Interfaces in random media
shocks and field theory

- model: elastic manifolds in random media

some examples

- statics
  - field theory : Functional RG
  - measuring R(u) : shocks
  - 2D periodic: CO model, fermions
- depinning transition
  - avalanches

• **PLD** **cond-mat/0605490** *Finite temperature Functional RG, droplets and decaying Burgers Turbulence*

• **PLD** **Phys. Rev. Lett. 96, 235702 (2006)** *Chaos and residual correlations in pinned disordered systems*

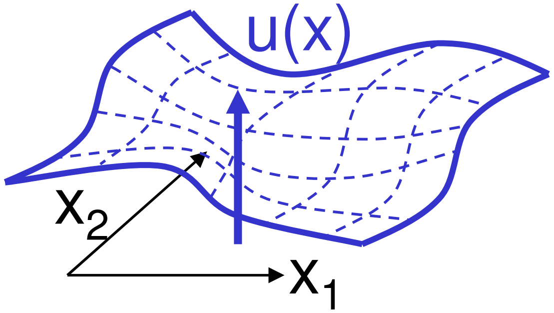
• **Alan Middleton, Kay Wiese, PLD** **cond-mat/0606160, PRL 07**
Measuring functional renormalization group fixed-point functions for pinned manifolds

• **Gregory Schehr, PLD** **cond-mat/0607657**

Previous/other FRG work: **Kay Wiese**
Leon Balents, T. Giamarchi,..
Andrei Fedorenko

Elastic manifolds in a random potential

- interface

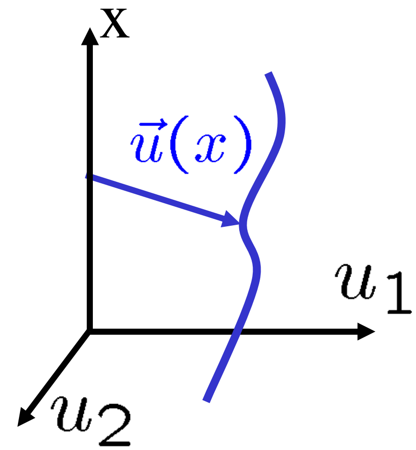


$\in \mathbb{R}^N$ target space

$\vec{u}(\vec{x})$

$\in \mathbb{R}^d$ internal space

- directed line



$$H = \int d^d x \quad \frac{c}{2} (\nabla u)^2$$

Elastic manifolds in a random potential

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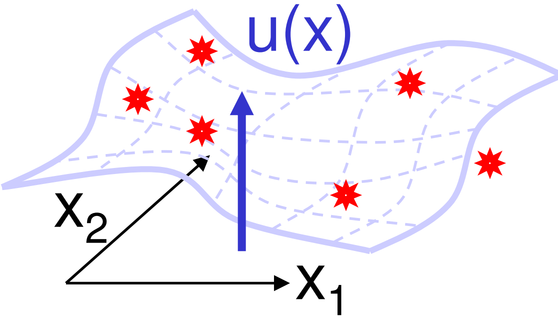


random potential

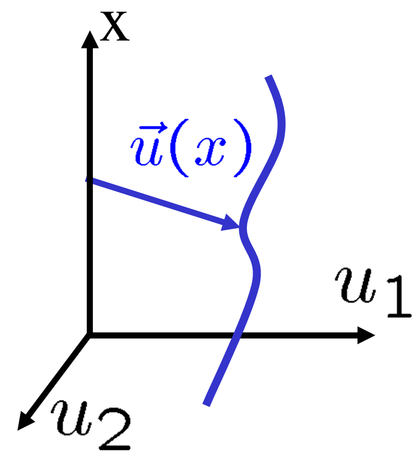
$V(x,u)$

$\vec{u}(\vec{x})$

$\in \mathbb{R}^N$ target space

$\in \mathbb{R}^d$ internal space

- directed line



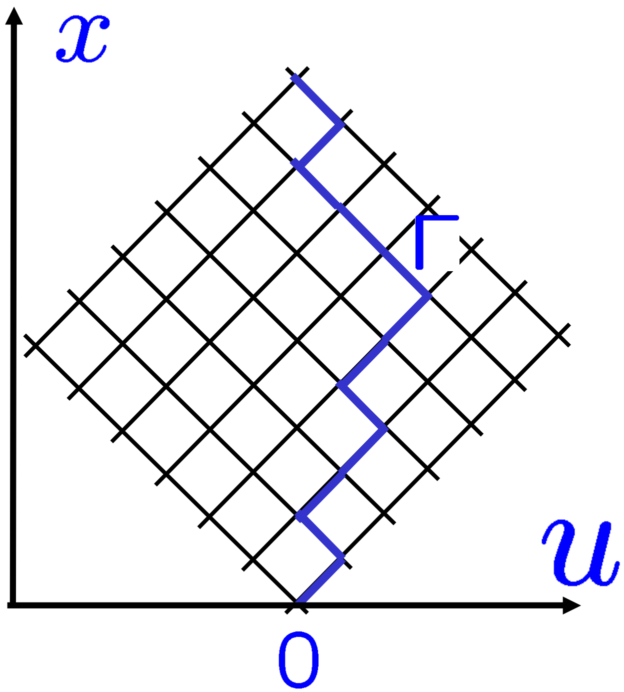
$$H = \int d^d x \quad \frac{c}{2}(\nabla u)^2 + V(x, u(x))$$

$$\overline{(u(x) - u(0))^2} \sim |x|^{2\zeta}$$

critical object

$$a \ll x \ll L$$

directed polymer $d = 1$



V_i random variables on each bond

$E_\Gamma = \sum_{i \in \Gamma} V_i$ energy of path Γ

find optimal path Γ_{min}

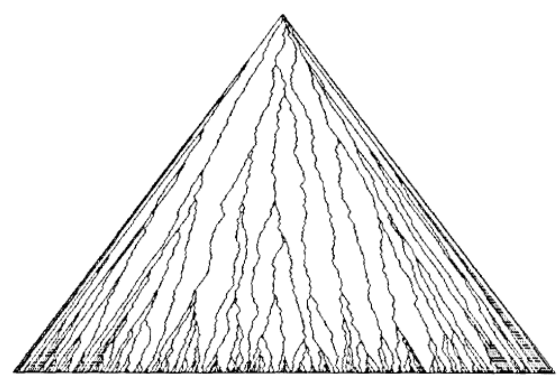
of minimal energy E_{min}

$$\overline{(E_{min} - \overline{E_{min}})^2} \sim x^{2\theta}$$

$$\overline{\langle (u(x) - u(0))^2 \rangle} \sim |x|^{2\zeta}$$

Exact results for N=1 $\zeta = 2/3$

$\theta = 1/3$



- Directed interfaces, no overhangs or bubble (large scale)

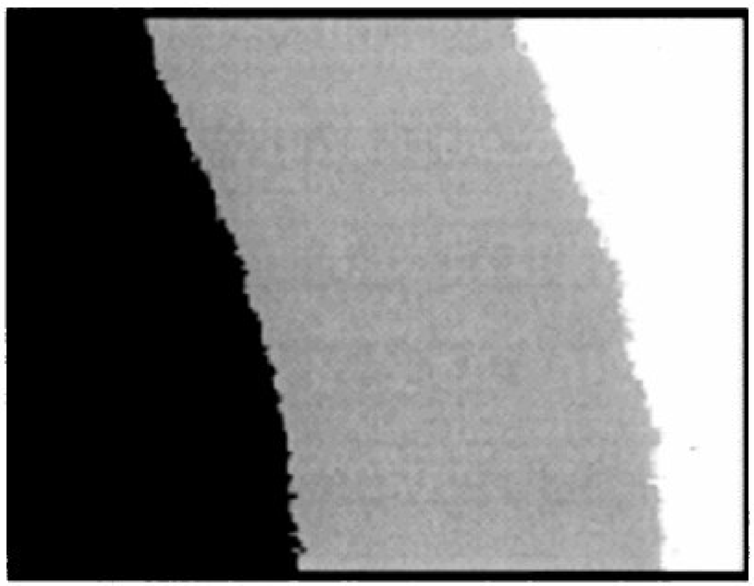
  One interface in the Ising model in ferromagnetic phase

  Random bond disorder $D=2,3$ $d=1,2$ $N=1$ D=d+N

  Random field disorder $D=3$ $d=2$, $N=1$

- NOT: 2D SAW with quenched random bonds

Magnetic interface

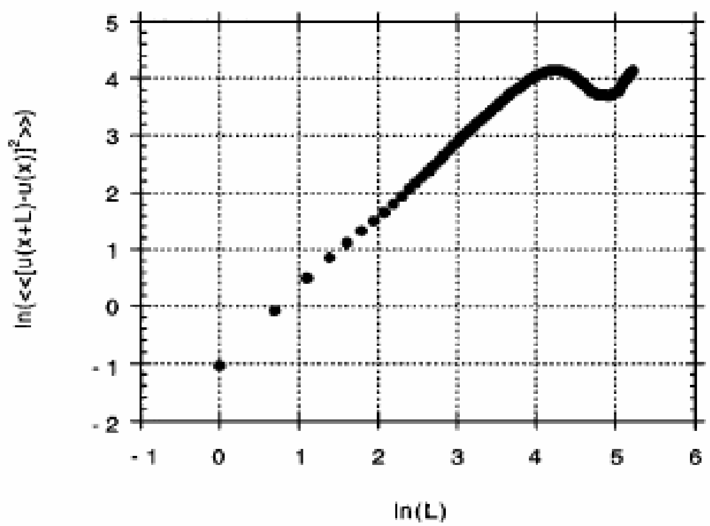


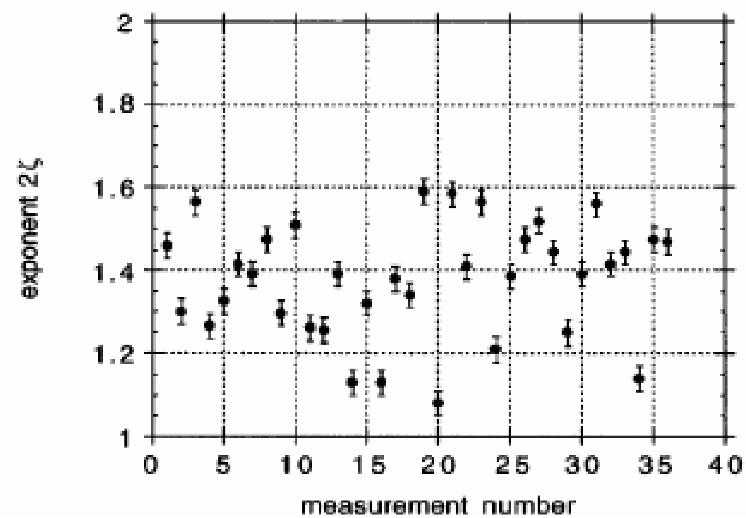
Lemerle Ferre et al. 98

Ising magnetic film Co

$$\overline{(u(x)-u(0))^2} \sim |x|^{2\zeta}$$

domain wall creep, D=1+1, RB





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  One interface in the Ising model in ferromagnetic phase

