Random shapes in water waves

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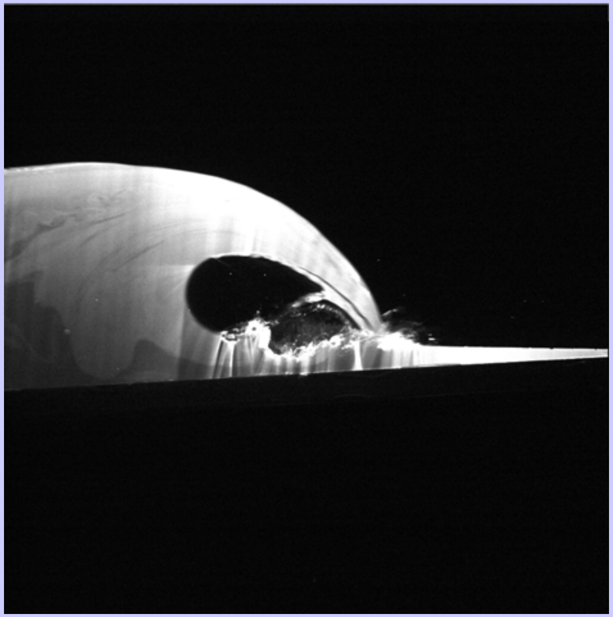
Image processing for random shapes

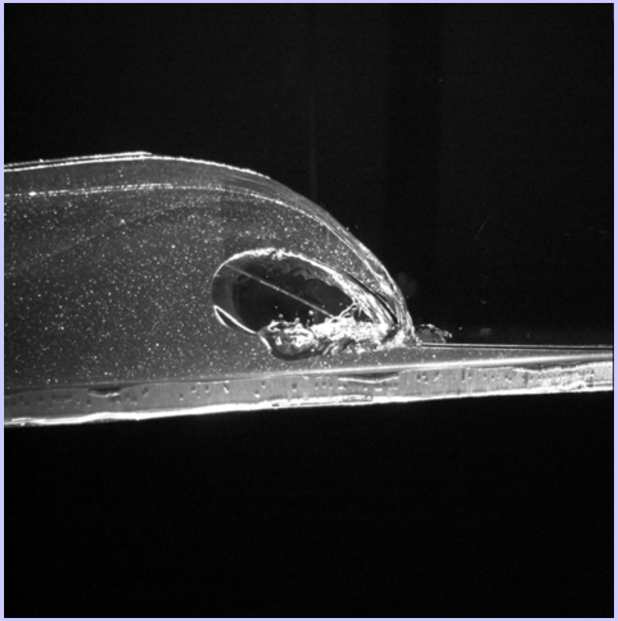
Outline

Acknowledgements

- J.-M. Morel, A. Trouvé, L. Moisan ($\in$ image team at CMLA, ENS-Cachan)
- B. Sapoval (CMLA, ENS-Cachan, and Ecole polytechnique)
- J.-M. Ghidaglia, D. Dutykh ($\in$ real fluid team at CMLA, ENS-Cachan)
- P. Bonneton (EPOC, Bordeaux, France)
- K. Melville (Scripps, UC San Diego)
- C. Synolakis (USC)
- A. Annunziato (Joint Research Center, EC, Italy)

Wave breaking





Laboratory experiments by O. Kimmoun (Marseille)

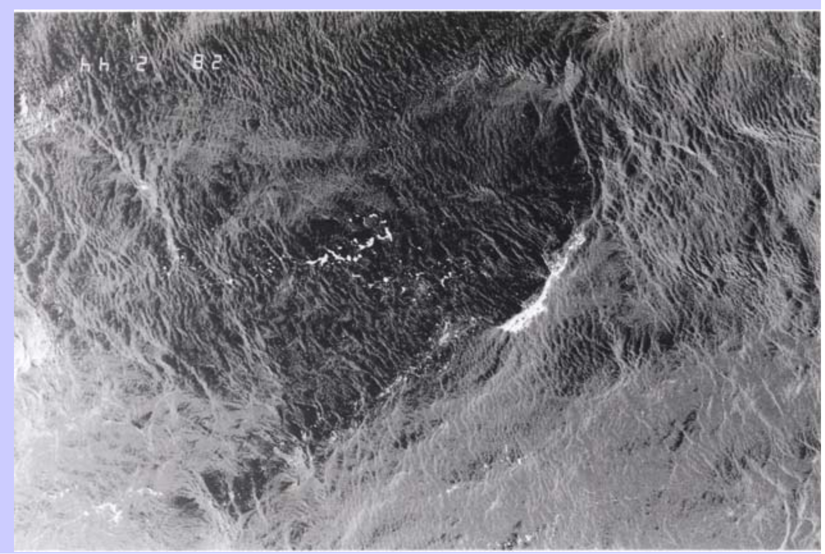
Picture taken by H. Peregrine

Wave breaking

Credit:

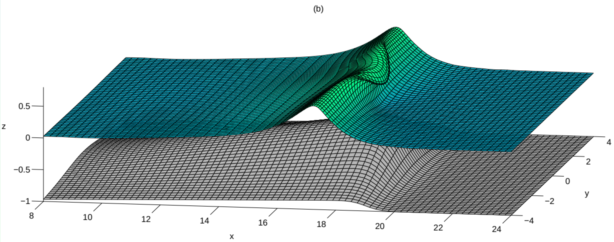
H. Peregrine

M. Banner



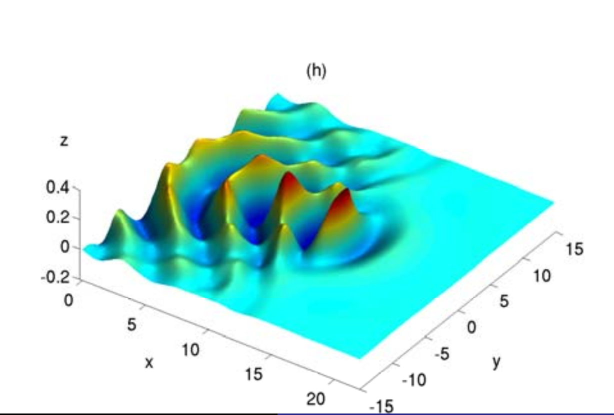
Computed water waves

- 3D overturning wave, state-of-the art computations based on boundary element method (Fochesato & FD, 2006), several hours of CPU for a single wave, several days of CPU for the equivalent two-phase flow (Lubin et al., in progress)

