The Waisman Laboratory
for Brain Imaging and Behavior

Encoding Surface Shape Asymmetry:
Weighted Spherical Harmonic Representation

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Joint work with

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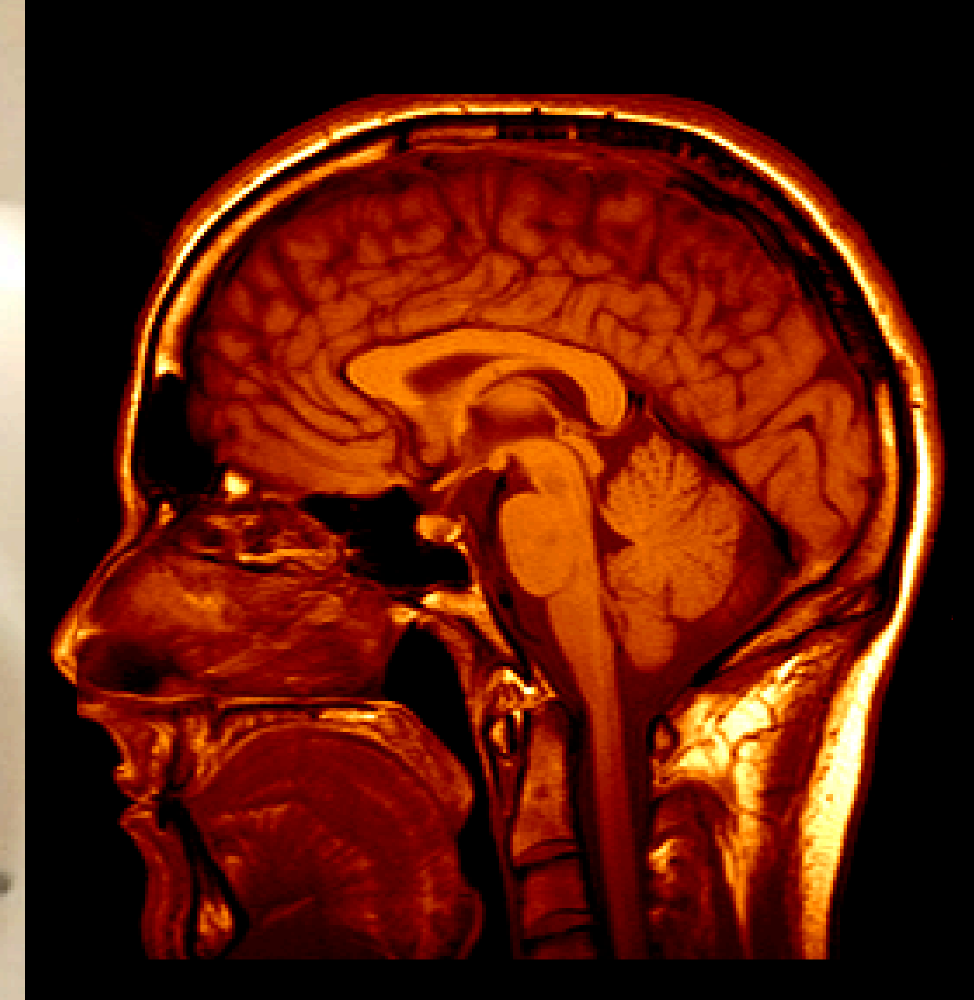
Abstracts

A recently developed weighted spherical harmonic (SPHARM) representation will be presented. The weighted-SPHARM is a partial differential equation (PDE) based shape representation technique that incorporates surface parameterization, surface data smoothing, and surface normalization in a unified framework. The weighted-SPHARM represents surface data as a weighted linear combination of spherical harmonics in such a way that the representation reduces the Gibbs phenomenon associated with Fourier series. Using the inherent angular symmetry of the spherical harmonics, surface shape can be decomposed into symmetric and asymmetric components. The resulting shape asymmetry index is given as the ratio of positive and negative order harmonics. As an illustration, the methodology is applied in characterizing and detecting abnormal cortical asymmetry pattern of autistic brain.

Outline

- Data
- Motivation
- Weighted Fourier Series Representation
- Shape Asymmetry Analysis
- Discussion



Data: 3T MRI

16 high functioning autistic subjects (15.93±4.71 years)

12 normal controls (17.08±2.78 years)

Right-handed males of compatible age range.

Aim:

Quantify abnormal cortical asymmetry pattern in the autistic subjects

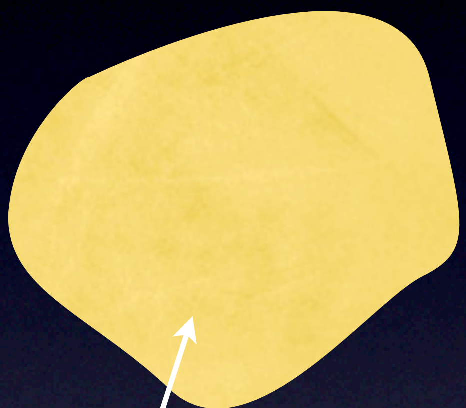
Motivation

Problem: Quantify cortical shape asymmetry across hemispheres

Challenge 1: Establishing hemispheric surface correspondence

Challenge 2: Establishing intersubject surface correspondence

Multiscale representation of anatomical surface and function defined on surface



Measurement $f(p)$ at position

$(u_1(p), \cdots, u_d(p))$

Anatomical boundary $\mathcal{M} \in \mathbb{R}^d$

Hilbert space $L^2(\mathcal{M})$ with

$$\langle g_1, g_2 \rangle = \int_{\mathcal{M}} g_1(p) g_2(p) \, d\mu(p)$$

For a given self-adjoint operator $\mathcal{L}$

measurement + coordinates as the initial value

$$\partial_t g + \mathcal{L} g = 0, g(p, t=0) = f(p)$$

time = scale, bandwidth of smoothing

Weighted Fourier series (WFS) representation

$$\mathcal{L}\psi_j = \lambda_j \psi_j$$

PDE

$$g(p,t) = \sum_{j=0}^{\infty} e^{-\lambda_j t} \langle f, \psi_j \rangle \psi_j(p)$$

Basis expansion

$$= \int_{\mathcal{M}} K_t(p,q) f(q) \, d\mu(q)$$

Kernel smoothing

$\mathcal{L}$ =Laplace-Beltrami operator

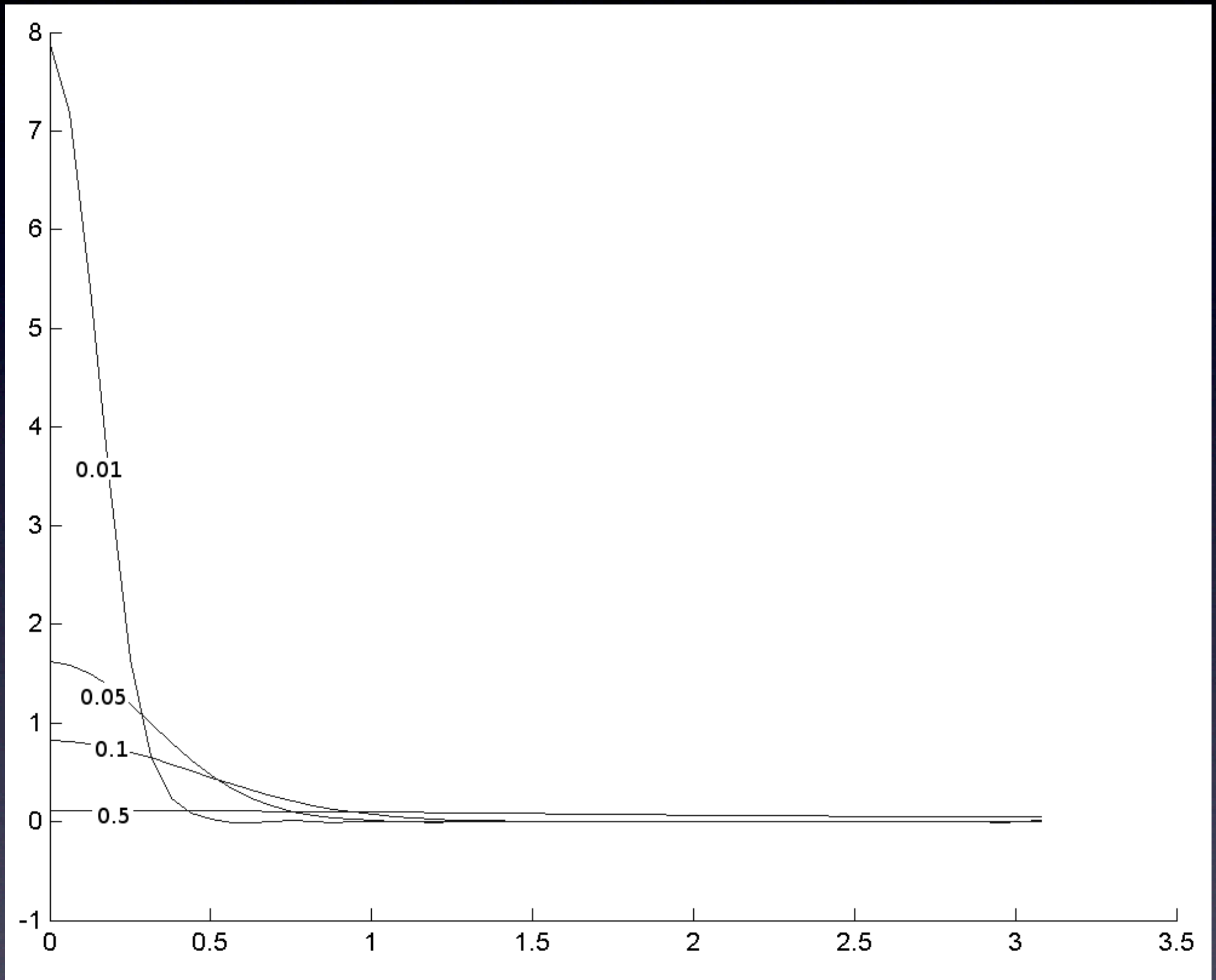
$\longrightarrow$ K_t =Heat kernel

Heat diffusion via FEM (NeuroImage, 2003)
= Heat kernel smoothing (NeuroImage, 2005)
= WFS representation (IEEE Trans. on Medical Imaging, 2007)