  Random bond disorder $D=2,3$ $d=1,2$ $N=1$

  Random field disorder $D=3$ $d=2$, $N=1$

- NOT: 2D SAW with quenched random bonds

- Crystal (lattice) with no topological defects (dislocations)

  Bragg glass phase ? $d=3$

  XY model with random field

Periodic object + weak disorder

Abrikosov vortex lattice

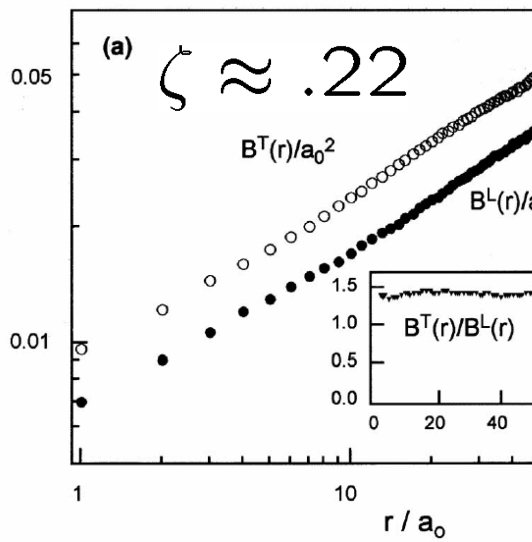
Bragg Glass: No dislocation
No translational order

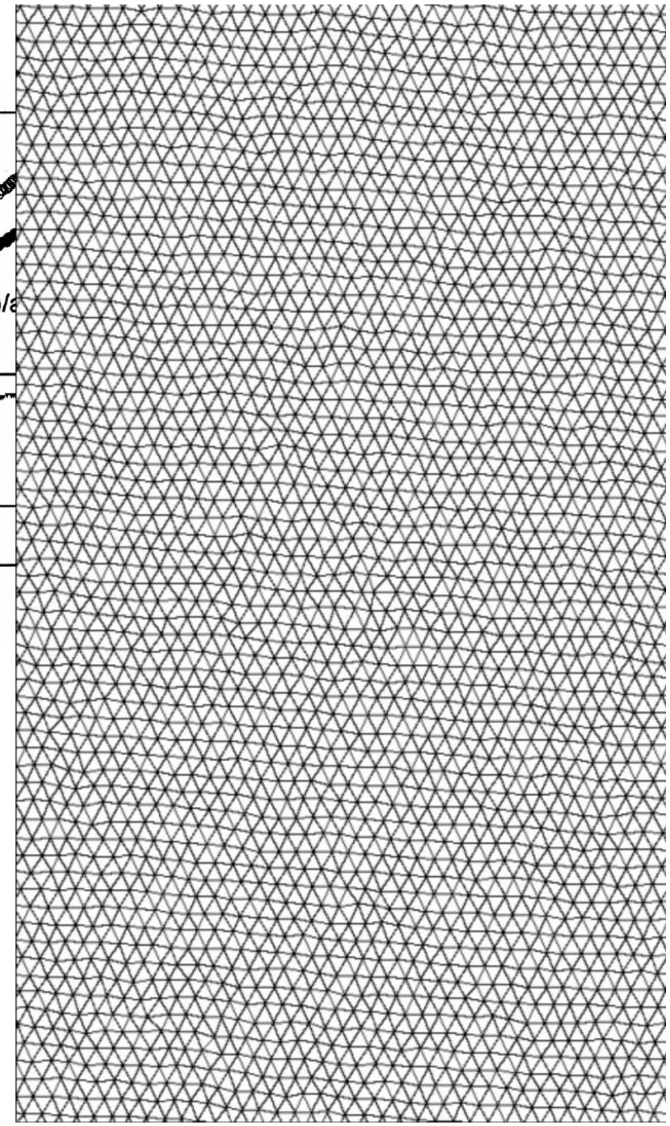
decoration

$$B(r) = \overline{(u(r) - u(0))^2}$$

$$B(r) \sim |r|^{2\zeta} \ll a_0^2$$

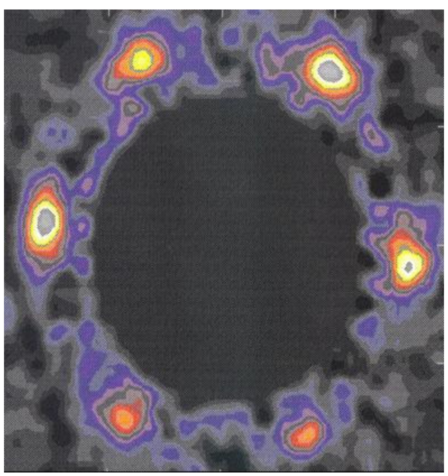
$$d = 3, N = 2$$





$$B(r) \sim A_d \ln |r| \geq a_0^2$$

Neutron diffraction



divergent Bragg peaks

$$\rho_K(x) = \rho_0 e^{iKu(x)}$$

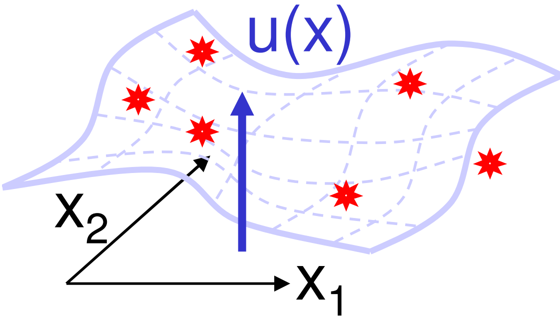
$$\overline{\rho_K(x)\rho_K^*(0)} \sim |x|^{-\eta}$$

T. Klein, TG,PLD
Nature 2001

Elastic Manifold in Random Potential

$u(x) \in \mathbb{R}^N$

$x \in \mathbb{R}^d$



$$H = \int d^d x \quad \frac{c}{2}(\nabla u)^2 + V(x, u(x))$$

$$\overline{V(x,u)V(x',u')} = \delta^d(x-x')\ R(u-u')$$

the function

$R(u)$ is

- short range — random bond — *magnetic DW*
- long range — random field — *interfaces*
- periodic — *vortex lattice (Bragg glass)*

$$P[u] \sim e^{-H[u]/T} \qquad \overline{\langle (u(x)-u(0))^2 \rangle} \sim |x|^{2\zeta}$$

$$T_l \sim L^{-\theta}$$

few univ class

$$\theta = d - 2 + 2\zeta$$ Pinned T=0 $$\zeta(N, d, ..)$$

How to treat the problem

- replicas
- dynamics

$$Z_V = \int Du e^{-\beta H_V[u]}$$

$$\beta = 1/T$$

$$H_V[u] = \int d^d x \frac{c}{2} (\nabla u)^2 + V(x, u(x))$$

$$\overline{V(x,u)V(x',u')} = \delta^d(x-x') R(u-u')$$

$$\overline{Z_V^n} = Tr e^{-\beta H_{rep}}$$

$$\overline{\langle u(x) u(0) \rangle} = \langle u_a(x) u_a(0) \rangle_{n=0}$$

$$\overline{\langle u(x) \rangle \langle u(0) \rangle} = \langle u_a(x) u_b(0) \rangle_{n=0}$$

$$H_{rep} = \int d^d x \, \frac{1}{2} \sum_{a=1}^{n} (\nabla u_a)^2 - \frac{1}{2T} \sum_{ab} R(u_a(x) - u_b(x))$$

starting model: interacting field theory

How to average over disorder

- statics

$$H[u] = \int d^d x [\frac{c}{2}(\nabla u)^2 + V(x, u(x))]$$

Replica Field Theory

$$R(u_a - u_b)$$

- dynamics

$$\eta \partial_t u(x,t) = c\nabla^2 u(x,t) + F(x, u(x,t)) + \xi(x,t) + f$$

friction | elastic force | random pinning force | thermal noise | external force

$$F(x,u) = \partial_u V(x,u)$$

$$\overline{F(x,u)F(x',u')} = \delta^d(x-x')\Delta(u-u')$$

$$\Delta(u) = -R''(u)$$

Dynamical Field Theory (MSR,DJ)

$$\Delta(u(t) - u(t'))$$

Peculiar features of field theory for glass

- coupling constant is a function of the field

$$\begin{matrix} R(u) \\ \Delta(u) \end{matrix} \leftrightarrow g\,\phi^4$$

infinity of relevant operators d<4: Functional RG

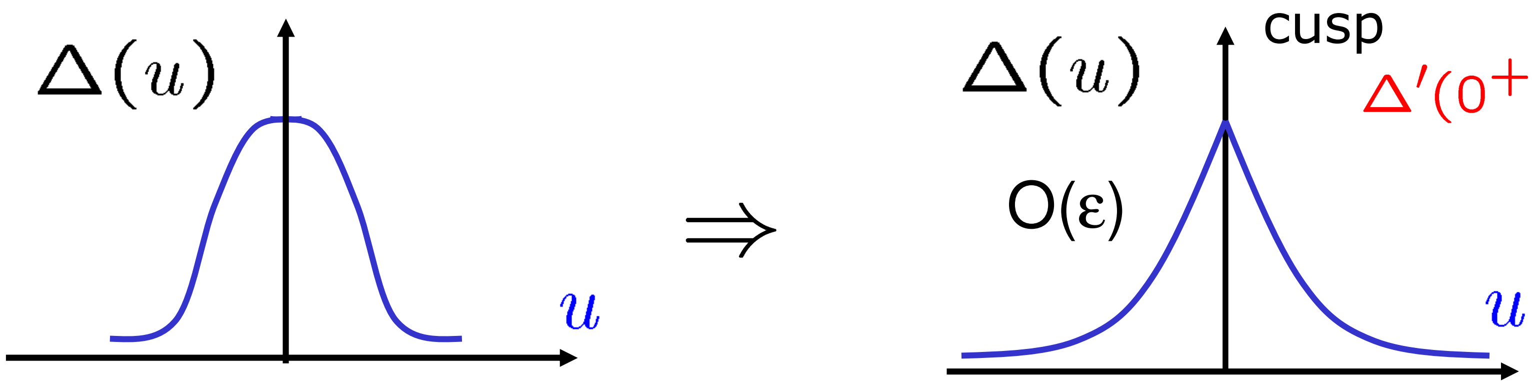
- since $T_l = L^{-\theta} \to 0$ there should be

