

# Continuous polarization variable QKD

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# discrete vs. continuous

discrete

dichotomic variable

$$\begin{aligned}\Psi &= \alpha |0\rangle + \beta |1\rangle = \\ &= \sum_{i=1}^2 \alpha_i |i\rangle\end{aligned}$$

→ click detectors

discrete &

$\infty$  dim Hilbert space

$$\Psi = \sum_{i=1}^{\infty} \alpha_i |i\rangle$$

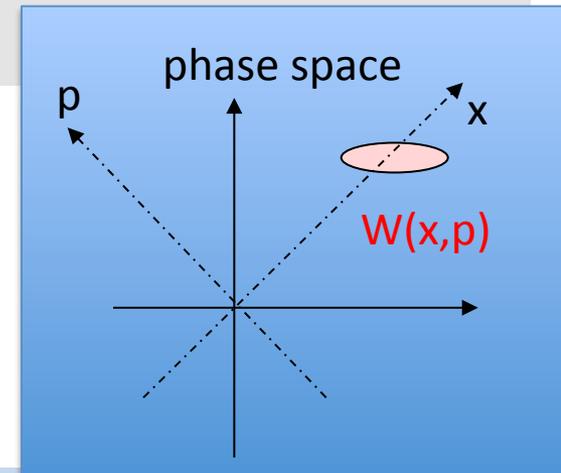
→ photon number resolving detectors

alternatively : **continuous variables**  $x, p$

$$W(x, p) = \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} d\xi \exp\left(-\frac{ip\xi}{\hbar}\right) \Psi^*\left(x - \frac{1}{2}\xi\right) \Psi\left(x + \frac{1}{2}\xi\right)$$

Wigner function

→ homodyne detection



types of continuous quantum variables

- field quadratures
- Stokes variables (polarization)

**QKD** – based on the impossibility of perfectly measuring non-orthogonal quantum states

two strategies:

**BB84**

Alice prepares states which are **eigenstates of Bob's measurement operator**  
 - need to switch between different bases

**B92**

Alice prepares non-orthogonal states and Bob uses **quantum state estimation**

Alice's state \ Bob's det.	click detector	PNR detector	homo-dyne detector
n=1, pol base	✓	✓	?
n>1, pol base	–	✓	?
coherent pol base	(⚡⚡)	(⚡) *	(⚡)
squeezed st.	–	–	(✓)

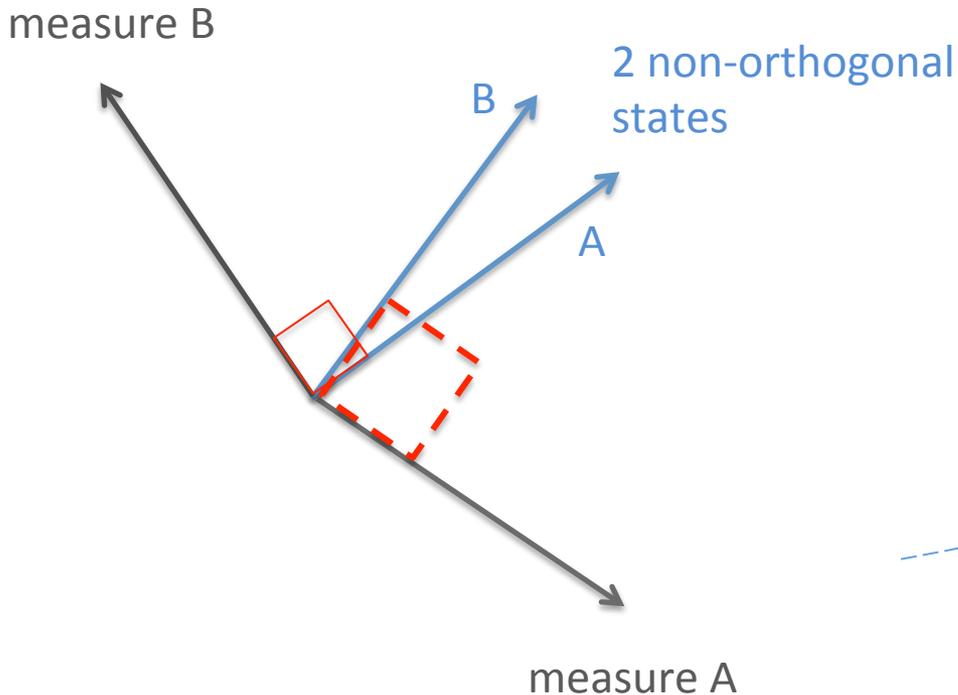
Alice's state \ Bob's det.	click detector	PNR detector	homo-dyne detector
n=1, diff pol	✓	✓	?
n>1, diff pol	–	✓	?
coherent two phases	✓	✓	✓
squeezed st.	–	–	✓

(⚡) = no eigenstate → not perfect  
 (✓) = unrealistic – only if squeezing is infinite