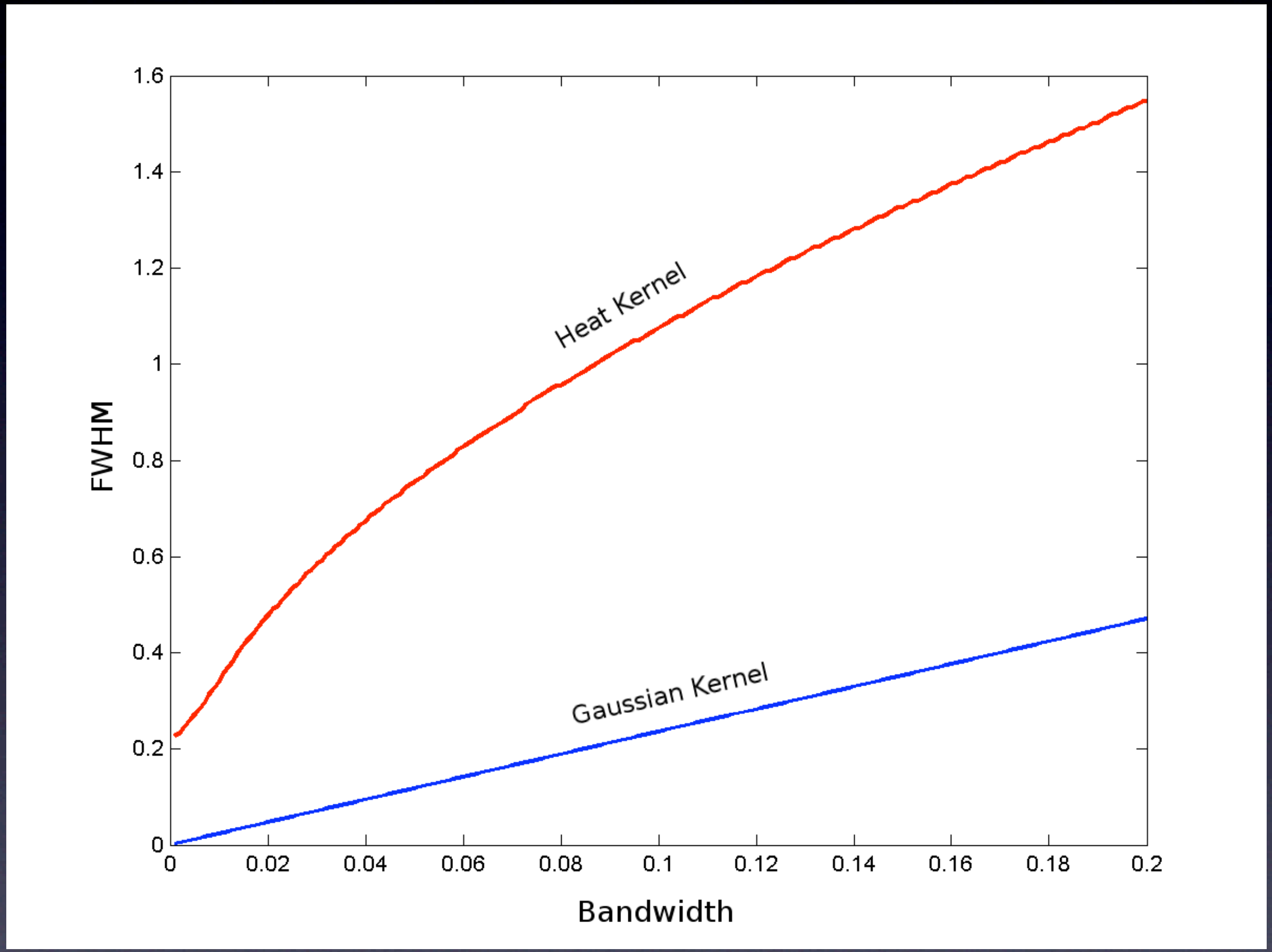
Implicit nonparametric approaches

Explicit parametric approach

Heat kernel on unit sphere



Shape



FWHM vs. bandwidth

Function estimation on manifold

Finite subspace: $$\mathcal{H}_l = \{\sum_{j=0}^{l} \beta_j \psi_j(p) : \beta_j \in \mathbb{R}\}$$

If the kernel is a probability distribution,

$$\sum_{j=0}^{l} e^{-\lambda_j t} \langle f, \psi_j \rangle \psi_j(p) = \arg\min_{h \in \mathcal{H}_l} \int_{\mathcal{M}} K_t(p,q) |f(q) - h(p)|^2 \, d\mu(q)$$

WFS is the 0th order local polynomial regression!

closer measurements are given more weights

Surface-to-surface registration (WFS-correspondence)

The performance of registration will be determined by the choice of a differential operator.

Consider two WFS surfaces v_{i1}, v_{i2}.

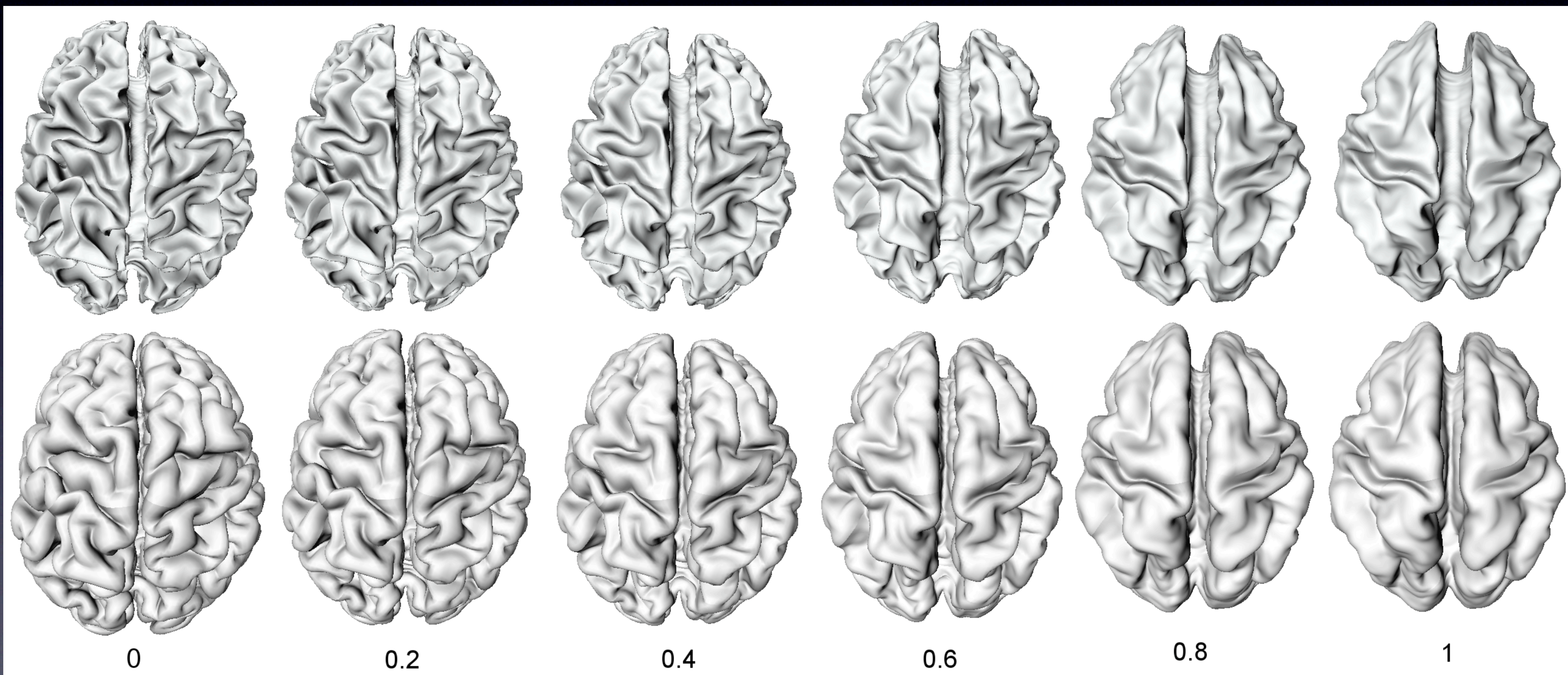
Find the displacement d_i that minimizes the discrepancy between two surfaces:

$$v_{i2} - v_{i1} = \arg\min_{d_i \in \mathcal{H}_l} \int_{\mathcal{M}} [v_{i1} + d_i(v_{i1}) - v_{i2}]^2 \, d\mu(p).$$

Surface registration is simply done by subtracting two WFS representations.

Trajectory of surface registration

(ex. Laplace-Beltrami operator: smoothing operator)



single
subject

average
surface

Is WFS-correspondence intuitively correct?

Intuition

For two algebraically defined surfaces, the optimal deformation should be obtained algebraically by hand without a numerical optimization.

What is going on?

Displacement field is estimated by matching the surface features of the same frequency while minimizing the goodness of fit.

Summary

WFS = surface representation
+ surface registration
+ surface smoothing (fairing)
+ surface data smoothing

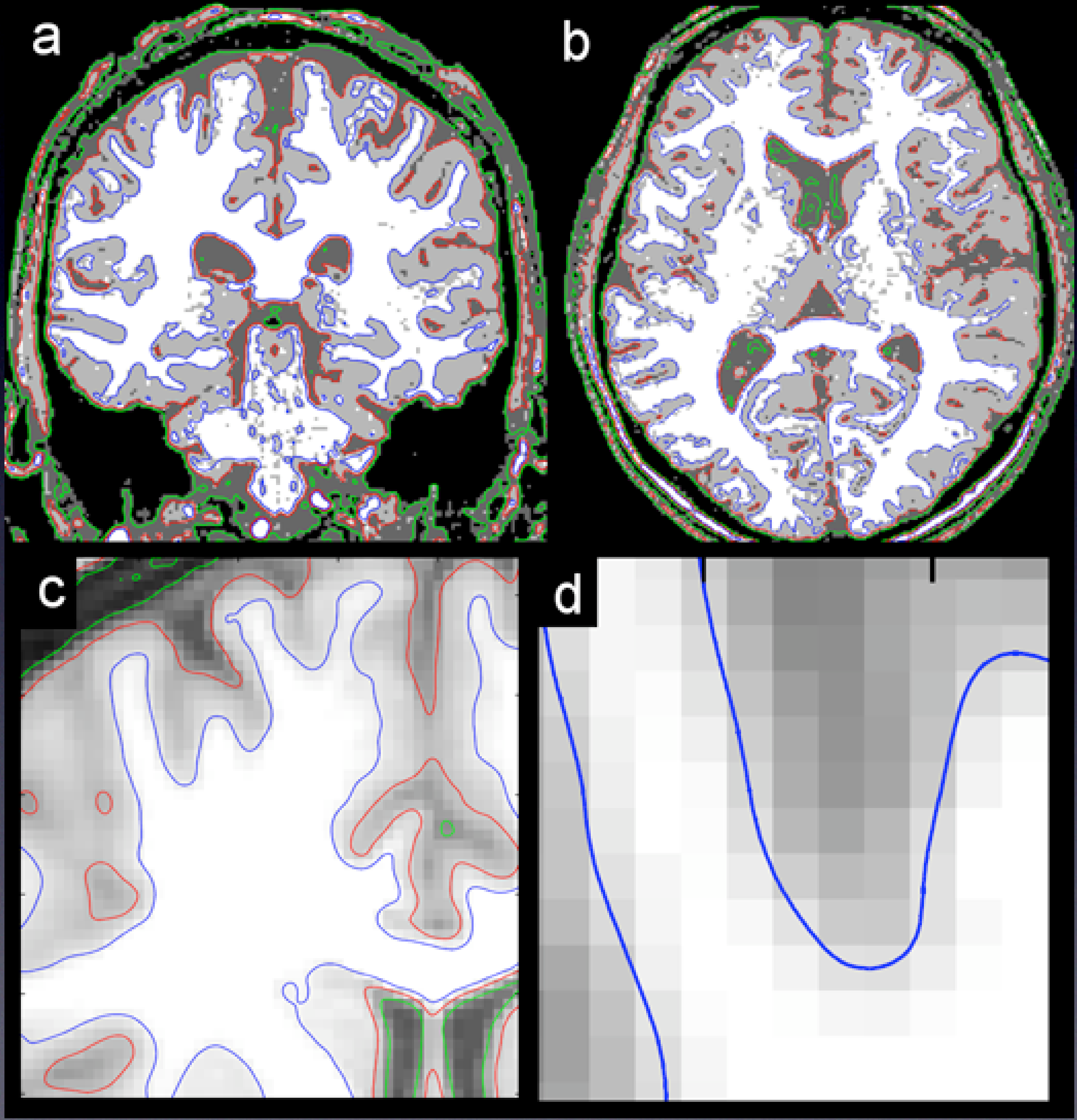
Is it possible to come up with a more unified basis function method that contains surface segmentation ?