  a T=0 fixed point theory, indeed $\Delta_l(u) \to \Delta^*(u) = O(\epsilon)$

  $\epsilon = 4 - d$

  BUT $\Delta^*(u)$ is non-analytic at u=0 $L > L_c$

why? mode minimization instead of integration : shocks



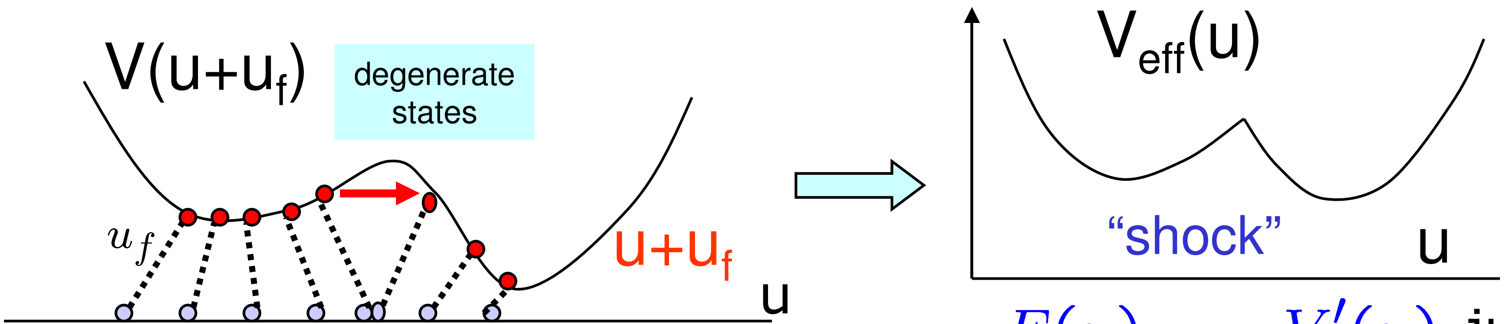
needs rescaling: yields ζ

Physics of the Cusp: metastability

- "Toy RG" $$H = \frac{q^2}{2}u^2 + \frac{\Lambda^2}{2}|u_f|^2 + V(u+u_f)$$

$$V_{eff}[u] = \mathrm{Min}_{u_f}\, H[u,u_f]$$

Balents, Bouchaud, Mezard



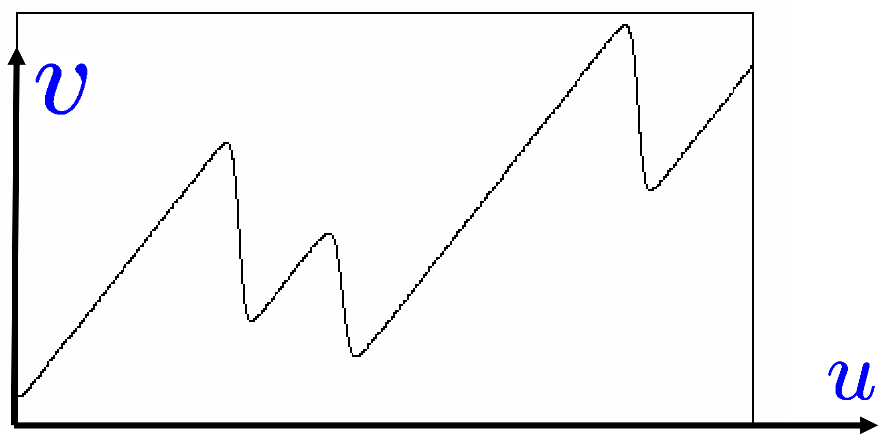
$F(u) = -V'(u)$ jumps

$F(u) - F(u')$ finite if shock between u and u' proba $p \sim |u-u'|$

$\Delta(u) - \Delta(0) \sim \overline{(F(u) - F(u'))^2} \sim p \sim |u-u'|$ (uniform density)

- Burgers d=1

$v(u,t)$



Functional RG

T=0

$$\frac{H_{rep}}{T} = S[u] = \int_x \frac{1}{2T} \sum_a (\nabla u_a)^2 + m^2 u_a^2 - \frac{1}{2T^2} \sum_{ab} R_0(u_a(x) - u_b(x))$$

$$\Gamma[u] = \frac{1}{2T} \int_x \sum_a (\nabla u_a)^2 + m^2 u_a^2 - \frac{1}{2T^2} \sum_{ab} \Gamma_2[u_a, u_b] + ..$$

$$R(u_a - u_b) := \Gamma_2[u_a(x) = u_a, u_b(x) = u_b]$$

$$\tilde{R}_l(u) = m^{-\epsilon+4\zeta} R(u m^{-\zeta})$$

$$-m\partial_m \tilde{R}(u) = (\epsilon - 4\zeta)\tilde{R}(u) + \zeta u \tilde{R}'(u)$$

$$= \partial_l \tilde{R}(u)$$

$$+\frac{1}{2}\tilde{R}''(u)^2 - \tilde{R}''(u)\tilde{R}''(0) + O(R^3)$$

$$l = \ln(1/m)$$

Analysis of one loop FRG equation T=0

D.Fisher 86

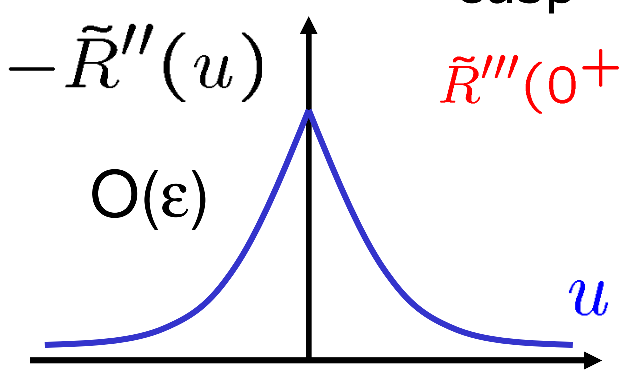
$$\partial_l \tilde{R}(u) = (\epsilon - 4\zeta)\tilde{R} + \zeta u \tilde{R}' + \frac{1}{2}\tilde{R}''(u)^2 - \tilde{R}''(u)\tilde{R}''(0)$$

start with R(u) analytic

$\partial_l \tilde{R}''''(0) = \epsilon \tilde{R}''''(0) + \tilde{R}''''(0)^2$ $\quad \tilde{R}''''(0) \to \infty \quad m = m_c^+$

R(u) becomes non-analytic at u=0 beyond Larkin scale

$\Rightarrow$



cusp

$\tilde{R}'''(0^+) \neq 0$

Unique solution each
Univ class (B.C)

$$\zeta_{RB} = 0.208\epsilon$$

$$\zeta_{RF} = \epsilon/3$$

$$\epsilon = 4 - d$$

cusp $\longrightarrow$ *difficulties at two loop*

renormalizable theory for statics at T=0

PLD,Wiese,Chauve 2000

procedure (2 loop): • one loop counterterms are non-ambiguous

• ask for renormalizability cancellation of $1/\epsilon$ poles

$\Longrightarrow$ lift ambiguities

$$\partial_l \Delta(u) = (-\frac{\Delta^2}{2} + \Delta\Delta(0))'' + \frac{1}{2}(\Delta'^2(\Delta - \Delta(0))''$$

$$+\frac{\lambda}{2}\Delta'(O^+)^2\Delta''(u)$$

$$\lambda_{stat} = -1$$

- preserves linear cusp $\Delta(u) = -R''(u)$ $\zeta_{RF} = \epsilon/3$

$$\zeta_{RB} = 0.20829804\epsilon + 0.006858\epsilon^2$$

$d=1$	one loop	two loop	exact
$\epsilon = 3$	0.625	0.687	0.666

- needs testing !

- what is R(u) ?

$$\frac{R''(u)}{R''(0)} = Y(u/\xi) \qquad \int dz Y(z) = 1$$

random field disorder

$$d = 4 - \epsilon$$

$$Y = Y(z) \leftrightarrow z = \frac{\sqrt{Y - 1 - \ln Y - \frac{\epsilon}{3} F(y)}}{\int_0^1 dy \sqrt{y - 1 - \ln y - \frac{\epsilon}{3} F(y)}}$$

$$F(y) = 2y - 1 + \frac{y \ln y}{1-y} - \frac{1}{2} \ln y + \mathrm{Li}_2(1-y)$$

- are there exactly solvable examples ?

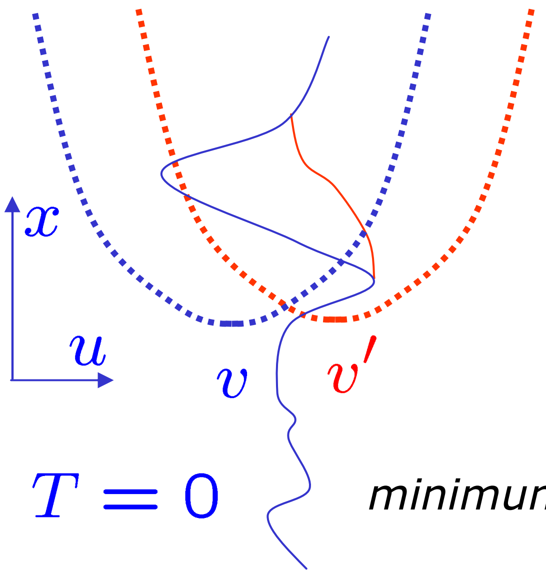
How to measure R(u)

PLD

cond-mat/0605490

$$\exp(-\frac{1}{T}\hat{V}(v)) =$$

$$\int Du \; e^{\frac{1}{T}\int d^dx \frac{1}{2}m^2(u(x)-v)^2+\frac{1}{2}(\nabla u)^2+V(x,u(x))}$$



$$\overline{\hat{V}(v)\hat{V}(v')} = L^d \hat{R}(v-v')$$

one shows that: $\hat{R}(v) = R(v)$ $\longleftrightarrow$ defined from $\Gamma[u]$