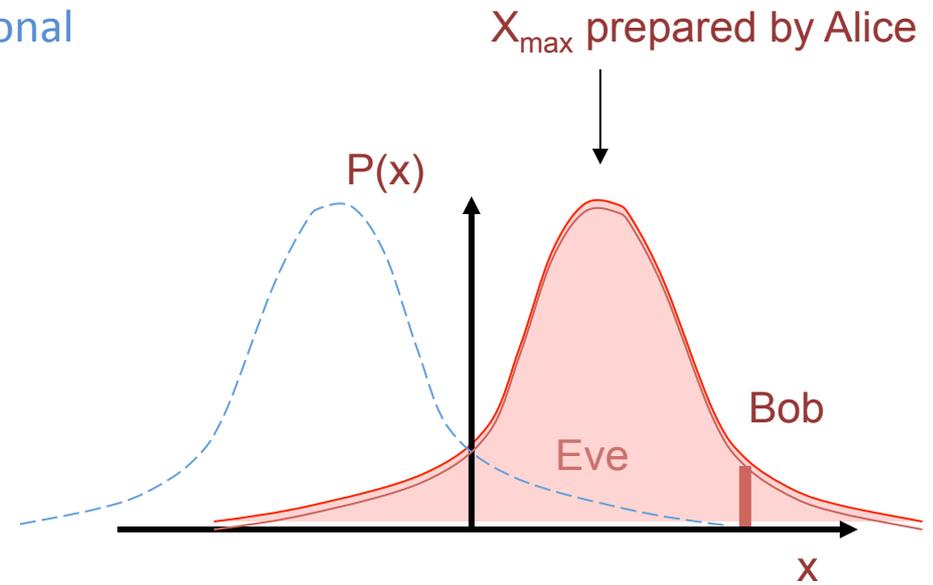
\* Norbert's comment yesterday  
 ? D. Bozyigit et al. Nat. Phys. 7,154 (2011)

# Quantum state estimation

a single photon in either of two non-orthogonal polarizations



the continuous quantum variable case



Ch. Silberhorn, T.C. Ralph, N. Lütkenhaus, and G. L., PRL 89, 167901 (2002)

# Quantum state estimation

PRL 96, 020409 (2006) **PHYSICAL REVIEW LETTERS** week ending 20 JANUARY 2006  
**Experimental Demonstration of Coherent State Estimation with Minimal Disturbance**  
1 \* **Metin Sabuncu**,<sup>1</sup> **Radim Filip**,<sup>1,2</sup> and **Gerd Leuchs**<sup>1</sup>  
<sup>1</sup>Physikalisches Institut, Universität Erlangen-Nürnberg,  
Erlangen, Germany

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PRL 101, 210501 (2008) **PHYSICAL REVIEW LETTERS** week ending 21 NOVEMBER 2008

**Demonstration of Near-Optimal Discrimination of Optical Coherent States**

Ch **Takeoka**,<sup>2,3</sup> **Katiúscia N. Cassemiro**,<sup>4</sup> **Masahide Sasaki**,<sup>5,1</sup> on 24 July 2012  
CE by: SDB US English NIP429857  
**New Journal of Physics**  
The open-access journal for physics

**Quadrature phase shift keying coherent state discrimination via a hybrid receiver**

**C R Müller**<sup>1,2</sup>, **M A Usuga**<sup>1,3</sup>, **C Wittmann**<sup>1,2</sup>, **M Takeoka**<sup>4</sup>,  
**Ch Marquardt**<sup>1,2</sup>, **U L Andersen**<sup>1,3,5</sup> and **G Leuchs**<sup>1,2,5</sup>  
<sup>1</sup>Max Planck Institute for the Science of Light, Erlangen, Germany

24 July 2012

$|\alpha|^2 \leq 0.75$   
100kHz

## Advantage of homo-/heterodyning in free space

→ measuring single mode

number of photons per mode in direct sun light:

$$N = 8\pi \frac{\nu^2}{c^3} \Delta\nu \frac{h\nu}{e^{kT} - 1} \frac{c}{6} \cdot \left( \frac{D_{sun}}{R_{S-E}} \right)^2 \frac{1}{2\pi \cdot \Delta\nu} \frac{\pi\lambda^2}{4 \left( \frac{D_{sun}}{R_{S-E}} \right)^2} \frac{1}{h\nu}$$
$$\approx \left( e^{\frac{h\nu}{kT}} - 1 \right)^{-1}$$

