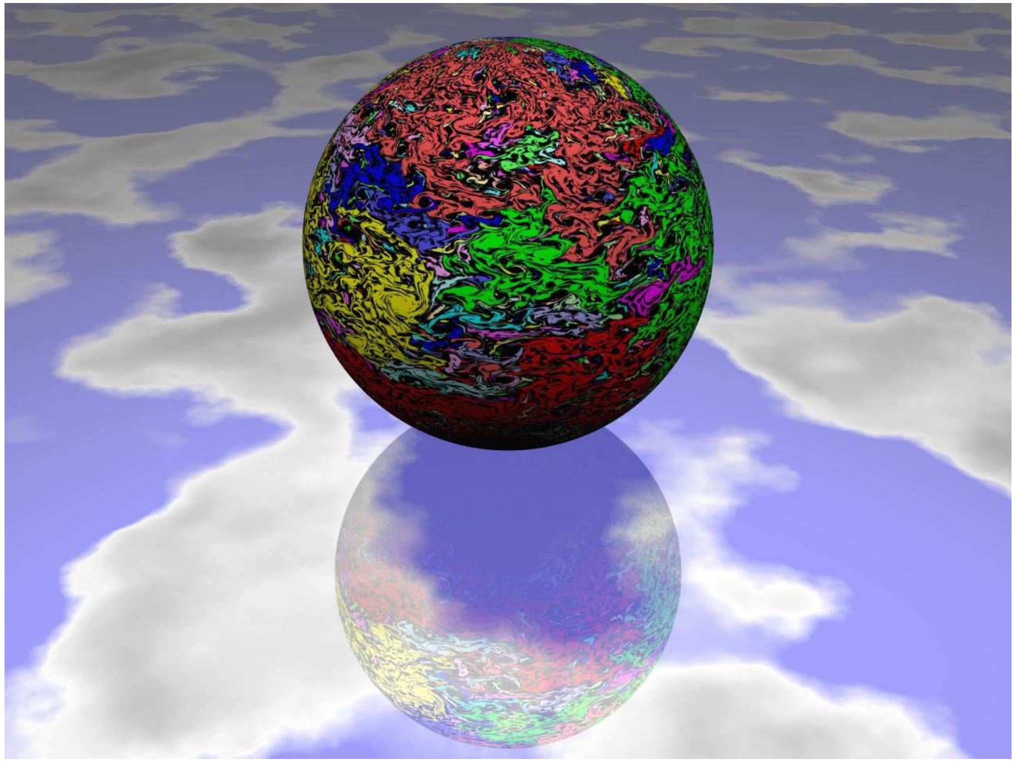
SLE in 2d Turbulent Systems

with G. Boffetta, A. Celani, G. Falkovich



Strong fluctuations + infinite many strongly interacting degrees of freedom
$\longrightarrow$ scale invariance...if in addition... + locality $\longrightarrow$ conformal invariance

Here... out of equilibrium systems + (non?) locality $\longrightarrow$???????

Basics of 2D turbulence.

Fluid Mechanics: Governed by the Navier-Stokes equation with (random) forcing (+ dissipation + friction). Non linearity.

$$\partial_t u + u \cdot \nabla u - \nu \nabla^2 u + u/\tau = -\nabla P + f, \quad \nabla . u = 0$$

Vorticity $\omega = \nabla \wedge u$ transported by the fluid (without ...)

$$\partial_t \omega + u \cdot \nabla \omega - \nu \nabla^2 \omega + \omega/\tau = F$$

All moments conserved (without...) $\rightarrow$ 2 quadratic conserved quantities:

$$\text{Energy } E = 1/2 \int d^2x\, u^2 \quad ; \quad \text{Enstrophy } Z = 1/2 \int d^2x\, \omega^2$$

In a stationnary state, balance between injection ($f.u$) /dissipation ($\nu(\nabla u)^2$).

Basics... the double cascade.

Fully developed turbulence; $Reynolds \to \infty$ ie. $\nu \to 0$.

Energy and enstrophy are injected at length L_f.