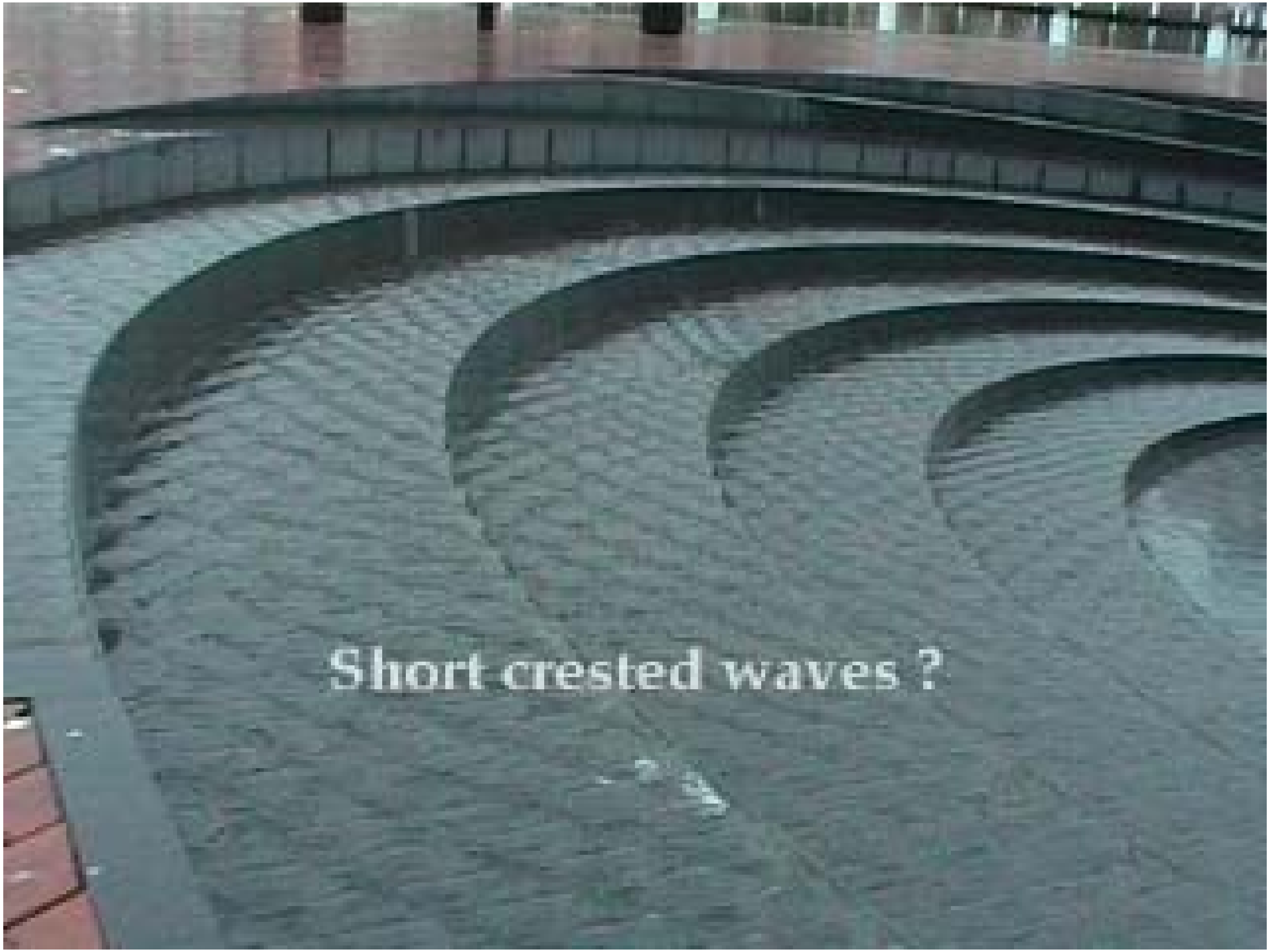


Computed water waves

- freak wave formation, state-of-the art computations based on boundary element method (Fochesato, Grilli & FD, 2007), several hours of CPU



Regular wave pattern

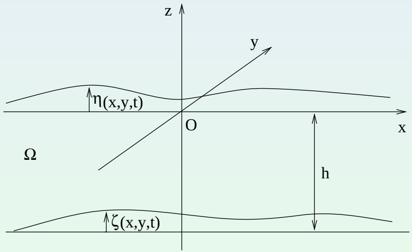


Rip currents

- As waves travel from deep to shallow water, they will break near the shoreline. When waves break strongly in some locations and weakly in others, this can cause circulation cells which are seen as rip currents: narrow, fast-moving belts of water traveling offshore.

Assumptions

- inviscid and incompressible liquid
- irrotational flow ($\mathbf{q} = \nabla \phi$), one-phase flow



$$\Omega = \mathbb{R}^2 \times [-h(x,y), \eta(x,y,t)]$$

Full water wave problem

Assumptions: potential flow, incompressible liquid

- Continuity equation

$$\Delta\phi = 0, \quad (x, y, z) \in \Omega,$$

- Kinematic bottom condition

$$\frac{\partial\phi}{\partial z} + \nabla\phi\cdot\nabla h = 0, \qquad z = -h(x, y),$$

- Kinematic free-surface condition

$$\frac{\partial\phi}{\partial z} = \frac{\partial\eta}{\partial t} + \nabla\phi\cdot\nabla\eta, \qquad z = \eta(x, y, t),$$

- Dynamic free surface condition

$$\frac{\partial\phi}{\partial t} + \frac{1}{2}\left|\nabla\phi\right|^2 + g\eta = 0, \qquad z = \eta(x, y, t).$$

Simplified versions of the water wave problem

- The full water-wave problem is extremely difficult to solve, due to the boundary conditions on the free surface.
- Mathematicians have been studying it for more than 200 years!
- Linearized water-wave problem
- Linear shallow-water equations
- Nonlinear shallow-water equations
- Gerstner wave : an exact solution which is not irrotational

Simplest model : Linearized water-wave equations with flat bottom

$$\begin{aligned} \Delta\phi &= 0, \quad (x,y,z)\in\mathbb{R}^2\times[-h,0], \\ \frac{\partial\phi}{\partial z}=\frac{\partial\eta}{\partial t}, \quad z &= 0, \quad \frac{\partial\phi}{\partial t}+g\eta=0, \\ \frac{\partial\phi}{\partial z} &= 0, \quad z=-h, \end{aligned}$$

- travelling as well as standing wave solutions ; waves are dispersive

$$\omega=\sqrt{gk\tanh(kh)}$$

- proof of existence of standing waves for the full water-wave equations provided by Iooss, Toland, Plotnikov in 2005!

Another simple model : Nonlinear shallow water equations

- Governing equations:

$$\begin{aligned} \frac{\partial \eta}{\partial t} + \nabla \cdot ((h+\eta)\vec{v}) &= 0, \\ \frac{\partial \vec{v}}{\partial t} + \frac{1}{2}\nabla |\vec{v}|^2 + g\nabla \eta &= 0. \end{aligned}$$

  η : free surface elevation with respect to still water level, $\vec{v}$: horizontal velocity, h: bathymetry

- Lots of different numerical schemes for integrating these equations (finite differences, finite volumes) (see for example G. Haro, R. LeVeque, D. Dutykh & FD among the hundreds (?) of codes)

Eulerian vs Lagrangian description

- Eulerian point of view: the observer stays at a fixed geographical point as the fluid evolves around him.
- Lagrangian point of view: the observer watches the fluid evolve while travelling with a fluid particle
- Mixed Eulerian-Lagrangian description for water waves
- Movie : trajectories for water waves

Gerstner's rotational waves

- Discovered by Gerstner in 1802!
- Waves of limited physical relevance because they are rotational. However geometrical structure of Gerstner's waves similar to the structure of Stokes' waves
- Simple solution. The coordinates (x, z) of a liquid particle depend on time τ as follows:

$$
\begin{aligned}
x &= x_0 - a e^{k z_0} \sin k(x_0 - c\tau), \\
z &= z_0 + a e^{k z_0} \cos k(x_0 - c\tau),
\end{aligned}
$$

where $c = \sqrt{g/k}$, a is the wave amplitude, (x_0, z_0) identify the particle