→ direct sun light : 0.05 coherent photons @ 800 nm

→ indirect sun light: even lower

propagation in turbulent medium → beam distortion

→ small homo-/heterodyne efficiency

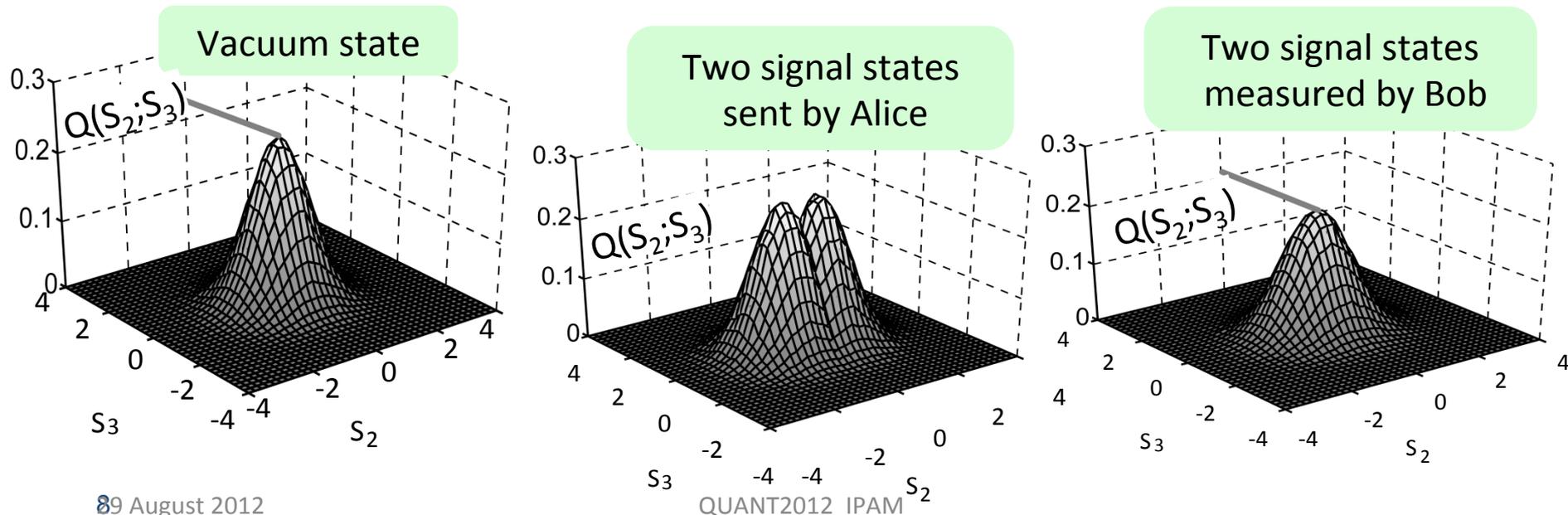
**the cure: polarisation variables**

# our approach

fiber/free space

# Our approach

- CV QKD with discrete modulation
- „prototype“ experiments:  
Lorenz et al., Appl. Phys. B **79**, 273 (2004) &  
Lorenz et al., PRA, **74** (4), 042326 (2006)
- polarization encoding
- heterodyne detection of conjugate Stokes variables



# Quantum Stokes operators and measurement

$$\hat{S}_0 = \hat{a}_x^\dagger \hat{a}_x + \hat{a}_y^\dagger \hat{a}_y \quad (\text{total intensity})$$

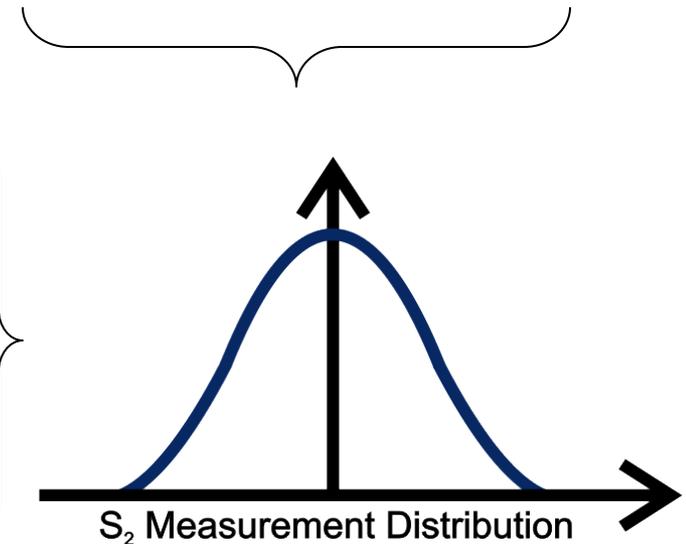
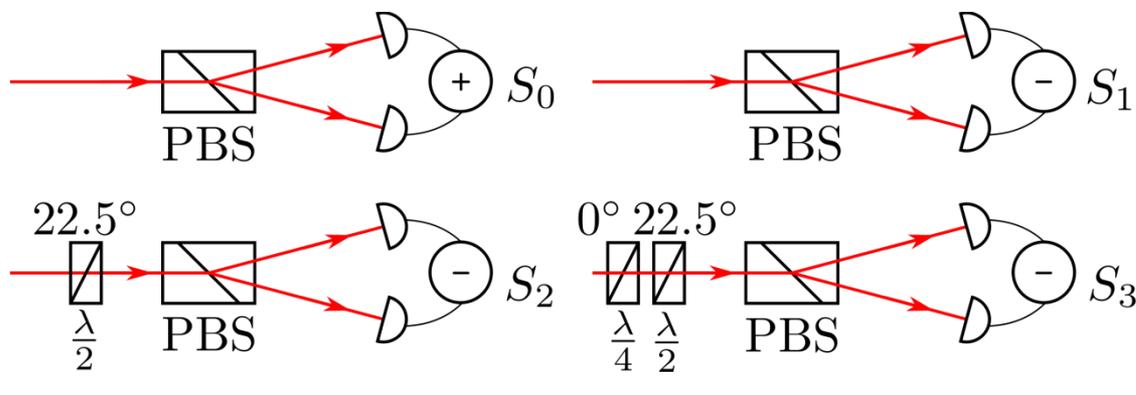
$$\hat{S}_1 = \hat{a}_x^\dagger \hat{a}_x - \hat{a}_y^\dagger \hat{a}_y \quad (\leftrightarrow - \updownarrow)$$

$$\hat{S}_2 = \hat{a}_x^\dagger \hat{a}_y + \hat{a}_y^\dagger \hat{a}_x \quad (\nearrow - \nwarrow)$$