Enstrophy dissipated at small scale but not energy $\longrightarrow$ double cascade

Direct cascade: enstrophy flux ζ

inertial range $L_f \gg \ell \gg L_d$.

scaling $u_\ell \sim \zeta^{1/3} \ell$

possibly anomalous

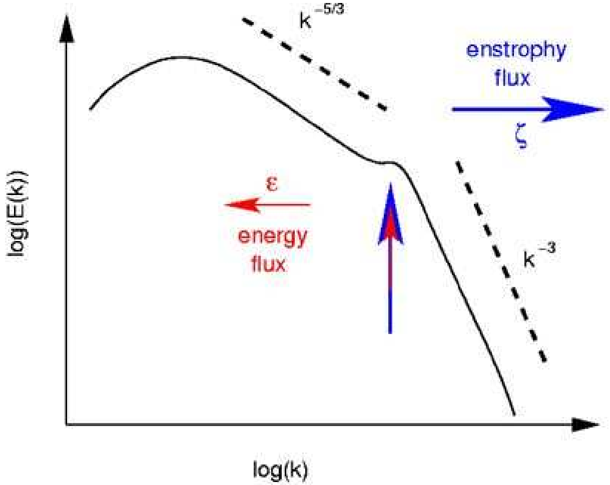
Inverse cascade: ensergy flux ε

scaling $u_\ell \sim \varepsilon^{1/3} \ell^{1/3}$

inertial range $L, L_\tau \gg \ell \gg L_f$.

possibly not anomalous

Kraichnan's double cascade.



$E(k) \sim k^{-3}$; $E(k) \sim k^{-5/3}$.

Basics... Numerical simulations.

Inverse cascade

Inertial range $10^{-2} < \ell < 1$

boxe size =1

Parameters: N spatial resolution, dx grid spacing, ν viscosity, α friction, ℓ_f forcing scale, ℓ_d=enstrophy dissipative scale, ...

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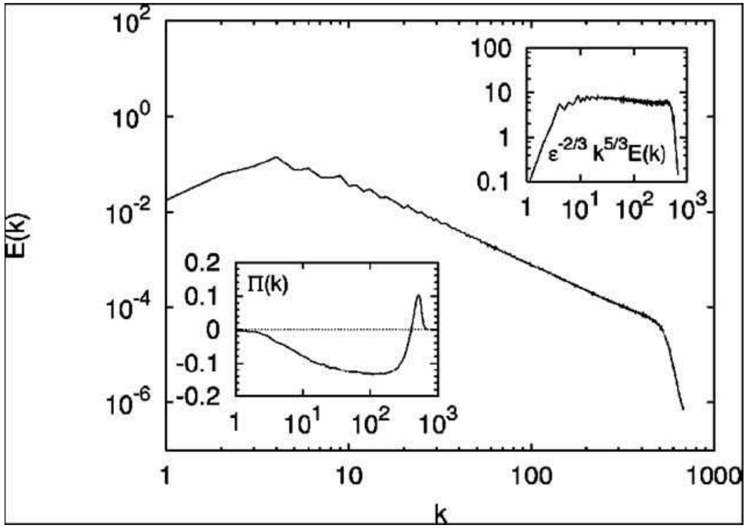
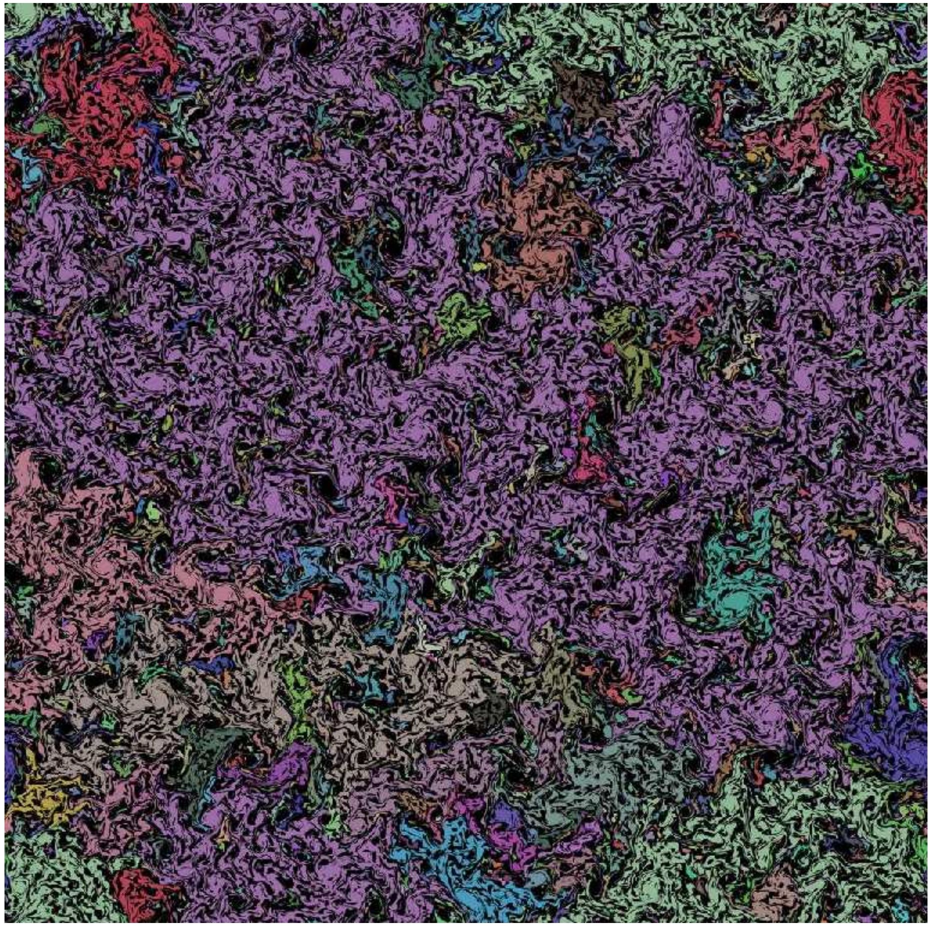