Our first attempt

Thin-plate spline (TPS) segmentation

Directly segment and represent the anatomical boundary as the linear combination of thin-plate basis functions

(joint work with Xie and Wahba)

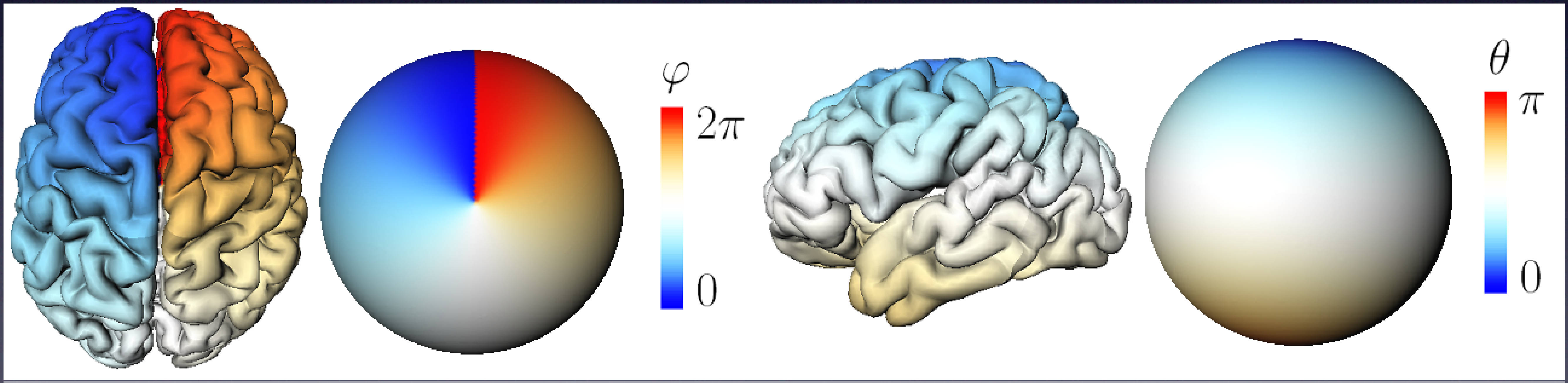


Weighted spherical harmonic (SPHRM) representation

When we choose the Laplace-Beltrami operator

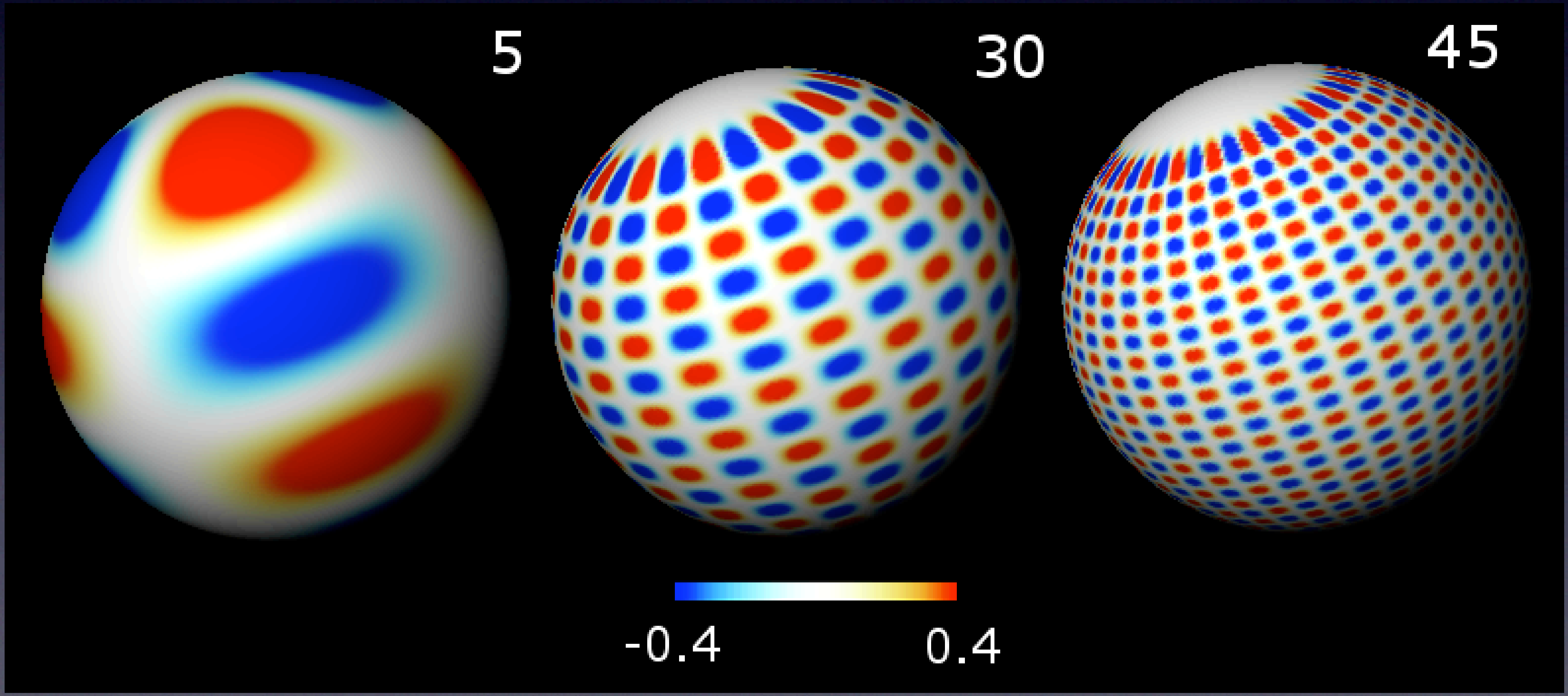
Spherical mapping

Deformable surface algorithm (McDonalds et al., 2001) is used to segment surfaces and obtain the mapping from a unit sphere to a cortical surface.



Spherical harmonic of degree *l* and order *m*

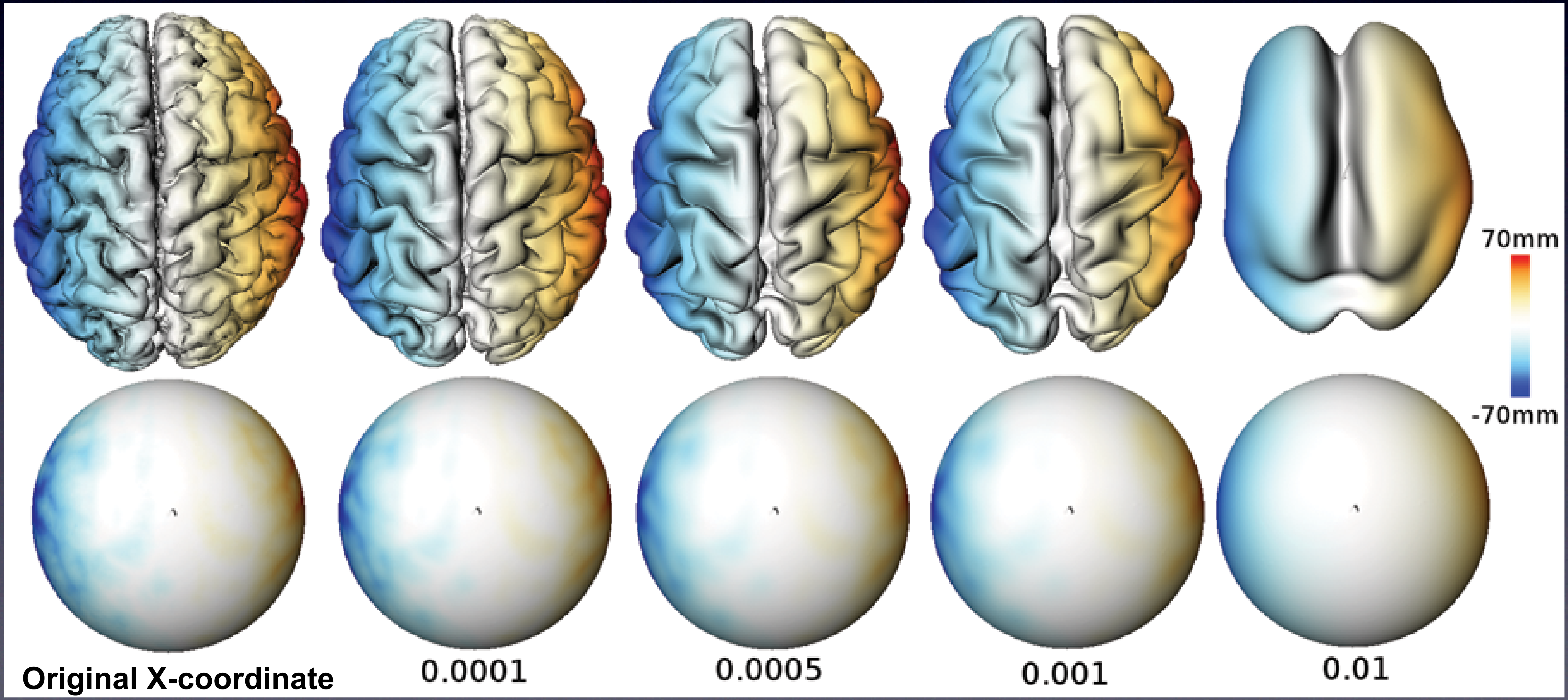
$$Y_{lm} = \begin{cases} c_{lm} P_l^{|m|}(\cos\theta)\sin(|m|\varphi), & -l \le m \le -1, \\ \frac{c_{lm}}{\sqrt{2}} P_l^0(\cos\theta), & m = 0, \\ c_{lm} P_l^{|m|}(\cos\theta)\cos(|m|\varphi), & 1 \le m \le l, \end{cases}$$



