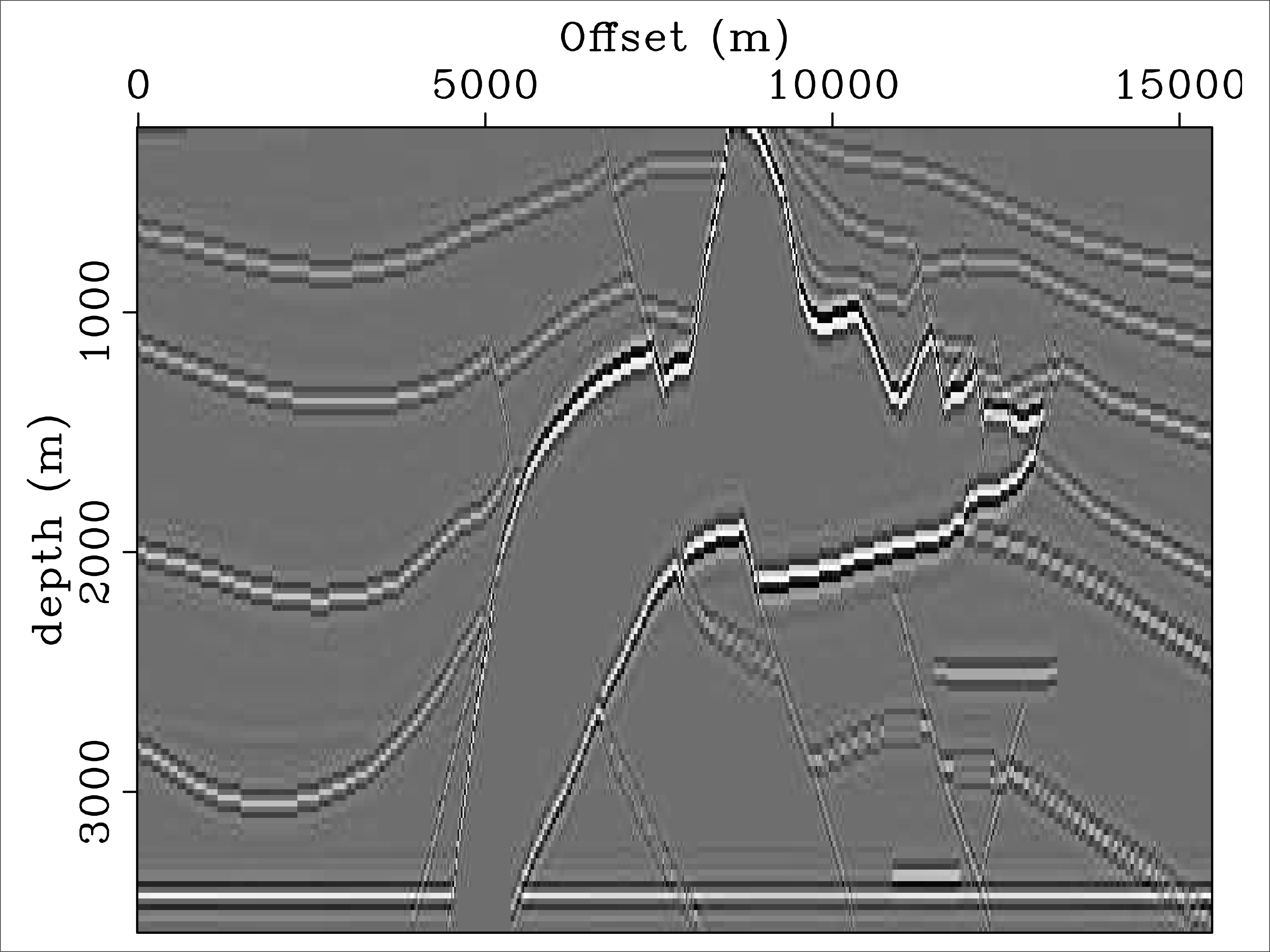

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depth (m)
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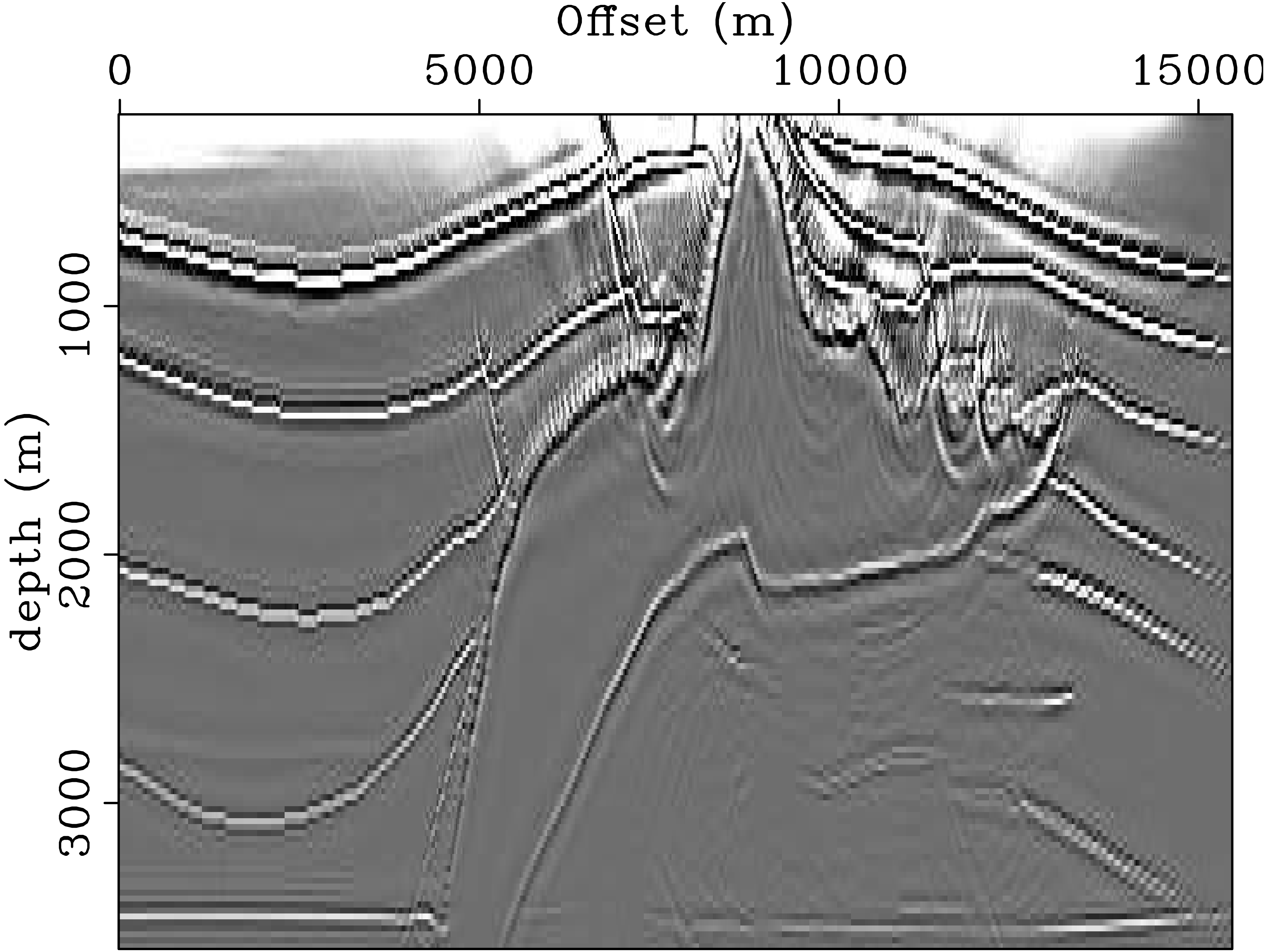
Offset (m)
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depth (m)
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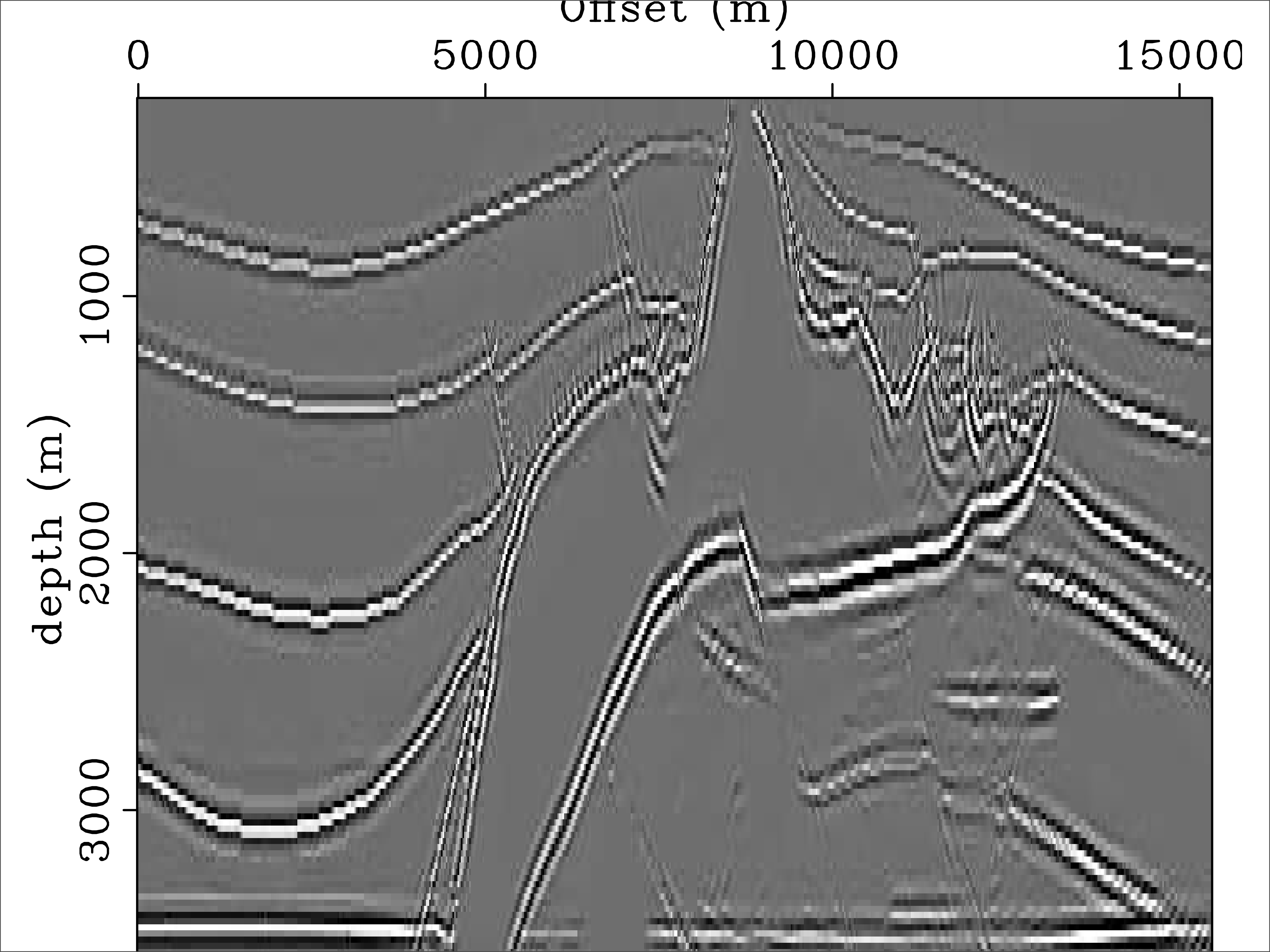

Offset (m)
0
5000
10000
15000
depth (m)
1000
2000
3000

Observations

- Curvelet invariance and sparsity leads to an improved recovery.
- Singularities are preserved during imaging.
- Aside from curvelet sparsity finding appropriate penalty functionals are an open problem.
- Synthetic examples have a singularity structure that is too restrictive.