N	dx	ν	α	u_{rms}	ℓ_f	ℓ_d	ε_I	ε_ν	
2048	4.9×10^{-4}	2×10^{-5}	0.015	0.26	0.01	2.4×10^{-3}	3.9×10^{-3}	1.8×10^{-3}	2.1
4096	2.4×10^{-4}	5×10^{-6}	0.024	0.26	0.01	1.2×10^{-3}	3.9×10^{-3}	0.7×10^{-3}	3.2
8192	1.2×10^{-4}	2×10^{-6}	0.025	0.27	0.01	7.8×10^{-4}	3.9×10^{-3}	0.3×10^{-3}	3.6
16384	0.6×10^{-4}	1×10^{-6}	0.0	0.24	0.01	5.5×10^{-4}	3.8×10^{-3}	0.2×10^{-3}	3.6



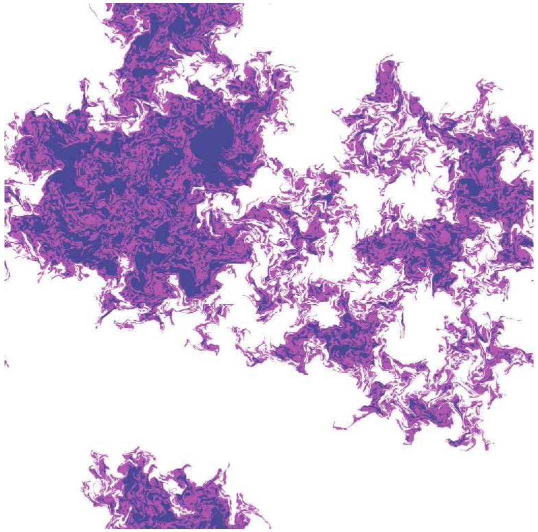
colored clusters of vorticity of given sign

2D (inverse) turbulence: vorticity clusters.

vorticity clusters: connected components of set of points with positive vorticity

cluster boundaries: macroscopic zero isovorticity lines (L_f = UV cutoff)

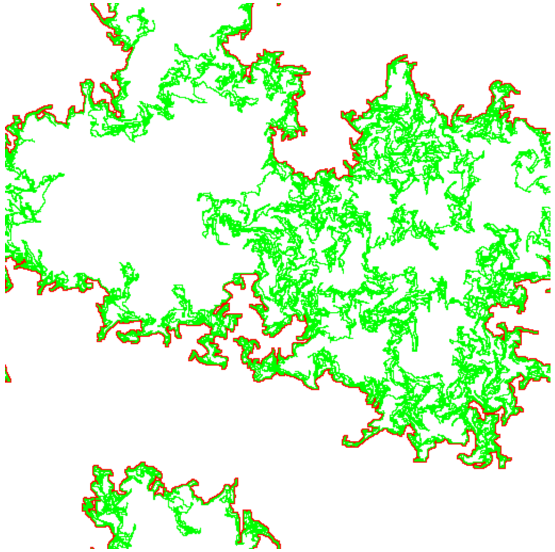
A large macroscopic filled vorticity cluster



dark violet = filled holes

Frontier of a vorticity cluster

Ext. perimeter of a vorticity cluster



exterior perimeter = closed fjords (L_f)

Fractal dimensions of vorticity clusters.