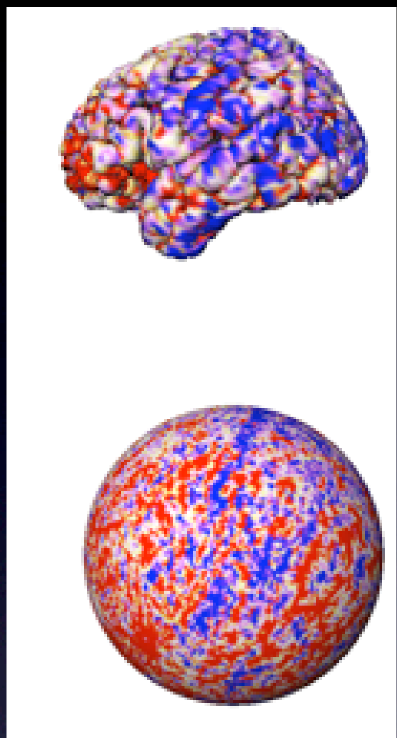
Lower degree
Coarse detail

Higher degree
Fine detail

Multiscale representation of surface and x-coordinate



Weighted-SPHARM of cortical thickness

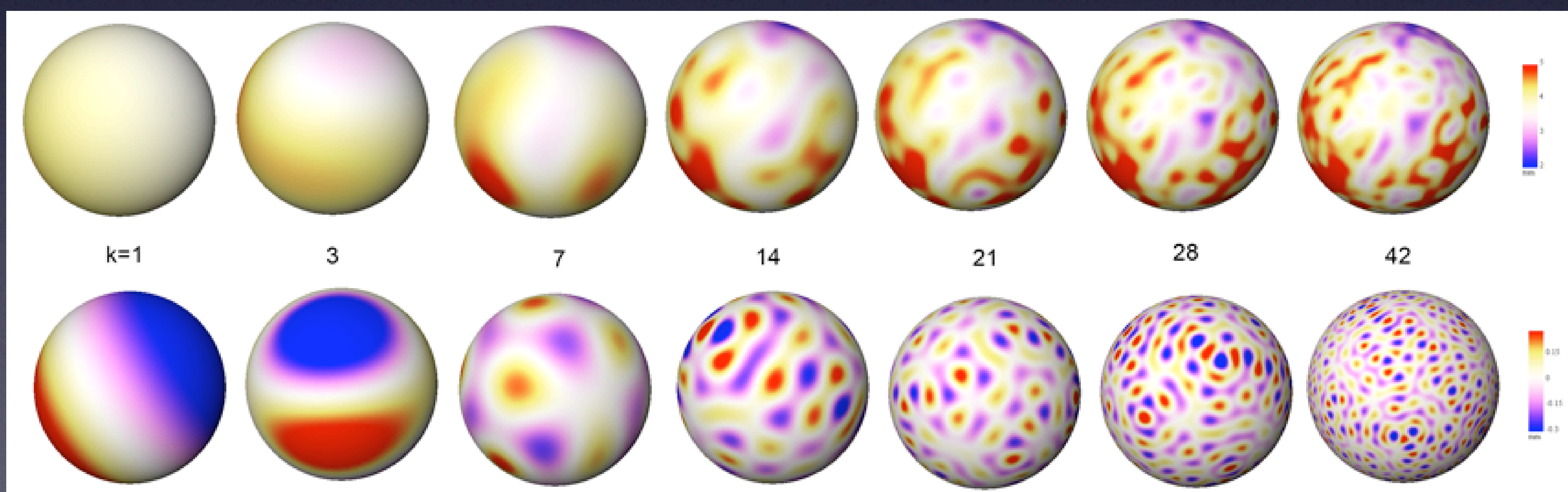


Cortical thickness

Spherical mapping

1st row: $$\sum_{m=-k}^{k} e^{-l(l+1)\sigma} \langle f, Y_{lm} \rangle Y_{lm}$$

2nd row: $$\sum_{m=-k}^{k} \langle f, Y_{lm} \rangle Y_{lm}$$

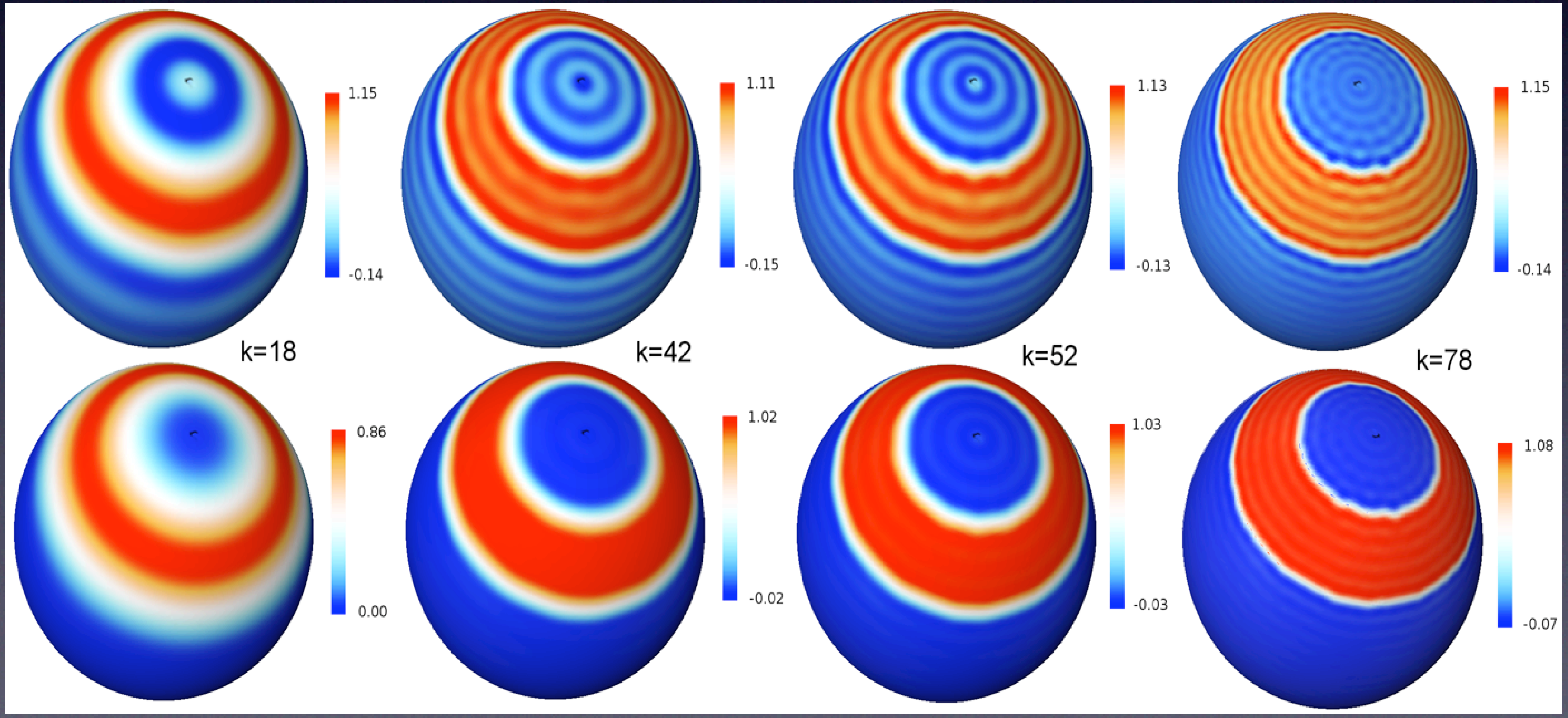


Property: reduction of Gibbs phenomenon (ringing artifacts)

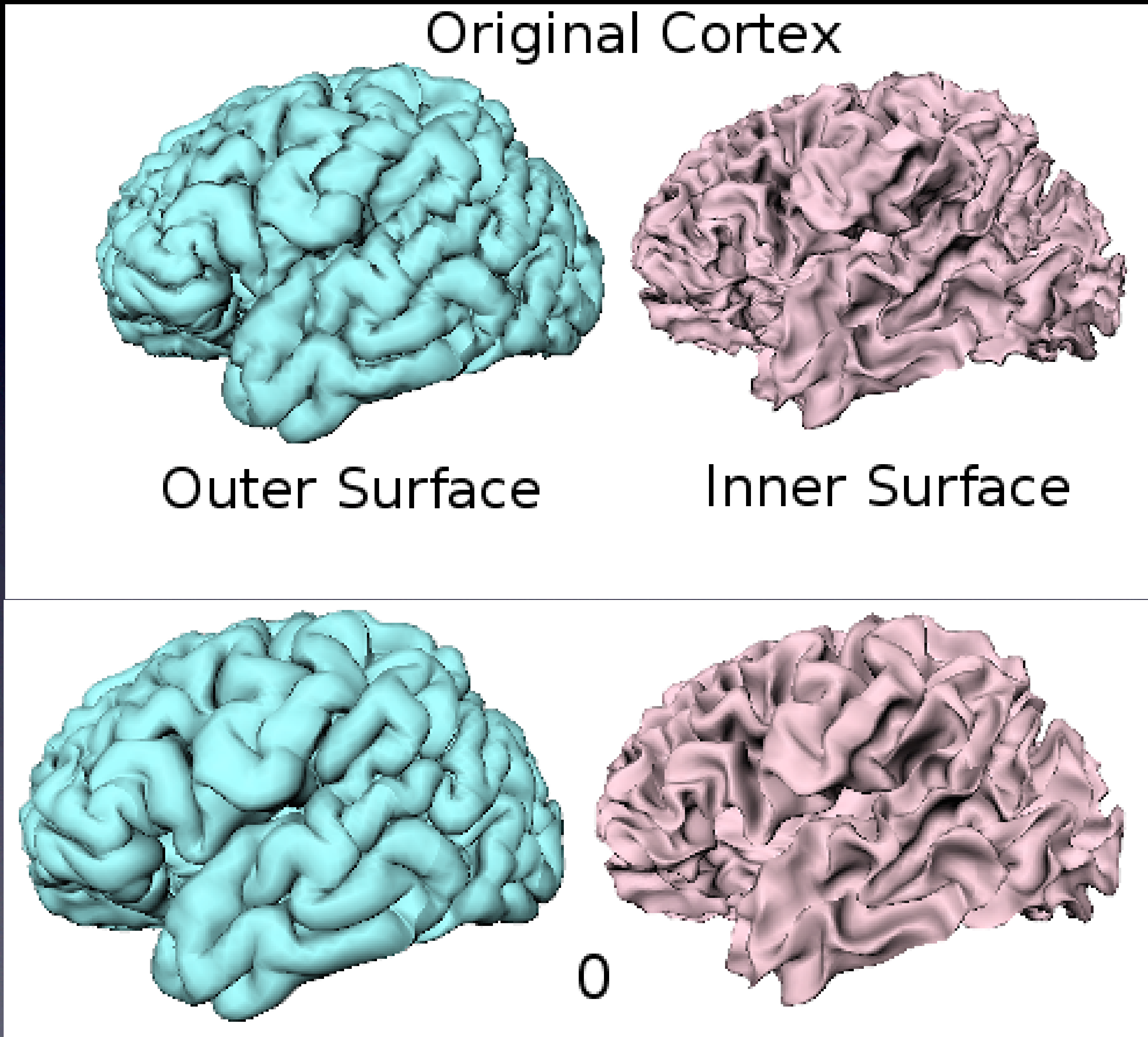
Value one in the circular band $\frac{1}{8} < \theta < \frac{1}{4}$