Gerstner's rotational waves

- For $z_0 = 0$ (free surface), in the coordinate system moving with the wave at velocity c,

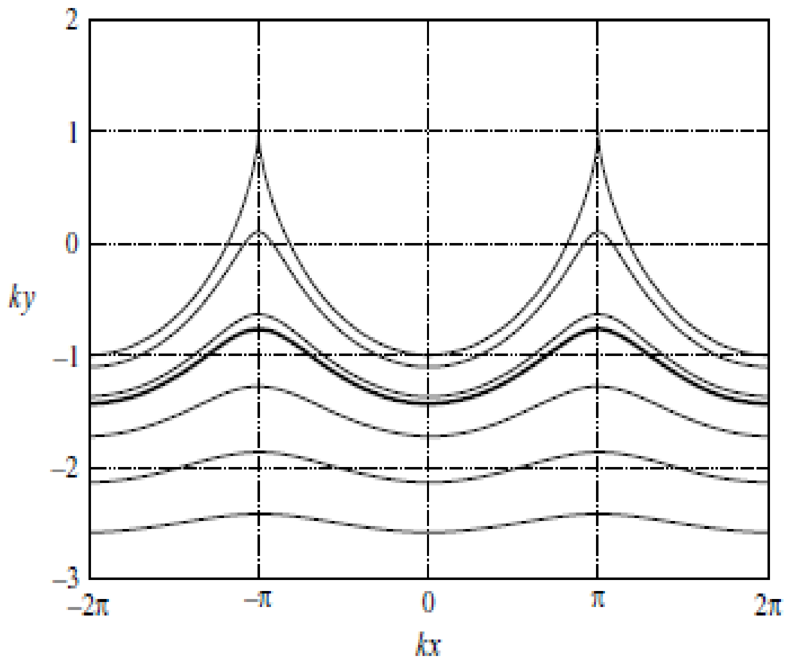
$$\begin{aligned} x &= t/k - a \sin t, \\ z &= a \cos t, \end{aligned}$$

where $t = k(x_0 - c\tau)$. This is a trochoid.

- surface slope with steepness parameter $b = ka$

$$\gamma = dz/dx = -\frac{b \sin t}{1 - b \cos t}$$

Gerstner's waves



Gerstner's waves at t=0, plotted for kz_0= - 2.5 (bottom curve) to kz_0 = 0 (top curve)

Leblanc (2004)

Examples of applications related to image processing

- Damage caused by tsunamis or storm surges
- Oil spills
- Algae blooms
- Surfactants
- Coastal erosion
- Sea-ice monitoring
- Entertainment industry
- . . .

Types of images

- Satellite surface remote sensing monitoring
  1. optical instruments
  2. infrared radiometers
  3. passive microwave radiometers
  4. active microwave systems (altimeter, scatterometer, SAR)
- video-based techniques

Synthetic Aperture Radar

SAR gives convincing images of swell waves propagating onto coasts, including the effects of depth refraction, shadowing, and diffraction (Johannessen et al. 2000).

- SAR in image mode provides rather detailed pictures of wave fields near shorelines
- SAR monitors wave refraction by bottom topography
- Oil slick dampens capillary and short gravity waves - appears as a dark slick in SAR images of the ocean surface

On the stilling of waves by means of oil

XLIV. *Of the ſtilling of Waves by means of Oil. Extracted from ſundry Letters between* Benjamin Franklin, *LL. D. F. R. S.* William Brownrigg, *M. D. F. R. S. and the Reverend Mr.* Fariſh.

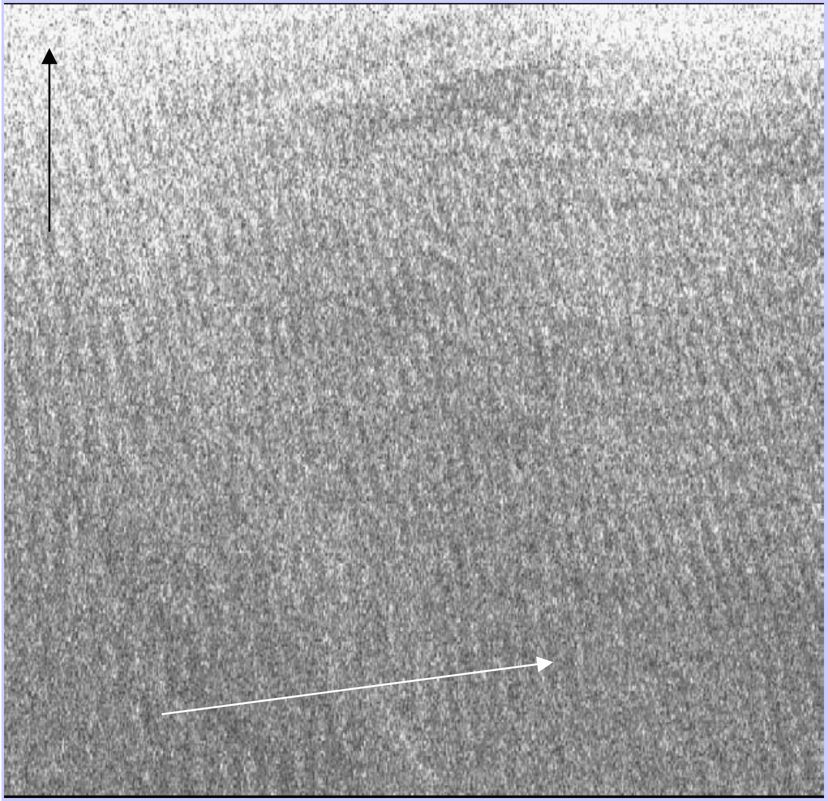
Extract of a Letter from Doctor BROWNRIGG to Dr. FRANKLIN, dated Ormathwait, January 27, 1773.

Redde, June 2, 1774.

BY the encloſed from an old friend, a worthy clergyman at Carliſle, whoſe

SAR image data

Left: Image of traveling swell in the Pacific Ocean off the coast of California near San Francisco (Schuler et al., Remote Sensing of Environment, 2004), Right: Oil spill



Coastal erosion

Shore-based video systems provide continuous and automated data collection (much greater range of time and spatial scales than with field surveys) - Aarninkhof et al. (2003)

- continuous (typically every daylight hour) collection of image data with a resolution of cm to m, extending along regions of hundreds of m to several km
- challenges: first identify shoreline and foreshore features by analysis of oblique images, secondly convert distorted image (2D) coordinates to their real-world (3D) position
- although research is progressing into sophisticated coupling of video data with numerical models, in simplest case video-derived data can at least form the boundary conditions for process models (e.g. bathymetry and wave parameters)