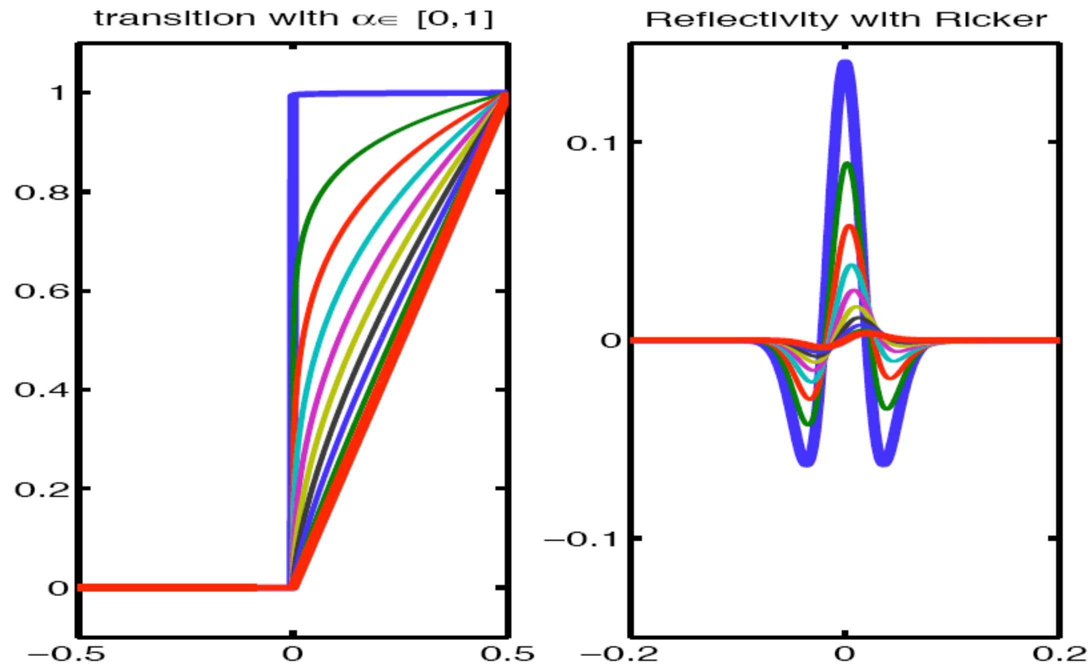
Characterizing singularities

Problem

- Delineate the structure (stratigraphy) from seismic images.
- Parameterize seismic transitions.
- Estimate the parameters from seismic images:
  - location
  - singularity order
  - instantaneous phase

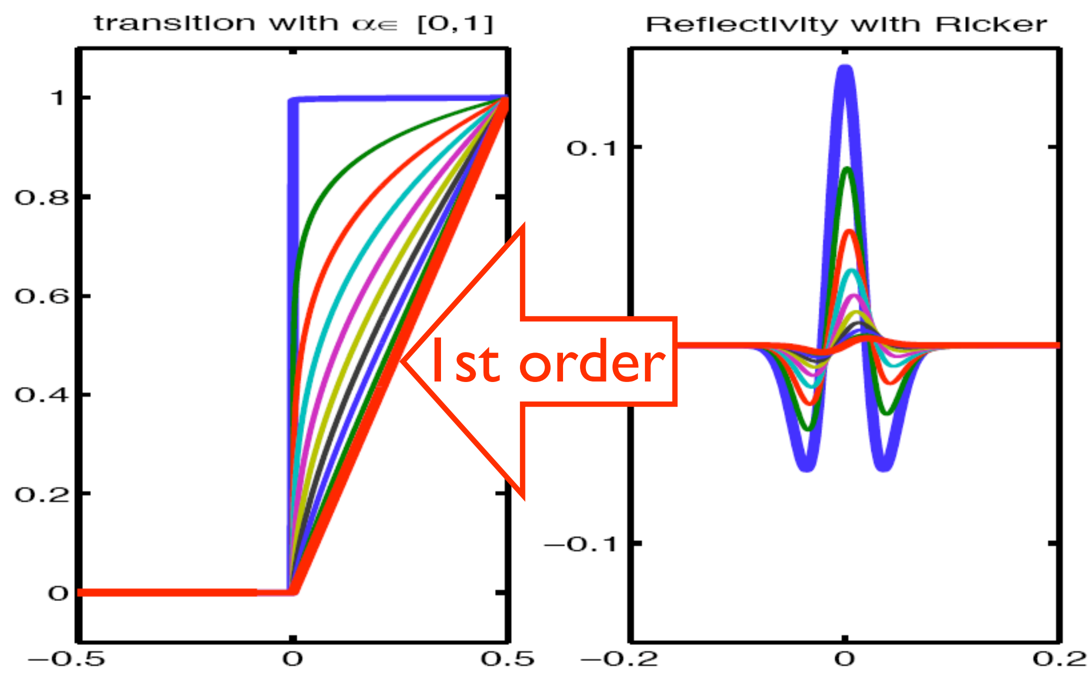
Singularity characterization

through waveforms



- **generalization of zero- & first-order discontinuities**
- **measures wigglyness / # oscilations / sharpness**

Singularity characterization
through waveforms



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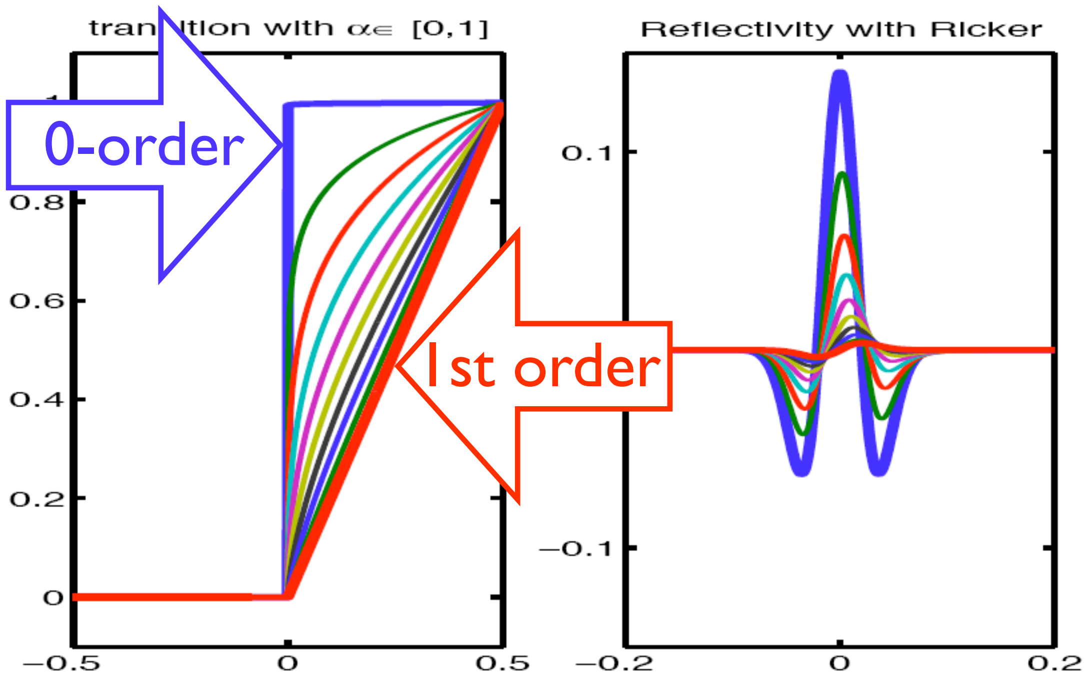
Singularity characterization
through waveforms



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- **measures wigglyness / # oscilations / sharpness**

Parameterization

Consider Earth as superposition of algebraic singularities

$$f(x) \triangleq \sum_{n \in N} c^n \chi^{\alpha_n}_{\pm,*}.(x - x_n)$$

with

$$\chi^{\alpha}_{+}(x) = \begin{cases} 0 & x \leq 0 \\ \frac{x^{\alpha}}{\Gamma(\alpha+1)} & x > 0 \end{cases}, \quad \chi^{\alpha}_{-}(x) = \begin{cases} 0 & x \geq 0 \\ \frac{x^{\alpha}}{\Gamma(\alpha+1)} & x < 0 \end{cases}$$

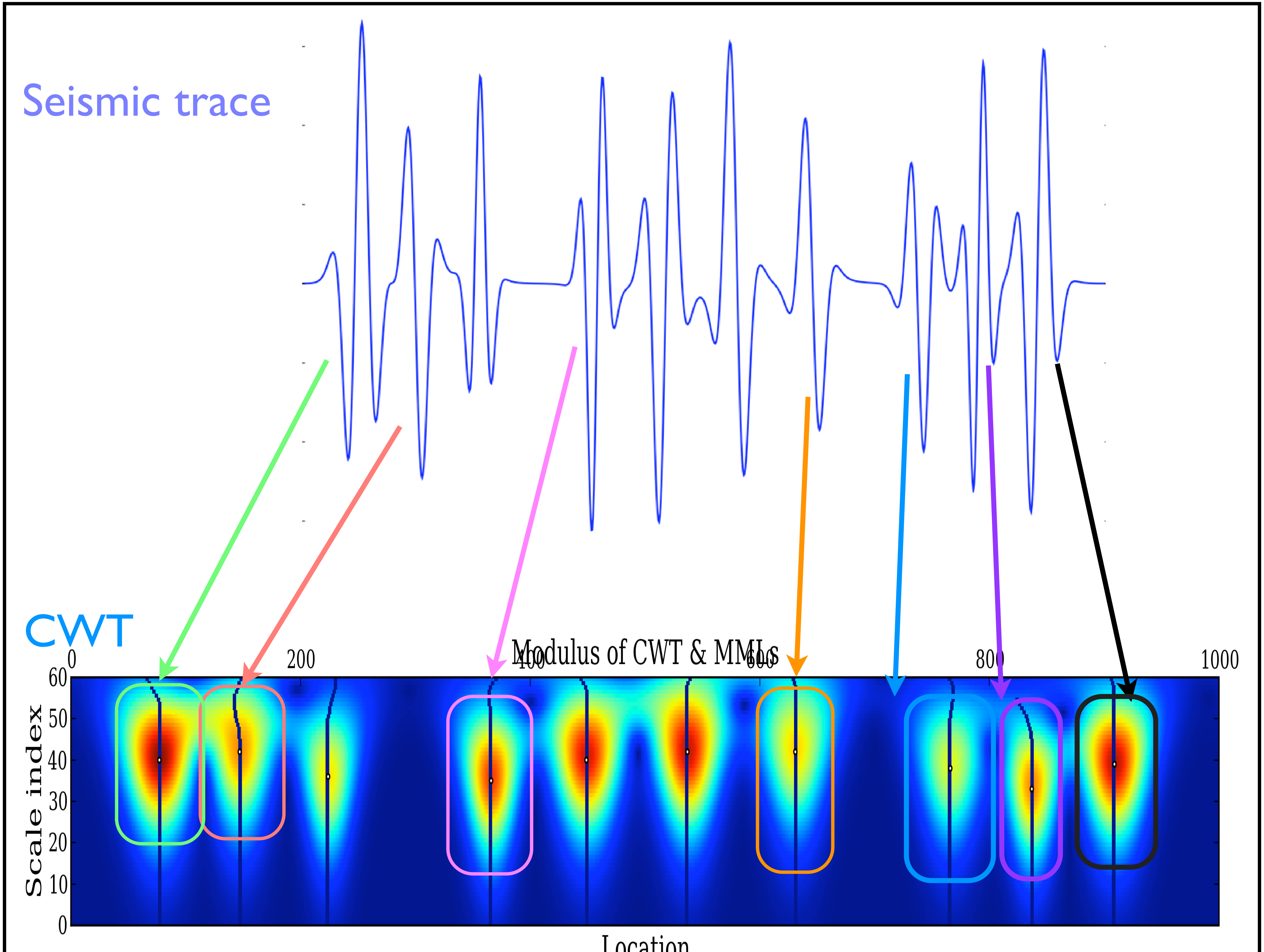
yielding (with $\varphi(x)$ the seismic wavelet)

$$d(x) = (r * \varphi)(x) \quad \text{with} \quad r(x) = \sum_{n \in N} c^n_{\alpha} \chi^{\alpha_n - 1}_{\pm,*}(x - x_n)$$

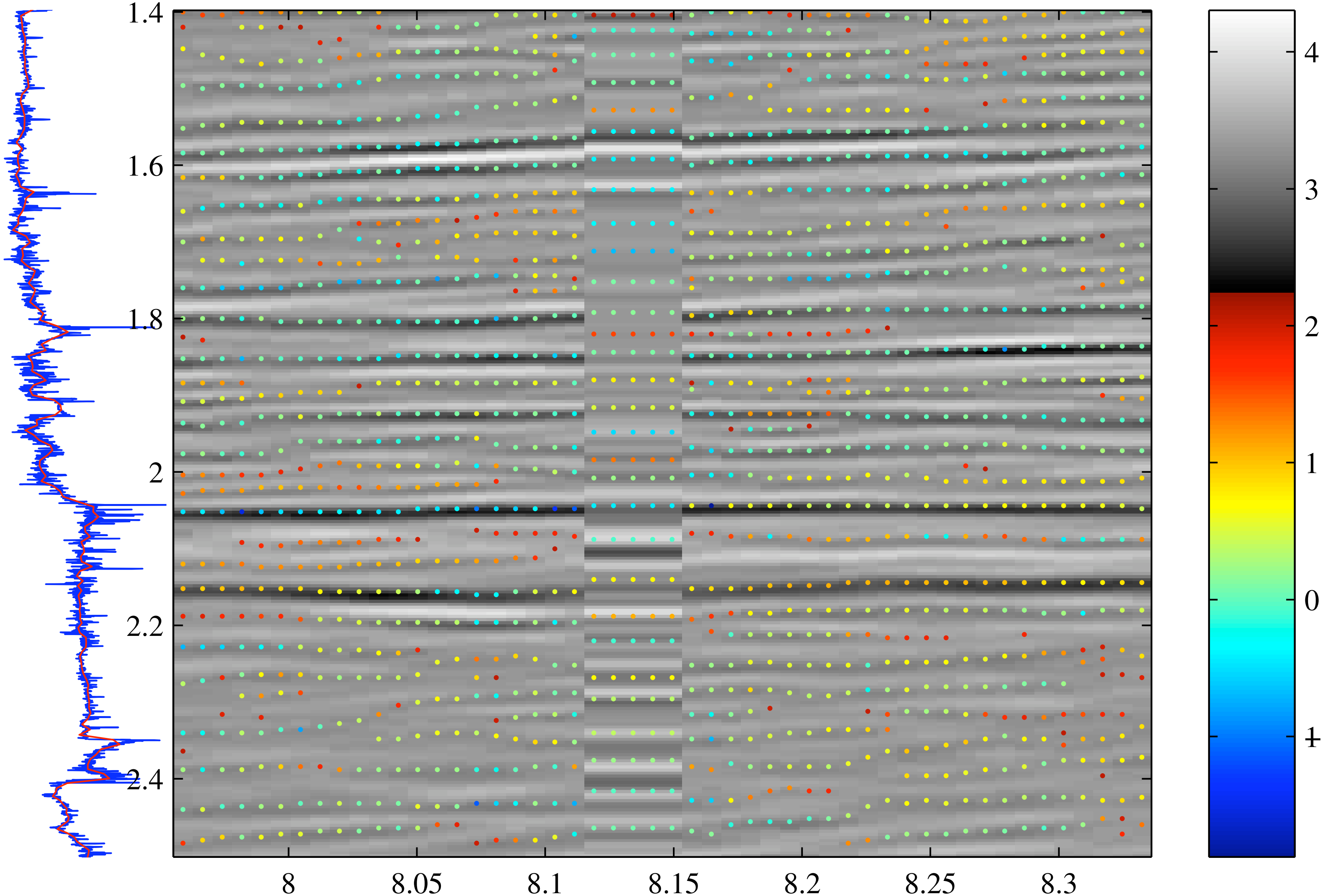
Approach

[Wakin et al ‘05-’-07, M&H ‘07]