differences in higher cumulants

$T=0$ *minimum energy configuration* $u_0(x;v)$

$$u(v) = L^{-d}\int d^dx \; u_0(x;v)$$

$v - u(v)$ *exhibits shocks*

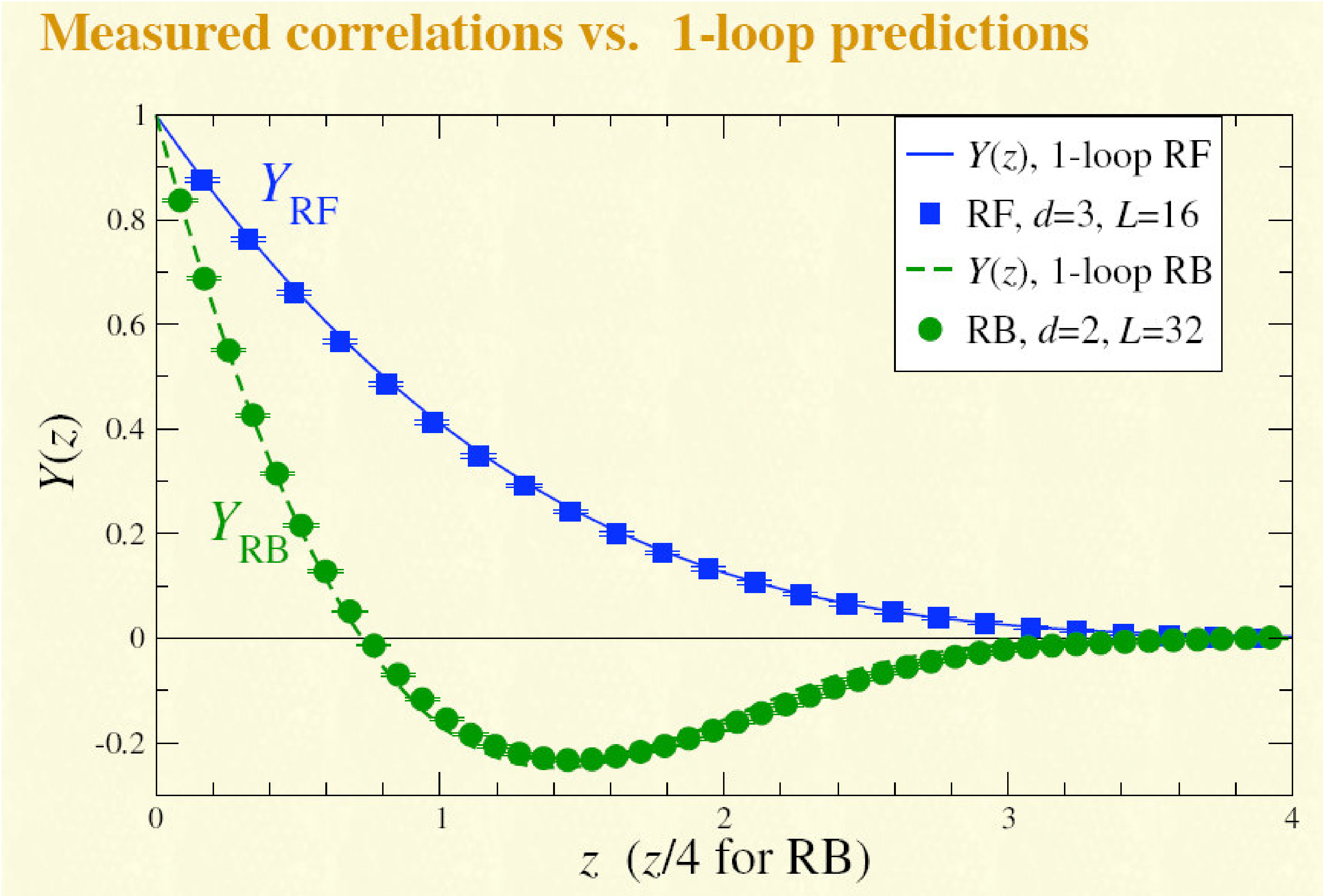
$$\Delta(u) = -R''(u)$$

$$\overline{(v-u(v))(v'-u(v'))} = L^{-d}m^{-4}\Delta(v-v')$$

Alan Middleton

PLD,KW

condmat/0606160

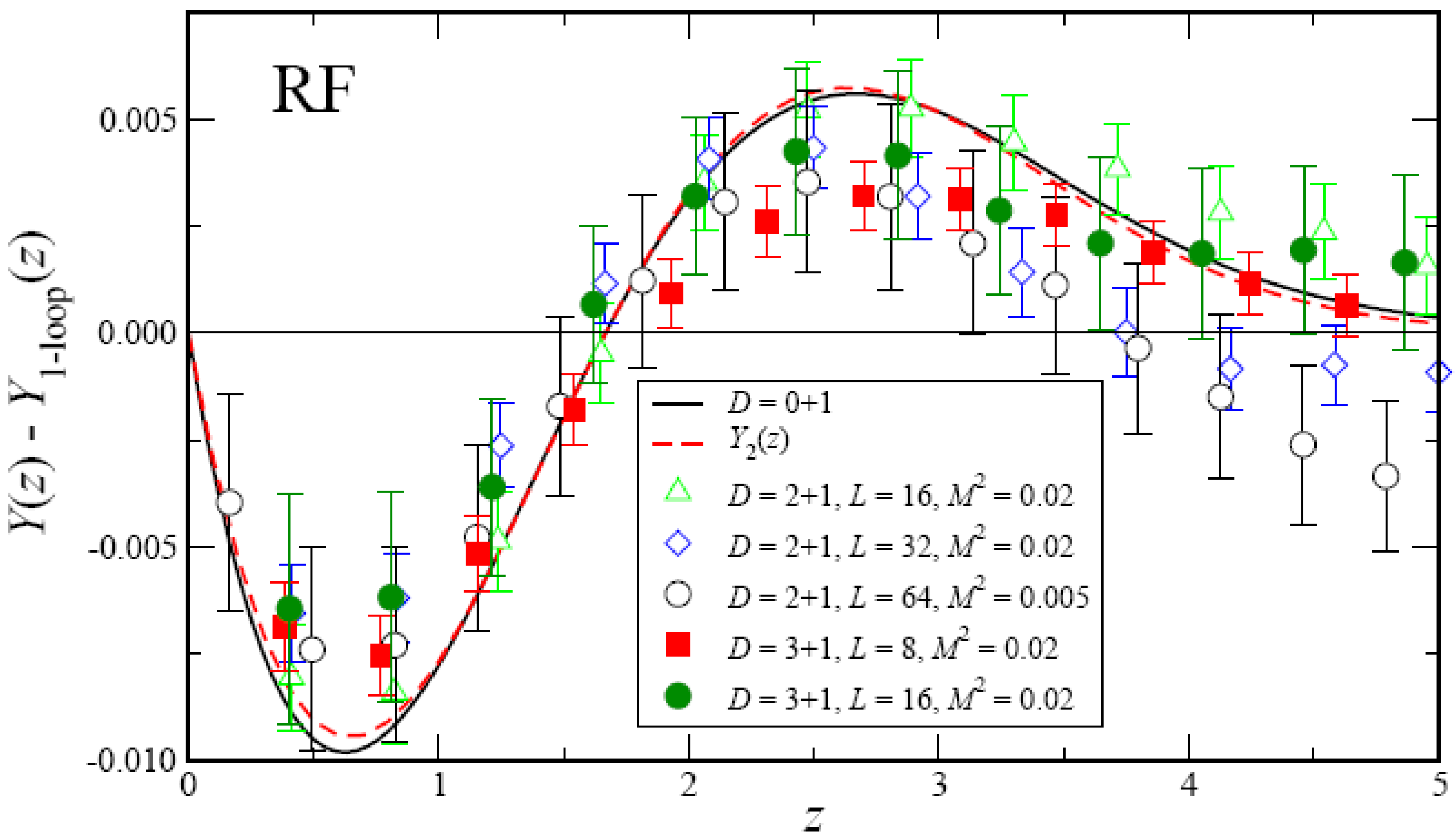


$$Y = \Delta/\Delta(0)$$

$$\overline{(v - u(v))(v' - u(v'))} = L^{-d} m^{-4} \Delta(v - v')$$

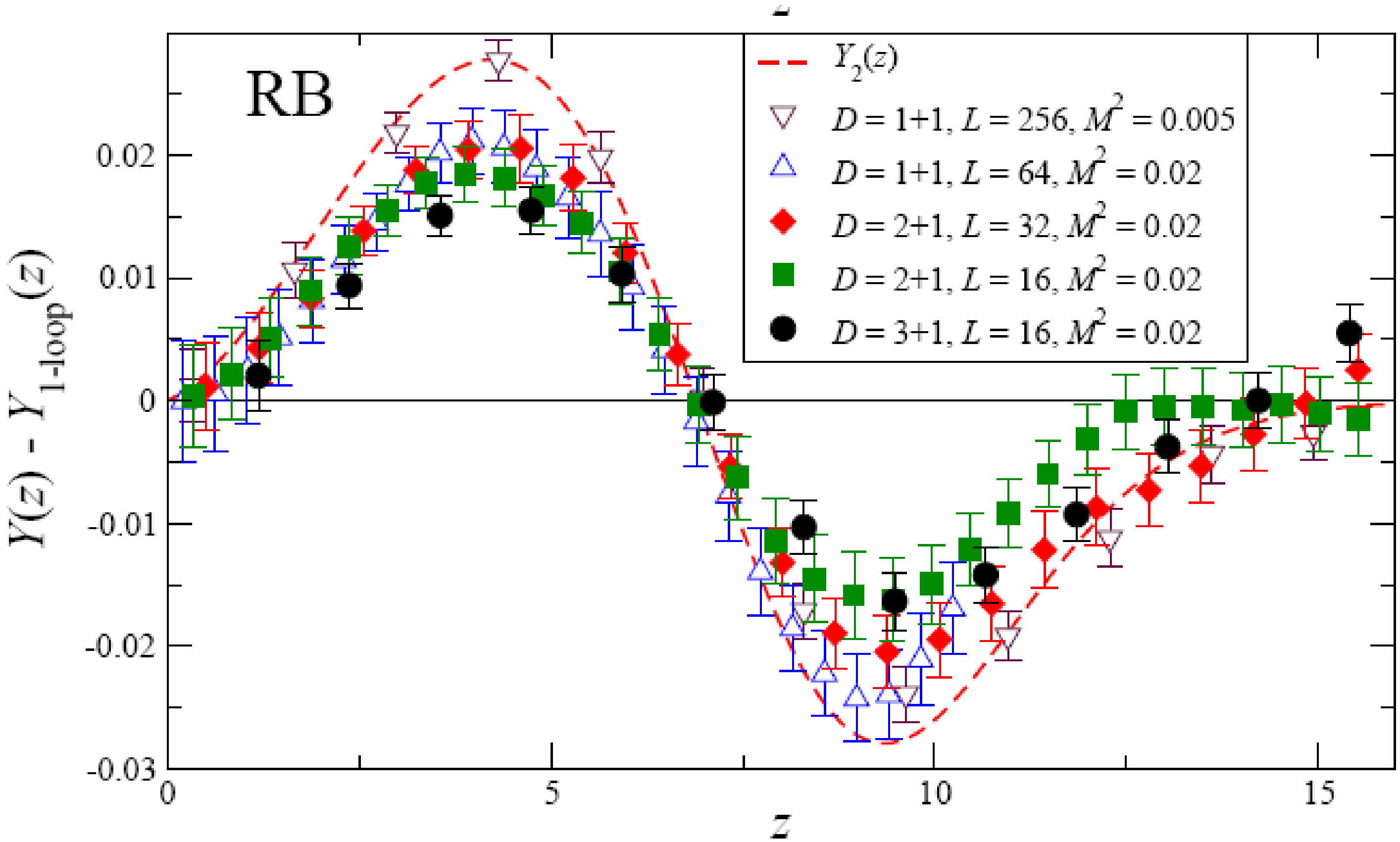
$$\Delta(u) = -R''(u)$$

Deviations from one loop: random field disorder



$$Y(z) = Y_1(z) + \epsilon Y_2(z) + ..$$

Deviations from one loop: random bond disorder



$$Y(z) = Y_1(z) + \epsilon Y_2(z) + ..$$

Decaying Burgers equation and shocks

- *d=0* $\exp(-\frac{1}{T}\hat{V}_m(v)) = \int du\ e^{\frac{1}{T}[\frac{1}{2}m^2(u-v)^2+V(u)]}$

particle in a random potential $\overline{V(u)V(0)} = R_0(u)$

Force $F(v) = \hat{V}'(v)$ *obeys Burgers equation* $F \leftrightarrow \mathsf{u}$