Naive KK scaling: $u_\ell^3 \sim \varepsilon\ell$

Macroscopic cluster size L

$\Gamma \equiv \int d^2x\,\omega \sim \omega_L L^2 \propto L^{4/3}$

$\Gamma = \oint u \cdot d\ell \sim N_{L_f} u_{L_f} L_f \propto P$

Perimeter P

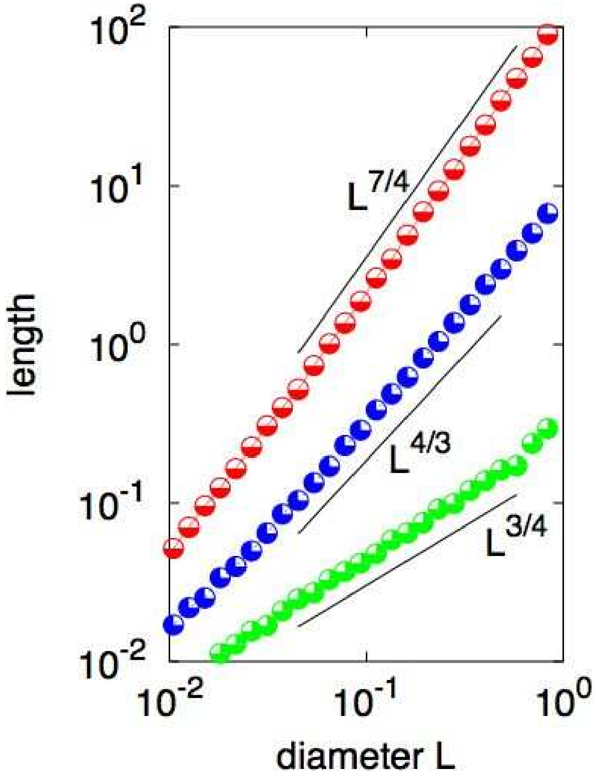
$P \propto L^{4/3}$

Dim. of frontier = 7/4, (as SLE_6);

Dim. of ext. perimeter = 4/3, (as $SLE_{8/3}$);

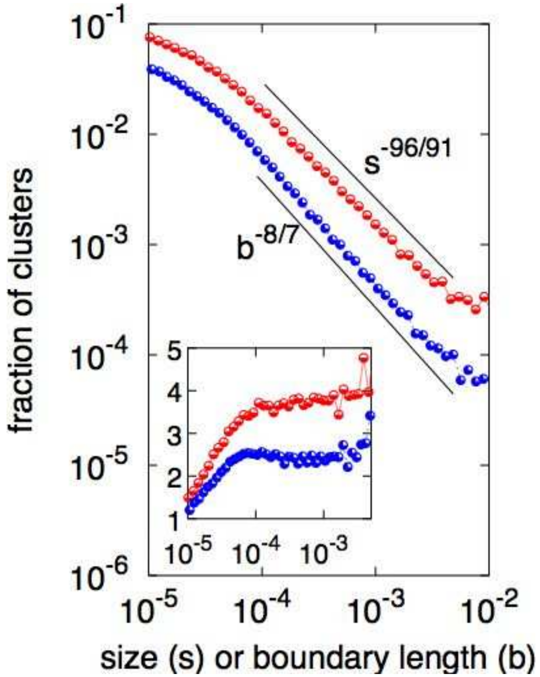
Dim. of double points = 3/4.

Box counting fractal dimension



More tests of conformal invariance.

Distribution of clusters

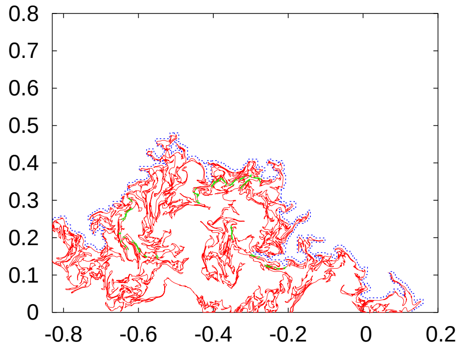


Fraction of clusters of size between s and $1.1s$

Fraction of clusters with boundary length between b and $1.1b$

CFT prediction.

Reconstructing (discrete) SLE in 2D turbulence.



(i) Extract contour samples from turbulent flows;

(ii) Code them into conformal maps;

(iii) Reconstruct the (discrete) Loewner driving source;

(iv) Analyse their statistics.