- Use a detection-estimation technique
  - multiscale detection => segmentation
  - multiscale Newton technique to estimate the parameterization
- Overlay the image with the parametrization



Seismic trace
CWT
Modulus of CWT & MMLs
Scale index
Location
0
200
400
600
800
1000
60
50
40
30
20
10
0


1.4
1.6
1.8
2
2.2
2.4
8
8.05
8.1
8.15
8.2
8.25
8.3
4
3
2
1
0
-1

Observations

- Stratigraphy is detected
- Parameterization provides information on the lithology
- Method suffers from curvature in the imaged reflectors
- Extension to higher dimensions necessary
- Model that explains different types of transitions

Modeling singularities

Problem

Earth subsurface is highly heterogeneous

- sedimentary crust
- upper-mantle transition zone
- core-mantle boundary

Smooth relation volume fractions and rock properties.

Homogenization/equivalent medium (EM) theory *smoothes* the singularities during *upscaling*

- relatively *easy* for *volumetric* properties (density)
- *notoriously difficult* for *transport* properties (velocity)

Q: How to model transitions in effective properties?

Our approach

Include *connectivity* in models for the *effective* properties of bi-compositional mixtures **<=> SWITCH**

Start with *binary* mixtures, e.g.

- sand-shale
- gas-hydrate, opal
- upper-mantle mineralogy

Studied two cases:

- elastic properties upper mantle (H & B '04)
- fluid-flow properties synthetic rock (B & H '04)

Mixing model

Homogeneous mixing (e.g., solid solution) of **two** *phases* (**LP** weak and **HP** strong) can only produce ***gradually*** varying *elastic* properties.

Heterogeneous (e.g. *random macroscopic* inclusions) mixing, then a **singularity** in the elastic properties ***must*** arise at the depth where the **strong**, **HP** phase becomes **connected** (observed in binary alloys).

Site-percolation model

Assume volume fractions p and $q = 1 - p$, are **linear** functions of depth z.

At a **critical** depth z_c, which corresponds to the percolation threshold $p_c = p(z_c)$, an "infinite", **connected HP** *cluster* is formed.

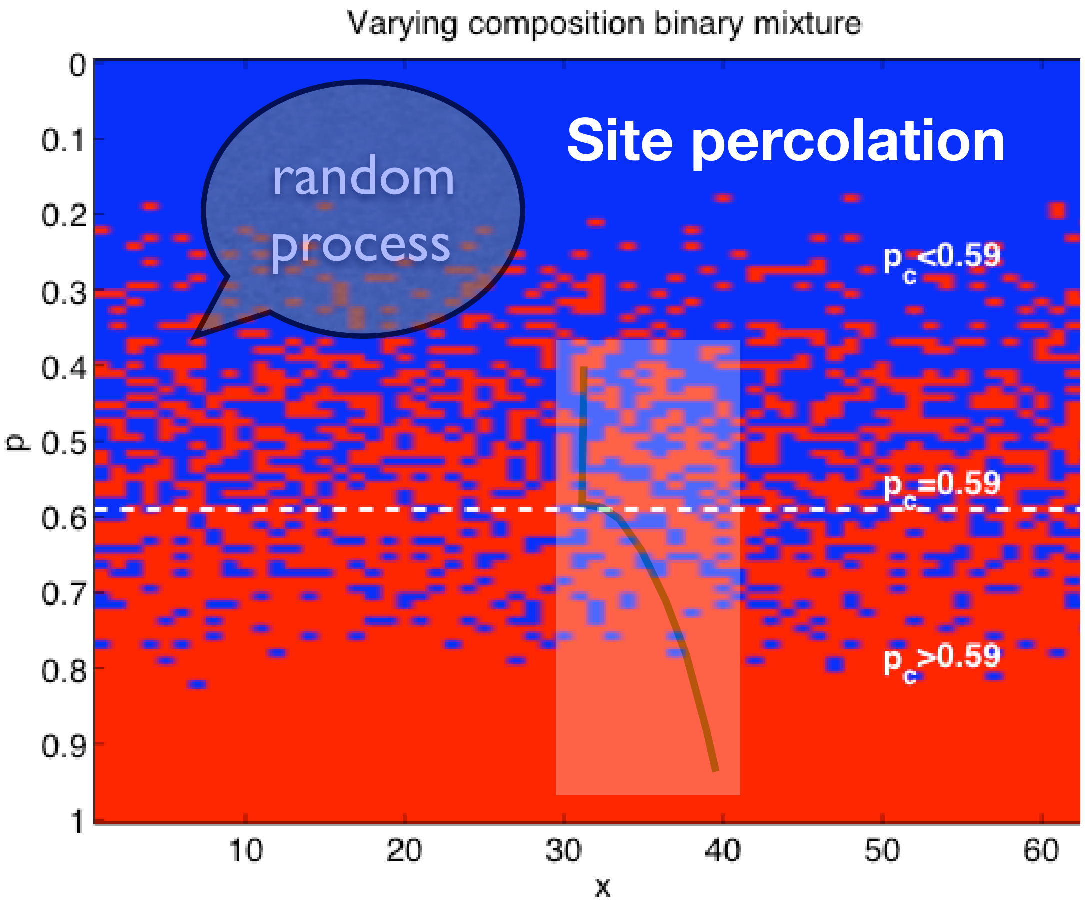
for $z \geq z_c$

—not all HP inclusions belong to the *infinite* cluster.

—isolated HP inclusions can still be found, embedded in the remaining LP material and forming with it a ***mixture*** (**M**).