Traditional SPHARM representation

Weighted-SPHARM representation



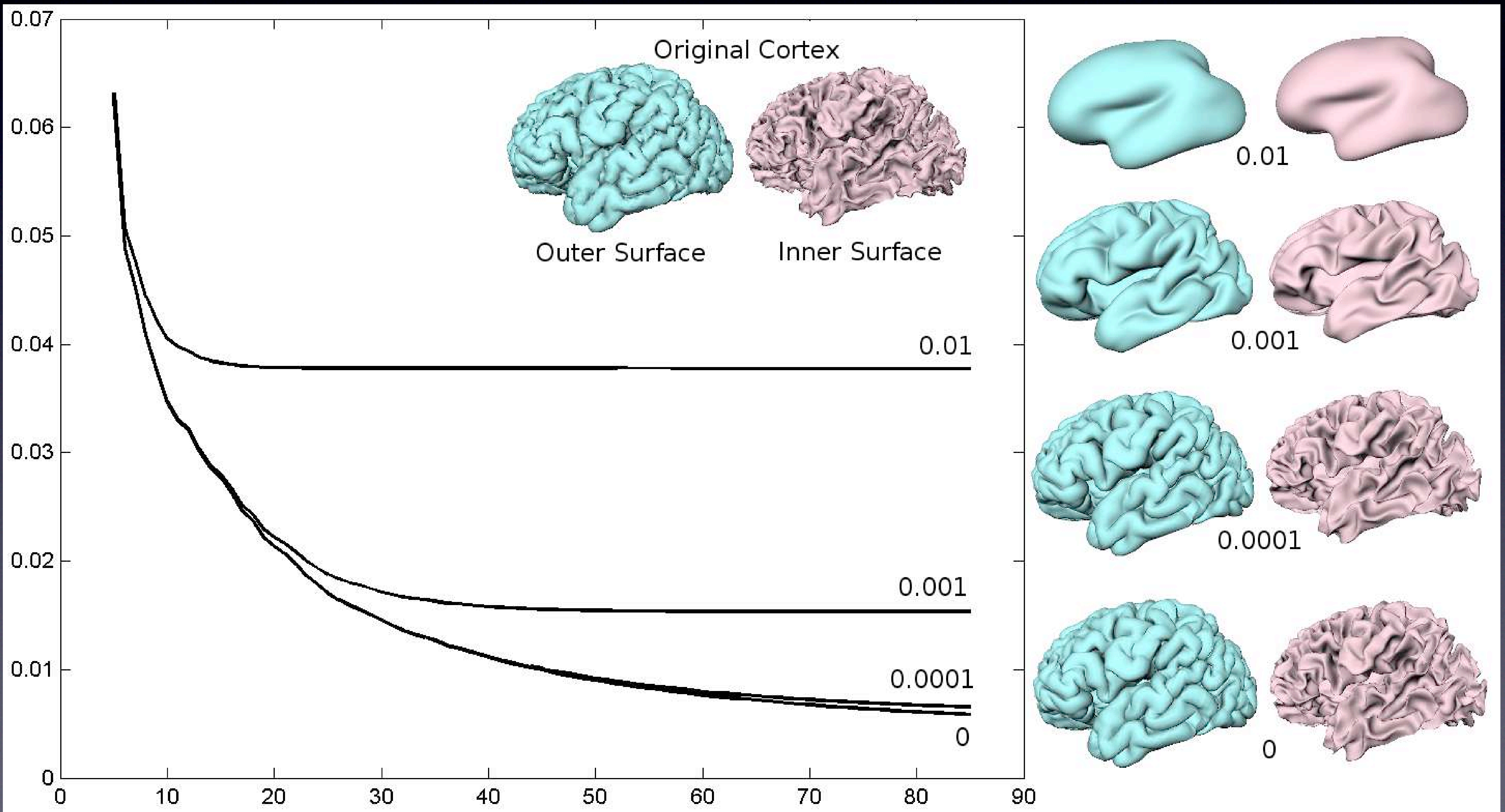
80th degree representation



What is the optimal degree?

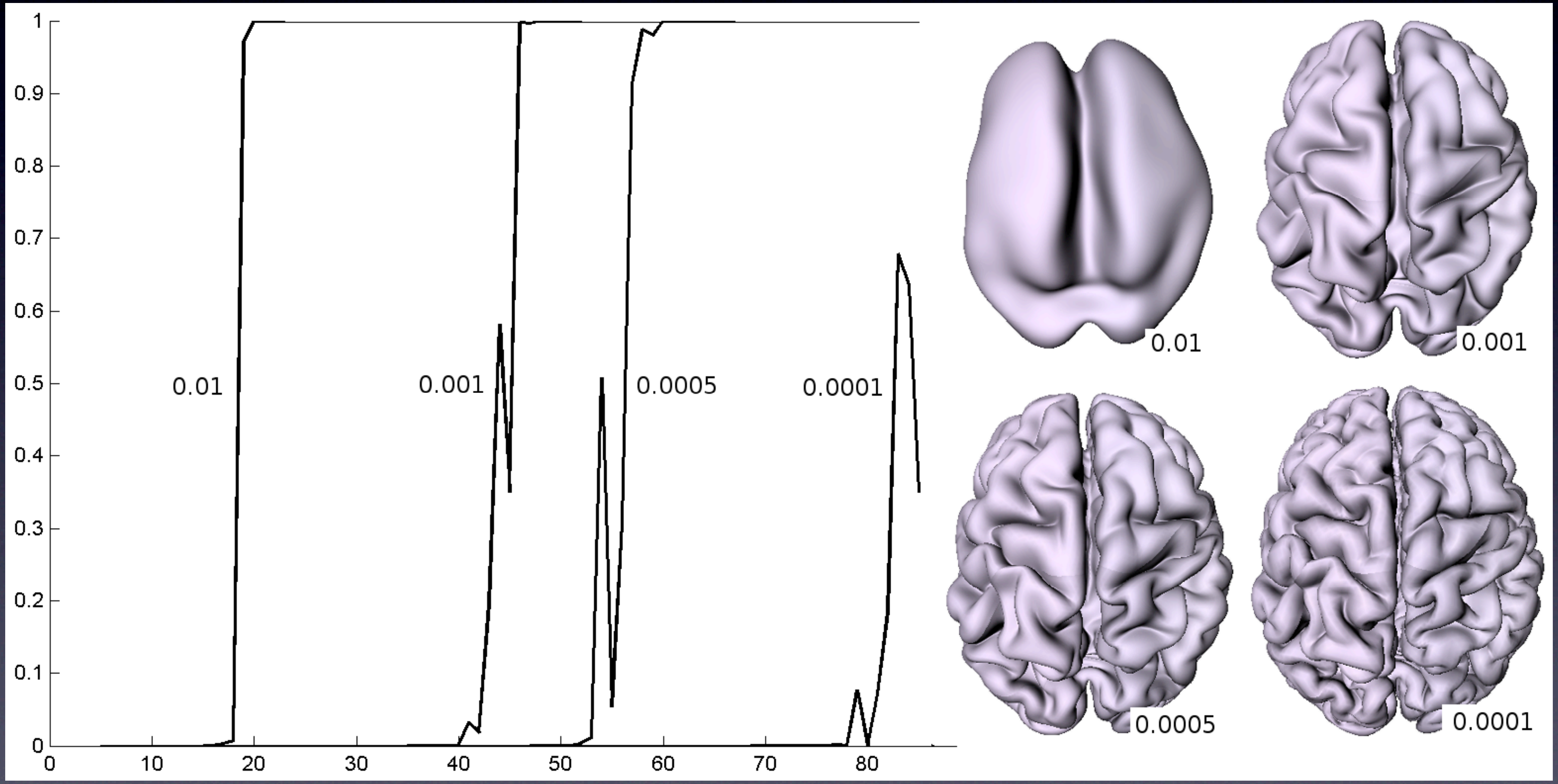
RMSE over degree

At certain degree, the reduction of residual error is no longer statistically signification



Automatic degree selection

1-Pvalue



degree

Numerical implementation

Estimating approximately 5,000 eigenvalues per each coordinates = 20,000 eigenvalues for each subjects

Iterative residual fitting (IRF) algorithm

Break one huge linear problem (=6GB) into smaller linear problems (<500MB, the memory limit of my old laptop) and solve the small problems iteratively. (joint work with Li Shen)

MATLAB code available:
http://www.stat.wisc.edu/~mchung/softwares/weighted-SPHARM/weighted-SPHARM.html

Iterative residual fitting (IRF) algorithm

Step 1. measurements $f(p_1), \cdots, f(p_n)$

Step 2. Set initial degree=0 $k = 0$

Step 3. Solve $f(p_i) = \sum_{m=-k}^{k} \beta_{km} Y_{km}(p_i)$

Project data into a finite subspace

Iterate

Step 3.5. $f \leftarrow f - \hat{f}$

Once low frequency parts are estimated, we throw them away

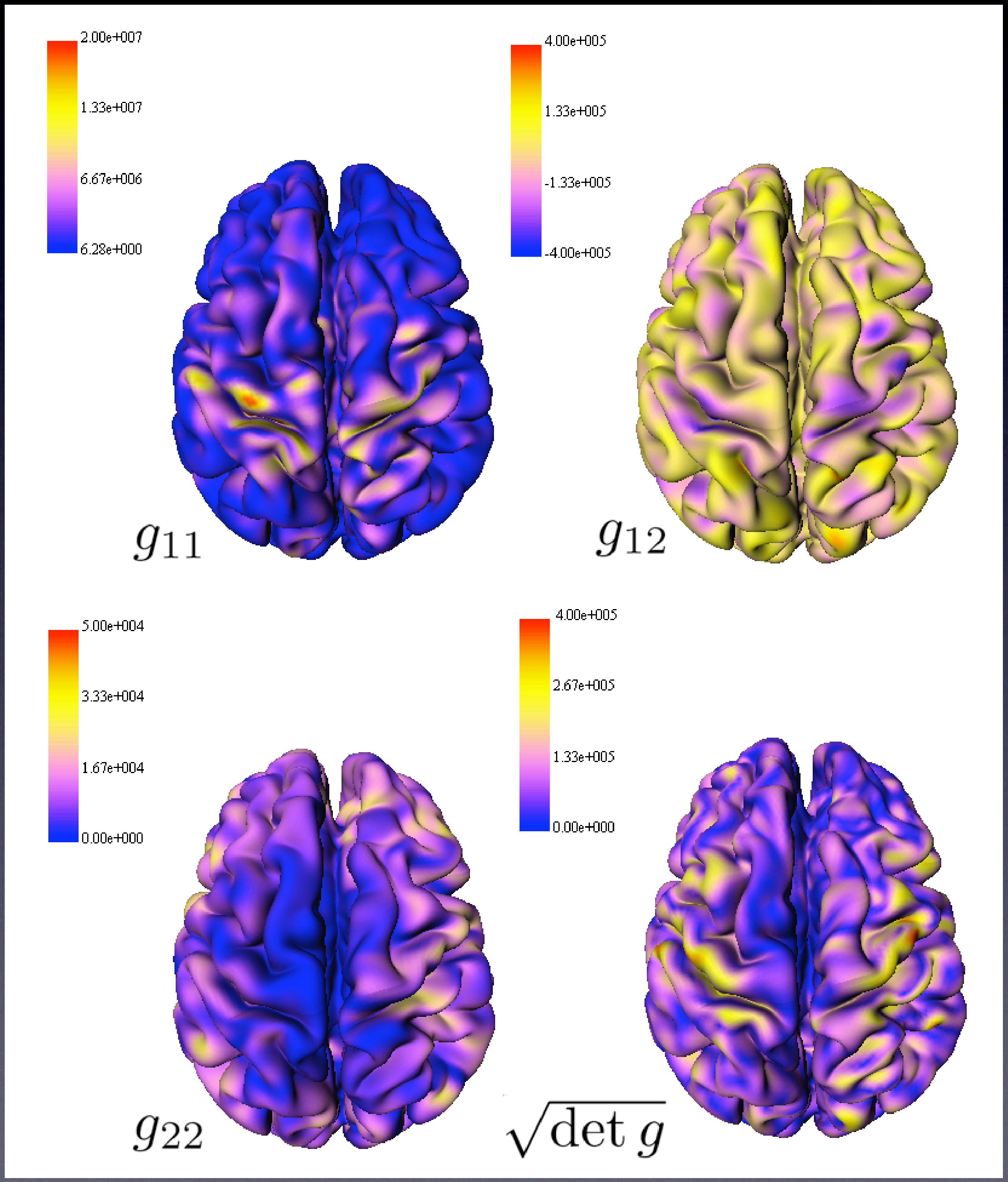
Step 4. Set degree $k \leftarrow k + 1$

Application: Tensor-based morphometry (TBM)