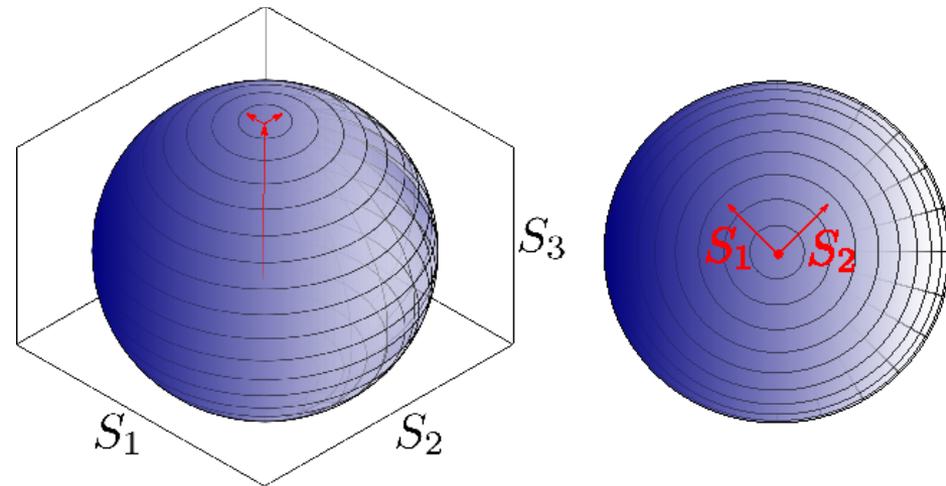
$$\hat{S}_3 = i(\hat{a}_y^\dagger \hat{a}_x - \hat{a}_x^\dagger \hat{a}_y) \quad (\circlearrowleft - \circlearrowright)$$

**Uncertainty Relation:**

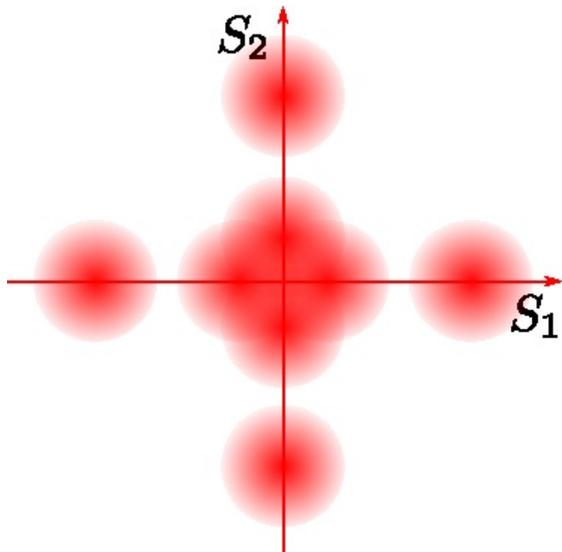
$$\text{Var}(\hat{S}_1) \cdot \text{Var}(\hat{S}_2) \geq |\langle \hat{S}_3 \rangle|^2$$



# Polarization encoding



- start with circularly polarized LO
- signal modulation in  $S_1$ - $S_2$ -darkplane using two commercial electro-optical modulators (EOMs)
- homodyne detection: weak signal detectable with help of bright LO ( $S_3$ )
- signal and LO travelling in one spatial mode  
→ further advantages:

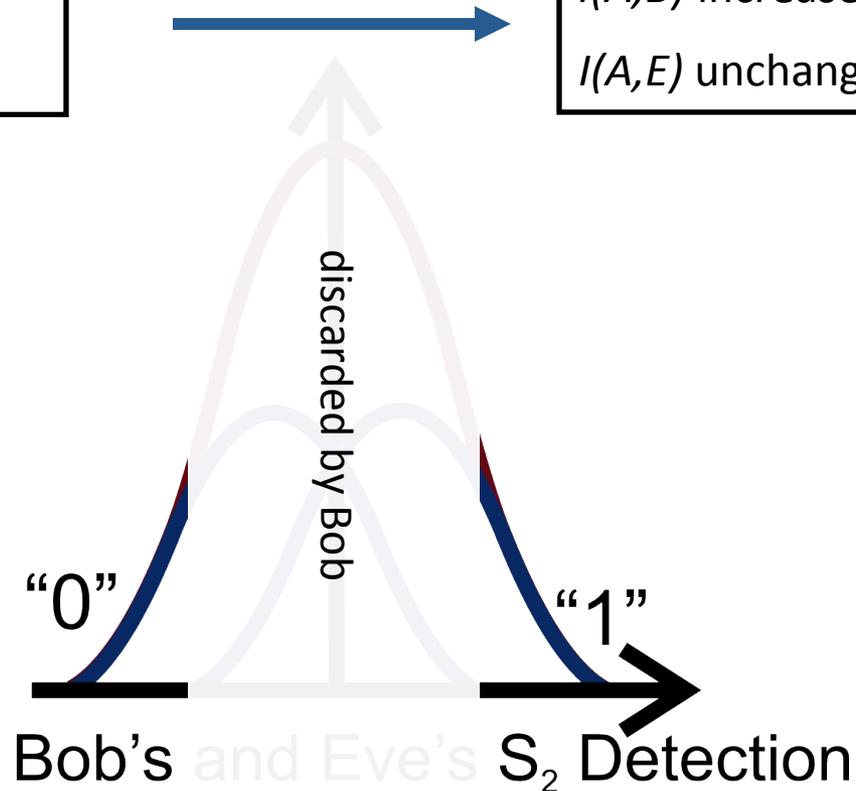


- excellent spatial interference
- no additional spatial filtering
- LO as spectral filter  
→ precise adjustment of detection bandwidth
- daylight operation
- easy monitoring of atmospheric effects