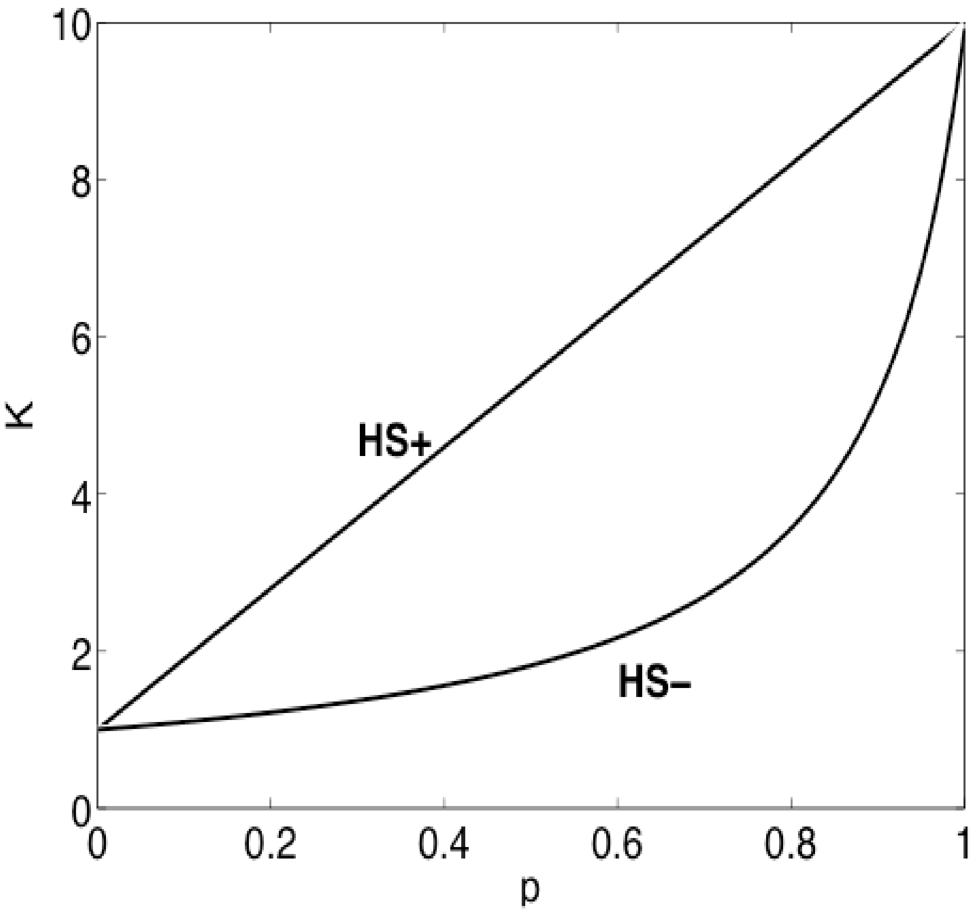
Site-percolation model

LP olivine



HP β-spinel

elastic properties

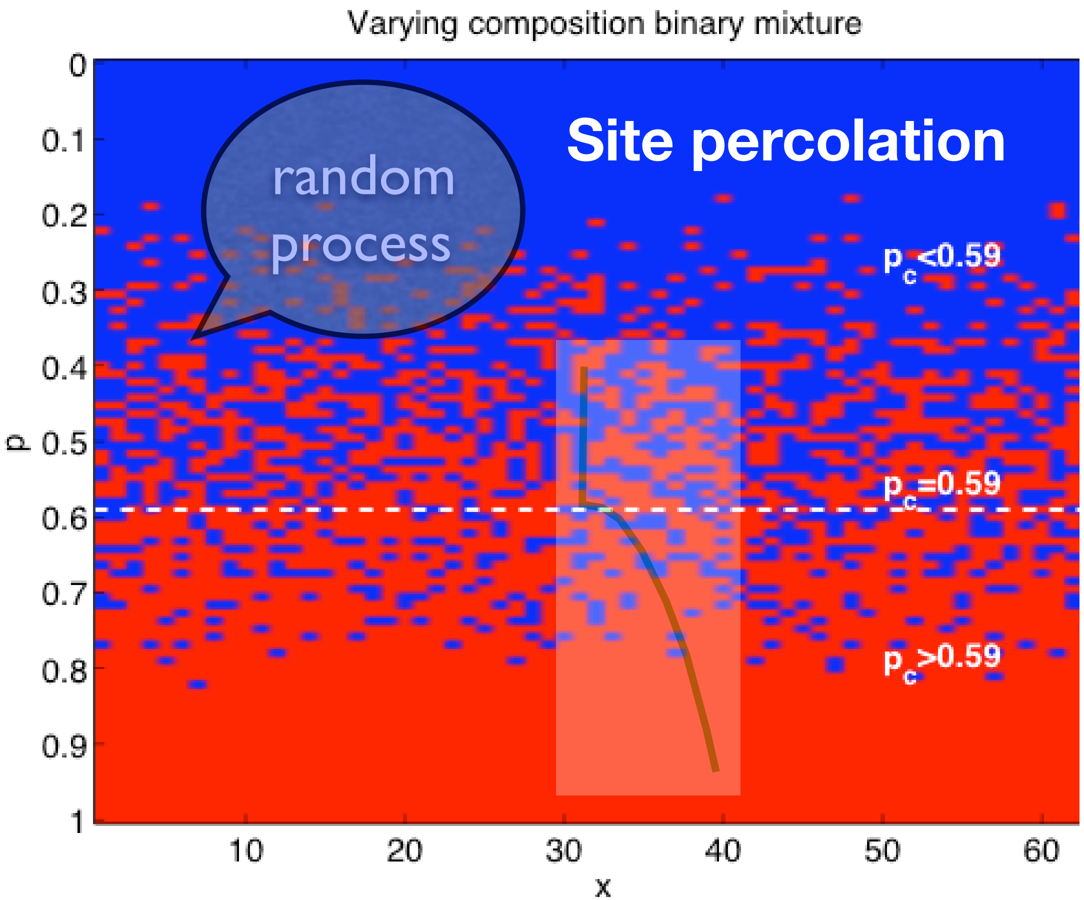


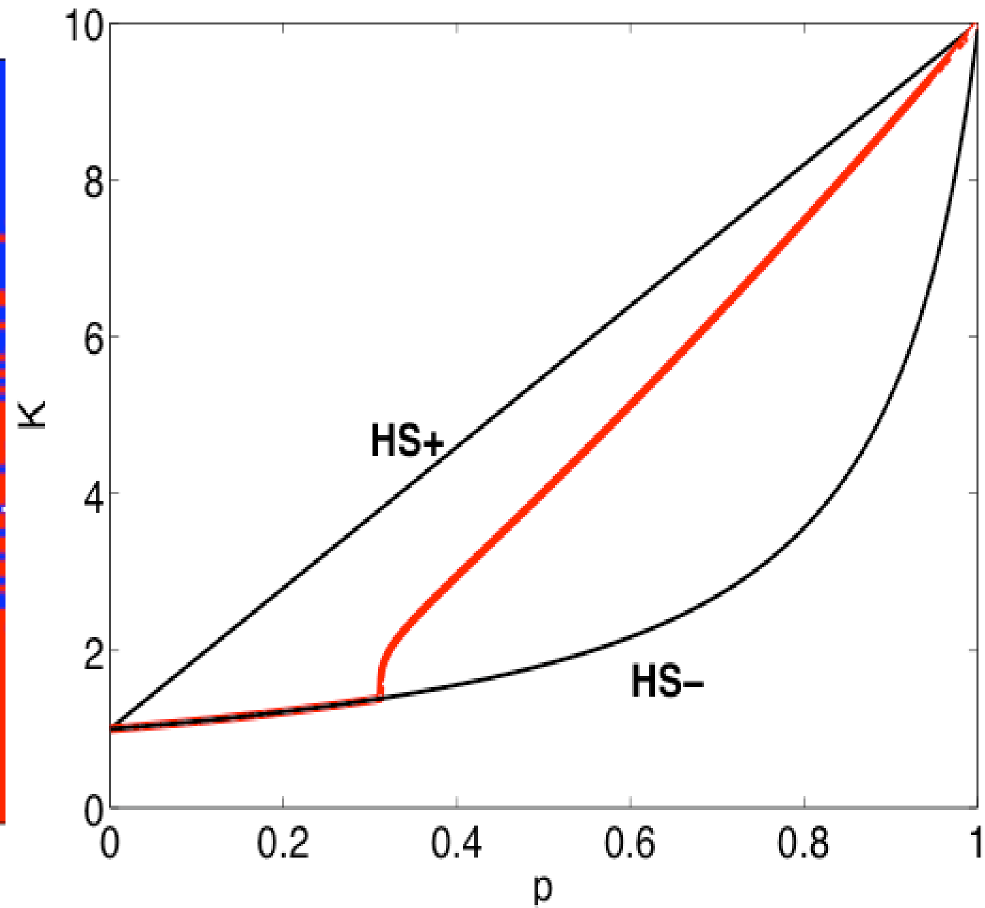
volume fraction

Site-percolation model

LP olivine

elastic properties

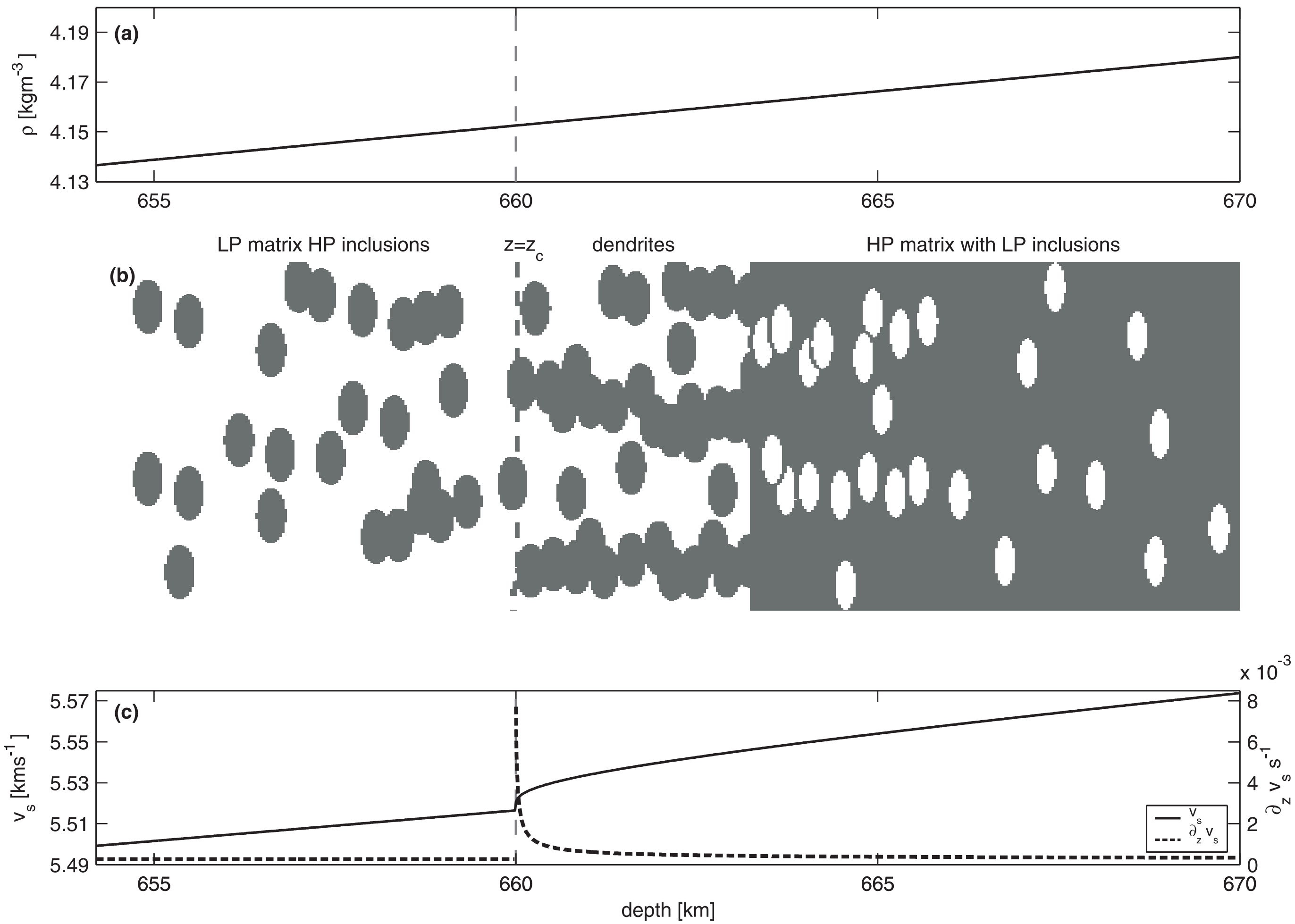




HP β-spinel

volume fraction

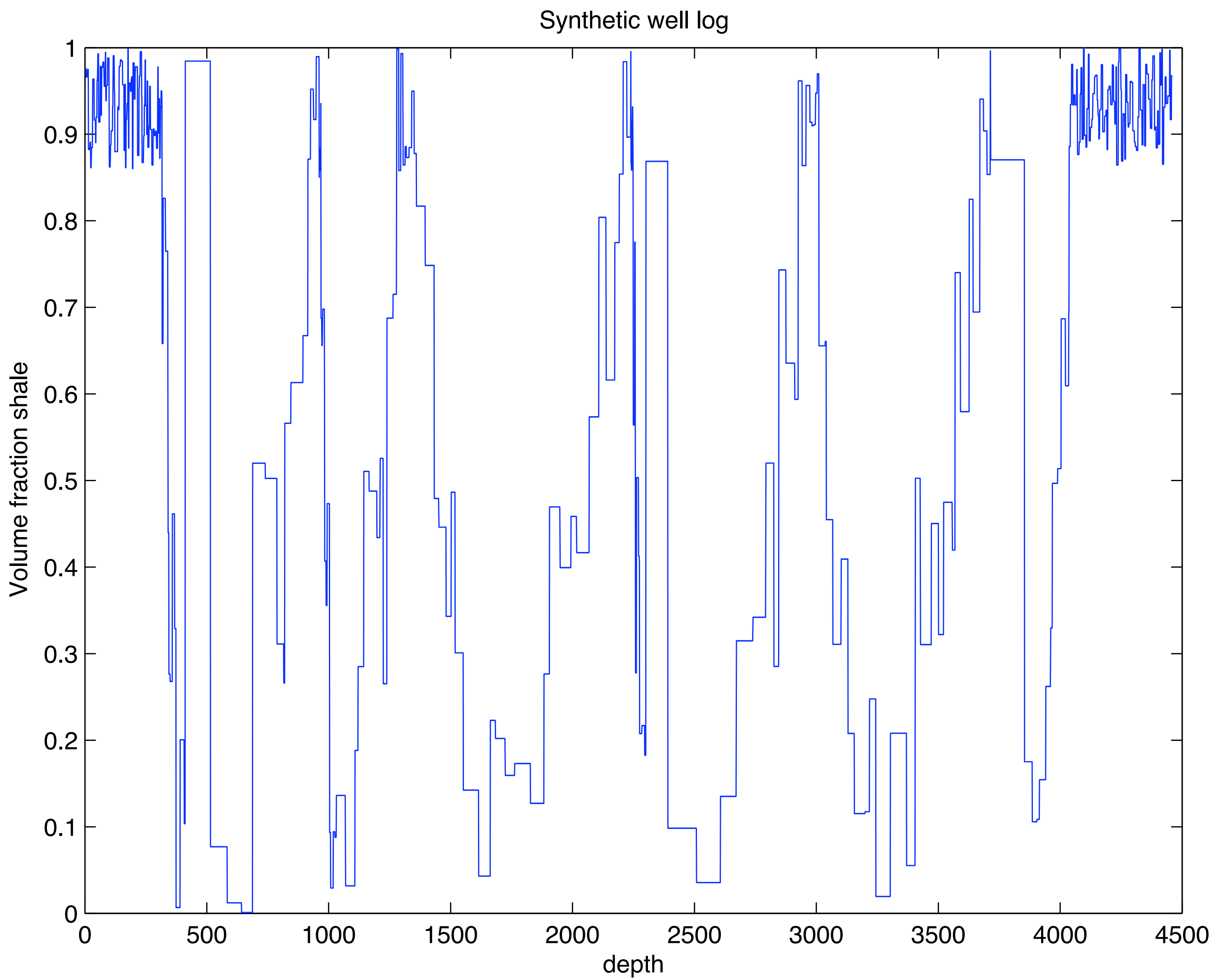
Singularity model



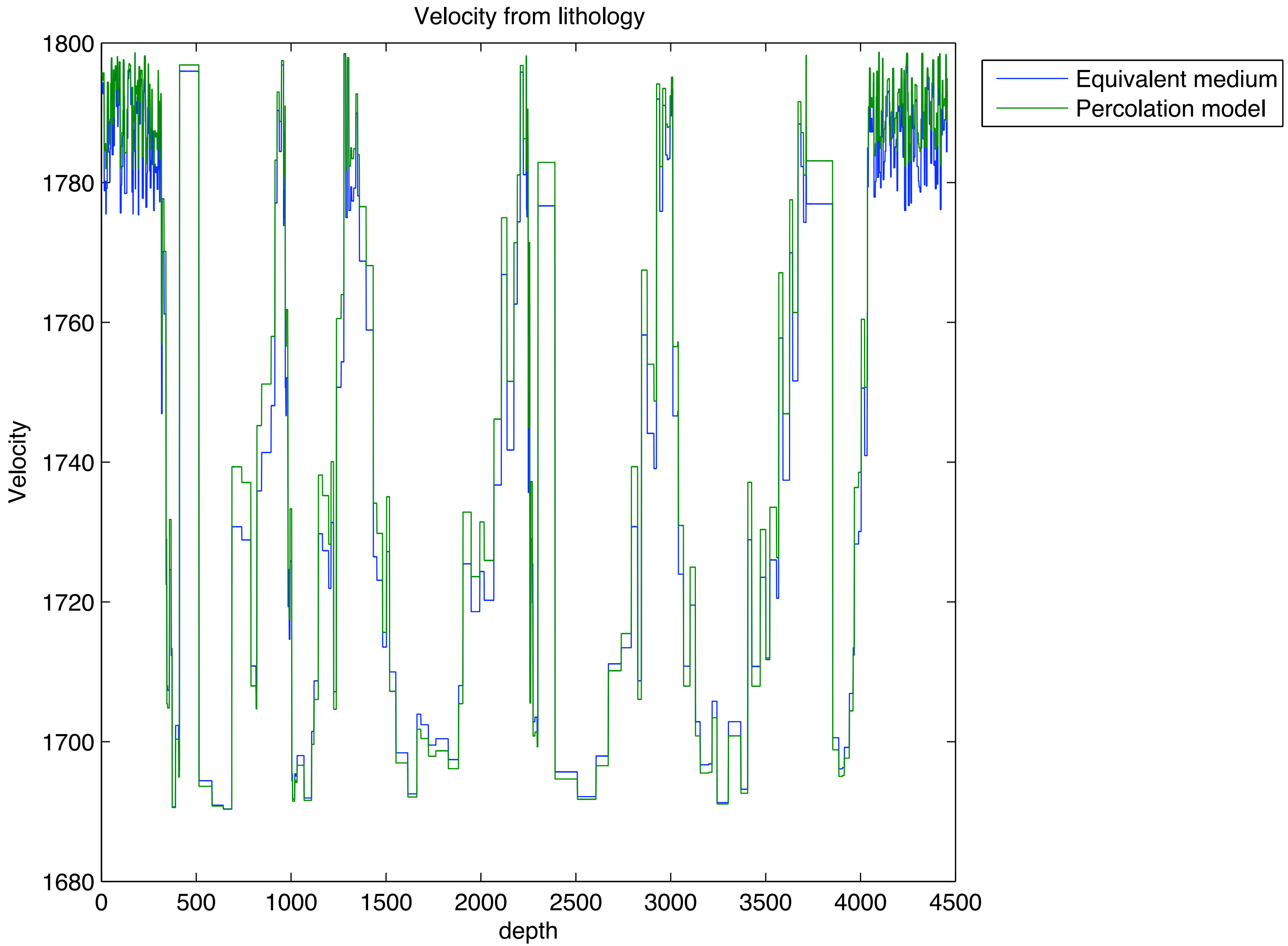
Percolation model

- Relation composition versus seismic contains ***now*** a critical point <=> **switch**
- Composition may vary **smoothly** but elastic moduli and velocity may **not**
- Use the switch to do a singularity-preserving upscaling by spatial smoothing the composition