Shoreline detection

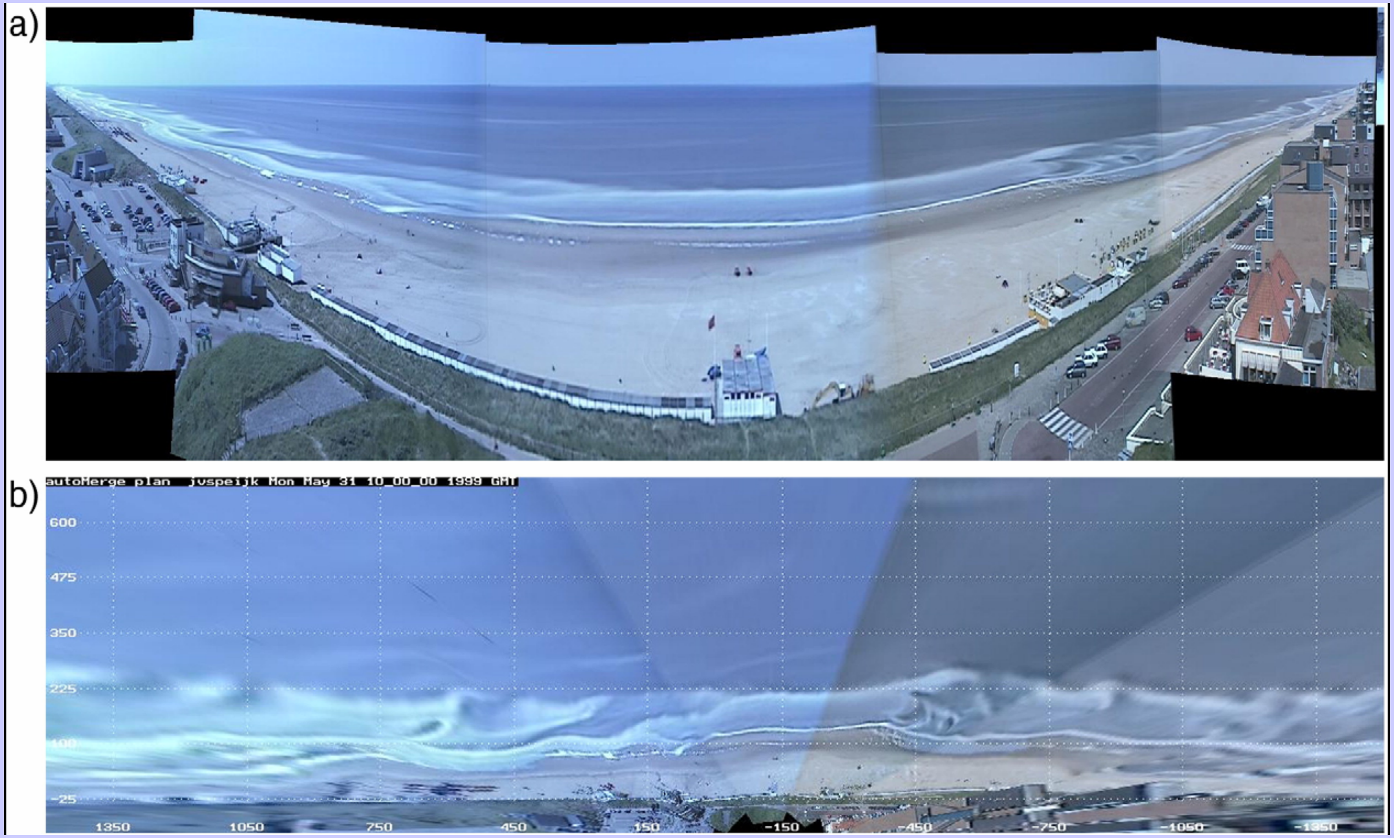
Shoreline detection from contrasting pixel intensity characteristics at the sub-aqueous and sub-aerial beach (Aarninkhof et al. 2003)

Left: from color clustering

Right: from luminance clustering

CoastView project (Europe)

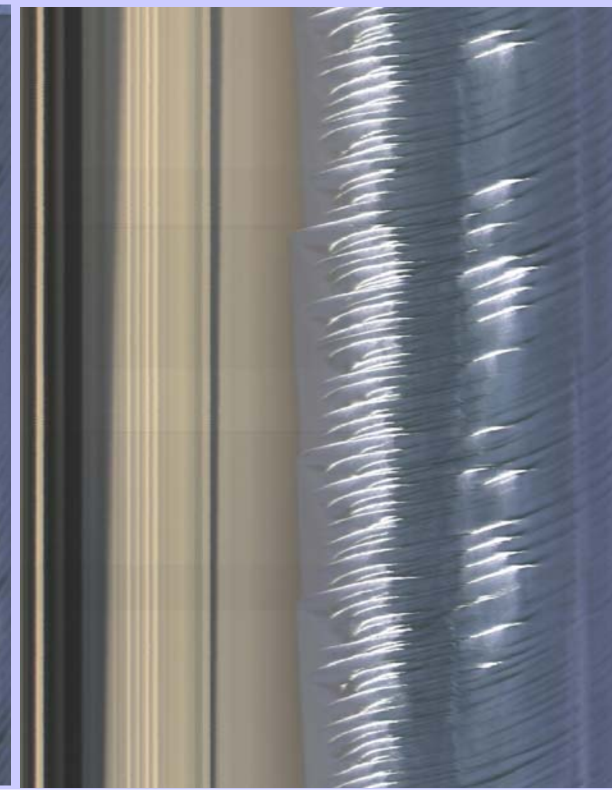
Top : Panoramic view + rectified plan-view of Egmond, Netherlands from 4 digitally merged Argus cameras (Davidson et al. 2007) – images are 10 minute time-averages



Coastal imaging

Credit: P. Bonneton, N. Sénéchal, R. Almar

South West Coast, France



Problems of video-based techniques (Aarninkhof et al. 2007)

- coastal video systems must be mounted as high as possible (at least 10 m)
- image degradation resulting from poor visibility due to adverse weather, accumulations on the camera lens
- ability to recognise and eliminate poor image data which might give rise to erroneous or misleading results
- robustness of the algorithms that deliver the video-derived variables
- develop image analysis algorithms that are stable and accurate, independent of location and conditions

Non intrusive wave measurement

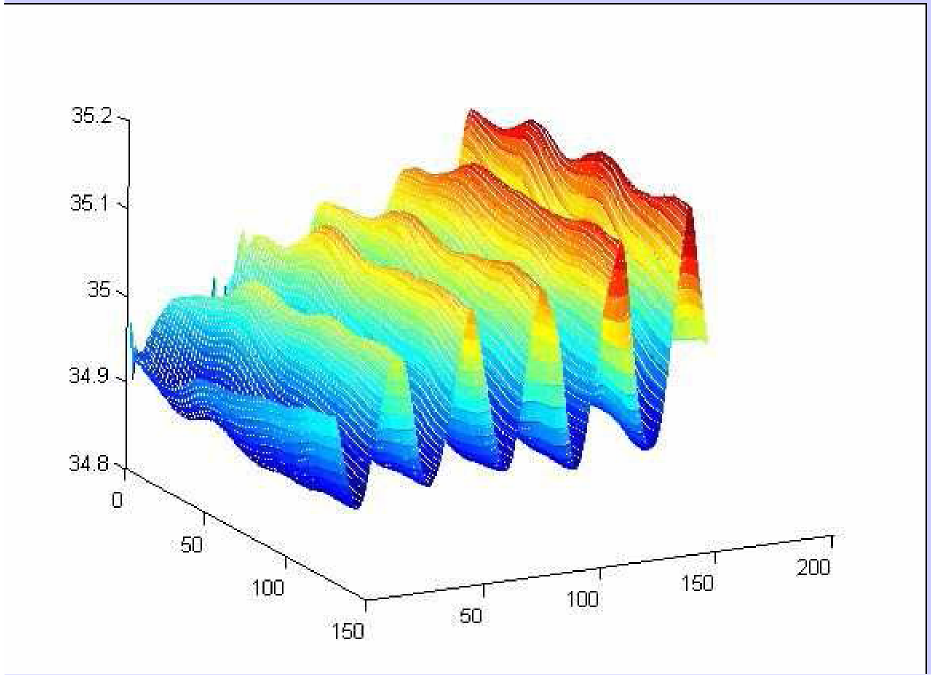
- Rabaud et al. (2007) recently proposed a non intrusive method to measure the instantaneous topography of a fluid interface.
- Method based on the analysis of the apparent displacement of a random pattern of points due to the refraction through the surface
- Correlation between instantaneous image and reference image obtained with flat surface allows reconstruction of local fluid elevation
- 20 cm $\times$ 20 cm area captured at frequency of 100 images/s by camera of 1280 $\times$ 1024 pixels