The curves are discretized (set of points w_j) but coded into a discrete Loewner equation via composition of maps (iteration of discrete slits):

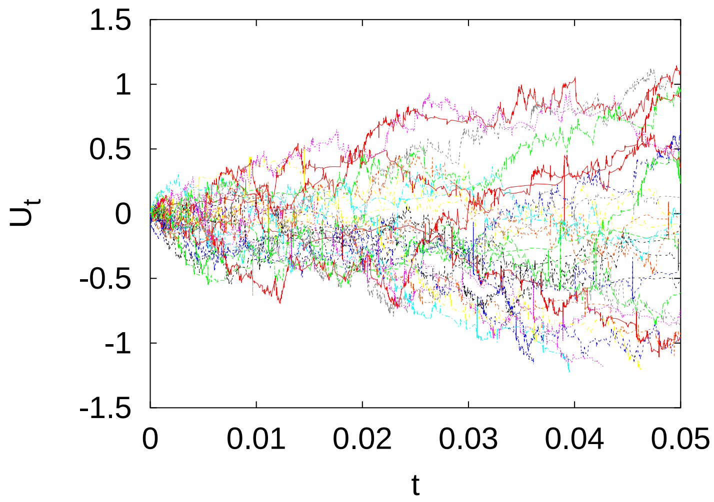
$$G^{(n)} = g_n \circ G^{(n-1)} \text{ where } g_n = \sqrt{(z-a_n)^2 + b_n} + a_n$$

with $b_n^2 = (Im\, w_n)^2 = 4\Delta t_n$ and $a_n = Re\, w_n = \xi(t_n)$.

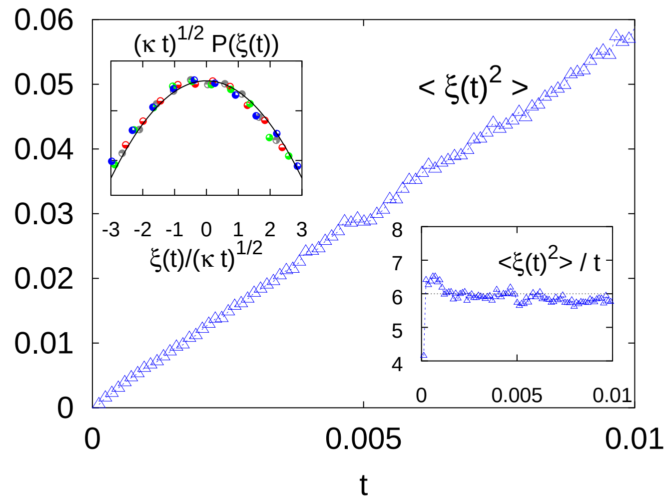
Then iterate... (the points move at each iteration)

Reconstructing (discrete) SLE in 2D turbulence $\longrightarrow$ SLE(6).

Reconstructed driving sources.



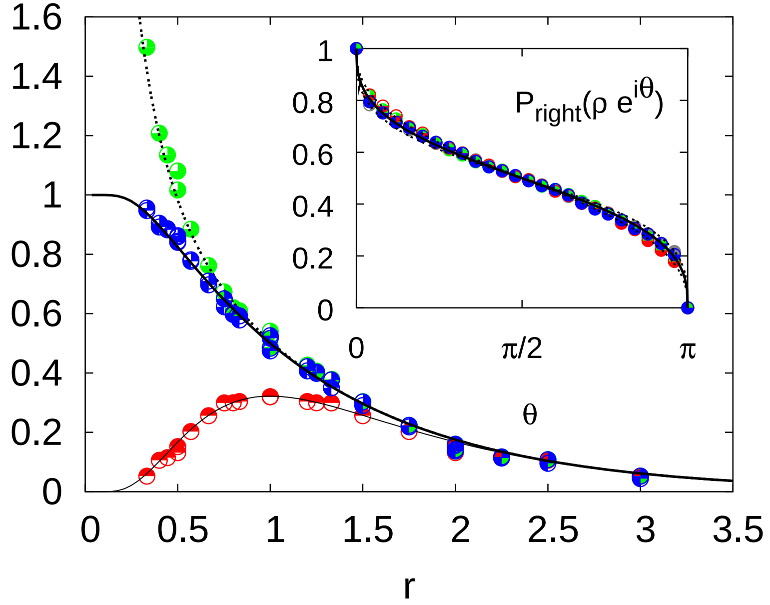
Distribution of the driving sources



Statistics of $\xi(t)$ is close to that of a 1D Brownian motion $\longrightarrow$ SLE(6)

but need for more tests...