$v \leftrightarrow x$ $\mathsf{u}(x,t)$

any N $m^{-2} \leftrightarrow t$ $\partial_t \mathsf{u} + \mathsf{u}'_x \mathsf{u} = \nu \mathsf{u}''_{xx}$ *velocity field*

$T \leftrightarrow 2\nu$

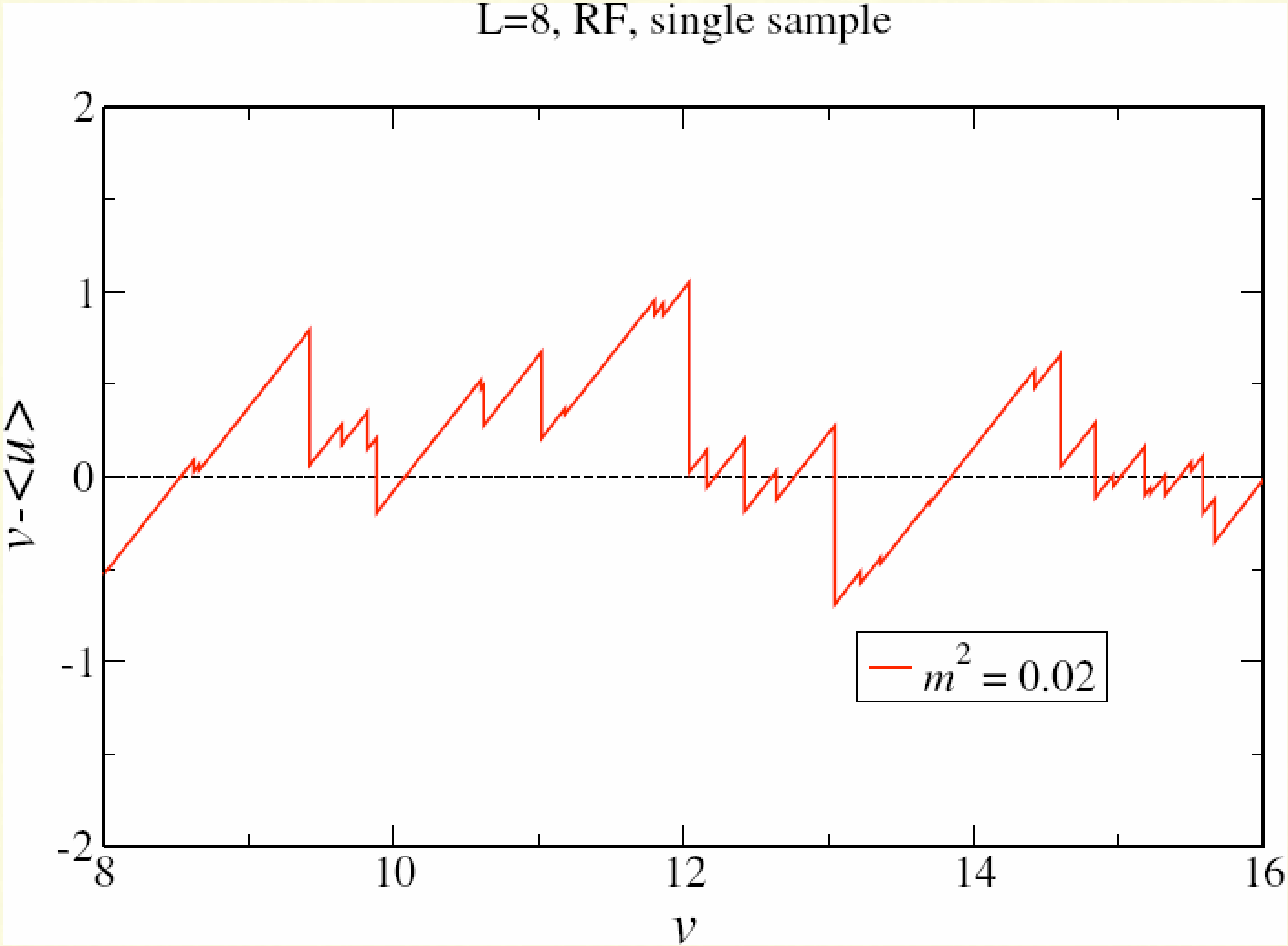
shocks form then merge (N=1: ballistic aggregation)

- *d>0*

Functional Decaying Burgers $\mathcal{F}_x = \hat{V}'_x[v] \equiv \delta\hat{V}[v]/\delta v_x$

$$-2m\partial_m \mathcal{F}_x[v] = \int_{yz} \partial g_{yz}(T\mathcal{F}''_{xyz}[v] - \mathcal{F}'_{xy}[v]\mathcal{F}_z[v])$$

Sequence of m^2 in a single sample



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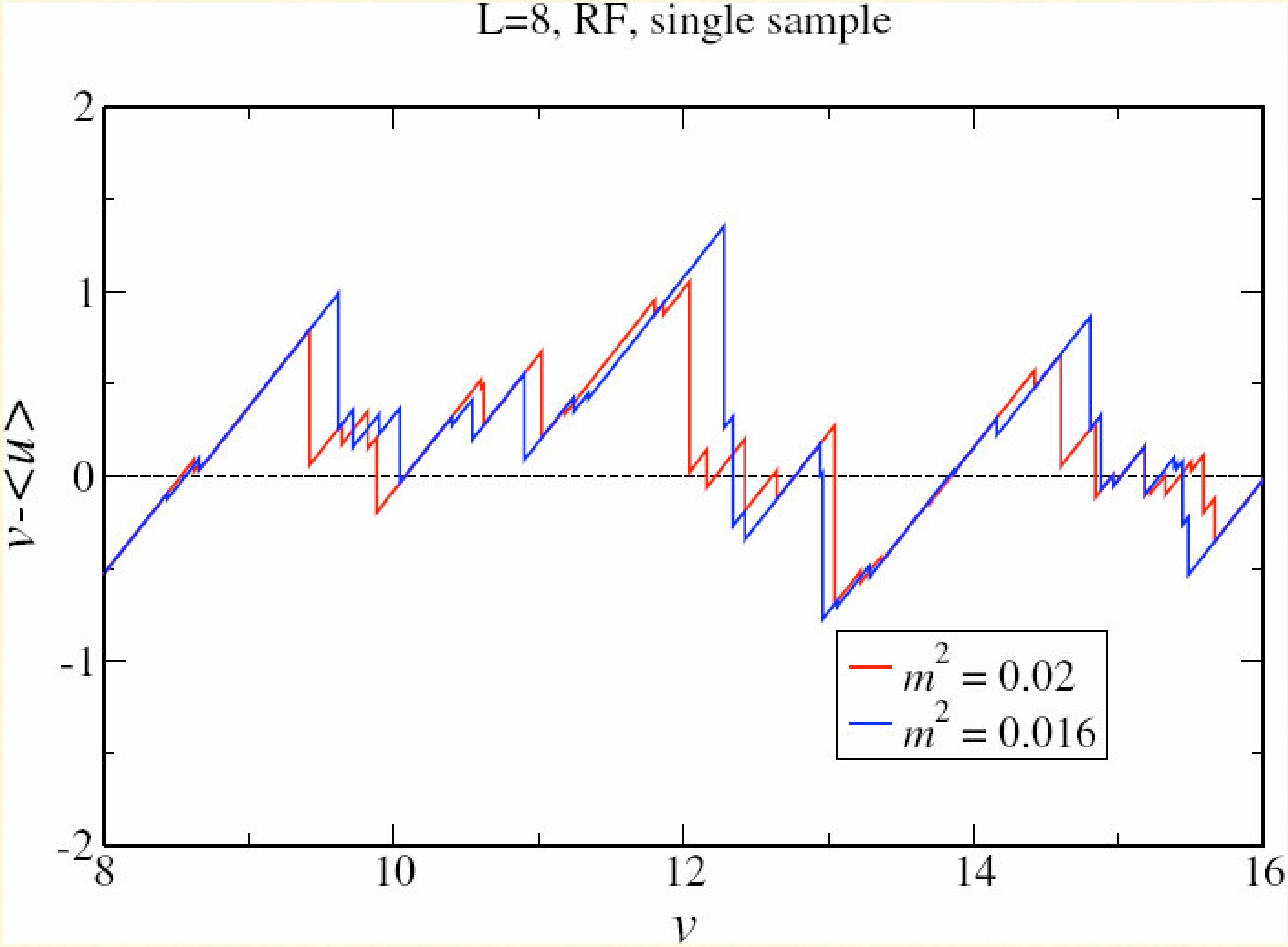