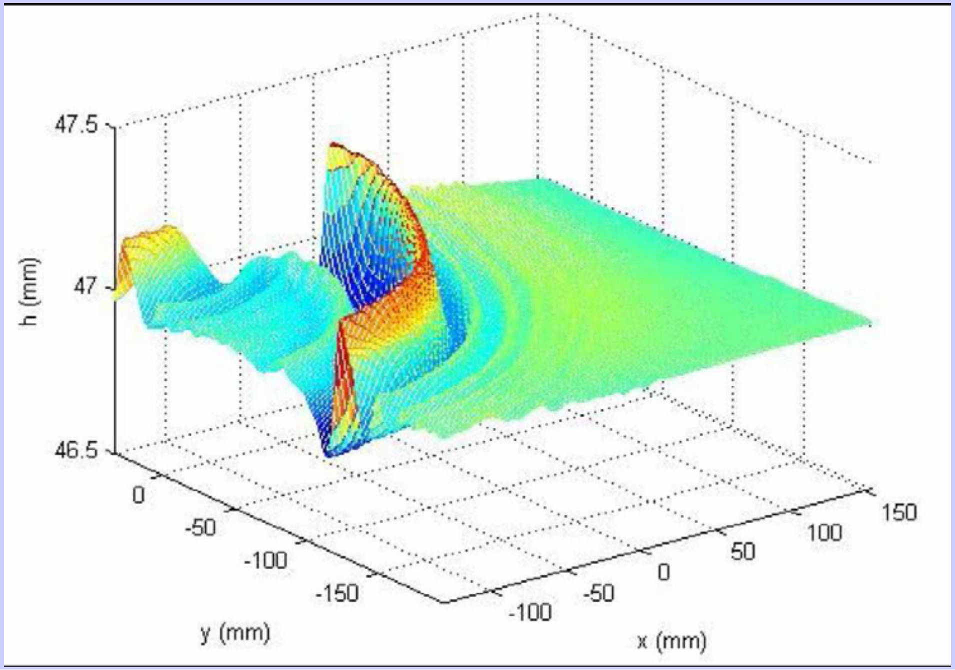
Non intrusive wave measurement

3D representation of the water surface (Rabaud et al. 2007)

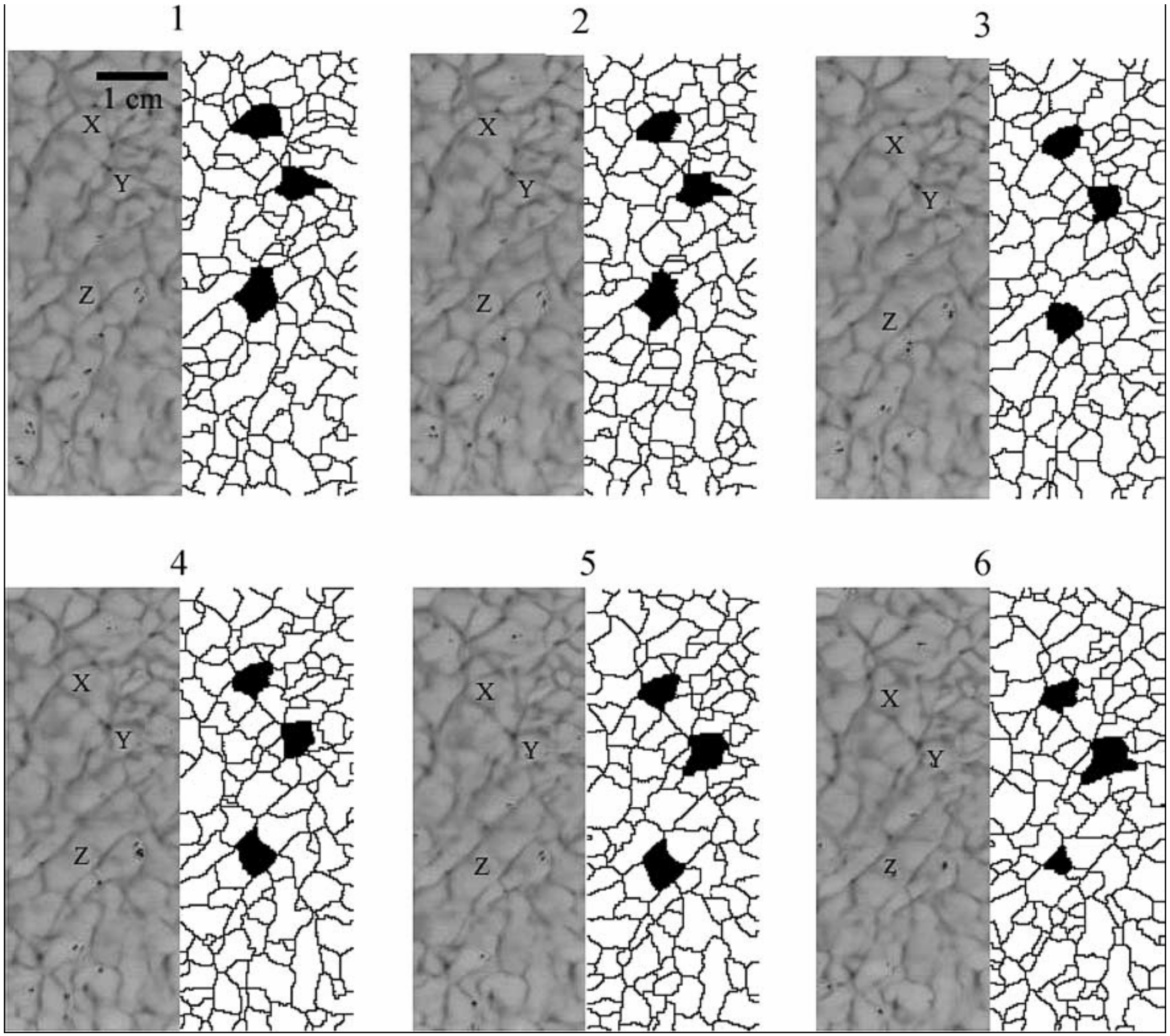
Left: wave maker oscillating at 7 Hz

Right: waves created by water drop impacting the free surface





Surface structures acquired with high-speed camera



Liquid-air interface in inclined open-channel water flows (Smolentsev, Miraghaie 2005)

Original (left) and converted (right) images of a 6.4 x 2.0 cm area of the free surface. The images are 1/2000 s apart.