Test of SLE in 2D turbulence.



– Cardy's formula (blue): probability for a cluster to cross a rectangle

– Watts-Dubedat formula (red): probability of 'four-legged' cluster joining all sides of a rectangle

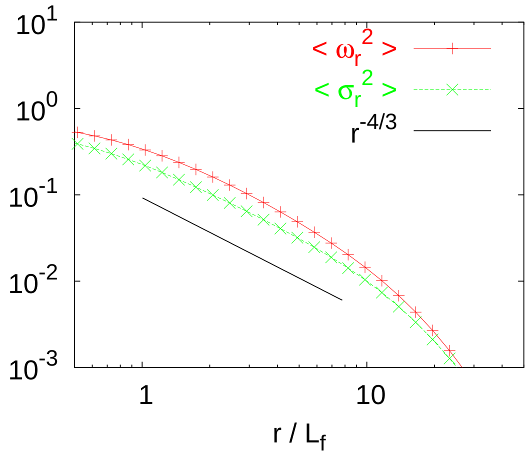
– Schramm's formula (insert): probability that isovorticity line leaves the point $\rho e^{i\theta}$ to its right

$\longrightarrow$ "some" conformal invariance in 2d turbulence....

Test of SLE in 2D turbulence.

Harris criteria......

Long range correlation of sign(ω)



Non Gaussianity

Randomized phase....

$\hat{\omega} = \int d^2k\, \omega_k e^{i\phi_k}$

Identical 2-point function

(but not pdf)

$< \xi(t)^2 >$ not linear in t.

$\rightarrow$ "some" conformal invariance in 2d turbulence....

Surface Quasi-Geostrophic (SQG) turbulence

Transport by the fluid (without ...) (T as ω):

$$\partial_t T + u \cdot \nabla T - \nu \nabla^2 T + T/\tau = F$$

Non linearity: velocity $u^i = \varepsilon^{ij} \partial_j \Psi$ with $\boldsymbol{\Psi_k = |k|^{-\alpha} T_k}$... (NS is $\alpha = 2$)

Two quadratic conserved quantities (without...):

$$E = 1/2 \int d^2x \, T\Psi \quad ; \quad Z = 1/2 \int d^2x \, T^2$$

Two cascades: (small scale Z) and (large scale E)

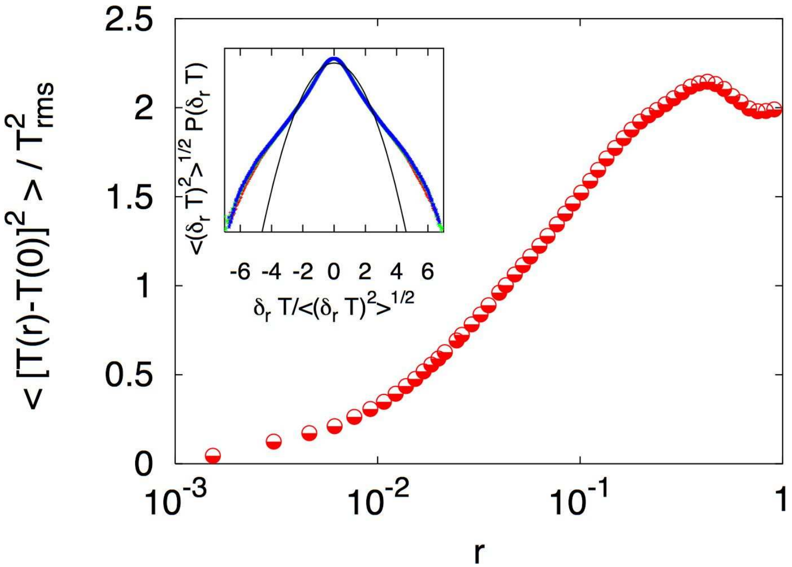
Scaling in the inverse cascade: $T_\ell \simeq \ell^H$ with $H = 2(1-\alpha)/3$