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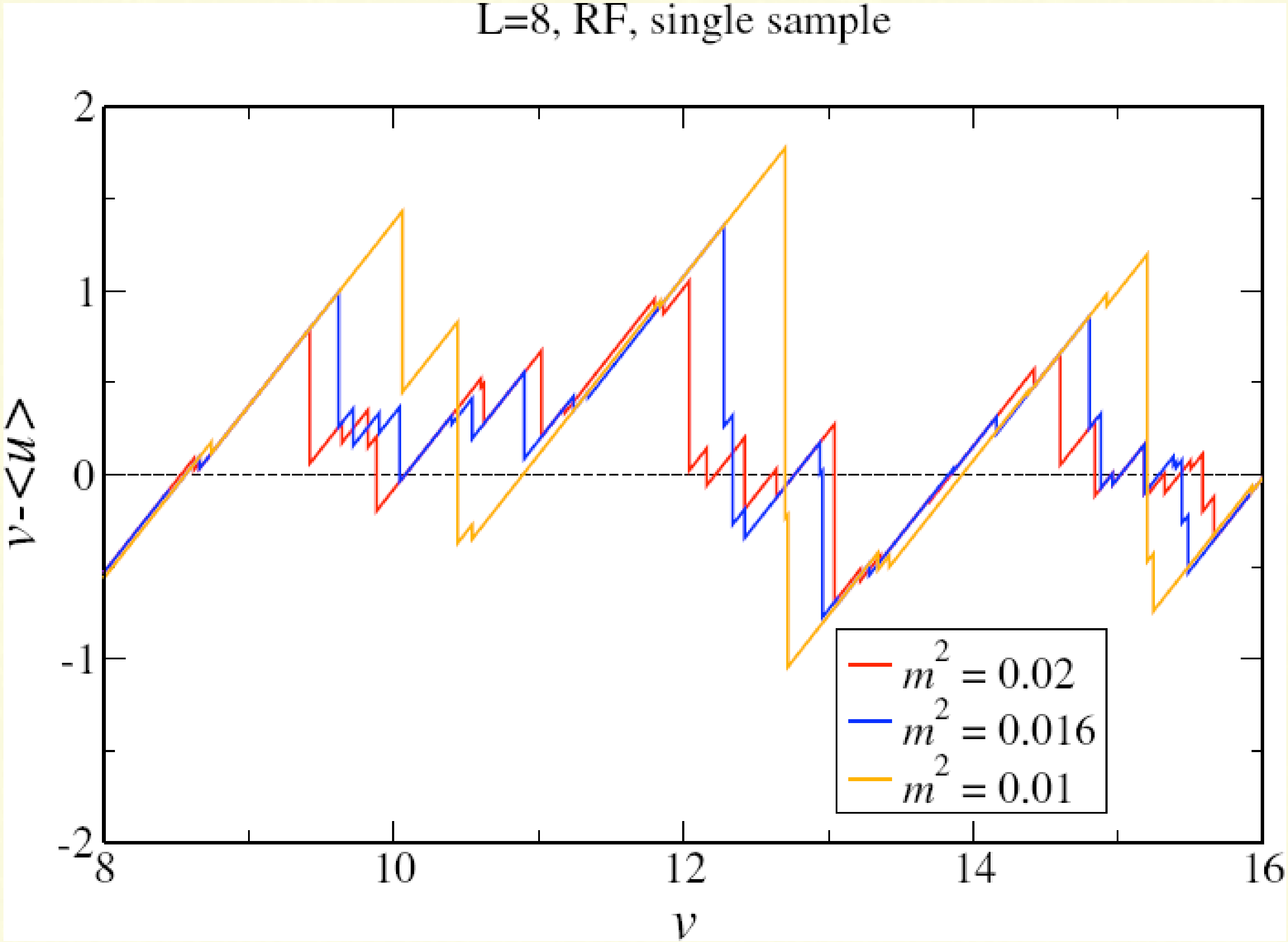
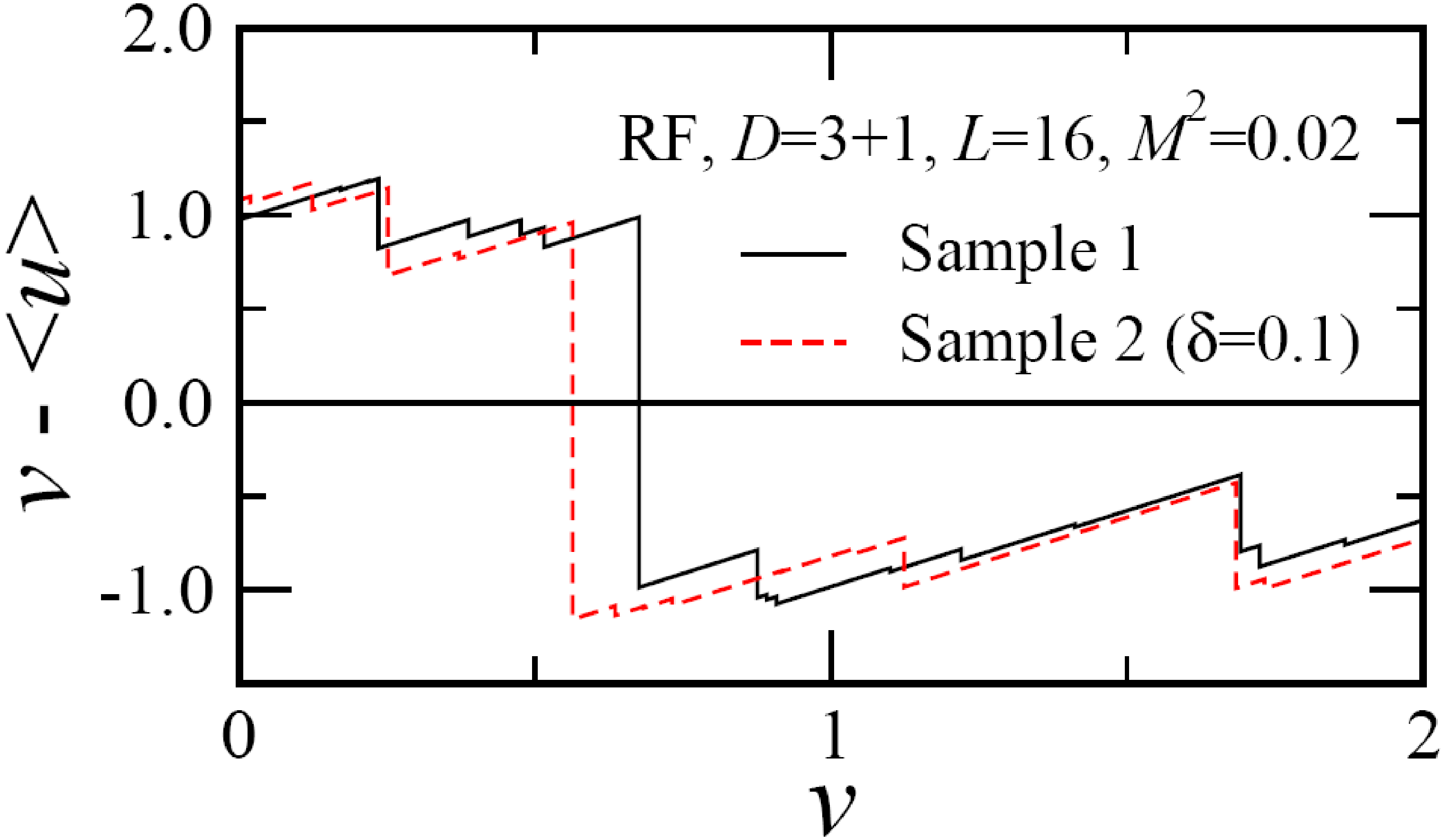
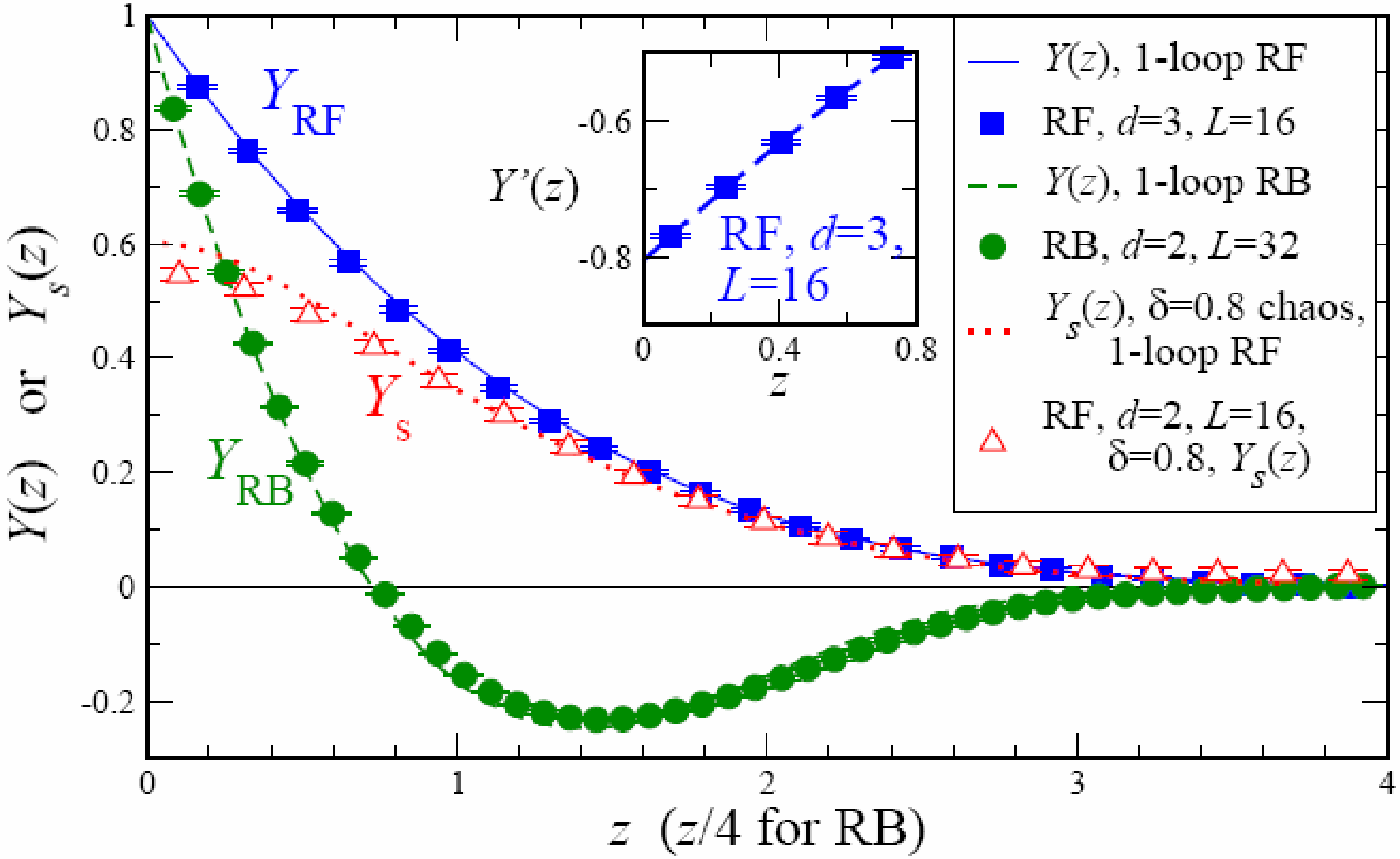


Chaos



chaos



Y_{RF}
Y_{s}
Y_{RB}
$Y'(z)$
RF, d=3, L=16
z
$Y(z)$, 1-loop RF
RF, d=3, L=16
$Y(z)$, 1-loop RB
RB, d=2, L=32
$Y_s(z)$, δ=0.8 chaos, 1-loop RF
RF, d=2, L=16, δ=0.8, $Y_s(z)$
$Y(z)$ or $Y_s(z)$
z (z/4 for RB)