# Detection and Postselection

Post Selection:  
Silberhorn et al.,  
PRL 89, 167901 (2002)

$I(A,B)$  increased  
 $I(A,E)$  unchanged



# Witnessing effective entanglement over a 2 km fiber channel

Christoffer Wittmann<sup>1,2,†</sup>, Josef Fürst<sup>1,2,†</sup>, Carlos Wiechers<sup>1,2,3</sup>,  
Dominique Elser<sup>1,2</sup>, Hauke Häselser<sup>2,4</sup>,  
Norbert Lütkenhaus<sup>1,2,4</sup>, and Gerd Leuchs<sup>1,2</sup>

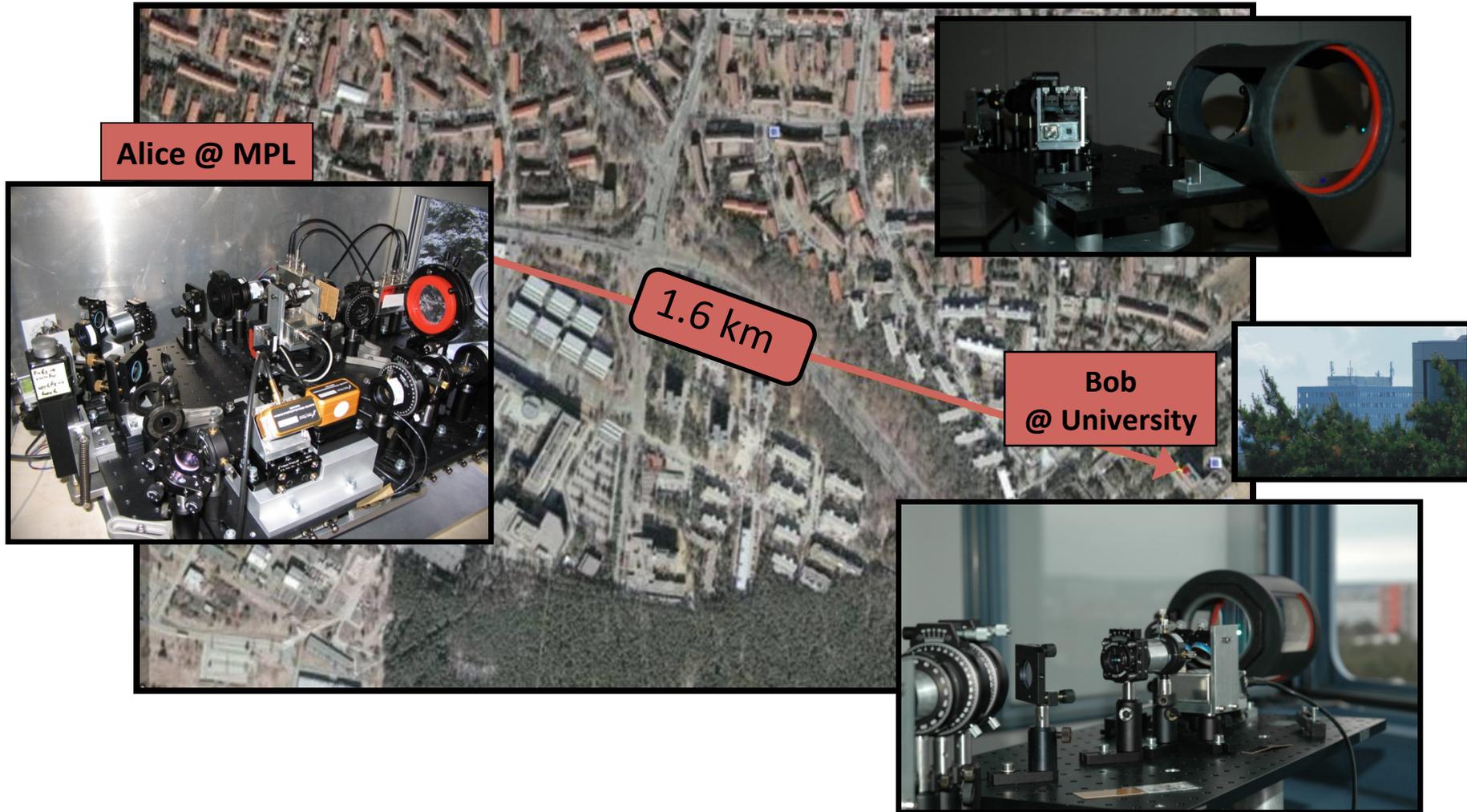
Optics Express **18**, 4499 (2010)

**Abstract:** We present a fiber-based continuous-variable quantum key distribution system. In the scheme, a quantum signal of two non-orthogonal weak optical coherent states is sent through a fiber-based quantum channel. The receiver simultaneously measures conjugate quadratures of the light using two homodyne detectors. From the measured Q-function of