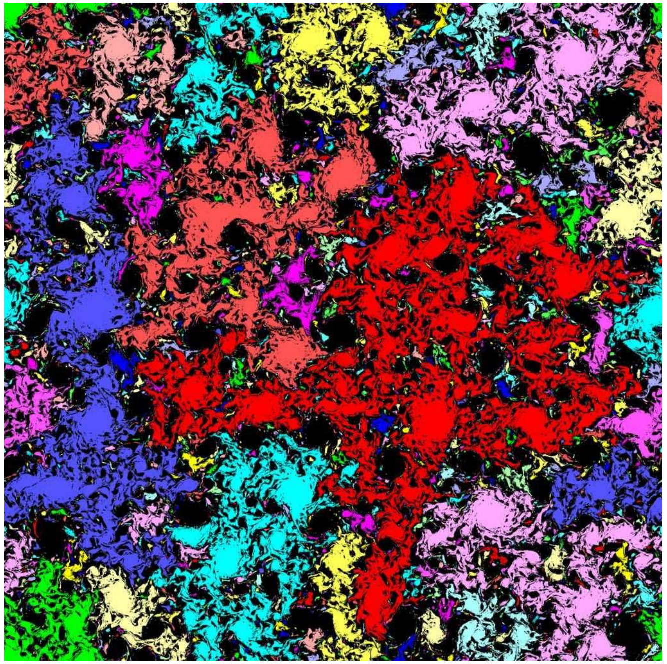
In the following $\alpha = 1$.

SQG inverse cascade

Numerical simulations...

Log scaling: $< T_0 T_r > \propto \log(r/L_f)$



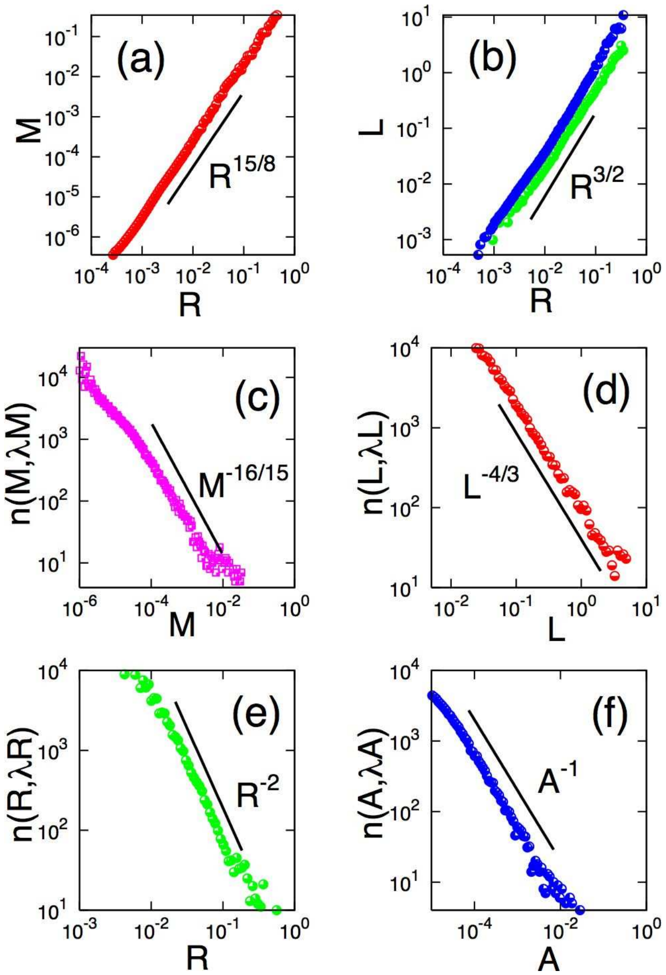


Non anomalous scaling.

$$\mathrm{Pdf}(T_r, r) \simeq T_r^{-1} f(T_r/\log(r/L_f))$$

$\rightarrow$ look at the T clusters and their zero iso-lines ($T_{rms} \gg T$)...................