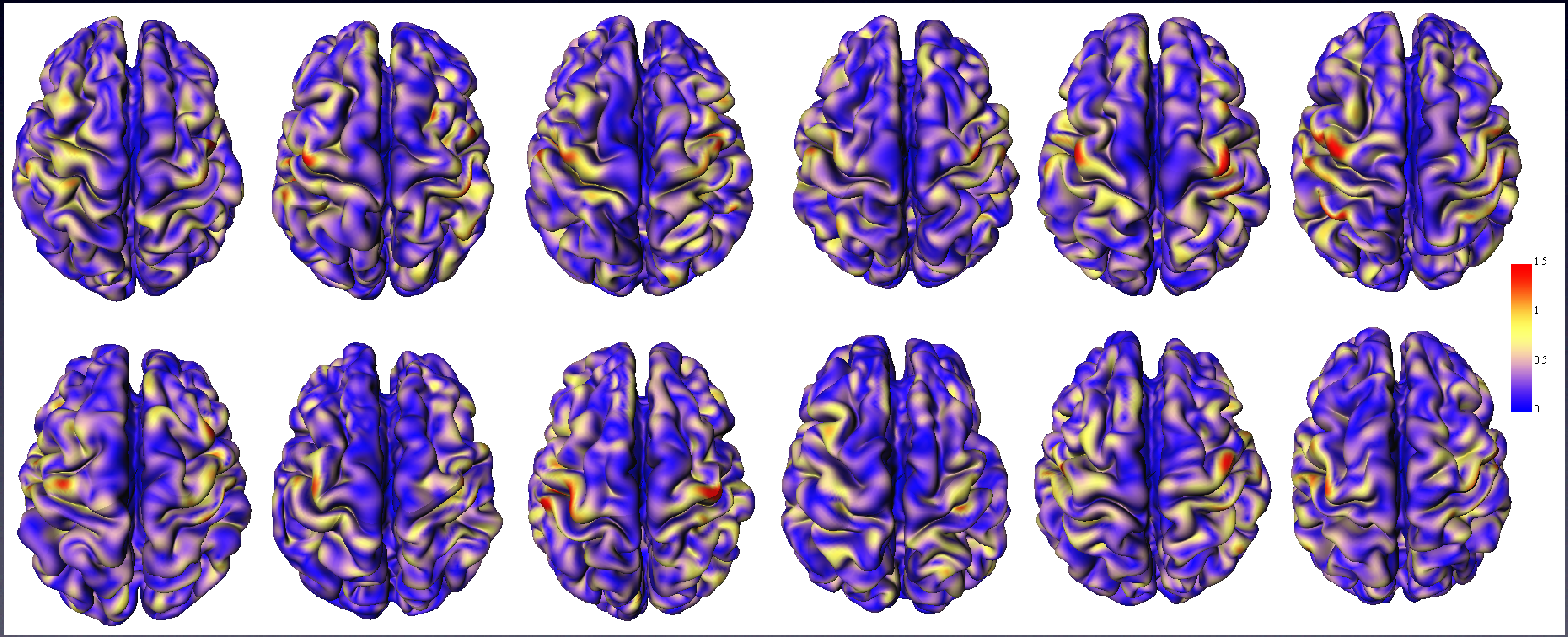
Previous approaches for estimating derivatives : local polynomial patch, discrete differential geometric operations, tensor voting

Weakness: not stable and introduce substantial mesh noise

Riemannian metric tensors and local area element

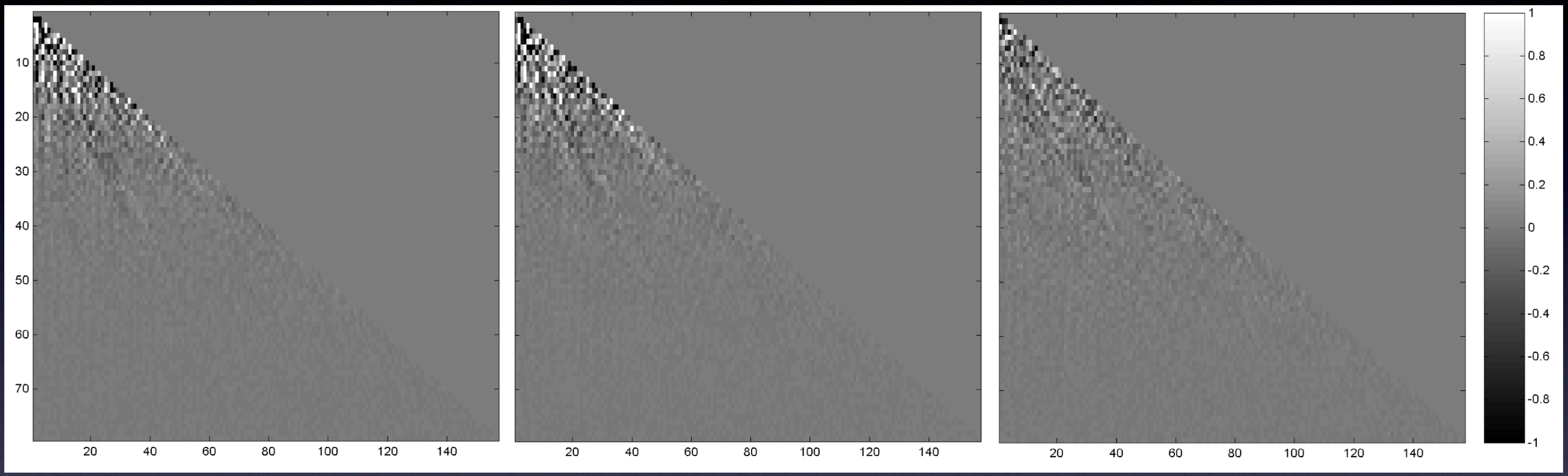


Local area expansion with respect to a template
(it ranges between 0 and 1.3)



Application:

78th degree representation = (2*78+1)^2 eigenvalues



Autistic

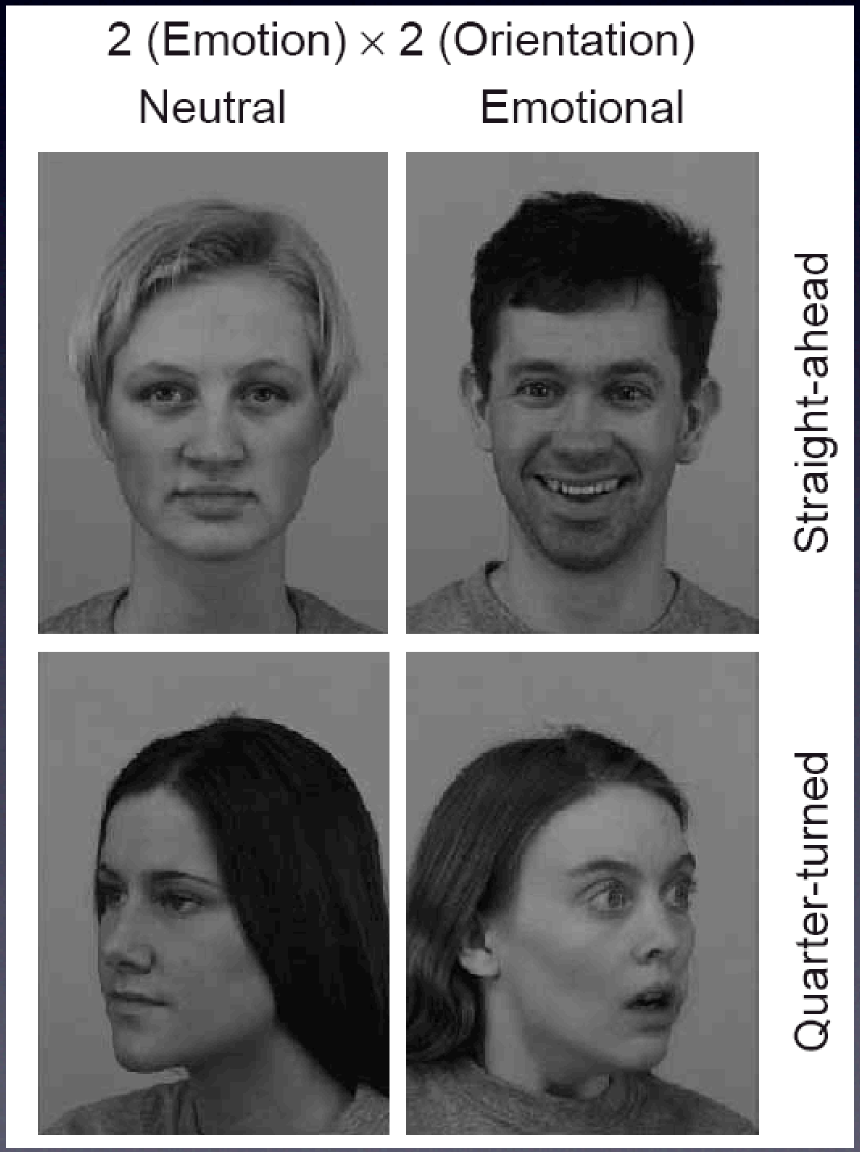
Control

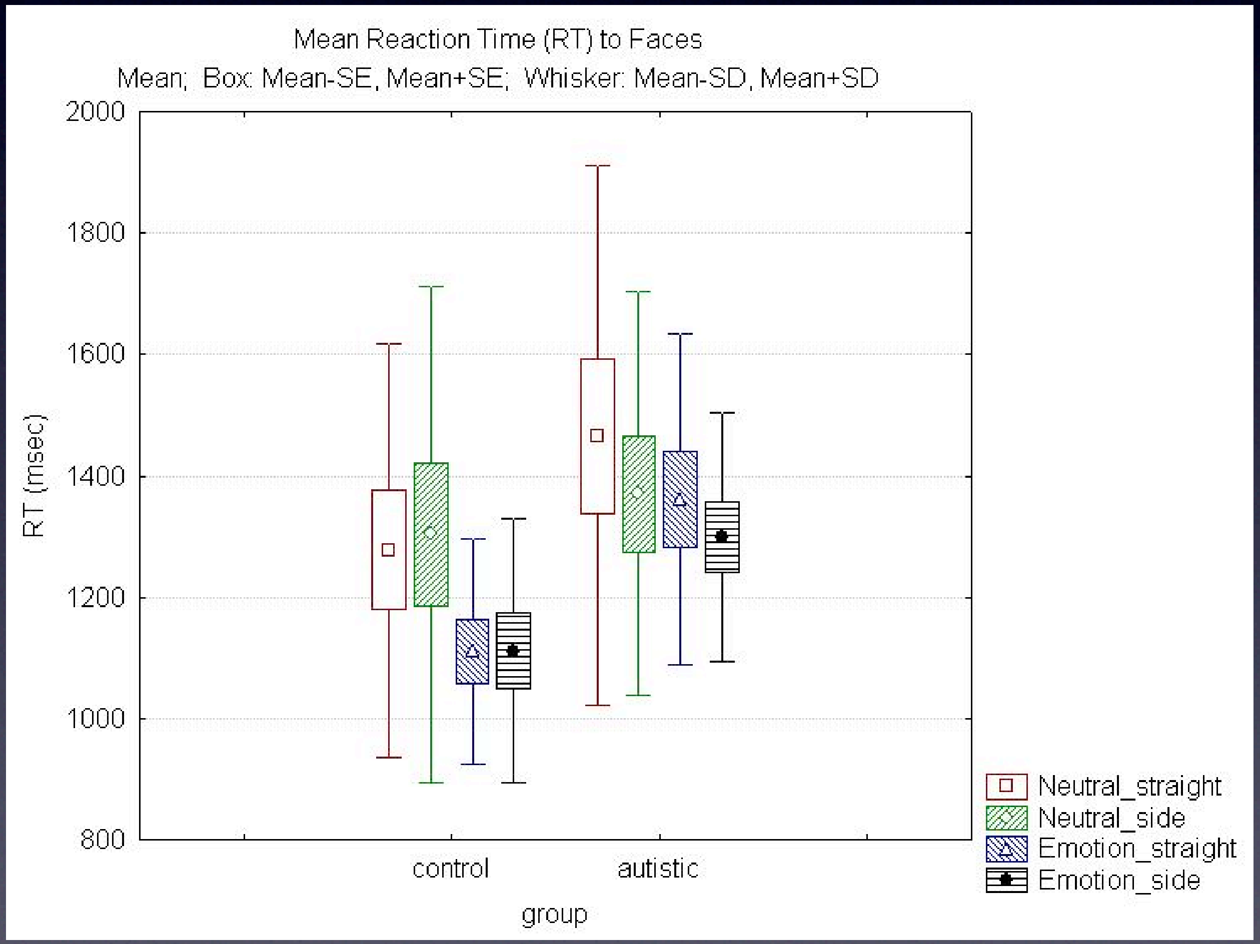
Difference

Classification techniques

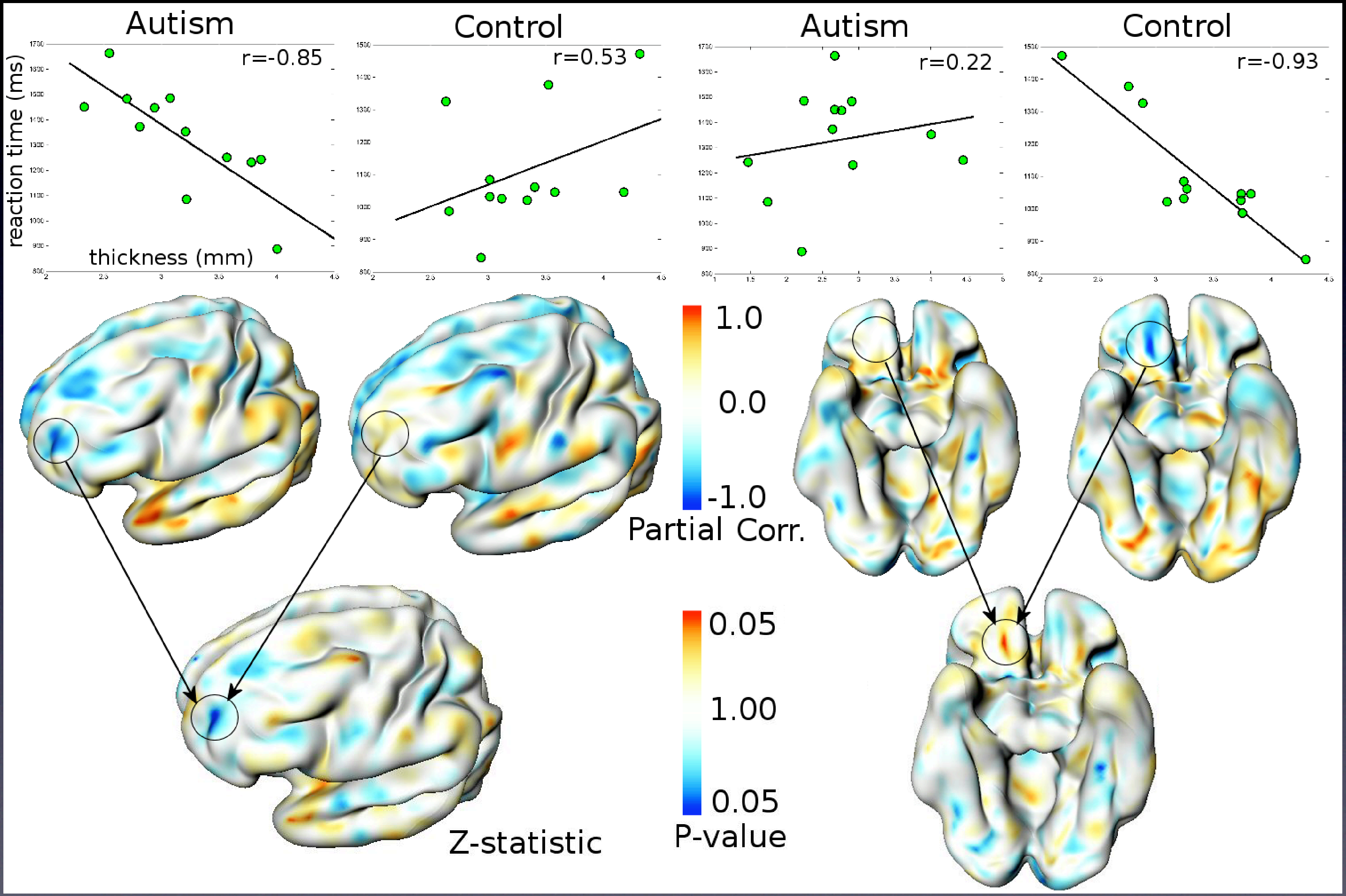
Application:
Brain-behavior correlation

Facial emotion discrimination task response time



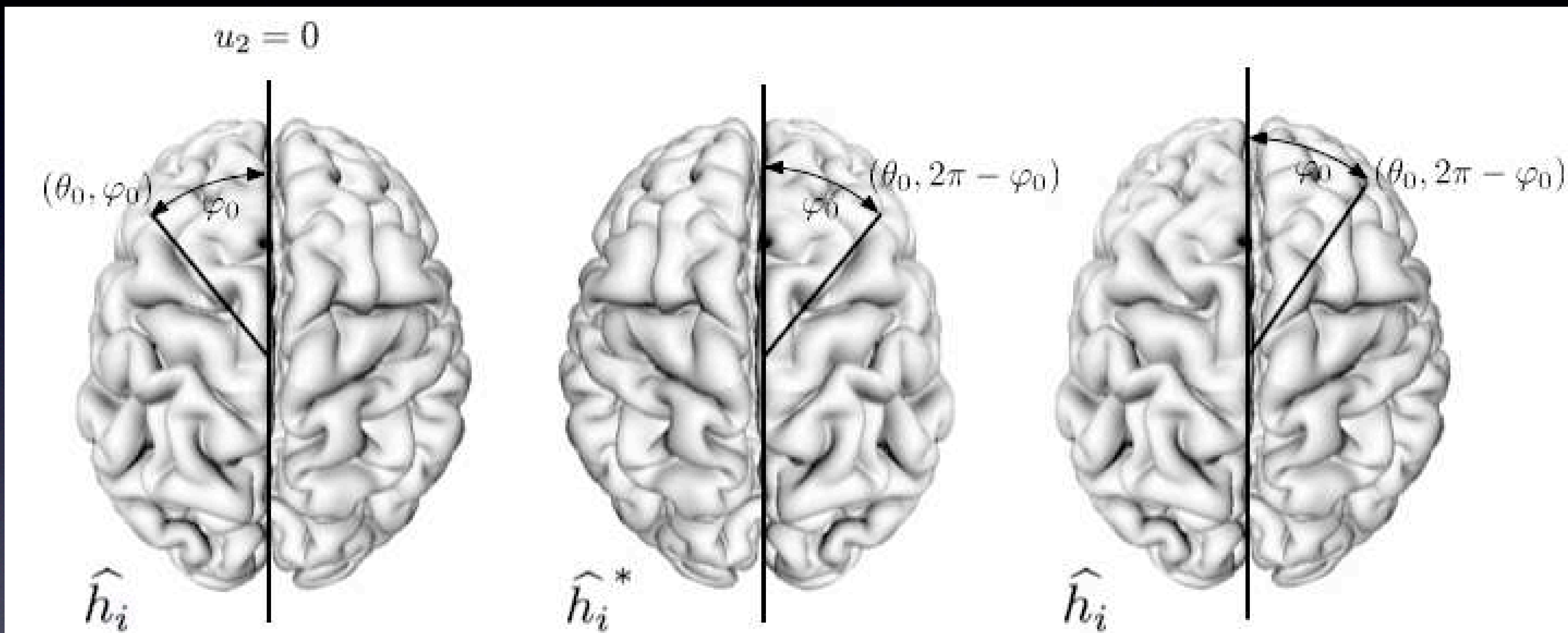


response time vs. cortical thickness



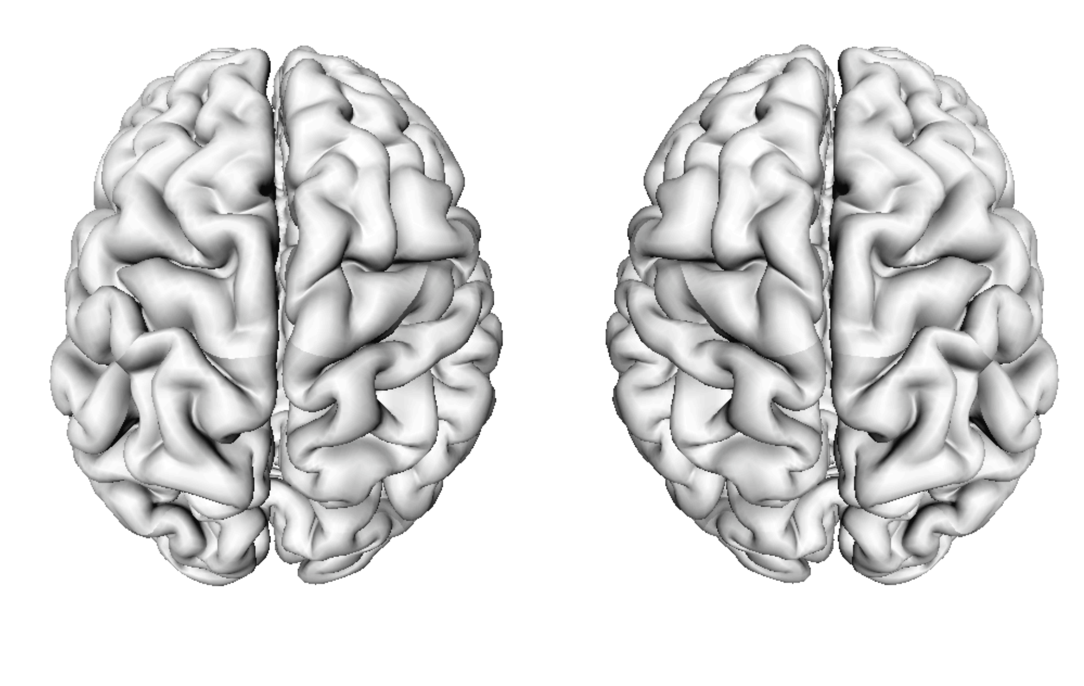
Shape asymmetry analysis via weighted-SPHARM