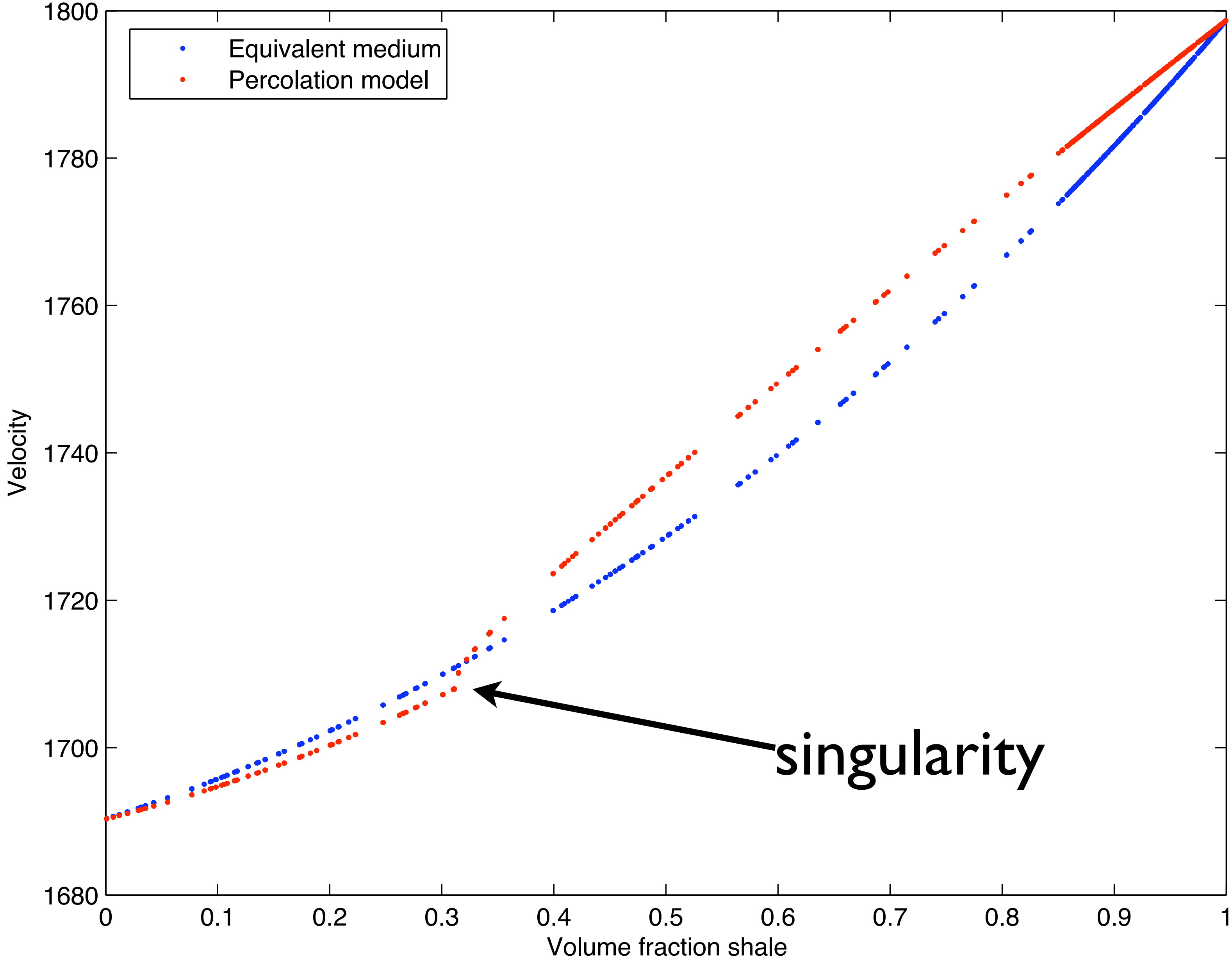
Volume fraction



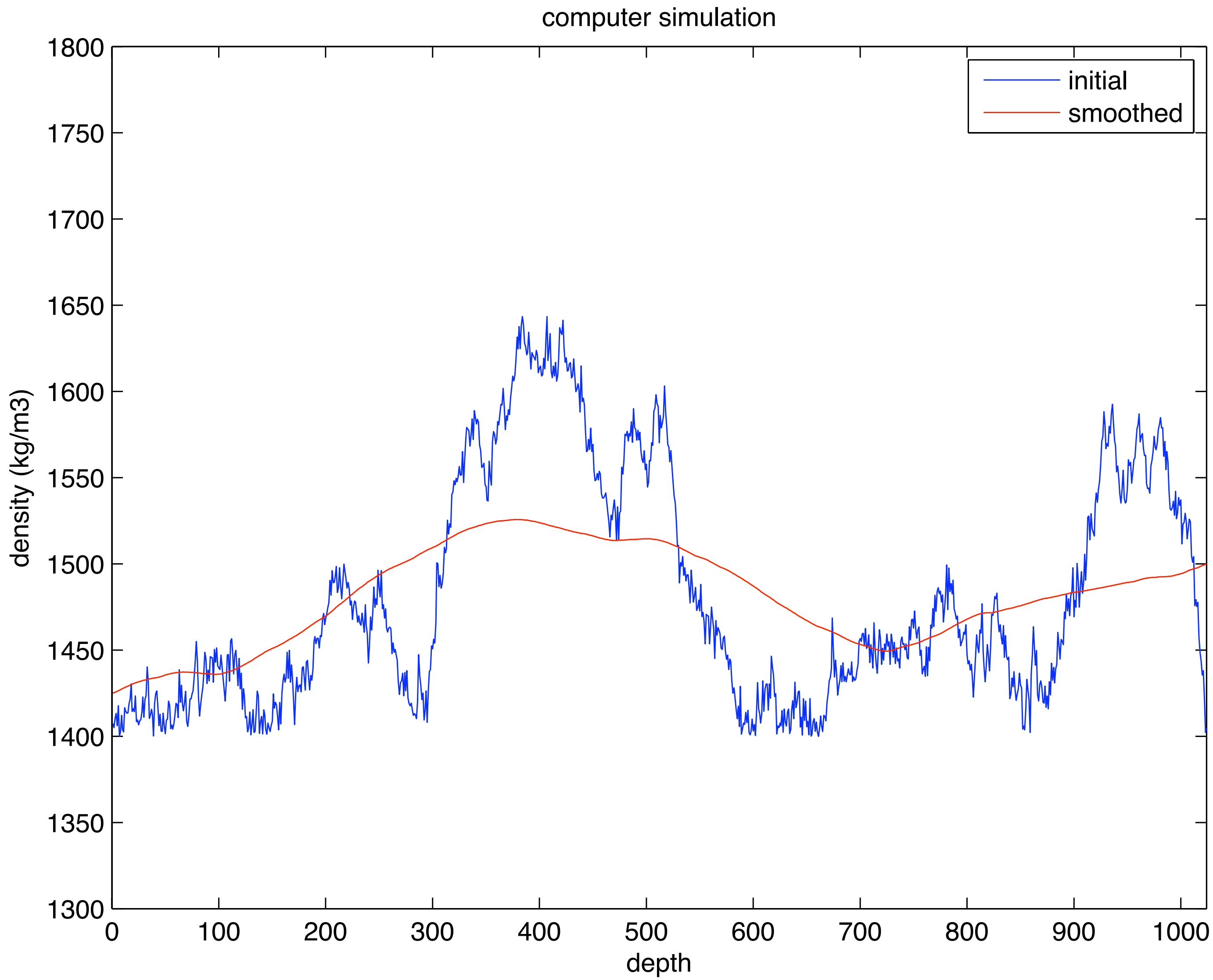
Switch vs no switch



Switch vs no switch

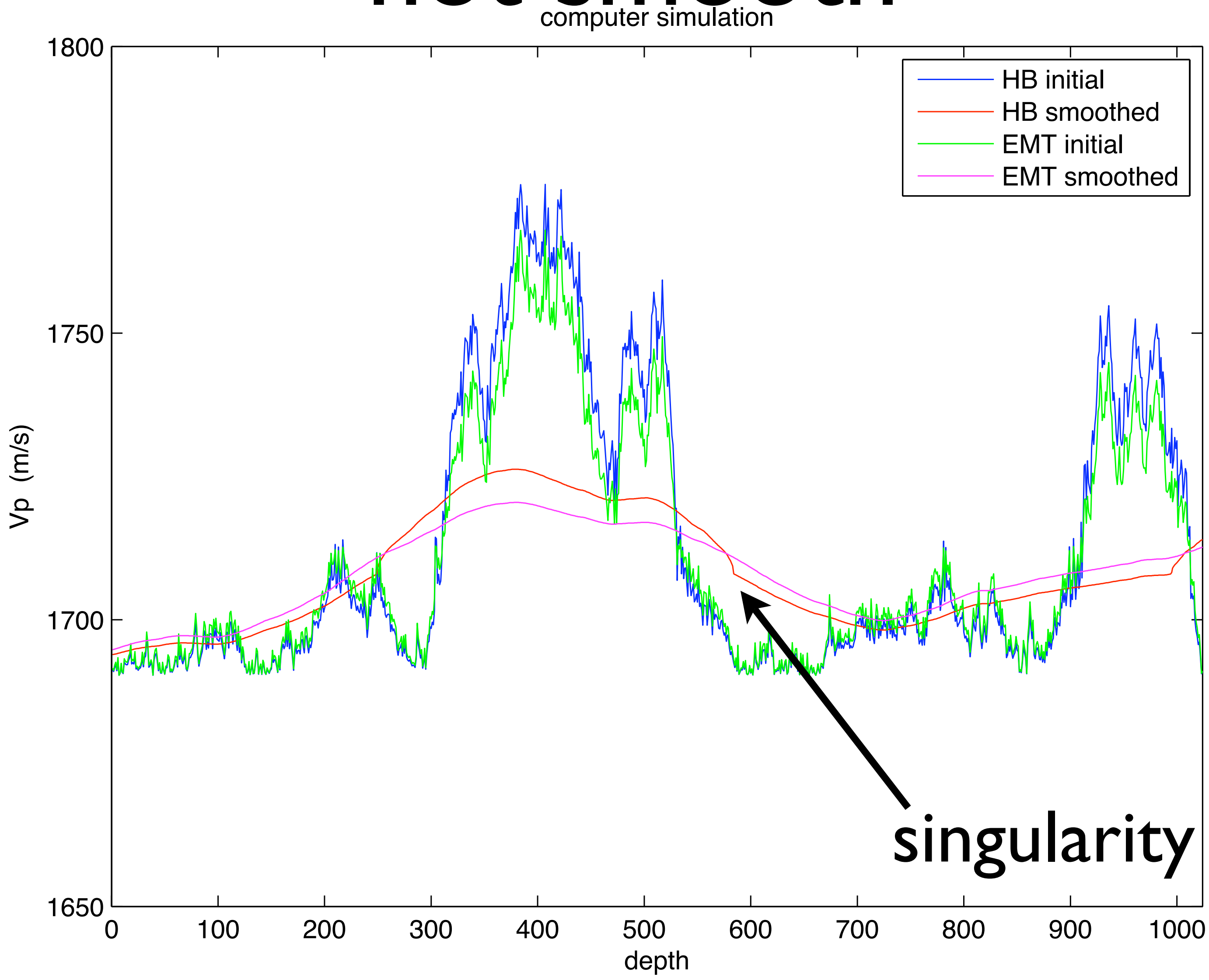


Upscaled density
“smooth”