Solution for Sinai model: *particle in Brownian energy lansdcape*

random field disorder d=0

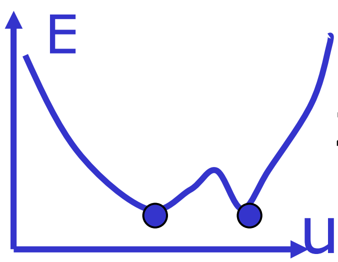
$$\overline{(V(u)-V(0))^2} \sim 2|u|$$

$$R_0(u) \sim -|u|$$

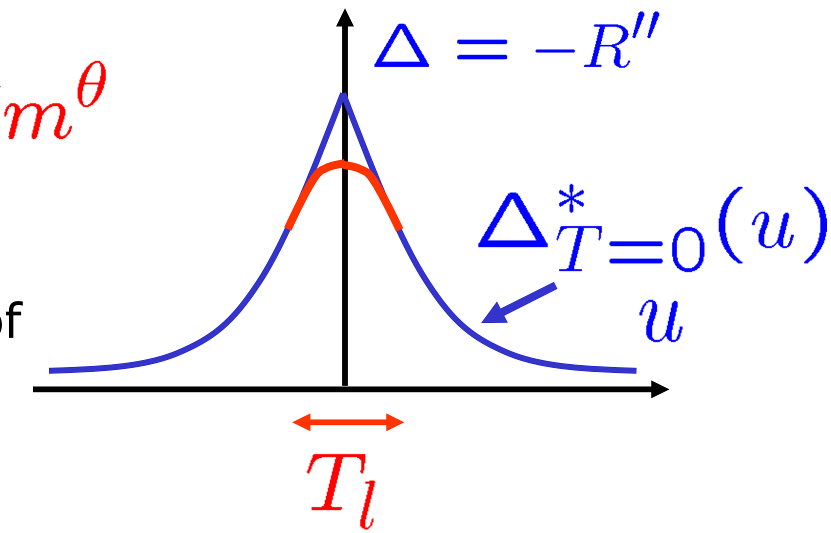
$$R^*_{T=0}(0) - R^*_{T=0}(v) = 2\sqrt{\pi v} e^{-\frac{1}{48}v^3} \int_{-\infty}^{+\infty} \frac{dz_1}{2\pi i} \int_{-\infty}^{+\infty} \frac{dz_2}{2\pi i} [v + 2(z_2 - z_1)^2]$$

$$\times e^{\frac{v}{2b}(z_1+z_2) + \frac{(z_2-z_1)^2}{v}} [\frac{1}{vAi(z_1)Ai(z_2)} + \frac{\int_0^\infty dV e^{\frac{v}{2}V} Ai(V+z_1)Ai(V+z_2)}{Ai(z_1)^2 Ai(z_2)^2}]$$

Thermal boundary layer $u \sim T_l = T m^\theta$



is probability density of
2 degen. minima distant of
u=y



$$D(y) = 2\int \frac{d\lambda}{2\pi} e^{i\lambda y} \frac{Ai'(i\lambda)}{Ai(i\lambda)} \int \frac{d\mu}{2\pi} e^{-i\mu y} \frac{1}{Ai(i\mu)^2}$$

$$\tilde{R}''(u) - \tilde{R}''(0) = T_l \int dy D(y) y^2 \frac{yu}{4T_l} (\coth \frac{yu}{2T_l} - \frac{1}{2})$$

Cardy Ostlund model (1982)

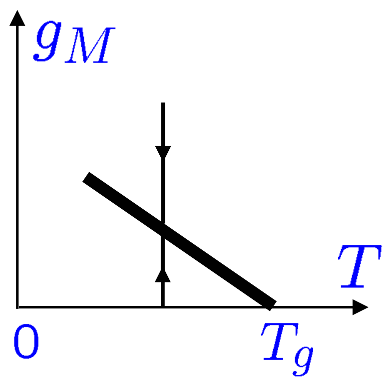
Periodic pinning d=2

Random field XY (excluded vortices) O(2)

$$\cos(u(r) - \chi(r))$$

$$\beta H_{CO} = \frac{1}{T} \int d^2r \; \frac{1}{2}(\nabla u)^2 + Re(\xi e^{iu(r)}) - h(r) \cdot \nabla u$$

$$\overline{\xi(r)\xi^*(r')} \sim g_M \delta_{rr'} \qquad \overline{h(r)h(r')} \sim g_A \delta_{rr'}$$



super-rough
glass phase

$$\overline{(u(r) - u(0))^2} \sim A(T)(\ln r)^2$$

$$A(T) = 2(1 - \frac{T}{T_g})^2 + ..$$

$$H_1 = \kappa \sum_{<i,j>} |h_i - h_j|$$

$$h_i = n_i + d_i$$

$$A(T=0) \approx 0.5$$

Zeng,Middleton,Shapir 96

Rieger Blasum 97

- *free fermions* + *disorder*

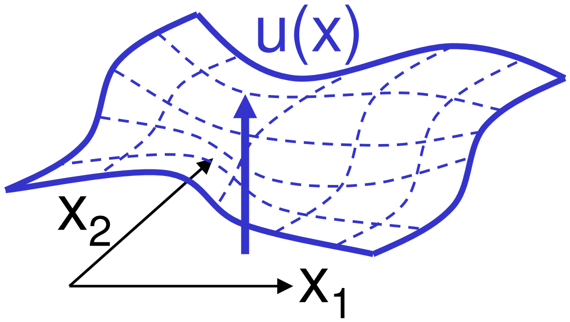
  Guruswamy,Ludwig,Leclair(2000)

- *Functional RG*

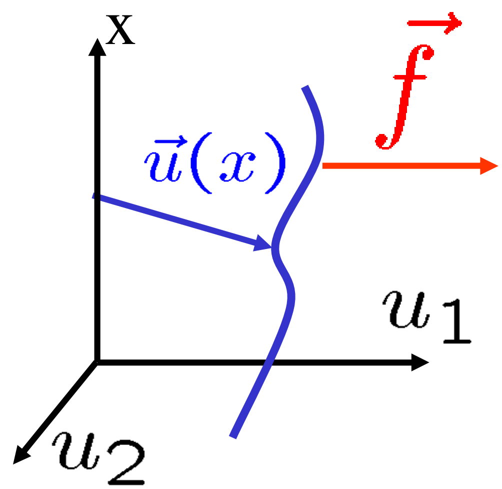
  Schehr,PLD (2006)

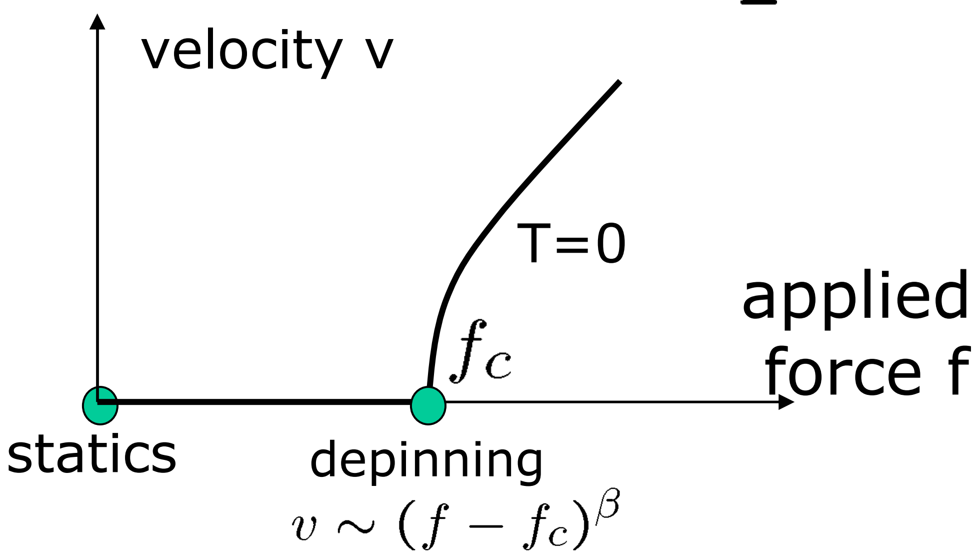
Driven elastic manifolds in random potential

interface

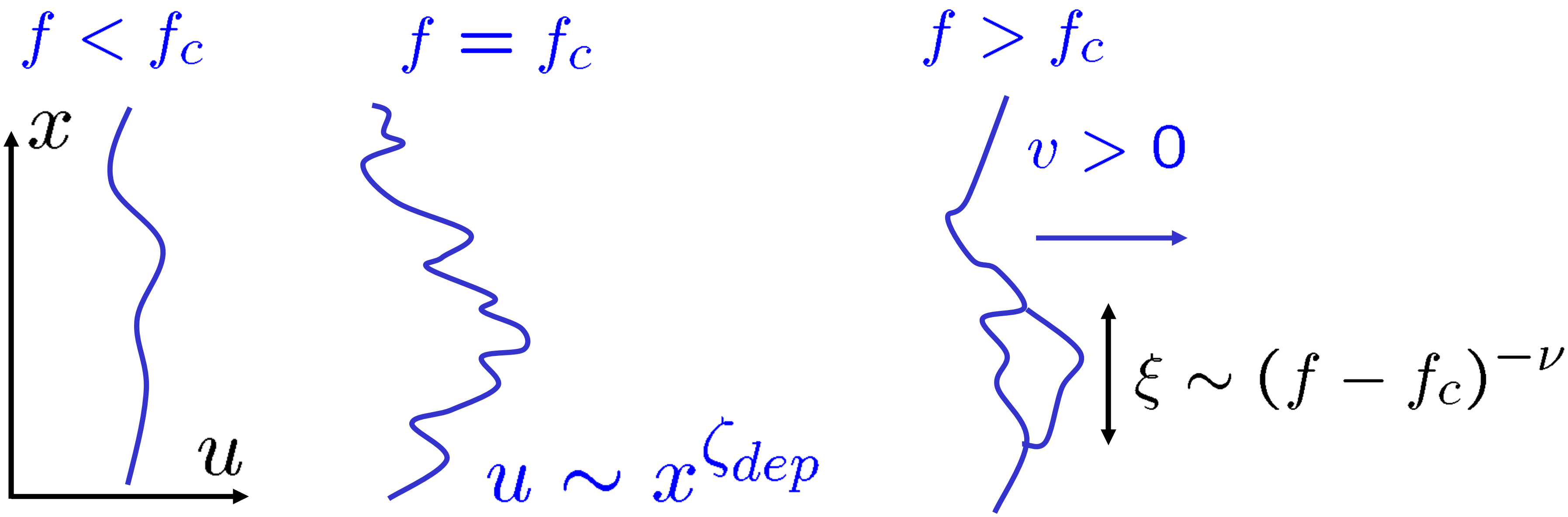


directed polymer





Scaling picture of depinning



critical blocked configuration

moving

$$v \sim u/\tau \sim \xi^{\zeta - z} \sim (f - f_c)^{\beta}$$

Two loop depinning

PLD,K.Wiese,P.Chauve 2001

$$\partial_l \Delta(u) = (\epsilon - 2\zeta)\Delta + \zeta \Delta' - (\frac{\Delta^2}{2} + \Delta\Delta(0))''$$
$$+\frac{1}{2}(\Delta'^2(\Delta - \Delta(0))'' + \frac{\lambda}{2}\Delta'(O^+)^2\Delta''(u)$$