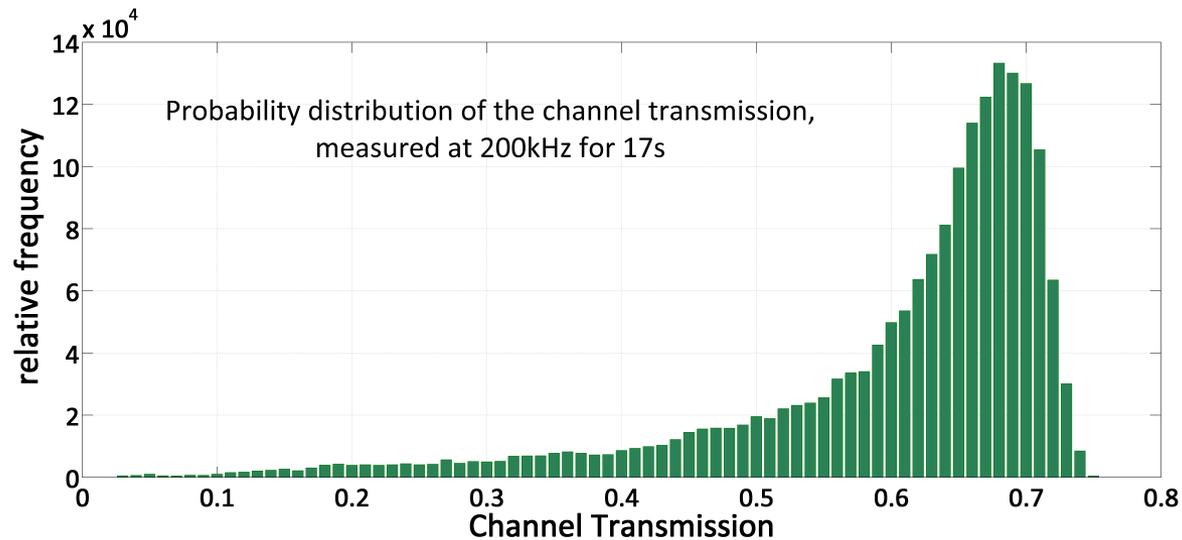
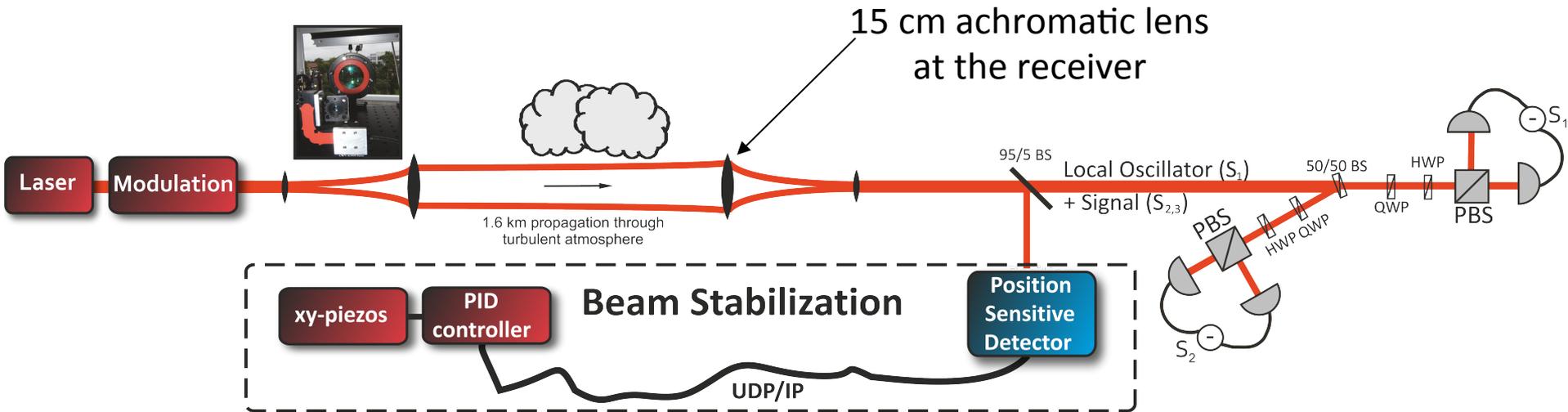
losses equivalent to > 15 km