Establishing hemispheric correspondence



Mirror reflection

WFS-correspondence



What is preserved and what is not preserved after mirror reflection ?

$$\widehat{g}(\theta,\varphi) = \sum_{l=0}^{k} \sum_{m=-l}^{l} e^{-l(l+1)\sigma} \langle f, Y_{lm} \rangle Y_{lm}(\theta,\varphi).$$

$$\begin{aligned}\widehat{g}(\theta,2\pi-\varphi) &= \sum_{l=0}^{k} \sum_{m=-l}^{-1} e^{-l(l+1)\sigma} \langle f, Y_{lm} \rangle Y_{lm}(\theta,\varphi) \\ &\quad - \sum_{l=0}^{k} \sum_{m=0}^{l} e^{-l(l+1)\sigma} \langle f, Y_{lm} \rangle Y_{lm}(\theta,\varphi)\end{aligned}$$

Shape decomposition into symmetric and asymmetric parts

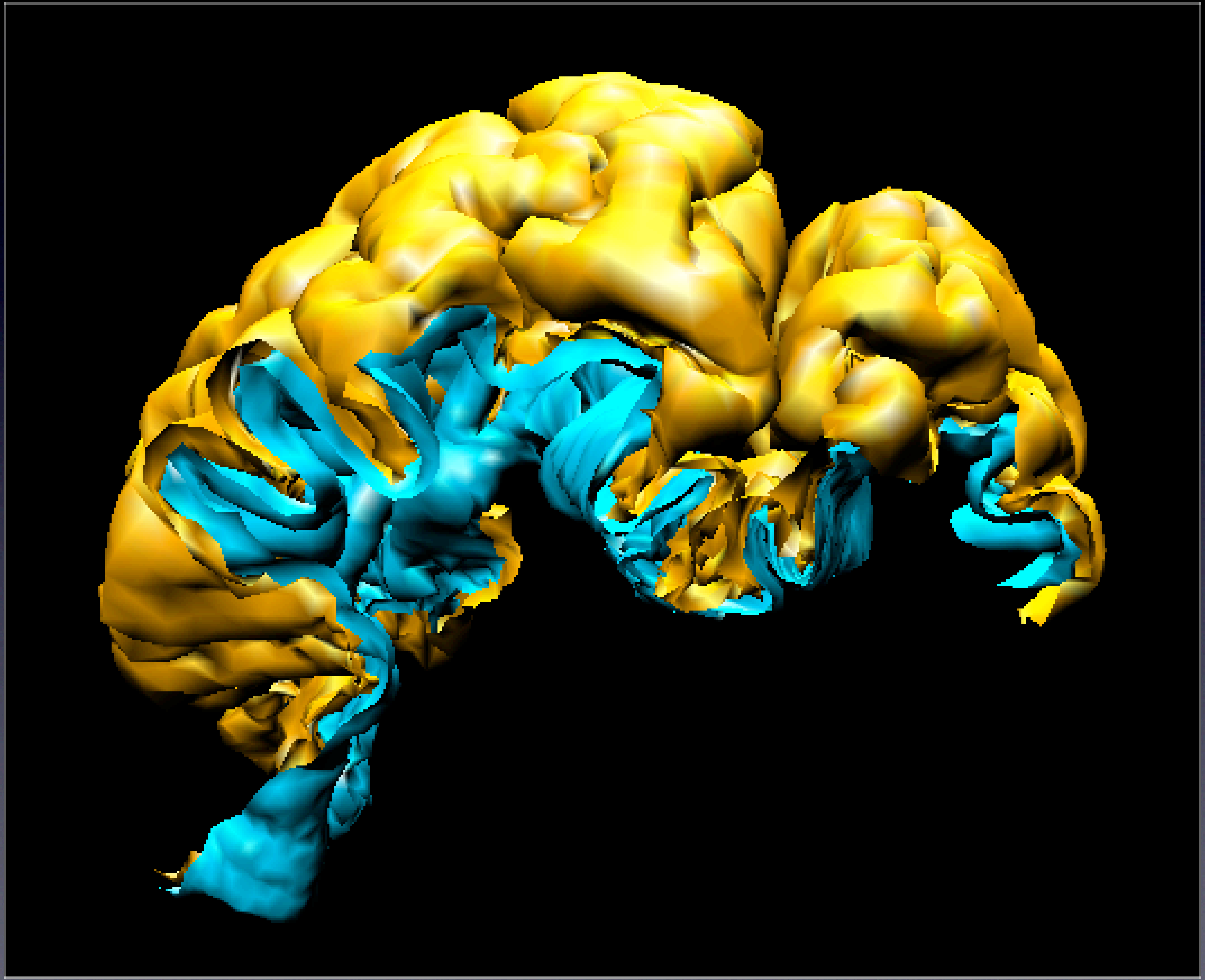
$$S(\theta,\varphi) = \frac{1}{2}\Big[\widehat{g}(\theta,\varphi) + \widehat{g}(\theta,2\pi-\varphi)\Big] = \sum_{l=0}^{k}\sum_{m=-l}^{-1} e^{-l(l+1)\sigma}\langle f, Y_{lm}\rangle Y_{lm}(\theta,\varphi)$$

$$A(\theta,\varphi) = \frac{1}{2}\Big[\widehat{g}(\theta,\varphi) - \widehat{g}(\theta,2\pi-\varphi)\Big] = \sum_{l=0}^{k}\sum_{m=0}^{l} e^{-l(l+1)\sigma}\langle f, Y_{lm}\rangle Y_{lm}(\theta,\varphi)$$

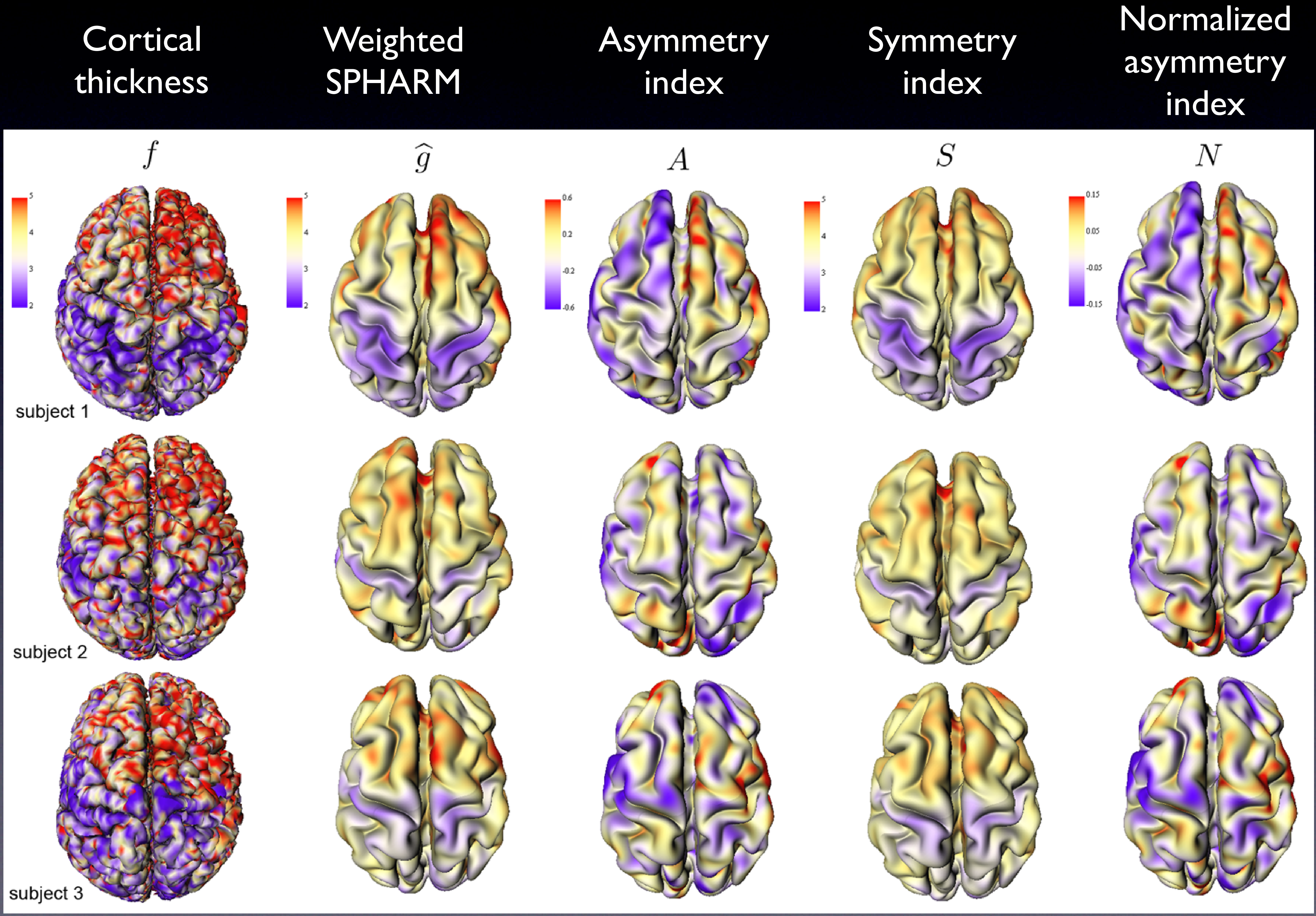
Normalized asymmetry index = (L-R)/(L+R)

$$N(\theta,\varphi) = \frac{\widehat{g}(\theta,\varphi) - \widehat{g}(\theta,2\pi-\varphi)}{\widehat{g}(\theta,\varphi) + \widehat{g}(\theta,2\pi-\varphi)} = \frac{\sum_{l=1}^{k}\sum_{m=-l}^{-1} e^{-1(l+1)\sigma}\langle f, Y_{lm}\rangle Y_{lm}(\theta,\varphi)}{\sum_{l=0}^{k}\sum_{m=0}^{l} e^{-l(l+1)\sigma}\langle f, Y_{lm}\rangle Y_{lm}(\theta,\varphi)}$$

Asymmetry analysis of cortical thickness



Asymmetry index



Final result: Statistical parametric map

multiple comparison correction via
random field theory (Worsley, Taylor)