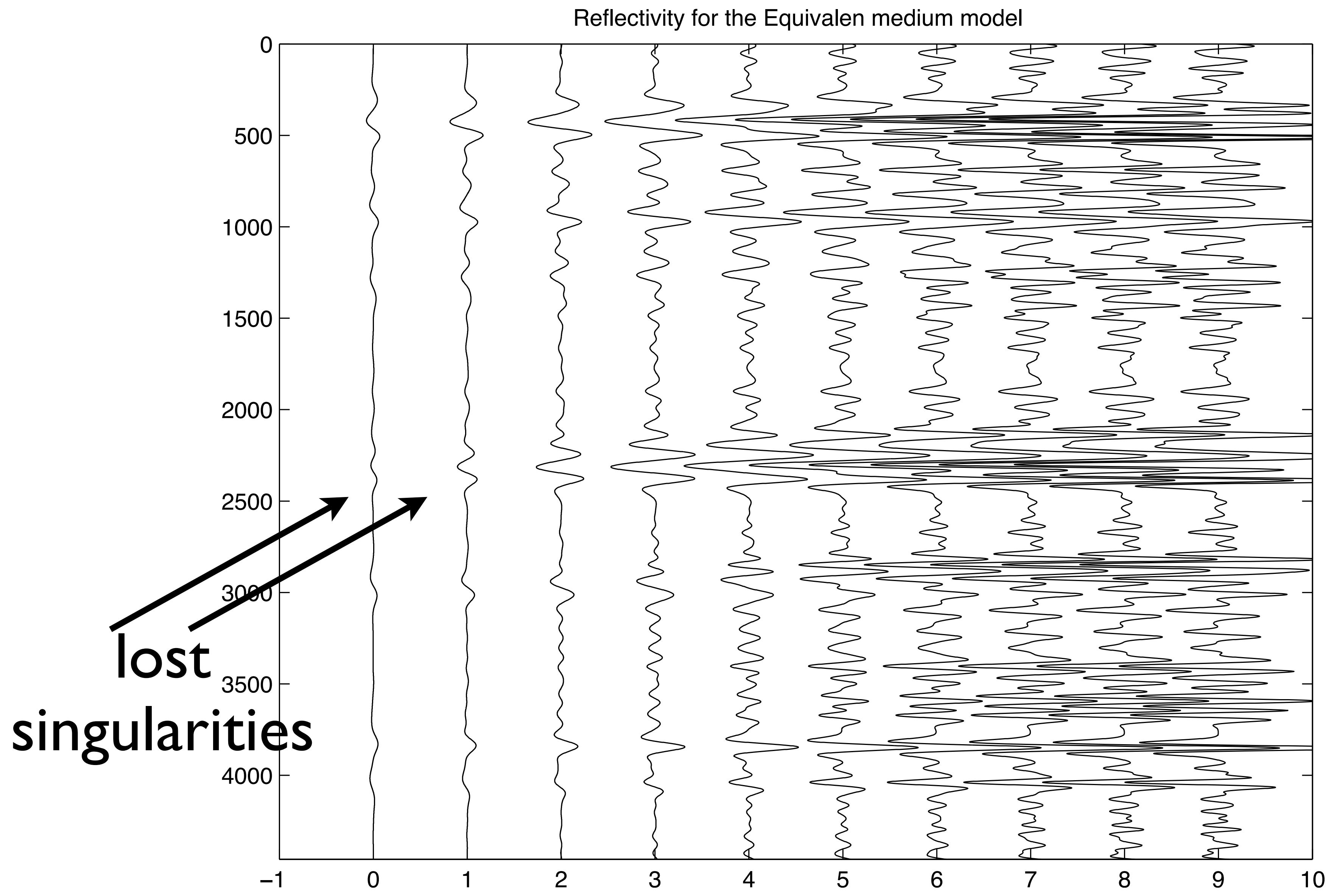


Upscaled velocity

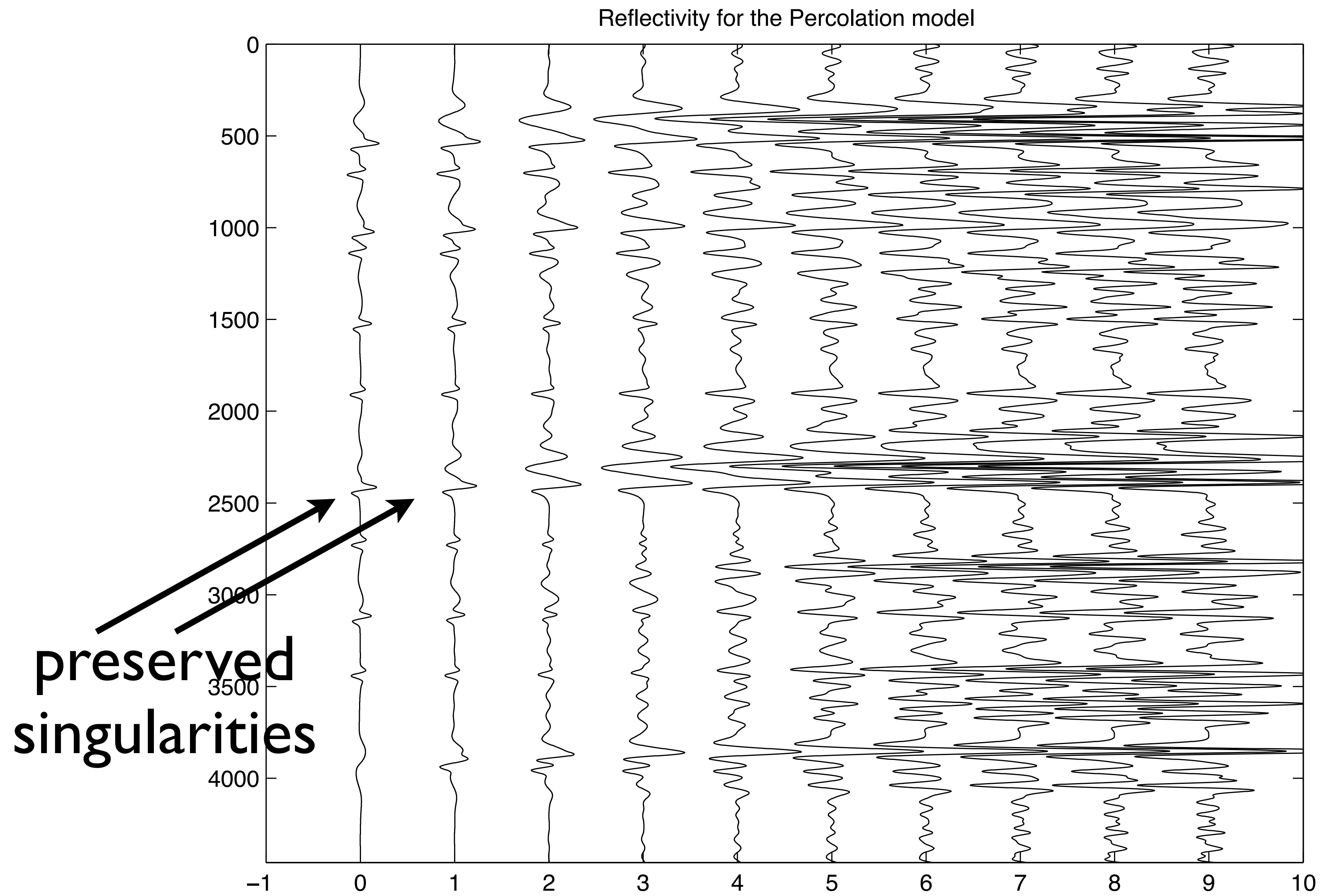
not smooth



EM upscaled reflectivity



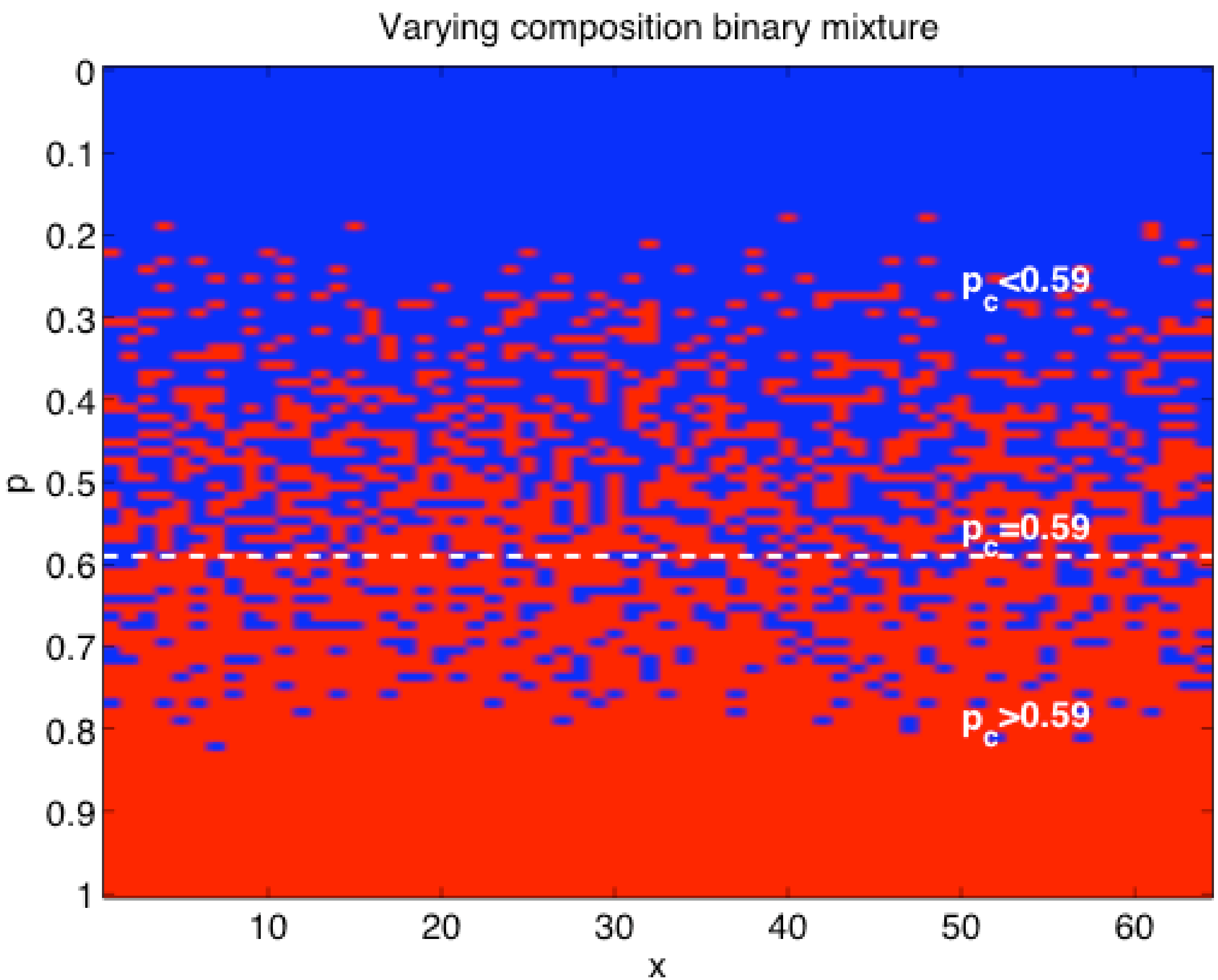
Percolation upscaled reflectivity



Observations

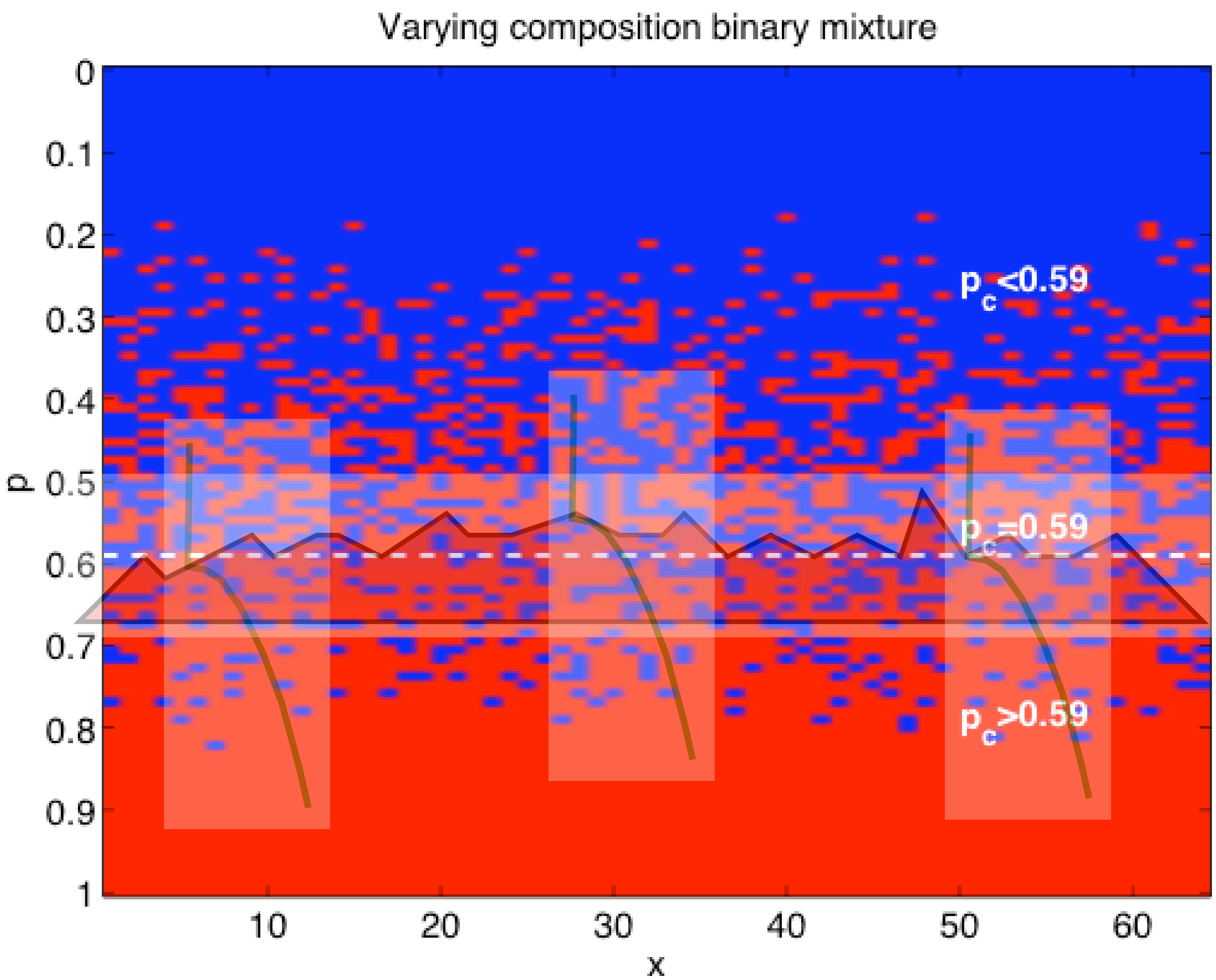
- Percolation model preserves the singularities
- Swich model provided "access" to the fine-structure (connectivity) from macroscopic waves
- Rigorous mathematical framework for the "shapes" of these percolation-induced transitions is an open problem

Morphology?