free space

# 1.6 km free space link – from MPL's roof to university tower

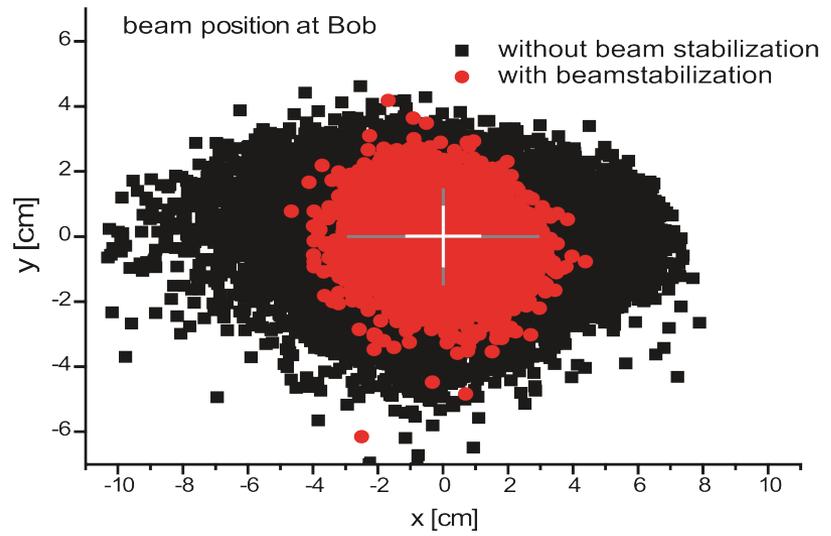
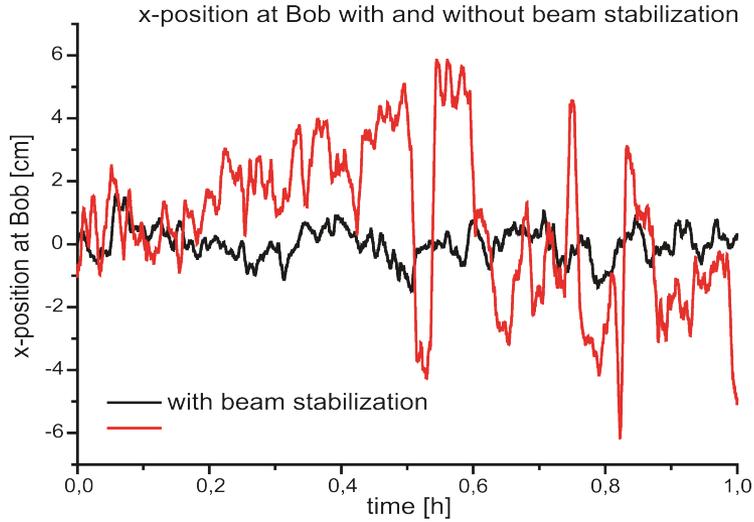
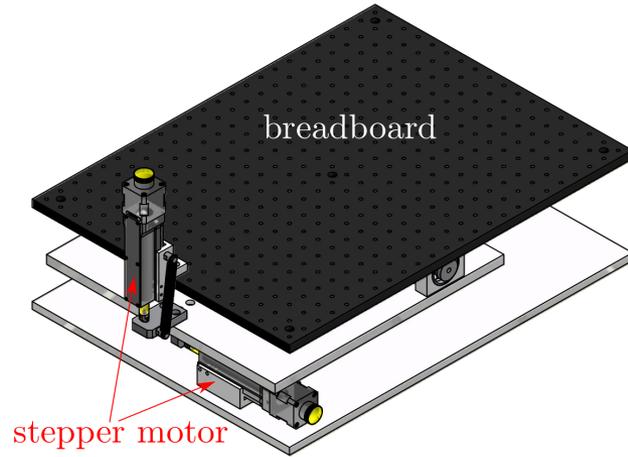


# Receiver setup



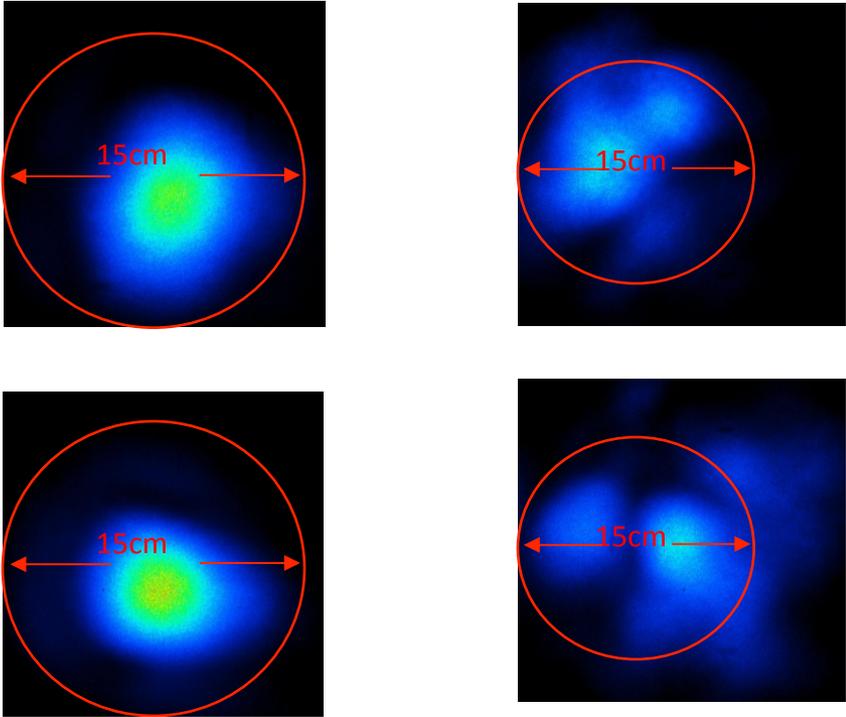
# Beam Stabilization system

- software-based PID-controller
- beam tap-off at Bob and measurement by a position sensitive detector (PSD)
- movement of Alice 's small telescope lens according to PSD feedback signal  
→ compensation of beam wandering
- $\pm 20$  cm beam displacement at Bob's side with an accuracy in the mm range
- additional movement of the whole breadboard by stepper motors