Test of conformal invariance of T-clusters in SQG



(a) Mass versus radius

(b) Length versus radius

Fractal dim = 3/2

(c) Number of clusters versus mass

(d) Number of loops versus length

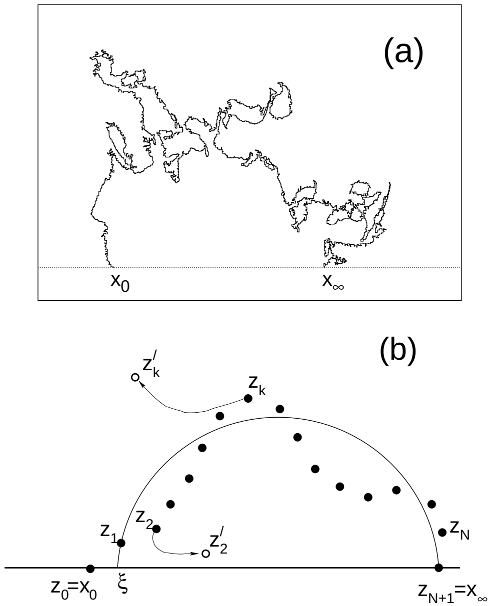
(e) Number of loops versus radius

(f) Number of loops versus area

Stat. idem as in $O(2)$ model

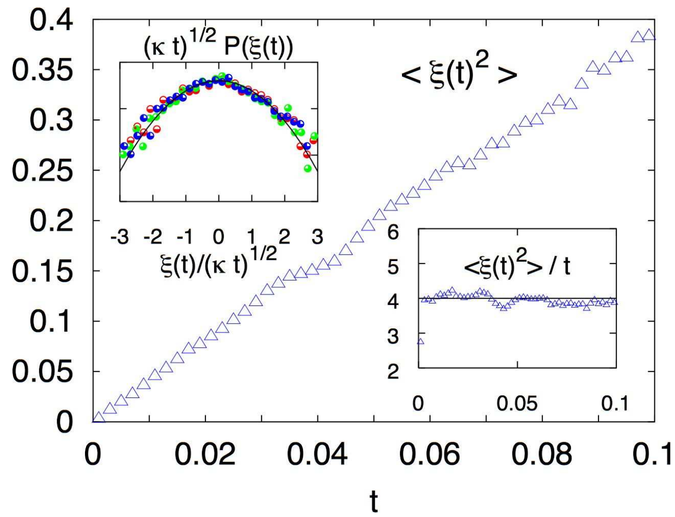
c=1 CFT.

SLE test of for T-clusters in SQG $\longrightarrow$ SLE(4)



Extract isoline traces

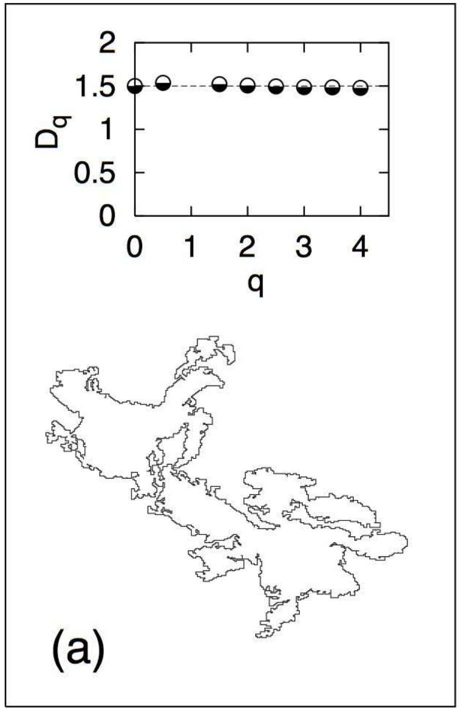
Check statistics.... $\kappa \simeq 4$

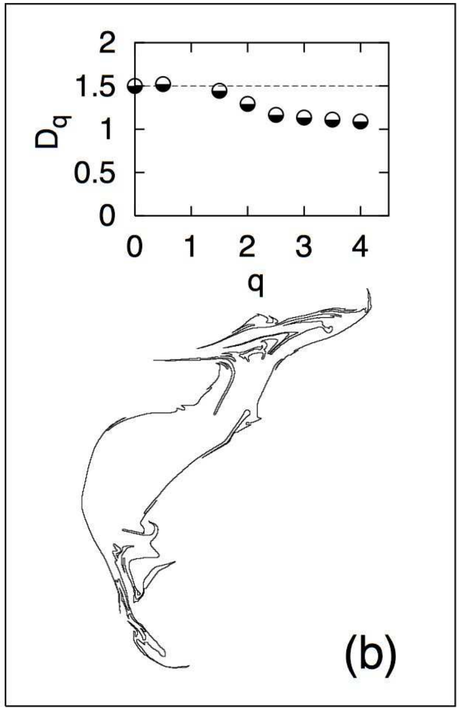


Statistics of T-clusters in the same universality class as $O(2)$ models

$\longrightarrow$ why conformal invariance? analytic confirmation? why inverse cascade? for which class of outoff equilibrium systems??........

Comparing with direct 2D turbulence.





Iso-loops in direct cascade are not SLEs

different structure, differents spectrum of fractal dimensions