Whitecaps

- Scientists would like to have an accurate ocean model that includes dissipation by wave breaking.
- What part of the energy provided by wind is dissipated by breaking?
- Energy balance: input by wind, energy transfer between resonant waves, viscous damping and breaking.
- Phenomenological description of wave breaking (does not exist yet to my knowledge)

Observation of wave breaking

cadreone

High-tech imagery for new insights into breaking wave dynamics

- In the past, wave breaking has been tracked by so-called “whitecap coverage,” in which still or video imagery was used to statically identify ocean whitecaps and the corresponding surface coverage by breaking waves.
- Fail to account for the motion of the breaking waves.

High-tech imagery for new insights into breaking wave dynamics

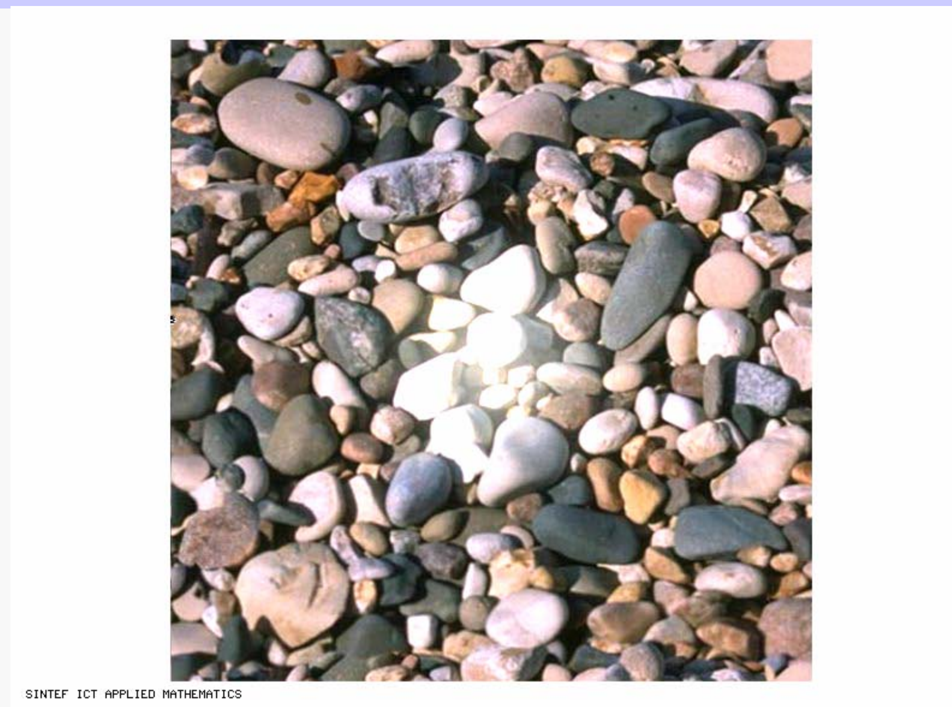
- New study (part of Shoaling Waves Experiment, or "SHOWEX"): K. Melville & P. Matusov (Scripps Institution) used high-tech instrumentation aboard a light aircraft to obtain detailed image sequences of breaking waves.
- Equipment included an airborne video system and differential global positioning system (GPS) technology to precisely characterize breaking processes.
- Remote imaging techniques help describe aspects of wave growth and decay in unique detail.

Entertainment industry

- Fournier (1986) used Gerstner's waves as building block. The overall "randomness" and "short-crestedness" of the ocean is achieved by a combination of small variations within a train and large variations between trains.
- Peachey (1986) used waveforms consisting of a phase function which correctly produces wave refraction and a wave profile which changes according to wave steepness and water depth.
- Layton & van de Panne (2002) solved the shallow water equations at the level of interactive simulations.
- Foster, Fedkiw & Osher (2001,2003) considered the problem of splashing water (particle method + level set method).

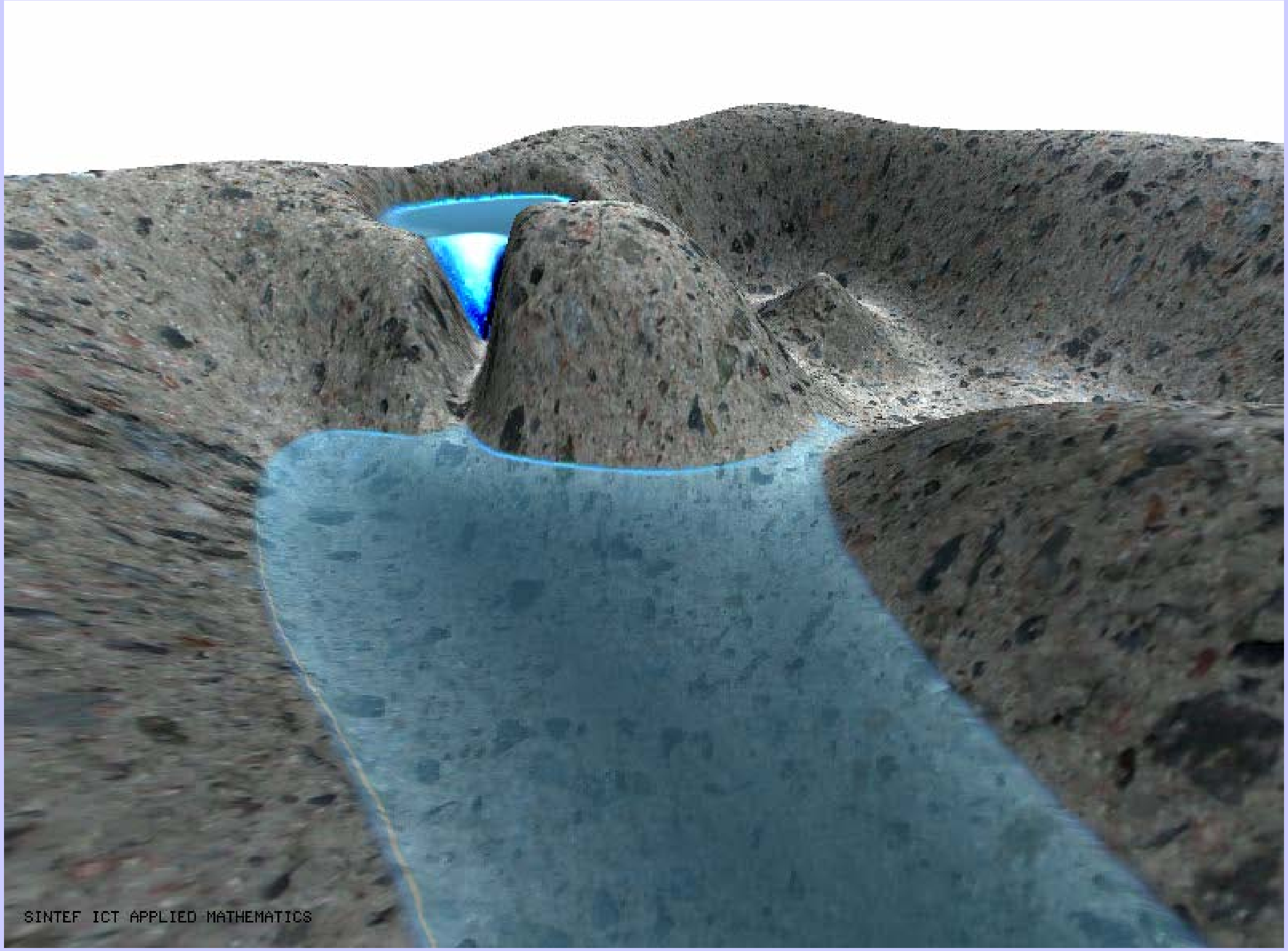
Animations

- Finite-difference scheme integrating the linear wave equation (Hagen et al. 2005, SINTEF, Norway)