Varying composition binary mixture
p
x
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
10
20
30
40
50
60
p_c<0.59
p_c=0.59
p_c>0.59

Morphology?



Varying composition binary mixture
p_c<0.59
p_c=0.59
p_c>0.59
p
x
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
10
20
30
40
50
60

Conclusions

- Multiscale compressible signal representations are indispensable for acquiring accurate information on the imaged waveforms.
- Imaged waveforms carry information on the fine structure of the reflectors.
- Multiscale detection-estimation provides estimates for the exponents.
- Percolation model provides an interesting perspective.

Acknowledgments

The authors of CurveLab (Demanet, Ying, Candes, Donoho)

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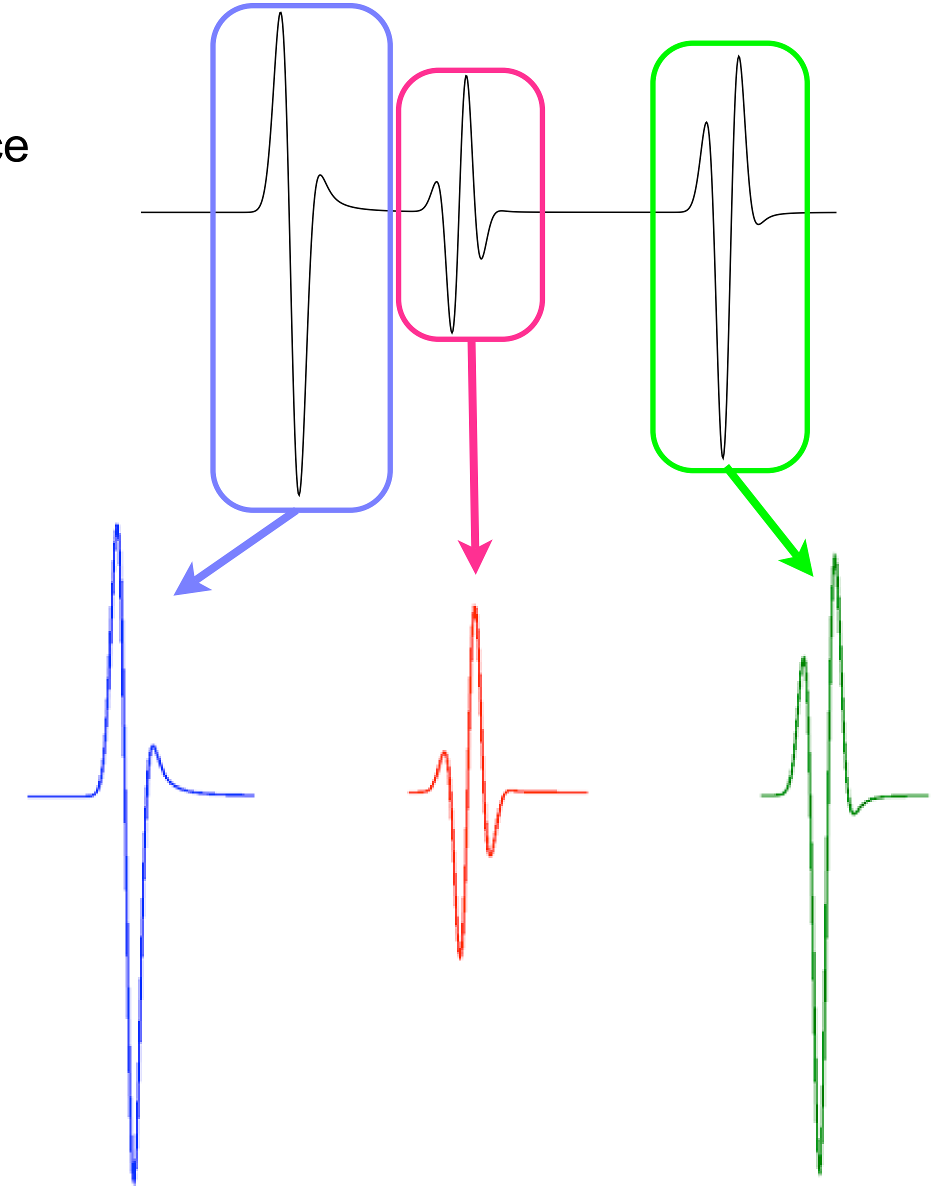
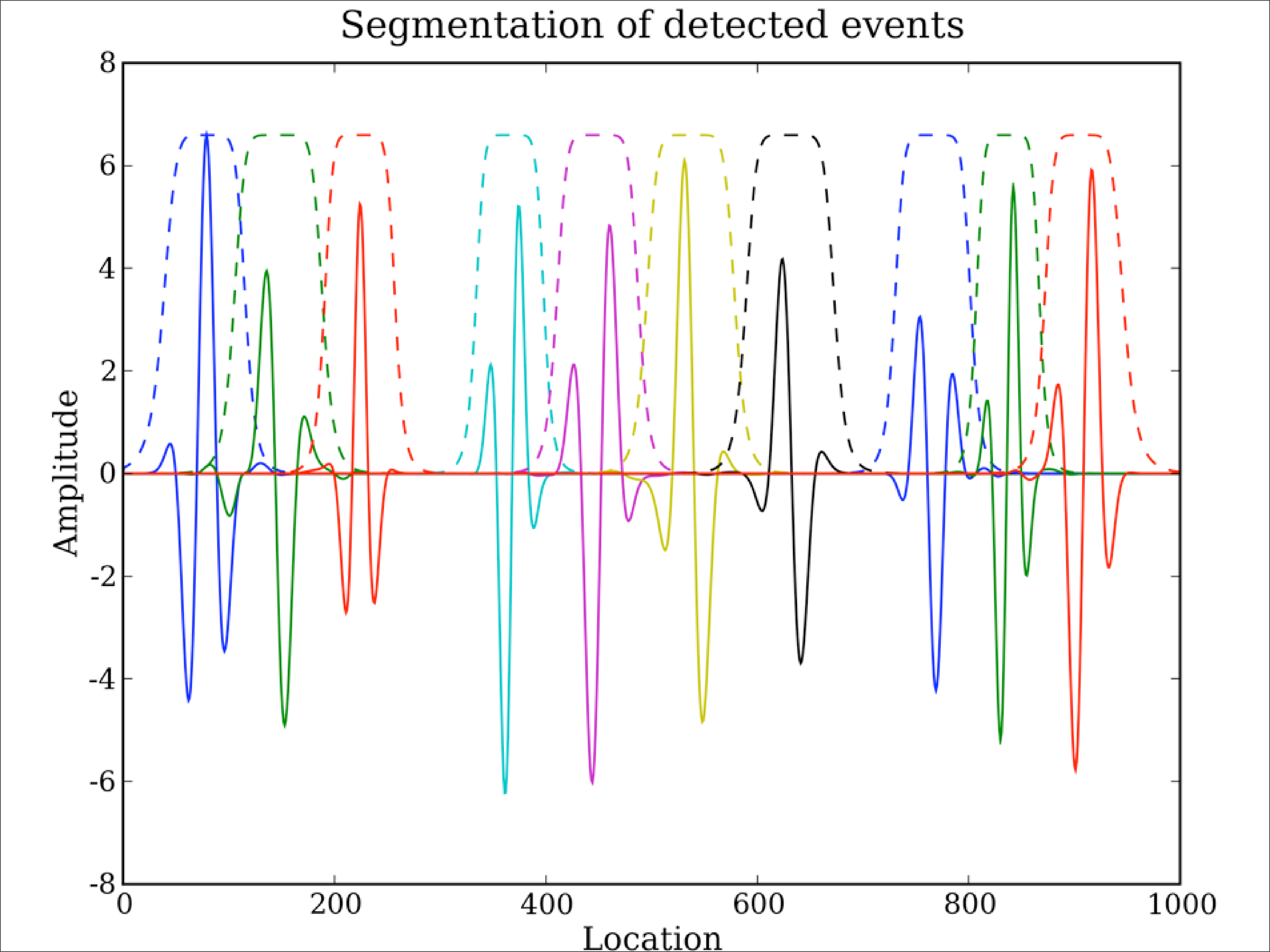
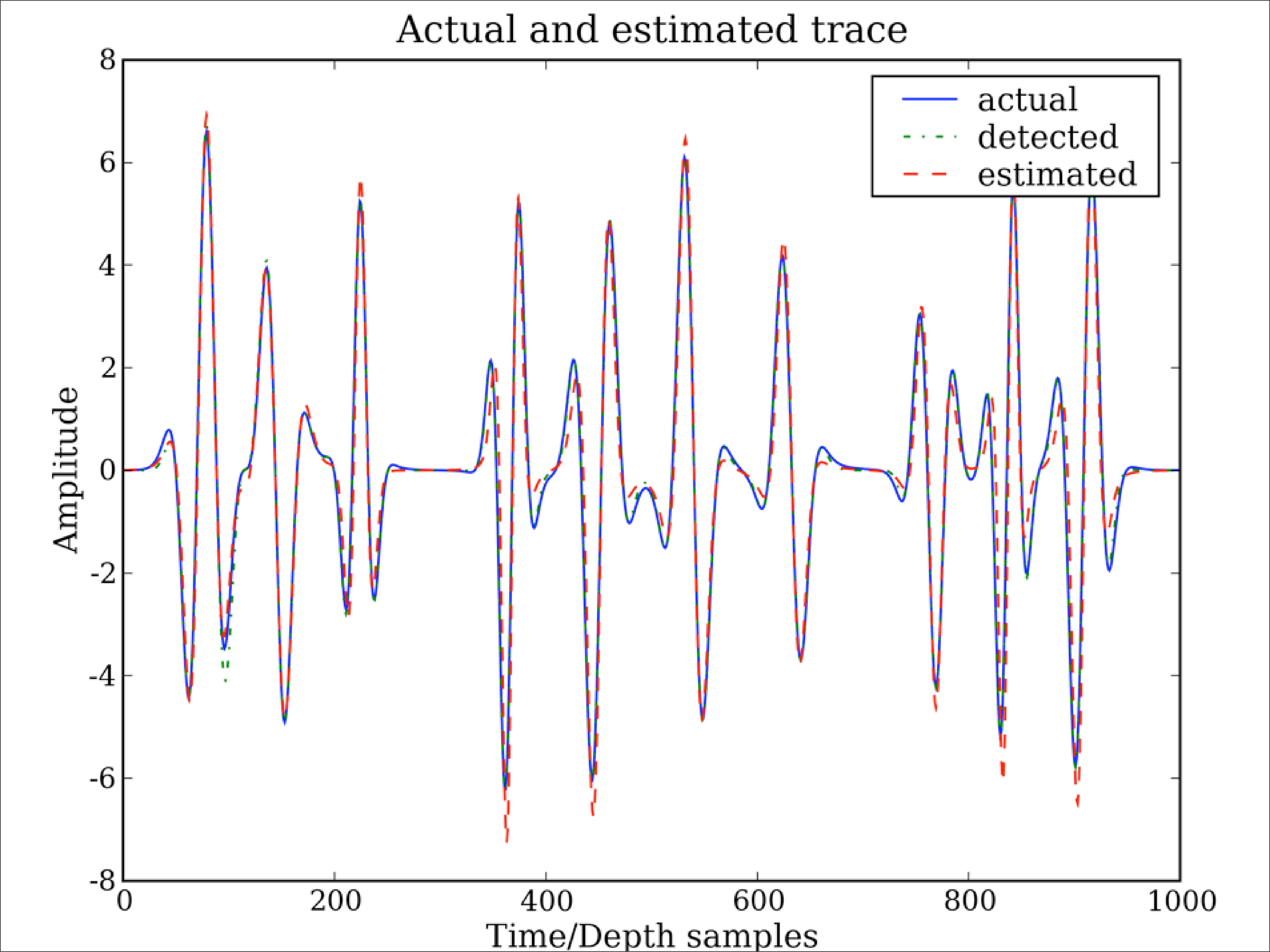
Detection-Estimation method