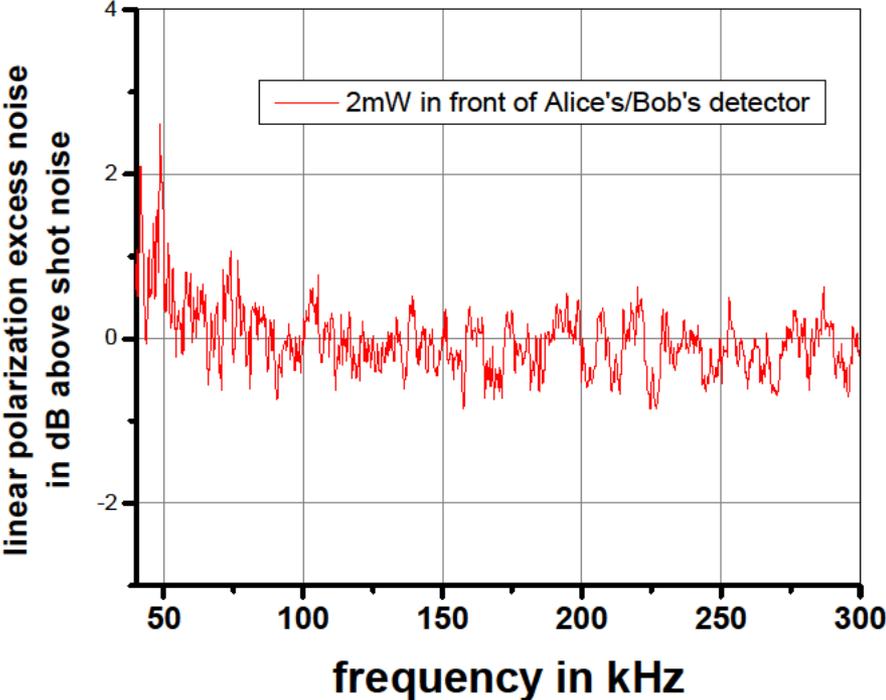


# Beam profiles and polarization noise

Spatial beam profiles after transmission



Less ...and more  
turbulent atmosphere



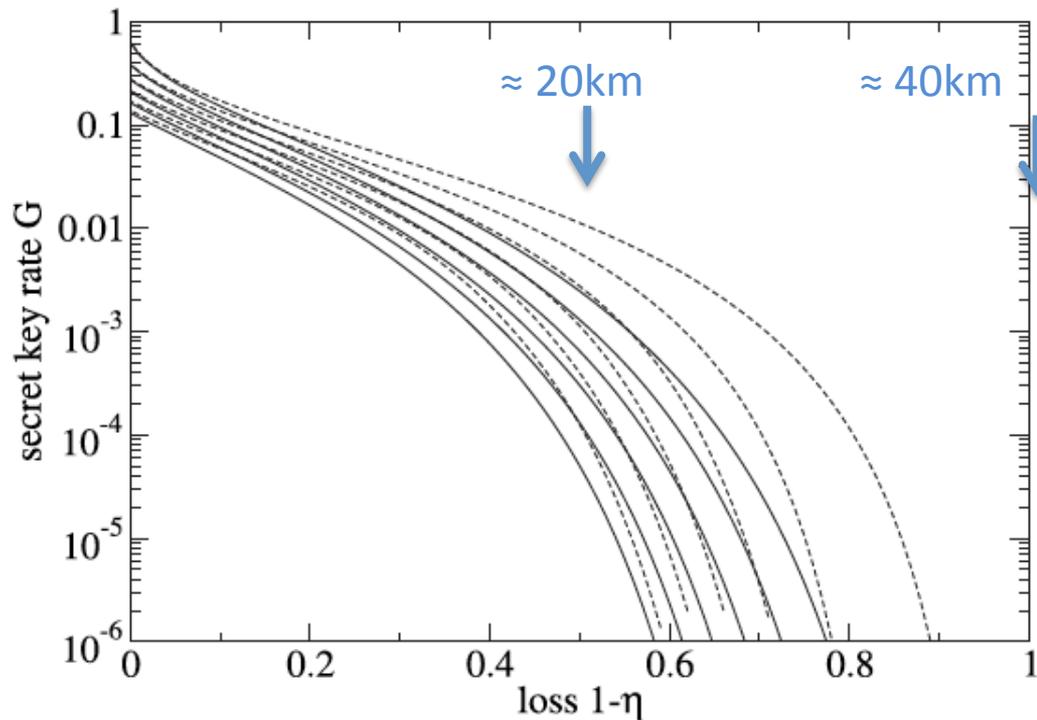


FIG. 5. Secret key rates  $G$  for post-selected protocols using the two-way error correction scheme CASCADE (solid lines). For comparison, key rates for the PS-RR protocol with one-way codes, that are as efficient as CASCADE are also shown (dashed lines). The excess noise  $\delta$  varies between 0 and 0.1 as in Fig. 2.

## SUMMARY

continuous quantum variables

sources : **coherent light** , change of wavelength straight forward

transmission : **loss sensitivity** > than for 1 photon technology

detectors : **direct detection** , shot noise sensitivity required

rates : **high** (no time filtering)

Higher order modes & multi level encoding → **rate even higher**

→→→ link to airplane