Animations

- Nonlinear flood waves (Hagen et al. 2005, SINTEF, Norway)



Entertainment industry: model vs speed

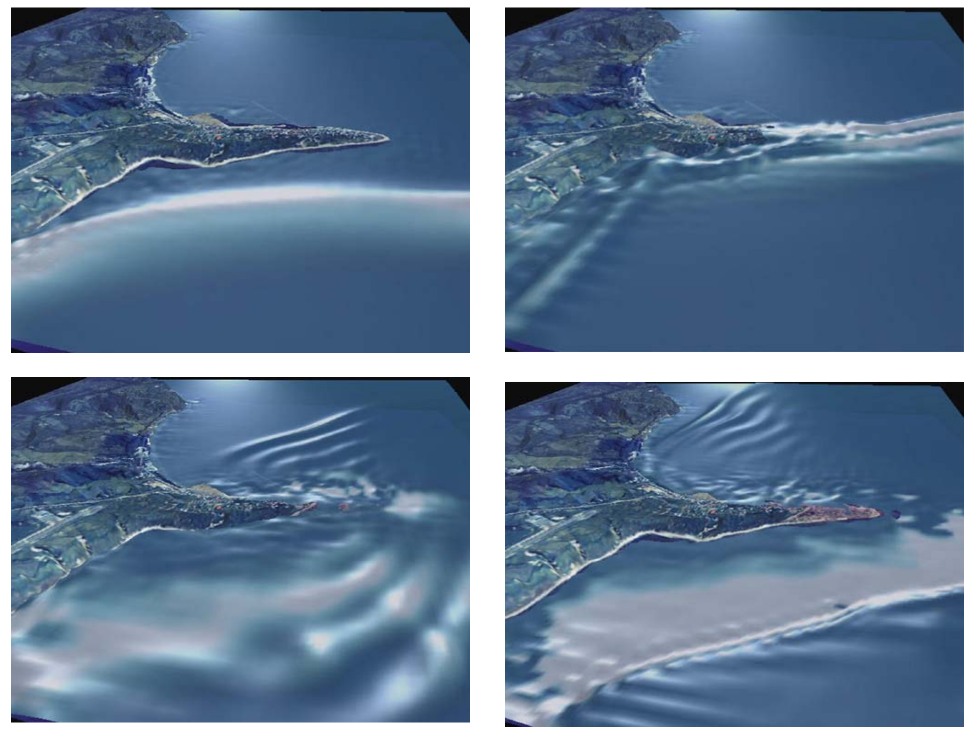
- Hagen et al. (SINTEF, Norway) are exploiting the possibilities for parallel processing within programmable graphics cards (GPU as opposed to CPU)
- Direct numerical simulation of mathematical models (linear wave equation, nonlinear shallow water equations) to produce realistic physical effects in real-time
- Simulations on the GPU found to be 15 – 30 times faster than on a CPU (Hagen et al., Simulation Modelling Practice and Theory, 2005)
- Use small grid models and simple schemes of low order
- Interactive simulations (fly-through mode that allows the user to inspect the solution while being computed)