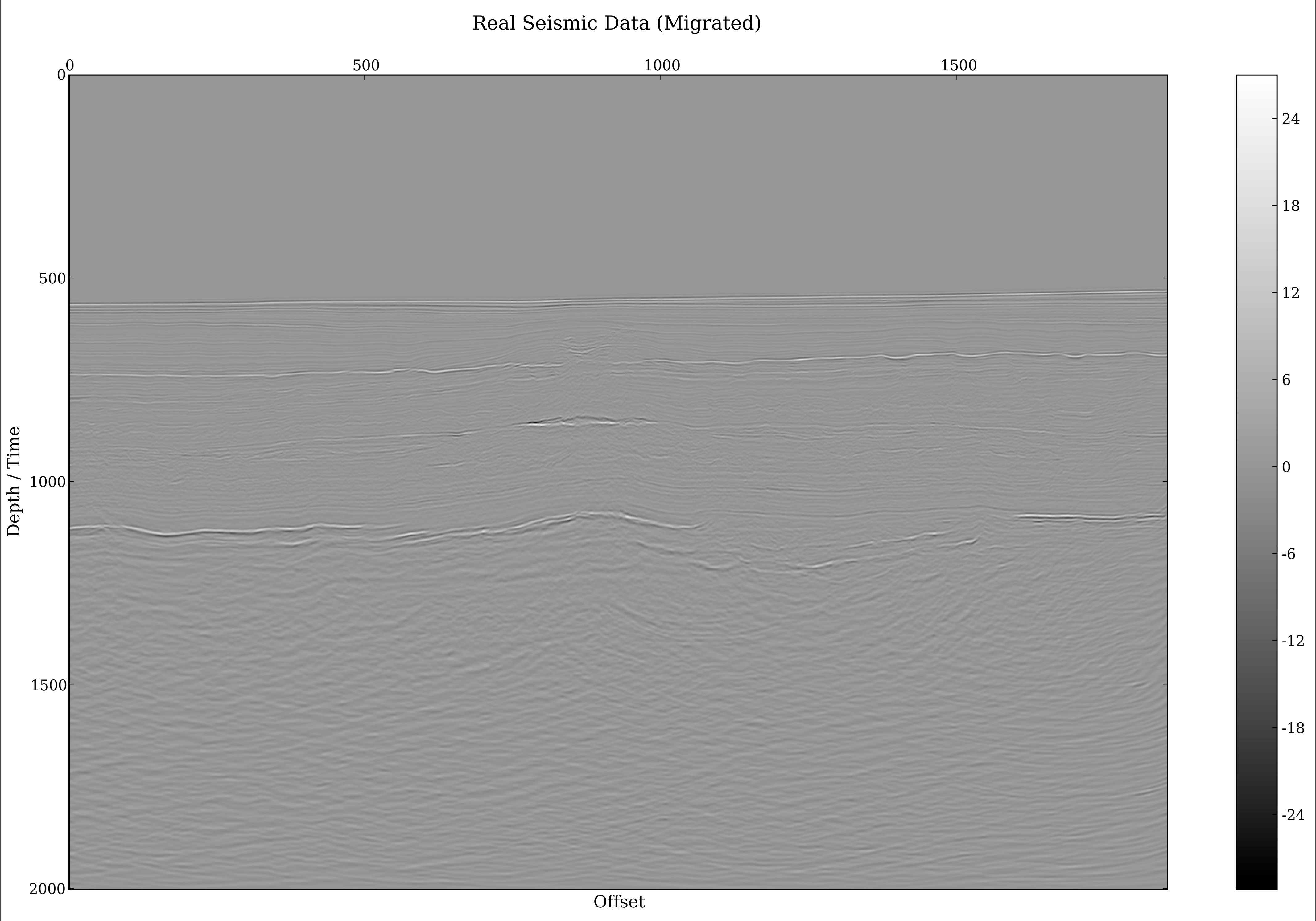
- Detect the events
  - 1D Complex CWT on seismic trace
  - Find local maxima on CWT plane
- Isolate the events
  - windowing based on location & scale of event
- Estimate characterization of windowed events

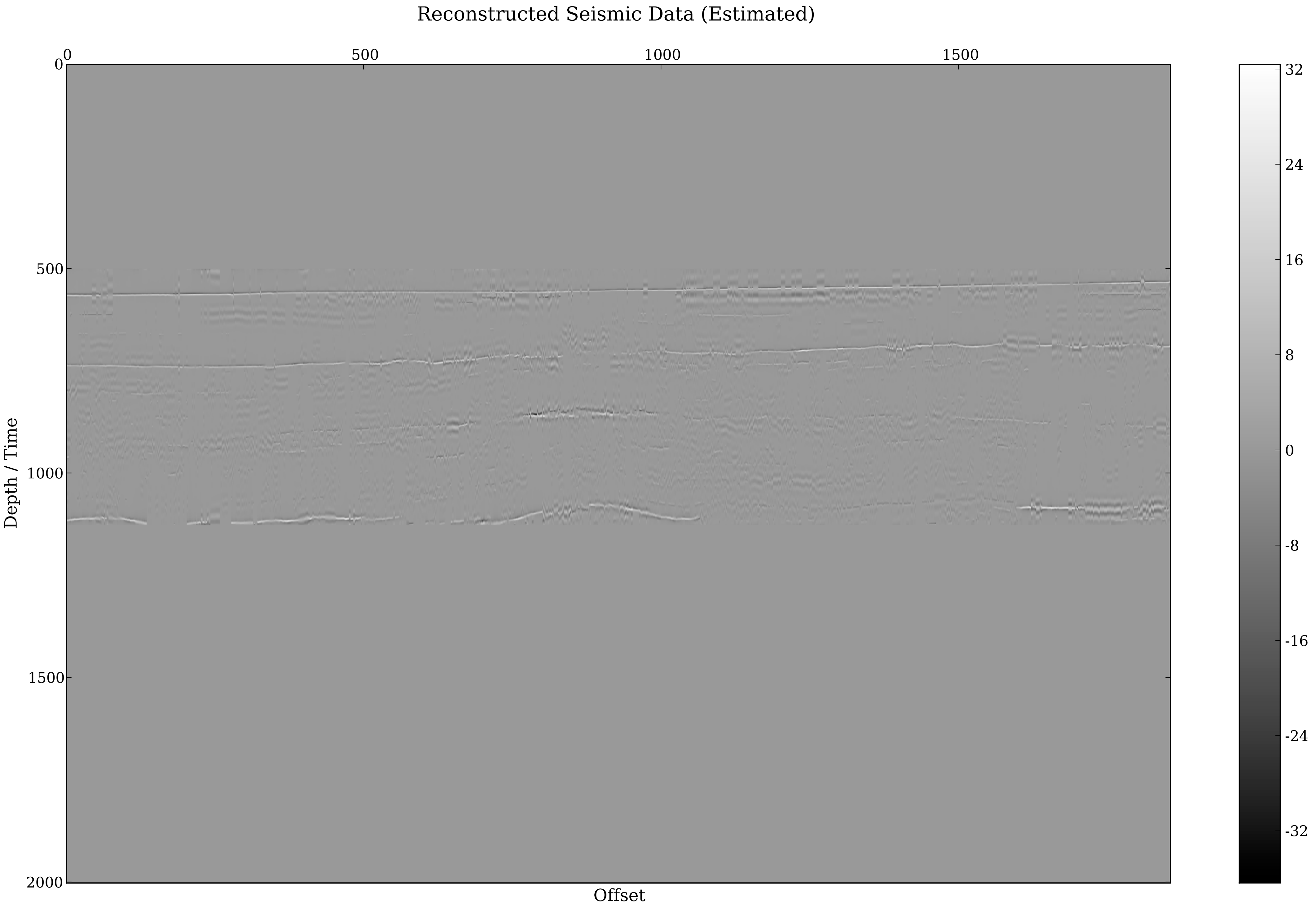




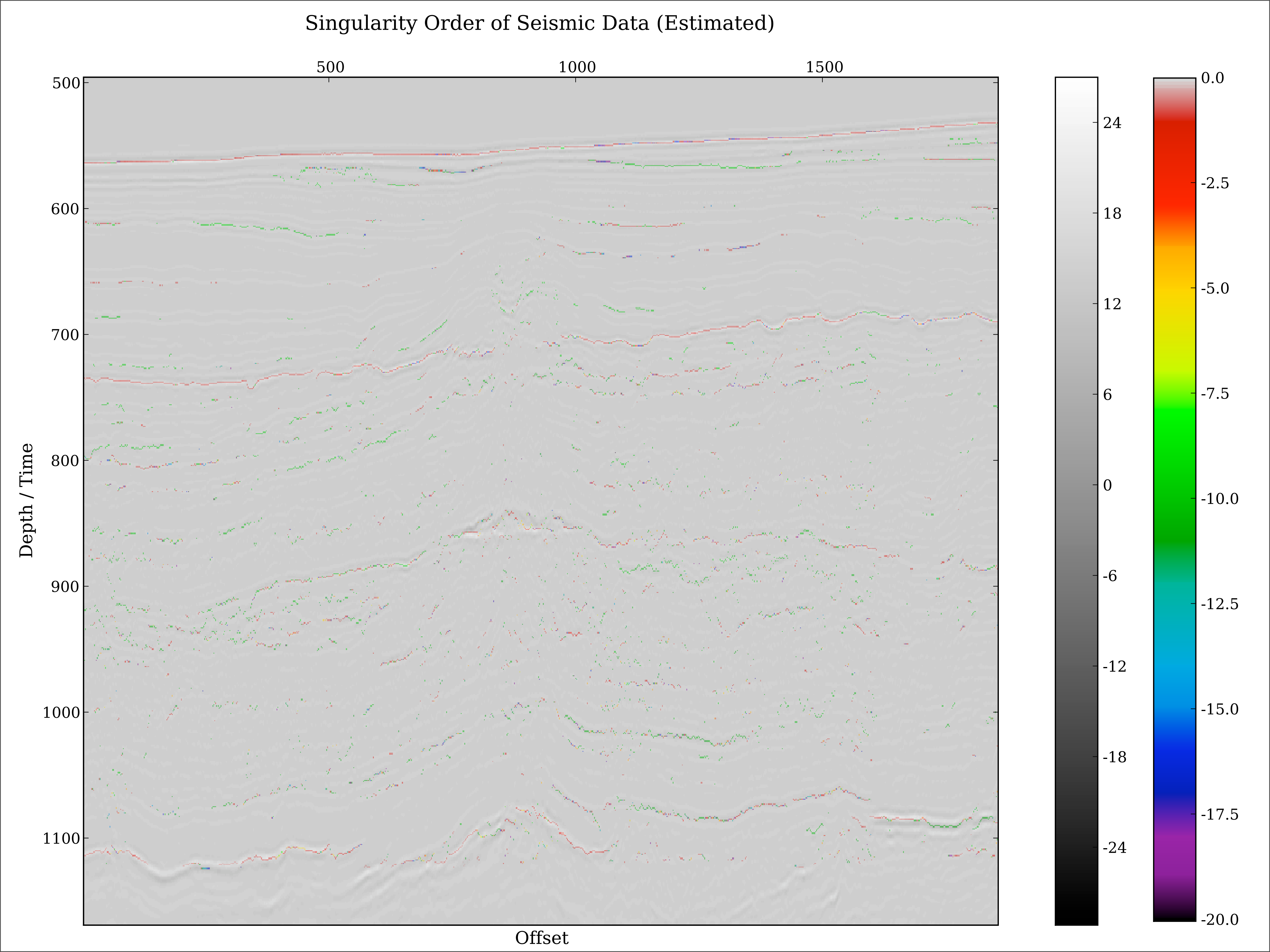
Segmentation of detected events
Amplitude
Location
8
6
4
2
0
-2
-4
-6
-8
0
200
400
600
800
1000


Actual and estimated trace
actual
detected
estimated
Amplitude
Time/Depth samples
8
6
4
2
0
-2
-4
-6
-8
0
200
400
600
800
1000


Real Seismic Data (Migrated)
0
500
1000
1500
0
500
1000
1500
2000
Depth / Time
Offset
24
18
12
6
0
-6
-12
-18
-24



Reconstructed Seismic Data (Estimated)
0
500
1000
1500
0
500
1000
1500
2000
Depth / Time
Offset
32
24
16
8
0
-8
-16
-24
-32



Singularity Order of Seismic Data (Estimated)
500
1000
1500
Depth / Time
500
600
700
800
900
1000
1100
Offset
24
18
12
6
0
-6
-12
-18
-24
0.0
-2.5
-5.0
-7.5
-10.0
-12.5
-15.0
-17.5
-20.0