$$\lambda_{dep} = 1$$

$$\lambda_{stat} = -1$$

- different from statics: irreversibility recovered

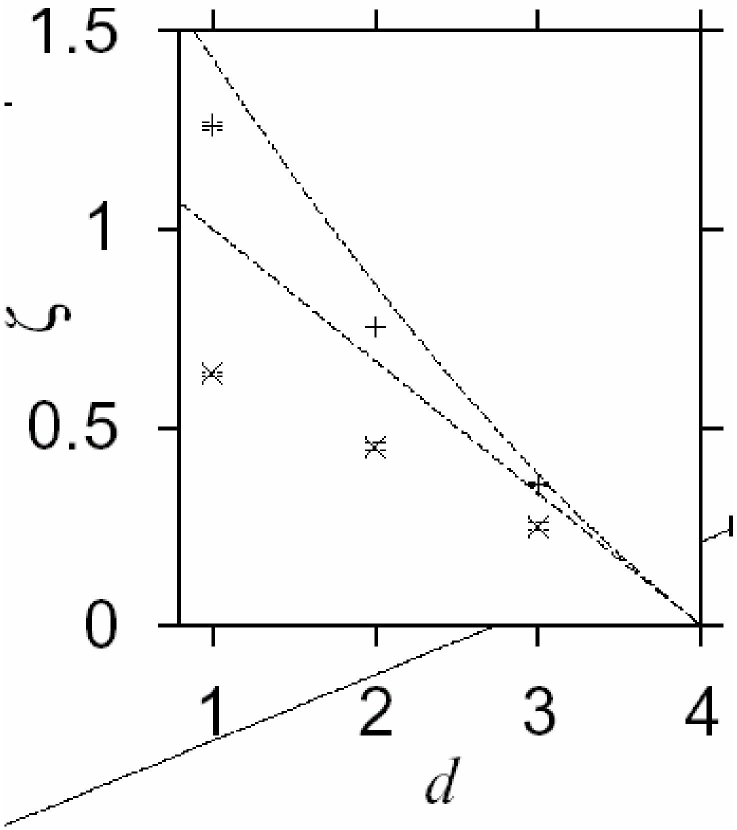
$$\zeta_{dep} = \frac{\epsilon}{3}(1 + 0.1433\epsilon + ..) > \zeta_{NF} = \frac{\epsilon}{3}$$

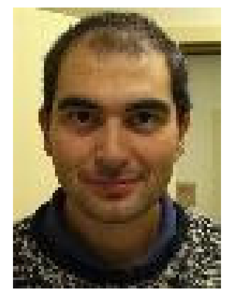
- single universality class, RB=RF

Numerics new high precision algorithm by Rosso and Krauth

Find exact critical string configuration on cylinder $L^d \times M$

	Analytic		
d	One-loop	Two-loop	ζ_{Δ^2}
1	1	1.44	1.26 ± 0.01
2	2/3	0.86	0.753 ± 0.002
3	1/3	0.38	0.355 ± 0.01





Numerical calculation of FRG fixed point at depinning

Alberto Rosso

PLD,KW

condmat/0610821

+ quadratic well

driven quasi-statically

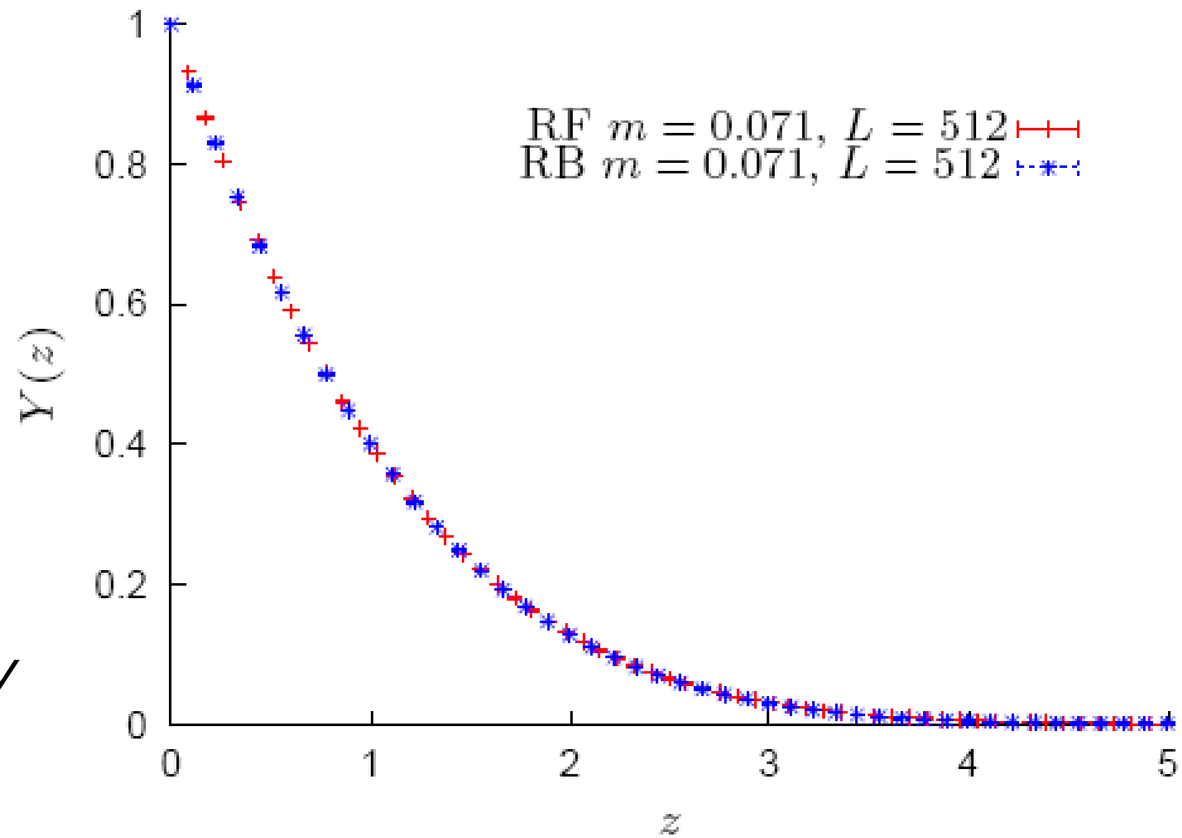
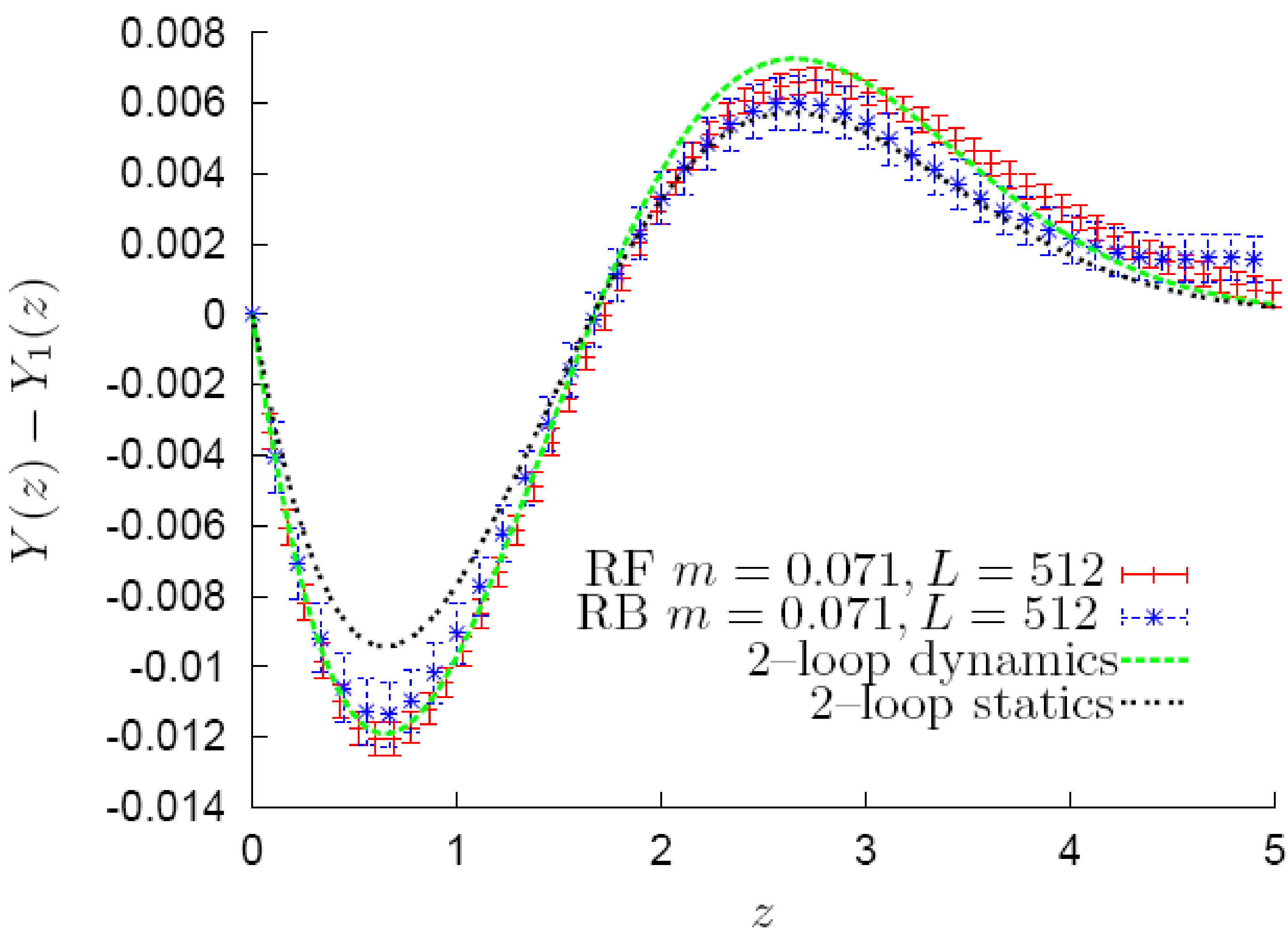


FIG. 3: Universal scaling form $Y(z)$ for $\Delta(u)$ for RB and RF disorder.

Deviations from one loop



Conclusion

- *field theory of pinning: statics, depinning and creep*

  Functional RG

- *the FRG allows to compute not only exponents but also*

  correlation of shocks and avalanches: good numerical test

  - *experimental tests ?*
    - *DW in magnetic film w/field gradient*
    - *contact line of fluid in partial wetting/capillarity*

      E. Rolley, S. Moulinet

  - *chaos*

  - *2D connections to fermions, nearly conformal FT*

- *can it be extended to describe INTERESTING random surfaces ??*