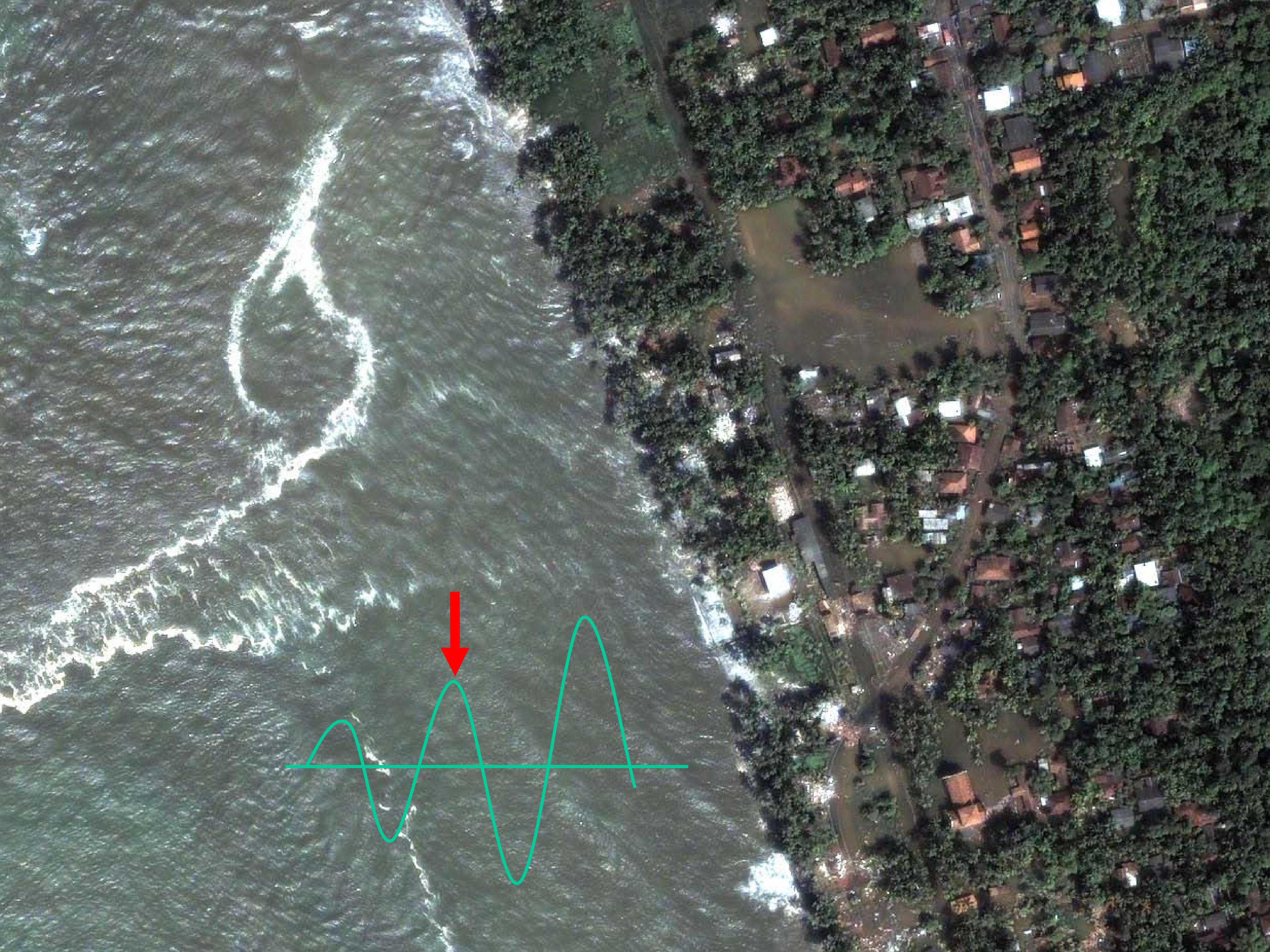
Tsunami evolution in real-time

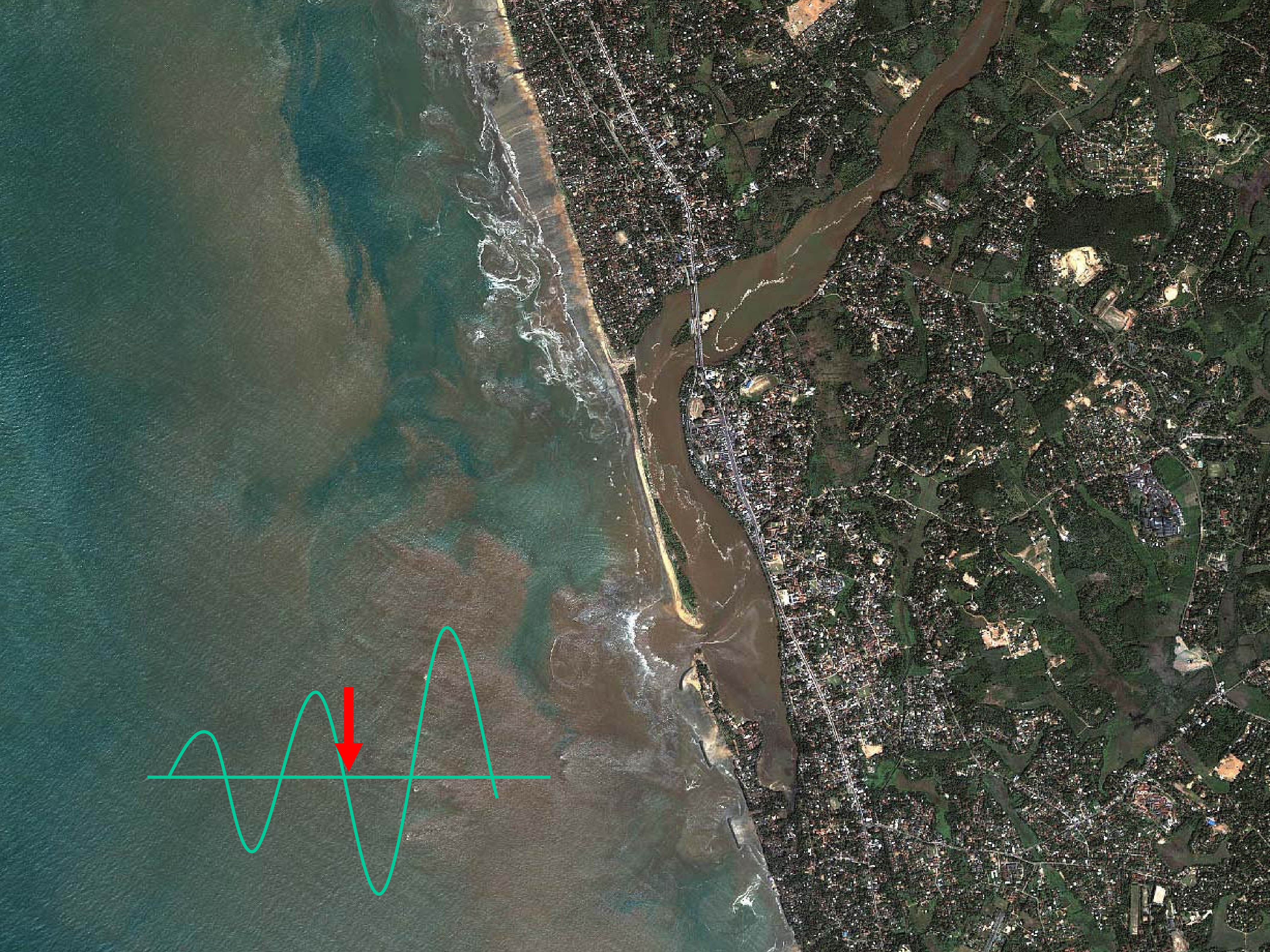
- Integrating the linear wave equation allows the determination of tsunami arrival times in less than 1 minute of computing
- Integrating the nonlinear shallow-water equations allows the determination of tsunami heights in less than 30 minutes of computing
- The start of the computations can be made automatic after receiving info. from USGS (for example)

Okushiri 1993

Damage in Aonae, during the 1993 Okushiri tsunami. *Notice the overland flow in the animation stills on the left.* (Synolakis 2006)





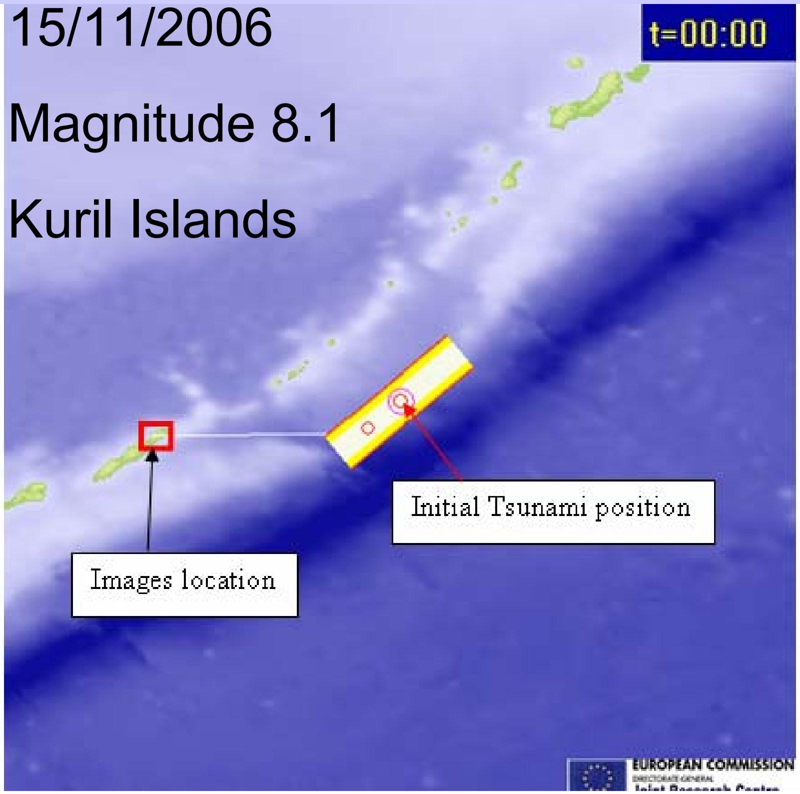


Satellite images of the tsunami area

15/11/2006

Magnitude 8.1

Kuril Islands



The analysis of satellite images of the area close to the epicenter indicate a damage to the vegetation which can only be explained with the effect of the tsunami (Annunziato 2006)

6 September 2006

20 November 2006